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Nonthermal cosmic neutrino backgroundMu-Chun Chen,^{1,*} Michael Ratz,^{2,†} and Andreas Trautner^{2,3,‡}¹*Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA*²*Physik-Department T30, Technische Universität München, James-Frank-Straße 1, 85748 Garching, Germany*³*Excellence Cluster Universe, Boltzmannstraße 2, 85748 Garching, Germany*

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We point out that, for Dirac neutrinos, in addition to the standard thermal cosmic neutrino background (CνB), there could also exist a nonthermal neutrino background with comparable number density. As the right-handed components are essentially decoupled from the thermal bath of standard model particles, relic neutrinos with a nonthermal distribution may exist until today. The relic density of the nonthermal (nt) background can be constrained by the usual observational bounds on the effective number of massless degrees of freedom N_{eff} and can be as large as $n_{\nu_{\text{nt}}} \lesssim 0.5n_\gamma$. In particular, N_{eff} can be larger than 3.046 in the absence of any exotic states. Nonthermal relic neutrinos constitute an irreducible contribution to the detection of the CνB and, hence, may be discovered by future experiments such as PTOLEMY. We also present a scenario of chaotic inflation in which a nonthermal background can naturally be generated by inflationary preheating. The nonthermal relic neutrinos, thus, may constitute a novel window into the very early Universe.

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I. INTRODUCTION

It is well established by the observation of neutrino oscillations that at least two neutrino species have non-vanishing mass. In the standard picture of early Universe cosmology, an upper bound on the sum of the neutrino masses can be inferred [1],

$$\sum m_\nu < 0.23 \text{ eV} \quad \text{at 95\% C.L.} \quad (1)$$

Arguably, the most straightforward way to reconcile neutrino masses with the standard model (SM) is to introduce three right-handed (RH) “neutrino” Weyl spinors ν_{R}^j ($1 \leq j \leq 3$) which are neutral under G_{SM} and only couple to the SM via Yukawa interactions of the form

$$\mathcal{L}_\nu = Y_\nu^{ij} \bar{\ell}_L^i \cdot \tilde{H} \nu_{\text{R}}^j + \text{H.c.} \quad (2)$$

Here, Y_ν denotes the matrix of Yukawa couplings, ℓ_L^i are the left-handed (LH) lepton doublets, and H is the Higgs doublet. After electroweak symmetry breaking, left- and right-handed Weyl spinors combine to a massive Dirac fermion. The Dirac neutrino masses are given by the singular values of Y_ν times the Higgs vacuum expectation value. In order to get masses of $\mathcal{O}(0.1)$ eV or below, the eigenvalues of Y_ν need to be 10^{-12} or smaller. A possible (lepton number violating) Majorana mass term $M_{\text{L}} \overline{\nu_{\text{R}}^c} \nu_{\text{R}}$

may be forbidden by a discrete subgroup of a baryon-minus-lepton number ($B-L$) symmetry or by other means. Throughout this work we will assume that there are no Majorana mass terms.

The smallness of the Yukawa couplings and the absence of other interactions implies that RH neutrinos are essentially decoupled from the thermal bath in the early Universe [2]. As is well known (cf. e.g. Ref. [3]), this means that RH neutrinos are not created in any significant number from the thermal bath. Another consequence, however, is that any previously existing abundance of RH neutrinos would not thermalize. RH neutrinos would only be affected by a redshifting of their kinetic energy as well as a dilution of their number density due to the Hubble expansion of the Universe.

In the early Universe, neutrinos are highly relativistic. Therefore, left- and right-handed components of the Dirac spinor do not mix significantly and can be thought of as individual species. Long after the decoupling of LH neutrinos from the thermal bath, neutrinos become nonrelativistic and thus form a massive Dirac fermion. We refer to this process as “left-right equilibration.” For the originally thermally coupled LH neutrinos, this implies that half of them are converted to the RH component, thus halving the experimental count rate [4] in neutrino capture experiments. For the originally decoupled RH neutrinos, on the other hand, this implies that half of them are converted to the LH component, thus allowing for their experimental detection through weak currents. The distinction between left- and right-handed neutrinos, therefore, is obsolete when the neutrinos are nonrelativistic. We will refer to the different massive Dirac neutrinos as “thermal” and “nonthermal” neutrinos, according to their origin.

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Let us briefly recall the usual “standard” scenario of a decoupled relativistic species with a thermal spectrum. Here, the crucial assumption is that the species has been in thermal equilibrium with the bath at some earlier epoch. At decoupling, the decoupled species exhibits an essentially thermal spectrum with a characteristic energy scale, i.e. the expectation value of the particle energy $\langle E \rangle$, given by the temperature of the bath at decoupling $\langle E \rangle \sim T_{\text{dec}}$. After decoupling, the decoupled species still obeys a thermal spectrum with a characteristic energy scale given by the redshifted decoupling temperature. This temperature will be lower than the temperature of the nondecoupled components of the bath by a factor of $(g_{*S}(T_{\text{today}})/g_{*S}(T_{\text{dec}}))^{1/3}$, with g_{*S} being the effective number of degrees of freedom in entropy, due to entropy conservation. Thus, if the decoupling happened sufficiently early, the number density of decoupled relics gets diluted below any detectable value. SM LH neutrinos have a relatively low decoupling temperature of $T \sim 1$ MeV from which an average number density of $n_{\nu_L} \sim 336 \text{ cm}^{-3}$ today can be anticipated. This is the well-known thermal cosmic neutrino background (CνB) which should be compared to the cosmic microwave background (CMB) of photons with an average relic abundance of $n_\gamma = 410 \text{ cm}^{-3}$. It will be interesting to see how the usual local density of neutrinos [5] changes in the presence of the additional, nonthermal degrees of freedom.

RH neutrinos may be populated through some beyond-SM interactions. Examples include scenarios with a gauged B - L symmetry which is broken at the TeV scale [6–8]. Here, one obtains a RH neutrino background with an almost thermal spectrum that is, at the time of big bang nucleosynthesis (BBN), colder than the SM particles. In case the thermal RH neutrinos decouple at temperatures above the electroweak phase transition, their relic number density is smaller than $\sim 36 \text{ cm}^{-3}$.

Let us now consider the case in which a species has never been in thermal equilibrium with the rest of the Universe. Then the spectrum of the decoupled species can be non-thermal. The energy of the individual quanta, of course, still gets redshifted, and their number density is still diluted by the Hubble expansion. Nevertheless, the typical energy scale of the spectrum is no longer required to be anywhere close to the temperature of the bath at the time the decoupled particles are created. This leaves some room to raise $\langle E \rangle$ above T , such that the subsequent “reheating” of the bath can be compensated, and one is left with a non-negligible relic density of the decoupled species. How much the relic density can be raised crucially depends on how much the total energy density of the Universe can be raised. In standard cosmology, the energy density is constrained by observational bounds on the Universe expansion rate during the epoch of BBN and during the formation of the CMB (cf. Refs. [9,10] for reviews). The constraints are typically quoted in terms of the number of effective

relativistic degrees of freedom N_{eff} that are thermally coupled to the bath corresponding to an energy density

$$\rho = N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_\gamma. \quad (3)$$

The current bounds still allow for an additional energy density corresponding to [1,11]

$$\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046 = 0.2 \pm 0.5 \quad \text{at 95\% C.L.} \quad (4)$$

Translated into a bound on the energy density of decoupled nonthermal RH neutrinos, we will show that a relic number density as high as $n_{\nu_{\text{nt}}} \sim 217 \text{ cm}^{-3}$ can be consistent with data.

An important question is whether the scenario of an existing nonthermal spectrum of RH neutrinos is well motivated, that is, if there exists a process in the early Universe that could produce such a spectrum. Interestingly, coherent oscillations of the inflaton field at the end of inflation can very efficiently produce particles nonthermally via the parametric resonance (cf. Ref. [12]). This so-called “preheating” occurs rapidly and far from thermal equilibrium. That is, it occurs before the perturbative decay of the inflaton and, thus, naively at a scale which is above the reheating temperature. We will see that already the simplest scenarios of “fermionic preheating” [13] can successfully produce nonthermal relic neutrinos. In this case, nonthermal relic neutrinos would constitute a probe of the Universe at the time of inflation.

II. NONEQUILIBRATION OF ν_R

It is well known that RH neutrinos are not thermally produced [2]. Let us briefly recall how this also implies that any existing abundance of RH neutrinos is not thermalized.

The only interaction of RH neutrinos with SM particles is the Yukawa coupling (2). The corresponding interaction rate at temperatures above the one of the electroweak phase transition can be estimated to be

$$\Gamma_{\text{RH}}(T \gtrsim v_{\text{EW}}) \sim |y_\nu|^2 T. \quad (5)$$

Comparing this to the expansion rate of the Universe, $H \sim T^2/M_{\text{P}}$, we find that this interaction is effective only for temperatures

$$T \lesssim 10^{-24} M_{\text{P}} \sim \text{keV}, \quad (6)$$

which is far below the range of validity of (5).

After the electroweak phase transition, interaction rates of RH neutrinos with the plasma are suppressed by a factor proportional to $(m_\nu/E)^2$ relative to the interaction rates of left handed neutrinos, $\Gamma_{\text{LH}} \sim G_{\text{F}}^2 T^5$. Therefore,

$$\Gamma_{\text{RH}}(T \lesssim v_{\text{EW}}) \sim G_{\text{F}}^2 m_{\nu}^2 T^3. \quad (7)$$

Comparing this to H , we find that this interaction is effective only for temperatures

$$T \gtrsim 10^{12} \text{ GeV}, \quad (8)$$

which is far above the range of validity of (7).

This shows that RH neutrinos do not equilibrate either above or below the electroweak phase transition.

III. RELIC ABUNDANCE OF NONTHERMAL ν_{R}

Let us assume that there is a population of nonthermal RH neutrinos in the early Universe. In general, the neutrinos may have a distribution f which depends nontrivially on the neutrino energy ε . In what follows, we discuss the extreme case in which the RH neutrinos form a degenerate Fermi gas at a (RH neutrino) temperature equal to zero. The RH neutrinos are completely decoupled from the thermal bath, which we assume to have temperature T_{R} at the time the RH neutrinos are produced. At this time, the number and energy densities of the RH neutrinos are given by

$$n_{\nu_{\text{R}}} = \frac{g}{6\pi^2} \varepsilon_{\text{F}}^3 \quad \text{and} \quad \rho_{\nu_{\text{R}}} = \frac{g}{8\pi^2} \varepsilon_{\text{F}}^4, \quad (9)$$

respectively. Here, ε_{F} is the Fermi energy, and g counts the degrees of freedom, and is 2 for a Weyl fermion.

Other nonthermal distributions f may be considered. We will also discuss a nondegenerate Fermi gas in which not all states below ε_{F} are occupied. This is accomplished by introducing a ‘‘filling factor’’ $0 \leq \eta \leq 1$ multiplying both densities (9). In order keep our presentation simple, we will set $\eta = 1$ here and comment on the case $\eta \neq 1$ below.

Besides g and η , the only free parameter of our scenario is $\xi := \varepsilon_{\text{F}}/T_{\text{R}}$. In order not to spoil the picture of a standard model radiation dominated cosmic evolution, we require that $\rho_{\nu_{\text{R}}} \ll \rho_{\text{rad}}$. As we find that all values of ξ which are consistent with observation always respect this requirement, there is no need to state the corresponding bound explicitly.

In a radiation dominated universe, entropy conservation implies that the scale factor is proportional to $R \propto g_{*S}^{-1/3} T^{-1}$. One may wonder whether entropy conservation is spoiled by the fact that there is a nonzero chemical potential for the RH neutrinos. This is not the case due to the fact that the RH neutrinos are essentially noninteracting, implying that their particle number is conserved and the standard form of entropy conservation is maintained.

The scaling of the particle number and that of the energy density then are given by

$$n_{\nu_{\text{R}}}(T) = \frac{g\xi^3}{6\pi^2} \frac{g_{*S}(T)}{g_{*S}(T_{\text{R}})} T^3 \quad (10)$$

and

$$\rho_{\nu_{\text{R}}}(T) = \frac{g\xi^4}{8\pi^2} \left(\frac{g_{*S}(T)}{g_{*S}(T_{\text{R}})} \right)^{4/3} T^4. \quad (11)$$

The relic density of nonthermal neutrinos today can be obtained from the scaled density of the originally RH neutrinos and is given by

$$\frac{n_{\nu_{\text{nt}}}}{n_{\gamma}} = \frac{n_{\nu_{\text{R}}}(T_{\gamma})}{n_{\gamma}} = \frac{g}{12\zeta(3)} \frac{g_{*S}(T_{\gamma})}{g_{*S}(T_{\text{R}})} \xi^3. \quad (12)$$

It is straightforward to translate the observational bounds on the energy density in the Universe into a constraint on the energy density of the degenerate RH neutrinos during BBN or after CMB formation. From (3) and (11) we obtain at BBN

$$\Delta N_{\text{eff}}^{(\nu_{\text{R}})} = \frac{8}{7} \frac{30}{8\pi^4} \frac{g\xi^4}{2} \left(\frac{g_{*S}(T_{\text{BBN}})}{g_{*S}(T_{\text{R}})} \right)^{4/3}. \quad (13)$$

The current observational limit (4) implies $\Delta N_{\text{eff}}^{(\nu_{\text{R}})} \lesssim 0.7$. In case there are three generations of relic nonthermal neutrinos with equal ξ , we take $g = 6$ and find

$$\xi \lesssim 3.26, \quad (14)$$

where we have used $g_{*S}(T_{\text{BBN}}) = 10.75$ and $g_{*S}(T_{\text{R}}) = 106.75$. This corresponds to a relic density of

$$n_{\nu_{\text{nt}}} \lesssim 0.53 n_{\gamma} \approx 217 \text{ cm}^{-3}. \quad (15)$$

This should be compared to the abundance of $\text{C}\nu\text{B}_{\text{th}}$ thermal relic neutrinos $n_{\nu_{\text{th}}} \sim 336 \text{ cm}^{-3}$.

In principle, there could also be different values of ξ for different generations of RH neutrinos. Irrespective of this assumption, however, the total relic density is bounded from above by (15).

Let us comment on how our results change if we allow for a nontrivial filling factor η . Assuming that ξ is equal for all generations, the relic density can be written in the form

$$\frac{n_{\nu_{\text{nt}}}}{n_{\gamma}} \approx 1.2 \frac{g_{*S}(T_{\gamma})}{g_{*S}(T_{\text{BBN}})} \eta^{1/4} g^{1/4} (\Delta N_{\text{eff}}^{(\nu_{\text{R}})})^{3/4}. \quad (16)$$

This allows us to determine the maximal relic density for some given value of η while keeping $\Delta N_{\text{eff}}^{(\nu_{\text{R}})}$ at the observational upper bound. We see that the relic density can be sizable even for low values of η .

Note that this discussion also shows that it is possible to explain sizable deviations in N_{eff} without introducing any exotic particles.

IV. A CONCRETE SCENARIO FOR ν_R PRODUCTION

In what follows, we will discuss a scenario in which a nonthermal neutrino background can be naturally generated. Let us stress that the main point of this paper does not rely on this specific possibility, which is just an existence proof of a scenario with the desired properties. In the simplest cases of fermionic preheating [14], the inflaton ϕ is assumed to have a potential $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$ and a Yukawa coupling $\lambda\phi\bar{\Psi}\Psi$, where Ψ is a Dirac fermion. Preheating via the parametric resonance [15] then produces fermions with a nondegenerate Fermi spectrum, i.e. with momenta stochastically filling a sphere of radius $\varepsilon_F \sim q^{1/4}m_\phi$, where $q := \lambda^2\phi_0^2/m_\phi^2$ is the so-called resonance parameter and ϕ_0 the initial displacement of ϕ . After a couple of inflaton oscillations, this process becomes ineffective, and the inflaton decays perturbatively, seeding the hot early Universe. In addition to the perturbative decay, reheating could also occur via a coupling $\phi^2 H^2$ to the Higgs portal and the scalar preheating mechanism [15], thus producing the usual SM particles.

Since q *a priori* is a free parameter, it is possible to obtain a characteristic energy of the nonthermal spectrum $\langle E \rangle \sim \varepsilon_F$ which can be much bigger than the naive reheating temperature of $T_R \sim m_\phi/2$. Thus, a nonthermal spectrum may be created with a number density which is non-negligible even today.

A possible scenario for the production of a nonthermal neutrino background is, therefore, based on the coupling

$$\mathcal{L} \supset \lambda\phi\bar{\nu}_R^c\nu_R + \text{H.c.} \quad (17)$$

For this coupling to be allowed, the inflaton must be appropriately charged under the symmetry that prohibits the Majorana mass term of ν_R . This also implies that the vacuum expectation value of ϕ must vanish such that the Majorana mass term is not reintroduced.

The non-thermal neutrino spectrum is then assumed to be created via fermionic preheating directly after inflation and, thus, can be approximated by a nondegenerate Fermi-Dirac distribution at zero temperature.

For values of $\xi \lesssim 3$ and with the naive reheating temperature given by $T_R \sim m_\phi/2$, we find $q \sim \xi^4 \lesssim 10^2$. Even though there has been no dedicated analysis in this direction, this value of q seems to easily allow for filling factors reaching $\eta \gtrsim 0.3$ [16].

V. DETECTION OF A NONTHERMAL NEUTRINO BACKGROUND

The scaling of the density (10) implies that the Fermi energy scales linearly with T ,

$$\varepsilon_F(T) = \left(\frac{g_{*S}(T)}{g_{*S}(T_R)} \right)^{1/3} \xi T. \quad (18)$$

In particular, the characteristic energy of the nonthermal neutrinos is $\langle E_{\nu_n} \rangle \sim \varepsilon_F$, and they are nonrelativistic at late times, just as the standard $C\nu B_{\text{th}}$. Thus, after left-right equilibration half of the initially RH neutrinos may be detected via an inverse beta-decay in neutrino capture experiments such as PTOLEMY [17]. Even though the projected energy resolution could resolve an ‘‘electron neutrino mass’’ close to its upper bound (1), it will not suffice to resolve the spectrum of relic neutrinos. For this reason the nonthermal neutrinos constitute an irreducible contribution to any planned experiment which is sensitive to the $C\nu B$. Far future experiments with a substantially improved neutrino energy resolution, however, could distinguish the contributions of thermal and nonthermal neutrinos.

Since the thermal neutrinos propagate as mass eigenstates, the different flavors will, to a good approximation, be equilibrated at late times [4]. We further assume that the flavor composition of the nonthermal neutrinos is also roughly 1 : 1 : 1. The maximal global number density of relic nonthermal neutrinos which is available for electron–neutrino capture is then given by $\sim 36 \text{ cm}^{-3}$. Comparing this with the detectable number density of $C\nu B_{\text{th}}$ neutrinos which is $\sim 56 \text{ cm}^{-3}$, we see that any experiment which aims for detecting the $C\nu B$ should be able to detect the non-thermal Dirac neutrino background.

Recently it has been suggested that measurements of the relic neutrino abundance could discriminate between Dirac and Majorana neutrinos due to their different projected count rates for PTOLEMY of $\sim 4 \text{ yr}^{-1}$ and $\sim 8 \text{ yr}^{-1}$, respectively [4]. We see that this proposal may not work in the presence of additional nonthermal Dirac neutrinos which could increase the respective count rate by 64%, thereby diminishing the difference between Dirac and Majorana neutrinos.

VI. SUMMARY

We depict the basic points of our scenario in Fig. 1. In the very early Universe, a significant number of ν_R states are created with a nonthermal spectrum. They are decoupled until very late and have during BBN an average energy below the one of the thermal neutrinos (Fig. 2). This leads to a situation where the contribution of nonthermal neutrinos to the energy density of the Universe is consistent with observation. Yet the relic abundance of nonthermal neutrinos can be as large as $\sim 0.5n_\gamma$ today. This has important implications for the prospects of discovering the $C\nu B$ as well as for the clustering of relic neutrinos. Note that our scenario can explain deviations of N_{eff} from its usual value 3.046 without the need to add any extra states to the SM apart from right-handed neutrinos and the inflaton, which is an ingredient of almost any realistic cosmology.

The CMB provides us with information on the Universe at the time of photon decoupling, which happened around

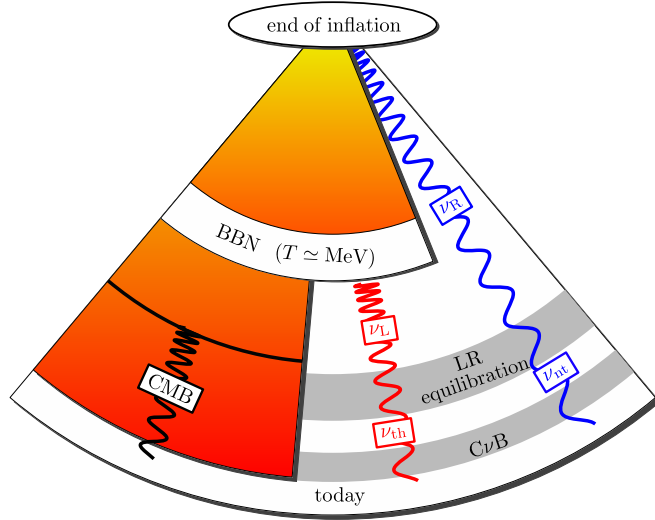


FIG. 1 (color online). Cartoon of the history of the Universe with nonthermal neutrinos.

380,000 years after the big bang. Likewise, the thermal neutrino spectrum is sensitive to how the Universe looked like at the time of BBN, when it was roughly 1 sec old. This is to be contrasted with what one could learn from the nonthermal neutrino background. As illustrated in the discussion of the preheating scenario, a future possible detection and subsequent careful examination of the nonthermal neutrino background may provide us with a possibility to directly probe features of inflation (Fig. 1). Assuming an inflation scale of the order 10^{16} GeV, nonthermally produced right-handed neutrinos may allow us to

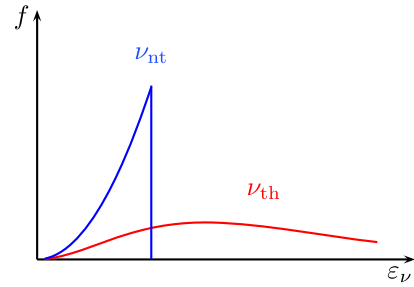


FIG. 2 (color online). Phase space distributions of thermal and nonthermal neutrinos during BBN.

probe the Universe when it was as young as 10^{-38} sec. The data gained this way will be complimentary to what one can learn from gravitational waves and a nontrivial tensor-to-scalar ratio.

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