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TRANSPORT PROPERTIES OF QUARK AND GLUON PLASMAS

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Abstract

The kinetic properties of relativistic quark-gluon and electron-photon plasmas are described in the weak coupling limit. The troublesome Rutherford divergence at small scattering angles is screened by Debye screening for the longitudinal or electric part of the interactions. The transverse or magnetic part of the interactions is effectively screened by Landau damping of the virtual photons and gluons transferred in the QED and QCD interactions respectively. Including screening a number of transport coefficients for QCD and QED plasmas can be calculated to leading order in the interaction strength, including rates of momentum and thermal relaxation, electrical conductivity, viscosities, flavor and spin diffusion of both high temperature and degenerate plasmas. Damping of quarks and gluons as well as color diffusion in quark-gluon plasmas is, however, shown not to be sufficiently screened and the rates depends on an infrared cut-off of order the "magnetic mass", $m_{\rm mag} \sim g^2T$.

1 Introduction

QCD plasmas consisting of quarks, antiquarks and gluons appear in a number of situations. In the early universe the matter consisted mainly of a quark-gluon plasma for the first microsecond before hadronization set in. Present day experiments at the CERN SPS, the Brookhaven AGS and future RHIC collider are searching for the quark-gluon plasma in relativistic heavy ion collisions. Cold degenerate plasmas, $T \ll \mu$, of quarks may exist in cores of neutron stars or in strangelets.

Relativistic QED and QCD plasmas have a lot of common features in the weak coupling limit such as the Rutherford divergence in the elastic differential cross section, the screening properties, and therefore also transport properties. In a plasma the typical momentum transfers are of order T or μ, whichever is the larger. For sufficiently high temperature or density, the running coupling constant $\alpha_s(Q)$ is small and we can treat the quark-gluon plasma as weakly interacting. In fact lattice gauge calculations indicate that the quark-gluon plasma behaves much like a free gas already at temperatures not far above the critical temperature, $T_c \simeq 160$ MeV, at which the phase transition between hadronic matter and quark-gluon plasma takes place. Anyway, since we are better at interpolating than extrapolating, knowledge of the behavior of weakly interacting quark-gluon plasmas at high temperatures or densities should be very useful.

We emphasize that it is the effect of Landau damping which effectively leads to screening of transverse interactions and give the characteristic relaxation rates in transport processes and some transport coefficients for weakly interacting electron-photon and quark-gluon plasmas for both thermal plasmas [1, 2] as well as degenerate ones [3]. In addition, we discuss the quark and gluon quasiparticle damping rates and the rates of color diffusion in which the transverse interactions are not sufficiently screened so that an infrared cut-off on the order of the "magnetic mass", $m_{\rm mag} \simeq g^2T$, is needed.

2 Screening in QCD and QED

The static chromodynamic interaction between two quarks in vacuum gives the matrix element for near-forward elastic scattering

$$M(q) = \frac{\alpha_s}{q^2}\left(1 - \frac{\mathbf{v}_1 \times \hat{\mathbf{q}}}{c}\frac{\mathbf{v}_2 \times \hat{\mathbf{q}}}{c}\right) , \qquad (1)$$

where $\alpha_s = g^2/4\pi$ is the QCD fine structure constant, $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the two interacting particles and $\mathbf{q}$ is the momentum transfer in the collision. The first part is the electric or longitudinal (+timelike) part of the interactions. The second part is the magnetic or transverse part of the interactions. In the Born approximation the corresponding potential is obtained by Fourier transform of (1) by which one obtains the standard Coulomb and Lorenzt interactions respectively. A weakly interacting QCD plasma is very similar to a QED plasma if one substitutes the fine structure constant α_s by $\alpha = e^2 \simeq 1/137$, the gluons by photons and the quarks by leptons with the associated statistical factors. The interaction via the gluon field is determined by gauge symmetry in much the same way as in QED and therefore scattering by a gluon exchange is very similar to that by photon exchange and the Feynman diagrams carry over. There is one crucial difference, namely that the gluon can couple directly to itself. This leads to confinement and the running coupling constant, $\alpha_s(Q)$. For a given coupling constant, however, the kinetic properties are very similar as we shall see.

In non-relativistic plasmas such as the electron plasma in terrestrial metals, the electron velocities are of order the Fermi velocity, which is much smaller than the speed of light, and therefore the transverse interactions are usually ignored. In relativistic plasmas they are, however, of similar magnitude and for degenerate plasmas the transverse interactions may in fact dominate the transport properties, as will be described below, because they are effectively less screened than the longitudinal interactions.

By squaring the matrix element we obtain the Rutherford differential cross section at small momentum transfer $\mathbf{q}$ in the center-of-mass system

$$\frac{d\sigma}{d\Omega} = 4E^2\frac{\alpha_s^2}{q^4}(1+v^2)\,. \tag{2}$$

The momentum transfer is related to the scattering angle θ by $q = 2E\sin(\theta/2)$ where E is the particle energy in the center-of-mass system. We observe that (2) diverges as θ^{-4} at small angles and the total cross section is infinite signifying a long-range interaction. In calculating transport properties one typically weights the differential cross section by a factor $q^2 = 2E^2(1-\cos\theta)$ but that still leads to a logarithmically diverging integral and therefore to vanishing transport coefficients.

In a medium this singularity is screened as given by the Dyson equation in which a gluon self energy $\Pi_{L,T}$ is added to the propagator $t^{-1} = \omega^2 - q^2$. For the matrix element this gives

$$t^{-1} \to \omega^2 - q^2 - \Pi_{L,T}\,, \tag{3}$$

(we refer to Weldon [4] for details on separating longitudinal and transverse parts of the interaction) where the longitudinal and transverse parts of the selfenergy in QED and QCD are for $\omega, q \ll T$ given by

$$\Pi_L(\omega,q) = q_D^2\left(1-\frac{x}{2}\ln\frac{x+1}{x-1}\right), \tag{4}$$

$$\Pi_T(\omega,q) = q_D^2\left[\frac{1}{2}x^2+\frac{1}{4}x(1-x^2)\ln\frac{x+1}{x-1}\right], \tag{5}$$

where $x = \omega/qv_p$ and $v_p = c$ for the relativistic plasmas considered here. The Debye screening wavenumbers in QCD is

$$q_D^2 = g^2\left((N_q+2N)\frac{T^2}{6}+N_q\frac{\mu_q^2}{2\pi^2}\right), \tag{6}$$

where $N = 3$ is the number of colors, N_q is the number of quark flavors, T the plasma temperature and μ_q the quark chemical potential. We refer to [12] for a detailed comparison to QED plasmas.

In the static limit, $\Pi_L(\omega=0,q) = q_D^2$, and the longitudinal interactions are Debye screened. Consequently, the typical elastic scattering and transport cross sections due to longitudinal interactions alone become

$$\sigma_{el}^L = \int\frac{d\sigma}{d\Omega}d\Omega \simeq \alpha_s^2\int\frac{d^2q}{(q^2+q_D^2)^2} \simeq \frac{\alpha_s}{T^2}\,. \tag{7}$$

$$\sigma_{tr}^L \simeq \int\frac{d\sigma}{d\Omega}(1-\cos\theta)d\Omega \simeq \frac{\alpha_s^2}{E^2}\int\frac{q^2d^2q}{(q^2+q_D^2)^2} \simeq \frac{\alpha_s^2}{E^2}\ln(T/q_D)\,. \tag{8}$$

in a high temperature quark-gluon plasma, where particles energies are $E \sim T$.

For the transverse interactions the selfenergy obeys the transversality condition $q^\mu \Pi_{\mu\nu} = 0$, which insures that the magnetic interactions are unscreened in the static limit, $\Pi_T(\omega = 0, q) = 0$. Consequently, the cross sections corresponding to (7) and (8) diverge leading to zero transport coefficients. It has therefore been suggested that the transverse interactions are cut off below the "magnetic mass", $m_{mag} \sim g^2 T$, where infrared divergences appear in the plasma [5]. However, as was shown in [1] and as will be shown below, dynamical screening due to Landau damping effectively screen the transverse interactions off in a number of situations at a length scale of order the Debye screening length $\sim 1/gT$ as in Debye screening. Nevertheless, there are three important length scales in the quark-gluon plasma. For a hot plasma they are, in increasing size, the interparticle spacing $\sim 1/T$, the Debye screening length $\sim 1/gT$, and the scale $1/m_{mag} \sim 1/g^2T$ where QCD effects come into play.

3 Transport Coefficients for Hot Plasmas

In this section we calculate a number of transport coefficients for weakly-interacting, high temperature plasmas. We work in the kinematic limit of massless particles, $m \ll T$, and zero chemical potential, $|\mu \ll T|$.

The characteristic timescales, τ, describing the rate at which a plasma tends towards equilibrium if it is initially produced out of equilibrium, as in a scattering process, or if driven by an external field, are determined by solving the kinetic equation. For a system with well defined quasiparticles, the Boltzmann transport equation is

$$
\begin{aligned}
(\frac{\partial}{\partial t} + \mathbf{v_p} \cdot \nabla_\mathbf{r} \;\; &+ \;\; \mathbf{F} \cdot \nabla_\mathbf{p}) n_\mathbf{p} = -2\pi\nu_2 \sum_{234} |M_{12\to34}|^2 \delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \\
&\times \;\; [n_{\mathbf{p}_1} n_{\mathbf{p}_2} (1 \pm n_{\mathbf{p}_3})(1 \pm n_{\mathbf{p}_4}) - n_{\mathbf{p}_3} n_{\mathbf{p}_4} (1 \pm n_{\mathbf{p}_1})(1 \pm n_{\mathbf{p}_2})] \,, \qquad (9)
\end{aligned}
$$

where $\mathbf{p}$ is the quasiparticle momentum, $n_\mathbf{p}$ the quasiparticle distribution function and $\mathbf{F}$ the force on a quasiparticle. The r.h.s. is the collision integral for scattering particles from initial states 1 and 2 to final states 3 and 4, respectively. The $(1 \pm n_p)$ factors correspond physically to the Pauli blocking of final states, in the case of fermions, and to (induced or) stimulated emission, in the case of bosons.

The matrix element for the scattering process $1 + 2 \to 3 + 4$ is $|M_{12\to34}|^2 = |\mathcal{M}_{12\to34}|^2/16\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4$ where $\mathcal{M}$ is the Lorentz invariant matrix element normalized in the usual manner in relativistic theories. When electrons, quarks and gluons scatter elastically through photon or gluon exchange in a vacuum, the matrix element squared averaged over initial and summed over final states is dominated by a $t^2 = (\omega^2 - q^2)^2$ singularity, for example for quark-gluon scattering

$$
|\mathcal{M}_{qg\to qg}|^2 = g^4(u^2 + s^2)/t^2. \qquad (10)
$$

The gluon-gluon and quark-quark matrix elements only differ by a factor 9/4 and 4/9 respectively near small momentum scattering.

3.1 Viscosities

In Ref. [1, 12] the first viscosity of a quark-gluon plasma was derived to leading logarithmic order in the QCD coupling strength by solving the Boltzmann kinetic equation. For gluons

$$\eta_g = \frac{2^9 15 \xi(5)^2}{\pi^5(1+11N_q/48)} \frac{T^3}{g^4 \ln(T/q_D)} = 0.34 \frac{T^3}{\alpha_s^2 \ln(1/\alpha_s)} . \tag{11}$$

Second, the quarks carry momentum, and therefore produce an increase in the total viscosity, $\eta = \eta_g + \eta_q$. The quark viscosity, η_q, is obtained analogously to the gluon one

$$\eta_q = 2.2 \frac{1+11N_q/48}{1+7N_q/33} N_q \eta_g , \tag{12}$$

which for $N_q = 2$ results in $\eta_q = 4.4\eta_g$, a quark viscosity that is larger than the gluon one because the gluons generally interact stronger than the quarks.

Writing each η_i ($i = q, g$) in terms of the viscous relaxation time, $\tau_{\eta,i}$, as

$$\eta_i = w_i \tau_{\eta,i}/5, \tag{13}$$

where w is the enthalpy, we obtain the viscous relaxation rate for gluons

$$\frac{1}{\tau_{\eta,g}} = 4.11(1+\frac{11N_q}{48}) T \alpha_s^2 \ln(1/\alpha_s), \tag{14}$$

and for quarks and antiquarks

$$\frac{1}{\tau_{\eta,q}} = 1.27(1+\frac{7N_q}{33}) T \alpha_s^2 \ln(1/\alpha_s) . \tag{15}$$

The second viscosity ζ is zero for a gas of massless relativistic particles. Thermal conduction is not a hydrodynamic mode in relativistic plasmas with zero chemical potential.

3.2 Momentum Relaxation Times

In [2] the time for transfer of momentum between two interpenetrating, spatially uniform plasmas in relative motion has been calculated. Elastic collisions between the two interpenetrating plasmas lead to a relative flow velocity decreasing as function of time and the characteristic stopping time is the "momentum relaxation time". We emphasize that we are only considering collisional phenomena in this calculation. It might be the case that collective phenomena, such as the two-stream instability could lead to relaxation faster than that due to collisions.

The resulting momentum relaxation rate $\tau_{\mathrm{mom},qg}^{-1}$ for gluons colliding with quarks and antiquarks ($\nu_2 = 12N_q$), exact to leading logarithmic order in $\alpha_s = g^2/4\pi$, is

$$1/\tau_{\mathrm{mom,qg}} = \frac{20\pi}{7}(1+\frac{21}{32}N_q)\alpha_s^2 \ln(1/\alpha_s) T . \tag{16}$$

Momentum relaxation rates for two plasmas with different quark flavors, spins, or colors, or for different gluon colors or spins have the same form, $\sim \alpha_s^2 \ln(1/\alpha_s)T$.

In QED plasma similar stopping times are obtained for lepton stopping on antileptons [2, 12] as in (16) when α_s is replaced by α. Photon stopping rates are, however, different, $\tau_{\mathrm{mom},\gamma\ell} \simeq \alpha_s^2 T$, because the photon does not couple to itself as does the gluon and so it does not interact with a lepton by exchanging a photon but only through Compton scattering.

3.3 Electrical Conductivity

Another transport coefficient of interest is the electrical conductivity, σ_{el}, of the early universe. The principal conduction process is flow of charged leptons, and the dominant scattering process is electromagnetic interaction with other charged particles; strongly interacting particles have much shorter mean-free paths. The infrared singularity of the transverse interaction in QED is treated as in QCD, only now $q_D^2 = N_l e^2 T^2/3$, where N_l is the number of charged lepton species present at temperature T. To calculate σ_{el} we consider the current of charged leptons (1) and antileptons (2) in a static electric field, $\mathbf{E}$. Taking the components to be thermally distributed with opposite fluid velocities, $\mathbf{u_1} = -\mathbf{u_2}$, the total electrical current is $\mathbf{j}_{\ell\bar{\ell}} = -en_{\ell\bar{\ell}}\mathbf{u}_1$, where the density of electric charge carriers is $n_{\ell\bar{\ell}} = 3\zeta(3)N_\ell T^3/\pi^2$. Solving the kinetic equation (9) we find the electrical conductivity

$$\sigma_{\ell\bar{\ell}} = j_{\ell\bar{\ell}}/E = \frac{3\zeta(3)}{\ln 2}\frac{1}{\alpha\ln(1/\alpha)}T\,. \tag{17}$$

The related electric current relaxation time is

$$\frac{1}{\tau_{\ell\bar{\ell}}} = \frac{4\pi N_\ell \ln 2}{27\zeta(3)}\alpha^2 \ln(1/\alpha)T, \tag{18}$$

which is very similar to the corresponding momentum relaxation time (16) in QED.

Although quarks (of charge $Q_q e$) contribute negligibly to the electrical current, their presence leads to additional stopping of the leptons and thus smaller conductivity. Adding contributions from ℓq and $\ell\bar{q}$ collisions we obtain the total electrical conductivity $\sigma_{el} = \sigma_{\ell\bar{\ell}}/(1 + 3\sum_{q=1}^{N_q} Q_q^2)$.

4 Transport Coefficients in Degenerate Matter

In degenerate QED and QCD plasmas there are practically no photons or gluons present respectively since $T \ll \mu$. The Debye screening length is according to Eq. (6) proportional to $\lambda_D \simeq 1/e\mu$ for an electron plasma and $\lambda_D \simeq 1/g\mu$ for a quark plasma.

In a degenerate plasma there are three momentum scales, namely μ, T, and q_D, whereas in the hot plasmas we could neglect the chemical potential. In the cold plasma $T \ll \mu$ and likewise for the weakly interacting plasma $q_D \ll \mu$, but it is important to consider the limits of $q_D \ll T$ and $T \ll q_D$ separately.

Momentum transfer processes in degenerate quark matter, $T \ll \mu$ (chemical potential), as for example in neutron stars, are also characterized by the rate of momentum relaxation for strong interactions, $\tau_{\rm mom}^{-1}$. For N_q quark flavors with the same chemical potentials we find, neglecting the strange quark mass, that [3]

$$\frac{1}{\tau_{\rm mom}} = \frac{8N_q}{3\pi} T\alpha_s^2 \left\{ \begin{array}{ll} (3/2)\ln(T/q_D), & T \gg q_D \\ a(T/q_D)^{2/3} + (\pi^3/12)(T/q_D), & T \ll q_D \end{array} \right\}, \tag{19}$$

where $a = (2\pi)^{2/3}\Gamma(8/3)\zeta(5/3)/6 \simeq 1.81$.

The two limiting cases can be qualitatively understood by noticing that, due to Pauli blocking and energy conservation, the energy of the particles before and after collisions must be near the Fermi surface, and therefore $\omega \lesssim T$. For $T \gg q_D$ the limitation $\omega \lesssim T$ does not affect the screening because $|\omega| \le q \sim q_D \ll T$ and consequently the result (19) is analogous to that for high temperatures, Eq. (14), but with $q_D^2 = 2N_q\alpha_s\mu^2/\pi$. The result for $T \ll q_D$ is qualitatively different due to Landau damping of modes with $q \lesssim (\omega q_D^2)^{1/3}$, where now $\omega \sim T$. The two terms in (19) correspond to the contributions from transverse and longitudinal interactions respectively and we note that transverse interactions dominate for $T \ll q_D$ because Landau damping is less effective than Debye screening in screening interactions at small q and ω.

The viscous relaxation time for quarks is defined analogously to (13) by

$$\eta = \frac{1}{5} n p_F v_F \tau_\eta \,, \tag{20}$$

where $p_F = \mu_q$ is the Fermi momentum, $v_F = c = 1$ is the Fermi velocity and $n = N_q n_q$ is the density of quarks. By solving the kinetic equation we find [3]

$$\frac{1}{\tau_\eta} = \frac{8}{\pi} N_q \alpha_s^2 \times \left\{ \begin{array}{ll} \frac{5}{3} T \ln(T/q_D)\,, & T \gg q_D \\ a\frac{T^{5/3}}{q_D^{2/3}} + \frac{\pi^3}{4}\frac{T^2}{q_D}\,, & T \ll q_D \end{array} \right\}. \tag{21}$$

We define a characteristic relaxation time for thermal conduction, τ_κ, by

$$\kappa = \frac{1}{3} v_F^2 c_v \tau_\kappa \,, \tag{22}$$

where $c_v = (N_q/2)T\mu_q^2$ is the specific heat per volume. Thus we find [3]

$$\frac{1}{\tau_\kappa} = \frac{4}{\pi^3} N_q \alpha_s^2 \mu_q^2 \times \left\{ \begin{array}{ll} \frac{\ln(T/q_D)}{3T}\,, & T \gg q_D \\ 2\zeta(3)T/q_D^2\,, & T \ll q_D \end{array} \right\}. \tag{23}$$

The relaxation time for thermal conduction has a different temperature dependence than both τ_s and τ_η because the thermal conduction was weighted by energy transfers (ω^2) instead of momentum transfers (q^2) as for momentum stopping, electrical conduction and viscous processes.

Applications to burning of nuclear matter into strange quark matter in the interior of a neutron star are described in Ref. [3]. In the above calculation the quark matter was assumed to be present in bulk. The transport properties may, however, be significantly different in a complex mixed phase of quark and nuclear matter in cores of neutron stars [7].

5 Damping of Quarks and Gluons

Historically, calculating damping rates of quasiparticles in a thermal quark-gluon plasma is a controversial subject, since early results indicated that the damping rate was negative, and gauge dependent (for a review, see Ref. [9]). Here we show how the lifetime may be calculated within the framework of the kinetic equation [8]. The quasiparticle decay rate for a gluon of momentum $\mathbf{p}_1$ scattering on other gluons is [6]

$$\frac{1}{\tau_{p_1}^{gg}} = 2\pi\nu_2 \sum_{\mathbf{q},\mathbf{p}_2} \frac{n_2(1+n_3)(1+n_4)}{1+n_1} |M_{gg\to gg}|^2 \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\,, \qquad (24)$$

where $\mathbf{p}_2$ is the momentum of the other gluon. When $p_1 \gg q_D \sim gT$ the integrals in (24) over the transverse part of the interaction diverge as shown in [8] because the factor $(1\cos\theta)$ or q^2 appearing in the earlier transport calculations is missing; i.e., Landau damping alone is insufficient to obtain a non-zero quasiparticle lifetime. Including an infrared cut-off, $\lambda \simeq m_{mag} \simeq g^2T$, which takes into account the failure of perturbative ideas at momentum scales of order the magnetic mass, the leading contribution comes from small momentum transfers $q \sim q_D \sim gT$ and we obtain

$$1/\tau_{p_1}^{(g)} = 3\alpha_s \ln(1/\alpha_s)\, T\,. \qquad (25)$$

The quasiparticle decay rate for quarks can be calculated analogously and is just 4/9 of that for the gluon, the factor coming from the matrix elements at small momentum transfer for quark scattering on quarks and gluons compared with those for gluon scattering.

Let us now compare this result with that obtained using field-theoretic techniques. Braaten & Pisarski [9] have developed a technique for resumming soft thermal loops which provides screening so that the damping, γ, at $p_1 = 0$ is positive and gauge independent. Recently, Burgess, and Marini & Rebhan [10] have obtained the leading logarithmic order for quarks and gluons with momenta $p_1 \gg gT$. They evaluate the gluon self energy, given by the gluon bubble, to leading order by including screening in the propagator of the soft gluon in the bubble and introducing the same cutoff. Their result for the imaginary part of the self energy, which is one half the quasiparticle decay rate, agrees with ours, Eq. (25), because exchange contributions (vertex corrections), which are automatically included in the kinetic equation, do not contribute to leading order.

The relaxation rates in transport processes [1] are of order $\sim \alpha_s^2 T$, i.e., suppressed by a factor α_s with respect to the damping rates. This is because in transport processes one has an extra factor q^2 in integrals like (9) which suppresses the rate by a factor $q_D^2/T^2 \simeq \alpha_s$.

6 Flavor, Spin and Color Diffusion

Let us first consider the case where the particle flavors have been separated spatially, i.e., the flavor chemical potential depends on position, $\mu_i(\mathbf{r})$. In a steady state scenario the flavor will then be flowing with flow velocity, u_i. If we make the standard ansatz for the distribution function (see, e.g., [6, 8])

$$n_{\mathbf{p},i} = \left(\exp(\frac{\epsilon_\mathbf{p} - \mu_i(\mathbf{r}) - \mathbf{u}_i \cdot \mathbf{p}}{T}) + 1\right)^{-1} \simeq \left(\exp(\frac{\epsilon_\mathbf{p} - \mu_i(\mathbf{r})}{T}) + 1\right)^{-1} - \frac{\partial n_p}{\partial \epsilon_p}\mathbf{u}_i \cdot \mathbf{p} \,, \quad (26)$$

the expansion is valid near global equilibrium where μ_i and therefore also $\mathbf{u}_i$ is small. The two terms are those of local isotropic equilibrium and the deviation from that. In general the deviation from local equilibrium has to be found selfconsistently by solving the Boltzmann equation. However, in the case of the viscosity the analogous ansatz was found to be very good [1]. The flavor current is then simply $\mathbf{j}_i = \sum_\mathbf{p} n_{\mathbf{p},i} = \mathbf{u}_i n_i$ where $n_i = \sum_p n_{p,i}$ is the density of a particular flavor i. Linearizing the Boltzmann equation we now find

$$\begin{aligned} -\frac{\partial n_1}{\partial \epsilon_1}\mathbf{v}_1 \cdot \nabla \mu_i &= 2\pi \sum_{234} n_1 n_2 (1-n_3)(1-n_4)\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) \\ &\times \; |M_{12\to 34}|^2 (\mathbf{u}_1 - \mathbf{u}_2)\cdot \mathbf{q} \end{aligned} \quad (27)$$

The resulting flavor diffusion coefficient defined by: $\mathbf{j}_i = -D_{flavor}\nabla\mu_i$, is now straightforward to evaluate when the screening is properly included as described above. The calculation is analogous to that of the momentum stopping or viscous times. We find

$$D_{flavor}^{-1} \simeq 2.0(1+\frac{N_f}{6})\alpha_s^2 \ln(1/\alpha_s) T \,. \quad (28)$$

Subsequently, let us consider the case where the particle spins have been polarized spatially, i.e., the spin chemical potential depends on position, $\mu_\sigma(\mathbf{r})$. With the analogous ansatz to (26) for the distribution functions with μ_σ instead of μ_i we find the spin current $\mathbf{j}_\sigma = \mathbf{u}_\sigma n_\sigma$. Linearizing the Boltzmann equation we find

$$\begin{aligned} -\frac{\partial n_1}{\partial \epsilon_1}\mathbf{v}_1 \cdot \nabla \mu_\sigma &= 2\pi \sum_{234} n_1 n_2 (1-n_3)(1\pm n_4)\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) \\ &\times \; \left[|M^{\uparrow\uparrow}_{12\to 34}|^2 (\mathbf{u}_1 - \mathbf{u}_2)\cdot \mathbf{q} + |M^{\uparrow\downarrow}_{12\to 34}|^2 (\mathbf{u}_1 - \mathbf{u}_2)\cdot (\mathbf{p}_1+\mathbf{p}_2)\right] \end{aligned} \quad (29)$$

where $M^{\uparrow\downarrow}$ and $M^{\uparrow\uparrow}$ refer to the spin flip and the non-spin flip parts of the amplitude.

The transition current can be decomposed into the non-spin flip and the spin flip parts by the Gordon decomposition rule

$$J_\mu \;\simeq\; g\bar{u}_f\gamma_\mu u_i \simeq \frac{g}{2m}\bar{u}_f\left[(p_f+p_i)_\mu + i\sigma_{\mu\nu}(p_f-p_i)^\nu\right]u_i \,. \quad (30)$$

We notice that the latter is suppressed by a factor $q = p_f - p_i$ which leads to a spin flip amplitude suppressed by a factor q^2. We then find that the spin flip interactions do not contribute to collisions to leading logarithmic order and the collision integral is similar to those evaluated above. Consequently, the corresponding quark spin diffuseness parameter is

$$D_\sigma^{(q)} = D_{flavor} \,. \tag{31}$$

Gluon diffusion is slower, $D_\sigma^{(g)} \simeq (4/9)^2 D_{\sigma,q}$, partly because they interact stronger.

Finally, let us, like for the spin diffusion, assume that color has been polarized spatially given by a color chemical potential, $\mu_c(\mathbf{r})$. The basic difference to spin diffusion is that *quarks and gluons can easily flip color directions* in forward scattering by color exchanges, i.e., one does not pay the extra q^2 penalty factor as in the case of spin flip. Consequently, the color flip interactions will dominate the collisions since they effectively reverse the color currents. As a consequence the collision term becomes similar to that for the quasiparticle scatterings (14) and we find for the color diffuseness parameter

$$D_{color}^{-1} \simeq 0.3\,\alpha_s \ln(1/\alpha_s)T \sim \alpha_s^{-1} D_\sigma^{-1} \,. \tag{32}$$

The color flip mechanism amplifies the collisions so the color cannot diffuse as easily as spin or flavor.

The color conductivity is found analogous to the electrical conductivity, where $\sigma_{el} \simeq q_D^2 \tau_{el}$ (see Eqs. (17) and (18)),

$$\sigma_c \sim q_D^2 D_{color} \sim T/\ln(1/\alpha_s) \,. \tag{33}$$

The characteristic relaxation times for conduction are very different in QCD, where $\tau_{color} \sim D_{color} \sim (\alpha_s \ln(1/\alpha_s)T)^{-1}$, as compared to QED, where $\tau_{el} = \tau_{mom} \simeq (\alpha^2 \ln(1/\alpha)T)^{-1}$. Consequently, QGP are much poorer color conductors than QED plasmas for the same coupling constant.

These surprising results for QCD are qualitatively in agreement with those found by Selikhov & Gyulassy [11] who have considered the diffusion of color in color space. They use the fluctuation-dissipation theorem to estimate the deviations from equilibrium and find the same two terms as in (32), which they denote the momentum and color diffusion terms, and they also find that the latter dominates. Inserting the same infrared cut-off they find a color diffusion coefficient in color space equal to (25)

$$d_c = \frac{1}{\tau_1^{(g)}} = 3\alpha_s \ln(1/\alpha_s)T \,. \tag{34}$$

Note that this quantity is proportional to the inverse of D_{color} of Eq. (32).

The color flip mechanism is not restricted to QCD but has analogues in other non-abelian gauge theories. In the very early universe when $T > T_{SSB} \simeq 250$ GeV, the $W^\pm$ bosons are massless and faces the same electroweak screening problems as QCD and QED. Since now the exchanged $W^\pm$ bosons carry charge (unlike the photon, but like the colored gluon), they can easily change the charge of, for example, an electron to a neutrino in forward scatterings. Thus the collision term will lack the usual factor q^2 as for the quasiparticle damping rate and the color diffusion. Since $SU(2) \times U(1)$

gauge fields should have the same infrared problems as SU(3) at the scale of the magnetic mass, $\sim e^2T$, we insert this infrared cutoff. Thus we find a diffusion time for charged electroweak particles in the very early universe of order

$$\tau_{diff} \sim (\alpha \ln(1/\alpha)T)^{-1}, \tag{35}$$

which is a factor α smaller than the momentum stopping time. The electrical conductivity will be smaller by the same factor as well.

7 Summary

Whereas the troublesome Rutherford divergence at small scattering angles is screened by Debye screening for the longitudinal or electric part of the interactions, the transverse or magnetic part of the interactions is effectively screened by Landau damping of the virtual photons and gluons transferred in the interactions. Including the screening, we calculated a number of transport coefficients for QED and QCD plasmas to leading order in the interaction strength. These included rates of momentum and thermal relaxation, electrical conductivity, and viscosities of quark-gluon plasmas for thermal as well as degenerate plasmas.

Our calculations above show that the transport properties of high temperature and degenerate QCD and QED plasmas of relativistic particles have several interesting features. In a degenerate plasmas there are three scales: the chemical potentials, the temperature, and the Debye screening wavenumber, whereas in thermal plasmas, where $\mu \ll T$, the chemical potentials can be neglected. In the degenerate case, where μ is much larger than both T and q_D, the physics changes between the two limiting cases of $T \gg q_D$ and $T \ll q_D$. When $T \gg q_D$ the characteristic relaxation rates are $1/\tau \simeq \alpha_s^2 \ln(T/q_D)T$ as in the high temperature plasma, and the contributions from longitudinal and transverse interactions are of comparable magnitudes. However, the Debye wavenumber is different, $q_D \sim gT$ at high temperatures and $q_D \sim g\mu$ in degenerate plasmas. When $T \ll q_D$ the transverse interactions dominate the scattering processes because they are screened by Landau damping only for energy transfers less than $\sim (q_D^2 T)^{1/3}$. The resulting relaxation rates for momentum transfer, electrical conduction, and viscous processes scale as $1/\tau \sim (\alpha_s T)^{5/3}/\mu_q^{2/3}$, while the relaxation rate for thermal conduction is $1/\tau_\kappa \sim \alpha_s T$. The qualitative reason for τ_κ behaving in a different way from the other relaxation times is related to the singular character of the interaction for small energy transfer and small momentum transfer. An important general conclusion of these studies of QCD and QED plasmas is that the the transport coefficients deviate from the standard results of Fermi liquid theory and relaxations times are different for different transport processes.

Color diffusion and the quark and gluon quasiparticle decay rates are not sufficiently screened and do depend on an infrared cut-off of order the magnetic mass, $m_{mag} \sim g^2T$; typically $\tau^{-1} \sim \alpha_s \ln(q_D/m_{mag})T \sim \alpha_s \ln(1/\alpha_s)T$. As a consequence, quasiparticle decay is fast, color diffusion is slow and the QGP is a poor color conductor.

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