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A Resource-Rational Process-Level Account of Violation of Stochastic Dominance

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Abstract

Dominance is widely considered a pillar of rational choice and has played a major role in the history of theorizing and developing models of human decision-making. A wealth of empirical evidence reveals that humans' violation of dominance is both substantial and systematic. But could violation of dominance be given a rational basis? Specifically, could it be understood in terms of the optimal use of limited cognitive resources? In this work, we present the first resource-rational account of stochastic dominance, the most empirically studied version of dominance. Concretely, we show that a resource-rational process model, *sample-based expected utility* (SbEU), provides a unified account of a broad range of empirical results on violation of stochastic dominance. We discuss the implications of our work for risky decision-making, and more broadly, human rationality.

Keywords: stochastic dominance; resource-rationality; risky choice; resource-rational process models

1 Introduction

A cornerstone of decision theory, *dominance* is arguably considered "the most obvious" principle of rational choice (Kahneman & Tversky, 1986), playing a major role in the history of modeling human decision-making (Tversky & Kahneman, 1992; Birnbaum, 2005).[1]

In this work, we focus on the most empirically studied version of dominance, called first-order *stochastic dominance* (SD). A wealth of empirical evidence reveals that humans' violation of SD (VoSD) is both substantial and systematic (e.g., Birnbaum & Navarrete, 1998; Birnbaum, 2004a, 2004b; Birnbaum et al., 1999; Birnbaum, 1999, Birnbaum & Martin, 2003; Birnbaum, 2005).

Although past work has suggested that bounded rationality plays a role in VoSD (e.g., Birnbaum, 1999; Huck & Müller, 2012; Levy, 2008; Choi et al., 2014; Kourouxous & Bauer, 2019), it has remained largely unknown how bounded rationality shapes the algorithmic foundations of VoSD. We ask whether VoSD could be given a resource-rational algorithmic basis (Griffiths, Lieder, & Goodman, 2015; Gershman, Horvitz, & Tenenbaum, 2015; Nobandegani, 2017; Bhui, Lai, & Gershman, 2021). Specifically, could VoSD be understood in terms of the optimal use of limited cognitive resources?

In this work, we present the first resource-rational account of VoSD. Concretely, we show that a resource-rational process model, *sample-based expected utility* (SbEU; Nobandegani et al., 2018), provides a unified account of a broad range of empirical results on VoSD. Here, we particularly focus on Birnbaum (2005) which is, to our knowledge, the most extensive empirical study of VoSD.

We begin by formally defining SD (Sec. 2) and discussing how SbEU works (Sec. 3). We then present our simulation results, quantitatively comparing SbEU model predictions to human data (Sec. 4). We conclude by discussing the implications of our work for risky decision-making, and more broadly, human rationality.

2 Stochastic Dominance

In simple terms, SD can be described as follows. Gamble A *stochastically dominates* gamble B, denoted by $A \succ_{SD} B$, if the probability of winning any given prize x or more is at least as high in A as in B, and this probability is strictly higher in A for at least one value of x. More formally, $A \succ_{SD} B$, if $\forall x$: $\Pr(\text{winning a prize} \geq x|A) \geq \Pr(\text{winning a prize} \geq x|B)$ and $\exists x$: $\Pr(\text{winning a prize} \geq x|A) > \Pr(\text{winning a prize} \geq x|B)$.

As an example (Birnbaum, 2005), gamble P stochastically dominates gamble Q (w.p. stands for "with probability") :

$$P = \begin{cases} \$96 & \text{w.p. } 90\% \\ \$14 & \text{w.p. } 5\% \\ \$12 & \text{w.p. } 5\% \end{cases} \tag{1}$$

$$Q = \begin{cases} \$96 & \text{w.p. } 85\% \\ \$90 & \text{w.p. } 5\% \\ \$12 & \text{w.p. } 10\% \end{cases} \tag{2}$$

The rationale is as follows. The probability to win \$96 or more is .90 in gamble P, and only .85 in gamble Q; the probability to win \$90 or more is the same in both gambles; the probability to win \$14 or more is .95 in gamble P and only .9 in gamble Q; and the probability to win \$12 or more is the same in both gambles. Hence, $P \succ_{SD} Q$.

3 Resource-Rational Process Model

Extending an earlier risky decision-making model (Lieder, Griffiths, & Hsu, 2018) to the realm of meta-reasoning, *sample-based expected utility* (SbEU; Nobandegani et al., 2018) is a resource-rational process model of risky choice that posits that people rationally adapt their strategy depending on

[1]Interestingly, due to its normative appeal, dominance was even used by some scholastics to argue in favor of Christianity (see Covello & Mumpower, 1985).

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the amount of time available for decision-making. Concretely, SbEU assumes that people estimate expected utility

$$\mathbb{E}[u(o)] = \int p(o)u(o)do, \tag{3}$$

using self-normalized importance sampling (Hammersley & Handscomb, 1964; Geweke, 1989), with its importance distribution q^* aiming to optimally minimize mean-squared error (MSE):

$$\hat{E} = \frac{1}{\sum_{j=1}^{s} w_j} \sum_{i=1}^{s} w_i u(o_i), \quad \forall i:\ o_i \sim q^*,\ w_i = \frac{p(o_i)}{q^*(o_i)}, \tag{4}$$

$$q^*(o) \propto p(o)|u(o)| \sqrt{\frac{1+|u(o)|\sqrt{s}}{|u(o)|\sqrt{s}}}. \tag{5}$$

MSE is a standard measure of estimation quality, widely used in decision theory and mathematical statistics (Poor, 2013). In Eqs. (3-5), o denotes an outcome of a risky gamble, $p(o)$ the objective probability of outcome o, $u(o)$ the subjective utility of outcome o, $\hat{E}$ the importance-sampling estimate of expected utility given in Eq. (3), q^* the importance-sampling distribution, o_i an outcome randomly sampled from q^*, and s the number of samples drawn from q^*.

SbEU assumes that, when choosing between a pair of risky gambles A, B, people consider whether the expected value of the utility difference $\Delta u(o)$ is positive or negative:

$$A = \begin{cases} o_A & \text{w.p. } P_A \\ 0 & \text{w.p. } 1 - P_A \end{cases} \tag{6}$$

$$B = \begin{cases} o_B & \text{w.p. } P_B \\ 0 & \text{w.p. } 1 - P_B \end{cases} \tag{7}$$

$$\Delta u(o) = \begin{cases} u(o_A) - u(o_B) & \text{w.p. } P_A P_B \\ u(o_A) - u(0) & \text{w.p. } P_A(1 - P_B) \\ u(0) - u(o_B) & \text{w.p. } (1 - P_A)P_B \\ 0 & \text{w.p. } (1 - P_A)(1 - P_B) \end{cases} \tag{8}$$

In Eq. (8), $u(\cdot)$ denotes the subjective utility function of a decision-maker. Fully consistent with past work (Nobandegani et al., 2018; Nobandegani et al., 2019a; Nobandegani, Destais, & Shultz, 2020a, Nobandegani & Shultz, 2020b), in this paper we use the following utility function:

$$u(x) = \begin{cases} x^{0.85} & \text{if } x \geq 0, \\ -|x|^{0.95} & \text{if } x < 0. \end{cases} \tag{9}$$

Also, in line with prospect theory (Kahneman & Tversky, 1979), we here assume that people perform a variant of *segregation*, as a form of editing, prior to evaluating the gambles. The purpose of editing is to obtain a simplified representation of gambles prior to further evaluation (Kahneman & Tversky, 1979).[2] In this variant of segregation, a risky gamble is decomposed into a sure thing (corresponding to the minimum outcome of that gamble) and the remaining risky gamble, with the branch corresponding to the sure thing fully removed. In our simulations (Sec. 4), we assume that 50% of participants adopt segregation.

In our simulations (Sec. 4), we also assume that people draw between 1 to 10 samples when deciding. Specifically, we adopt a uniform distribution and assume that one-tenth of the population draw one sample (i.e., $s = 1$; see Eqs. (4-5)), one-tenth of the population draw two samples (i.e., $s = 2$), one-tenth of the population draw three samples and so on. This is consistent with mounting evidence suggesting that people draw only a few samples in probabilistic judgment and reasoning (e.g., Vul et al., 2014; Battaglia et al., 2013; Lake et al., 2017; Gershman, Horvitz, & Tenenbaum, 2015; Hertwig & Pleskac, 2010; Griffiths et al., 2012; Gershman, Vul, & Tenenbaum, 2012; Bonawitz et al., 2014; Nobandegani et al., 2018; Nobandegani et al., 2020a).

Recent work has shown that SbEU provides a unified account of a broad range of major empirical findings across risky, value-based, and strategic decision-making (Nobandegani et al., 2018; Nobandegani et al., 2019a, 2019b; Nobandegani et al., 2020a; Nobandegani & Shultz, 2020b, 2020c, 2020d; Lizotte, Nobandegani, & Shultz, 2021), and also bridges between decision-making under risk and decision-making under uncertainty (Nobandegani et al., 2021). Notably, SbEU is the first rational process model to score near-perfectly in optimality, economical use of limited cognitive resources, and robustness, all at the same time (see Nobandegani et al., 2018; Nobandegani et al., 2019c).

4 Simulation Results

In this section, we simulate Experiments 1-7 in Birnbaum (2005), conducted with a total of 1,802 human participants.

4.1 Experiments 1-2

In Experiments 1-2, Birnbaum (2005) empirically investigated how split of probabilities among the branches of risky gambles affects VoSD. Experiments 1-2 involved 7 pairs of 3-branch gambles with positive outcomes, with each pair comprising a stochastically dominant gamble G^+ and a stochastically dominated gamble G^- (see Appendix for the gambles). Hence, choosing G^- over G^+ indicates VoSD.

Fig. 1(a) shows SbEU model predictions for Birnbaum's (2005) Experiments 1-2, along with the empirical data. The model predictions correlate highly with the empirical data (Pearson $r = .9523$, $p < .001$).

[2] As such, editing is broadly consistent with resource-rationality as it correctly acknowledges the representational constraints that people are naturally faced with (see Bhui & Gershman, 2018). To show that editing is fully consistent with resource-rationality, future work should investigate whether people boundedly-optimally allocate their representational bandwidth in editing.

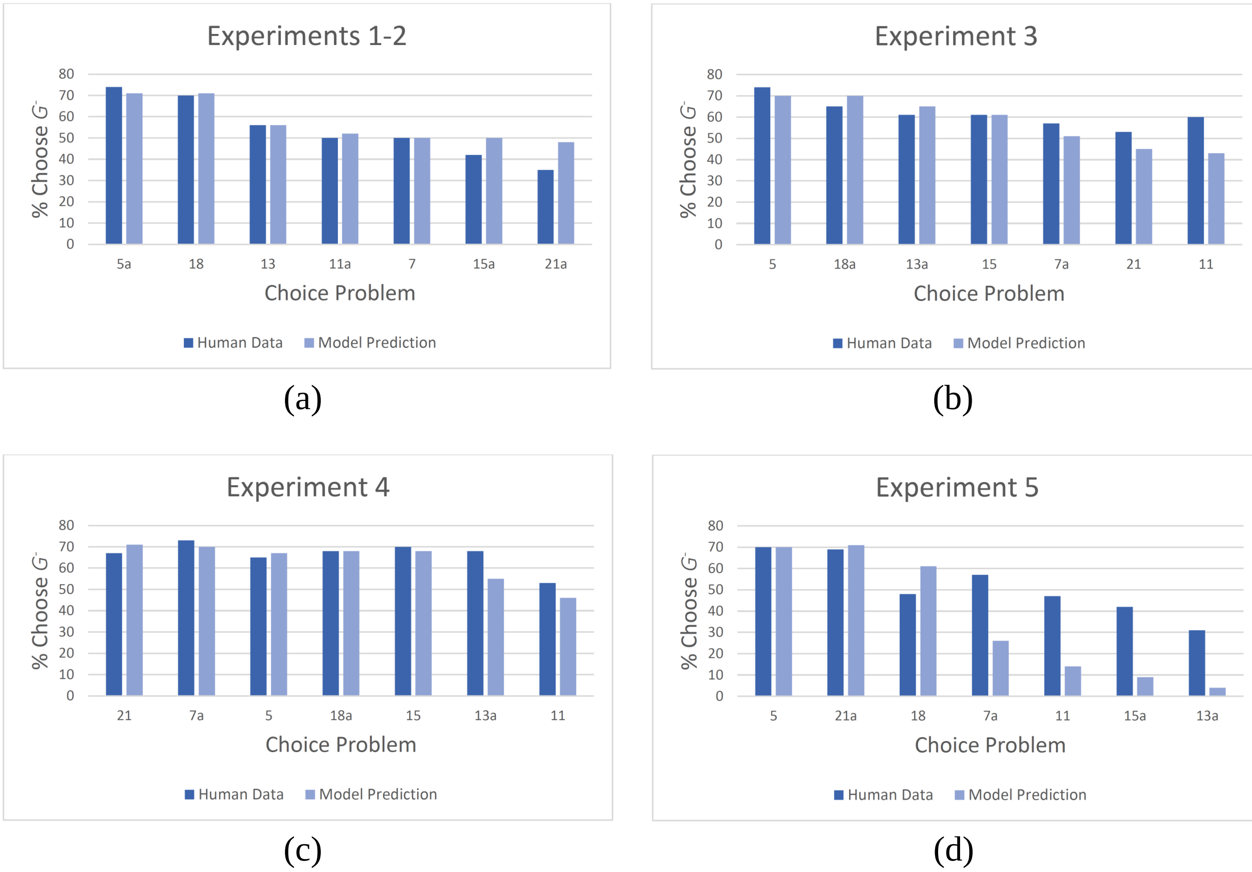

Figure 1: **Comparing human data (Birnbaum, 2005) with SbEU model predictions.** In each subplot, the x-axis indicates the choice problem number (see Appendix for details), and the y-axis shows the percentage of participants choosing G^-over G^+, hence violating stochastic dominance. **(a)** Birnbaum's (2005) Experiments 1-2, **(b)** Birnbaum's Experiment 3, **(c)** Birnbaum's Experiment 4, and **(d)** Birnbaum's Experiment 5. We simulated $10,000$ participants in each condition of each experiment.

4.2 Experiment 3

In Experiment 3, Birnbaum (2005) empirically investigated the effect of redistribution of probabilities on VoSD, previously predicted in Birnbaum (1997). Experiment 3 involved 7 pairs of 3-branch gambles with positive outcomes, with each pair comprising a stochastically dominant gamble $G^+$and a stochastically dominated gamble G^-(see Appendix for the gambles). Hence, choosing G^-over G^+indicates VoSD.

Fig. 1(b) shows SbEU model predictions for Birnbaum's (2005) Experiment 3, along with the empirical data. The model predictions correlate highly with the empirical data (Pearson $r = .7708$, $p < .05$).

4.3 Experiment 4

In Experiment 4, Birnbaum (2005) empirically ruled out a heuristic account of VoSD, the *consequence counting heuristic*, according to which people should pay excess attention to outcome values in risky choice. Experiment 4 involved 7 pairs of 3-branch gambles with positive outcomes, with each pair comprising a stochastically dominant gamble $G^+$and a stochastically dominated gamble G^-(see Appendix for the gambles). Hence, choosing G^-over G^+indicates VoSD.

Fig. 1(c) shows SbEU model predictions for Birnbaum's (2005) Experiment 4, along with the empirical data. The model predictions correlate highly with the empirical data (Pearson $r = .7988$, $p < .05$).

4.4 Experiments 5-7

In Experiment 5, Birnbaum (2005) empirically investigated the effect of manipulating the outcomes of the dominated gamble G^-on VoSD. Experiment 5 involved 7 pairs of 3-branch gambles with positive outcomes, with each pair comprising a stochastically dominant gamble $G^+$and a stochastically dominated gamble G^-(see Appendix for the gambles). Hence, choosing G^-over G^+indicates VoSD.

Fig. 1(d) shows SbEU model predictions for Birnbaum's (2005) Experiment 5, along with the empirical data. The model predictions again correlate highly with the empirical data (Pearson $r = .8261$, $p < .05$). As can be seen in Fig. 1(d), although the model prediction is quantitatively off in Choice Problems 7a, 11, 15a, and 13a, the model nevertheless accurately captures the qualitative trend of the empirical data for those Choice Problems, with both model predictions and empirically observed data gradually decreasing when moving from Choice Problem 7a to 11, then to 15a, and finally to 13a.

Birnbaum's (2005) Experiment 6 contained only a single test of VoSD using a pair of 5-branch mixed gambles (empirically observed VoSD = 63% vs. model prediction = 54.76%). Birnbaum's Experiment 7 contained two tests of VoSD again using pairs of 5-branch mixed gambles (empirically observed VoSD = 66.5% vs. model prediction = 54.62%; empirically observed VoSD = 65.5% vs. model prediction = 57.66%).

5 Discussion

Considered as arguably "the most obvious" principle of rational choice (Kahneman & Tversky, 1986), *dominance* has played a major role in the history of theorizing and developing models of human decision-making (Tversky & Kahneman, 1992; Birnbaum, 2005; Kourouxous & Bauer, 2019).

Interestingly, a wealth of empirical evidence reveals that humans' violation of dominance is both substantial and systematic (e.g., Birnbaum & Navarrete, 1998; Birnbaum, 2004a, 2004b; Birnbaum et al., 1999; Birnbaum, 1999, Birnbaum & Martin, 2003; Birnbaum, 2005). For example, by examining the health plan choices of 23,894 employees at a U.S. firm, recent work has shown that the majority of employees chose dominated plans, which resulted in excess spending equivalent to 24% of chosen plan premiums (Bhargava, Loewenstein, & Sydnor, 2017; see also Handel, 2013).

Here, we ask whether violation of dominance could be given a rational basis. Specifically, could it be understood in terms of the optimal use of limited computational and cognitive resources? In this work, we focus on the most empirically studied version of dominance, first-order stochastic dominance (SD), and provide the first resource-rational account of violation of SD (VoSD). We show that a single parameterization of SbEU, a resource-rational process model of risky choice, provides a unified account of a broad range of empirical results on VoSD (Birnbaum, 2005).

To be consistent, we use the exact same utility function (Eq. 9) used in past work, without optimizing it to improve model fit. Future work should optimize model fits to empirical data and make comparisons with other prominent models (e.g., the transfer of attention exchange model (TAX), Birnbaum, 2005).

Although this work particularly focuses on violation of dominance in single-agent risky decision-making, there is evidence that this modeling approach can also explain violation of dominance in *multi-agent* settings. Recently, Nobandegani et al. (2019a) provided a resource-rational account of (ostensibly irrational) cooperation in one-shot Prisoner's Dilemma (OPD). In OPD, defection is the dominant strategy whereas cooperation is the dominated strategy. Hence, choosing cooperation over defection is a violation of dominance in OPD — a violation that Nobandegani et al.'s (2019a) SbEU-based, resource-rational model accounted for. Future work should investigate whether a resource-rational account of violation of dominance that goes beyond the specific game of OPD could be developed in multi-agent decision-making.

An intimately related concept to dominance is the *sure-thing principle* (STP) (Savage, 1954). According to STP, a decision-maker who takes action $\mathcal{A}$ both when event E has occurred and when the negation of E has occurred, should take the same action $\mathcal{A}$ even when they know nothing about E. Although Savage (1954) provided a strong normative basis for STP by appealing to dominance, substantial empirical evidence revealed that people violate STP across a range of domains (e.g., Khrennikov & Haven, 2009; Tversky & Shafir, 1992; Li et al., 2010; Croson, 1999). Given this intimate link between dominance and STP, the work presented here suggests that resource-rationality might hold the key for developing a normative basis for violation of STP. The observation that the Allais paradox and the Ellsberg paradox, as two notable instances of violation of STP, can both be given a resource-rational account further elevates this possibility (Nobandegani et al., 2021). Future work should investigate whether STP could be given a resource-rational foundation.

In this work, we look at VoSD through the lens of modern psychological theories of bounded rationality (see Bhui et al., 2021), providing a resource-rational, algorithmic foundation for VoSD. Given the broad empirical coverage of SbEU across risky, value-based, and strategic decision-making (see Sec. 3), this result is particularly interesting as it brings us a step closer to developing a unified, boundedly-optimal account of human decision-making. The work presented here is a step in this important direction.

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Appendix

For brevity, we use the following shorthand, à la Birnbaum (2005). Gamble P given in (1) in the main text can be alternatively represented as follows: $P = (\$96, .9; \$14, .05; \$12, .05)$.

Experiments 1-2:

Choice Problem 5a:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$90, .05; \$12, .1)$

Choice Problem 18:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .85; \$91, .05; \$13, .1)$

Choice Problem 13:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .75; \$91, .05; \$13, .2)$

Choice Problem 11a:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .65; \$90, .05; \$12, .3)$

Choice Problem 7:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .55; \$91, .05; \$13, .4)$

Choice Problem 15a:

$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .45; \$90, .05; \$12, .5)$

Choice Problem 21a:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .25; \$90, .05; \$12, .7)$

Experiment 3:

Choice Problem 5:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$90, .05; \$12, .1)$

Choice Problem 18a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .85; \$91, .05; \$13, .1)$

Choice Problem 13a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .65; \$91, .25; \$13, .1)$

Choice Problem 15:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .55; \$90, .35; \$12, .1)$

Choice Problem 7a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .65; \$91, .25; \$13, .1)$

Choice Problem 21:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .25; \$90, .65; \$12, .1)$

Choice Problem 11:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .15; \$90, .75; \$12, .1)$

Experiment 4:

Choice Problem 21:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$90, .05; \$12, .1)$

Choice Problem 7a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .85; \$91, .05; \$13, .1)$

Choice Problem 5:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$94, .85; \$90, .05; \$10, .1)$

Choice Problem 18a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$95, .85; \$91, .05; \$11, .1)$

Choice Problem 15:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$70, .05; \$12, .1)$

Choice Problem 13a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .85; \$41, .05; \$13, .1)$

Choice Problem 11:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$20, .05; \$12, .1)$

Experiment 5:

Choice Problem 5:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$96, .85; \$90, .05; \$12, .1)$

Choice Problem 21a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$97, .85; \$91, .05; \$13, .1)$

Choice Problem 18:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$90, .85; \$84, .05; \$6, .1)$

Choice Problem 7a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$90, .85; \$80, .05; \$10, .1)$

Choice Problem 11:
$G^+ = (\$96, .9; \$14, .05; \$12, .05)$
$G^- = (\$85, .85; \$75, .05; \$4, .1)$

Choice Problem 15a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$80, .85; \$70, .05; \$5, .1)$

Choice Problem 13a:
$G^+ = (\$97, .9; \$15, .05; \$13, .05)$
$G^- = (\$70, .85; \$60, .05; \$2, .1)$

Experiment 6:

Choice Problem:
$G^+ = (\$100, .35; \$0, .37; -\$95, .04; -\$97, .04; -\$100, .20)$
$G^- = (\$100, .10; \$99, .10; \$96, .10; \$0, .40; -\$100, .30)$

Experiment 7:

Choice Problem 4:
$G^+ = (\$100, .35; \$0, .37; -\$95, .04; -\$97, .04; -\$100, .20)$
$G^- = (\$100, .10; \$99, .10; \$96, .10; \$0, .40; -\$100, .30)$

Choice Problem 21:
$G^+ = (\$100, .35; \$0, .37; -\$90, .04; -\$95, .04; -\$100, .20)$
$G^- = (\$100, .12; \$99, .10; \$97, .10; \$0, .38; -\$100, .30)$

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