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Author
He, Songyu

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Essays in Preferences under Uncertainty

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by

Songyu He

Committee in charge:

Professor Charles Sprenger, Co-Chair
Professor Emanuel Vespa, Co-Chair
Professor Songzi Du
Professor Craig McKenzie
Professor Joel Sobel

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The Dissertation of Songyu He is approved, and it is acceptable in quality and form for publication on microfilm and electronically.

University of California San Diego

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DEDICATION

To family and Friends.

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VITA

2017	Bachelor of Science, Texas A & M University
2023	Master of Arts, University of California San Diego
2024	Doctor of Philosophy, University of California San Diego

FIELDS OF STUDY

Behavioral Economics, Experimental Economics, and Micro-economic Theory

ABSTRACT OF THE DISSERTATION

Essays in Preferences under Uncertainty

by

Songyu He

Doctor of Philosophy in Economics

University of California San Diego, 2024

Professor Charles Sprenger, Co-Chair
Professor Emanuel Vespa, Co-Chair

In this dissertation I axiomatize and extend current behavioral economic models for risk preferences. Based on my theoretical findings, I also design experiment to test these theories.

In Chapter 1, I provide axiomatic foundations for salience and regret theory, and then evaluate their empirical validity. In Chapter 2, which is co-authored with Professor Charles Sprenger, we extend the model of salience theory into a multi-dimensional setting and test the

new model using both existed dataset and novel experiments. In Chapter 3, I propose a method to test cumulative prospect theory.

Chapter 1

An Axiomatic Test for Salience and Regret Theory

Abstract

Salience and regret theories describe how the manipulation of relative differences between potential outcomes of risky options affects individuals' decisions. In this project, we present axiomatizations for both theories. The axioms for salience theory formulate hypotheses concerning preferences for a large difference in outcomes compared to the summation of smaller differences that partition it. In contrast, axioms for regret theory ensure that comparisons between outcome pairs maintain the ordinal measure of their utility differences. To test these axioms, we conduct an online experiment with 800 participants. The current experiment finds supportive evidence for both theories at the aggregate level. However, there's notable heterogeneity in the results: The violation rates of our main hypotheses are between 39% to 72%.

1.1 Introduction

It is well known in economic and psychological studies that when decision-makers choose among alternatives, they may not evaluate each option independently. Instead, they tend to compare these options jointly and focus on their relative differences. This phenomenon of relative comparison underpins a multitude of significant behavioral phenomena, including reference dependence, loss aversion, the endowment effect, and categorical thinking, as evidenced in various studies.[1]

[1] Relative comparisons can explain various interesting phenomena such as reference dependence and loss aversion (Kahneman and Tversky, 1979a; Gul, 1991; Kőszegi and Rabin, 2006), endowment effect (Kahneman et al., 1990), and categorical thinking (Koszegi and Szeidl, 2012; Bordalo et al., 2012, 2013b; Bushong et al., 2021).

Within the context of decisions under uncertainty, the process of choosing a preferred option often involves contrasting the potential outcomes of different lotteries and weighting these gains and losses based on their probabilities. In response to this phenomenon, theories such as regret theory (Bell, 1982; Loomes and Sugden, 1982) and salience theory Bordalo et al. (2012) have been introduced. These theories, grounded in psychological principles, have been instrumental in explaining stylized patterns observed across various domains, including auction bidding, insurance purchasing, and investment.[2]

Despite their widespread application and influence, a comprehensive qualitative understanding of these theories remains elusive. This gap raises several concerns, such as the ambiguity of the validity of their foundational assumptions, the difficulty in distinguishing them from other models, and the challenges in identifying which aspects of the theory require modification when empirical data contradicts their predictions.

This project addresses these issues by exploring these two models from a qualitative standpoint. To this end, we adopt an axiomatic approach to characterize both theories under the framework introduced by Fishburn (1990a) and Lanzani (2022). Additionally, we conduct a novel experiment to empirically test these axioms.

To illustrate the mechanisms of salience and regret theory, their generalizations from the canonical expected utility theory, and the intuitions behind their key axioms, let's consider the following stylized example. Suppose a decision-maker is deciding whether to purchase an insurance at a price of \$20. An accident happens with probability 10%. In case the the accident occurs, the decision-maker loses \$100 if she has insurance and \$200 otherwise. The following table describes the two possible scenarios and payoffs of the corresponding options.

	Accident (10%)	No Accident (90%)
Purchase insurance	−\$120	−\$20
Forfeit insurance	−\$200	\$0

[2] See Filiz-Ozbay and Ozbay (2007) for auction bidding, Braun and Muermann (2004) for insurance purchasing, Bordalo et al. (2013a) for investment.

Expected utility posits that the decision-maker prefers purchasing the insurance if it leads to a higher expected utility value than forfeiting the insurance. Putting it mathematically, we have $0.1u(-120)+0.9u(-20) \geq 0.1u(-200)$, where $u(\cdot)$ is the decision-maker's utility function over monetary outcomes with $u(0)=0$.[3]

Regret theory introduces a nuanced perspective to decision-making under uncertainty, emphasizing the role of anticipated regret – decision-makers may experience such feelings thinking about how they could have been better off had they chosen differently. Specifically, in the previous example, according to regret theory the decision-maker is willing to purchase the insurance if

$$0.1Q\Big(u(-120)-u(-200)\Big)+0.9Q\Big(u(-20)\Big) \geq 0,$$

where $Q(\cdot)$, named as the *regret function*, is strictly increasing and skew-symmetric. It quantifies the individual's emotional response to gains and losses in utility. A critical aspect of this theory is its proportionality assumption: the intensity of regret (or joy) is directly proportional to the magnitude of the utility difference. Therefore, our axiom demands that the regret experienced from two different outcome pairs should be identical if their respective utility differences are perceived as equivalent.

Salience theory, on the other hand, suggests that the decision-maker's cognitive ability is limited. As a result, she focuses on states in which the alternatives are strikingly distinct. Using the previous example again, salience theory posits that the decision-maker prefers the insurance if

$$0.1f\Big(\sigma(-120,-200)\Big)\Big(u(-120)-u(-200)\Big)+0.9f\Big(\sigma(-20,0)\Big)\Big(u(-20)\Big) \geq 0,$$

where $\sigma(\cdot,\cdot)$ is the *salience function* measuring the distinctiveness between outcomes and f is some positive monotonic transformations. Bordalo et al. (2012) propose several properties of

[3] Since this equation can be expressed as $0.1(u(-120)-u(-200))+0.9u(-20) \geq 0$, one way to interpret the rationale is that the decision-maker is performing a state-contingent evaluation of the utility difference between the two possible options and choose the one with a higher utility on average.

the salience function, and these regulations formulate the essence of salience theory. For this reason, our axioms for salience theory closely reflect the hypotheses of the salience function. In particular, the axioms identify lotteries such that expected utility theory provides strict and *identical* behavioral predictions regardless of the shape of the utility function $u(\cdot)$. Further, the magnitude of deviations from expected utility theory constitute non-parametric bounds on the value of $f \circ \sigma$. Lastly, appropriately regulating these bounds completes characterization of salience theory.

Additional to the main axioms, we also characterize an empirically important class of preferences wherein the utility function is concave. Further, we provide a mild restriction on the salience function that facilitates our subsequent experimental investigation. Lastly, applying our representation theorems, we extend the prior work by Herweg and Müller (2021) and determine a precise delineation between these models based on parameterizations. Essentially, a preference aligns with both regret and salience theories if it exhibits a regret representation with a concave utility, its regret function is almost convex, and its average regret value from 0 to x remains distinct from 0. Figure 1.1 visualizes our characterization.

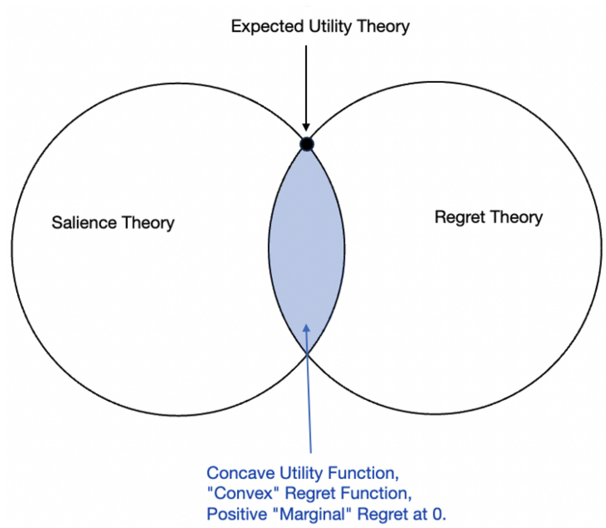

Figure 1.1. Characterization of Salience and Regret Theories

Figure 1 presents an overview of this project's theoretical results. It provides separate characterizations for salience and regret theories. In addition, besides being expected utility theory, it shows the exact situation in which these two models overlap: roughly speaking the requirements are the decision-maker's preference has a concave utility and convex regret function with a positive derivative.

With the current axioms, we conduct a novel experiment to qualitatively investigate regret and salience theories. Due to their distinct axioms, these two theories require different experimental formats. For this reason, we divide our experiment into two parts. Questions in the first part are called *payment-variation tasks* wherein we fix the state probabilities and vary the outcome pairs under each state. This part aims at testing the axiom for regret theory. In the second part, subjects are asked to complete *probability-variation tasks* wherein we fix the outcome pairs under each state and manipulate states' probabilities. This part tries to test salience theory.

The experimental results suggest that subjects exhibit correlation-sensitive behaviors. On the aggregate level, our results support both regret and salience theory. Nevertheless, for each axiom, the experiment identifies a significant proportion of violations. For regret theory, in each test, around 40% of subjects follow its predictions strictly. In contrast, for salience theory, in each test, around 55% of subjects follow its hypothesis. Furthermore, the results also suggest that there are positive correlations between the violations of different hypotheses within each theory, but only weak correlations of violations across the two theories.

This project closely relates to literature that studies representations of correlation-sensitive preferences, regret, and salience theory. Fishburn (1989, 1990b); Sugden (1993); Quiggin (1994); Bikhchandani and Segal (2014); Lanzani (2022) study the general representation. Diecidue and Somasundaram (2017) give an axiomatization for regret theory under the subjective probability setting.[4] Lanzani (2022) formulates axioms for salience theory with linear utility.[5] Our current work extends these works by axiomizing regret and salience theory with general functional forms under the preference set setting introduced in Fishburn (1990a).

Further, this project is related to the parameterization of these two models. While these

[4]Contemporary works such as Liu (2023) also provides axiomatizations of regret theory in the subjective setting.

[5]To illustrate the limitation of imposing linear utility on salience theory, let's consider the following example. Suppose the decision-maker is betting on a basketball game. Team 1 wins with probability 75% while Team 2 wins with probability 25%. Option A provides $100 in case team 1 wins while option B pays $300 if team 2 wins. Under salience theory with linear utility, the decision maker will always choose lottery B. However, since option B is a mean preserving spread of option A, the decision-maker prefers option A if she is risk averse.

theories have distinct structures, they share some common intuitions and predictions. For instance, Herweg and Müller (2021) show that under certain structural assumptions, regret theory is a special case of salience theory while in general each has its own merit. As a direct application of our characterization results, we provide the precise parametric boundaries between these two models. Our results offer qualitative guidance for the future application and estimation of these theories in a parametric context.

This project also contributes to the debate over the empirical validity of correlation-sensitive preferences. Previous experimental literature finds controversial results. On the one hand, Loomes et al. (1991); Zeelenberg et al. (1996); Zeelenberg (1999); Bleichrodt et al. (2010); Frydman and Mormann (2018); Königsheim et al. (2019); Dertwinkel-Kalt and Köster (2020) find supportive evidence for correlation-sensitive behaviors that can be rationalized by these two theories. On the other hand, Starmer and Sugden (1993a); Humphrey (1995); Loewenfeld and Zheng (2021); Ostermair (2021); Loewenfeld and Zheng (2023) argue that these findings are subject to "*event-splitting*" effect, which stems from nonlinear probability weightings (Kahneman and Tversky, 1979a; Tversky and Kahneman, 1992). To control for the *event-splitting* effect, it is important to restrict variations in probability when testing correlation-sensitive effects. In addition to these concerns, in order for relative differences to have bite, the experiment needs to elicit state-by-state comparisons, which can be cognitively demanding (Esponda and Vespa, 2019; Niederle and Vespa, 2023). We introduce a new presentation format that guides subjects to make contingent reasoning. Furthermore, our tasks control for variations in probabilities and the total number of states. In addition, our tests also alleviate the potential biases introduced by varying numbers of winning states for each option. Therefore, our current design provides a more robust evaluation over correlation-sensitive behaviors.

More broadly, the current project is also related to works analyzing the impact of attention on a decision maker's choices. Previous literature has proposed various mechanisms that can shift people's attention towards different options or certain features of the options (Koszegi and Szeidl, 2012; Bordalo et al., 2020; Bushong et al., 2021; Landry and Webb, 2021). Further, it has been shown that manipulating subjects' attentions toward different options can elicit preference reversals (Krajbich and Rangel, 2011; Li and Camerer, 2022). Our qualitative analysis provide direct evidence for the effectiveness of channels proposed in regret and salience theory.

The manuscript organizes as follows. Section 2 lays out the background. Section 3 characterizes salience theory and corresponding extensions. Section 4 axiomatizes regret theory and summarizes its parametric connections with salience theory. Section 5 describes the experimental design. Section 6 presents our results. Section 7 concludes.

1.2 Preliminaries: Correlation-Sensitive Preferences

For completeness, we now review the setting in Lanzani (2022). Let $X \subset \mathbb{R}$ be the set of possible outcomes. We denote $\Delta(X \times X)$ the set of finitely supported bivariate joint distributions on $X \times X$. We now introduce some useful notation. For every π in $\Delta(X \times X)$, let $\pi_1 = \sum_y \pi(x,y)$ be the marginal distribution of the first argument of π while $\pi_2 = \sum_x \pi(x,y)$ be the marginal distribution of the second argument. For every $\pi \in \Delta X \times X$, we define the *conjugate* of π as $\bar{\pi}$, where $\bar{\pi}(x,y) = \pi(y,x)$. For every pair of outcomes (x,y), we denote $\delta_{(x,y)}$ the Dirac distribution where $\delta_{(x,y)}(x,y) = 1$. For every π, π' in $\Delta(X \times X)$ and $\alpha \in [0,1]$, we have the mixed distribution $\tilde{\pi} = \alpha\pi + (1-\alpha)\pi'$ such that for every (x,y) in $(X \times X)$, $\tilde{\pi}(x,y) = \alpha\pi(x,y) + (1-\alpha)\pi'(x,y)$.

Every $\pi \in \Delta X \times X$ describes a binary choice problem: a decision maker is choosing between two options: option A and option B. Option A gives the monetary distribution π_1 while option B gives the distribution π_2, and together they form a distribution π. To express a decision-maker's preference, we denote a nonempty subset Π of $\Delta(X \times X)$ as the *preference set* wherein $\pi \in \Pi$ implies the decision-maker (weakly) prefers option A in the decision problem described by π. In addition, we also denote the *strict preference set* by $\widehat{\Pi} = \{\pi \in \Pi | \bar{\pi} \notin \Pi\}$. Comparing to the classical settings in von Neumann and Morgenstern (1944) where a preference order is imposed on the univariate distribution, the current setting is more natural to address the issue of correlation sensitivity. For instance, given two lotteries π_1 and π_2, there can exist multiple joint distributions to describe correlations between them. However, if the preference is imposed directly on π_1 and π_2, it suggests that π_1 is preferred to π_2 under all correlation structures.

Finally, a function $\phi(x,y) : X \times X \to \mathbb{R}$ is skew-symmetric if $\phi(x,y) = -\phi(y,x)$. In addition, $\phi(x,y)$ is monotonic if it increases in the first argument while decreases in the second.

A decision maker has a correlation-sensitive preference if

$$\pi \in \Pi \Leftrightarrow \sum_{(x,y)\in \mathrm{supp}\pi} \pi(x,y)\phi(x,y) \geq 0.$$

Lanzani (2022) proves that the following five axioms provide equivalent conditions for a correlation-sensitive preferences with a continuous and monotonic ϕ.

Axiom 1 (Completeness)**.** *For every $\pi \in \Delta(X \times X)$, $\pi \in \Pi$ or $\bar{\pi} \in \Pi$.*

Axiom 2 (Strong Independence)**.** *For every π, $\pi' \in \Pi$ and $\alpha \in [0,1]$, we have $\alpha\pi + (1-\alpha)\pi' \in \Pi$. In addition, if $\pi \in \widehat{\Pi}$, $\alpha\pi + (1-\alpha)\pi' \in \widehat{\Pi}$.*

Axiom 3 (Archimedean Continuity)**.** *For every $\pi \in \widehat{\Pi}$ and $\pi' \notin \Pi$, we can find $\alpha, \beta \in [0,1]$ such that $\alpha\pi + (1-\alpha)\pi' \in \widehat{\Pi}$ and $\beta\pi + (1-\beta)\pi' \notin \Pi$*

Axiom 4 (Monotonicity)**.** *For every $x,y,z \in X$ such that $x > y$, $\pi \in \Delta(X \times X)$, and $\alpha \in (0,1]$ if $\alpha\delta_{(y,z)} + (1-\alpha)\pi \in \Pi$, $\alpha\delta_{(x,z)} + (1-\alpha)\pi \in \widehat{\Pi}$.*

Axiom 5 (Coordinate-wise Continuity)**.** *Let $\{x_n\}_{n\in\mathbb{N}} \to x$. Then for every $\alpha \in [0,1]$, $yinX$, $\pi \in \Delta(X \times X)$*

$$\alpha\delta_{x_n,y} + (1-\alpha)\pi \in \Pi, \forall n \in \mathbb{N} \Rightarrow \alpha\delta_{x,y} + (1-\alpha)\pi \in \Pi$$

, and

$$\alpha\delta_{y,x_n} + (1-\alpha)\pi \in \Pi, \forall n \in \mathbb{N} \Rightarrow \alpha\delta_{y,x} + (1-\alpha)\pi \in \Pi$$

.

Lemma 1. Π *induces a correlation-sensitive preference representation with monotonic and continuous ϕ if and only if Axiom 1-5 are satisfied.*[6]

Proof. See Lanzani (2022) theorem 1. □

Since regret and salience theories can be thought as special cases for the correlation-sensitive preference, our subsequent characterizations are built upon the existence of a correlation-sensitive preference.

1.3 Axioms for Salience Theory

1.3.1 Baseline Characterization

First, we review the key ingredients for salience theory in Bordalo et al. (2012). A function $\sigma : X \times X \mapsto \mathbb{R}^+$ is a *salience function* if it satisfies the following properties:

- Symmetry: For every $x, y \in X$, $\sigma(x,y) = \sigma(y,x)$ and $\sigma(x,x) = 0$.
- Weak Continuity: $\sigma(x,y)$ is continuous in each argument.
- *Ordering*: For every $x \leq y$ and $\varepsilon > 0$, $\sigma(x,y) \leq \sigma(x-\varepsilon, y+\varepsilon)$.
- *Diminishing Sensitivity*: For every $0 \leq x \leq y$ and $\varepsilon > 0$, $\sigma(x,y) \geq \sigma(x+\varepsilon, y+\varepsilon)$.
- *Weak Reflexivity:* For all $x, y, x', y' \in \mathbb{R}^+$ with $|x-y| = |x'-y'|$,

$$\sigma(x,y) \geq \sigma(x',y') \Leftrightarrow \sigma(-x,-y) \geq \sigma(-x',-y').$$

Π induces a salience representation if

$$\pi \in \Pi \Leftrightarrow \sum_{x,y} f(\sigma(x,y))(u(x) - u(y))\pi(x,y) \geq 0,$$

[6]Here, continuous means continuous in each argument.

where u and f are both continuous and strictly increasing, with f also being positive. [7]

Readers familiar with salience theory (Bordalo et al., 2012) may observe the discrepancy between our current setting and the original one. In their original work, two parametric versions are introduced. The first is the *rank-based* version wherein decision-makers rank the states based on their salience levels. Roughly, states with larger salience levels are assigned higher ranks, and states with higher ranks are more likely to be overweighted than their objective probabilities. Since the ranking of one state will depend on salience level of other states, strictly speaking, this version is not nested in our current consideration. Nevertheless, as pointed out in Bordalo et al. (2012), the rank-based version is mainly a simplified parametric model that provides tractability and intuition[8], and hence, we leave it out of the current discussion. The second version introduced in Bordalo et al. (2012) is a solution to deal with the potential concerns surrounding the rank-based version. Specifically, they suggest that $f(\sigma(x,y)) = \delta^{\sigma(x,y)}$ with $\delta > 1$. This parameterization is included in our setting. In summary, our current version abstracts away from the actual effect size of salience distortion and focus on the qualitative properties of salience function.

In the current baseline analysis, we restrict the outcome space to a compact set within non-negative real numbers since this environment is sufficient for our following experimental investigation.[9] To simplify the description of axioms, we introduce some convenient definitions. $\{x_i\}_{i=0}^n$ is an *increasing arithmetic sequence* (IAS) if it is increasing, and $x_{i+1} - x_i = x_i - x_{i-1}$ for all i with $0 < i < n$. Given a sequence $\{x_i\}_{i=0}^n$, define the joint distribution $U(\{x_i\}_{i=1}^n)$ as

$$\{(x_1, x_0), \frac{1}{n}; (x_2, x_1), \frac{1}{n}; \ldots; (x_n, x_{n-1}), \frac{1}{n}\},$$

[7]Since σ is symmetric and $u(x) - u(y)$ is skew symmetric, their product is skew symmetric. Therefore, salience theory is a special case in correlation-sensitive preference. Notice that the above equation is additive separable: $\pi \in \Pi \Leftrightarrow \sum_{x,y} f(\sigma(x,y))u(x)\pi(x,y) \geq \sum_{x,y} f(\sigma(x,y))u(y)\pi(x,y)$. Due to this separability, Bordalo et al. (2012) defines $v(\pi_1) = \sum_{x,y} f(\sigma(x,y))u(x)\pi(x,y)$ the *local utility* of marginal distribution π_1.

[8]See footnote 9 in Bordalo et al. (2012)

[9]For the complete characterization, please see Appendix A.2.

and define $\overline{U(\{x_i\}_{i=1}^n)}$ as its conjugate. Intuitively, $U(\{x_i\}_{i=1}^n)$ is the uniform joint distribution such that under each state the first option will always provide an additional bonus defined by the difference between two adjacent terms in $\{x_i\}_{i=1}^n$. Furthermore, for the sake of economic notation, we denote $\alpha\pi + \beta\pi'$ the mixed distribution $\frac{\alpha}{\alpha+\beta}\pi + \frac{\beta}{\alpha+\beta}\pi'$ for every $\alpha, \beta > 0$. In other words, to ensure that the mixture is describing a distribution, we scale down the likelihood of each state by a constant factor $\alpha + \beta$.

Before introducing our axioms, we now briefly describe their underlying principles. It's important to note that the regularities of salience theory are closely tied to the properties of salience functions. As a result, most axioms of the theory should impose behavioral regulations to meet the requirements of salience functions. However, as we just hinted, decision-makers' perceptions of outcome differences are a composite of both salience function distortions and utility differences between the outcomes. Such convolution prevents us from directly regulating the salience function. To address this challenge, we explore a particular class of bivariate distributions wherein expected utility theory has specific behavioral predictions. Further, within these bivariate distributions, deviations from expected utility theory arise solely from variations in the salience function.

Our first axiom guarantees the existence of an increasing utility function and the identification of the salience function.

Axiom 6 (Increasing Utility). *For every $x, y \in X$ with $y < x$, there exists $p \in (0,1]$ such that for all IAS $\{x_i\}_{i=0}^n$ with $x_0 = y$ and $x_n = x$, we have*

$$(1-p)U(\{x_i\}_{i=1}^n) + \frac{p}{n}\delta_{(y,x)} \in \widehat{\Pi}.$$

Furthermore, $p \to 0.5$ as $x \to y$.

To fix ideas, consider the bivariate distribution in the above axiom with $p = 0.5$, or equivalently:

$$U(\{x_i\}_{i=1}^n) + \frac{1}{n}\delta_{(y,x)}.$$

Within the framework of expected utility theory, since the marginal distributions are identical,[10], decision-makers should be indifferent between these two options. However, salience theory posits that decision-makers prefer the second option in this bivariate distribution.[11] To see this, notice that when the second option provides a bonus, its amount is huge: $x-y$. In contrast, when the first option provides a bonus, $y-x$ is segmented into small pieces – y to x_1, x_1 to x_2, and so on. According to the *ordering* property of the salience function, partitioning the difference from y to x into smaller pieces will diminish the perceived value of x compared to directly contrasting it with y. This is because, when comparing y and x directly, decision-makers experience not only the pleasure of this additional $x-y$ amount but also a stark contrast between x and y that can amplify such pleasure. Conversely, this distortion from contrast diminishes when the difference from x to y is broken down into small pieces. As a result, such discrepancies in value distortions lead to a preference for the second option. Further, the magnitude of the distortion can be captured by $\frac{1-p}{p}$ where p is the probability at which the two marginal distributions of $(1-p)U(\{x_i\}_{i=1}^n) + \frac{p}{n}\delta_{(y,x)}$ are indifferent to the decision-maker. Increasing utility asserts that $\frac{1-p}{p}$ is bounded across all IAS, which implies that there is a positive *inherent value* of x relative to y, one that remains unaffected by any such partitioning. Within the model, this inherent value is represented by the utility difference $u(x)-u(y)$. Moreover, when x and y are close, comparing them only endures a marginal contrast distortion, so approximately, the preference behaves similar to expected utility theory.

The next two axioms provide hypotheses that implies *ordering* and *diminishing sensitivity*.

Axiom 7 (Skewness Preference)**.** *For all IAS sequences* $\{x_i\}_{i=0}^n$ *with* $0 \leq x_0$*, we have*

$$\frac{p}{n-1}\delta_{(x_{n-1},x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n-1})} \in \Pi \Rightarrow \frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_0\}_{i=0}^{n})} \in \Pi,$$

[10]Both marginal distributions are $\{x_0, \frac{1}{n}; x_1, \frac{1}{n}; \ldots, x_n, \frac{1}{n}\}$.

[11]A similar observation is used in Loewenfeld and Zheng (2021).

for all $p \in [0,1]$.

To get intuition, consider the two bivariate distributions in the axiom:

$$\frac{p}{n-1}\delta_{(x_{n-1},x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n-1})} \text{ and } \frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_0\}_{i=0}^{n})}.$$

While for both of these distributions expected utility theory posits that the first option is preferred if and only if $p \geq 0.5$, salience theory suggests that decision-makers may prefer the first option for $p \leq 0.5$ because it provides a bonus in the most salient state. Further, salience theory also predicts that the attention will be distorted more severely in $\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_0\}_{i=0}^{n})}$. The reason is that the first option in this bivariate distribution offers a higher bonus compared to the first option in the other one. By *ordering* of salience function, such differences move decision-makers' attention in favor of the first option. In addition, due to *diminishing sensitivity*, adding a state in which the first option returns the additional bonus to the second option can barely move the attention. Taking these two factors together, keeping p constant, the *Skewness Preference* axiom suggests that if a decision-maker favors the first option in $\frac{p}{n-1}\delta_{(x_{n-1},x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n-1})}$, she will also prefer the first option in $\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_0\}_{i=0}^{n})}$.

Axiom 8 (Diminishing Relative Sensitivity)**.** *For every IAS* $\{x_i\}_{i=0}^{n}$, $p \in [0,1]$, *and* $0 \leq y_0 \leq x_0$, *there is an IAS* $\{y_i\}_{i=0}^{m}$ *with* $y_m - y_0 = x_n - x_0$ *such that*

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n})} \in \Pi \Rightarrow \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^{m})} \in \Pi.$$

According to the constructions, the endpoints of the sequence $\{x_i\}_{i=0}^{n}$ are derived by shifting the endpoints of the sequence $\{y_i\}_{i=1}^{m}$ parallelly to the right on the real line. The *Relative Diminishing Sensitivity* axiom dictates that, compared to x_n and x_0, the perceived difference of y_m and y_0 is inflated more. This restriction aligns precisely with the *diminishing sensitivity* property of the salience function. We state our representation theorem below. All proofs can be found in Appendix A.1.

Theorem 1. *A decision-maker satisfies Axiom 1-8 if and only if her preference has s a salience representation. Furthermore, the salience function and utility are unique up to affine transformations.*[12]

One implication from our representation result is that although salience function imposes various restrictions on the behaviors, salience theory is still close to the general correlation-sensitive preference. Axiom 6 implies that breaking down a large profit into small pieces cannot vanquish its value. Furthermore, Axioms 7 and 8 only concern a special type of joint distributions wherein the two options have an identical support and the occurrence order of their outcomes under different states are merely a change of rotation. Salience theory's flexibility has been reflected in relevant literature. For instance, Landry and Webb (2021) shows that under different parameterizations salience theory is able to rationalize a large range of behavioral patterns. Experimental predictions related to salience theory (Frydman and Mormann, 2018; Dertwinkel-Kalt and Köster, 2020; Somerville, 2022) usually impose additional structural assumptions. In the next subsession, we discuss some extensions to further regulate the model.

1.3.2 Extensions

K-Regular Salience Function

In salience theory, there are two competing forces that regulate the salience function. First is *ordering*, which requires the salience function's value to increase as the two outcomes move further apart. The second one is *diminishing sensitivity*, which posits that the salience function decreases if both outcomes are shifted to the right while their distance is kept constant. However, salience theory imposes no restriction on the salience function if the outcomes' distance is expanding and shifting to the right simultaneously. For instance, a salience function may

[12]Requiring $f(\cdot)$ to be strictly positive is crucial to achieve the uniqueness result. For instance, Lanzani (2022) requires the salience representation to be $\sigma(x,y)(x-y)$. Consider the example $|\sqrt{x}-\sqrt{y}|(x-y)$. With the outcome space being non-negative real numbers, one can show that $|\sqrt{x}-\sqrt{y}|$ is indeed a salience function. However, $|\sqrt{x}-\sqrt{y}|(x-y) = |x-y|(\sqrt{x}-\sqrt{y})$. Therefore, the representation is not unique.

posit that $\sigma(1,0) > \sigma(x,0.01)$ for arbitrarily large x.[13] Indeed, one can argue that although the difference between $1 and 0$ is smaller than the difference between $1000 and 0.01$, this disparity mainly comes from the utility difference rather than the salience difference. Consequently, it might be true that $\sigma(1,0) > \sigma(1000,0.01)$. However, $\sigma(1,0) > \sigma(1000,0.01)$ implies that ordering is weak, and if diminishing sensitivity is the only dominating force, there is an easy way to characterize salience theory.[14] We now state a new condition that avoids such an extreme case. Given $k \geq 1$, we call a salience function *k-regular* if:

- For all $(x,y),(x',y') \in X \times X$ such that $k|x-y| \leq |x'-y'|$, $\sigma(x,y) \leq \sigma(x',y')$.

k-regular imposes a lower bound on the magnitude of *ordering* effect. It suggests that as long as the distance between two outcomes is extended k times, no matter how their locations are shifted, they will now have a higher salience level. As an example, let us consider the salience function proposed by Bordalo et al. (2012):

$$\sigma(x,y) = \frac{|x-y|}{x+y+\theta},$$

where $x, y \geq 0$, and $\theta > 0$. Suppose the maximal value of x and y is 30, $\sigma(x,y)$ is k-regular for all $k \geq \frac{60+\theta}{\theta}$.

Before we provide additional axioms, we first analyze the restrictiveness of *k-regular*. Roughly speaking, every salience function is *k-regular* except for outcome pairs with almost identical elements. Conversely, given $k > 1$, every salience function, is *k-regular* on some subset of the outcome space. We summarize the results below:

[13] In some research, including Bordalo et al. (2013b), $\sigma(x,y) = \frac{|x-y|}{x+y}, x,y \geq 0$ while $\sigma(0,0) = 0$ is used to formulate salience theory predictions. According to this form, $\sigma(x,0) > \sigma(x',y)$ for all x and x' unless $y = 0$. Nevertheless, this functional form is ruled out in the current analysis because it is not continuous at $(0,0)$.

[14] The reason is that since $\sigma(\cdot,\cdot)$ is continuous, $\sigma(1,0) \approx \sigma(1.01,0.01)$. As a result, while ordering indicates $\sigma(1.01,0.01) \leq \sigma(x,0.01)$, we also have $\sigma(1.01,0.01) \geq \sigma(x,0.01)$ for arbitrarily large x. Consequently, extending the distance between outcomes hardly affects their salience value. Therefore, $\phi(x,0.01) \approx u(x) - u(0.01)$, and hence there is a natural way to separate utility from the salience function.

lemma 1a. *Given any compact outcome space $X \subset \mathbb{R}$, salience function $\sigma(\cdot,\cdot)$, and $\varepsilon > 0$, for every $x, y, x', y' \in X$ with $|x-y| > \varepsilon$, $|x'-y'| > \varepsilon$, there exists $k > 1$ such that $\sigma(x,y) \leq \sigma(x',y')$ whenever $k|x-y| \leq |x'-y'|$.*

lemma 1b. *Given any compact outcome space $X \subset \mathbb{R}$, salience function $\sigma(\cdot,\cdot)$, and $k > 1$, there exists $\delta > 0$ such that for every $x, y, x', y' \in X$ with $|x-y| > \delta$, $|x'-y'| > \delta$, $\sigma(x,y) \leq \sigma(x',y')$ whenever $k|x-y| \leq |x'-y'|$.*

Lemma 1a and 1b have two implications. First, as k increases to infinity, the set of *k-regular* salience functions converges to the set of salience functions. Second, for suitable k, even if we mistake a salience thinker for a *k-regular* salience thinker, her violations of axioms related to *k-regular* should not be significant. The reason is that the violations can occur only when outcome pairs with small differences are considered in the choice problem. However, the salience distortions over such outcome pairs are negligible. Therefore, the deviations from predictions should also be marginal. We now state our new axiom.

Axiom 9 (k-regular)**.** *For every $p \in [0,1]$ and IAS $\{x_i\}_{i=0}^n$,*

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^n)} \in \Pi \;\Rightarrow\; \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^m)} \in \Pi.$$

for all IAS $\{y_i\}_{i=0}^m$ such that $y_1 - y_0 = x_1 - x_0$, $y_0 \geq x_n$, and $y_m - y_0 \geq k(x_n - x_0)$.

The current axiom is similar to skewness preference. To see this, instead of requiring the length of the IAS $\{x_i\}_{i=0}^n$ to be extended k times, skewness preference requires that the starting location of the IAS stays fixed while its length is stretched. We now state the representation result.

Proposition 1. *A decision-maker's preference has s a k-regular salience representation if and only if she satisfies axiom 1 - 9.*

Our next result suggests that by imposing *k-regular* condition, we can strengthen diminishing relative sensitivity.[15] Consequently, *k-regular* not only constitutes a self-contained restriction but also facilitates empirical analysis for other properties of the salience function.

Proposition 2. *Suppose a decision-maker's preference has a k-regular salience representation, then for every* $p > 0$ *and IAS* $\{x_i\}_{i=0}^n$

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^n)} \in \Pi \;\Rightarrow\; \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^n)} \in \Pi.$$

for all IAS $\{y_i\}_{i=0}^m$ *such that* $y_m - y_0 = x_n - x_0$, $0 \leq y_0 \leq x_0$, *and* $k(y_1 - y_0) \leq x_1 - x_0$.

According to the condition in proposition 2, given an IAS $\{x_i\}_{i=0}^n$, we can precisely identify a set of IAS such that diminishing relative sensitivity holds. Compared to the existence condition in diminishing relative sensitivity, such identifications facilitates our latter experimental analysis regarding the axiom.

Salience Theory with Concave Utility

As suggested by the example from our introduction, *diminishing sensitivity* alone cannot capture some prevail patterns of risk aversion. A natural way to accommodate such behaviors is to assume that decision-makers are inherently risk averse. We now briefly discuss the characterization.

Axiom 10 (Concavity)**.** *For every* $h > 0$ *and* $x, y \in X$ *with* $x \geq y \geq 0$ *and* $x \geq h \geq 0$,

$$\{(x, x-h), 0.5;\ (x, x+h), 0.5\} \in \Pi.$$

Proposition 3. *The utility function of a decision-maker with a salience representation is concave if and only if she follows axiom 10.*

[15] Furthermore, by introducing k-regular, the requirement in increasing utility that $p \to 1$ as $x \to 0$ can be dropped.

An example of the decision problem in the axiom is asking decision-makers to choose between options such as (\$$x$, 1) vs. (\$$x_1$, 0.5; \$$x_2$, 0.5) with $x_1 - x = x - x_2 > 0$.[16] Concavity predicts that decision-makers choose the safe option, (\$$x$, 1).

Interestingly, concavity is similar but weaker than the condition in Lanzani (2022) that characterizes diminishing sensitivity of the salience function assuming that utility is linear. In this sense, these results suggest that diminishing sensitivity and classical risk aversion share some common features. Indeed, it is straightforward to show that for an expected utility maximizer, the regularity in the above axiom is equivalent to risk aversion. Nevertheless, comparing the current restriction with Diminishing Relative Sensitivity, salience theory suggests that risk aversion would be exacerbated when comparing large outcome differences. Such postulation can address the inconsistency between realistic utility curvatures and observed magnitudes of risk aversion (Rabin, 2000) and is aligned with the assumption of "local linearity" that prevails in the experimental and empirical literature.

Negative and Mixed Outcomes

Since we restricted the outcome space to be nonnegative, the *weak reflexivity* requirement of salience function is not relevant. Notice that for any four nonnegative outcomes x, y, x', y', if $|x-y| = |x'-y'|$, their salience function value is purely governed by diminishing sensitivity. Therefore, in the negative outcome space, *weak reflexivity* is equivalent to a reversed diminishing sensitivity. That is, for every $y \leq x \leq 0$, and $\varepsilon > 0$, $\sigma(x,y) \geq \sigma(x-\varepsilon, y-\varepsilon)$. Consequently, we can adjust Skewness Preference and Diminishing Relative Sensitivity to provide an equivalent set of axioms.

In the presence of mixed outcomes,[17] Skewness Preference doesn't hold, and it needs to be weakened to an existence statement analogous to Diminishing Relative Sensitivity. The reason is that the salience function doesn't impose any restriction on the relation of magnitudes

[16] Such a decision problem belongs to the well-known certainty equivalent elicitation tasks (Tversky and Kahneman, 1992; Tversky and Fox, 1995).

[17] That is, for some $x < 0$ and $y > 0$, $\pi(x,y) > 0$.

between negative outcome pairs and positive outcome pairs. For instance, a decision maker is allowed to be an expected utility maximizer on lotteries with negative outcomes while a salience thinker on lotteries with positive outcomes. This flexibility prevents us from inferring changes in salience levels as IAS alternates. Nevertheless, the investigation of non-mixed outcomes leads to a unique utility function representation, which is preserved if we extend the analysis to the mixed outcome space.

1.4 Axioms for Regret Theory

Regret theory is another special case of correlation-sensitive preference. In Bell (1982); Loomes and Sugden (1982), the functional form is defined as $\phi(x,y) = Q(u(x) - u(y))$, where $u(\cdot)$ is a continuously increasing utility function and $Q(\cdot)$ is an increasing, continuous, and skew-symmetric function. Recall the underlying psychological foundation for regret theory is that decision-makers tend to avoid options that may result in large losses in utility. This phenomenon is captured by the functional specification $Q(\cdot)$ if it is convex on the positive domain and concave on the negative domain.

To understand the intuition of our axiom, notice that since $\phi(x,y)$ under regret theory is a monotonic transformation of the utility difference between x and y, the magnitudes of utility differences are proportional to the magnitudes of regret levels. For instance, $u(5)-u(4) \geq u(2)-u(1)$ if and only if $Q(u(5)-u(4)) \geq Q(u(2)-u(1))$. Moreover, $u(5)-u(4) \geq u(2)-u(1)$ is equivalent to $u(5)-u(2) \geq u(4)-u(1)$, so it implies $Q(u(5)-u(2)) \geq Q(u(4)-u(1))$. In other words, swapping payoffs from \$2 and \$4 shouldn't change the the order of these regret levels. The next axiom captures this principle.

Axiom 11 (Swapping Independence)**.** *For all* $(x_1,y_1),(x_2,y_2) \in X \times X$*:*

$$\{(x_1,y_1),0.5;\ (x_2,y_2),0.5\} \in \Pi \ \Rightarrow\ \{(x_1,y_2),0.5;\ (x_2,y_1),0.5\} \in \Pi.$$

Theorem 2. *A decision-maker's preference has a regret representation if and only if she satisfies axiom 1-5, and 11. Furthermore, the utility is unique up to affine transformations.*[18]

Swapping independence requires the behavior to be correlation-insensitive for joint distributions with two equiprobable states. With two states, correlation insensitivity only requires the ordinal differences between outcomes are preserved after correlation distortions. In contrast, correlation insensitivity for joint distributions with three states requires the *cardinal* differences are unchanges. In appendix A.3, we prove that if the decision-maker is "swapping independent" for lotteries with three states, she is an expected utility maximizer.

Our current analysis provides different axioms for salience and regret theories. This suggests that without further structural assumption, these two models are generally different. Nevertheless, as shown in Herweg and Müller (2021), salience and regret theory share some similarities. We conclude the current session with some complementary results. Our first result suggests that requiring the utility function to be concave imposes identical restrictions on both salience and regret theory.

Proposition 4. *The utility function of a decision-maker with a regret representation is concave if and only if she satisfies axiom 10.*

Since (weak) risk aversion is ubiquitous in theoretical analysis and verified by extensive empirical and experimental literature, it is reasonable to impose this assumption when calibrating salience and regret theories. In fact, having a concave utility is a necessary consequence if the two theories intersect with each other (other than being expected utility theory), our next result provides an exact boundary between these two theories in terms of parameterizations.

Proposition 5. *Suppose the outcome space is a compact subset of $\mathbb{R}_+$, a decision-maker's preference either has an expected utility representation or has a regret representation with*

[18] The proof uses a similar technique from Diecidue and Somasundaram (2017), which propose a behavioral foundation for regret theory under Savage's subjective environment (Savage, 1954). However, their axiom are not sufficient for *Swapping Independence*. In terms of *trade-off consistency*, a sufficient condition is $\alpha_1\alpha_2 \sim_t \gamma_1\gamma_2, \alpha_2\alpha_3 \sim_t \gamma_2\gamma_3$, then $\alpha_1\alpha_3 \sim_t \gamma_1\gamma_3$. See Köbberling and Wakker (2004); Diecidue and Somasundaram (2017) for more details.

concave $u(\cdot)$, increasing $\frac{Q(x)}{x}$ for $x \geq 0$, and $\lim_{x\to 0} \frac{Q(x)}{x} > 0$ if and only if she satisfies axioms 1-8, and 11.

Proposition 5 is an extension of Theorem 2 in Herweg and Müller (2021). Their result establishes that convexity in $Q(\cdot)$ and concavity in $u(\cdot)$ are sufficient for achieving the equivalence between salience and regret theories. They construct the salience theory representation as $\frac{Q(u(x)-u(y))}{u(x)-u(y)}(u(x)-u(y))$ and verify that all requirements of salience function are satisfied. Proposition 5 implies that their construction represents the only candidate for salience representation and provides equivalent structural conditions between these two theories.[19]

Additionally, in case $u(\cdot)$ is concave and has a strictly positive derivative, if both salience and regret theories are satisfied, the induced salience representation is also *k-regular*. To see the reason, let $\bar{x}$ denote the maximal outcome on space X. Because the decision-maker has a concave utility, for all $x, y \in X$, we have $\frac{u'(x)}{u'(y)} \leq \frac{u'(0)}{u'(\bar{x})}$. Due to the Mean Value Theorem, $\frac{u(h)-u(0)}{u(\bar{x})-u(\bar{x}-d)} \leq \frac{u'(0)h}{u'(\bar{x})d}$ where $h, d > 0$. Furthermore, for a decision-maker following both salience and regret theories, her salience function's value $\sigma(x,y)$ is proportional to $|u(x)-u(y)|$. Consequently, since $\frac{u(x+h)-u(x)}{u(y)-u(y-kh)} \leq \frac{u'(0)}{u'(\bar{x})k}$ for all $x, y \in X$, $\sigma(x, x+d) < \sigma(y, y-kh)$ if $\frac{u'(0)}{u'(\bar{x})k} \leq 1$. Therefore, the decision-maker's preference satisfies *k-regular* for $k \geq \frac{u'(0)}{u'(\bar{x})}$.

1.5 Experimental Methodology

1.5.1 General Designs

Our experiment aims to test the validity of the axioms underpinning salience and regret theory. The central axioms of salience are skewness prereference, diminishing relative sensitivity, and k-regular. The central axiom of regret is swapping independence. In addition to these axioms, our experiment is also designed to test more basic properties like monotonicity and

[19] It is important to note that there are two differences between the settings in our current analysis and those in Herweg and Müller (2021). First, we require the monotonic transformation f of salience function to be strictly positive while they only require it to be non-negative. The key distinction is that we require $Q'(0) > 0$. Second, we regard the shape of $Q(\cdot)$ as a modeling choice while they restrict $Q(\cdot)$ to be convex.

independence.[20]

The experiment is conducted online through Prolific (Palan and Schitter, 2018). Subjects are asked to complete 16 tasks. Each task consists of multiple decisions. Each decision problem requires subjects to choose between two lotteries marked by option A and option B. Subjects are told that payments from both options depend on the color of a ball that is randomly drawn from a box at the end of the experiment.

The experimental interface is designed using Otree (Chen et al., 2016a), and Figure 1.2 presents an example decision problem. The presentation consists of three parts: the table, the bar chart, and the pie chart. The table records all relevant information regarding possible colors of balls, the probability of selecting a ball in each color, and payoffs from options A and B. In Figure 1.2, there are four possible ball colors: red, black, blue, and orange. These colors are recorded in the first row of the table. Next to each color is the probability of a ball in that color being selected. In this example, all colors have an equal chance (25%) to be picked. Under each color, there are two columns that record payments from option A and B in case a ball in that color is drawn. In Figure 1.2, if a red ball is drawn, option A pays $13.5 while option B pays $15. The bar chart above the table represents payments from options A and B under each ball color, while the pie chart shows the probabilities of selecting each color. Subjects make their decisions by selecting the radio buttons at the bottom.

The experiment consists of two parts. Tasks in the first part are called "payment-variation tasks," and tasks in the second part are called "probability-variation tasks." Before the experiment starts, participants are randomly divided into two groups. Questions from payment-variation tasks differ between groups. Subjects are asked to finish the payment-variation tasks first.[21] Once participants finish all tasks, their payments will be determined.

[20]The experiment is pre-registered in the AEA RCT Registry in July 2023 with ID AEARCTR-0011753.

[21]The reason is that all decision problems in this type contain four possible states while all remaining decision problems contain five possible states. In order to help subjects understand the questions, we design the flow in a way with progressive difficulty levels.

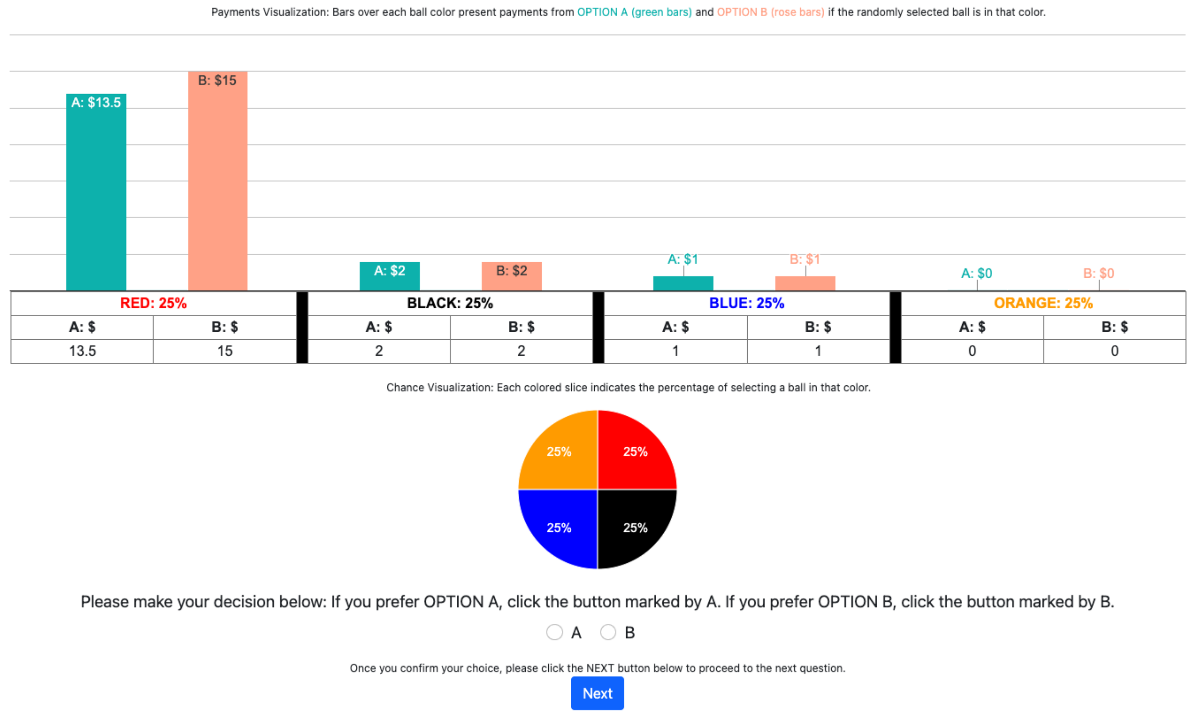

Figure 1.2. Example Decision Problem

Note: This is a presentation of decision problems. The first row of the table records all possible colors and their probabilities. The second and third rows record payments from options A and B under each ball color. The bar chart on top of the table represents the amount of payments from each option under every state. The pie chart under the table represents the chance of each ball color being picked.

1.5.2 Details and Hypotheses of Payment-Variation Tasks

Subjects are asked to complete 7 tasks in payment-variation tasks. One task serves as an attention check, the remaining tasks are designed to test swapping independence, independence, and correlation sensitivity in general. In each of the tasks, there are eight decision problems. Across these eight decision problems, one option's payments stay constant while the payment from the other option under a specific ball color is increasing from problem one to eight. Furthermore, for every decision problem in part 1, there are always four *equiprobable* states.

The first task asks subjects to choose between options A and B under the joint distribution

$$\{(15,X),\frac{1}{4};\,(2,2),\frac{1}{4};\,(1,1),\frac{1}{4};\,(0,0),\frac{1}{4}\},$$

where $X \in \{12, 13.5, 14, 14.5, 15.5, 16, 16.5, 17\}$. In this task, options A and B only provide different payments under one state. Therefore, this task serves as a comprehension check to see whether subjects are able to choose the option with statewise weakly dominant payments. The corresponding hypothesis is that subjects choose option A if and only if $X <= 14.5$.

The remaining six tasks can be grouped into three pairs. Between the two tasks within the same pair, each option's marginal distributions stays constant while the correlation between the options' payments changes. In one task, the payments are positively correlated, and in the other task they are negatively correlated.[22] Table 1.1 provides an overview of each pair. Since probabilities of states stay constant in payment-variation tasks, they are omitted in the joint distribution descriptions of table 1.1.

Table 1.1. Summary of payment-variation Tasks

	Positively Correlated	Negatively Correlated
Pair 1:		
Group 1	$\{(22,X); (1.5,1.5); (1.5,1.5); (5,0.5)\}$	$\{(22,0.5); (1,1); (1,1); (5,X)\}$
Group 2	$\{(22,X); (1,1); (1,1); (5,0.5)\}$	$\{(22,5); (1.5,1.5); (1.5,1.5); (0.5,X)\}$
$X \in \{25, 26.5, 27, 27.5, 28.5, 29, 29.5, 30\}$		
Pair 2:		
Group 1	$\{(20,X); (1,1); (1,1); (0,5)\}$	$\{(20,5); (1.5,1.5); (1.5,1.5); (0,X)\}$
Group 2	$\{(20,X); (1.5,1.5); (1.5,1.5); (0,5)\}$	$\{(20,5); (1,1); (1,1); (0,X)\}$
$X \in \{12, 13.5, 14, 14.5, 15.5, 16, 16.5, 17\}$		
Pair 3:		
	$\{(20,20); (18,10); (0,0); (2,X)\}$	$\{(20,0); (0,X); (2,10); (18,20)\}$
$X \in \{7, 8.5, 9, 9.5, 10.5, 11, 11.5, 12\}$		

Note: Summary of all payment-variation tasks excluding the comprehension check. Participants are randomly divided into two different groups. Tasks in pairs 1 and 2 are slightly different between groups. In each task, there are four equiprobable states. Since the states' chances are fixed, they are omitted in the descriptions of joint distributions.

Pairs 1 and 2 test swapping independence. To see this, let us consider the two tasks in

[22] For questions in pair 3, if $X = 7$, options in both versions of tasks are, in fact, positively correlated. Nevertheless, their correlation is still higher if the joint distribution is $\{(20,20); (18,10); (0,0); (2,7)\}$.

pair 1:

$$\{(22,X),\frac{1}{4};(a,a),\frac{1}{4};(a,a),\frac{1}{4};(5,0.5),\frac{1}{4}\} \text{ and } \{(22,0.5),\frac{1}{4}(b,b),\frac{1}{4};(b,b),\frac{1}{4};(5,X),\frac{1}{4}\},$$

where $X \in \{25, 26.5, 27, 27.5, 28.5, 29, 29.5, 30\}$, and $a = 1.5, b = 1$ for subjects in group 1 while $a = 1, b = 1.5$ for subjects in group 2. Notice that for both tasks, the options only provide different payments in two states. According to correlation-sensitive preferences, states with identical payments from both options are irrelevant for the decision. As a result, mixing lotteries with such states or alternating payment amounts under these states will not change the decision-maker's preference towards to two option. Therefore, for both groups, pair 1's decision problems is equivalent to decision problems $\{(22,X),\frac{1}{2};\ (5,0.5),\frac{1}{2}\}$ and $\{(22,0.5),\frac{1}{2};\ (5,X),\frac{1}{2}\}$. Swapping independence posits that decision-makers will make identical decisions between these two tasks, and hence, it also predicts that subjects make consistent choices between the 2 tasks in pair 1. Following a similar argument, we can get the same hypothesis for tasks in pair 2.

It is important to note that, however, according to some theories, the irrelevancy of states with identical payments can be violated.[23] Realizing such possibilities, we provide a between-subject analysis to test the aggregate behaviors over tasks that are only distinct in states with identical payments.

Since both regret theory and expected utility theory make the same prediction regarding choices for pairs 1 and 2, it is indistinguishable whether the choice consistencies in these tasks originate from swapping independence of regret theory or correlation independence of expected utility theory. To further differentiate these models, we include pair 3 to test whether participants' choices respond to the general correlation manipulations. The difference between the two tasks in pair 3 lies in the occurrence order of payments from option B. While salience and regret theories do not provide specific predictions, correlation-insensitive preference still predicts

[23]One example is expectation-based reference dependence preference (Gul, 1991; Kőszegi and Rabin, 2006). The reason is that such manipulation will change the options' distributions.

choice consistency for the tasks in pair 3.

1.5.3 Details and Hypotheses of Probability-Variation Tasks

The second part of our experiment consists of nine tasks. In each task, subjects are asked to complete nine decision problems. Within each task, the payments of both options are fixed while the probabilities of states vary. The nine tasks in this part can be separated into three classes. Each class contains three tasks and tests one of the three axioms for salience theory – skewness prereference, diminishing relative sensitivity, and k-regular with $k = 2$. It is for this reason, we name the three classes by the names of the axioms they're investigating. Within each class, we denote the three tasks as High Contrast task, Normal Contrast task, and Low Contrast task. These tasks are summarized in table 1.2.

For all probability-variation tasks, the first option "wins" (provides a higher payment) in one state with probability p while the second option (weakly) wins in the remaining four equiprobable states. Under such structure, monotonicity of preferences predict that the first option is becoming more attractive as p increases. It follows that for each task, there is a unique probability p such that the decision-maker is indifferent. At this specific probability p, we call $\frac{q}{p}$ the *switching odds*, and denote $\frac{q}{p}_H$, $\frac{q}{p}_N$, and $\frac{q}{p}_L$, the switching odds for High, Normal, and Low Contrast tasks respectively.

Axioms of salience theory formulate hypotheses regarding the relative magnitudes of the switching odds among the three tasks within each class. As an example, consider the High Contrast and Normal Contrast tasks testing skewness preference:

$$\text{High Contrast Task: } \{(30,0),p;\ (0,7),q;\ (7,7.5),q;\ (7.5,14.5),q;\ (14.5,15),q\},$$
$$\text{Normal Contrast Task: } \{(30,0),p;\ (0,15),q;\ (1,1),q;\ (1,1),q;\ (1,1),q\}.$$

At the switching odds, we obtain the following identities:

$$\frac{q}{p_H} = \frac{f(\sigma(30,0))}{\mu_1 f(\sigma(7,0)) + \mu_2 f(\sigma(7.5,7)) + \mu_3 f(\sigma(14.5,7.5)) + \mu_4 f(\sigma(15,14.5))} \frac{u(30)}{u(15)},$$

where $\mu_1 = \frac{u(7)}{u(15)}, \mu_2 = \frac{u(7.5)-u(7)}{u(15)}, \mu_3 = \frac{u(14.5)-u(7.5)}{u(15)}, \mu_4 = \frac{u(15)-u(14.5)}{u(15)}$.

$$\frac{q}{p_N} = \frac{f(\sigma(30,0))}{f(\sigma(15,0))} \frac{u(30)}{u(15)}.$$

$$\text{Hence, } \frac{q}{p_H} / \frac{q}{p_N} = \frac{f(\sigma(15,0))}{\mu_1 f(\sigma(7,0)) + \mu_2 f(\sigma(7.5,7)) + \mu_3 f(\sigma(14.5,7.5)) + \mu_4 f(\sigma(15,14.5))}. \quad (1.1)$$

Table 1.2. Summary of probability-variation Tasks

Skewness Preference:	
High Contrast	$\{(30,0), p;\ (0,7), q;\ (7,7.5), q;\ (7.5,14.5), q;\ (14.5,15), q\}$
Normal Contrast	$\{(30,0), p;\ (0,15), q;\ (1,1), q;\ (1,1), q;\ (1,1), q\}$
Low Contrast	$\{(15,0), p;\ (0,7.5), q;\ (7.5,15), q;\ (15,22.5), q;\ (22.5,30), q\}$
Diminishing Relative Sensitivity:	
High Contrast	$\{(15,0), p;\ (15,23), q;\ (1,1), q;\ (1,1), q;\ (23,30), q\}$
Normal Contrast	$\{(15,0), p;\ (15,30), q;\ (1,1), q;\ (1,1), q;\ (1,1), q\}$
Low Contrast	$\{(30,15), p;\ (0,4), q;\ (4,8), q;\ (8,11.5), q;\ (11.5,15), q\}$
2-Regular:	
High Contrast	$\{(30,10), p;\ (0,4.5), q;\ (4.5,5), q;\ (1,1), q;\ (5,10), q\}$
Normal Contrast	$\{(30,10), p;\ (0,10), q;\ (1,1), q;\ (1,1), q;\ (1,1), q\}$
Low Contrast	$\{(10,0), p;\ (10,15), q;\ (15,20), q;\ (20,25), q;\ (25,30), q\}$

Note: Summary of all probability-variation tasks. In each task, $p + 4q = 1$ and $p \in \{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%, 32\%, 36\%\}$.

Notice that $\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1$ for all monotonic utility functions. Under expected utility theoy, $f \circ \sigma$ is a constant function, so $\frac{q}{p_H} / \frac{q}{p_N} = 1$. Consequently, deviations of winning odds from 1 can only stem from the salience distortion difference between comparing \$15 to \$0 directly and comparing them in four smaller pieces.

Furthermore, consider the Low Contrast task:

$$\{(15,0),p;\,(0,7.5),q;\,(7.5,15),q;\,(15,22.5),q;(22.5,30),q\}.$$

$$\frac{q}{p_L}=\frac{f(\sigma(15,0))}{\nu_1 f(\sigma(7.5,0))+\nu_2 f(\sigma(15,7.5))+\nu_3 f(\sigma(22.5,15))+\nu_4 f(\sigma(30,22.5))}\frac{u(15)}{u(30)},$$

where $\nu_1=\dfrac{u(7.5)}{u(30)},\nu_2=\dfrac{u(15)-u(7.5)}{u(30)},\nu_3=\dfrac{u(22.5)-u(15)}{u(30)},\nu_4=\dfrac{u(30)-u(22.5)}{u(30)}$.

Therefore,

$$\frac{q}{p_N}\cdot\frac{q}{p_L}=\frac{f(\sigma(30,0))}{\nu_1 f(\sigma(7.5,0))+\nu_2 f(\sigma(15,7.5))+\nu_3 f(\sigma(22.5,15))+\nu_4 f(\sigma(30,22.5))}. \quad (1.2)$$

On one hand, under certain smoothness conditions, skewness preference predicts $\frac{q}{p_H}/\frac{q}{p_N}\le\frac{q}{p_N}\cdot\frac{q}{p_L}$.[24] On the other hand, $\frac{q}{p_H}/\frac{q}{p_N}\le\frac{q}{p_N}\cdot\frac{q}{p_L}$ implies skewness preference.[25] Therefore, refuting skewness preference by rejecting $\frac{q}{p_H}/\frac{q}{p_N}\le\frac{q}{p_N}\cdot\frac{q}{p_L}$ constitutes a marginally stronger test for the axiom. A simple way to interpret $\frac{\frac{q}{p_H}/\frac{q}{p_N}}{\frac{q}{p_N}\cdot\frac{q}{p_L}}$ is that it imposes an upper bound on $\frac{f(\sigma(15,0))}{f(\sigma(30,0))}$.[26] Using similar arguments, $\frac{q}{p_H}/\frac{q}{p_N}\le\frac{q}{p_N}\cdot\frac{q}{p_L}$ is also a slightly stronger prediction for diminishing relative sensitivity and 2-regular.

Our design also contains some other auxiliary tests for salience theory. First, notice that for the Normal Contrast task testing skewness prereference, monotonicity of correlation-sensitive preferences predicts that the first option is preferred whenever $p\ge q$. Therefore, we also use

[24]The requirements are $\mu_1 f(\sigma(7,0))+\mu_2 f(\sigma(7.5,7))\approx\frac{u(7.5)}{u(15)}f(\sigma(7.5,0)$ and $\mu_3 f(\sigma(14.5,7.5))+\mu_4 f(\sigma(14.5,7.5))\approx\frac{u(15)-u(7.5)}{u(15)}f(\sigma(15,7.5)$. These conditions hold if $f\circ\sigma(\cdot,\cdot)$ and $u(\cdot)$ have locally stable curvatures.

[25]Consider IAS $\{0,7.5,15,22.5\}$ and $\{0,7.5,15,22.5,30\}$, by applying skewness preference twice, the exact prediction is $\frac{f\left(\sigma(15,0)\right)}{(\mu_1+\mu_2)f\left(\sigma(7.5,0)\right)+(\mu_3+\mu_4)f\left(\sigma(15,7.5)\right)}<\frac{q}{p_N}\cdot\frac{q}{p_L}$. Moreover, by ordering of salience function, we have $\sigma(7.5,0)\ge\max\{\sigma(7,0),\sigma(7.5,7)\}$ and $\sigma(15,7.5)\ge\max\{\sigma(15,14.5),\sigma(14.5,7.5)\}$, so $\frac{f\left(\sigma(15,0)\right)}{(\mu_1+\mu_2)f\left(\sigma(7.5,0)\right)+(\mu_3+\mu_4)f\left(\sigma(15,7.5)\right)}<\frac{q}{p_H}/\frac{q}{p_N}$.

[26]To fix ideas, notice that under the same smoothness condition, the denominator in equation 1.1 is a weighted average between the denominator in equation 1.2 and $\nu_3 f(\sigma(22.5,15))+\nu_4 f(\sigma(30,22.5))$. Due to diminishing sensitivity of salience function, the latter term must be smaller than the denominator in equation 1.2, and hence leading to a smaller average between them.

this task as a comprehension check, and subjects "pass" this test if they switches to option A when $p \geq q$. Second, it is straightforward to show that salience theory implies $\frac{q}{p_H} / \frac{q}{p_N} \geq 0$, $\frac{q}{p_N} * \frac{q}{p_L} \geq 0$, and $\frac{q}{p_H} * \frac{q}{p_L} \geq 0$.[27]

Table 1.3 summarizes every hypothesis's targeting axioms, involving tasks, and statements. In total, there are nine tests, and we categorize them by the specific models they are investigating: four for correlation-sensitive preference, two for regret theory, and three for salience theory.

1.5.4 Recruitment and Attention Requirements

We recruited 800 participants through Prolific who list English as their first language and maintain a high approval rating on Prolific (detailed summaries in Table A.9). We recruited an equal number of male and female participants. Participants receive a $4.5 payment upon completion. Every subject also has 25% chance to receive an additional bonus payment based on their decisions in the study. Each task as an equal to chance to determine subjects' bonus payment.[28] Participants in current sample took on average 37 minutes to complete the experiments. 190 of them received a bonus payments with the average amount equaling $8.53. In addition, the average completion times for each task are recorded in Table A.10. On average, they took 74.1 seconds to complete each task.

Since this experiment is conducted online, it is challenging to measure subjects' attentions throughout the experiment. To ameliorate this issue, we require subjects to stay in full screen mode and do not switch to other tabs throughout the experiment. Violating these rules more than four times will terminate the experiment and drop the subject from the sample.[29] After the instructions, there are four comprehension questions to ensure that subjects understand the experimental design. For each question, participants have three chances to provide the correct

[27] However, as we will explain in sections 1.5.5 and 6.2, these tests may be influenced by a well-known confounder called *event-splitting effect* in a way that is against salience theory.

[28] Once the task is randomly selected, one decision problem within that task will be randomly chosen to determine the payment amount.

[29] Only 3 subjects dropped out because of violating more than four times.

answer. The experiment will also terminate in case they fail to provide the correct answers.[30] Subjects are informed about these rules before the experiment starts.

The order of tasks, options, and the colors of balls are all conditionally randomized. Regarding the task order, the first task every subject encounters is the comprehension check within the payment-variation tasks. After this task, the remaining six payment-variation tasks are conditionally randomized. We randomize their sequence while ensuring that tasks from the same pair are always separated by two tasks from different pairs. This randomization standard is also applied to the probability-variation tasks: Between any two tasks from the same class, there are always two tasks from different classes.

For the order of options in the payment-variation tasks: if a lottery is labeled as 'A' in one task, it will be labeled as 'B' in its paired task. In the probability-variation tasks, for each subject, either all options are listed according to the joint distributions in table 1.2, or all of them are based on the conjugate joint distributions in table 1.2.

Lastly, regarding color assignments: for each subject, the colors of all questions either match those in Figure 1.2[31] or they are reversed. For instance, in payment variation tasks, the sequence from left to right becomes orange, blue, black, and red if the occurrence is reversed.

Failure of Contingent Reasoning

Failure of contingent reasoning refers to the phenomenon wherein decision makers have trouble to explicitly and separately account for potential outcomes under each possible state. This pattern is observed in various domains, such as decisions under uncertainty (Ellsberg, 1961), auction (Li, 2017), and voting (Esponda and Vespa, 2019). In the current experiment, contingent reasoning is the basic underlying assumption since it is the very foundation of correlation-sensitive preference. Therefore, failure of contingent reasoning imposes serious challenges on experiments testing correlation manipulations.

To encourage subjects to perform state-wise comparisons instead of other heuristics. We

[30] No subjects dropped out from failing to provide correct asnwers.

[31] For probability variation tasks, there is an additional color, marked by purple, on the right.

use a presentation frame different from the conventional matrix presentation. In our design, individual lotteries are never listed in isolation. Instead, they are jointly presented under each state. Due to juxtaposition effect, we believe that not only our presentation helps subjects to perform a state-wise comparison, but also prevent them from using other heuristics, such as calculating expected values, for doing so requires them to read the table several times back and forth.

Another potential source for the failure of contingent reasoning comes from the similarity between tasks. Given limited time and energy, if two questions deem similar to a subject, they may not think about these two questions individually. To accommodate this effect, for regret tasks, we add states with different common payments. For salience theory, we use a three-question format instead of two-question format.[32]

1.5.5 Discussion of Design

We now discuss some potential confounders documented in previous literature, and how the current design addresses them.

Event-Splitting Effects

One of the most common confounders for testing correlation-sensitive preference is the *event-splitting* effect noted by Starmer and Sugden (1993a); Humphrey (1995). It posits that changing the number of states without altering its underlying distribution can impact subjects' behaviors. For instance consider the following two joint distributions:

$$\{(1,0),50\%;\,(3,4),50\%\} \text{ and } \{(1,0),25\%;\,(1,0),25\%;(3,4),50\%\}.$$

[32]Consider the three tasks testing skewness preference, instead of using the three tasks. One can just test $\{(30,0),p;\ (0,7.5),q;\ (7.5,15),q;\ (15,22.5),q;\ (22.5,30),q\}$ and $\{(15,0),p;\ (0,7.5),q;\ (7.5,15),q;\ (1,1),q;\ (1,1),q\}$. Nevertheless, in case subjects realize that these two marginal distributions are just permutations to each other, they may not be willing to conduct state-wise comparisons, but instead, just compare probabilities.

Table 1.3. Summary of Hypotheses

(A) Axioms	(B) Tasks	(C) Predictions
	Panel A: Regret Theory	
Swapping Independence	pairs 1 and 2 in payment-variation tasks	Subject's choices are consistent between the two tasks in a same pair.
Swapping Independence	pairs 1 and 2 in payment-variation tasks	Choice patterns of the paired tasks are similar between subjects' groups.
	Panel B: Salience Theory	
Skewness Preference Diminishing Relative Sensitivity 2-Regular	3 tasks testing Skewness Preference 3 tasks testing Diminishing Relative Sensitivity 3 tasks testing 2-Regular	$\frac{q}{p_H} / \frac{q}{p_N} \leq \frac{q}{p_L} \cdot \frac{q}{p_N}$
	Panel C: Correlation-Sensitive Preference	
Monotonicity	1^{st} payment-variation tasks	Subjects choose option A $\Leftrightarrow X < 15$.
Monotonicity	Normal Contrast Task in Skewness Preference	Subjects choose option A $\Leftarrow p \geq 0.2$.
Independence	pairs 1 and 2 in payment-variation tasks	Choice patterns for a same task are similar between subjects' groups.
Correlation Independence	pair 3 in payment-variation tasks	Subject's choices are consistent between the two tasks.

Note: Summary of all major hypotheses from the current experiment. Column A states the axioms' names involved in each hypothesis. Column B reports the tasks that are involved in each hypothesis. Column C records each hypothesis's statement. Panel A lists hypotheses from regret theory. Panel B lists hypotheses from salience theory. Panel C lists hypotheses for the general correlation-sensitive preference.

Although these two joint distributions are identical in the sense that they can be expressed by $\pi \in \Delta(X \times X)$ with $\pi(1,0) = 0.5$ and $\pi(3,4) = 0.5$, event-splitting effect predicts that more subjects will choose option A in the latter joint distribution.

To address this issue, some contemporary research aims at controlling event-splitting effect while testing salience and regret theory (Loewenfeld and Zheng, 2021; Ostermair, 2021; Dertwinkel-Kalt and Köster, 2020). The common approach they adopted is to increase the number of states to guarantee that the relevant tasks have identical state space. Controlling the total number of states is crucial to eliminate certain confounders. For instance, varying quantities of states usually come with changing of states' probabilities. If such probabilistic alternation is not well-regulated, observed preference reversals may caused by nonlinear probability weightings (Kahneman and Tversky, 1979a) other than correlation sensitivity. Although controlling the total number of states is efficient to manage probabilistic variations among tasks, it ignores an important underlying principle of event-splitting effect. As pointed out in Humphrey (2001), a prominent factor driving the event-splitting effect is the favoring of a higher number of winning states. In the previous example, splitting $(1,0)$ into two states increases the number of winning states for option A, so more subjects prefer option A under the latter joint distribution, as predicted by the event-splitting effect. If this factor is the dominant force, equalizing the number of total states instead of each option's winning state smay even exacerbate the event-splitting effect.[33].

Our current experimental design controls for both the total number of states and the quantity of winning states for each option. For tasks investigating regret theory, each option

[33]As an illustration, let us consider tasks relevant to MAO pairs (Mao, 1970) used in both Dertwinkel-Kalt and Köster (2020); Loewenfeld and Zheng (2021). First, they find evidence for preferential reversal in the task pairs such that the total number of states is not controlled. In this case, one example of the paired tasks are $\{(135,81), 64\%; (60,156), 36\%\}$ and $\{(135,156), 36\%; (135,81), 28\%; (60,81), 36\%\}$. On the other hand, they find no evidence after fixing the number of total states across tasks. However, to control the total number of states, the former task with 2 states is changed to $\{(135,81), 36\%; (135,81), 28\%; (60,156), 36\%\}$. Notice that, when the number of states is not controlled, the ratio of winnings states from each option changes from 1 to 2 between tasks. In contrast, after controlling the states' number, this ratio changes from $\frac{1}{2}$ to 2 between tasks. In this particular example the forces between event-splitting effect and salience effect are working in opposite directions, so the absence of preference reversal can be caused by the cancellation of these two forces after strengthening even-splitting effect.

has exactly one winning state. Furthermore, for salience theory's tasks, we keep the ratio of winning states between the two options similar across High and Low contrasts problem. Since the main hypotheses perform a subtraction between the switching odds of these two tasks, the event-splitting effect influencing the switching odds for individual questions is canceled out. And hence, the potential confounding issue can be at least alleviated.

Random Choices

Overall, the current experiment can be thought of as price lists questions assembled by binary choices. This design has some advantages over tasks that consist of single binary choices, which is a prevalent format in previous experimental literature studying correlation-sensitive preferences. First, the current design allows us to obtain a precise measure of valuation changes caused by correlation manipulations. Furthermore, recent work by McGranaghan et al. (2022) suggests that in the presence of random noise in decisions, single binary choices problems may suffer from the varying error magnitudes among tasks and produce unintended predictions. To see how the same issue can contaminate the current experiment, consider the pair 1 payment-variation tasks. In the positively correlated version, regret theory posits that option A is chosen if and only if $Q(u(X)-u(22)) \geq Q(u(5)-u(0.5))$. Regret theory suggests that in this case $Q(u(X)-u(5)) \geq Q(u(22)-u(0.5))$, and hence predicts subjects to make the consistent choice in the negatively correlated version. However, regret theory stays silent on the cardinal difference between the options. That is, it is possible that $Q(u(X)-u(22))-Q(u(5)-u(0.5)) \neq Q(u(X)-u(5))-Q(u(22)-u(0.5))$. Following the rationale from random utility models (McFadden, 1974), subjects' choices may be more disbursed for the task with the smaller regret difference. Consequently, random errors may lead to false rejection of regret theory.

According McGranaghan et al. (2022), adopting a price list design can ameliorate this issue because at the switching pair, the choice probabilities of each option are identical, and random noise cannot change this balance. Nevertheless, it is important to note that adopting price lists directly in the current experiment can introduce other biases. The reason is that options in price lists may become references while subjects make decisions, and hence influence their choices. Therefore, in our experimental design, we still present each decision problem as binary choice.

1.6 Main Experimental Results

We begin by investigating the two hypotheses regrading monotonicity. 284 out of 800 (35.5%) subjects violate at least one of the hypotheses.[34] Since these two tasks are relatively easier than remaining ones, their violations may indicate subjects' confusion or inattention. Therefore, we exclude these subjects from subsequent analysis.

Additionally, since each tasks consist of multiple binary choices, it is possible that subjects switch back and forth between the two options as they complete the decision problems, and hence exhibit the so-called "multiple switching points". For each of the 16 tasks, the occurrence rates of multiple switching points range from 9.3% - 36.9%.[35] On one hand, multiple switch points indicate subjects' confusion or attentional noise, so include these subjects may incur additional randomness in our analysis. On the other hand, given that most subjects exhibit multiple switching points at least once in our experiment, mechanically excluding these subjects may result in selection bias. Due to these conflicting rationales, in current analysis we evaluate each hypothesis over two different populations – Po_a and Po_s – as described below.

1. Po_a contains all subjects that follow monotonicity. [36]

2. Po_s contains all subjects following monotonicity and do not exhibit multiple switching points in all tasks that are relevant to the hypothesis.[37]

[34]159 subjects violate the hypothesis of the first payment-variation tasks while 174 subjects violate the hypothesis of the Normal Contrast task testing skewness preference.

[35]The average rate of multiple switching points among the tasks is 20.8%. On average, subjects exhibit this behavior 3.1 times.

[36]The current analysis focus on the switch point in each task. For subjects with multiple switching points, their first and last switching points provide a range of the payments or probabilities at which they are indifferent between the options.

[37]For instance, for regret theory analysis, subjects in this population never exhibit multiple switching points in the pairs 1 and 2 payment variation tasks and follow monotonicity.

We focus on individual-level data for tasks relevant to regret and salience theory in our main analysis. Roughly, based on subjects' responses, we can at least identify a range of the payment amounts or probability magnitudes that make subjects indifferent between the two options. As a result, we can conduct our hypotheses by comparing the differences between these switching choices across tasks. In Appendix A.4.3, we provide aggregate results and tests for the general correlation-sensitive preferences.

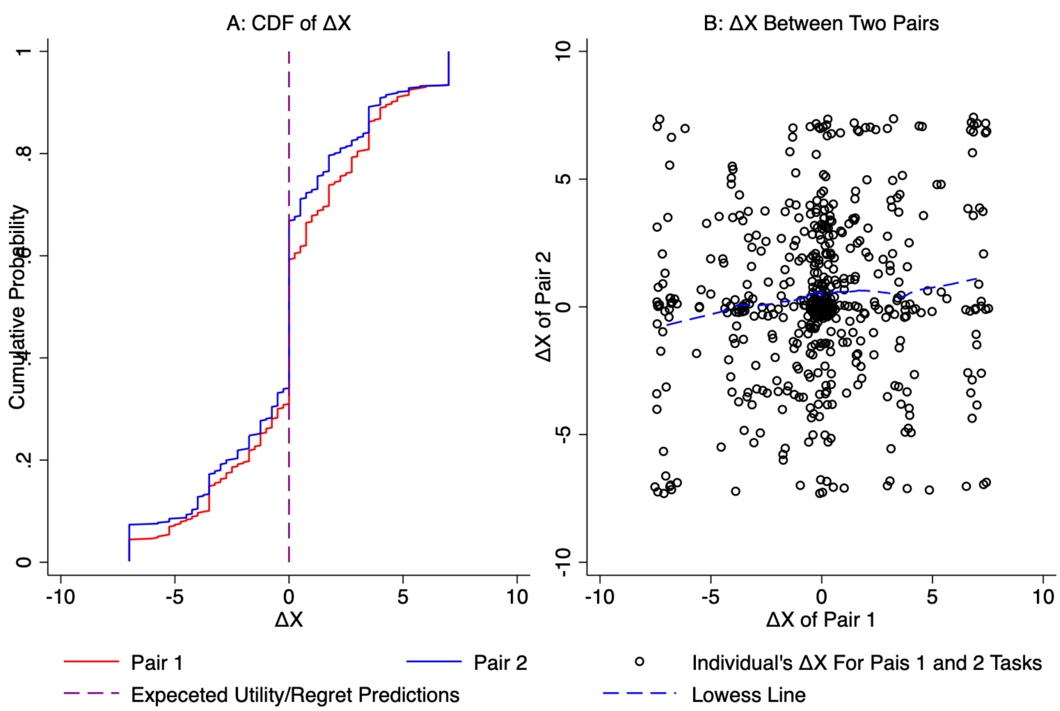

Figure 1.3. Regret Theory – ΔX

Note: This is the summary of Po_s population behaviors regarding payment-variation tasks pertaining to regret theory. Panel A draws the CDF for $\Delta X \equiv X_{pos} - X_{neg}$ for pair 1 and 2 in the payment variation tasks. Panel B plots ΔX from pair 1 against pair 2, with the scatters jittering at a 3 percent level. Panel B also includes the Lowess line that fits the data excluding subjects with both ΔX equaling to 0.

1.6.1 Results for Regret Theory

For each subject, we calculate the switching payments X_{pos} and X_{neg} in every task pair and analyze $\Delta X \equiv X_{pos} - X_{neg}$.[38] Figure 1.3 presents the result. Panel A presents the CDFs for ΔX in both pair 1 and 2. Two features stand out in the figure. First, the two CDFs look strikingly similar to each other. Although, based on the figure, the CDF of ΔX for pair 1 appears to first order stochastic dominates that of pair 2, this trend is not significant.[39] Second, both CDFs imply that a large proportion of ΔX is centered around 0. [40] Table 1.4 presents the mean tests using interval regressions (Stewart, 1983). Panel A in Table 1.4 represents the average of ΔX in Pairs 1 and 2 tasks under populations Po_a and Po_s. The results are mixed for different task pairs. For pair 1 task, a significant difference between X_{pos} and X_{neg} is detected at the aggregate level. In both populations Po_a and Po_s, the evidence suggests that subjects switch to option B at lower X values when the outcomes of options are negatively correlated. In contrast, no significant difference in switching payments is found for pair 2 tasks. Panel A also presents ΔX at group level. The deviation is slightly larger from group 1 subjects.[41]

The significant deviation from swapping independence in the mean test may be driven by a minority of subjects who make decisions at two *distinct corners* in a paired tasks.[42] To investigate such possibility, Panel B in Table 1.4 presents the sign test. For subjects with multiple switch points, we take the midpoint of their largest and smallest switch decision as an indicator.

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[38] The calculation is based on midpoints formulated by the maximal X value the subject choosing option A and the minimal X value that she chooses option B. Furthermore, there is a major proportion of subjects who make at least one corner decision – choosing a same option throughout all decision problems in a task. For the current analysis, we use $\min X - 2$ and $\max X + 2$ as the lower and upper bounds. Altering these two endpoints in a symmetric way doesn't affect our current conclusion.

[39] A two-sample Kolmogorov–Smirnov test cannot reject that ΔX from pair 1 is larger than ΔX from pair 2. Excluding subjects with $\Delta X = 0$ in both pairs, the p-values are 0.16 for Po_a subjects and 0.23 for Po_s subjects.

[40] $\Delta X = 0$ for 37.6% of the Po_a subjects in pair 1 and 40.9% in pair 2. These numbers 39.3% and 42.2% for Po_s subjects.

[41] Options A and B between the two groups are different in the amount of their common payments. Appendix A.4.3 provides a between-group analysis controlling this factor, and the results still indicate violations of swapping independence.

[42] A similar phenomenon is documented in Loewenfeld and Zheng (2023). In the current experiment, there are in total 131 subjects that exhibit such behaviors at least once. On average, these subjects complete tasks in which they made corner decisions around 9 seconds faster than the remaining subjects.

The results are largely aligned with Panel A.

Table 1.4. Swapping Independence

	Panel A: Mean Test				Panel B: Sign Test			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Task Pair	1	2	1	2	1	2	1	2
ΔX	0.37**	-0.10	0.45*	-0.10	+	-	+	-
	(0.17)	(0.16)	(0.26)	(0.26)	0.02	0.62	0.08	0.69
$\Delta X_{group=1}$	0.63***	-0.10	0.93**	-0.15	+	-	+	+
	(0.24)	(0.23)	(0.38)	(0.38)	0.02	0.78	0.01	0.62
$\Delta X_{group=2}$	0.11	-0.09	0.02	-0.05	+	-	-	-
	(0.24)	(0.23)	(0.36)	(0.35)	0.33	0.67	0.87	0.88
Population	Po_a	Po_a	Po_s	Po_s	Po_a	Po_a	Po_s	Po_s
N	516	516	211	211	369	346	128	122

Note: Summary of tests for swapping independence. Each test is performed under Po_s and Po_a. Panel A provides estimates for the mean differences in the switching value $\Delta X \equiv X_{pos} - X_{neg}$ obtained from interval regressions (Stewart, 1983). In addition, Panel A also records ΔX at group level. The standard errors are included in parentheses. Panel B presents the sign test at both aggregate and group levels. The dominate group is presented. "+" suggests a majority of subjects has $\Delta X > 0$ while "-" suggests a majority of subjects has $\Delta X < 0$. P-values are recorded under each test. When Performing each sign test, subjects with $\Delta X = 0$ are excluded.

We now investigate potential heterogeneity at subject-level. Panel B in Figure 1.3 plots the relative values of ΔX between the two paired tasks. It suggests that there only exists a weak correlation between magnitudes of deviations across pairs. The correlations between ΔX of Pairs 1 and 2 tasks are 0.13 ($p = 0.004$) and 0.18 ($p = 0.008$) for Po_a and Po_s population, which are considered as significant but weak. Further, there is no evidence suggests the heterogeneity regarding average values of ΔX between groups. Nevertheless, although we don't find a strong correlation between the values of ΔX across different pairs, Panel B also suggests a significant proportion of subjects are clustering at $(0,0)$. Indeed, conditional on $\Delta X = 0$ in pair 1 leads to a 18.6% higher chance to have $\Delta X = 0$ in pair 2 for Po_a subjects and a 23.8% higher chance for subjects in Po_s.[43] Therefore, the results suggest a consistency of respecting swapping

[43] Denote ΔX_1 the value of ΔX at Pair 1 and ΔX_2 the value of ΔX at Pair 2. For Po_a subjects, $\mathbb{P}(\Delta X_2 = 0 | \Delta X_1 = 0) =$

independence at subject level.

Taking stock, the current results find evidence that subjects violate swapping independence. However, the violation is not robust across questions parameters, populations, and groups. Furthermore, the deviation magnitudes on average are under \$1. In addition, we find a significant correlation for violations among tasks at subject-level. Specifically, the result suggests that a subject violating the axiom in one task pair has a higher chance to violate again in the other pair.

1.6.2 Results for Salience Theory

We now conduct a similar exercise regarding probability-variation tasks. The number of subjects in Po_s for this part of experiment is 350. Among these subjects, only 148 of them never submit a corner decision. To accommodate the majority of subjects, we assume an upper bound for switching winning odds at $exp(3.5)$ and a lower bound at $exp(-3.5)$. These bounds are not restrictive because for a switching winning odds to be $exp(3.5)$, the decision maker is expected switch to the choice with one winning state at winning probability less than 1%. Furthermore, in order for the winning odds to be $exp(-3.5)$, the decision maker is expected to choose the option with four winning states when of the total winning probability is smaller than 0.12 (each winning states has a chance smaller than 0.03).

Fiugre 1.4 provides CDFs of $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$ from Po_s subjects for each of the three task groups.[44] Their CDFs share two common features. First, a majority of subjects have a negative difference in winning odds. Under current boundary specifications, 207 out of 350 (59.1%) subjects follow skewness prereference, 196 out of 350 (56%) subjects follow diminishing relative sensitivity, 193 out of 350 (55.1%) subjects follow 2-regular, and overall 80

$\frac{68}{147} \approx 46.26\%$ and $\mathbb{P}(\Delta X_2 = 0 | \Delta X_1 \neq 0) = \frac{102}{369} \approx 27.64\%$. For Po_s subjects, $\mathbb{P}(\Delta X_2 = 0 | \Delta X_1 = 0) = \frac{47}{83} \approx 56.63\%$ and $\mathbb{P}(\Delta X_2 = 0 | \Delta X_1 \neq 0) = \frac{42}{128} \approx 32.81\%$. The p-values of the two-sample Kolmogorov-Smirnov test are 0.001 for the 18.62% increasing chance and 0.007 for the 23.8% increasing chance. Additionally, conditional on violating swapping independence, $\Delta X < 0$ in pair 1 leads to a 17% (0.042) increasing chance to have $\Delta X < 0$ in pair 2 for subjects in Po_a and 16.3% (0.618) increasing chance for subjects in Po_s.

[44] For more intuitive visualization, the logarithm of winning odds are generated by taking the midpoint of the intervals in which they lie.

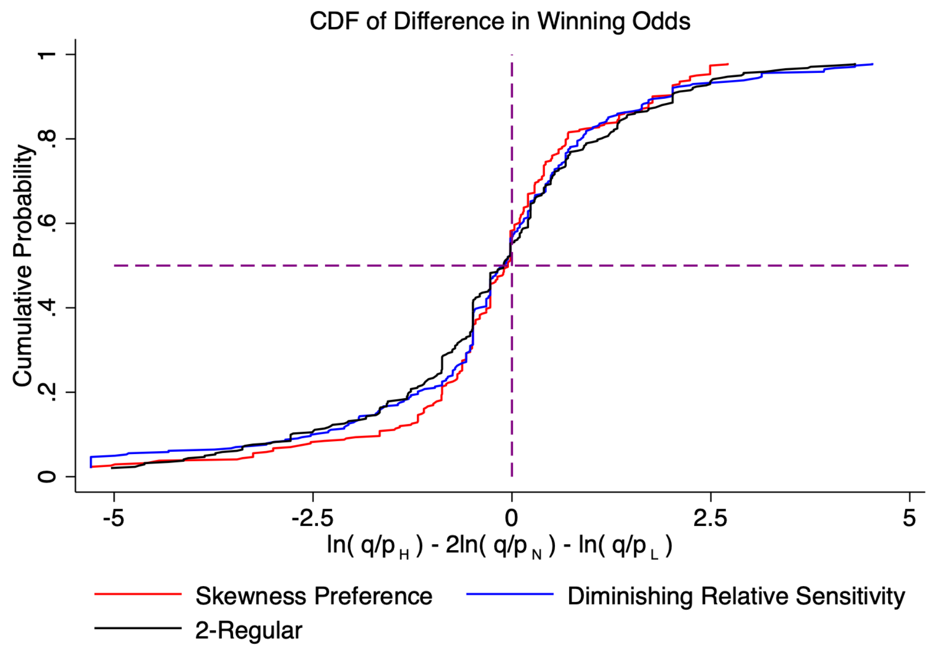


Figure 1.4. Salience Theory: $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$

Note: Figure 1.4 presents the cumulative distribution functions of $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$ for task groups related to each of the three axioms. Each CDF ignores the bottom and top 2% subjects for better visualization. The red curve presents data from tasks testing skewness preference.The blue curve presents data from investigating diminishing relative sensitivity. The black curve presents data from investigating 2-Regular. The purple dash line serves as a reference for $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L}) = 0$.

out of 350 (22.8%) subjects are consistent with all three axioms.[45]

Secondly, all CDFs in Figure 1.4 have long left tails. This indicates that subjects following salience theory deviate more severely from expected utility theory. To fix ideas, Figure 1.5 draws individual-level data testing the corresponding axiom. In Figure 1.5, subjects are separated into followers, meaning their behaviors are consistent with predictions from salience theorem, and violators. For the followers, it's expected that $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ to be smaller than $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$.[46] Table 1.5 Panel A presents the results. For the main hypotheses, we find supportive evidence for salience theory's prediction that is robust across populations and

[45] In our design, each axiom can rationalize 52% of all responses assuming single switch. Therefore, around 14% of responses are considered to be consistent with all axioms.

[46] In terms of visualization, scatter points from followers should lie above the 45 degree line. Furthermore, a subject behaves more like a expected utility maximizer if her scatter point is located closer to the 45 degree line.

different axioms. To interpret the magnitudes, consider the statistics under column (1), which presents results for tasks investigating skewness preference for popuation Po_a. Roughly, the result suggests that $\frac{f(\sigma(15,0))}{f(\sigma(30,0))} \leq exp(-0.19) \approx 0.83\,(0.05)$. For Po_s subjects, the bound is 0.85 (0.06). Moving on to relative diminishing sensitivity, the upper bound of $\frac{f(30,15)}{f(15,0)}$ is 0.83 (0.06) for population Po_a and 0.84 (0.08) for population Po_s. Lastly, for 2-Regular, the upper bound of $\frac{f(10,0)}{f(30,10)}$ is 0.75 (0.06) and 0.82 (0.07) for Po_a and Po_s respectively.

Table 1.5. Salience Axioms

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Main Hypotheses						
	-0.19***	-0.16**	-0.18**	-0.18*	-0.29***	-0.20**
	(0.06)	(0.07)	(0.08)	(0.09)	(0.08)	(0.09)
Panel B: Auxiliary Hypotheses						
Mean Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-0.11***	-0.10**	0.04	0.04	0.07*	0.12**
	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	0.09*	0.06	0.24***	0.24***	0.46***	0.44***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Population:	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s
N	516	350	516	350	516	350
Sign Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-	-	+	+	+	+
p-value	0.036	0.005	0.851	0.353	0.069	0.024
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	+	+	+	+	+	+
p-value	0.489	0.682	0.000	0.000	0.000	0.000

Note: This table summarizes the mean values of differences in logarithm winning odds using interval regression. Columns (1) and (2) present results from skewness preference for both Po_a and Po_s populations. Columns (3) and (4) present results from diminishing relative sensitivity. Columns (5) and (6) present results from 2-Regular. The axiom test records result for $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$. In Auxiliary tests, $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ and $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$ are presented using interval regression. Furthermore, sign tests are also performed. In the table, sign tests record the sign of dominant group and corresponding p-value.

One potential confounder of the results is nonlinear probability weighting. Specifically,

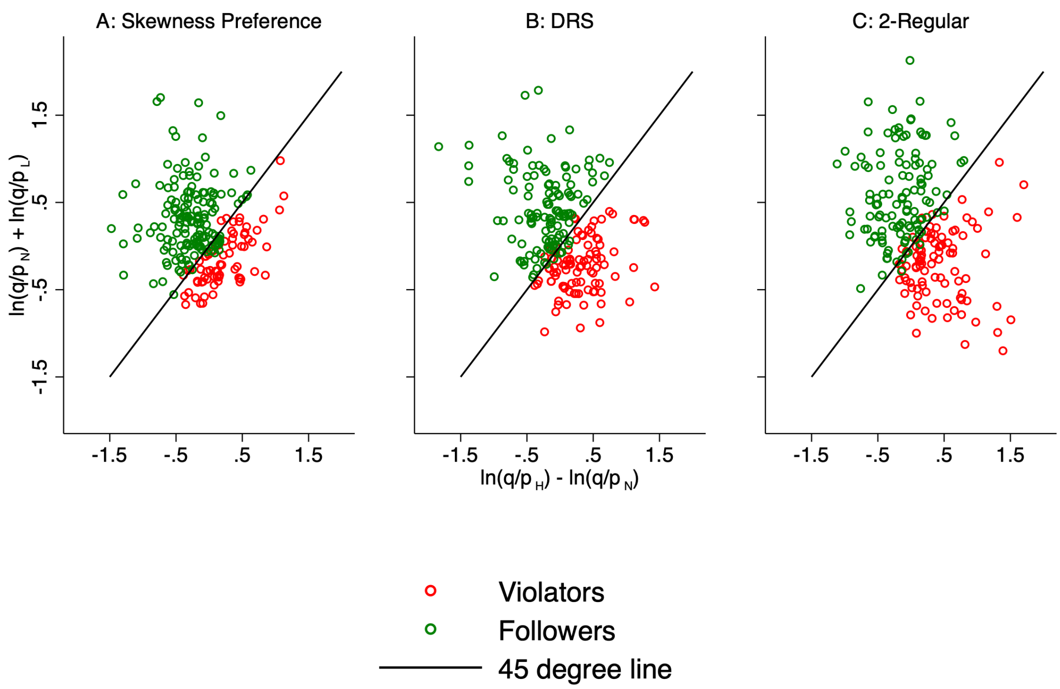


Figure 1.5. Salience Theory Subject-Level Analysis

Note: The graph summarizes individual level differences in switching winning odds. X-axis indicates values of $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$, and Y-axis presents $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$. The data for each of the three axioms are plotted individually. Salience theory posits that $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L}) \geq \ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$. The green scatters represent subjects who follows the predictions while red scatters represent subjects who violates the predictions.

since we compare the difference between $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ and $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$, if probabilities are nonlinear, it is possible that their magnitudes are distorted and the estimates in Table 1.5 can be misleading. To circumvent this issue, we analyze the auxiliary tests from probability-variation tasks. Unlike the main hypotheses, these auxiliary tests are not influenced by nonlinear probability weighting because they compare the winning odds between different tasks directly and analyze the ordinal ranking between them. Since we keep the probabilistic variations identical across tasks, the winning ratio can serve as a measure of this ordinal ranking. Specifically, the tests we consider are $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N}) > 0$ and $\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L}) > 0$. As mentioned before, the results of this direct comparison may be contaminated by the fact that options with identical marginal distributions have different numbers of winning states. Nevertheless, we argue that the impact from event-splitting effect is working against salience theory in our setting. To illustrate, consider the test $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N}) > 0$. The event-splitting effect will decrease the value of $\ln(\frac{q}{p_H})$ because option A only has a single winning state with probability p while option B has four winning states each with probability q. In order for the decision-maker to consider switching the option, she demands a higher p, and hence drags down the value of $\ln(\frac{q}{p_H})$. In contrast, $\ln(\frac{q}{p_N})$ is not influenced by event-splitting effect since both options have exactly one winning state. Therefore, $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ decreases in the presence of event-splitting effect. The idea works similarly for $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$. Table 1.5 Panel B reports the mean tests and sign tests for $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ and $\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$.[47] The results between the main and auxiliary hypotheses are largely aligned. Nevertheless, notice that salience theory's predictions are only rejected at $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N}) \geq 0$ in tasks testing skewness preference. The cause of rejection, other than the failure of theory, can also originate from the fact that we exlude subjects with $\ln(\frac{q}{p_N}) \leq 0$. The reason we exclude these subjects is that their behaviors constitute violations from monotonicity and indicate misunderstanding of the experiment.[48] However, through this process, subjects' noise in the normal contrast task are eliminated while keep the noise in others. As a result, the discrepancy in how noise is excluded may contribute to the violation. One way to ameliorate this issue is to also exclude certain noises from the high contrast problem. We present the corresponding results in Table A.5.

We conclude current analysis with a brief discussion about subject-level heterogeneity. For each axiom there is a significant proportion of subjects who violate its hypothesis. First, we find significant but weak positive correlations (around 0.13 for Po_a and 0.19 for Po_s) among the magnitudes of the responses across the three task groups. Table A.6 presents the correlations. Furthermore, we find a significant correlation between the indicators of violating k-regular and diminishing relative sensitivity. Specifically, conditional on following k-regular, there is a 17% (0.01) increasing chance for subjects to follow diminishing relative sensitivity compared to

[47] When conduct the sign tests, the proportion with 0 differences are excluded.

[48] $\ln(\frac{q}{p_H}) < 0$ also violates monotonicity. However, it may also be caused by event splitting effect.

conditional on violating k-regular.

In summary, we find supportive evidence for all three axioms of salience theory at the average level. However, it is important to note that there is a significant amount of violations for each of our main tests. In addition, there exists positive correlations among violating different axioms.

1.6.3 Model Evaluation

We now briefly assess each theory's predictive performances. To this end, we implement two measures. First, we use a deterministic approach to evaluate their abilities to rationalize behaviors by categorizing subjects into followers and violators for each axiom. Second, we use the completeness measure introduced in Fudenberg et al. (2022) to appraise their performances to fit out-of-sample data.

Table 1.6 presents categorizations of subjects. Out of 516 subjects who pass our attention checks, 68 (13.2%) follow regret theory, 121 (23.4%) follow salience theory, and 24 (4.7%) follow both. Table 1.6 also presents the results for the 107 subjects who never exhibit multiple switches throughout the experiment. The performances of most tests become better under this more conservative subject pool.[49] It is important to notice that axioms for regret and salience theories provide distinct hypotheses on behaviors. Therefore, one should account for the differences in restrictiveness when assessing their predictive successes. Specifically, for regret theory, Pairs 1 and 2 in payment-variation tasks provide two tests for the same hypothesis that $X_{pos} - X_{neg} = 0$. For each test, there are in total 81 different possible choice combinations and regret theory can rationalize 9 of them. In contrast, for salience theory, each hypothesis can be rationalized by 53.7% of arbitrary choice combinations.[50]

[49]Only 1 person followed Expected utility theory based on our main tests. For probability-variation tasks, expected utility theory posits that $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L}) = 0$. However, there is no behavioral pattern in our design can hit exact 0. Therefore, we count subjects as expected utility followers if the absolute value of differences in winning odds is smaller than 0.04.

[50]One way to assess the predictive power of these theories is through the Selten measure (Selten, 1991), which can be expressed as $r - a$. Here, r is the fraction of observations that can be rationalized by the theory, and a is the proportion of choice combinations that are consistent with the theory. Considering monotonicity as an

Table 1.6. Model Evaluations

	Po_a	Po_s
N:	516	107
Regret Theory:		
Pair 1	147	42
	(28.5%)	(39.3%)
Pair 2	170	49
	(32.9%)	(45.8%)
Overall	68	29
	(13.2%)	(27.1%)
Salience Theory:		
Skewness Preference	315	64
	(61%)	(59.8%)
Diminishing Relative Sensitivity	290	63
	(56.2%)	(58.9%)
2-Regular	292	66
	(56.6%)	(61.7%)
Overall	121	33
	(23.4%)	(30.9%)
Overall	24	8
	(4.7%)	(7.5%)

Note: This table summarizes the number of subjects following each hypothesis. Po_a is the population that passed the attention checks. Po_s is the sub-population of Po_a that exhibit single switch behaviors in all tasks.

Now we measure their out-of-sample performances. To give a suitable assessment, we use the "completeness" measure introduced in Fudenberg et al. (2022).[51] The completeness measure requires a cost function and three prediction rules: the best rule, the theory's rule, and the baseline rule.[52] We choose squared errors as our cost function. For regret theory, we use observations from negatively correlated tasks to predict behaviors in positively correlated tasks. In this case, the best prediction rule is $\mathbb{E}(X_{pos}|X_{neg})$. The theory's prediction rule is

additional layer of requirement, the Selten measures are 0.082 for regret theory and 0.092 for salience theory over Po_a population.

[51] Section A.4.1 lays out the detailed construction.

[52] The best prediction rule presents the most accurate model one can obtain based on the explanatory data. The theory's prediction rule is specified by the model of interests. The baseline prediction rule represents the "worst" prediction rule we consider.

X_{neg}. In addition, we set the baseline rule as the minimal possible value of X_{pos}.[53] Under these specifications, the completeness measure for regret theory is 15.1% for Pair 1 tasks and 19.1% for Pair 2 tasks. One driving factor for these low values is the minority of subjects providing responses at two different extreme corners in the paired tasks. Excluding these subjects, we find that the completeness measure is 75.5% for Pair 1 and 90% for Pair 2.

For salience theory,we predict $Y = \ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ based on $X = \ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$. The best prediction rule is $\mathbb{E}(Y|X)$. Since salience theory posits that $Y \leq X$, its prediction rule is $\min\{\mathbb{E}(Y|X), X\}$. Additionally, the baseline prediction rule is X. Based on these specifications, the completeness measures of salience theory are 80% for skewness prereference, 63.7% for diminishing relative sensitivity, and 73.9% for k-regular.

1.6.4 Robustness Checks and Other Results

Our previous main analysis suggests that a significant proportion of subjects exhibit correlation-sensitive behaviors. Furthermore, the current results also imply that on average both salience and regret theories perform reasonably well. Nevertheless, in each of our tests, we observe individual heterogeneity. Since there are significant correlations between individuals' violations across relevant axioms, the observed correlation-sensitive behaviors cannot be fully attributed to randomness. Here we discuss some robustness checks to further validate our main conclusions.

The first issue we consider is the ordering effect. Since relevant tasks in our experiment have similar parameters, it is possible that subjects realize this and forfeit statewise comparisons in each question.[54] To address this issue, for each relevant test, we conduct a between-subject analysis wherein the data of each task only consists of subjects who encounter that task before

[53]Since regret theory's prediction involves no parameter specification, we set the baseline rule to be extremely uninformative. In fact, since X_{neg} and X_{pos} are only weakly correlated and there is significant heterogeneity between subjects, the out-of-sample performance of both X_{neg} and $\mathbb{E}(X_{pos}|X_{neg})$ are both worse than taking the mean of possible X_{neg} values.

[54]The direction of biases is unclear. On one hand, subjects may realize the similarity and choose consistently. On the other hand, subjects may deliberately randomize between similar questions (Machina, 1985; Cerreia-Vioglio et al., 2019).

other relevant ones. Table A.7 presents the results. Panel A corresponds to Table 1.4 while Panel B corresponds to Table 1.5.[55] The results are reasonably stable and hence qualitatively unchanged among these tables.

Another potential concern regarding salience theory is the unbalanced winning odds at two endpoints. In order to help subjects comprehend the probabilistic variations, the parameters are chosen in a way that the changes of p and q are constant between two subsequently questions. However, this leads to the max $\frac{q}{p} = \frac{24}{4} = 6$ and min $\frac{q}{p} = \frac{16}{36} = 0.44$. Ideally, these two winning odds ratios should be reciprocals. In this way, if a subject switches from the beginning in high or normal contrast tasks while never switches in the low contrast task is considered to be uninformative.[56] To address this issue, we trim the data such that subjects switches at or before $p = 0.12$ and subjects switches at or after $p = 0.32$ are considered to be censored.[57] Table A.8 presents the results, and they are in line with our previous analysis.

We now briefly describe some other results omitted in the main discussion. In appendix A.4.2, we conduct a similar analysis towards the pair 3 payment-variation tasks, which aims at uncovering general correlation sensitive behaviors. We find a significant proportion of subjects exhibiting correlation sensitivity behaviors (72.5% for Po_a and 62% for Po_s). At the average level, there is a significant tendency to switch to option B at a smaller X value in the negatively correlated task. Nevertheless, the magnitude of ΔX is less than \$1 for both Po_a and Po_s. Additionally, we find no significant correlation between violation of swapping independence and correlation insensitivity.

In Appendix A.4.3, we perform a test for strong independence. To this end, we compare the values of ΔX between two subjects groups for each of the payment-variation tasks in pairs 1 and 2. For each of the tasks, the only difference between groups 1 and 2 is the common payments from option A and B – while one group gets \$1 under states in which the two options

[55] The sign test in Table 1.5 is ignored since we are comparing averages obtained from different subjects.

[56] The number of subjects that exhibit such behaviors is 53 for skewness prefrence, 45 for diminishing relative sensitivity, and 35 for 2-regular.

[57] $\frac{0.22}{0.12} \approx 1.83$ and $\frac{0.17}{0.32} \approx 1.88^{-1}$, they are almost reciprocals.

provide identical payments, the other group gets \$1.5. We find group heterogeneity for Po_s subjects in pair 1 positively correlated task. Since the heterogeneity is not persistent across different correlation structures, we conclude that the violation of strong independence is at most mild in our experiment, and it is unlikely such violations can alternate our results qualitatively. Additionally, we conduct a between-group analysis for ΔX in pairs 1 and 2 controlling the amount of common payments. The results are almost identical with our main analysis.[58]

1.7 Conclusion

In this study, we undertake a qualitative analysis of salience and regret theory, identifying independent axioms that characterize each. Salience theory's axioms enable us to examine the salience function independently of the utility function's shape by focusing on special lotteries. In contrast, regret theory's axiom hinges on correlation-insensitive preferences in scenarios involving lotteries with two equiprobable outcomes. Additionally, we describe the parametric boundaries distinguishing salience from regret theory. Further, the current experiment, designed to control confounders such as event-splitting effects, failures in contingent reasoning, and random errors, yields two primary insights. Firstly, our findings lend aggregate-level support to both theories. Secondly, a noteworthy rate of axiom violations in each theory suggests their limitations in accounting for certain heterogeneous factors, even though they provide valuable benchmarks for predicting average behaviors.

This project paves the way for future explorations in salience and regret theory. From a theoretical standpoint, it would be beneficial to investigate decision-making patterns in the context of arbitrary choice sets and to understand how deviations from standard optimization can be influenced by salience and regret. Furthermore, while regret theory has been axiomatized within a subjective framework as per Savage (1954), to our knowledge such comprehensive

[58]Additionally, to test salience and regret theory's abilities to rationalize behaviors in the aggregate level, we perform a similar analysis by comparing the average switching values (winning odds) across relevant tasks. These additional results are largely in line with our main analysis.

treatment appears scarce for salience theory.[59]

For future experimental research, a key objective is to identify functional forms that allow for a quantitative evaluation of these theories. Moreover, it's critical to distinguish between the effects of correlation sensitivity and nonlinear probability weighting, as they can sometimes lead to similar outcomes.[60] Disentangling these influences remains a vital challenge for advancing our understanding of decision-making processes.

[59] We postulate that different characterizations may be necessary for finite state spaces.

[60] See Abdellaoui (2000) and Bleichrodt et al. (2010) for an example.

Chapter 2

Multidimensional Salience: Theoretical Foundations and Experimental Tests

Abstract

Economic decisions frequently involve uncertainty along multiple dimensions. This manuscript proposes and characterizes an extension of salience theory (Bordalo et al., 2012, 2013a) for the treatment of such multidimensional lotteries. The model's predictions are explored in three existing data sets and a novel experiment focused on a canonical example of multidimensional risk, intertemporal risky choice, where prior data differ markedly from the benchmark of Discounted Expected Utility. Around 70-80% of the prior data are consistent with multidimensional salience, and new experimental data largely confirm the predictions of the theory.

2.1 Introduction

Many economic decisions involve trade-offs across different dimensions: buyers trade off price and quality for different goods; workers trade off effort and pay for different jobs; investors trade off streams of returns across states of nature for different assets. Importantly, in many of these cases, the outcomes in each dimension may be uncertain: quality may be uncertain, the costs of effort may not be deterministic, and states of nature are realized probabilistically. Thus, many canonical decisions are both multidimensional and risky.

Within behavioral economics two largely separate literatures have emerged for analyzing context dependence in either multidimensional deterministic choice or one-dimensional risky choice.[1] In this manuscript, we attempt to bridge these two approaches to provide a multidimensional model of context-dependent risky choice that can be applied to a broad range of economic environments. We extend the salience model of Bordalo et al. (2012), henceforth BGS, to choices over multidimensional lotteries. The model provides a new perspective on how attention drives risky decisions and leads to deviations from neoclassical, context-independent theories.

The mechanism we consider follows from a simple two-dimensional example. Consider the choice between a pair of deterministic two-dimensional options, $[x_1, x_2] \in \{[100,0],[50,50]\}$ where the objects of choice provide payoffs relevant to the decision maker in two generic monotonic dimensions.[2] Suppose a decision-maker is indifferent between $[100,0]$ and $[50,50]$. What happens when the dimension payoffs, x_1 and x_2, are subject to risk, each being paid with an independent probability, $p = 0.5$?[3] Such multidimensional risk delivers a choice set over lotteries, $\{L_{[100,0]}, L_{[50,50]}\}$, each with four equiprobable states — B(oth) dimension payoffs are made, N(either) dimension payoff is made, only the F(irst) dimension payoff is made, or only the S(econd) dimension payoff is made. The payoffs in each state for the two options are thus:

Option	State			
	B	N	F	S
$L_{[100,0]}$	$[100,0]$	$[0,0]$	$[100,0]$	$[0,0]$
$L_{[50,50]}$	$[50,50]$	$[0,0]$	$[50,0]$	$[0,50]$.

In State B, the two lotteries yield outcomes which are of equal value by the assumption of

[1]One theoretical exception is the contemporaneous work of Köster (2021), who considers a two-layer salience model both within deterministic dimensions and then across states of nature. The fundamental distinction between our baseline model and that of Köster (2021) is that while he aims to integrate Bordalo et al. (2012) and Bordalo et al. (2013a) into a single decision-making process, we directly extend the salience comparisons of Bordalo et al. (2012) from single to multi-dimensional outcomes. In addition, our baseline model separates behavioral effects from inter-dimensional comparisons and from risk. For other recent theoretical advances on 'correlation-sensitive' preferences see Lanzani (2022); Diecidue and Somasundaram (2017). For experimental results confirming correlation-sensitive risk preference, see Frydman and Mormann (2018); Dertwinkel-Kalt and Köster (2020). For experimental results against it, see Loewenfeld and Zheng (2021). For a summary on different context-dependent effects, see Landry and Webb (2021). For an experimental investigation in a riskless environment, see Somerville (2022).

[2]Here, we leave the dimensions unspecified. Our theoretical analysis is general; dimensions could be quality and minus price, leisure and earnings, or different time periods' returns.

[3]Throughout this manuscript we assume that all risks are resolved immediately after choice, eliminating any mechanism related to the timing of the resolution of uncertainty.

indifference in the deterministic case. In State N, the two lotteries yield the same outcome, while in States F and S, $L_{[100,0]}$ yields a better or worse outcome than $L_{[50,50]}$, respectively. The canonical model of Expected Utility (EU) with separable utility across dimensions requires that a decision-maker who is indifferent in the deterministic setting remain indifferent under these risks.[4] Salience posits that attention is drawn to states of nature depending on the absolute payoffs and payoff differences between lotteries in each state. Attention leads to distortions of state probabilities away from their objective likelihoods and drives behavior away from EU predictions. Under our extension, attention will be drawn to state S in this example, distorting its probability upward and driving the preference away from indifference towards $L_{[50,50]}$. In effect, the decision-maker behaves as if she is disproportionately worried about the chance of receiving nothing in State S from $L_{[100,0]}$ when she could have received 50 (in dimension 2) in the same state from $L_{[50,50]}$.

In Section 2 of the manuscript we provide our formal theoretical extension of BGS to multidimensional risk. The central component of our theory is a salience function, which maps from multidimensional options to attention, and determines exactly how different two options are perceived to be. In our example, the salience function captures the psychologically plausible pattern of attending disproportionately to the state where one option pays something while the other pays nothing at all.

To generate multidimensional salience, our theoretical development begins with a lattice construction and a set of assumptions connecting differences between options to perceived salience. Unlike the original BGS model, our construction implies that the salience level between options is not always proportional to their geometric distance. The previous example illustrates this distinction and highlights the central challenge of applying the original salience model in the multidimensional lottery setting. With only a single payoff dimension, BGS provides principles connecting the Euclidean distance between two lottery payoffs in a state to the perceived

[4] In this example, we assume a separable utility across dimensions only to illustrate intuitions behind our model. In general, multidimensional salience theory doesn't require such separability.

difference of the pair, and hence, the level of salience. Consequently, the salience of a state is positively correlated with the Euclidean distance between lottery payments in that state. In multidimensional lottery choice this is plausibly not the case: State B has the greatest Euclidean distance between the two lotteries, differing by 50 in both dimensions, but the decision-maker has already expressed indifference between the two outcomes obtained in State B in the deterministic case. Hence, it seems unlikely this state would be most salient under uncertainty. Our extension overcomes this implausible implication.

With our extension's measure of salience in hand, we posit that the decision-maker's attention is drawn to states in which the options differ most substantively, as in the original BGS model. Following from the general construction, we connect our model with BGS and discuss structural simplifications to facilitate empirical analysis. We end our theoretical development with an axiomatization and organize our empirical analysis around testing the model's corresponding implications for behavior.

In Section 3 of the manuscript, we consider a canonical decision environment to which multidimensional salience applies: intertemporal risk. By regarding time periods as different dimensions, we apply our theory to intertemporal settings in which decision-makers choose between multi-period streams of payments and each period's payments may be uncertain. Relevant examples that correspond to this application include human capital formation, consumption-savings problems, and insurance choice. Our model provides some intuitive predictions. The first prediction, "Salience-Based Present Bias," makes a connection between multidimensional salience and apparently present-biased behaviors (Laibson, 1997; O'Donoghue and Rabin, 1999). If the present is certain while the future is uncertain, our theory predicts that individuals will be relatively impatient for decisions involving certain, present payments and relatively patient for decisions involving only uncertain, future payments.[5] The second prediction, which we

[5]As in work on temporal probability weighting (Halevy, 2008), our theory associates present-biased behaviors with violations of DEU rather than non-exponential discounting. This is not to say that non-exponential discounting is not an important driver of behavior overall, only that apparently present-biased behaviors can be exacerbated by the failure to keep risks constant over time.

call "Intertemporal Hedging," predicts that individuals will be more likely to smooth their intertemporal allocations when facing risk than when facing certainty. For instance, Intertemporal Hedging predicts that present bias should be reduced in the presence of risk, which is indeed documented by previous experiments (Keren and Roelofsma, 1995; Weber and Chapman, 2005; Baucells and Heukamp, 2010; Reddinger, 2020). The third prediction, which we call "Correlation Dependence," is that sensations of salience, and, hence, behaviors will depend on the correlation of intertemporal risks.[6] If risks over time are positively correlated, opportunities for Intertemporal Hedging are reduced; while if risks over time are perfectly negatively correlated, hedging remains an attractive possibility.[7]

A number of experimental studies have been conducted to study choices in intertemporal risky environments. These publicly-available data provide an opportunity to test the specific predictions of multidimensional salience for intertemporal risks. We conclude Section 3 with an an exploration of prior data from Andreoni and Sprenger (2012b), Miao and Zhong (2015), and Cheung (2015). We show that the data in all three data sets deviate from Discounted Expected Utility (DEU) with high frequency, up to around 80% in some comparisons. Multidimensional salience is able to rationalize many of these observed deviations. In tests of Salience-Based Present Bias, 70% of DEU deviations are consistent with our multidimensional model. In tests of Intertemporal Hedging, 78% of DEU deviations can be rationalized. Additionally, 80% of observations with positive correlation and 74% of observations with negative correlation are consistent with the model's predictions for Correlation Dependence.

[6]The idea of Correlation Dependence, which fixes the marginal distributions of every choice and manipulates the joint distribution, is a crucial consequence of salience theory and has been exploited in the previous literature on atemporal salience. For instance, Bordalo et al. (2012) show that the "common consequence effect" of Allais (1953) is not robust to changing the correlation between lottery outcomes; Frydman and Mormann (2018) conduct an experiment altering correlations to explore salience-based sensitivity to the joint distribution of lottery outcomes; and Dertwinkel-Kalt and Köster (2020) also provide an experiment that manipulates correlation structures in an atemporal setting.

[7]Consider the example above and regard x_1 and x_2 as payments in two time periods. Suppose intertemporal risks are perfectly *positively* correlated – if the first payment is received, the second will be as well. This correlation structure eliminates State F and State S from consideration. If only State B or State N can obtain, our model predicts State B will be salient, and the decision-maker will adhere to their deterministic statement of indifference. In contrast, if intertemporal risks are perfectly *negatively* correlated, such that only State F or State S will obtain, State S will be salient, and the individual will prefer $L_{[50,50]}$.

In addition to exploring the predictions of multidimensional salience in prior data sets, in Section 4 of the manuscript we consider a new experimental test, also in the intertemporal risk setting. The new experiment focuses on replicating evidence on Intertemporal Hedging in a simpler decision environment, and testing a final prediction of the theory when applied to intertemporal risks, called "Reordering Dependence." Reordering Dependence differentiates multidimensional salience from other non-attention based explanations for some DEU deviations (Miao and Zhong, 2015; Epper and Fehr-Duda, 2015). Holding fixed two lotteries' marginal distributions (i.e., the number of distinct payoff states,[8] and the probability of each state), decision-makers' sensations of salience depend on the stream differences between lotteries in each state. If salience depends on the difference between streams in each state, then total sensations of salience and corresponding choices can be altered by reordering; changing which outcomes are compared to each other under each state.[9]

Our new experiment asks subjects to choose between pairs of lotteries $L_{[\$c_1,\$c_2]}$ on two-period monetary streams $[\$c_1,\$c_2]$ such that, subject to payment uncertainty, $\$c_1$ arrives one week after the experiment while $\$c_2$ arrives four weeks after the experiment. In our primary design with 105 subjects, individuals make decisions involving the monetary streams [\$18, \$2] and [\$10, \$10] under three different risk structures. First, in the deterministic choice between the two streams, 55% of subjects choose $[\$10,\$10]$ over $[\$18,\$2]$. Second, with common and independent payment probability of $p = 0.5$ in each period, 75% of subjects chose $L_{[\$10,\$10]}$

[8]Two states are distinct in payoffs if there exists some lottery in the choice set that yields different outcomes in the two states. Without controlling this, behavioral patterns may be subject to an *event-splitting* effect (Starmer and Sugden, 1989; Humphrey, 1995).

[9]Consider the independent equiprobable case above, but re-order the outcomes of $L_{[100,0]}$ by exchanging the State N and F outcomes to make $L'_{[100,0]}$. Leaving $L_{[50,50]}$ unchanged, the outcomes in each state are:

Option	State			
	B	N	F	S
$L'_{[100,0]}$	$[100,0]$	$[100,0]$	$[0,0]$	$[0,0]$
$L_{[50,50]}$	$[50,50]$	$[0,0]$	$[50,0]$	$[0,50]$

Now attention will be disproportionately drawn to State N, where $L'_{[100,0]}$ pays $[100,0]$ while $L_{[50,50]}$ pays $[0,0]$. A decision-maker whose state probabilities are sufficiently distorted by salience will prefer $L'_{[100,0]}$ to $L_{[50,50]}$, prefer $L_{[50,50]}$ to $L_{[100,0]}$, and be indifferent between $L_{[100,0]}$ and $L'_{[100,0]}$ in pairwise choices.

over $L_{[\$18,\$2]}$, a significant deviation from the DEU prediction of equal choice across these first two conditions ($F_{1,104} = 9.69, p < 0.01$). This deviation replicates the findings of Intertemporal Hedging from the prior literature in a simple choice environment. Third, *only* reordering the streams of $L_{[\$10,\$10]}$, but maintaining the marginal distribution of each lottery leads subjects to choose $L_{[\$18,\$2]}$ more frequently: 64% of subjects chose the reordered $L'_{[\$10,\$10]}$ over $L_{[\$18,\$2]}$, a significant decrease from the condition with independent risks ($F_{1,104} = 4.95, p < 0.05$). This sensitivity of behavior to the joint distribution of outcomes corresponds to the prediction of Reordering Dependence. Our experiment shows deviations from DEU in line with the predictions of multidimensional salience. With the remaining subjects in our design we also document some experimental effects of the ordering of questions for deviations from DEU.[10]

Our theoretical and empirical results connect to several literatures. First, our theory provides a uniform environment for the analysis of salience-based attention effects in both deterministic and risky settings when the objects of choice are multidimensional. This permits researchers to analyze salience in richer, potentially more ecologically relevant choice environments. We provide functional forms and corresponding assumptions that facilitate this future analysis. Further, though our baseline model focuses on salience-based context dependence between multidimensional objects, our extensions permit the incorporation of other attentional forces such as focusing or relative thinking within dimensions (Koszegi and Szeidl, 2012; Bushong et al., 2021), as well (see Appendix B.2). Second, we show that multidimensional salience is able to explain a substantial portion of behavior in prior and new data sets on intertemporal risky choice. Thus, the theory provides a useful, novel account for the pronounced DEU deviations in such data sets. The central logic of Intertemporal Hedging, and sensitivity to correlation structure and reordering could have broad economic implications. For example, in

[10]In our development of this study we forecasted that order effects could be substantial for choices between intertemporal lotteries. Subjects who face a choice between deterministic streams as their first choice are more likely to both pass interim attention checks and exhibit the effects of intertemporal salience; subjects who face the more complicated independently risky streams or re-ordered risky streams as their first choice are more likely to fail the interim attention checks, choose apparently randomly, and exhibit consistency in choice probabilities across conditions. See Section 4 for additional detail.

Appendix B.3 we show multidimensional salience's Intertemporal Hedging can manifest a form of precautionary savings beyond what standard economic models would imply based on third derivatives of utility (Leland, 1968; Kimball, 1990; Eeckhoudt and Schlesinger, 2006). Future behavioral applications in areas like retirement savings and human capital formation are readily apparent from such results.

The paper proceeds as follows. Section 2 lays out our theory, axiomatization, simplifications, and connections with other models. Section 3 provides examples tailored to the intertemporal risky setting and examines prior published data. Section 4 presents our new experimental design. Section 5 concludes.

2.2 General Model for Multidimensional Salience

In this section, we present our formal model of multidimensional salience. The presentation is carried out in three subsections: Subsection 2.2.1 provides model primitives and definitions of salience for multidimensional lotteries; Subsection 2.2.2 discusses properties of our model, provides connections to the single dimension lottery setting of BGS, and describes additional assumptions to facilitate applications in the multidimensional lottery setting; Subsection 2.2.3 gives a complete axiomatization for our baseline model. Additionally, Appendix B.2 discusses potential extensions and connections to other context-dependent models.

2.2.1 Primitives and Conditions for Multidimensional Salience

We consider a world in which there is a collection S of finitely many possible states. Every state, s, has a positive objective probability, $p_s > 0$, with $\sum_{s \in S} p_s = 1$. A decision-maker chooses between M lotteries $\{L_i\}_{i=1}^{M}$. Each lottery, L_i, yields an n-dimensional payoff $X_s^i = [x_{s1}^i, x_{s2}^i, \ldots, x_{sn}^i]$ in each state, s. $x_{sk}^i \in \mathbb{R}$ for all $k \leq n$. We assume that the decision-maker processes a utility function $U(X_s^i)$ over n-dimensional payoffs.

In the evaluation of L_i, a decision-maker assigns each state, s, with a non-negative decision weight $\pi_s^i \geq 0$ such that $\sum_{s \in S} \pi_s^i = 1$. We assume the decision-maker chooses the lottery from $\{L_i\}_{i=1}^{M}$ with the highest expectation, $U(L_i) = \sum_{s \in S} \pi_s^i U(X_s^i)$. If the decision weight, π_s^i, is equal to the objective probability, p_s, for all states and lotteries, $U(L_i)$ corresponds to the neoclassical Expected Utility (EU) model.

Under multidimensional salience, decision-makers will potentially deviate from EU by applying a decision weight that differs from a state's objective probability. This deviation from EU is driven psychologically by attention. For a given lottery, attention is drawn to states where the resulting outcome "differs most substantively" from other lotteries in the choice set under the same state. These states become salient for the given lottery and receive disproportionate weights. The requirement $\sum_{s \in S} \pi_s^i = 1$ implies that overweighting of one state generates underweighting of another. Our definition of salience below lays out precisely what is meant by the phrase "differs most substantively" in the multidimensional context.

Before discussing the main construction of our model, we introduce some useful notations, borrowing from the lattice literature. Let $X = [x_1, x_2, \ldots, x_n]$ and $Y = [y_1, y_2, \ldots, y_n]$ in $\mathbb{R}^n$ be two multidimensional outcomes. We say $X \geq (\leq) Y$ if $x_i \geq (\leq)\ y_i$ for all i. Similarly, $X > (<) Y$ if $X \geq (\leq) Y$ and $x_i > (<) y_i$ for some i. Additionally, $X + Y = [x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n]$, and $X \in \mathbb{R}_+^n$ if $x_i \geq 0$ for all i. Define $X \wedge Y \equiv [\min\{x_1, y_1\}, \min\{x_2, y_2\}, \ldots, \min\{x_n, y_n\}]$; $X \vee Y \equiv [\max\{x_1, y_1\}, \max\{x_2, y_2\}, \ldots, \max\{x_n, y_n\}]$. Letting ρ be a *permutation* on $\{1, 2, \ldots, n\}$, define $X_\rho \equiv [x_{\rho(1)}, x_{\rho(2)}, \ldots, x_{\rho(n)}]$. The values $X \wedge Y$ and $X \vee Y$ consider minimal and maximal elements between two outcomes in each dimension, while X_ρ considers a re-ordering of a given outcome. These values will provide a structure for defining multidimensional salience.

Definition 1. *A* ***multidimensional salience function*** *is a continuous map* $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}_+$ *satisfying the following five conditions:*

1. *Ordering: For every X, Y, Z in* $\mathbb{R}^n$*,*

    (a) *Upper Ordering:* $\sigma(X, Y) \leq \max\{\sigma(X \vee Z, Y), \sigma(X, Y \vee Z)\}$.

    (b) *Lower Ordering:* $\sigma(X, Y) \leq \max\{\sigma(X \wedge Z, Y), \sigma(X, Y \wedge Z)\}$.

(c) *Inclusion: Suppose* $X \leq Y \leq Z$, $\sigma(X,Y) \leq \sigma(X,Z)$ *and* $\sigma(Y,Z) \leq \sigma(X,Z)$. *In addition,* $\sigma(X,Y) = 0$ *if* $X = Y$.[11]

2. *Diminishing sensitivity: For every* X, Y *in* $\mathbb{R}^n_+$ *such that* $X \geq Y$, *we have* $\sigma(X,Y) \geq \sigma(X+\varepsilon, Y+\varepsilon)$ *for all* $\varepsilon > 0$.

3. *Reflection: For every* X, Y, Z, H *in* $\mathbb{R}^n_+$,

   $\sigma(X,Y) \leq \sigma(Z,H)$ *if and only if* $\sigma(-X,-Y) \leq \sigma(-Z,-H)$.

4. *Symmetry: For every* X, Y *in* $\mathbb{R}^n_+$, $\sigma(X,Y) = \sigma(Y,X)$

5. *Compatibility: For every* X, Y, *and permutation* ρ, $\sigma(X,Y) = \sigma(X_\rho, Y_\rho)$.

We call a multidimensional salience function ***free*** *if it only satisfies the first four conditions.*

The conditions on $\sigma(\cdot,\cdot)$ establish what it means for two outcomes to be perceived as different. These conditions represent analogs of the BGS assumptions for the multidimensional setting but carry some nuance relative to the one-dimensional lottery setting, which is discussed below in Subsection 2.2.1

Having described how a decision-maker evaluates differences between two outcomes, we explore how a decision-maker evaluates the differences among M alternative lotteries, each yielding an outcome in each state.

Definition 2. *Let* $\Delta^i_s = \dfrac{1}{M-1}\sum_{j\neq i}\sigma(X^i_s, X^j_s)$ *be the multidimensional salience level of a state* s *when evaluating lottery* L_i.

The *multidimensional salience level* of a state for a given lottery is derived from the average perceived difference of that lottery's outcome from all other outcomes in the choice set in the corresponding state.[12]

[11] Under BGS formulation, for the single dimension salience function we have $\sigma(X,Y) = 0$ if and only if $X = Y$.

[12] This criterion is proposed in Bordalo (2011). Other related works usually assume that when facing multiple lotteries, the salience level is defined by comparing the outcome of each option with *the average of remaining*

Our theory posits that decision-makers put more decision weight on states with higher salience levels. The deviation of the decision weight of a state from its objective probability is quantified by the following mechanism.

Definition 3. *Let $f : \mathbb{R}_+ \mapsto \mathbb{R}_{++}$ be an increasing function, the decision weight of state s when evaluating lottery L_i is*

$$\pi_s^i = \frac{f(\Delta_s^i)}{\sum_{s' \in S} p_{s'} f(\Delta_{s'}^i)} p_s$$

Note that $\sum_s \pi_s^i = 1$ and $\frac{\pi_s^i}{\pi_{s'}^i} = \frac{p_s}{p_{s'}} \frac{f(\Delta_s^i)}{f(\Delta_{s'}^i)}$. As a result, the ratio of decision weights between states s and s' is related to both the ratio of their objective probabilities and the ratio of the salience levels of these two states.

BGS propose a simplification of $f(\cdot)$, which is called *rank-based* salience.[13] It proceeds in two steps. First, given a lottery, L_i, one ranks every state by its salience level from high to low. Second, if a state, s, has rank k,

$$f(\Delta_s^i) = \theta^{k-1},\ \theta \in [0,1].^{14} \tag{2.1}$$

Consequently, in this *rank-based* salience representation, a state with lower salience rank will be weighted less than its objective probability. This simplification is favored by most of the salience literature since it doesn't rely on a cardinal measure of salience level. Instead, it

options. Despite its simplicity, the "compare to the average" criterion may lead to counterintuitive predictions. Consider the following example: suppose there are three equiprobable states: a, b, and c, and there are three lotteries, X, Y, and Z. In state a, all lotteries yield \$0; in state b, X yields \$8, Y yields \$5, and Z yields \$2; in state c, X yields \$2, Y yields \$5, and Z yields \$8. When evaluating Y, the averages of payoffs from the remaining options are \$0 in state a, \$5 in state b, and \$5 in state c. Thus, if one compares Y to the remaining averages, no state is more salient than others as the distance to the average of alternative is zero in every state. Consequently, each state receives a weight of 1/3. When evaluating X and Z, states b and c will be more salient than state a, where the distance to the average remains zero. Hence, state a will receive less than proportionate weight in the evaluation of X and Z. Ignoring the zero payment in state a may lead to a preference for X or Z over the less risky option, Y. This pattern seems to be unlikely (see the *isolation effect* in Kahneman and Tversky (1979b)). To circumvent such implausible predictions, we adopt the alternate criterion proposed in Bordalo (2011).

[13]This name is used in Herweg and Müller (2021).

[14]Strictly speaking, under rank-based salience representation, the value of $f(\cdot)$ depends on information from multidimensional salience levels of all states when evaluating a lottery.

provides qualitative intuitions from an ordinal perspective. Nevertheless, *rank-based* salience may introduce some difficulties (**?**Lanzani, 2022). To resolve these issues, BGS also propose a *continuous* salience representation with $f(\Delta_s^i) = \gamma^{\Delta_s^i}$ where $\gamma \geq 1$.

Discussion of Conditions and Assumptions for Multidimensional Salience

While our assumptions for $\sigma(\cdot,\cdot)$ are analogs of the BGS assumptions in the multidimensional setting, they carry a somewhat different logic than the single-dimension lottery case. The BGS assumptions in the single-dimension lottery case generate a positive correlation between the Euclidean distance between two lotteries in a given state and the salience level of that state. In the multidimensional lottery setting, this linkage between Euclidean distance and salience levels seems more tenuous. Consider the outcome $[3,9]$ in $\mathbb{R}^2$ and contrast it with either $[9,3]$ or $[3,3]$. The corresponding three vectors are drawn in Figure 2.1. Note that the Euclidean distance between $[3,9]$ and $[9,3]$ is larger than between $[3,9]$ and $[3,3]$. Nonetheless, the former difference seems plausibly smaller: the decision-maker receives twelve in total from $[3,9]$ and $[9,3]$, and six from the outcome $[3,3]$. Our conditions ensure that this plausible relation maintains in such examples.[15] Consequently, ordering rules out certain forms for multidimensional salience functions. For instance, $\frac{\sum_{i=1}^n |x_i - y_i|}{\sum_{i=1}^n (|x_i|+|y_i|)}$ records the sum of *distances* in each dimension but completely ignores the distinction between *distance* and *difference*, so it violates the ordering property. As shown in Figure 2.1, intuitively speaking, the difference in lengths is more relevant than the difference in directions when evaluating salience. In Subsection 2.2.2, we also explore the restrictions on the salience function if the direction is completely irrelevant (Proposition 7).

Diminishing sensitivity states that the perceived difference between two outcomes is diminishing in the baseline level of the outcomes. Two outcomes which differ by one in a single dimension would appear more similar if baseline values were one thousand rather than one. Note that *diminishing sensitivity* does not imply that all increases in outcomes reduce salience. As in the previous example, outcomes [9,3] and [3,9] are similar to a decision-maker since in

[15]To see this, *lower ordering* implies that $\sigma([9,3],[3,9]) \leq \max\{\sigma([9,3],[3,3]), \sigma([3,9],[3,3])\}$. Then, *compatibility* suggests that $\sigma([9,3],[3,3]) = \sigma([3,9],[3,3])$.

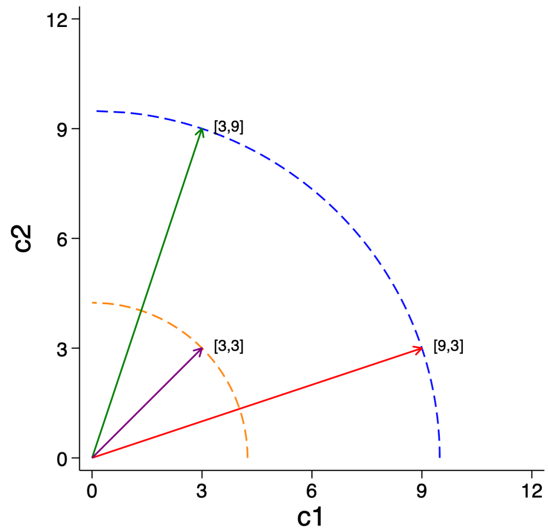


Figure 2.1. Difference vs. Distance

Notes: The Figure presents three two-dimensional outcomes $[c_1, c_2]$ as vectors in $\mathbb{R}^2_+$: $[3,9]$ (green), $[3,3]$ (purple), and $[9,3]$ (red). The blue dash line represents nonnegative vectors having a same Euclidean length with $[3,9]$. The orange dash line represents nonnegative vectors having a same Euclidean length with $[3,3]$.

each dimension one outcome gives nine while the other gives three. If we increase only each outcome's first dimensional payment by 1000, they become [1009,3] and [1003,9]. It is possible that the two outcomes actually become more different after the increment of 1000 in the first dimension, as the second dimension difference could loom large in this case. Acknowledging such possibilities, *diminishing sensitivity* only requires that the salience function is monotonically decreasing when comparisons are "trivial" in the sense that one outcome is uniformly larger than the other in *all* dimensions. As we will show in Subsection 2.2.2, combined with *reflection*, requiring such *diminishing sensitivity* results in a close connection between multidimensional salience theory and the diminishing sensitivity for gains and losses discussed in applications of prospect theory. BGS make a similar diminishing sensitivity assumption in their one-dimensional lottery application[16], and explicitly assume monetary amounts are the drivers of salience.

While the first four conditions of a salience function describe how the difference between

[16]When there is only a single dimension, every outcome is either larger of smaller than others, so *diminishing sensitivity* holds for all pairs of non-negative outcomes.

two outcomes pertains to their numerical values, the last condition, *compatibility*, discusses how the unit of outcomes affects salience. As a simple example, when *compatiblity* holds we have $\sigma([1,0],[0,0]) = \sigma([0,1],[0,0])$. The outcomes $[1,0]$ and $[0,0]$ are different in the first dimension, while $[0,1]$ and $[0,0]$ yield an identical difference in the second dimension. As a result, *compatibility* implies that the marginal effects of differences are independent of the dimension in which they occur. It is for this reason that perhaps the most natural way to think of the entries of X and Y is as utilities reduced to some common unit. In practical applications, we recognize that outcomes would be initially comprised of physical outcomes rather than separable utilities. One can use functional form assumptions, such as constant relative risk aversion in each dimension, and corresponding empirical estimates, to map physical values to utilities. More abstractly, as we axiomatize the model in Subsection 2.2.3, the salience level between two outcomes is not necessarily proportional to their utility difference. Therefore, under the full generality of our model, compatibility can also be considered as a requirement for comparable salience units independent from utility.

Under some circumstances *compatibility* may not be appropriate. For instance, as in Bordalo et al. (2013b), when choosing from several commodities decision-makers compare the options in the dimensions of prices and qualities. In this case, $X = [\text{price}, \text{quality}]$. Without an assumption under which price differences can be substituted for quality differences, *compatibility* does not hold. Therefore, the requirement of *compatibility* is optional. While we keep the assumption in most subsequent analyses, we explore some extensions in Appendix B.2.

2.2.2 Lottery Dimensions, Reduction to Standard BGS, and Facilitating Applications

In Subsection 2.2.1, we described the fundamental ingredients of multidimensional salience. The current section describes how these general properties are connected to the previous literature. The results in this section provide a way to simplify empirical analysis by reducing the problem to the standard BGS framework of one-dimensional lottery choice.

In addition, the results establish a connection between BGS salience theory and diminishing sensitivity to gains and losses often discussed in prospect theory applicationss.

To begin, note that if there is only a single dimension, our definition of multidimensional salience coincides with that of BGS. Thus, a one-dimensional salience function in our setting is a BGS salience function (and defined as such). To see this, note that *inclusion* implies that if $x_1, x_2, y_1, y_2 \in \mathbb{R}$ and the interval (x_1, x_2) is a subset of (y_1, y_2), then $\sigma(x_1, x_2) \leq \sigma(y_1, y_2)$. This corresponds to the ordering property of BGS, that the salience function is increasing with respect to the difference between two outcomes. *Diminishing sensitivity* retains its prior meaning and *reflection* describes the similarity between comparing gains and comparing losses. *Compatibility* does not have practical meaning in the one-dimensional case as no dimensions can be exchanged. In contrast, in the presence of multiple dimensions, our definition is more demanding than BGS. To better understand the effects of the properties in definition 1, we present the following proposition:

Proposition 6. *Consider $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}_+$ such that for all $X, Y \in \mathbb{R}^n$, $\sigma(X,Y) = \tilde{\sigma}(h(X), h(Y))$ where $h : \mathbb{R}^n \mapsto \mathbb{R}$ is continuously differentiable and additively separable, and $\tilde{\sigma} : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}_+$ is a BGS salience function. Then:*

1. *If $\sigma(X,Y) = \tilde{\sigma}(h(X), h(Y))$ is a multidimensional salience function, then $h(X) = \sum_{i=1}^{n} g(x_i)$ where $g(\cdot)$ is some strictly monotonic function.*

2. *If $g(x_i)$ is concave for $x_i > 0$, convex for $x_i < 0$, and $g(0) = 0$, then $\tilde{\sigma}(\sum_{i=1}^{n} g(x_i), \sum_{i=1}^{n} g(y_i))$ is a multidimensional salience function for all BGS salience functions $\tilde{\sigma}$.*

3. *Furthermore, if we assume that in addition for every X, Y in $\mathbb{R}^n_+$, we have $\sigma(X,Y) \geq \sigma(X+\varepsilon, Y+\varepsilon)$ for all $\varepsilon > 0$. Then $\sigma(X,Y) = \tilde{\sigma}(\sum_{i=1}^{n} g(x_i), \sum_{i=1}^{n} g(x_i))$ is a multidimensional salience function if and only if $g(\cdot)$ is a non-constant linear function.*

Proof. See Appendix B.5.1. □

The first part of Proposition 6 suggests that as a special case, one can operationalize multidimensional salience theory in two steps. First, for outcomes $[X,Y]$, the values $h(X)$ and $h(Y)$ can serve as "utility indices." The ordering property for salience in definition 1 implicitly requires that each dimension of an outcome is an economic good to the decsion-maker. Therefore, the utility indices, $h(\cdot)$, should be monotonic. Second, use the BGS salience functional form, $\tilde{\sigma}(\cdot)$, to evaluate the difference created by these utility indices.[17] This result provides a simplification to greatly facilitate empirical analysis. For example, if $h(X) = \sum_{i=1}^{n} x_i$, one can construct a natural extension of the salience function proposed by Bordalo et al. (2012):

$$\sigma(X,Y) = \frac{|\{\sum_{i=1}^{n}(x_i - y_i)\}|}{|\sum_{i=1}^{n} x_i| + |\sum_{i=1}^{n} y_i| + \beta}, \text{ where } \beta \text{ is a positive real number.} \tag{2.2}$$

In many applications, including the empirical exercise in this project, using the simplified salience function of equation (2.2) seems quite reasonable. Indeed, if each entry of the outcome represents an appropriately normalized flow utility, its sum should represent a sufficient statistic to capture the difference between outcomes.

The second part of Proposition 6 provides a more stringent connection between multidimensional salience, gain-loss attitudes, and BGS salience. Since $\sigma(X,Y) = \tilde{\sigma}(\sum_{i=1}^{n} g(x_i), \sum_{i=1}^{n} g(y_i))$, the properties of function $g(\cdot)$ flow through to $\sigma(\cdot)$. Hence, the salience function takes into account the differential effect of gains and losses and transfers them to salience-based probability distortions. In general, $g(\cdot)$ need not be consistent with the underlying utility over deterministic outcomes, so multidimensional salience can accommodate differential attitudes towards gains and losses without additional assumptions on utility.

The last part of Proposition 6 reflects that by requiring different levels of *diminishing sensitivity*, one restricts allowable curvatures on $g(\cdot)$. The statement shows that with the strongest

[17]Notice that the additively separable condition is necessary for the result. For any nonlinear function $h(\cdot)$, there is some BGS salience function that can use the tension between ordering and diminishing sensitivity to induce indeterminable salience comparisons.

version of *diminishing sensitivity*, there is no room for curvature at all for $g(\cdot)$.[18]

Moreover, note that under equation (2.2), each entry of an outcome is comparable to *all* entries of another. For instance, the difference between $[1,1]$ and $[1,0]$ is identical to the difference between $[2,0]$ and $[1,0]$. In this case, the directional difference between outcomes is totally irrelevant. This results in a stronger version of compatibility, which turns out to be the necessary and sufficient condition to appropriately simplify the index $h(\cdot)$ from a sum over monotonic transformations of outcome entries to the exact sum. Formally speaking:

Strong Compatibility: A function $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ satisfies strong compatibility if for every $X, Y, Z \in \mathbb{R}^n$ and permutation ρ, $\sigma(X+Z,Y) = \sigma(X+Z_\rho, Y)$.

The next proposition gives the more restrictive result.

Proposition 7. *A multidimensional salience function $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}_+$ satisfies strong compatibility if and only if $\sigma(X,Y) = \tilde{\sigma}(\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} y_i)$ where $\tilde{\sigma} : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}_+$ is a BGS salience function.*

Proof. See Appendix B.5.1. □

Proposition 6 and 7 also provide an appealing path for choosing an appropriate salience function.[19] One can choose a reasonable salience function from decisions under uncertainty regarding *one-dimensional* risk, such as that of Bordalo et al. (2012). Then, we can expand the salience function to a *multidimensional* environment by keeping the functional form and integrating each outcome with a linear sufficient statistic.

[18] When there are multiple dimensions, under *compatibility*, summarizing X and Y into $h(X)$ and $h(Y)$ guarantees the existence of distinct X and Y that generate no difference, $\sigma(X,Y) = 0$. Imposing *diminishing sensitivity* upon these two outcomes requires that adding the same amount to both does not make them different. This implies the rate of increment is constant on $g(\cdot)$.

[19] In addition, proposition 7 bridges our model with the *additively integrable representation* in Köster (2021) and **?**. However, our model is different from the general form in Köster (2021). According to his model, decision-makers first categorize dimensions into different groups, and within each group, they perform a salience analysis using the form in Proposition 7.

2.2.3 Axioms for Multidimensional Salience Theory

In this subsection, we give a representation theorem for multidimensional salience theory when decision makers are choosing from two options. For the main text we present results under the maintained assumption that a well-defined preference relation over deterministic outcomes and a corresponding utility function exists. We provide axioms related to the difference between potentially observable choices and those implied by EU given their known preference over deterministic outcomes. The primary result of this subsection demonstrates that the axioms on choice are equivalent to a multidimensional salience representation. To smooth our main analysis, we impose a differentiability assumption. Both a technical axiom to approximate differentiability and a discussion of how differentiability separates utilities from salience distortions are presented in Appendix B.1, along with other proofs.

Our axiomatic environment can be considered as a generalization from Lanzani (2022) for "correlation-sensitive" preferences. Thus, we divide this subsection into two parts. First, we review the choice environment and provide the axioms for our generalization of correlation-sensitive preferences. Then, we analyze the axioms for multidimensional salience theory.

Axioms for Correlation-Sensitive Preferences

For completeness, we first review the setting in Lanzani (2022). Let $X = \mathbb{R}^n$ be the set of outcomes, and X_+, X_{++} the sets of positive and strictly positive outcomes respectively. We denote by $\Delta(X \times X)$ the set of finitely supported bivariate joint distributions on X. For every π in $\Delta(X \times X)$, let $\pi_1 = \sum_y \pi(x,y)$ be the marginal distribution of the first argument, and $\pi_2 = \sum_x \pi(x,y)$ be the marginal distribution of the second argument. For every π in $\Delta(X \times X)$, denote the conjugate distribution by $\bar{\pi}$ where $\bar{\pi}(x,y) = \pi(y,x)$. For every π, π' in $\Delta(X \times X)$ and $\alpha \in [0,1]$, we have the usual mixed distribution $\tilde{\pi} = \alpha\pi + (1-\alpha)\pi'$. That is, for every (x,y) in $(X \times X)$, $\tilde{\pi}(x,y) = \alpha\pi(x,y) + (1-\alpha)\pi'(x,y)$. For every pair of outcomes (x,y), we denote by $\delta_{(x,y)}$ the Dirac distribution such that $\delta_{(x,y)}(x,y) = 1$. We write a specific element in $\Delta(X \times X)$ as $\{(x_1,y_1), p_1; (x_2,y_2), p_2; \ldots; (x_n,y_n), p_n\}$, which means that the outcome pair x_1 from option 1 and y_1 from option 2 happens with probability p_1, x_2 from option 1 and y_2 from option 2 happens with probability p_2, etc.

Let $\Pi \subset \Delta(X \times X)$ be a nonempty subset. We call Π the preference set in the following sense: a decision maker is choosing between two lotteries 1 and 2. Lottery 1 has marginal distribution π_1, lottery 2 has marginal distribution π_2, and together they form a joint distribution, π. If the decision-maker weakly prefers lottery 1 to lottery 2 under π, we have $\pi \in \Pi$. We also denote the strict preference by $\widehat{\Pi} = \{\pi \in \Pi | \bar{\pi} \notin \Pi\}$. Since we focus on objective lotteries, using the preference set is more convenient than working on orderings or choice functions over ΔX. While capturing choices over paired lotteries, the preference set Π also summarizes any preference reversal caused by manipulating correlations and thus altering the joint distribution of lottery 1 and lottery 2. Such context-dependence in choice can only be illustrated by orderings or choice functions after adding a further layer of flexibility.[20]

A function $\phi(x,y) : X \times X \to \mathbb{R}$ is called skew-symmetric if $\phi(x,y) = -\phi(y,x)$. In addition, we call $\phi(x,y)$ is monotonic if it increases in the first argument and decreases in the second.[21] We say that Π induces a smooth correlation-sensitive presentation if $\pi \in \Pi \Leftrightarrow \sum_{x,y} \phi(x,y)\pi(x,y) \geq 0$ for some continuous ϕ. Using techniques similar to Lanzani (2022) for the one-dimensional case, the following five axioms provide equivalent conditions for a smooth correlation-sensitive representation with monotonic ϕ.

- (Completeness) For every $\pi \in \Delta(X \times X)$, $\pi \in \Pi$ or $\bar{\pi} \in \Pi$.
- (Strong Independence) For every π, $\pi' \in \Pi$ and $\alpha \in [0,1]$, we have $\alpha\pi + (1-\alpha)\pi' \in \Pi$. In addition, if $\pi \in \widehat{\Pi}$, $\alpha\pi + (1-\alpha)\pi' \in \widehat{\Pi}$.
- (Archimedean Continuity) For every $\pi \in \widehat{\Pi}$ and $\pi' \notin \Pi$, we can find $\alpha, \beta \in [0,1]$ such that $\alpha\pi + (1-\alpha)\pi' \in \widehat{\Pi}$ and $\beta\pi + (1-\beta)\pi' \notin \Pi$.

[20] In addition, as mentioned by Lanzani (2022), this method avoids an arbitrary state space. Since choice sets come from the whole set of objective lotteries, this state space must have an infinite cardinality.

[21] Recall that $x > y$ if $x_i \geq y_i$ for all $i \leq n$. By increasing we mean that if $x \geq \tilde{x}$, $\phi(x,y) \geq \phi(\tilde{x}y)$.

- (Monotonicity) For every $x,y,z \in X$ such that $x > y$, $\pi \in \Delta(X \times X)$, and $\alpha \in (0,1]$ if $\alpha\delta_{(y,z)} + (1-\alpha)\pi \in \Pi$, $\alpha\delta_{(x,z)} + (1-\alpha)\pi \in \widehat{\Pi}$.

- (Continuity) For every sequence (x_n, y_n) converges to (x,y), $\alpha \in [0,1]$, and $\pi \in \Delta(X \times X)$, if $\alpha\delta_{(x_n,y_n)} + (1-\alpha)\pi \in \Pi$ for all but finitely many n, $\alpha\delta_{(x,y)} + (1-\alpha)\pi \in \Pi$.

Lemma 2. Π *induces a smooth correlation-sensitive presentation with monotonic* ϕ *if and only if completeness, strong independence, Archimedean continuity, monotonicity, and continuity are satisfied.*[22]

Proof. See Appendix B.5.1 □

We end the current review with an assumption on $\phi(\cdot,\cdot)$.

Assumption 1. $\phi(\cdot,\cdot)$ *is uniformly differentiable with positive partial derivatives with respect to x at all points* (x,y) *such that* $\phi(x,y) = 0$.[23]

In Appendix B.1, we provide a technical axiom for this assumption. For multidimensional salience theory, this assumption is satisfied if the utility over deterministic outcomes has continuous and positive partial derivatives. Requiring partial derivatives to be positive is minor given the preference is strictly monotonic. Furthermore, as we show in Appendix B.1, having a continuous differentiable utility allows us to separate the salience function, $\sigma(\cdot,\cdot)$, from the utility function, $u(\cdot)$.

Axioms for Multidimensional Salience Theory

Our goal now is to provide further axioms for multidimensional salience theory. We begin by simplifying the presentation for decisions with two alternatives.

[22]The only difference between current setting and one in Lanzani (2022) is that here we require ϕ has a stronger version of continuity, pointwise continuity, while Lanzani (2022) only requires continuity in each argument.

[23]We define uniform differentiability as: For all $\varepsilon \in \mathbb{R}_+$ and compact set $K \in X$, there is $\delta > 0$ that $\forall x,y \in K$ with $x \sim_u y$, $|\frac{\phi(x+t,y+l) - D(x,y)\cdot(t,l)}{||(t,l)||}| < \varepsilon$ for all $(t,l) \in X \times X$ such that $||(t,l)|| \leq \delta$, where $D(x,y)$ is the derivative of $\phi(\cdot,\cdot)$ at (x,y). In addition, we can relax this assumption by just requiring $\phi(\cdot,\cdot)$ to be uniformly partial differentiable. See axiom 10 in appendix B.1 for further discussion.

Definition 4. Π *induces a smooth salience presentation if* $\pi \in \Pi$ *implies*

$$\sum_{x,y} \left(u(x) - u(y) \right) f(\sigma(x,y))\pi(x,y) \geq 0,$$

for some salience function $\sigma(\cdot,\cdot)$*, utility function* $u(\cdot)$ *with strictly positive and continuous partial derivatives, and* $f(\cdot) > 0$ *that is strictly increasing and continuous.*

Definition 4 is different from Lanzani (2022) in two respects.[24] First, we propose that salience distortions are characterized by a positive monotonic transformation of the salience level instead of by the salience level itself. Both the rank-based and continuous version of BGS salience are rooted in this fashion. Second, we require the utility function to be differentiable with positive partial derivatives. Both differences are technical, but they enable us to separate salience distortions over outcome pairs from their utility differences using hypotheses on potentially observable choice instead of structural assumptions on utility.[25] We begin our axiomatization with the existence of a utility representation over deterministic outcomes.

Axiom 12 (Deterministic Transitivity)**.** *For every* $x,y,z \in X$*, if* $\delta_{(x,y)} \in \Pi$ *and* $\delta_{(y,z)} \in \Pi$*,* $\delta_{(x,z)} \in \Pi$*. In addition if* $\delta_{(x,y)} \in \widehat{\Pi}$*,* $\delta_{(x,z)} \in \widehat{\Pi}$*.*

Consider function $u : X \to \mathbb{R}$, we say Π induces utility u on X if for every $x, y \in X$, $\delta_{(x,y)} \in \Pi \Leftrightarrow u(x) - u(y) \geq 0$, and $\delta_{(x,y)} \in \widehat{\Pi} \Leftrightarrow u(x) - u(y) > 0$.

Lemma 3. *If a correlation sensitive preference satisfies axiom 1, it induces a continuous utility* $u(\cdot)$ *on X that is strictly increasing.*

Proof. See Appendix B.5.1 □

We are now ready to give the axioms for multidimensional salience theory. The characterization is closely related to the identification of $u(\cdot)$. When there are multiple dimensions, $u(\cdot)$

[24]Lanzani (2022) defines a smooth salience representation as $\pi \in \Pi$ if $\sum_{x,y} \left(u(x) - u(y) \right) \sigma(x,y)\pi(x,y) \geq 0$ for some continuous utility function u.

[25]Both Lanzani (2022) and Köster (2021) assume utility to be linear.

can be quite complex. In practice, studies often impose additional structural assumptions on this utility, such as additive separability, constant elasticity of substitution, or constant relative risk aversion. For now, we assume that $u(\cdot)$ is known and leave its characterization to the last axiom.

Our main axioms characterize all properties in definition 1 except for compatibility. They share a common intuition. On one hand, given the preference $u(\cdot)$ over deterministic outcomes, we can induce the expected utility preference over the lotteries. This preference takes into account the effect of the shape of $u(\cdot)$, while removing the influence of salience. On the other hand, definitions of the salience function regulate its variation along specific directions of outcome changes. Therefore, comparing the deviations between the proposed expected utility and potentially observable choices provide testable axioms for salience.

To simplify subsequent descriptions, we introduce a new notation. Given two outcomes $x,y \in X$, let $|(x,y)| \in X \times X$ be (x,y) if $x \succeq_u y$ and (y,x) otherwise, where $x \succeq_u y$ is the implied deterministic preference according to the assumed utility $u(\cdot)$. Similarly, let $-|(x,y)|$ be (y,x) if $x \succeq_u y$ and (x,y) otherwise. In other words, $|\cdot|$ puts the more preferred outcome in the first the argument. Furthermore, it is also useful for later analysis to introduce $\Pi_{\mathbb{E}(u)}$, the preference set over $\Delta(X \times X)$ induced by the expected utility application of $u(\cdot)$. For instance, $\pi \in \Pi_{\mathbb{E}(u)}$ implies $\sum_x \pi_1(x)u(x) \geq \sum_x \pi_2(x)u(x)$. [26]The strict preference set $\hat{\Pi}_{\mathbb{E}(u)}$ is defined similarly.

Axiom 13 (Upper Ordering)**.** *$\forall x,y,z \in X$ s.t. $x \succeq_u y$, at least one of the following is true.*

1. $\forall p \in [0,1]$ s.t. $\{(y,x),p;(x \vee z,y),(1-p)\} \in \Pi_{\mathbb{E}(u)}$, $\{(y,x),p;(x \vee z,y),(1-p)\} \in \Pi$.

2. $\forall p \in [0,1]$ s.t. $\{(y,x),p;|(y \vee z,x)|,(1-p)\} \in \Pi_{\mathbb{E}(u)}$, $\{(y,x),p;|(y \vee z,x)|,(1-p)\} \in \Pi$.

To understand the two cases in the above axiom, let us consider a two-dimensional environment. For the first case, with $x \succeq_u y$ and $x \vee z \geq x$, the difference between $x \vee z$ and y is more extreme than the difference between x and y. The decision-maker's attention will be drawn to the state under which the first option gives $x \vee z$, and this distortion favors of the first option.

[26]In appendix B.1, we argue that $\Pi_{\mathbb{E}(u)}$ can be approximated by preferences over bivariate joint lotteries with almost identical outcome pairs under each state.

In this case, the axiom states that if the first option were favored on EU grounds, it would be favored on salience grounds as well. The other case has a similar basis.

The axiom for lower ordering follows the same logic.

Axiom 14 (Lower Ordering). *$\forall x,y,z \in X$ s.t. $x \succeq_u y$, at least one of the following is true.*

1. $\forall p \in [0,1]$ s.t. $\{(y,x),p;(x,y\wedge z),(1-p)\} \in \Pi_{\mathbb{E}(u)}$, $\{(y,x),p;(x,y\wedge z),(1-p)\} \in \Pi$.

2. $\forall p \in [0,1]$ s.t. $\{(y,x),p;|(x\wedge z,y)|,(1-p)\} \in \Pi_{\mathbb{E}(u)}$, $\{(y,x),p;|(x\wedge z,y)|,(1-p)\} \in \Pi$.

Our next two axioms govern inclusion and diminishing sensitivity. Since these two properties are straightforward extensions from BGS salience theory, their corresponding axioms share some common features with those of Lanzani (2022). For inclusion, the axiom suggests that compared to EU predictions, decision-makers will behave as if they're paying more attention to the state with larger outcome differences. For diminishing sensitivity, the axiom implies that when facing two states with the same outcome difference but at different outcome levels, decision-makers will focus more on the state in which the outcome levels are closer to zero.

Axiom 15 (Inclusion). *$\forall p \in [0,1]$ and $\forall x,y,z,t \in X$ with $z \geq y > x \geq t$, if $\{(x,y),p;(z,t),1-p\} \in \Pi_{\mathbb{E}(u)}$, $\{(x,y),p;(z,t),1-p\} \in \Pi$.*

Axiom 16 (Diminishing Sensitivity). *For every $x,y \in X$ s.t. $x > y \geq 0$, $\varepsilon > 0$, and $\alpha \in (0,1)$ s.t. $\{(x,y),p;(y+\varepsilon,x+\varepsilon),1-p\} \in \Pi_{\mathbb{E}(u)}$, $\{(x,y),p;(y+\varepsilon,x+\varepsilon),1-p\} \in \Pi$.*

Our next axiom regulates reflection. It suggests that fixing the supports of lotteries to $|(x,y)|$ and $-|(x',y')|$, if the actual preference set, Π, is expanded relative to the expected utility preference set $\Pi_{\mathbb{E}(u)}$, the actual preference set should also be expanded if we change their supports to $|(-x,-y)|$ and $-|(-x',-y')|$.

Axiom 17 (Reflection). *For every nonnegative outcomes $x,y,x',y' \in X$ with $-x' \succeq_u -y'$, if $\forall p \in [0,1]$, $\{|(x,y)|,p;-|(x',y')|,1-p\} \in \Pi_{\mathbb{E}(u)} \Rightarrow \{|(x,y)|,p;-|(x',y')|,1-p\} \in \Pi$, then $\forall q \in [0,1]$, $\{|(-x,-y)|,q;(-y',-x'),1-q\} \in \Pi_{\mathbb{E}(u)} \Rightarrow \{|(-x,-y)|,q;(-y',-x'),1-q\} \in \Pi$.*

We end the main analysis with two technical axioms. They both impose restrictions on preferences regarding lotteries supporting outcome pairs that exhibit minimal utility differences. The first axiom addresses variation in attention. It suggests that decision-makers will behave as if they pay less attention to states in which outcome pairs are almost identical. One important implication from this axiom is that our model doesn't require the utility difference between two outcomes to be perfectly aligned with their salience level. Specifically, even when two outcomes yield an identical utility, they can still be perceived as different.

Axiom 18 (Local Ignorance)**.** *$\forall x, y \in X$ s.t. $x \not\sim_u y$, if for every natural number n we have $\{|(x,y)|, p_n; (x - \frac{e_1}{n}, x), 1 - p_n\} \in \hat{\Pi}_{\mathbb{E}(u)}$, then $\{|(x,y)|, p_n; (x - \frac{e_1}{n}, x), 1 - p_n\} \in \hat{\Pi}$ for all but finitely many n.*

Our last axiom connects the utility function $u(\cdot)$ with preference set Π from a cardinal perspective. It requires the sets Π and $\Pi_{\mathbb{E}(u)}$ to be identical if we restrict them to lotteries supporting outcome pairs with vanishing distances. Together with skew-symmetry of ϕ, axiom 8 implies that when two outcomes, x and y, are equally preferred, the percentage change in the marginal utility of x when y is present must be the same as the percentage change in the marginal utility of y when x is present. Furthermore, axiom 8 imposes limitations on the existence of $u(\cdot)$. Roughly, it requires that local behaviors of $\phi(x,y)$ at $x = y$ to coincide with some utility $u(x)$ representing the preference over deterministic outcomes.[27]

Axiom 19 (Local Identity)**.** *For all $p \in [0,1]$ and $x, h, t \in X$ with $h, t > 0$, we have $\{(x + \frac{h}{n}, x), p; (0, \frac{t}{n}), 1 - p\} \in \Pi$ for all but finitely many $n \in \mathbb{N}$ if and only if $\{(x + \frac{h}{m}, x), p; (0, \frac{t}{m}), 1 - p\} \in \Pi_{\mathbb{E}(u)}$ for all but finitely many $m \in \mathbb{N}$.*

We now state our main representation result.

[27] In appendix B.1, we provide an alternative for axiom 8 addressing the existence of utility function in smooth salience representation. Technically, it implies that the partial derivatives of x, $\frac{\partial \phi(x,y)}{\partial x_i}$, at points $x = y$ must be a vector field.

Proposition 8. *Under assumption 1, a correlation sensitive preference induces a smooth salience representation with utility function, $u(\cdot)$, and some free salience function, $\sigma(\cdot,\cdot)$, if and only if* Π *satisfies axioms 1 to 8.*

Proof. See Appendix B.5.1. □

In addition, compatibility requires that permuting coordinates of outcome pairs doesn't change their salience levels. The following axiom governs this restriction.

Axiom 20 (Compatibility)**.** *For every (x,y) in $X \times X$ s.t. $x \succeq_u y$, $\alpha \in [0,1]$, and permutation τ for $\{1,2,\ldots,n\}$, we have*

$$\{(y,x),\alpha;|(x_\tau,y_\tau)|,1-\alpha\} \in \Pi_{\mathbb{E}(u)} \Leftrightarrow \{(y,x),\alpha;|(x_\tau,y_\tau)|,1-\alpha\} \in \Pi,$$

Corollary 1. *Under assumption 1, a correlation sensitive preference induces a smooth salience representation with utility function $u(\cdot)$, and some salience function, $\sigma(\cdot,\cdot)$, if and only if* Π *satisfies axioms 1 to 9.*

Proof. See Appendix B.5.1. □

As an extension, since we are working with multiple dimenstions, one could imagine that preferences over deterministic payoffs are context-dependent as well. This would add a further layer of context dependence to that analyzed here.[28] Following work from the literature on context-dependent preferences (Koszegi and Szeidl, 2012; Bordalo et al., 2013b; Bushong et al., 2021; Landry and Webb, 2021), in Appendix B.2, we allow decision-makers to have salience functions convoluted with these effects by providing a general formulation that encompasses models of focusing Koszegi and Szeidl (2012), deterministic relative thinking (Bushong et al., 2021), salience for consumer choice (Bordalo et al., 2013b), and pairwise normalization (Landry

[28]For models in this fashion, see Bushong et al. (2021); Köster (2021).

and Webb, 2021).[29] In the end, we provide conditions in a two-option setting under which the difference in context-dependent utilities can serve as the basis of a free salience function. Consequently, these results also indicate that definition 1 is general enough to encompass a large class of models.[30]

2.3 Application: Intertemporal Risky Choice

We now turn to a prominent potential application for multidimensional salience: intertemporal risky choice. In many economic problems, risk and time are intertwined. By considering different time periods as dimensions, our model can explain a variety of behavioral phenomena in the study of intertemporal risky choice. In this section, we present three stylized examples with four predictions, and test their validity using data from previous experiments.

2.3.1 Examples

Our examples consider a market in which there are two (risky) assets: I_1 and I_2 and two periods. For every dollar invested in I_1, it either returns \$0 or \$1 in period 1 while for every dollar invested in I_2, it either returns \$0 or \$(1+r) in period 2 with $1 > r > 0$. The probabilities and correlations between their returns will vary between examples. A decision-maker chooses between two portfolios, F_1 and F_2, over these two assets. Under each state, a portfolio yields a two-period stream, $[x_1, x_2]$. To simplify the exposition, we assume that the decision-maker follows the rank-based multidimensional salience model with an underlying discounted utility function $u([x_1, x_2]) = v(x_1) + \delta v(x_2)$ and $\delta \approx 1$.[31] We denote the decision-maker's salience-based utility for portfolio F, $U(F)$. In each example, the canonical model of Discounted Expected

[29]Such extensions can provide further insights on certain behavioral patterns. For example, in an intertemporal consumption model, by considering each period's payment as a separate attribute, the Bordalo et al. (2013b) salience theory can rationalize the well-known *magnitude effect* (Thaler, 1981; Prelec and Loewenstein, 1991). By analyzing preferences on lotteries over large and small stakes, one can further differentiate these context-dependent preference models with previous ones addressing the *magnitude effect* (Noor, 2009; Ericson and Noor, 2015).

[30]In a relevant work, **?** introduce "categorical thinking model" under which the decision-maker view options or attributes differently according to the categories they belong to. As the results in Appendix B.2 suggest, their model in general cannot serve as a basis unless certain monotonicity conditions across different categories are imposed.

[31]Therefore, we preclude the case of extreme time discounting.

Utility (DEU), predicts no sensitivity of behavior in the setting studied.

Example 1. *Salience-Based Present Bias:*

Suppose that a decision-maker is choosing from two portfolios (F_1, F_2)*:* F_1 *invests a dollar in* I_1 *while* F_2 *invest the dollar in* I_2*. It follows that there are four potential states, and in each state a portfolio yields the two-period stream,* $[x_1, x_2]$*, summarized in the following table:*

Option	*State*			
	HH	*LL*	*HL*	*LH*
F_1	$[1,0]$	$[0,0]$	$[1,0]$	$[0,0]$
F_2	$[0,1+r]$	$[0,0]$	$[0,0]$	$[0,1+r]$

Regardless of how probabilities are assigned to the four states, for relatively small interest rates, r, the salience level ranking of the four states is $\Delta_{LH} > \Delta_{HL} > \Delta_{HH} > \Delta_{LL}$*; the decision-maker disproportionately attends to state LH, where* F_2 *pays* $[0, 1+r]$ *while* F_1 *pays* $[0,0]$.

One central distinction between intertemporal and atemporal allocations is that the future is inherently uncertain whereas the present is plausibly certain (absent exogenous risk). Consequently, risk structures for trade-offs between today and tomorrow are different from tomorrow and the next day. We capture such insight as follows: let I_1*'s return be paid with probability p, while* I_2*'s return is paid with an independent probability* $p \cdot q$*. In this case then the state probabilities in the table above are* $p^2q, (1-p)(1-q), p(1-pq), (1-p)pq$*. Note that* $p = 1$ *corresponds to the trade-off between today and tomorrow, while* $p < 1$ *corresponds to the trade-off between two future dates. Under multidimensional salience, we have that* $\frac{U(F_2)}{U(F_1)}$ *is a decreasing function of p.*[32] *This corresponds to the decision-maker more greatly preferring the portfolio with the more delayed reward if all reward dates are pushed into the future: a Salience-Based Present Bias. The mechanism for the result is that the most salient state, LH, never obtains for* $p = 1$*; while for* $p < 1$ *it does, and it substantively influences the relative preference. When* $p < 1$*, the decision-maker attends to the salient LH risk of getting their second*

[32]Formal derivations of results in this and following examples are presented in Appendix B.3.

payment but not getting their first in a manner conducive to greater patience. As noted above DEU predicts that $\frac{U(F_2)}{U(F_1)}$ *is a constant with respect to p.*

Example 2. *Intertemporal Hedging and Correlation Dependence:*

Suppose that a decision-maker is choosing from two portfolios (F_3, F_4)*:* F_3 *invests a dollar in* I_2 *while* F_4 *evenly splits the dollar between* I_1 *and* I_2*. It follows that there are four potential states, and in each state a portfolio yields the two-dimensional stream summarized in the following table:*

Option	*State*			
	HH	*LL*	*HL*	*LH*
F_3	$[0, 1+r]$	$[0,0]$	$[0,0]$	$[0, 1+r]$
F_4	$[\frac{1}{2}, \frac{1+r}{2}]$	$[0,0]$	$[\frac{1}{2}, 0]$	$[0, \frac{1+r}{2}]$

1. *Intertemporal Hedging : Suppose returns of* I_1 *and* I_2 *are paid independently with an identical probability* $p \in (0,1)$*, then the state probabilities are* $p^2, (1-p)^2, p(1-p), p(1-p)$*. In addition, suppose r is moderate such that the salience level ranking of the four states is* $\Delta_{HL} > \Delta_{LH} > \Delta_{HH} > \Delta_{LL}$*: the decision-maker disproportionately attends to state HL, where* F_4 *pays* $[\frac{1}{2}, 0]$ *while* F_3 *pays* $[0,0]$*. Under multidimensional salience,* $\frac{U(F_3)}{U(F_4)}$ *is increasing in p. Therefore, with* $p = 1$*, the relative preference is more likely to favor* F_3*, while for* $p < 1$ *the relative preference is more likely to favor* F_4*. The mechanism is as follows: for* $p = 1$ *only state HH will obtain and the decision-maker may choose the temporally unhedged portfolio,* F_3*, for return and discounting reasons. When* $p < 1$*, the decision-maker now considers the additional states LH and HL, the latter of which is likely to be more salient for moderate r. State HL corresponds to the decisionmaker receiving nothing from the intertemporally unhedged portfolio,* F_3*, but receiving* $[\frac{1}{2}, 0]$ *from the intertemporally hedged portfolio,* F_4*. This hedging benefit leads to a great relative preference for* F_4*. As noted above DEU predicts that* $\frac{U(F_3)}{U(F_4)}$ *is a constant with respect to p.*

2. *Correlation Dependence: building on the previous example, fix the return probability at* $p = 0.5$ *and change the states' probabilities to* $0.25+\gamma$, $0.25+\gamma$, $0.25-\gamma$, $0.25-\gamma$ *where* $\gamma \in [-0.25, 0.25]$. *In other words,* γ *is a measure for correlation between the assets. In this case,* $\frac{U(F_3)}{U(F_4)}$ *is an increasing function of* γ. *The intuition is similar to Intertemporal Hedging : when* I_1 *and* I_2 *are negatively correlated(*$\gamma < 0$*), hedging between them counters the risk. On the other hand, when the assets are positively correlated(*$\gamma > 0$*), hedging between them can no longer eliminate the risk. As noted above DEU predicts that* $\frac{U(F_3)}{U(F_4)}$ *is a constant with respect to* γ.

Example 3. *Reordering Dependence:*

Suppose that a decision-maker is choosing from two portfolios (F_3, F_4') *where* F_3 *is the same as in the previous example, while the stream payoffs of* F_4' *come from the permutation of* F_4 *summarized in the following table:*

Option	*State*			
	HH	*LL*	*HL*	*LH*
F_3	$[0, 1+r]$	$[0,0]$	$[0,0]$	$[0, 1+r]$
F_4'	$[0,0]$	$[0, \frac{1+r}{2}]$	$[\frac{1}{2}, 0]$	$[\frac{1}{2}, \frac{1+r}{2}]$

Compare utility differences between F_3 *vs.* F_4 *and* F_3 *vs.* F_4' *evaluated when the decision-maker's choice set is* (F_3, F_4) *and* (F_3, F_4')*, respectively. Multidimensional salience predicts that* $U(F_3) - U(F_4) \leq U(F_3) - U(F_4')$. *The underlying reason is that a permutation of one portfolio's streams changes the joint distribution of streams and, hence, changes all relevant salience distortions. When the choice set is* (F_3, F_4)*, the intertemporally hedged portfolio* F_4 *is attractive because it insures against the salient state HL in which* F_3 *pays nothing,* $[0,0]$ *while* F_4 *pays* $[\frac{1}{2}, 0]$. *After the permutation,* F_4' *loses some attractiveness because it cannot insure against the salient state HH, in which* F_4' *itself pays nothing,* $[0,0]$*, while* F_3 *pays* $[0, 1+r]$. *As noted above DEU predicts that* $U(F_3) - U(F_4) = U(F_3) - U(F_4')$. *In addition, note that because* F_4 *and* F_4' *are simple reorderings, any context-independent theory of choice where*

$U(F_4)$ is independent of the choice set or environment in which it is embedded will also imply $U(F_3)-U(F_4)=U(F_3)-U(F_4')$.[33]

Salience-Based Present Bias, *Intertemporal Hedging* , and *Correlation Dependence* all speak to the effects of changing joint distributions' probabilities on decision-makers' behaviors. On the other hand, *Reordering Dependence* is a special correlation manipulation in which states' probabilities stay constant but streams are swapped across states. While there is no data on *Reordering Dependence* in the intertemporal setting (and so a new experiment is presented in Section 4), previous experimental studies using a common Convex Time Budget design (Andreoni and Sprenger, 2012a,b; Miao and Zhong, 2015; Cheung, 2015) implement conditions analogous to the first three predictions. We now review the experimental design and test corresponding predictions of multidimensional salience using these data.

2.3.2 CTB Design and Predictions of Multidimensional Salience

The Convex Time Budget (CTB) design was introduced in Andreoni and Sprenger (2012a) to measure individual time preferences. Subjects are given a budget of $\$m$ and are asked to allocate this budget over two periods, t and $t+k$, at a given gross interest rate, $1+r$. In effect, the task asks subjects to maximize the utility of sooner payment, c_t, and later payment, c_{t+k}, subject to the linear budget constraint

$$(1+r)c_t + c_{t+k} = m.$$

The experimental budget establishes a menu of potential streams $[c_t, c_{t+k}]$, from $[\frac{m}{(1+r)}, 0]$ to $[0, m]$.[34] The CTB design has been widely adopted for the study of time preferences in the lab and the field (for a meta-analysis of the CTB literature, see Imai et al., 2020).

[33] One could, of course, imagine pathological examples where $U(F_4) \neq U(F_4')$ because the individual prefers to win on HH and lose on LL rather than lose on HH and win on LH, but the labeling of the states should be inconsequential.

[34] The term "convex" in CTB derives from the fact that the prior standard Multiple Price List design (Coller and Williams, 1999) common in the experimental literature asked subjects to decide between the two budget end-points: a sooner payment versus a later payment.

In Andreoni and Sprenger (2012b), there are two extensions of the standard CTB, which we name as $MULT$ and $SING$. Under $MULT$, c_t is paid with probability 0.5 while c_{t+k} is paid with probability 0.4. As a result, in $MULT$, the possible streams are $[c_t, c_{t+k}]$, $[c_t, 0]$, $[0, c_{t+k}]$, and $[0,0]$. On the other hand, under $SING$, c_t is paid with probability 1 while c_{t+k} is paid with probability 0.8. Consequently, in this case, the possible streams are $[c_t, c_{t+k}]$, $[c_t, 0]$. $SING$ and $MULT$ correspond to conditions appropriate for examining Salience-Based Present Bias (Example 1), as the former has $p = 1$, the latter has $p = 0.5$, and both have $q = 0.8$ for the additional risks on future payments.

Andreoni and Sprenger (2012b), Cheung (2015), and Miao and Zhong (2015) also implement both the standard risk-free CTB and an additional condition in which c_t is paid with probability 0.5 and c_{t+k} is paid with independent probability 0.5. We term the former condition $CERT$, with only one possible stream associated with a given choice: $[c_t, c_{t+k}]$. We term the latter condition IND, and there are four streams identical to those in $MULT$, but paid equiprobably. $CERT$ and IND correspond to conditions appropriate for examining Intertemporal Hedging (Example 2), as the former has $p = 1$, and the latter has independent payment risk of $p = 0.5$ in each time period.

Miao and Zhong (2015) provide an additional extension of the CTB with two conditions termed POS and NEG. In both of these conditions, payments are subject to a probability of 0.5. In POS either both payments are paid or both payments are not paid. In NEG if the first payment is paid, the second is not and vice versa. In both POS and NEG there are two equiprobable streams associated with a given choice: for POS, they are $[c_t, c_{t+k}]$ and $[0,0]$; for NEG, they are $[c_t, 0]$ and $[0, c_{t+k}]$. POS and NEG correspond to conditions appropriate for examining Correlation Dependence (Example 2), as the former has $\gamma = 0.25$, and the latter has $\gamma = -0.25$, and both have $p = 0.5$ in each time period.

Appendix B.4 formalizes the predictions of multidimensional salience within the CTB environment for sooner allocations c_t^j with $j \in \{SING, MULT, CERT, IND, POS, NEG\}$ representing all relevant conditions.[35] For each of type of comparison, predictions for c_t^j are organized around a threshhold, c^*. The value of c^* depends on parameters of the assumed salience function for our application, i.e., equation (2.2), and the interest rate, r. For reasonable parameterizations of the salience function, $c^* \approx \frac{m}{2(1+r)}$, i.e., exactly half the budget. The predictions can be summarized as follows:

- Salience-Based Present Bias:
  - (1). If $c_t^{SING} \geq c^*$, $c_t^{MULT} \in [c^*, c_t^{SING}]$.
  - (2). If $c_t^{SING} \leq c^*$, $c_t^{MULT} \in [c_t^{SING}, c^*]$.
  - (3). If $c_t^{SING} = c^*$, $c_t^{MULT} = c^*$.
- Intertemporal Hedging :
  - (1). If $c_t^{CERT} \geq c^*$, $c_t^{IND} \in [c^*, c_t^{CERT}]$.
  - (2). If $c_t^{CERT} \leq c^*$, $c_t^{IND} \in [c_t^{CERT}, c^*]$.
  - (3). If $c_t^{CERT} = c^*$, $c_t^{IND} = c^*$.
- Correlation Dependence:
  - (1). If $c_t^{IND} \geq c^*$, $c_t^{NEG} \in [c^*, c_t^{IND}]$.
  - (2). If $c_t^{IND} \leq c^*$, $c_t^{NEG} \in [c_t^{IND}, c^*]$.
  - (3). If $c_t^{IND} = c^*$, $c_t^{NEG} = c^*$.
  - (4). $c_t^{CERT} = c_t^{POS}$.

Note that, with the exception of $c_t^{CERT} = c_t^{POS}$, in each case the predictions of multidimensional salience admit deviations from DEU, which predicts equality across all comparisons. Testing these predictions of multidimensional salience on the aggregate and individual data of Andreoni and Sprenger (2012b), Cheung (2015), and Miao and Zhong (2015) is the focus of the next subsection.

[35]The corresponding prediction for c_{t+k} on the budget constrained is implicitly established.

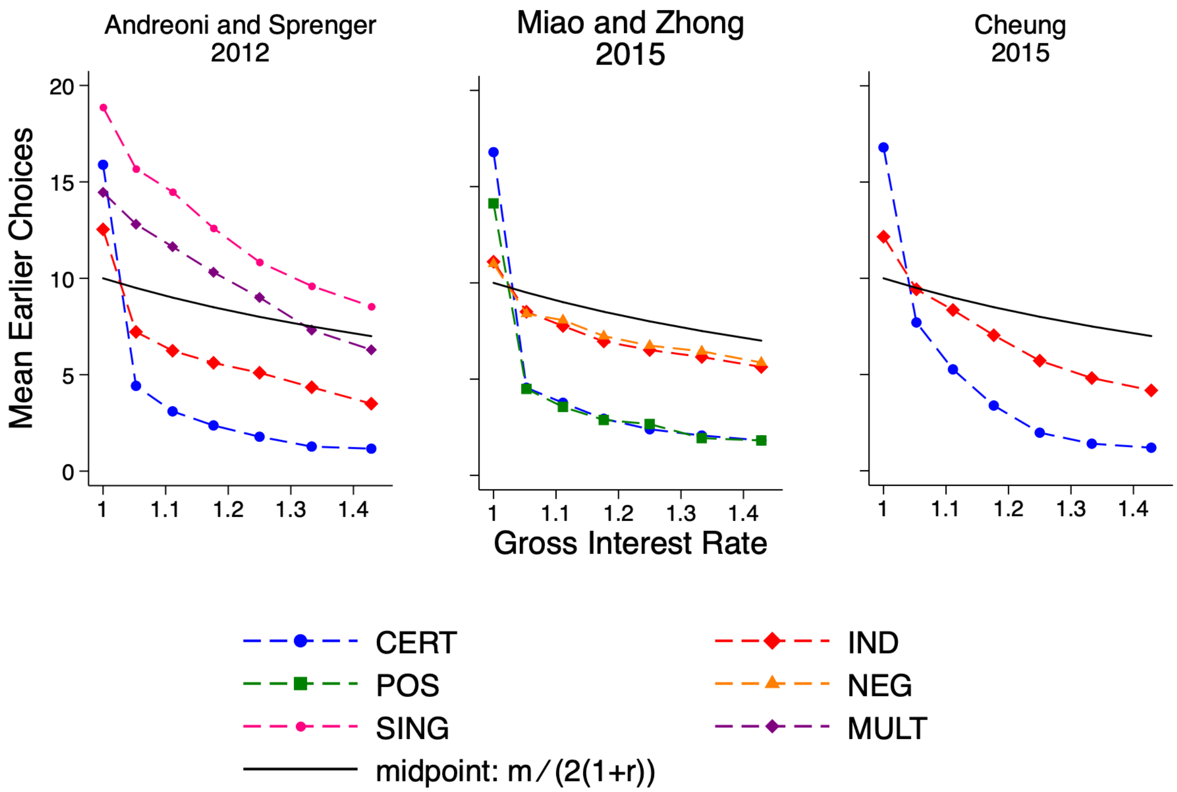


Figure 2.2. Mean Behavior

2.3.3 Prior Experimental Results

Figure 2.2 provides the average data from Andreoni and Sprenger (2012b), Cheung (2015), and Miao and Zhong (2015) in the conditions where $t = 7$ days and $k = 28$ or 35 days. The experimental gross interest rate, $1+r$, is graphed against the average sooner menu choice, c_t^j, for all relevant conditions $j \in \{SING, MULT, CERT, IND, POS, NEG\}$. Figure 2.2 also provides the midpoint of the CTB budget, $\frac{m}{2(1+r)}$, as an approximation of c^* in each panel.

Testing Salience-Based Present Bias: *SING* vs. *MULT*

While DEU predicts that behaviors should be identical between *SING* and *MULT*, multidimensional salience theory generates deviations from the DEU benchmark. In Figure 2.2, on average, all earlier choices under *SING* in Andreoni and Sprenger (2012b) lie above budget midpoints. Consequently, multidimensional salience predicts c_t^{MULT} should be weakly below c_t^{SING} and weakly above c^*. At every interest rate the average $c_t^{MULT} < c_t^{SING}$, but in two cases c_t^{MULT} falls slightly below our approximation of $c^* = \frac{m}{2(1+r)}$.

Figure 2.3, Panel A provides disaggregated individual data. For every observation we calculate $d^j = c_t^j - c^*$, for $j \in \{SING, MULT\}$, assuming $c^* = \frac{m}{2(1+r)}$. d^{SING} is graphed against d^{MULT} for every observation, along with the DEU benchmark, $d^{SING} = d^{MULT}$. Prediction regions for multidimensional salience established in Appendix B.4 are highlighted in purple. Out of 1120 total observations, 872 violate DEU (77.86%). Among violations, 612 of 872 (70.18%) are consistent with the predictions of our theory. The accuracy of prediction for multidimensional salience for these prior data is notably high. As indicated by the area of the prediction regions in Figure 2.3, Panel A, random data would yield only 25% of observations consistent with multidimensional salience.

Testing Intertemporal Hedging : *CERT* vs. *IND*

In all three data sets a clear deviation is observed between the data and the DEU prediction of $c_t^{CERT} = c_t^{IND}$ in Figure 2.2. For the lowest value of $1+r$, c_t^{CERT} lies above the budget midpoint, $\frac{m}{2(1+r)}$; and c_t^{IND} lies below c_t^{CERT}. At precisely the point c_t^{CERT} crosses our approximation for the threshold $c^* \approx \frac{m}{2(1+r)}$, c_t^{IND} passes above c_t^{CERT}. This cross-over in intertemporal demands between the conditions *CERT* and *IND* follows exactly the pattern predicted by multidimensional salience. And, the location of the cross-over is also where the theory predicts it should be at $c^* \approx \frac{m}{2(1+r)}$.

The aggregate data displayed in all three data sets is consistent with multidimensional salience. Figure 2.3, Panel B graphs d^{CERT} against d^{IND} from every observation in the three data sets. The data deviates frequently from DEU: only 857 of 3556 (24.10%) observations correspond to the DEU prediction, $d^{CERT} = d^{IND}$. The remaining 2699 observations deviate from DEU in a manner that is largely consistent with the predictions of multidimensional salience:

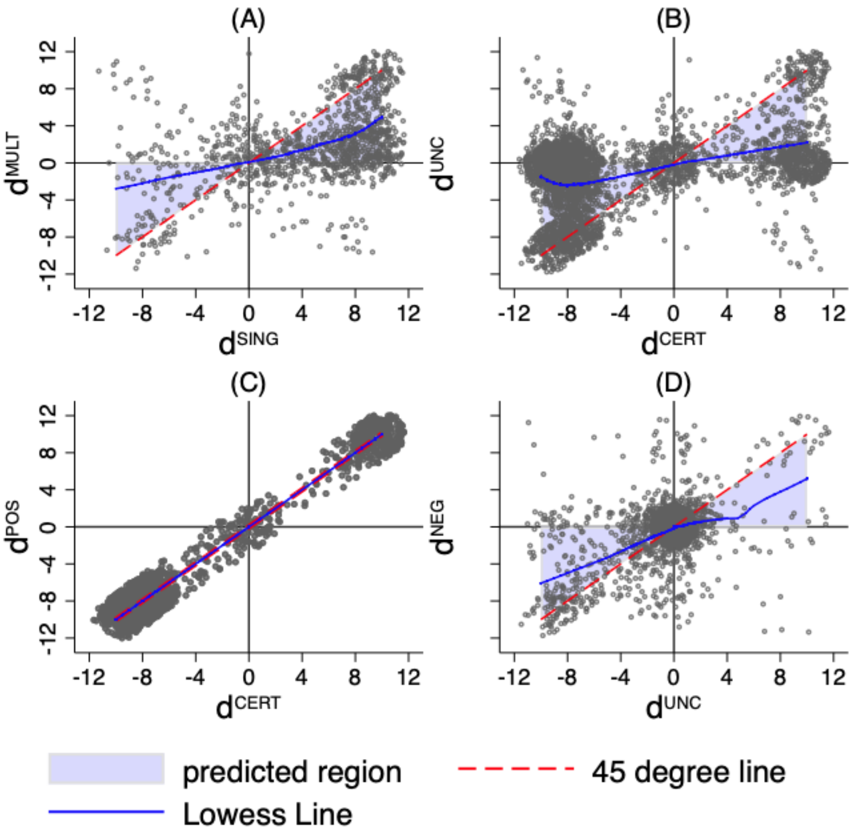


Figure 2.3. Individual Data

Notes: The Figure represents the performance of multidimensional salience theory on individual level data. Predicted areas by Proposition 12 assuming $\beta = 0$. Panel A uses data from Andreoni and Sprenger (2012b) under either *SING* or *MULT*. In panel A, $d^{SING} = c_t^{SING} - \frac{m}{2(1+r)}$ is plotted against $d^{MULT} = c_t^{MULT} - \frac{m}{2(1+r)}$. Panel B uses all data from three previous experiments with $(p_1, p_2) = (0.5, 0.5)$. In panel B, $d^{CERT} = c_t^{CERT} - \frac{m}{2(1+r)}$ is plotted against $d^{IND} = c_t^{IND} - \frac{m}{2(1+r)}$. Panel C uses data from Miao and Zhong (2015) under either *POS* or *CERT* condition. In Panel C, $d^{POS} = c_t^{POS} - \frac{m}{2(1+r)}$ is plotted against d^{CERT}. Panel D uses data from Miao and Zhong (2015) under either *NEG* or *IND* condition. In Panel D, $d^{NEG} = c_t^{NEG} - \frac{m}{2(1+r)}$ is plotted against d^{IND}.

2158 of 2699 (79.96%) DEU deviations are consistent with multidimensional salience. As in panel A, random data would yield only 25% of observations consistent with the theory.

Testing Correlation Dependence: ***POS*** **vs.** ***NEG***

Figure 2.2 also provides data for Miao and Zhong (2015) in conditions *POS* and *NEG*. Aggregate choices again deviate from the DEU prediction, $c_t^{POS} = c_t^{NEG}$. Interestingly, the intertemporal demand for c_t^{POS} is virtually identical to that of c_t^{CERT}, while that for c_t^{NEG} is virtually identical to that of c_t^{IND}. The correspondence between c_t^{POS} and c_t^{CERT} is exactly predicted by multidimensional salience. Figure 2.3, Panel C plots the individual observations of c_t^{POS} against c_t^{CERT}. For 1248 of 1554 (80.03%) observations $c_t^{POS} = c_t^{CERT}$.

The relation between c_t^{NEG} and c_t^{IND} is also predicted by our theory. Figure 2.3, Panel D graphs d^{NEG} against d^{IND}. The data again largely sit in the prediction region for multidimensional salience: 1151 of 1554 (74.07%) observations are consistent with the predictions of multidimensional salience.[36]

Overall within the Miao and Zhong (2015) data, 791 of 1554 (50.09%) observations are consistent with the combined predictions of multidimensional salience: $c_t^{CERT} = c_t^{POS} \geq c_t^{IND} \geq c_t^{NEG} \geq c^*$ or $c^* \geq c_t^{NEG} \geq c_t^{IND} \geq c_t^{POS} = c_t^{CERT}$. In contrast, only 189 of 1554 (12.16%) observations satisfy DEU: $c_t^{CERT} = c_t^{POS} = c_t^{IND} = c_t^{NEG}$. Given 16 possible orderings of these four observations, random data would be consistent with these predictions only 8.33% of the time (conditional on the equality $c_t^{CERT} = c_t^{POS}$).[37]

To quantitatively measure the multidimensional salience effects in prior experiments, in Appendix B.6 we provide structural estimates under the assumed functional form of equation (2). The results are broadly consistent across data sets and show a quantitatively important role for salience. For the c_t^{IND} data, we estimate the most salient state's probability is upweighted from 0.25 to 0.36-0.49 across data sets. These distortions deliver the behavioral phenomena associated with multidimensional salience observed in the data.

2.4 Experimental Test of Multidimensional Salience

The prior section demonstrated that a substantial portion of the DEU deviations documented by Andreoni and Sprenger (2012b), Cheung (2015), and Miao and Zhong (2015) are naturally accommodated by multidimensional salience. These data, however, are also consistent

[36]Strictly speaking, the development of Appendix B.4 requires that decision-makers violating DEU must have $d^{NEG} \neq d^{IND}$ unless $d^{NEG} = d^{IND} = 0$. Overall, 1026 of 1237 (82.94%) observations violating DEU hypothesis are consistent under this more stringent requirement.

[37]Normalize $c \in [0,1]$ and $c^* = 0.5$, the probability is obtained as $2 * \int_0^{0.5} \int_0^{c^{NEG}} \int_0^{c^{IND}} 1 dc^{POS} dc^{IND} dc^{NEG}$.

with alternative theories proposed in those manuscripts and others. For example, Andreoni and Sprenger (2012b) note the possibility of differences between riskless and risky utility functions as an explanation of the data; Miao and Zhong (2015) describe formulations that separate risk aversion from intertemporal elasticities of substitution; and Epper and Fehr-Duda (2015) argue the data are driven by a specific form of intertemporal prospect theory. These alternative theories all posit that the value of an intertemporal lottery depends only upon that lottery's marginal distribution. In contrast, multidimensional salience posits that salience levels are determined by comparing potential outcomes in each state *across* different options in the choice set. Consequently, reordering one option's outcomes, as suggested in example 3 for Reordering Dependence, may provide a basis for distinguishing multidimensional salience from these context-independent alternatives.

An experiment using reordering provides three advantages to distinguish salience theory from other models. First, reordering outcomes keeps each option's marginal distribution constant, so the above context-independent models predict null effects. Second, reordering fixes the number of nonidentical states and their probability in each comparison. Therefore, unlike our tests of Correlation Dependence, which change the number of states that may be realized and their probabilities across comparisons, reordering doesn't introduce the possibility of event-splitting or differential probability distortions across comparisons. (Starmer and Sugden, 1993b; Humphrey, 1995).[38]

We designed our experiment to test the multidimensional salience prediction of Reordering Dependence and assess replicability of Intertemporal Hedging in a simple decision environment. In our experiment subjects faced the three choices presented in Figure 2.4. Each choice was presented as a decision between two options: Option A and Option B. In Choice 1, subjects chose between the deterministic monetary streams $[x_1, x_2] \in \{[\$18, \$2], [\$10, \$10]\}$, where x_1 would be paid one week from the study date and x_2 would be paid four weeks from the

[38]Recent experimental literature find evidence suggesting event-splitting effect may contaminate previous results. See Loewenfeld and Zheng (2021) and a replication for Dertwinkel-Kalt and Köster (2020) for details.

Choice 1: Riskless Payments

Decision

	Event							
	HH		HT		TH		TT	
	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $
Option A	$18	$2	$18	$2	$18	$2	$18	$2
Option B	$10	$10	$10	$10	$10	$10	$10	$10

Choice 2: Independent Risks

Decision

	Event							
	HH		HT		TH		TT	
	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $
Option A	$18	$2	$18	$0	$0	$2	$0	$0
Option B	$10	$10	$10	$0	$0	$10	$0	$0

Choice 3: Re-ordered Risks

Decision

	Event							
	HH		HT		TH		TT	
	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $
Option A	$18	$2	$18	$0	$0	$2	$0	$0
Option B	$0	$0	$10	$10	$0	$10	$10	$0

Figure 2.4. Experimental Decisions

study date. In Choice 2, subjects chose between the temporal lotteries $\{L_{[\$18,\$2]}, L_{[\$10,\$10]}\}$. Four equiprobable states were implemented by flipping two virtual coins with potential outcomes $\{HH, HT, TH, TT\}$. In state HH both x_1 and x_2 would be received; in state HT only x_1 would be received; in state TH only x_2 would be received; and in state TT neither would be received. This mapping from states to outcomes was identical for both options in Choice 2. In Choice 1, the states were presented but the streams were identical in each state for each option.

Under multidimensional salience's Intertemporal Hedging , individuals would be predicted to deviate from DEU in a particular way between Choice 1 and Choice 2. Due to diminishing sensitivity, state TH should receive the greatest salience weight and have its probability distorted upwards. As Option B is superior in state TH, a salience theory decision-maker is predicted to be more likely to choose Option B in Choice 2 relative to Choice 1. Appendix B.7 formalizes this prediction under our proposed functional form assumptions for rank-based multidimensional salience.

In Choice 3, $L_{[\$10,\$10]}$ is re-ordered to construct $L'_{[\$10,\$10]}$ by moving the state payments of HH to HT, HT to TT and TT to HH. $L_{[\$18,\$2]}$ is left unchanged. This re-ordering, while irrelevant in prior non-DEU models, is relevant for our model's prediction of Reordering Dependence. State HH should now receive the greatest salience weight and have its probability distorted upwards. As Option A is superior in state HH, a decision-maker is predicted to be less likely to choose Option B in Choice 3 relative to Choice 2.[39] Appendix B.7 formalizes this prediction under our proposed functional form assumptions for rank-based multidimensional salience.

In addition to these three choices, subjects also faced one attention check between choices. This question asked subjects to make a choice between temporal lotteries *conditional* on a given first coin outcome, H or T. Conditional on a first coin outcome, the choices of the attention check have a dominance relation and so a clearly superior option. Figure 2.5 provides an example attention check question. We also asked two questions without a temporal dimension at the end of the study, which are not discussed here. At the end of the study, one random question was chosen for each subject, the two coins were flipped, and outcomes were conveyed. Subjects were given a minimum payment of \$5 both 1 and 4 weeks from the study date in addition to their experimental payments to overcome differential transaction costs as in Andreoni and Sprenger (2012a). All payments were made via Amazon gift card.

A total of 240 subjects participated in our experiment. Sessions were conducted via Otree (Chen et al., 2016b) using the Zoom video conference interface to simulate a laboratory

[39]Moreover, according to rank-based multidimensional salience, the elimination of Intertemporal Hedging in Choice 3 should be complete. That is, Option A is more preferable in Choice 3 than in Choice 1. This is a sharper prediction than others in the sense that it's only valid under rank-based salience model. On the contrary, other predictions should also hold under a more general class of preferences related to regret theory given our propositions over differences (see definition 1). See Herweg and Müller (2021) for a detailed exposition of the relation between salience theory and regret theory.

	Event							
	HH		HT		TH		TT	
	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $	1 Week $	4 Week $
Option A	$18	$2	$0	$0	$0	$2	$18	$2
Option B	$18	$2	$0	$2	$0	$0	$18	$2

- If HH occurs, OPTION A pays $18 in one week and $2 in four weeks. OPTION B pays $18 in one week and $2 in four weeks.
- If HT occurs, OPTION A pays $0 in one week and $0 in four weeks. OPTION B pays $0 in one week and $2 in four weeks.
- If TH occurs, OPTION A pays $0 in one week and $2 in four weeks. OPTION B pays $0 in one week and $0 in four weeks.
- If TT occurs, OPTION A pays $18 in one week and $2 in four weeks. OPTION B pays $18 in one week and $2 in four weeks.

If Coin 1 lands on H, please make your decision:

○ Option A

○ Option B

If Coin 1 lands on T, please make your decision:

○ Option A

○ Option B

Figure 2.5. Attention Check

experience.[40] Of the 240 subjects, 206 (85.8%) passed the attention checks. In order to examine order effects all six orders of the three choices were implemented. In developing our design, we forecasted that it would be particularly relevant to have Choice 1 first to organize a subject's understanding of the task, and so we oversampled to ensure that roughly half of subjects would receive Choice 1 first. In total 117 subjects (48.9%) faced Choice 1 first, 56 (23.3%) faced Choice 2 first, and 67 (27.9%) faced Choice 3 first. The principle of giving subjects a simple task first to guide their understanding is supported in attention check and choice data. Individuals who receive the deterministic Choice 1 first are somewhat more likely to pass the attention checks (90% vs. 82%, $F_{1,239} = 2.92, p < 0.10$). Individuals who fail the attention checks exhibit choices that are consistent with a random choice benchmark, choosing Option B 51% of the time; those who pass choose Option B 67% of the time ($F_{1,239} = 6.11, p < 0.05$). Given the

[40] Due to technical videoconferencing issues, we exclude one session. In addition, two subjects are excluded because they were not able to follow instructions for the online experiment.

relationship between order, attention, and apparent random choice, we focus on the 105 subjects who received Choice 1 first and passed the attention checks as our primary sample. We use the remaining subjects to examine order effects.

Table 2.1 presents the results of our study as linear probability models with standard errors clustered at the individual level. In columns (1) and (2) we examine our primary sample with and without those subjects who failed the attention checks. For those who passed the attention checks, 55% of subjects chose Option B, [\$10, \$10], in Choice 1. Moving to Choice 2, with common independent risks, 75% of subjects chose Option B, $L_{[\$10,\$10]}$. This 20%-age point change in responses represents a significant deviation from DEU relative to Choice 1, $F_{1,104} = 9.69, p < 0.01$. The direction of this effect is consistent with the predictions of Intertemporal Hedging in our model and reproduces the findings of Andreoni and Sprenger (2012a), Cheung (2015), and Miao and Zhong (2015) in a different choice environment.

Moving to Choice 3, 64% of subjects choose Option B, $L'_{[\$10,\$10]}$. The difference in choice proportions between Choice 2 and Choice 3 identifies the extent of Reordering Dependence in our study. Overall, this effect is measured at 11%-age points in our primary sample and is statistically significant at the 5% level ($F_{1,104} = 4.95, p < 0.05$). Under our theory, the salience of state TH in Choice 2 increases the attractiveness of Option B, and the salience of state HH in Choice 3 decreases the attractiveness of Option B. This Reordering Dependence is precisely what is observed in our primary sample.[41] Overall the data in our primary sample reject DEU ($F_{2,104} = 5.39, p < 0.01$), and deliver results closely consistent with multidimensional salience.

In columns (3) and (4) of Table 2.1 we analyze the results for subjects who faced Choice 2 or Choice 3 first. While the order matters little on average for Choice 1 and Choice 2, behavior in Choice 3 appears quite sensitive to the order. Without prior deterministic choices, subjects seem somewhat more likely to choose Option B in Choice 3. While the effects of Intertemporal

[41] As discussed above, in our experiment the predictions of multidimensional salience are directional. For completeness, we include p-values of the various one-sided tests: 1) for the null hypothesis that more subjects choose Option B in Choice 1 than in Choice 2, $p < 0.01$; 2) for the hypothesis that more subjects choose Option B in Choice 3 than in Choice 2, $p = 0.014$; for the hypothesis that more subjects choose Option B in Choice 1 than in Choice 3, $p = 0.091$.

Table 2.1. Experimental Results

	Primary Sample		Alternate Orders		
	(1)	(2)	(3)	(4)	(5)
	Dependent variable: Chose Option B				
Choice 2	0.17	0.20	0.11	0.18	0.17
	(0.06)	(0.06)	(0.05)	(0.06)	(0.06)
Choice 3	0.06	0.09	0.16	0.24	0.06
	(0.06)	(0.06)	(0.05)	(0.06)	(0.06)
Alternate Order					0.03
					(0.07)
Alternate Order × Choice 2					-0.06
					(0.08)
Alternate Order × Choice 3					0.10
					(0.08)
Failed Attention Check					-0.17
					(0.06)
Constant (Choice 1)	0.56	0.55	0.57	0.55	0.57
	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)
Intertemporal Hedging (H_0: Choice 1 = Choice 2):	$F_{1,116} = 7.50$	$F_{1,104} = 9.69$	$F_{1,122} = 4.79$	$F_{1,100} = 10.35$	$F_{1,239} = 7.51$
	$(p < 0.01)$	$(p < 0.01)$	$(p = 0.03)$	$(p < 0.01)$	$(p < 0.01)$
Reordering Dependence (H_0: Choice 2 = Choice 3):	$F_{1,116} = 5.28$	$F_{1,104} = 4.95$	$F_{1,122} = 1.05$	$F_{1,100} = 1.28$	$F_{1,239} = 5.29$
	$(p = 0.02)$	$(p = 0.03)$	$(p = 0.31)$	$(p = 0.26)$	$(p = 0.02)$
DEU (H_0: Choice 1 = Choice 3):	$F_{2,116} = 0.92$	$F_{2,104} = 1.80$	$F_{2,122} = 8.82$	$F_{2,100} = 15.61$	$F_{2,239} = 0.92$
	$(p = 0.34)$	$(p = 0.18)$	$(p < 0.01)$	$(p < 0.01)$	$(p = 0.34)$
DEU (H_0: Choice 1 = Choice 2 = Choice 3):	$F_{2,116} = 4.65$	$F_{2,104} = 5.39$	$F_{2,122} = 4.53$	$F_{2,100} = 8.41$	$F_{2,239} = 4.66$
	$(p = 0.01)$	$(p < 0.01)$	$(p = 0.01)$	$(p < 0.01)$	$(p = 0.01)$
Order Effect (H_0: Alternate Order and Interactions = 0):					$F_{3,239} = 2.22$
					$(p = 0.09)$
# Observations	351	315	369	303	720
# Clusters	117	105	123	101	240
Attn Check Failure Removed	No	Yes	No	Yes	No

Notes: Ordinary least squares regressions. Standard errors clustered at individual level in parentheses. Hypotheses tested as restrictions on regression coefficients.

Hedging and overall deviations from DEU are maintained for these alternate orders, we no longer find empirical support for Reordering Dependence. In column (5) of Table 2.1, we include all of the data and account for the correlation between order, failure of the attention checks, and choice in multiple regression. The results are largely unchanged: Intertemporal Hedging and Reordering Dependence are observed in our primary sample leading to an overall rejection of DEU ($F_{2,239} = 4.66, p = 0.01$), and an order effect that borders on statistical significance is estimated ($F_{3,239} = 2.22, p = 0.09$)

Our results indicate support for the multidimensional salience, but they also indicate some important sensitivity with respect to the context of choice. In our primary design where subjects face a deterministic choice over streams prior to choosing between temporal lotteries, subjects validate the predictions of Intertemporal Hedging and Reordering Dependence. However, if subjects do not face a deterministic choice first, their choices over temporal lotteries are more consistent across conditions, eliminating Reordering Dependence. We suspect that subjects' confusion may be at play in producing this order effect as individuals are more likely to fail attention checks in these alternate orders, and such attention failures are linked to apparent randomness in choices.

2.5 Discussion and Conclusion

Multidimensional risky choice is a common decision environment. We extend the seminal (Bordalo et al., 2012) (BGS) salience model to this environment to provide a theory of multidimensional salience. We then provide examples for predictions of multidimensional salience in a canonical decision environment: intertemporal risky choice. We show the model deviates from Discounted Expected Utility (DEU) in several intuitive ways, including delivering a form of Salience-Based Present Bias and Intertemporal Hedging, a disproportionate willingness to smooth allocations when facing risk. We take these predictions to prior data sets and demonstrate robust DEU deviations consistent with multidimensional salience 70-80% of the time. To distinguish multidimensional salience from context-independent alternatives, we also provide a novel experiment of a prediction called Reordering Dependence; simply swapping state payments for one option in a choice set. We show deviations from DEU and other context-independent theories, consistent with multidimensional salience.

Our theoretical development provides useful methodology for the analysis of multidimensional risky choice, bridging prior applications of salience in either mutildimensional deterministic choice or one-dimensional risky choice. This permits researchers to analyze salience in richer, potentially more ecologically relevant choice environments. Though such environments are fundamentally complex, we show that salience-based analysis is tractable. We illustrate conditions under which analysis is as straightforward as BGS when one can reduce the dimensionality of options into scalars. This simplified formulation is used in our analysis of prior and new data, and is capable of capturing a number of meaningful behaviors.

Within our formulation attention is drawn to states of nature where multidimensional options are perceived to differ most substantially. One could imagine cases where the multidimensional objects themselves would be subject to attention effects, even with only one state of nature. For example, when considering the deterministic options, $[x_1, x_2]$, of $[100, 0]$ and $[50, 50]$, dimension x_2 may draw more attention. In Appendix B.2, we consider such an extension incorporating salience or other attentional forces, such as focusing or relative thinking, within dimensions (Koszegi and Szeidl, 2012; Bushong et al., 2021). This extension demonstrates that even with such forces within states, the predictions of multidimensional salience for risky choice are generally maintained.

Multidimensional salience makes a number of interesting predictions that could be explored in future work. For example, the prediction of Intertemporal Hedging is closely related to a form of precautionary savings generated by the model. In Appendix B.3, we provide a corresponding example. Standard rationalizations of precautionary savings have been developed which associate "prudent" behavior with the sign of third derivative of an expected utility function (Leland, 1968; Kimball, 1990; Eeckhoudt and Schlesinger, 2006). Multidimensional salience delivers prudent behavior without appeal to higher order derivatives, and so can potentially provide an alternative mechanism for precautionary savings motives. Behavioral applications in areas like retirement savings and human capital formation may be generated from this observation.

Chapter 2, in full, is a working paper coauthored with Professor Charles Sprenger. The dissertation author was the primary author of this paper.

Chapter 3

A Nonparametric Test for Cumulative Prospect Theory

Abstract

As a leading non-expected utility model, Cumulative Prospect Theory (CPT) rationalizes wide range of deviations from expected utility theory while keeps most of its theoretical axioms. This project proposes an experiment to jointly analyze the two major factors in CPT: probability weighting function and rank dependence. Specifically, under a nonparametric framework, we test the validity of rank dependence and estimate the first-order derivative of probability weighting functions.

3.1 Introduction

Modeling individual's behaviors under uncertainty has long been a fundamental task in economic studies. Under the paradigm of objective uncertainties, expected utility theory (EUT) serves as a central foundation to various economic analyses across most fields. While EUT provides a elegant mathematical tool for normative analysis, numerous experimental studies, such as (Allais, 1953), bring evidence of systematically violations from EUT predictions and cast doubts on its descriptive validity. In response to these issues, researchers have proposed various modifications and extensions to the classical EUT framework. Among these, cumulative prospect theory (CPT), introduced by Tversky and Kahneman (1992), has emerged as one of the most influential models addressing deviations from EUT. CPT is promising for both theoretical and empirical reasons. Theoretically, CPT maintains a robust mathematical foundation by only relaxing the highly debated and criticized *independence* axiom of EUT. Further, CPT can rationalize a wide span of empirical regularities including health economics (Bleichrodt and Pinto, 2000),finance (Barberis et al., 2001), and environmental economics (Heutel, 2019).

To describe the essence of CPT, let us consider the following example. Suppose the decision maker is choosing between a one dollar cash \$1 and a lottery L, which is denoted by $(p : \$X; q : \$Y; 1-p-q : \$Z)$ with $X > Y > Z \geq 0$, $p, q > 0$, and $p+q < 1$. In this case, if the decision maker chooses lottery L, she will receive \$X with probability p, \$Y with probability q, and \$X with probability $1-p-q$. CPT posits that the decision maker's preference for both the one-dollar cash \$1 and lottery L can be summarized by a functional $V(\cdot)$ such that $V(\$1) = u(1|r)$ and

$$V(L) = \pi(p)u(X|r) + \Big(\pi(p+q) - \pi(p)\Big)u(Y|r) + \Big(1 - \pi(p+q)\Big)u(Z|r) \tag{3.1}$$

Following from CPT, the decision maker chooses lottery L if and only if $V(L) \geq V(\$1)$.

There are three central components in the presentation functional $V(\cdot)$: the nonlinear probability weighting function $\pi(\cdot)$, rank dependence, and the intrinsic utility function $u(\cdot|r)$. First, $\pi : [0,1] \to [0,1]$ is a strictly increasing and bijective function. It predicts that given objective probability p, instead of perceiving it directly, the decision maker distorts it to $\pi(p)$ and treats the probability as if it was $\pi(p)$. Second, rank dependence suggests that decision weights of outcomes depend not only on their objective probabilities but also on the ranking of their magnitudes. For instance, in the above example, while the lottery offers \$Y with probability q , the decision weight putted on \$Y is not $\pi(q)$. Instead, it is the weight difference between getting at least $\$Y$ and receiving strictly higher than $\$Y$. Third, $u(\cdot|r)$ presents the decision maker's preference over deterministic monetary outcomes with an exogenously given reference point r.

While the probability weighting function, utility function, and rank dependence collectively determine the decision-maker's risk attitude, few studies have systematically investigated these factors within a unified framework (see section 3.2 for a review). The lack of such analysis imposes certain challenges for CPT. For instance, the effect of rank dependence is closely related to the shape of the probability weighting function. Consequently, it is difficult to judge the strength of rank dependence without estimations of the probability weighting function. Furthermore, since preference parameters are rarely stable across different paradigms, testing these factors separately undermines the credibility of CPT as a robust model for predictions. Finally, given the interdependent characteristics of CPT, it is unclear to what extent changes in one aspect can influence the performance of others.

In this chapter, we design a novel experiment to provide a measurement for probability weighting function and test for rank dependence while controlling effects from utility function. [1] The estimations only require a structural assumption on smoothness: for CPT preference we assume that both $\pi(\cdot)$ and $u(\cdot|r)$ have strictly positive and continuous derivatives.

We now describe the intuition of our experiment. Given a lottery $L = (p : \$X; q : \$Y; 1 - p - q : \$Z)$, we can decompose it into two components: winning *at least* \$$Y$ with in total of $p+q$ probability and winning \$$Z$ with probability $1-p-q$. Following the rationale from Choquet integration (Choquet, 1954; Schmeidler, 1989), so long as the cumulative probability of winning at least \$$Y$ remains constant, the individual probabilities of winning higher amounts *do not* affect the decision weight put on winning \$$Z$. Consequently, for lottery L, keeping $p+q$ constant, the marginal effect of increasing probability of winning \$$X$ on lottery L's value is linear to the first order derivative of probability weighting function $\pi(\cdot)$ and is independent with \$$Z$.[2] On the other hand, still keeping $p+q$ constant, the marginal effect of increasing amount \$$Z$ on lottery L's value is independent from the probability of winning X. Therefore, the ratio between these two marginal effects, their marginal rate of substitution (*MRS*), is linear to the first order derivative $\pi'(\cdot)$. Furthermore, the ratio between *MRS* at different probabilities of winning \$$X$ cancels the marginal effect from increasing \$$Z$. To estimate *MRS*, we use a technique that is similar to "equalizing reduction" from Bernheim and Sprenger (2020). Consider lottery

[1]The experiment can also give a measurement for the intrinsic utility function $u(\cdot|r)$, but this is not the focus of current project.

[2]According to equation 3.1, $\frac{\partial V}{\partial p}|_{p+q=k} = \pi'(p)\left(u(x|r) - u(y|r)\right)$.

$L' = (p - \Delta : \$X; q + \Delta : \$Y; 1 - p - q : \$Z + k)$ with $\Delta, k > 0$. Fixing Δ, for each decision maker there is a k such that L and L' are indifferent. For small Δ, $\frac{k}{\Delta}$ provides an approximation for the *MRS*. Notice that, estimations on the first derivative provides sufficient information on the probability weighting. For instance, in the absence of nonlinear probability weighting, the ratio of $\pi'(\cdot)$ should constantly be one. For a probability weighting function with the "reverted-S" shape, the ratio defined by $\frac{\pi'(p)}{\pi'(0.5)}$ should roughly be a U-shape across different values of p.

For our test of rank dependence, notice that there is another way to decompose lottery L: winning a large amount \$$X$ with probability p and winning some amount strictly smaller than \$$X$ with probability $1 - p$. Keeping the probability of winning \$$X$ constant, by a similar argument, the *MRS* between the marginal effect of increasing amount of \$$X$ and of increasing probability of winning \$$Y$ is linear in $\pi'(p+q)$.[3] Therefore, rank dependence has strong predictions for these two different types of *MRS* at specific probabilities p and q.

This chapter organizes as follows. Section 3.2 provides a brief literature review on related literatures testing CPT. Section 3.3 lays out the theoretical background. Section 3.4 introduces the experimental design in detail. Lastly, section 3.5 illustrates powers of our hypotheses using simulations.

3.2 Literature Review

Our project is relevant to a large literature discussing rank dependent utility theories. Wu (1994) tests *ordinal independence* (Green and Jullien, 1988), which roughly states that for every pair of lotteries sharing a common right tail, identically alternating that tail for the two lotteries doesn't affect the preference. More recently, Machina (2009) provides a thought experiment for rank dependent preferences under the subjective uncertainty framework. Machina (2009) uses *tail-separability* to address the fact that changing outcome magnitudes in either left or right tail

[3] $\frac{\partial V}{\partial q}|_{p=p} = \pi'(p+q)\Big(u(y|r) - u(z|r)\Big)$ and $\frac{\partial V}{\partial X} = \pi(p)u'(X|r)$.

without interfering overall rankings does not affect preference under rank dependent utility.[4] We use a parallel rationale that changing conditional distributions or outcomes within one tail (without alternating rankings) doesn't affect value of the other. We then take an additional step by realizing the fact that the trade-off rate between values in two tails provides measures that are linear to either the increment rate of decision weights or the marginal utilities of outcomes. Other related works include Wakker et al. (1994) and Weber and Kirsner (1997) in which they test the descriptive validity of independence axiom and comonotonic independence axiom. Diecidue and Wakker (2001) provide a psychological intuitive argument for rank dependence. Birnbaum (2008) compares the performances of CPT and configural weighted models.

Our work also speaks to the literature on measuring nonlinear probability weighting. Tversky and Fox (1995) estimates probability weighting using certainty equivalence assuming linear utility.[5] Wu and Gonzalez (1996) nonparametrically test the convexity/concavity of probability weighting at different probability levels and provide a structural estimation for both probability weighting and utility function.[6] Abdellaoui (2000) proposes a two-stage experiment to nonparametrically estimate the shape of utility function and probability weighting in both gains and losses domains. Our method can be treated as a "local version" of their experiment. Moreover, employing lotteries with three outcomes allow us to integrate their two stages into a uniform question form.

This paper differs from aforementioned researches in three aspects. First, our experiment provides a uniform framework to simultaneously investigate the validity of rank dependence and measure of probability weighting. In this way, we minimize the problem brought from preference alternations across different settings. Second, under a single environment, we eliminate or control

[4] Since the analysis in Machina (2009) works on subjective uncertainty, the outcomes are associated with "events" instead of objective probabilities. The ranking should be considered as one over states based upon the outcome magnitudes in each state.

[5] Specifically, Tversky and Fox (1995) denote $C(x,A)$ the certainty equivalence of the lottery in which one wins x dollars if event A happens. In case $C(x,A) = y$, under linear utility the decision weight on the probability of event A is $\frac{y}{x}$. They justify linear utility by showing that $2C(75,A)$ and $C(150,A)$ are not significantly different.

[6] In addition, Gonzalez and Wu (1999) nonparametircally estimate the shape of probability weighting and utility function by eliciting certainty equivalents for lotteries with different outcomes and probabilities.

correlations among probability weighting, utility function, and rank dependence. Therefore, we can provide independent analysis regarding each subject. Third, our measurement of probability weighting is novel in the sense that it directly reflects the first order derivative at different probability points. As we mentioned in the introduction, information on the first order derivative is sufficient to analyze properties of the weighting function itself. Furthermore, this information provide a direct investigation over debates on whether the probability weighting function is approximately linear in the interior of $[0,1]$.[7]

Two recent works (Bernheim and Sprenger, 2020; Bernheim et al., 2022) are closely related to ours. They consider the same lottery L as we described in equation 3.2 and elicit *MRS* between outcome $\$Y$ and $\$Z$. They name this method "equalizing reduction". By changing the magnitude of $\$X$, as the rank between $\$X$ and $\$Y$ alternates, CPT predicts a discontinuity in *MRS*.[8] Current work has two major differences. First, in addition to test rank dependency, we measure local curvatures of probability weighting function while Bernheim and Sprenger (2020); Bernheim et al. (2022) estimate the average change rates of $\pi(\cdot)$ within several probability regions.[9] Second, and more importantly, while our hypothesis states that rank dependence results in no difference in *MRS* ratios, "equalizing reduction" investigates the existence of discontinuity. The magnitude of the discontinuity, however, depends on the decision weight $\frac{\pi(p+q)-\pi(p)}{\pi(q)}$. Consequently, the shape of the decision weight function $\pi(\cdot)$ can lead to small discontinuity.[10] Further, heterogeneous weighting functions can affect signs of discontinuity.

[7]Tversky and Fox (1995) provide an argument for *boundary subadditivity*, Camerer (1992) shows that the indifference curve in the interior of Machina's Triangle is fairly linear, and Andreoni and Harbaugh (2009) suggest that EUT performs reasonable well for probabilities away from boundaries. On the other hand, Wu and Gonzalez (1996) suggest that a nonlinear probability weighting function fits the data better.

[8]Importantly, like the current method, "equalizing reduction" also provides a nonparmaetric test by only requiring the utility function to be locally stable. This feature contrasts "equalizing reduction" from other earlier work trying to elicit *MRS* between different outcomes (Diecidue et al., 2007)

[9]In principle, by refining parameter changes among experimental tasks, "equalizing reduction" also can estimate the curvature of $\pi(\cdot)$. However, for this measuring purpose, the current method is more data efficient.

[10]As an artificial illustration, in Bernheim and Sprenger (2020), they estimate that $\ln(\frac{\pi(0.7)-\pi(0.4)}{0.3}) - \ln(\frac{\pi(0.9)-\pi(0.7)}{0.2}) = -0.21$ with standard error 0.06. To get an estimate about $\ln(\frac{\pi(0.7)-\pi(0.4)}{\pi(0.9)-\pi(0.6)})$, let's use the functional form $\pi(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}}$. Then, it is reasonable to assume that γ is between 0.73 and 0.87 (two numbers are both within the 95% confidence interval), so $\ln(\frac{\pi(0.7)-\pi(0.4)}{\pi(0.9)-\pi(0.6)})$ is between -0.22 and -0.09. Consider the situation

These elements may post challenges on the statistical power for “equalizing reduction”.

3.3 Theoretical Background

Let $(p:\$X; q:\$Y; 1-p-q:\$Z)$ with $X > Y > Z \geq 0$ denote the objective lottery that gives \$X with probability p, \$Y with probability q, and \$Z with probability $1-p-q$. Hence under CPT, the preference functional is

$$\pi(p)u(X) + \Big(\pi(p+q) - \pi(p)\Big)u(Y) + \Big(1 - \pi(p+q)\Big)u(Z),$$

where $\pi(\cdot)$ is the probability weighting function, and $u(\cdot)$ is the utility function from monetary outcomes.[11] Through out this project, we assume that both $\pi(\cdot)$ and $u(\cdot)$ have strictly positive and continuous derivatives. Fixing payments X, Y, and probability $p+q$ to be some constant k between 0 and 1, the trade-off between p and Z is an additive separable function:

$$U(p,Z) = \underbrace{\pi(p)\Big(u(X) - u(Y)\Big)}_{\text{term of } p} + \underbrace{\Big(1 - \pi(k)\Big)u(Z)}_{\text{term of Z}} + \underbrace{\pi(k)u(Y)}_{\text{constant}}. \tag{3.2}$$

The corresponding *MRS* is

$$MRS(p,Z) = -\frac{\pi'(p)\Big(u(X) - u(Y)\Big)}{\Big(1 - \pi(k)\Big)u'(Z)}$$

in which the previous functional form doesn’t fit well with small probabilities and that the true $\pi(0.3)$ lies in the middle of $\pi(0.9) - \pi(0.6)$ and $\pi(0.7) - \pi(0.4)$. In case γ is around 0.87, we can have $\ln(\frac{\pi(0.9)-\pi(0.6)}{\pi(0.3)}) \approx 0.065$ and $\ln(\frac{\pi(0.7)-\pi(0.4)}{\pi(0.3)}) \approx -0.028$, which are both in estimated 95% confidence intervals in Bernheim and Sprenger (2020) (table III) and unlikely to be rejected.

[11] We can generalize the utility function by including a reference point, that is $u(\cdot|r)$. Our current results stay valid as long as r is fixed.

Now, consider the ratio between two *MRS* measured at two points with different p and identical Z:

$$\frac{MRS(p_1,Z)}{MRS(p_2,Z)} = \frac{\pi'(p_1)}{\pi'(p_2)},$$

similarly for the pair with identical p and different Z, we have:

$$\frac{MRS(p,Z_1)}{MRS(p,Z_2)} = \frac{u'(Z_2)}{u'(Z_1)}.$$

Therefore, by varying p we obtain information on the shape of probability weighting function while by varying Z we obtain information on the shape of individual utility.

So far we discussed the potential of using *MRS* and additive separable functional structure to identify both probability weighting and utility function up to affine transformations. However, *rank dependence* is left out of discussion. We now show that by altering the format slightly, we obtain a test for *rank dependence*. Consider instead of varying p and Z while holding $p+q$ constant, we now vary q and X while holding p constant. Rewrite $U(p,Z)$ in equation 3.2 as

$$V(q,X) = \underbrace{\pi(p)u(X)}_{\text{term of X}} + \underbrace{\pi(p+q)\Big(u(Y)-u(Z)\Big)}_{\text{term of } q} + \underbrace{u(Z)-\pi(p)u(Y)}_{constant}. \tag{3.3}$$

The corresponding *MRS* is

$$MRS(q,X) = -\frac{\pi'(p+q)\Big(u(Y)-u(Z)\Big)}{\pi(p)u'(X)}.$$

Its ratio between two different probability q is

$$\frac{MRS(q_1,X)}{MRS(q_2,X)} = \frac{\pi'(p+q_1)}{\pi'(p+q_2)}.$$

This gives us an opportunity to test rank dependence. For instance, by fixing z and manipulating p, we observe data from some probability points p_1 and p_2. Next, when switching to manipulate q and X, we fix the chance of getting the highest amount X at 15% and observe data from $q_1 = p_1 - 0.15$ and $q_2 = p_2 - 0.15$. According to CPT, we have a null hypothesis:

$$\frac{MRS(q_1, X)}{MRS(q_2, X)} = \frac{MRS(p_1, Z)}{MRS(p_2, Z)}.$$

Preceding analyses are based solely on the information from *MRS*. Therefore, any monotonic transformations of $U(p,Z)$ and $V(q,X)$ shares identical *MRS*. Nevertheless, recalling that both functional forms are additive separable. As a result, monotonic transformations that are additive separable can only be affine transformations. Therefore, the results in this section remain valid if we replace $U(p,Z)$ and $V(q,X)$ with their monotonic transforms.

3.4 Experimental Design and Hypotheses

Our experiment has two treatments. (p,Z) treatment collects *MRS* information following equation 3.2, and (q,X) treatment collects *MRS* information following equation 3.3. To elicit *MRS*, both treatments adopt multiple price lists (Holt and Laury, 2002). In (p,Z) treatment, the price lists reveal the monetary bonus over the smallest prize \$$Z$ that is equivalent to a 5% bonus of receiving the highest payment \$$X$ instead of the middle payment \$$Y$. In (q,X) treatment, the price lists induce the bonus over \$$Y$ that is equivalent to a 5% bonus of receiving \$$Y$ rather than the lowest payment \$$Z$. To simplify the tasks, we enforces a unique switching point. At the beginning of the experiment, subjects are randomly assigned into two groups. Subjects in group 1 complete (p,Z) treatment first while group 2 subjects complete (q,X) treatment first.

In (p,Z) treatment, subjects face two options :

$$\text{Option A: } (\$10, p - 0.03; \$7, 0.95 - p + 0.03, \$0.5 + e, 0.05),$$

where e ranges from \$0 to \$5 with 25 cents per increment, and

$$\text{Option B: } (\$10, p+0.02; \$7, 0.93-p, \$0.5, 0.05).$$

The labeling of options A and B are randomized at subject-level.[12] An example task with $p = 0.8$ is illustrated in panel A of figure 3.1. In the task presentation, the probabilistic differences between options A and B are highlighted in blue while the monetary differences are highlighted in red. Subjects decide at what value of e she would like to switch from option B to A, and they inform the decisions by clicking the "switch" button on the row of the corresponding e value. Once subjects clicked, the option they will receive for different values of e are highlighted as shown in figure 3.1.

For every subject, we elicit their preferences at seven different values of p: $p_i \in \{0.05, 0.2, 0.35, 0.5, 0.65, 0.8, 0.9\}$. Consequently, we can approximate $MRS(p_i, 0.5)$ by $\frac{e_i}{0.05}$ where e_i is subjects switching value of p_i. And hence, we obtain $\frac{\pi'(p_i)}{\pi'(0.5)} \equiv \frac{MRS(p_i, 0.5)}{MRS(0.5, 0.5)}$ at $i = 1, \ldots, 7$. If subjects process a linear weighting function, we should observe that $\frac{\pi'(p_i)}{\pi'(0.5)} = 1$ for all i. We summarize the hypothesis below.

Hypothesis 1. *Subjects have a linear weighting function:* $e_i = e_4$ *for all* $i \neq 4$.

In (q, X) treatment, the two options are

$$\text{Option A: } (\$15+v, 0.15; \$3, q-0.03, \$0, 0.88-q),$$

where v also ranges from \$0 to \$5 with 25 cents per increment, and

$$\text{Option B: } (\$15, 0.15; \$3, q+0.02, \$0, 0.83-q).$$

[12] That is, for half of the subjects option A is $(\$10, p; \$7, 0.95-p, \$0.5, 0.05)$ while option B is $(\$10, p-0.05; \$7, 0.95-p+0.05, \$0.5+e, 0.05)$. However, to reduce complexity, for each subjects either she is deciding when to switch from option A to B in all tasks, or she is deciding when to switch from option B to A in all tasks.

The task presentation of (q,X) treatment is similar to that of (p,Z) treatment. panel B of figure 3.1 presents an example task with $q = 0.05$.

We elicit every subject's preference at $q_i \in \{0.05, 0.2, 0.35, 0.5, 0.65, 0.75\}$. Under the current parameters $q_i = p_i - 0.15$, $i = 1, \dots, 6$. Since $\frac{\pi'(q_i + 0.15)}{\pi'(0.35 + 0.15)} = \frac{\pi'(p_i)}{\pi'(0.5)}$, we shouldn't observe any difference between the MRS ratios across these two treatments. We summarize the hypothesis for rank dependence below.

Hypothesis 2. *Subjects follow rank dependence:* $\frac{e_i}{e_4} = \frac{v_{i-1}}{v_3}$ *for* $i = 2, \cdots, 7$.

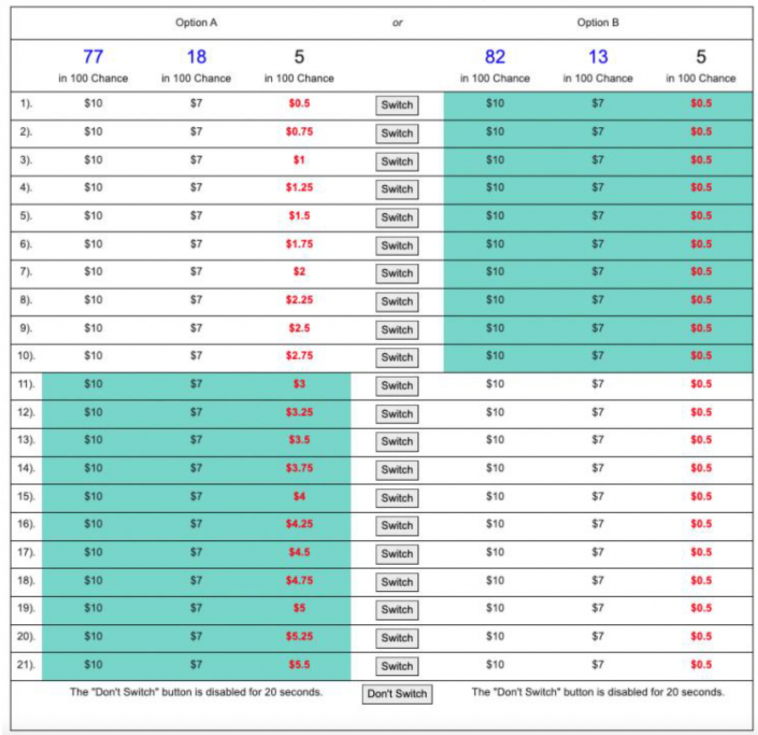
3.5 Simulations

While *MRS* constitutes foundations of our hypotheses, it is not empirically observable given its infinitely small nature. In the current experiment, we use $\frac{e_i}{e_4}$ and $\frac{v_j}{v_3}$ to approximate *MRS* at targeted probability levels. The accuracy of such approximation depends on local change rates of curvatures of $\pi(\cdot)$ and $u(\cdot)$. On one hand, following Rabin (2000) one can argue that $u(\cdot)$ should have locally stable curvatures. On the other hand, little is known regarding local changes of the probability weighting function. Consequently, it is not guaranteed *a priori* that the current approximation will reflect the true parameters or provide sharp tests. In this section, we address these concerns by simulating feedback from subjects and test performance of proposed tests.

We simulate 250 subjects and assume they are CPT utility maximizers endowed with a CRRA utility function $u(x) = x^\alpha, \alpha \sim \mathscr{U}(0.7, 1.1)$ as well as a probability weighting function $\pi(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma}, \gamma \sim \mathscr{U}(0.4, 0.9)$.[13] Since our results may be sensitive to the parameter regions, we provide separate simulations for γ under three distributions $\mathscr{U}(0.5, 1), \mathscr{U}(0.8, 1)$, and $\mathscr{U}(0.9, 1)$.

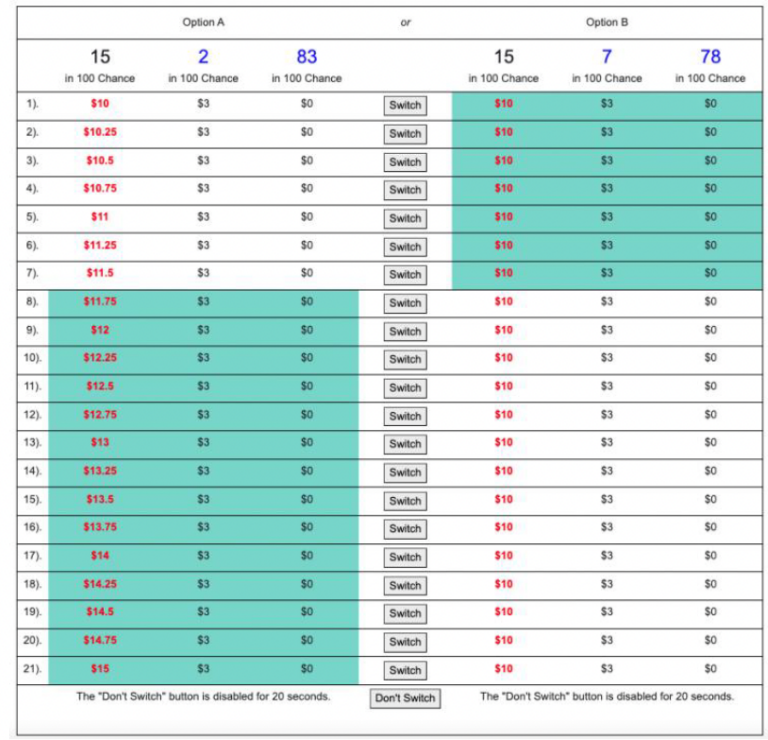
[13] $\mathscr{U}(a, b)$ represents the uniform distribution from a to b.

Panel A: (p, Z) treatment



	Option A			or	Option B		
	77 in 100 Chance	18 in 100 Chance	5 in 100 Chance		82 in 100 Chance	13 in 100 Chance	5 in 100 Chance
1).	$10	$7	$0.5	Switch	$10	$7	$0.5
2).	$10	$7	$0.75	Switch	$10	$7	$0.5
3).	$10	$7	$1	Switch	$10	$7	$0.5
4).	$10	$7	$1.25	Switch	$10	$7	$0.5
5).	$10	$7	$1.5	Switch	$10	$7	$0.5
6).	$10	$7	$1.75	Switch	$10	$7	$0.5
7).	$10	$7	$2	Switch	$10	$7	$0.5
8).	$10	$7	$2.25	Switch	$10	$7	$0.5
9).	$10	$7	$2.5	Switch	$10	$7	$0.5
10).	$10	$7	$2.75	Switch	$10	$7	$0.5
11).	$10	$7	$3	Switch	$10	$7	$0.5
12).	$10	$7	$3.25	Switch	$10	$7	$0.5
13).	$10	$7	$3.5	Switch	$10	$7	$0.5
14).	$10	$7	$3.75	Switch	$10	$7	$0.5
15).	$10	$7	$4	Switch	$10	$7	$0.5
16).	$10	$7	$4.25	Switch	$10	$7	$0.5
17).	$10	$7	$4.5	Switch	$10	$7	$0.5
18).	$10	$7	$4.75	Switch	$10	$7	$0.5
19).	$10	$7	$5	Switch	$10	$7	$0.5
20).	$10	$7	$5.25	Switch	$10	$7	$0.5
21).	$10	$7	$5.5	Switch	$10	$7	$0.5
	The "Don't Switch" button is disabled for 20 seconds.			Don't Switch	The "Don't Switch" button is disabled for 20 seconds.		

Panel B: (q, X) treatment



	Option A			or	Option B		
	15 in 100 Chance	2 in 100 Chance	83 in 100 Chance		15 in 100 Chance	7 in 100 Chance	78 in 100 Chance
1).	$10	$3	$0	Switch	$10	$3	$0
2).	$10.25	$3	$0	Switch	$10	$3	$0
3).	$10.5	$3	$0	Switch	$10	$3	$0
4).	$10.75	$3	$0	Switch	$10	$3	$0
5).	$11	$3	$0	Switch	$10	$3	$0
6).	$11.25	$3	$0	Switch	$10	$3	$0
7).	$11.5	$3	$0	Switch	$10	$3	$0
8).	$11.75	$3	$0	Switch	$10	$3	$0
9).	$12	$3	$0	Switch	$10	$3	$0
10).	$12.25	$3	$0	Switch	$10	$3	$0
11).	$12.5	$3	$0	Switch	$10	$3	$0
12).	$12.75	$3	$0	Switch	$10	$3	$0
13).	$13	$3	$0	Switch	$10	$3	$0
14).	$13.25	$3	$0	Switch	$10	$3	$0
15).	$13.5	$3	$0	Switch	$10	$3	$0
16).	$13.75	$3	$0	Switch	$10	$3	$0
17).	$14	$3	$0	Switch	$10	$3	$0
18).	$14.25	$3	$0	Switch	$10	$3	$0
19).	$14.5	$3	$0	Switch	$10	$3	$0
20).	$14.75	$3	$0	Switch	$10	$3	$0
21).	$15	$3	$0	Switch	$10	$3	$0
	The "Don't Switch" button is disabled for 20 seconds.			Don't Switch	The "Don't Switch" button is disabled for 20 seconds.		

Figure 3.1. Price Lists

Table 3.1. Estimations of Probability Weighting Curvature

Distribution of γ	$\mathscr{U}(0.5,1)$		$\mathscr{U}(0.8,1)$		$\mathscr{U}(0.9,1)$	
	(1)	(2)	(3)	(4)	(5)	(6)
$p = 0.05$	0.958***	1.018***	0.347***	0.315***	0.156***	0.143***
	(0.042)	(0.042)	(0.015)	(0.013)	(0.007)	(0.005)
$p = 0.2$	0.184***	0.159***	0.094***	0.080***	0.049***	0.041***
	(0.019)	(0.004)	(0.007)	(0.003)	(0.004)	(0.001)
$p = 0.35$	0.016***	0.007***	0.016***	0.015***	0.008***	0.008***
	(0.004)	(0.001)	(0.004)	(0.000)	(0.002)	(0.000)
$p = 0.65$	0.111***	0.101***	0.031***	0.024***	0.008***	0.011***
	(0.017)	(0.005)	(0.005)	(0.001)	(0.002)	(0.000)
$p = 0.8$	0.341***	0.403***	0.108***	0.104***	0.050***	0.046***
	(0.023)	(0.019)	(0.007)	(0.004)	(0.004)	(0.002)
$p = 0.92$	0.951***	1.234***	0.271***	0.275***	0.120***	0.117***
	(0.047)	(0.064)	(0.012)	(0.012)	(0.006)	(0.005)
N	250	250	250	250	250	250

Notes: Conditional Expectations of curvatures $\pi'(p) - 1$ are recorded at $p = 0.05, 0.2, 0.35, 0.65, 0.8$, and 0.92. Standard errors clustered at individual level in parentheses. Columns (1), (3), and (5) reports estimations from simulated subjects' responses while columns (2), (4), and (6) records true averages of $\pi'(p) - 1$ at corresponding probability levels. Columns (1) and (2) are results for $\gamma \sim \mathscr{U}(0.5,1)$. Columns (3) and (4) are results for $\gamma \sim \mathscr{U}(0.8,1)$. Columns (5) and (6) are results for $\gamma \sim \mathscr{U}(0.9,1)$.

The first object of this study is to investigate the shape of probability weighting functions. To provide valid conclusions, the estimations from simulated subjects' responses should be able to detect nonlinearity, and their value should be close to the true parameter values. Table 3.1 reports estimations and true sample averages of $\pi'(p) - 1$ at different probability levels across the three distributions of γ. As the results suggest, we can accurately test nonlinearity from simulated subjects' feedback. Even when the overall probability weighting function is close to linear ($\gamma \sim \mathscr{U}(0.9,1)$), with the 250 simulated subjects, linearity is rejected at $p = 0.35$ and $p = 0.65$. Further, the estimates are fairly close to the true average since their most differences are within 0.1. The largest discrepancy, 0.28, occurs at $p = 0.92$ when $\gamma \sim \mathscr{U}(0.5,1)$. This indication does reflect the fact that our estimation can be noisy if the curvature of probability weighting function changes dramatically. Nevertheless, according to previous results, subjects' probability weighting functions are likely to be smooth, which implies that they shouldn't change dramatically at all probabilities. Further, the quantitative estimation of γ, as shown in table 3.2, can reflect the true parameter reasonably well. The comparison between true and estimated curvatures with $\gamma \sim \mathscr{U}(0.5,1)$ at individual level is shown in figure 3.2. The result aligns with our previous observation.

Table 3.2. Nonlinear Least Square Estimation

	(1)	(2)	(3)
γ Distribution:	$\mathscr{U}(0.5,1)$	$\mathscr{U}(0.8,1)$	$\mathscr{U}(0.9,1)$
True γ average:	0.75	0.90	0.95
$\hat{\gamma}$	0.747	0.895	0.947
	(0.008)	(0.004)	(0.002)
N	250	250	250

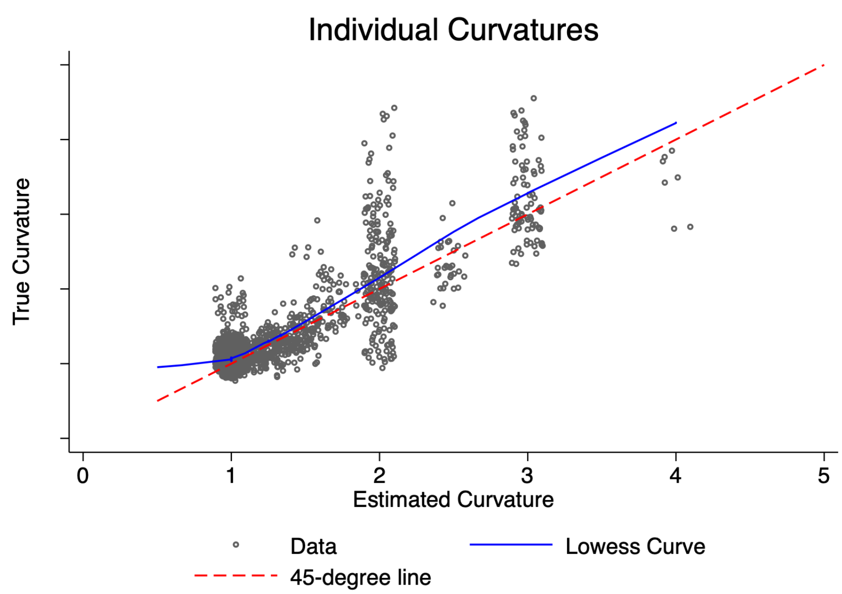


Figure 3.2. Individual Level Curvatures

The current experiment also aims at testing validity of rank dependence. Hypothesis 2 suggests that subjects following CPT should yield similar *MRS* between (p,Z) and (q,X) treatments. Since our current simulated samples follow CPT, our *MRS* estimations should be close to each other. Table 3.3 provides the results. It records $\frac{\pi'(p)}{\pi'(0.5)}$ between the two treatments under various γ distributions. Overall, the results indicates that our current method reflects rank dependence for most of the time. However, when curvature of $\pi(\cdot)$ is changing fast, there can be some noises in our estimation. In table 3.3, when $p+q=0.92$ in (q,X) treatment, the result rejects rank dependence. Nevertheless, the magnitude of (all) deviations are less than 0.1. As a result, the magnitudes also implies that rank dependence can explain the current data reasonably well.

Table 3.3. Rank Dependence with CTP Subjects

	(1)	(2)	(3)
$p+q=0.2$	-0.013	-0.005	-0.001
	(0.011)	(0.01)	(0.003)
$p+q=0.35$	0.001	0.007	0.004
	(0.08)	(0.008)	(0.004)
$p+q=0.65$	-0.02	0.002	0.004
	(0.02)	(0.018)	(0.004)
$p+q=0.8$	-0.01	-0.014	0.005
	(0.023)	(0.021)	(0.006)
$p+q=0.92$	-0.083**	-0.058	0.006
	(0.04)	(0.034)	(0.006)
N	250	250	250

Notes: The average difference between estimated curvatures between (p,Z) and (q,X) treatments conditional on cumulative probabilities are reported. Standard errors clustered at individual level in parentheses. Columns (1) records results for $\gamma \sim \mathscr{U}(0.5,1)$. Columns (2) presents results for $\gamma \sim \mathscr{U}(0.8,1)$. Columns (3) shows results for $\gamma \sim \mathscr{U}(0.9,1)$.

Previous simulation results suggest that if subjects' are CPT thinkers, our data and tests fail to reject the null hypothesis. We now investigate the case in which subjects are not CPT thinkers. To this end, we keep the parameters' settings – subjects are endowed with a utility function $u(x) = x^{\alpha}$ as well as a probability weighting function $\pi(p) = p^{\gamma}/(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}$. We now simulate the results of previous tests if the subjects are acting according to Prospect

Theory (Kahneman and Tversky, 1979a). Specifically we have

$$V(p,Z) = V(q,X) = \pi(p)u(X) + \pi(q)u(Y) + \pi(1-p-q)u(Z).$$

In both cases, the additive separable feature is preserved. Therefore, results from section 3.3 follow through. However, the difference is that now $\frac{\pi'(q_i+0.15)}{\pi'(\tilde{q}+0.15)} \neq \frac{\pi'(p_i)}{\pi'(\tilde{p})}$ in general.Table 3.4 summarizes the results. As expected, the differences are mostly significantly different from 0. Further, comparing magnitudes of deviations between Table 3.3 and 3.4, when subjects follow Prospect Theory instead of CPT, the magnitudes of deviations are at least 50% larger, and they go up to 200%. Therefore, the current method is sufficiently powerful to reject rank dependence.

Table 3.4. Rank Dependence with Non-CTP Subjects

	(1)	(2)	(3)
$p+q=0.2$	-0.477***	-0.078***	-0.034***
	(0.028)	(0.006)	(0.003)
$p+q=0.35$	-0.212***	-0.034***	-0.017***
	(0.018)	(0.008)	(0.006)
$p+q=0.65$	0.196***	0.022***	0.000
	(0.017)	(0.005)	(0.002)
$p+q=0.8$	0.460***	0.032***	0.004*
	(0.040)	(0.005)	(0.003)
$p+q=0.92$	-0.120***	-0.164***	-0.096***
	(0.027)	(0.009)	(0.007)
N	250	250	250

Notes: The average difference between estimated curvatures between (p,Z) and (q,X) treatments conditional on cumulative probabilities are reported. The behaviors are simulated from subjects following Propspect Theory instead of CPT. Standard errors clustered at individual level in parentheses. Columns (1) records results for $\gamma \sim \mathscr{U}(0.5,1)$. Columns (2) presents results for $\gamma \sim \mathscr{U}(0.8,1)$. Columns (3) shows results for $\gamma \sim \mathscr{U}(0.9,1)$.

Appendix A

Supplement Materials for Chapter 1

A.1 Proofs

Theorem 1: Π satisfies Axiom 1-8 if and only if it has s a salience representation. Furthermore, the salience function and utility are unique up to affine transformations.

Proof. ($\Rightarrow$) According to Lanzani (2022), Π satisfies axiom 1-5 if and only if it has a correlation-sensitive preference representation with monotonic and continuous ϕ. We start from this ϕ function and achieve a salience representation.

First , we restrict the outcome space to be the non-negative rational numbers. For all $x, x' \geq 0$ with $x' > x$, $n \in \mathbb{N}$, we consider the sequence defined by $\{x_i\}_{i=0}^{n}$ with $x_i = \frac{i}{n}(x' - x) + x$ and define $u^n_{x \to x'} = \sum_{i=1}^{n} \phi(x_{i+1}, x_i)$. Basically, $\{x_i\}_{i=0}^{n}$ is the IAS from x to x' with n-1 terms in between, and $u^n_{x \to x'}$ is the sum of its step differences. Define $\nu(x', x) = \liminf_{n \to \infty} u^n_{x \to x'}$, we begin by showing $\nu(x, x')$ is linear: $\nu(x', x) = \nu(x', 0) - \nu(x, 0)$.

To this end, for all $k \in \mathbb{N}$, consider the subsequence of $\{u^n_{x \to x'}\}_{n=1}^{\infty}$ defined by $\{u^{kn}_{x \to x'}\}_{n=1}^{\infty}$. We now show $\liminf_{n \to \infty} u^{kn}_{x \to x'} = \nu(x', x)$. By definition, subsequences will have higher limit inferiors than the sequence itself, so $\liminf_{n \to \infty} u^{kn}_{x \to x'} \geq \nu(x', x)$. On the other hand, for every $n \in \mathbb{N}$, $u^{kn}_{x \to x'} \leq u^n_{x \to x'}$. To see the reason, consider a special case of skewness prereference: one with the IAS

$\{x, \frac{x+x'}{2}, x'\}$, it follows by the axiom that

$$p\phi(\frac{x+x'}{2},x)+(1-p)\phi(x,\frac{x+x'}{2})\geq 0,$$
$$\Rightarrow \frac{p}{2}\phi(x',x)+\frac{(1-p)}{2}(\phi(x,\frac{x+x'}{2})+\phi(\frac{x+x'}{2},x'))\geq 0.$$

According to monotonicity, the first inequality holds whenever $p \geq 0.5$. Consequently, when $p = 0.5$, the second inequality becomes, $\phi(x',x)+(\phi(x,\frac{x+x'}{2})+\phi(\frac{x+x'}{2},x'))\geq 0$, or just $\phi(x',x)\geq \phi(x',\frac{x+x'}{2})+\phi(\frac{x+x'}{2},x) = u^2_{x\to x'}$ because ϕ is skew-symmetric. By iterating this argument, one can get $u^n_{x\to x'} \leq \phi(x',x)$ for all $n \geq 1$. Notice that $u^{kn}_{x\to x'} = \sum_{i=1}^n u^k_{(x+\frac{i}{n}(x'-x),x+\frac{i-1}{n}(x'-x))}$, and $u^n_{x\to x'} = \sum_{i=1}^n \phi(x+\frac{i}{n}(x'-x), x+\frac{i-1}{n}(x'-x))$. Therefore, $u^{kn}_{x\to x'} \leq u^n_{x\to x'}$, so

$$\liminf_{n\to\infty} u^{kn}_{x\to x'} = \nu(x',x).$$

If $x,x' \in \mathbb{Q}$, we can identify $k \in \mathbb{N}$ such that $\{x_i\}_{i=0}^{kn}$ with $x_i = \frac{i}{kn}x'$, and $x = x_j$ for some $j \leq n$.[1] Consider $u^{kn}_{0\to x'}$, we have the identity:

$$u^{kn}_{0\to x'} = u^{k'n}_{0\to x} + u^{k''n}_{x\to x'}, \tag{A.1}$$

for some $k',k'' \in \mathbb{N}$.[2]

Hence, by sub-additive of limit inferior, $\liminf\limits_{n\to\infty} u^{kn}_{0\to x'} \geq \liminf\limits_{n\to\infty} u^{k'n}_{0\to x} + \liminf\limits_{n\to\infty} u^{k''n}_{x\to x'}$. Together with $\liminf_{n\to\infty} u^{kn}_{x\to x'} = \nu(x',x)$, we have

$$\nu(x',0) \geq \nu(x,0)+\nu(x',x).$$

[1]If $x = \frac{a}{b}$, $x' = \frac{c}{d}$ with $a,b,c,d \in \mathbb{N}$. define $k = bc$ and $j = adn$ Since $x < x'$, $bc > ad$, so $j < kn$, and we have $\frac{j}{kn}x' = \frac{a}{b}$.

[2]If $x = \frac{a}{b}$, $x' = \frac{c}{d}$, we have $k' = ad, k'' = bc - ad$.

On the other hand, for every $\varepsilon > 0$, we can identify $n_1, n_2 \in \mathbb{N}$ such that

$$0 \leq u_{0\to x}^{k'n_1} - \liminf_{n\to\infty} u_{0\to x}^{k'n} < 0.5\varepsilon,$$

and

$$0 \leq u_{x\to x'}^{k''n_2} - \liminf_{n\to\infty} u_{x\to x'}^{k''n} < 0.5\varepsilon,$$

Let $n^* = n_1 \times n_2$, and use the identity in equation A.1, we have $u_{0\to x'}^{kn_3} = u_{0\to x}^{k'n_1n_2} + u_{x\to x'}^{k''n_1n_2}$. Since $u_{x\to x'}^{kn} \leq u_{x\to x'}^{n}$, so we have $u_{0\to x}^{k'n_1n_2} \leq u_{0\to x}^{k'n_1}$ and $u_{0\to x}^{k''n_1n_2} \leq u_{0\to x}^{k''n_2}$. Thus,

$$u_{0\to x'}^{kn_3} \leq \liminf_{n\to\infty} u_{0\to x}^{k'n} + \liminf_{n\to\infty} u_{x\to x'}^{k''n} + \varepsilon.$$

Furthermore, $u_{0\to x'}^{kn_3} \geq \liminf_{n\to\infty} u_{0\to x'}^{kn}$. The reason is that $u_{0\to x'}^{mkn_3} \leq u_{0\to x'}^{kn_3}$ for all $m \in \mathbb{N}$, so there are infinitely many terms in $\{u_{0\to x'}^{kn}\}_{n=1}^{\infty}$ with $u_{0\to x'}^{kn} \leq u_{0\to x'}^{kn_3}$. Consequently, we achieve

$$\liminf_{n\to\infty} u_{0\to x'}^{kn} \leq \liminf_{n\to\infty} u_{0\to x}^{k'n} + \liminf_{n\to\infty} u_{x\to x'}^{k''n} + \varepsilon.$$

Since ε is arbitrary, we have

$$\liminf_{n\to\infty} u_{0\to x'}^{kn} \leq \liminf_{n\to\infty} u_{0\to x}^{k'n} + \liminf_{n\to\infty} u_{x\to x'}^{k''n},$$

which is equivalent to:

$$\nu(x', 0) \leq \nu(x, 0) + \nu(x', x).$$

And hence, $\nu(x', 0) = \nu(x, 0) + \nu(x', x)$.

Our next step is to define the utility function $u(x)$ for $x \in \mathbb{R}_+$. For $x \in \mathbb{Q}$, let $u(x) = \nu(x, 0)$, otherwise let $u(x) = \lim_{x' \in \mathbb{Q}, x' \to x} \nu(x', 0)$. We now show that $u(x)$ is strictly increasing and continuous on rational numbers.

We first deal with continuity. Let $x, x' \in \mathbb{Q}$ with $x' > x$. Notice, $u(x') - u(x) = \nu(x', 0) -$

$\nu(x,0) = \nu(x',x)$ by the linearity we just showed. From skewness preference, we know that $0 \le \nu(x',x) \le \phi(x',x)$, hence as $x' \to x$, by continuity of ϕ, $\phi(x',x) \to 0$, so $u(x') \to u(x)$.

We now show monotonicity, again, $u(x') - u(x) = \nu(x',0) - \nu(x,0) = \nu(x',x)$, so monotonicity can be achieved if $\nu(x',x) > 0$. Consider axiom increasing utility, its statement directly translates to $\frac{1-p}{n} u^n_{x \to x'} + \frac{p}{n}\phi(x,x') > 0$. Or just, $u^n_{x \to x'} \ge \frac{p}{1-p}\phi(x',x)$ for all $n \in \mathbb{N}$. Therefore, $\nu(x',x) \ge \frac{p}{1-p}\phi(x',x)$. With $p > 0$, $\nu(x',x) > 0$.

Since $\mathbb{Q}$ is dense on real numbers, continuity and monotonicity follow directly by a standard convergence argument. It is omitted here.

Next, we define the salience function as:

$$f(\sigma(x,y)) = \begin{cases} \dfrac{\phi(x,y)}{u(x) - u(y)} \text{ if } x \neq y; \\ \\ 1 \text{ otherwise.} \end{cases} \tag{A.2}$$

For expression A.2 to be a valid salience function, we check four things: it satisfies ordering, it satisfies diminishing sensitivity, it satisfies continuity, and it obtains minimal at $x = y$. We start with checking these four properties at rational outcome pairs.

Let, $x, x', x'' \in \mathbb{Q}$ with $0 \le x < x' < x''$. We show that $f(\sigma(x',x)) \le f(\sigma(x'',x))$. Using a similar argument as before, we can find $k \in \mathbb{N}$ such that for all IAS $\{x_i\}_{i=0}^{kn}$ with $x_0 = x$ and $x_{kn} = x''$, we can identify $x_{i^*} = x'$ for some i^*. According to skewness prereference, for all $p \in (0,1]$, $\frac{p}{i^*}\phi(x',x) + \frac{1-p}{i^*}\sum_{j=1}^{i^*}\phi(x_{j-1},x_j) \ge 0$ implies $\frac{p}{kn}\phi(x'',x) + \frac{1-p}{kn}\sum_{j=1}^{kn}\phi(x_{j-1},x_j) \ge 0$.[3] Therefore,

$$\frac{\phi(x',x)}{\sum_{j=1}^{i^*}\phi(x_j,x_{j-1})} \ge \frac{1-p}{p} \implies \frac{\phi(x'',x)}{\sum_{j=1}^{n}\phi(x_j,x_{j-1})} \ge \frac{1-p}{p}.$$

[3]Starting with $\{x, x_1\}$ and extend this IAS one step at a time until it reaches x''. Skewness preference posits that

$$1 = \frac{\phi(x_1,x_0)}{\phi(x_1,x)} \le \frac{\phi(x_2,x)}{\phi(x_2,x_1) + \phi(x_1,x_0)} \le \frac{\phi(x_3,x)}{\phi(x_3,x_2) + \phi(x_2,x_1) + \phi(x_1,x_0)} \le \dots \le \frac{\phi(x',x)}{u^{i^*}_{x \to x'}} \dots \le \frac{\phi(x'',x)}{u^n_{x \to x''}}.$$

In other words,

$$\frac{\phi(x',x)}{u^{k'n}_{x\to x'}} \le \frac{\phi(x'',x)}{u^{kn}_{x\to x''}}, \text{ for some } k' \in \mathbb{N}.$$

As a result, $\liminf_{n\to\infty} \frac{\phi(x',x)}{u^{k'n}_{x\to x'}} \le \liminf_{n\to\infty} \frac{\phi(x'',x)}{u^{kn}_{x\to x''}}$. Therefore, $\frac{\phi(x',x)}{\nu(x',x)} \le \frac{\phi(x'',x)}{\nu(x'',x)}$. By the linearity of $\nu(\cdot)$, the axiom implies that

$$\frac{\phi(x',x)}{u(x')-u(x)} \le \frac{\phi(x'',x)}{u(x'')-u(x)} \Rightarrow f(\sigma(x',x)) \le f(\sigma(x'',x)).$$

Therefore, Ordering is satisfied on all rational outcomes.

For diminishing sensitivity, let's consider axiom diminishing relative sensitivity.

Given $\phi(x'+\varepsilon, x+\varepsilon)$ and $u^n_{x+\varepsilon\to x'+\varepsilon}$, there exists $p \in [0,1]$ such that $p\phi(x'+\varepsilon,x+\varepsilon) = (1-p)u^n_{x+\varepsilon\to x'+\varepsilon}$.[4] By a similar argument, we have

$$\frac{\phi(x'+\varepsilon,x+\varepsilon)}{u^n_{x+\varepsilon\to,x'+\varepsilon}} = \frac{1-p}{p} \text{ and } \frac{\phi(x',x)}{u^m_{x\to,x'}} \ge \frac{1-p}{p} \text{ for some } m \in \mathbb{N}.$$

Therefore,

$$\frac{\phi(x'+\varepsilon,x+\varepsilon)}{u^n_{x+\varepsilon\to x'+\varepsilon}} \le \frac{\phi(x',x)}{u^m_{x\to,x'}} \le \frac{\phi(x',x)}{v(x',x)}.$$

The second inequality holds because $u^m_{x\to,x'} \ge u^{km}_{x\to,x'}$ for all $k>1$, and it implies $u^m_{x\to,x'} > v(x',x)$. Hence,

$$\frac{\phi(x'+\varepsilon,x+\varepsilon)}{v(x'+\varepsilon,x+\varepsilon)} \le \frac{\phi(x',x)}{v(x',x)}.$$

Therefore, diminishing sensitivity also holds for rational outcomes.

Next, let us show that $f(\sigma(x,y))$ is continuous at all outcome pairs (x,y). For all pairs such that $x \ne y$, continuity of the salience function is implied by continuity of $u(\cdot)$ and $\phi(\cdot,\cdot)$. Now we show, as $x' \to x$, $\frac{\phi(x',x)}{u(x')-u(x)} \to 1$. According to increasing utility, there exists p such that $\frac{\phi(x',x)}{v(x',x)} \ge \frac{p}{1-p}$. Since the axiom also states that as $x' \to x$, $p \to 0.5$. The result follows.

Lastly, using skewness preference as in footnote 41, we have $1 \le \frac{\phi(x',x)}{v(x',x)}$ (here we don't

[4]The statement results from median value theorem.

need to restrict the outcome to be rationals). Therefore, $f\circ\sigma$ obtains minimal at $x=y$. Since $f(\sigma(x,y))$ is continuous, using a standard convergence argument, $f(\sigma(x,y))$ satisfies ordering and diminishing sensitivity at all outcome pairs.

($\Leftarrow$) Given a salience representation $\phi(x,y)=f(\sigma(x,y))(u(x)-u(y))$, we now show it must satisfies, increasing utility, skewness prereference, and diminishing relative sensitivity. Without losing the generality, let's assume that $f(\sigma(0,0))=1$.

For increasing utility, given $x>y\geq 0$, let $p=\frac{1}{f(\sigma(x,y))}$. For all IAS $\{x_i\}_{i=0}^n$ we have

$$
\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}\phi(x_i,x_{i-1})-\frac{p}{n}\phi(y,x),\\
=&\frac{1}{n}\left(\sum_{i=1}^{n}f(\sigma(x_i,x_{i-1}))u(x_i,x_{i-1})-pf(\sigma(x,y))(u(y)-u(x))\right),\\
\geq&\frac{1}{n}\left(\sum_{i=1}^{n}(u(x_i)-u(x_{i-1}))-(u(y)-u(x))\right)=0.
\end{aligned}
$$

Therefore, $(1-p)U(\{x_i\}_{i=0}^n)+\frac{p}{n}\delta_{y,x}\in\widehat{\Pi}$. And the axiom follows.

Now we show the necessity of skewness preference. Given an IAS $\{x_i\}_{i=0}^n$, we have

$$
\frac{f(\sigma(x_{n-1},x_0))}{\sum_{i=1}^{n-1}\frac{u(x_i)-u(x_{i-1})}{u(x_{n-1})-u(x_0)}f(\sigma(x_i,x_{i-1}))}\leq\frac{f(\sigma(x_n,x_0))}{\sum_{i=1}^{n}\frac{u(x_i)-u(x_{i-1})}{u(x_n)-u(x_0)}f(\sigma(x_i,x_{i-1}))}.
$$

To see this, first notice that for the numerators, by the ordering of salience theory, we have $f(\sigma(x_{n-1},x_0))\leq f(\sigma(x_n,x_0))$. Furthermore, for the denominators, denote

$$
\sum_{i=1}^{n-1}\frac{(u(x_i)-u(x_{i-1})}{u(x_n-1)-u(x_0)}f(\sigma(x_i,x_{i-1}))=\Lambda.
$$

Intuitively, Λ is an weighted average of $f(\sigma(x_i,x_{i-1}))$ where the corresponding weights are defined by $\frac{u(x_i)-u(x_{i-1})}{u(x_{n-1})-u(x_0)}$. Then it follows that

$$
\sum_{i=1}^{n}\frac{(u(x_i)-u(x_{i-1})}{u(x_n-1)-u(x_0)}f(\sigma(x_i,x_{i-1}))=\frac{u(x_{n-1})-u(x_0)}{u(x_n)-u(x_0)}\Lambda+\frac{u(x_n)-u(x_{n-1})}{u(x_n)-u(x_0)}f(\sigma(x_n,x_{n-1})).
$$

Since $|x_i - x_{i-1}|$ is constant for an IAS, $f(\sigma(x_n, x_{n-1})) \leq f(\sigma(x_i, x_{i-1}))$ for all $i < n$ due to diminishing sensitivity of salience function. Consequently, $f(\sigma(x_n, x_{n-1}))$ is smaller than any form of their average. Hence, $\sum_{i=1}^{n} \frac{(u(x_i)-u(x_{i-1})}{u(x_n-1)-u(x_0)} f(\sigma(x_i, x_{i-1})) < \Lambda$. And the proposed inequality follows. In skewness prereference, $\frac{p}{n-1}\delta_{(x_{n-1},x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n-1})} \in \Pi$ is equivalent to $\frac{f(\sigma(x_{n-1},x_0))}{\sum_{i=1}^{n-1} \frac{u(x_i)-u(x_{i-1})}{u(x_{n-1})-u(x_0)} f(\sigma(x_i,x_{i-1}))} \geq \frac{1-p}{p}$. Hence, $\frac{f(\sigma(x_n,x_0))}{\sum_{i=1}^{n} \frac{u(x_i)-u(x_{i-1})}{u(x_n)-u(x_0)} f(\sigma(x_i,x_{i-1}))} \geq \frac{1-p}{p}$, and it suggests that $\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_0\}_{i=0}^{n})} \in \Pi$.

Lastly, given $\{x_i\}_{i=0}^{n}$ and $y_0 \leq x_0$, find $y_1 > y_0$ such that $f(\sigma(y_1, y_0)) \leq f(\sigma(x_n, x_{n-1}))$, and $|x_1 - x_0|$ is a multiple of $|y_1 - y_0|$. Next construct the IAS $\{y_i\}_{i=0}^{m}$ based on y_0 and y_1 with $y_m - y_0 = x_n - x_0$. We have

$$\begin{aligned}\frac{f(\sigma(x_n,x_0))}{\sum_{i=1}^{n} \frac{u(x_i)-u(x_{i-1})}{u(x_n)-u(x_0)} f(\sigma(x_i,x_{i-1}))} &\leq \frac{f(\sigma(x_n,x_0))}{f(\sigma(x_n,x_{n-1}))}, \\ &\leq \frac{f(\sigma(y_m,y_0))}{f(\sigma(y_1,y_0))}, \\ &\leq \frac{f(\sigma(y_m,y_0))}{\sum_{i=1}^{m} \frac{u(y_i)-u(y_{i-1})}{u(y_n)-u(y_0)} f(\sigma(y_i,y_{i-1}))}.\end{aligned}$$

Following a similar argument as in skewness prereference, the necessity of diminishing relative sensitivity is established.

For uniqueness, if $\phi(x,y) = \hat{f}(\hat{\sigma}(x,y))h(x) - h(y) = f(\sigma(x,y))(u(x) - u(y))$. Set $y = 0$, without losing generality, we can assume $u(0) = h(0) = 0$. Fix x, we can identify the supreme of probability p such that

$$(1-p)U(\{x_i\}_{i=1}^{n}) + \frac{p}{n}\delta_{(y,x)} \in \widehat{\Pi}.$$

From increasing utility, we know that this supreme is positive, let's denote it p^*. In this case, we have

$$\frac{\phi(x,0)}{f(\sigma(0,0))u(x)} = \frac{\phi(x,0)}{\hat{f}(\hat{\sigma}(0,0))h(x)} = \frac{1-p^*}{p^*}.$$

As a result, $u(x) = \frac{\hat{f}(\hat{\sigma}(0,0))}{f(\sigma(0,0))} h(x)$. Hence, they are at most affine transformations. Here they are linear transformations to each other because we set them to have 0 utility at x = 0. □

Lemma 1a: Given any compact outcome space $X \subset \mathbb{R}$, salience function $\sigma(\cdot,\cdot)$, and $\varepsilon > 0$, for every $x, y, x', y' \in X$ with $|x-y| \geq \varepsilon$, $|x'-y'| \geq \varepsilon$, there exists $k > 1$ such that $\sigma(x,y) \leq \sigma(x',y')$ whenever $k|x-y| \leq |x'-y'|$.

Proof. For simplicity, let $0 \in X$. Notice that the set $X_\varepsilon = X \times X \bigcap \{(x,y) : |x-y| \geq \varepsilon\}$. Since X is compact and $\{(x,y) : |x-y| \geq \varepsilon\}$ is closed, X_ε is compact. Denote $\bar{x}$ the maximal value on X. For every $x \in X$ and $x \geq \varepsilon$, we can identify $T(x) \subset X$ such that $f(\sigma(x,0)) \leq f(\sigma(\bar{x},y))$ for all $y \in T(x)$.[5] Let $t(x) = \sup\{T(x)\}$. Due to *ordering* of salience function, $f(\sigma(x,0))$ is increasing in x, so $t(x)$ is decreasing in x. Further, $t(x)$ is continuous because $f(\sigma(\cdot,\cdot))$ is continuous. In summary, we have $\frac{\bar{x}-t(x)}{x}$ continuous on $\{x \in X | x \geq \varepsilon\}$. In addition, $\frac{\bar{x}-t(x)}{x}$ continuous on $\{x \in X | x \geq \varepsilon\}$ is compact, so $\frac{\bar{x}-t(x)}{x}$ obtains it maximal value, which we denoted as k.

To complete the analysis, let $(x,y),(x',y') \in X_\varepsilon$ with $k|x-y| \leq |x'-y'|$. By *diminishing sensitivity* of salience function, we have $f(\sigma(x,y)) \leq f(\sigma(|x-y|,0))$ and $f(\sigma(x',y')) \geq f(\sigma(\bar{x}-|x'-y'|,\bar{x}))$. Since $\frac{|x'-y'|}{|x-y|} \geq k$, $f(\sigma(|x-y|,0)) \leq f(\sigma(\bar{x}-|x'-y'|,\bar{x}))$. Hence, $f(\sigma(x,y)) \leq f(\sigma(x',y'))$. □

Lemma 1b: Given any compact outcome space $X \subset \mathbb{R}$, salience function $\sigma(\cdot,\cdot)$, and $k > 1$, there exists $\delta > 0$ such that for every $x, y, x', y' \in X$ with $|x-y| > \delta$, $|x'-y'| > \delta$, $\sigma(x,y) \leq \sigma(x',y')$ whenever $k|x-y| \leq |x'-y'|$.

Proof. Following the notations in the previous lemma, we have as $\varepsilon \to diam(X)$,[6] $\frac{\bar{x}-t(x)}{x} \to 1$. To see it, we have $f(\sigma(x,0)) \to f(\sigma(\bar{x},0))$ as $x \to \bar{x}$. In addition, as $\varepsilon \to diam(X)$, if $(x,y) \in X_\varepsilon$, $|x-y| \to \bar{x}$. The result follows. □

Proposition 1: A decision-maker's preference has s a k-regular salience representation if and only if she satisfies axiom 1 - 9.

[5] $T(x)$ is not empty because $f(\sigma(\bar{x},\bar{x})) \leq f(\sigma(x,0)) \leq f(\sigma(0,\bar{x}))$ and $f(\sigma(\cdot,\cdot))$ is continuous.
[6] $diam(X) = \max_{x \in X} x - \min_{x \in X} x$

Proof. (⇒) Given IAS $\{x_i\}_{i=0}^n$, we still denote $u^n_{x_0\to x_n}$ the summation $\sum\limits_{i=1}^{n}\phi(x_i,x_{i-1})$. First, assume $y_0 \geq x_n$, by the design of axiom k-regular, we have $\min\limits_{i\leq n} f(\sigma(x_i,x_{i-1})) \geq \max\limits_{i\leq m} f(\sigma(y_i,y_{i-1}))$, and $y_m - y_0 \geq k(x_n - x_0)$. Therefore, if the decision-maker has a k-regular salience representation, we have

$$\frac{f(\sigma(x_n,x_0))}{\sum\limits_{i=1}^{n}\frac{u(x_i)-u(x_{i-1})}{u(x_n)-u(x_0)}f(\sigma(x_i,x_{i-1}))} \leq \frac{f(\sigma(y_m,y_0))}{\sum\limits_{i=1}^{m}\frac{u(y_i)-u(y_{i-1})}{u(y_n)-u(y_0)}f(\sigma(y_i,y_{i-1}))}.$$

Therefore, the axiom is true.

(⇐) If axiom is true, we have

$$\frac{\phi(x_n,x_0)}{u^n_{x_0\to x_n}} \leq \frac{\phi(y_m,y_0)}{u^m_{y_0\to y_m}},$$

For all m,n such that $\frac{x_n-x_0}{n} = \frac{y_m-y_0}{m}$. Therefore,

$$\begin{aligned}
&\liminf_{n\to\infty}\frac{\phi(x_n,x_0)}{u^n_{x_0\to x_n}} \leq \liminf_{m\to\infty}\frac{\phi(y_m,y_0)}{u^m_{y_0\to y_m}},\\
&\Rightarrow \frac{f(\sigma(x_n,x_0))(u(x_n)-u(x_0))}{u(x_n)-u(x_0)} \leq \frac{f(\sigma(y_m,y_0))(u(y_m)-u(y_0))}{u(y_m)-u(y_0)},\\
&\Rightarrow f(\sigma(x_n,x_0)) \leq f(\sigma(y_m,y_0)).
\end{aligned}$$

The second line holds because the salience representation is unique, and $u(x) = \liminf\limits_{n\to\infty} u^n_{0\to x}$.

The axiom posits that $f(\sigma(x_n,x_0)) \leq f(\sigma(y_m,y_0))$ if $k(x_n - x_0) \leq y_m - y_0$ and $y_0 \geq x_n$. According to diminishing sensitivity, for all $\varepsilon \geq 0$, $f(\sigma(y_m-\varepsilon,y_0-\varepsilon)) \geq f(\sigma(y_m,y_0))$ given that $y_0 - \varepsilon \geq 0$. Therefore, k-regular of salience function follows. □

Proposition 2: Suppose a decision-maker's preference has a k-regular salience representation, then for every $p > 0$ and IAS $\{x_i\}_{i=0}^n$

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^n)} \in \Pi \;\Rightarrow\; \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^n)} \in \Pi.$$

for all IAS $\{y_i\}_{i=0}^{m}$ such that $y_m - y_0 = x_n - x_0$, $0 \le y_0 \le x_0$, and $k(y_1 - y_0) \le x_1 - x_0$.

Proof. This is a direct check of the definitions. □

Proposition 3: The utility function of a decision-maker with a salience representation is concave if and only if she follows axiom 10.

Proof. (⇒) $\phi(x, x-h) = f(\sigma(x, x-h))(u(x) - u(x-h))$ and $\phi(x+h, x) = f(\sigma(x+h, x))u(x+h) - u(x)$. By diminishing sensitivity of salience function, $f(\sigma(x, x-h)) \ge f(\sigma(x+h, x))$. Furthermore, by concavity of u, we have $u(x) - u(x-h) \ge u(x+h) - u(x)$. The result follows.

(⇐) Since utility of the salience representation is continuous, according to Sierpinski's theorem, it is sufficient to show that for all $x > 0$, $u(x+h) - u(x) < u(x) - u(x-h)$ given $x \ge h$. Since salience representation is unique up to affine transformations, from the proof of theorem 1, $u(x) - u(x-h) = \nu(x, x-h) = \liminf_{n\to\infty} u^n_{x-h\to x}$, and $u(x+h) - u(x) = \nu(x+h, x) = \liminf_{n\to\infty} u^n_{x\to x+h}$.

For all $\varepsilon > 0$, we can identify n^* such that $|u(x) - u(x-h) - u^{n^*}_{x-h\to x}| \le \varepsilon$ and $|u(x+h) - u(x-h) - u^{2n^*}_{x-h\to x+h}| \le \varepsilon$. Hence, $|u(x+h) - u(x) - (u^{2n^*}_{x-h\to x+h} - u^{n^*}_{x-h\to x})| < 2\varepsilon$. Notice that, $u^{2n^*}_{x-h\to x+h} - u^{n^*}_{x-h\to x} = \sum_{i=n^*+1}^{2n^*} \phi(x_i, x_{i-1})$ and $u^{n^*}_{x-h\to x} = \sum_{i=1}^{n^*} \phi(x_i, x_{i-1})$ where $\{x_i\}_{i=0}^{2n^*}$ is the IAS from $x-h$ to $x+h$. By axiom concavity, there is an order post upon these differences:

$$\phi(x_1, x_0) \ge \phi(x_2, x_1) \ge \phi(x_3, x_2) \ge \ldots \ge \phi(x_{2n^*}, x_{2n^*-1}).$$

Therefore, $u^{2n^*}_{x-h\to x+h} - 2u^{n^*}_{x-h\to x} \le 0$, so $(u(x+h) - u(x)) - (u(x) - u(x-h)) \le 3\varepsilon$. Since ε is arbitrary, we have $u(x+h) - u(x) \le u(x) - u(x-h)$. □

Theorem 2: Π has a regret representation if and only if it satisfies axiom 1-5, and 11. Furthermore, the utility is unique up to affine transformations.

Proof. (⇒) $\{(x_1, y_1), 0.5; (x_2, y_2), 0.5\} \in \Pi$ implies that $Q(u(x_1) - u(y_1)) \ge Q(u(x_2) - u(y_2))$. Since $Q(\cdot)$ is monotonic, $u(x_1) - u(y_1) \ge u(y_2) - u(x_2)$. It follows that $u(x_1) - u(y_2) \ge u(y_1) -$

$u(x_2)$. Therefore, $Q(u(x_1)-u(y_2)) \geq Q(u(y_1)-u(x_2))$, which implies

$$\{(x_1,y_2),0.5;\,(x_2,y_1),0.5\} \in \Pi.$$

($\Leftarrow$) The technique here is similar to Diecidue and Somasundaram (2017). We start by find a sequence $A_1 = \{a_0, a_1, a_2, \ldots\}$ (can be finite) such that $a_0 = 0$, and $\phi(a_{i+1}, a_i) > 0$ is constant. To this end, set $a_0 = 0$, and $a_1 = 1$. Suppose we identified a_i for some $i \geq 1$. Since ϕ is continuous, one of the following must be true:

A. There is a_{i+1} such that $\phi(a_{i+1}, a_i) = \phi(1,0)$.

B. For all $x > a_i$, $\phi(x, a_i) < \phi(1,0)$.

If B is true, terminate. If A is true, record a_{i+1} and continue.

Now, between a_0 and a_1, we define $a_{0.5}$, where $\phi(a_{0.5}, a_0) = \phi(a_1, a_{0.5})$. This $a_{0.5}$ exists because ϕ is continuous. Consider $T(x) = \phi(x, a_0) - \phi(a_1, x)$, so T is continuous. $T(0) = -\phi(a_1, a_0)$ and $T(1) = \phi(a_1, a_0)$. By median value theorem, there is $x \in (0,1)$ such that $T(x) = 0$. Set $a_{0.5} = x$.

Now we find $a_{1.5}$ such that $\phi(a_{1.5}, a_1) = \phi(a_{0.5}, a_0)$. Because $\phi(a_1, a_1) < \phi(a_{0.5}, a_0) < \phi(a_2, a_1)$. We can find $a_{1.5} \in (a_1, a_2)$ satisfies the requirement. We now show $\phi(a_2, a_{1.5}) = \phi(a_{0.5}, a_0)$. $\phi(a_{1.5}, a_1) = \phi(a_{0.5}, a_0)$ implies $\{(a_{1.5}, a_1), 0.5;\,(a_0, a_{0.5}), 0.5\} \in \Pi$. By swapping independence, $\{(a_{1.5}, a_{0.5}), 0.5;\,(a_0, a_1), 0.5\} \in \Pi$, so $\phi(a_{1.5}, a_{0.5}) \geq \phi(a_1, a_0)$. Using a similar argument, we can get $\phi(a_{1.5}, a_{0.5}) \leq \phi(a_1, a_0)$, and hence, $\phi(a_{1.5}, a_{0.5}) = \phi(a_1, a_0)$. Therefore, $\phi(a_{1.5}, a_{0.5}) = \phi(a_2, a_1)$. Apply swapping independence again, we have $\phi(a_2, a_{1.5}) = \phi(a_1, a_{0.5})$. Hence, $\phi(a_2, a_{1.5}) = \phi(a_{0.5}, a_0)$. We continue the process, for each $a_i \in A_1$, we find $a_{i+0.5}$. Notice, if A_1 is finite and ends at a_n, we may not find $a_{n+0.5}$, since it may be the case that $\phi(x, a_n) < \phi(a_{0.5}, a_0)$ for all $x > a_n$. In that case, we terminate.

The kew property of this sequence is that the value of $\phi(a_{i+0.5k}, a_i)$ only depends on k. To see this, we just showed that adjacent pair has a constant difference: $\phi(a_{0.5+0.5k}, a_{0.5k}) =$

$\phi(a_{0.5}, a_0)$, by swapping independence, $\phi(a_{0.5+0.5k}, a_{0.5}) = \phi(a_{0.5k}, a_0)$.

Next, we have $\phi(a_{1+0.5k}, a_{0.5+0.5k}) = \phi(a_1, a_{0.5})$. Hence, $\phi(a_{1+0.5k}, a_1) = \phi(a_{0.5+0.5k} - a_{0.5})$.

Keep iterating, we get $\phi(a_{i+0.5k}, a_i) = \phi(a_{0.5k}, a_0)$ for all i.

Denote $A_2 = \{a_0, a_{0.5}, a_1, a_{1.5,...}\}$.

Similarly, for each a_i in A_2, we find $a_{i+0.25}$ and get A_3. The process proceeds inductively, after getting A_n, we find $a_{i+0.5^n}$ for each $a_i \in A_n$ and get A_{n+1}. In the end, we get $\cup_{n=1}^{\infty} A_n$. By a similar argument, $\phi(a_j, a_i)$ only depends on the differences between j and i. For every $a_i \in \cup_{n=1}^{\infty} A_n$, we define $u(a_i) = i$.

We now claim that $a_{0.5^n} \to 0$ as $n \to \infty$. Since $a_{0.5^n}$ is monotonically decreasing and positive, the limit exists, denoted it by a and we have $a \leq a_{0.5^n}$ for all n. On the other hand, by construction, $\phi(a_{0.5^n}, 0) = \phi(a_{0.5^{n-1}}, a_{0.5^n})$, for all n. so $\phi(a_{0.5^n}, 0) \leq \phi(a_{0.5^{n-1}}, a)$. Therefore, $\phi(a_{0.5^n}, 0)$ converges to 0, and hence $a_{0.5^n} \to 0$ as $n \to \infty$.

For every generic $x \in \mathbb{R}_+$ such that $x \notin A_n$ for all n, we claim there is a sequence $\{b_i\}_{i=1}^{\infty}$ s.t. $b_i \in A_i$ for all i and $|b_i - x| \to 0$ as i goes to infinity. For each $x \geq 0$, we can find $a_{j_1} \in A_1$ s.t. $0 < \phi(x, a_{j_1}) < \phi(a_1, a_0)$. Because $x \notin A_1$, so we can ignore equality on both ends. Set $b_1 = a_{j_1}$. If $\phi(x, a_{j_1}) < \phi(a_{0.5}, a_0)$, set $b_2 = a_{j_1}$. Otherwise, set $b_2 = a_{j_1+0.5}$. The process continuous. In the end, we have $0 < \phi(x, b_i) < \phi(a_{0.5^{(i-1)}}, a_0)$ for all i. Since fix x, $\phi(x, y)$ is continuously increasing in y for $y \leq x$ and $\phi(a_{0.5^{(i-1)}}, a_0) \to 0$ as $i \to \infty$, we have $b_i \to x$. Define $u(x) = \lim_{i \to \infty} u(b_i)$. By design, u is continuous.

Similarly, we can perform the same process over negative outcomes, the detailed construction is omitted here. Expand $A_1 = \{a_0, a_1, a_2, \ldots\} \cup \{a_{-1}, a_{-2}, \ldots\}$ and expand A_i accordingly. Since for every $a_i, a_j \in \cup_{n=1}^{\infty} A_n$, we have $\phi(a_i, a_j)$ only depends on the differences between i and j, so we can find a function Q such that $\phi(a_i, a_j) = Q(i - j) = Q(u(a_i) - u(a_j))$.

By previous argument, for all $x, y \in \mathbb{R}$, we can find sequences $\{b_i^x\}_{i=1}^{\infty}$ and $\{b_i^y\}_{i=1}^{\infty}$, s.t

$b_i^x, b_i^y \in A_i$ for all i and $b_i^x \to x$, $b_i^y \to y$. Notice

$$\phi(x,y) = \lim_{i\to\infty} \phi(b_i^x, b_i^y) = \lim_{i\to\infty} Q(u(b_i^x) - u(b_i^y)) = Q(u(x) - u(y)).$$

It is direct to verify that Q is continuous, skew-symmetric, and increasing. Therefore, the preference can be presented by regret theory.

For uniqueness, notice that for every utility function that can represent the preference, $\cup_{n=1}^{\infty} A_n$ stays invariant. Therefore, if $u(\cdot)$ and $v(\cdot)$ both represent the same preference, we have

$$u(x_1) - u(y_1) \geq u(x_2) - u(y_2) \implies v(x_1) - v(y_1) \geq v(x_2) - v(y_2)\, \forall x_1, x_2, y_1, y_2 \in \cup_{n=1}^{\infty} A_n.$$

This implies that u and v must be affine transformations to each other on $\cup_{n=1}^{\infty} A_n$. Since $\cup_{n=1}^{\infty} A_n$ is dense, the result follows. □

Proposition 4: The utility function of a decision-maker with a regret representation is concave if and only if her preference set satisfies axiom 10.

Proof. The conclusion follows from the following equivalent statements.

$$\begin{aligned}
&\{(x, x-h), 0.5;\ (x, x+h), 0.5\} \in \Pi, \\
&\Leftrightarrow Q(u(x) - u(x-h)) \geq Q(u(x+h) - u(x)), \\
&\Leftrightarrow u(x) - u(x-h) \geq u(x+h) - u(x), \\
&\Leftrightarrow u(\cdot) \text{ is concave at } x.
\end{aligned}$$

□

Proposition 5: Suppose the outcome space is a compact subset of $\mathbb{R}_+$, a decision-maker's preference either has an expected utility representation or has a regret representation with concave $u(\cdot)$, increasing $\frac{Q(x)}{x}$, and $\lim_{x\to 0} \frac{Q(x)}{x} > 0$ if and only if she satisfies axioms 1-8, and 11.

Proof. (⇒) Without losing generality, let $\lim_{x\to 0} \frac{Q(x)}{x} = 1$. In this case, we show that the regret representation can be rewritten as a salience representation. Following Herweg and Müller (2021), let

$$f(\sigma(x,y)) = \begin{cases} \frac{Q(u(x)-u(y))}{u(x)-u(y)} \text{ if } x \neq y \\ \\ 1 \text{ otherwise} \end{cases}$$

Since increasing the distance between outcomes will enlarge their utility differences and $\frac{Q(x)}{x}$ is increasing in x, $f(\sigma(x,y))$ satisfies ordering. In addition, with $u(\cdot)$ being concave, we have $u(x)-u(y) \geq u(x+\varepsilon)-u(y+\varepsilon)$ for all $x > y \geq 0$. Again, with $\frac{Q(x)}{x}$ being increasing in x, we have $f(\sigma(x,y)) \geq f(\sigma(x+\varepsilon, y+\varepsilon))$. For continuity, by continuity of $u(\cdot)$ and $Q(\cdot)$, the proposed salience function is continuous at points with $x \neq y$. As for $x = y$, the continuity follows from $\lim_{x\to 0} \frac{Q(x)}{x} > 0$ and $\frac{Q(x)}{x}$ is increasing in x.

(⇐) Since axioms 1-8 and 11 are satisfied, the preference can be presented by $\phi(x,y) = Q(u(x)-u(y))$. It also has a *unique* salience representation $f(\sigma(x,y))(h(x)-h(y))$. In principle, $u(\cdot) \neq h(\cdot)$. However, as we will show, they are indeed equal to each other. Without losing generality, we assume $h(0) = u(0) = 0$.

Consider a sequence $\{y_j\}_{j=0}^{n}$ wherein $u(y_j)-u(y_{j-1})$ stays constant for all i, and define $\tau^n_{y_0\to y_n} = \sum_{j=1}^{n} Q(u(y_j)-u(y_{j-1}))$. Comparing to IAS, the current sequence cuts utility from y_0 to y_n into equal pieces. Notice that if the preference has a salience representation, fixing $y_0 = 0$ and $y_n = y$, we have $\lim_{n\to\infty} \tau^n_{0\to y} = f(\sigma(0,0))h(y)$. In other words,

$$\lim_{n\to\infty} nQ(\frac{u(y)}{n}) = f(\sigma(0,0))h(y),$$

$$\Rightarrow u(y) \lim_{n\to\infty} \frac{Q(\frac{1}{n})}{\frac{1}{n}} = f(\sigma(0,0))h(y).$$

Since Q is monotonic and continuous, we just showed $\lim_{x\to 0} \frac{Q(x)}{x} > 0$. In addtion, the result also suggests that $u(\cdot)$ and $h(\cdot)$ are identical up to linear transformations. For simplicity, we proceed with $u(x) = h(x)$. In this case, we have $f(\sigma(x,0)) = \frac{Q(u(x))}{u(x)}$. By ordering, $\frac{Q(x)}{x}$ must be increasing in x. By diminishing sensitivity, $u(\cdot)$ must be concave. □

A.2 Characterizations with Mixed Outcomes

When the outcome space is non-negative, straight forward extensions of *skewness preference* and *diminishing relative sensitivity* provide a basis for salience representation. However, when the outcome space X contains mixed outcomes, *skewness preference* needs to be slightly relaxed.

Axiom 21 (Strong Increasing Utility)**.** *For every $x, y \in X$ with $y < x$, there exists $p \in (0, 0.5]$ such that for all IAS $\{x_i\}_{i=0}^{n}$ with $x_0 = y$ and $x_n = x$, we have*

$$(1-p)U(\{x_i\}_{i=1}^{n}) + \frac{p}{n}\delta_{(y,x)} \in \widehat{\Pi}.$$

Furthermore, $p \to 0.5$ as $x \to y$.

The difference between strong increasing utility and increasing utility is now, p is restricted to be at most 0.5. This condition is originally implied by skewness preference. Since we are about to relax the axiom, we add the restriction back. Now, we state the generalized skewness preference.

Axiom 22 (Generalized Skewness Preference)**.** *For all IAS $\{x_i\}_{i=0}^{n}$ and $y_0 \leq x_0 < x_n \leq y_n$ there exists an IAS $\{y_i\}_{i=0}^{m}$ such that*

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^{n})} \in \Pi \Rightarrow \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^{m})} \in \Pi.$$

Axiom 23 (Generalized Diminishing Relative Sensitivity)**.** *For every non-mixed IAS $\{x_i\}_{i=0}^{n}$,*

$|y_0| \leq |x_0|$, *and* $y_0 x_0 \geq 0$, *there is an non-mixed IAS* $\{y_i\}_{i=0}^m$ *with* $y_m - y_0 = x_n - x_0$ *such that*

$$\frac{p}{n}\delta_{(x_n,x_0)} + (1-p)\overline{U(\{x_i\}_{i=0}^n)} \in \Pi \Rightarrow \frac{p}{m}\delta_{(y_m,y_0)} + (1-p)\overline{U(\{y_i\}_{i=0}^m)} \in \Pi.$$

Crudely, generalized diminishing relative sensitivity adds a mirrored restriction for non-positive IAS. Here we state and prove the representation theorem.

Proposition 9. Π *induces a salience representation if and only if it satisfies axiom 1-5, A.1, A.2, and A.3.*

Proof. Similar to Theorem 1. □

A.3 Characterization of Expected Utility Theory

We now characterize expected utility theory. From general correlation-sensitive preference to regret theory, we introduce swapping independence to ensure that decision-makers' behaviors respect the ordinal order upon utility differences. Once this ordinal order is preserved, the representation becomes some monotonic transformation of the utility difference. It is natural to ask whether a stronger version of swapping independence can preserve the *cardinal order* between utility differences and hence recover the expected utility representation. We present the appropriate version below.

Axiom 24 (Conditional Swapping Independence)**.** *For all* $(x_1,y_1),(x_2,y_2).(x_3,y_3) \in X \times X$*:*

$$\{(x_1,y_1),\frac{1}{3};\,(x_2,y_2),\frac{1}{3};\,(x_3,y_3),\frac{1}{3}\} \in \Pi \;\Rightarrow\; \{(x_1,y_2),\frac{1}{3};\,(x_2,y_1),\frac{1}{3};\,(x_3,y_3),\frac{1}{3}\} \in \Pi.$$

To see how conditional swapping independence can preserve the cardinal information, let us denote $\{(x_1,y_1),\frac{1}{2};\,(x_2,y_2),\frac{1}{2}\}$ as π and $\{(x_1,y_2),\frac{1}{2};\,(x_2,y_1),\frac{1}{2}\}$ as π_{swap}. Conditional swapping independence requires that $\frac{2}{3}\pi + \frac{1}{3}\delta_{(x_3,y_3)} \in \Pi$ implies $\frac{2}{3}\pi_{swap} + \frac{1}{3}\delta_{(x_3,y_3)} \in \Pi$. Therefore, the valuation of π and π_{swap} must be identical in order for the conclusion to hold.

Proposition 10. *A decision-maker's preference has a expected utility representation if and only if she satisfies axiom 1-5, and A.4.*

Proof. ($\Rightarrow$) Since conditional swapping independence implies preference consistency under a certain type of correlation manipulation, if the decision-maker is an expected utility maximizer, she will follow the axiom.

($\Leftarrow$) Since conditional swapping independence implies swapping independence, the decision maker has a regret representation $\phi(x,y) = Q(u(x) - u(y))$. We now show that her regret function $Q(\cdot)$ has to be linear.

To this end, it is sufficient to show $Q(x+y) = Q(x) + Q(y)$. Let us find $Qx, x', x'' \in X$ with $x > x' > x''$, then according to monotonicity we have $\{(x,x''), \frac{1}{3}; (x',x'), \frac{1}{3}; (x'',x), \frac{1}{3}\} \in \Pi/\widehat{\Pi}$. By conditional swapping independence, we have $\{(x,x'), \frac{1}{3}; (x',x''), \frac{1}{3}; (x'',x), \frac{1}{3}\} \in \Pi/\widehat{\Pi}$.[7] Therefore, $Q(u(x') - u(x)) + Q(u(x'') - u(x')) = Q(u(x'') - u(x))$. Therefore, $Q(\cdot)$ must be linear. Since it is also monotonically increasing, $\phi(x,y) = c(u(x) - u(y))$ for some $c > 0$. □

[7]The design of lottery reflects proposition 1 in Loewenfeld and Zheng (2023)

A.4 Additional Experimental Results

A.4.1 Completeness Measure

Fudenberg et al. (2022) introduce the completeness measure of a model as follows:

$$\frac{\mathscr{E}_{\text{base}} - \mathscr{E}_{\text{model}}}{\mathscr{E}_{\text{base}} - \mathscr{E}_{\text{irreducible}}},$$

where $\mathscr{E}_{\text{base}}$, $\mathscr{E}_{\text{model}}$, and $\mathscr{E}_{\text{irreducible}}$ are the out-of-sample prediction errors from the baseline model, model of interest, and the best model. Specifically, given observations $(x,y) \in X \times Y$, a prediction rule f is a mapping from explanatory variables X to dependent variables Y. Given a prediction rule f, a cost function $l : Y \times Y \mapsto \mathbb{R}_{+}$, and a joint distribution P over $X \times Y$, the expected prediction error of f is $\mathbb{E}_P[l(f(x),y)]$. Let us denote the baseline model's prediction rule as f_b and the best prediction rule as f^*. For current exercise, $l(x,y) = (x-y)^2$. Hence $f^*(x) = \mathbb{E}[Y|X=x]$. Since current experiment only concerns qualitative properties the prediction rules for regret and salience theories, f_r and f_s, are uniquely defined. For regret theory, X is subjects' choices from negatively correlated tasks and Y is their decisions in positively correlated tasks. Based on swapping independence, $f_r(x) = x$. For salience theory, X is the value of $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$ based on subjects' choices from normal and low contrast tasks while Y is the value of $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ based on their decisions from high and normal contrast tasks. Since all axioms of salience theory posits that $y \leq x$, we define $f_s(x)$ as $\min\{\mathbb{E}[Y|X=x],x\}$, that is they are only able to rationalize y given it is smaller than x. Lastly, because the experimental settings for regret and salience are different, we define separate baseline prediction rules for each theory. First, for regret theory, the baseline $f_b = \min_{y\in Y} y$, which is the smallest possible value in positively correlated tasks. In contrast, for salience theory, we let $f_b(x) = x$, which reflects the hypothesis from Expected Utility Theory: $x = y$.

To estimated the out-of-sample performance for each theory, we follow the procedure proposed in Fudenberg et al. (2020):

1. Given our 516 sample, we randomly divide into 10 groups with group 1 has 66 subjects, and remaining groups have 50 subjects. Denote these groups by $G_1, G_2, \ldots, G_{10}$.

2. Fix a group G_i, we train the prediction rules based on subjects that are not in this group. In our specification, $\mathbb{E}[Y|X=x] = \frac{1}{|G_{-ix}|} \sum_{j \in G_{-ix}} y_j$, where G_{-ix} represents the subjects not in group i that has $X = x$.

3. Calculate the average out-of-sample errors: $e_i(f) = \frac{1}{|G_i|} l(f(x_i), y_i)$.

4. Repeat steps 2 and 3 for each group and estimate

$$\mathscr{E}_{\text{base}} = \frac{1}{10}\sum_{i=1}^{10} e_i(f_b),\ \mathscr{E}_{\text{model}} = \frac{1}{10}\sum_{i=1}^{10} e_i(f_m),\ \text{and } \mathscr{E}_{\text{irreducible}} = \frac{1}{10}\sum_{i=1}^{10} e_i(f^*).$$

A.4.2 Correlation Dependence

We use pair 3 payment-variation tasks to test the general correlation sensitivity. Figure A.1 presents the CDF of ΔX. The results suggest that there is a significant choice reversal between the two questions. For Po_a population, 27.5% subjects exhibit correlation-insensitive behaviors in this pair while for Po_s population, 38% subjects exhibit such behaviors. The percentage of preference reversals for this pair is indeed lower than the percentages in both pairs 1 and 2. This result implies that preference consistency in pairs 1 and 2 cannot be fully explained by correlation insensitivity. Table A.1 presents the aggregate results for ΔX in pair 3 payment-variation task. Therefore, from an aggregate level, even the deviations are significant, their magnitudes on average are small (less than \$1). Additionally, we find no significant correlation between violation of swapping independence and correlation insensitivity.[8]

It is important to note that, however, unlike pairs 1 and 2, in pair 3 the ratio of winning states from options A and B are significantly distinct: in positively correlated task, the ratio is 1

[8] Let us denote ΔX_1, ΔX_2, and ΔX_3 as the ΔX levels for pairs 1, 2, and, 3 respectively. $\mathbb{P}(\Delta X_1 = 0|\Delta X_3 = 0) - \mathbb{P}(\Delta X_1 = 0|\Delta X_3 \neq 0) = 11.22\%$ and $\mathbb{P}(\Delta X_2 = 0|\Delta X_3 = 0) - \mathbb{P}(\Delta X_2 = 0|\Delta X_3 \neq 0) = 4.1\%$ Both increments are not significant according to Kolmogorov-Smirnov tests.

while in the negatively correlated task, the ratio is $\frac{1}{3}$. Therefore, the observed violations may be a combined effect from both correlation sensitivity and even-splitting.

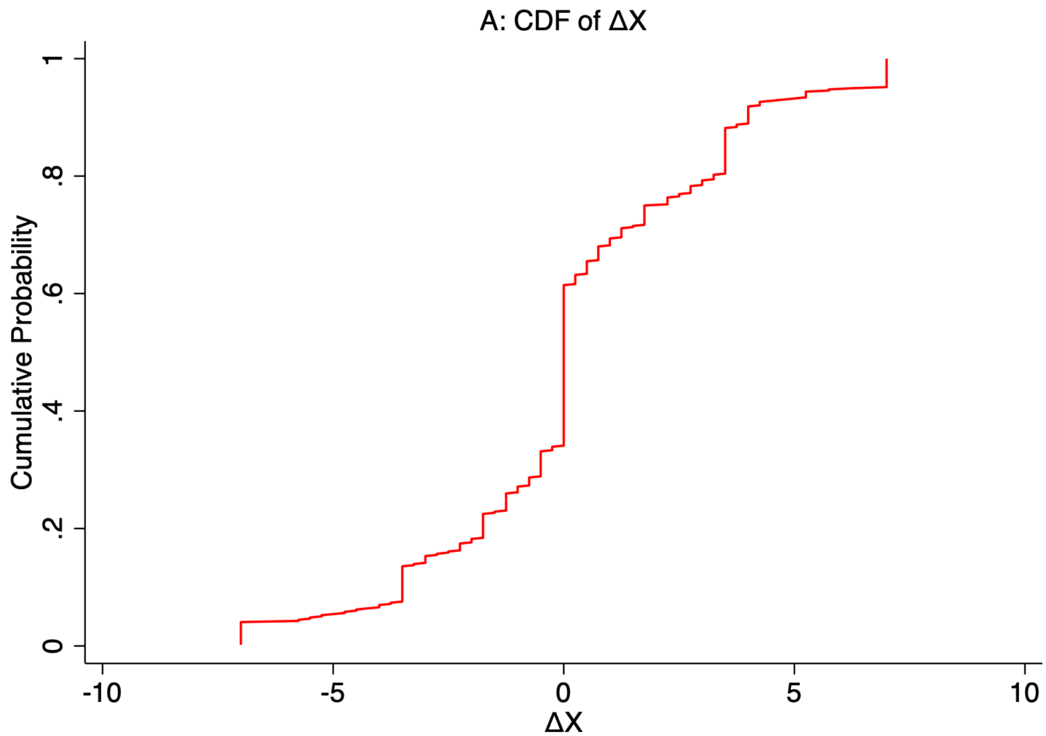


Figure A.1. CDF of ΔX from Pair 3 payment-variation Task

Note: This graph presents CDF of $\Delta X = X_{pos} - X_{neg}$ for Po_a subjects in Pair 3 payment-variation tasks.

Table A.1. Results for Pair 3 payment-variation Tasks

	(1)	(2)	(3)	(4)
ΔX	0.39**	0.53**	0.63***	0.85***
	(0.16)	(0.21)	(0.22)	(0.30)
$\mathbb{1}_{group=2}$			-0.50	-0.67
			(0.32)	(0.43)
N	516	263	516	263

Note: Summary of tests for correlation independence. Each test is performed under Po_s and Po_a and provides estimates for the mean differences in the switching value X (X in positively correlated task - X in negatively correlated task) obtained from interval regressions (Stewart, 1983). In addition, columns 3 - 4 present ΔX at group level with $\mathbb{1}_{group=2}$ being the indicator variable for being in the second group. The standard errors are included in parentheses.

A.4.3 Aggregate Experimental Results

Before we dive into the axioms for salience and regret theories, we summarize the results regarding strong independence.[9] The current experiment provides mixed results. On one hand, for both pairs of tasks, when the outcomes are negatively correlated, manipulating both options' identical payments does not impact the aggregate behavior. On the other hand, for both task pairs, when the outcomes are positively correlated, such manipulations may elicit significant differences between the two groups with subjects from Po_S. Figure A.2 summarizes their behaviors. What stands out from the figure is that for subjects in Po_S, increasing payments in states seems to encourage them to choose option B, which is the option with a changing payment, across different X values. Table A.2 reports the mean difference of the switching payments values between the two groups.

We now briefly discuss the mixed results. This pattern is in line with the failure of contingent reasoning (Niederle and Vespa, 2023). Nevertheless, this abnormality is usually observed when the underlying problem has subjective uncertainty (Ellsberg, 1961) and the state space is implicitly constructed by subjects (Esponda and Vespa, 2019), so it is surprising that we still observe this effect when all states are explicitly specified and highlighted in the representation. Furthermore, it is important to note that this discrepancy doesn't behave as noise. First, the violations are both observed from the subjects who never make multiple switches in our experiment and pass the two comprehension checks regarding monotonicity. Therefore, their choices are considered to have the highest qualities among our samples. Second, we only observe violations in negatively correlated tasks, so noise cannot explain such heterogeneity in treatments.

We believe that its impact on our subsequent results is minor and can be addressed. First, the significant violations are only observed among subjects in Po_S. Although these two populations are the subjects with least noise, they only consist of around 16% of the whole

[9]The violation of transitivity will become self-evident from other tasks. Therefore, we delegate the discussion of pair 3 tasks to the Appendix.

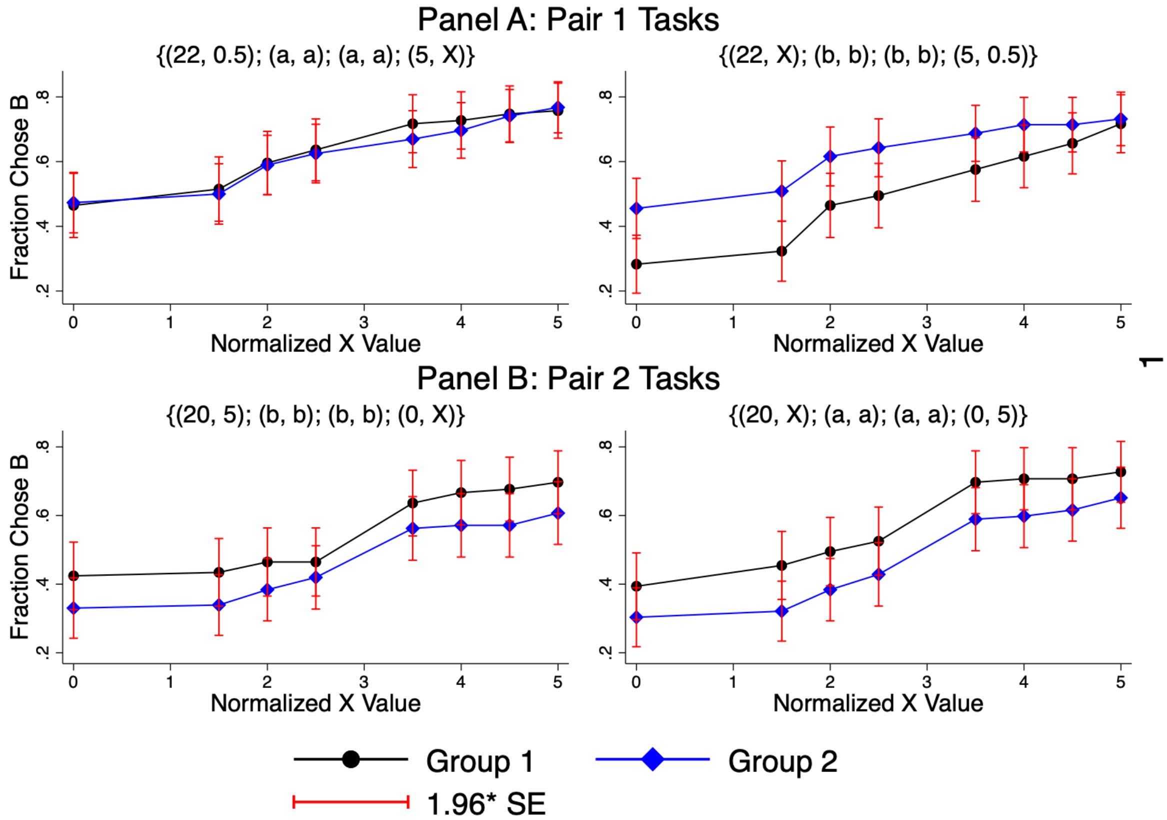


Figure A.2. Independence Axiom

Note: The current figure records group-wise average behaviors for the four payment-variation tasks. Subjects used in this figure follow monotonicity and never exhibit multiple switching points in payment-variation tasks (N = 211). Group 1 has 99 subjects while group 2 has 112 subjects. The subtitles represent the pair numbers of joint distributions. For all four tasks, $a = 1.5$ for subjects in group 1 while $a = 1$ for subjects in group 2. $b = 1$ for subjects in group 1 while $b = 1.5$ for subjects in group 2. Horizontal-axis represents values of X in each joint distribution normalized by subtracting the smallest X value in each task. Vertical-axis presents proportion of subjects choosing option B. 95% CI are constructed at each point.

population. Therefore, it is possible that this is not a representative sample group for the large population. Second, this effect, if it indeed has a persistent impact, works in contradictory directions for subjects from different groups. Since subjects in our experiment are randomly assigned to the groups, the average treatment effect should offset this potential confounder. In addition, for relevant tasks, we can still perform a between-group analysis fixing the common payments amount. Therefore, while we acknowledge its existence, our current experiment will remain largely unaffected. Since independence is not our main focus, we leave further discussions to future research.

Table A.2. Independence

	Panel A Negatively Correlated				Panel B Positively Correlated			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
ΔX	0.07	0.06	0.40	0.59	-0.30	-0.85**	0.38	0.69*
	(0.25)	(0.40)	(0.25)	(0.41)	(0.25)	(0.39)	(0.25)	(0.39)
Common Payments:								
Group 1:	1.5	1.5	1	1	1	1	1.5	1.5
Group 2:	1	1	1.5	1.5	1.5	1.5	1	1
Population:	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s
Task Pair	1	1	2	2	1	1	2	2
N	516	211	516	211	516	211	516	211

Note: Summary of tests for strong independence. The results are estimates for the mean between-group differences (Group 2 - Group 1) in the switching value X obtained from interval regressions (Stewart, 1983) of subjects' decisions on the indicator of group identity. Each test is performed on the two populations. Panel A records between-group results negatively correlated tasks from pairs 1 and 2 while Panel B provides results for positively correlated tasks. Standard errors clustered at individual level and are shown in parentheses.

A.4.4 Aggregate Results for Regret Theory

We now turn our attention to swapping independence, and the estimations are recorded in table A.3. Panel A performs a between-group test over $\Delta X = X_{pos} - X_{neg}$ fixing the amount of payments under states in which the two options are identical. From the results, we observe a suggestive pattern that subjects exhibit higher risk tolerance if the options are negatively correlated. To see this, In pair 1, $\{(22,X);\ (a,a);\ (a,a);\ (5,0.5)\}$, option B has higher variances for X values in the tasks. Since $\Delta X = X_{pos} - X_{neg}$, where X_{pos} and X_{neg} represent the switching X values when the two options are positively and negatively correlated, a positive ΔX value implies that subjects switch to option B at lower X values when the two options are negatively correlated, and hence they exhibit higher risk tolerance. In contrast, for pair 2 questions, so option B has lower variances, so a negative ΔX would reach the same conclusion. Nevertheless, such pattern is not robustly significant across all population groups and identical payments values.

Table A.3. Swapping Independence

	Panel A				Panel B			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
ΔX								
Pair 1:	0.48*	0.88**	0.25	0.08	0.63***	0.93**	0.11	0.02
	(0.25)	(0.39)	(0.25)	(0.40)	(0.42)	(0.74)	(0.41)	(0.90)
Pair 2:	0.31	0.54	-0.47*	-0.74*	-0.10	-0.15	-0.09	-0.05
	(0.25)	(0.40)	(0.25)	(0.40)	(0.24)	(0.39)	(0.22)	(0.34)
Common Payments:								
Group 1:	1	1	1.5	1.5	-	-	-	-
Group 2:	1	1	1.5	1.5	-	-	-	-
Subjects' Group:	-	-	-	-	1	1	2	2
Population:	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s
N	516	211	516	211	260	99	256	112

Note: Summary of tests for swapping independence. Each test is performed under the four populations. The results are estimates for the mean differences in the switching value X ($X_{pos} - X_{neg}$) obtained from interval regressions (Stewart, 1983) of subjects' decisions on either the indicator of group identity (Panels A) or the indicator of payments correlations (Panel B). Standard errors are clustered at individual level.

To complete the investigation for swapping independence, panel B in Table A.3 shows the results on the impact of correlation changes on the switching X values within each group. The results indicate that there exist heterogeneous treatment effects between groups. This discrepancy can be explained by the combined effects from both violations of strong independence and swapping independence. The relations among the results are summarized in Figure A.3. As an illustration, let us consider pair 1 tasks for subjects in group 1 and group 2. Since option B is the more risky option in these two tasks, according to the suggestive pattern that subjects are more risk tolerant in negatively correlated tasks, we have $\Delta X = X_{pos} - X_{neg} > 0$. Furthermore, according to the other suggestive pattern wherein subjects are more willing to switch if the common payments from both options are higher, for subjects in group 1 this will increase ΔX because for them the common payment is \$1.5 in pair 1 negatively correlated task while \$1 in current positively correlated tasks. In contrast, this pattern suggests Δ decreases for subjects from group 2 because the common payment is \$1.5 for them in the current positively correlated task while \$1 in negatively correlated task. As the result, the two forces are working collectively for subjects in group 1 while they are working inconsistently for subjects in group 2, and hence, result in heterogeneous treatment effect. In addition, the average treatment effect is documented in table A.3. In general the conclusion from average treatment effects and the previous between-group analysis are consistent: At the aggregate level, while there is evidence for violation of swapping independence, but the violation is not robustly significant across different scenarios.

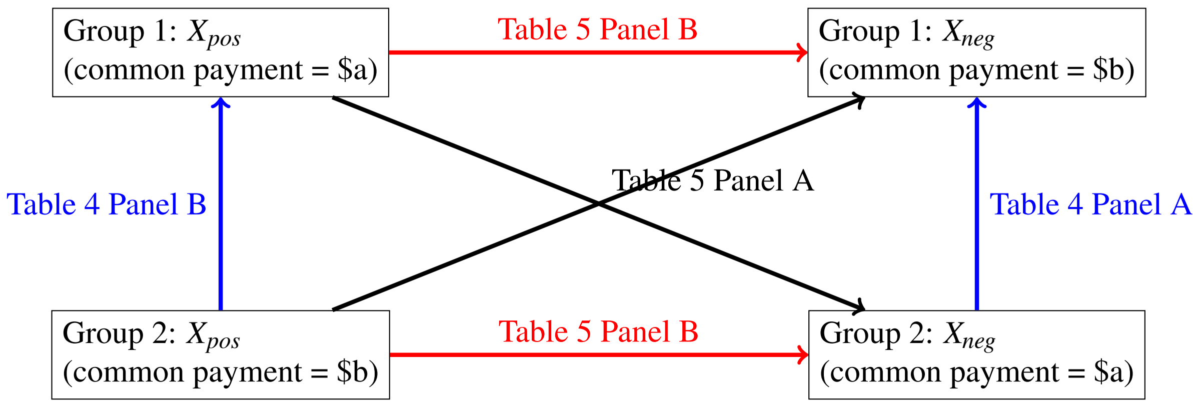


Figure A.3. Results Diagram

Note: X_{pos} is the switching X value for tasks with positively correlated outcomes. X_{neg} is the switching X value for tasks with negatively correlated outcomes. The arrows presenting a subtracting relation between the boxes. For instance the blue arrows always point from Group 2 to Group 1, so they suggest that table A.2 panel B reports ΔX for Group 2 - Group 1. For the common payments of pair 1 tasks, $a = 1$, $b = 1.5$ for subjects in group 1 while $a = 1.5$, $b = 1$ for subjects in group 2. For the common payments of pair 2 tasks, $a = 1.5$, $b = 1$ for subjects in group 1 while $a = 1$, $b = 1.5$ for subjects in group 2.

A.4.5 Aggregate Results for Salience Theory

We now move on to the three axioms for salience theory: skewness prereference, diminishing relative sensitivity, and k-regular. Figure A.4 plots the CDF of $\ln(\frac{q}{p}_H)$, $\ln(\frac{q}{p}_N)$, and $\ln(\frac{p}{q}_L)$, which is the negative of $\ln(\frac{q}{p}_L)$, for Po_a subjects. To intuitively visualize the hypothesis, notice that the higher the CDF's position, the lower the average value of the variable. Therefore, if the CDF of $\ln(\frac{p}{q}_L)$ is higher than that of $\ln(\frac{q}{p}_H)$, the inequality roughly requires that the CDF of $\ln(\frac{q}{p}_N)$ is closer to the CDF of $\ln(\frac{q}{p}_H)$. In contrast, the requirement would be reversed in case the relative position between CDF of $\ln(\frac{p}{q}_L)$ and $\ln(\frac{q}{p}_H)$ switches.

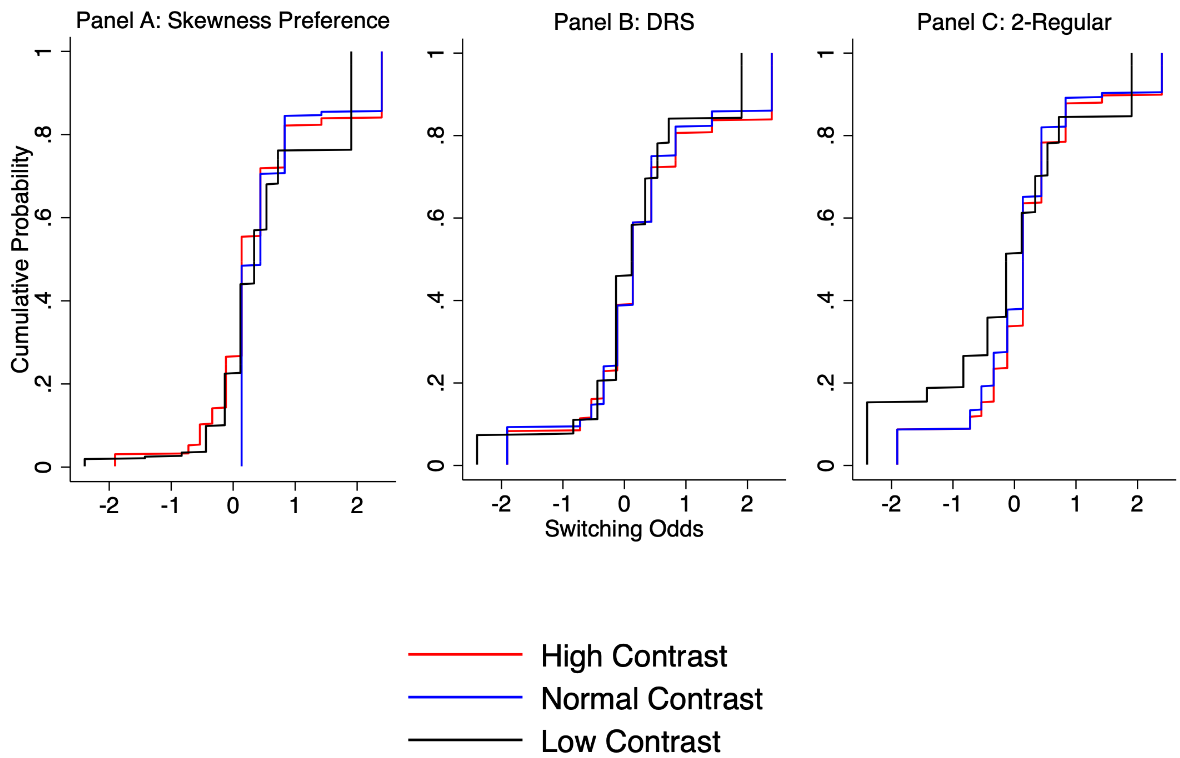


Figure A.4. CDF of Switching Odds

Note: The *cumulative distribution functions* of $\ln(\frac{q}{p}_H)$ (red), $\ln(\frac{q}{p}_N)$ (blue), and $\ln(\frac{p}{q}_L)$ (black), which is the negative of $\ln(\frac{q}{p}_L)$, are shown in the graph. Panel A depicts the three CDF for tasks testing Skewness Preference. Panel B draws the CDF for tasks relevant to diminishing relative sensitivity. Pancel C shows the CDF from 2-Regular tasks.

Table A.4 presents the results for the three axioms of salience theory. In general, the aggregate behaviors follow the salience theory's predictions. To understand the magnitudes, let us consider the tasks testing skewness prereference. Recall the analysis from subsection 5.1, $\frac{\frac{q}{p}_H / \frac{q}{p}_N}{\frac{q}{p}_N * \frac{q}{p}_L}$ provides an upper bound for $\frac{f(\sigma(30,0))}{f(\sigma(15,0))}$. Taking statistics from Po_s as an illustration, the results suggest that the salience level between \$15 and \$0 is *at most* $exp(-0.29) \approx 0.75$ (0.05) of the salience level between \$30 and \$0. Similarly, for population P2, the salience level ratio $\frac{f(\sigma(30,15))}{f(\sigma(15,0))}$ is bound above by $exp(-0.28) \approx 0.76$ (0.06), and $\frac{f(\sigma(10,0))}{f(\sigma(30,10))}$ is bounded by $exp(-0.26) \approx 0.77$ (0.07). Therefore, at the aggregate level, current experiment's tests find supportive evidence for salience theory's axioms.

Table A.4. Salience Axioms

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln \frac{q}{p}_H$	0.52	0.43	0.65	0.43	0.12	0.23
	(0.04)	(0.04)	(0.13)	(0.05)	(0.11)	(0.04)
$\ln \frac{q}{p}_N$	0.62	0.52	0.55	0.39	0.12	0.12
	(0.03)	(0.03)	(0.12)	(0.05)	(0.11)	(0.04)
$\ln \frac{q}{p}_L$	-0.35	-0.33	-0.10	-0.08	0.38	0.25
	(0.03)	(0.03)	(0.12)	(0.05)	(0.13)	(0.06)
Main Test:	-0.38***	-0.29***	-0.34*	-0.28***	-0.49***	-0.26***
	(0.07)	(0.08)	(0.19)	(0.09)	(0.17)	(0.08)
$\ln \frac{q}{p}_H - \ln \frac{q}{p}_N$:	-0.10**	-0.09*	0.11	0.03	0.00	0.11**
	(0.04)	(0.05)	(0.11)	(0.05)	(0.10)	(0.05)
$\ln \frac{q}{p}_H + \ln \frac{q}{p}_L$:	0.17***	0.10*	0.56***	0.35***	0.50***	0.49***
	(0.05)	(0.06)	(0.14)	(0.07)	(0.12)	(0.07)
N	516	350	516	350	516	350

Note: The results are estimates for the mean ln values of switching winning odds obtained from interval regressions (Stewart, 1983) of subjects' decisions on indicators of question type. Columns (1) and (2) report results for skewness preference. Columns (3) and (4) show results for diminishing relative sensitivity. Columns (5) and (6) record results for 2-regular. $\ln(\frac{q}{p}_H)$, $\ln(\frac{q}{p}_N)$, and $\ln(\frac{q}{p}_L)$ for each axiom are estimated. Statistics for $\ln(\frac{q}{p}_H) - \ln(\frac{q}{p}_N) - \ln(\frac{q}{p}_N) - \ln(\frac{q}{p}_L)$ are obtained by delta method and reported under main test. Statistics fo $\ln(\frac{q}{p}_H) - \ln(\frac{q}{p}_N)$ and $\ln(\frac{q}{p}_H) + \ln(\frac{q}{p}_L)$ are also documented. Standard errors are cluster at individual level in parentheses.

We now briefly discuss other auxiliary tests. As shown in subsection 5.1, salience theory also suggests that in all three classes of tasks, we should observe $\frac{q}{p}_H \geq \frac{q}{p}_N \geq \frac{p}{q}_L$. The relevant results are also included in Table A.4. In large, the aggregate level results are in line with our individual analysis.

A.5 Additional Tables

Table A.5. Salience Axioms Excluding Noise in High Contrast Problem

	(1)	(2)	(3)	(4)	(5)	(6)
Axiom Test:	-0.18***	-0.14**	-0.18**	-0.17*	-0.29***	-0.20**
	(0.06)	(0.07)	(0.08)	(0.09)	(0.08)	(0.08)
Auxiliary Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-0.08*	-0.07	0.05	0.05	0.09**	0.13***
	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	0.13***	0.11**	0.27***	0.27***	0.48***	0.47***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
N	506	342	506	342	506	342
Sign Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-	-	+	+	+	+
p-value	0.094	0.053	0.488	0.423	0.021	0.005
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	+	+	+	+	+	+
p-value	0.040	0.372	0.000	0.000	0.000	0.000

Note: This table summarizes the mean values of differences in ln winning odds using interval regression. Columns (1) and (2) present results from skewness preference for both Po_a and Po_s populations. Columns (3) and (4) present results from diminishing relative sensitivity. Columns (5) and (6) present results from 2-Regular. The axiom test records result for $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$. In Auxiliary tests, $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ and $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$ are presented using interval regression. Furthermore, sign tests are also performed. In the table, sign tests record the sign of dominant group and corresponding p-value.

Table A.6. Correlations of Salience Tasks Responses

Panel A: Po_a Population

	Skewness Preference	DRS	2-Regular
Skewness Preference	1		
DRS	0.1285 (0.0038)	1	
2-Regular	0.1329 (0.0027)	0.1329 (0.0027)	1

Panel B: Po_s Population

	Skewness Preference	DRS	2-Regular
Skewness Preference	1		
DRS	0.2103 (0.0001)	1	
2-Regular	0.1875 (0.0005)	0.14334 (0.0079)	1

Note: Pearson correlations among values of $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$ across the tests for three axioms of salience theory. Panel A presents correlations for Po_a population while Panel B presents correlations for Po_s population.

Table A.7. Robustness Check: Ordering Effect

Panel A: Regret Theory

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Task Pair	1	2	1	2	1	2	1	2
ΔX	0.72***	0.28	0.81**	0.18	0.88**	0.51	1.04*	0.15
	(0.25)	(0.25)	(0.40)	(0.40)	(0.36)	(0.35)	(0.58)	(0.60)
$\Delta X \times \mathbb{1}_{group=2}$					-0.32	-0.46	-0.43	0.03
					(0.50)	(0.50)	(0.79)	(0.81)
Population	Po_a	Po_a	Po_s	Po_s	Po_a	Po_a	Po_s	Po_s
N	516	516	211	211	516	516	211	211

Panel B: Salience Theory

	(1)	(2)	(3)	(4)	(5)	(6)
Main Test	-0.18	-0.03	-1.27***	-0.83***	-0.94*	-0.85***
	(0.16)	(0.22)	(0.42)	(0.21)	(0.54)	(0.23)
$\ln(\frac{q}{p}_H) - \ln(\frac{q}{p}_N)$	0.08	0.42***	-0.22	-0.14	0.02	-0.17
	(0.10)	(0.13)	(0.24)	(0.11)	(0.30)	(0.12)
$\ln(\frac{q}{p}_H) + \ln(\frac{q}{p}_L)$	0.33***	0.86***	0.84***	0.55***	0.98***	0.52***
	(0.09)	(0.13)	(0.32)	(0.13)	(0.30)	(0.13)
Population:	Po_a	Po_s	Po_a	Po_s	Po_a	Po_s
N	516	350	516	350	516	350

Note: This table is the replication of Table 1.4 and 1.5 controlling ordering effect. In Panel A, the average difference of ΔX is presented for pairs 1 and 2. For each pair, ΔX is computed by subtracting the average of X_{neg} over subjects who encountered the negatively correlated task first from the average of X_{pos} over subjects who answered the positively correlated task first. In Panel B, for each task group, the averages of winning odds from High, Normal, and Low contrast tasks are computed from subjects who countered the corresponding versions first.

Table A.8. Salience Axioms With Censored Data

	(1)	(2)	(3)	(4)	(5)	(6)
Axiom Test:	-0.25**	-0.21*	-0.35***	-0.37***	-0.41***	-0.32**
	(0.10)	(0.11)	(0.11)	(0.14)	(0.12)	(0.13)
Auxiliary Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-0.22***	-0.24***	-0.01	-0.03	0.12*	0.18**
	(0.06)	(0.07)	(0.06)	(0.07)	(0.06)	(0.07)
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	0.13	0.08	0.30***	0.25***	0.69***	0.72***
	(0.08)	(0.09)	(0.08)	(0.09)	(0.09)	(0.10)
N	516	350	516	350	516	350
Sign Tests:						
$\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$	-	-	+	+	+	+
p-value	0.027	0.005	0.859	0.962	0.012	0.002
$\ln(\frac{q}{p_H}) + \ln(\frac{q}{p_L})$	+	+	+	+	+	+
p-value	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table summarizes the mean values of differences in ln winning odds using interval regression. Columns (1) and (2) present results from skewness preference for both Po_a and Po_s populations. Columns (3) and (4) present results from diminishing relative sensitivity. Columns (5) and (6) present results from 2-Regular. The axiom test records result for $\ln(\frac{q}{p_H}) - 2\ln(\frac{q}{p_N}) - \ln(\frac{q}{p_L})$. In Auxiliary tests, $\ln(\frac{q}{p_H}) - \ln(\frac{q}{p_N})$ and $\ln(\frac{q}{p_N}) + \ln(\frac{q}{p_L})$ are presented using interval regression. Furthermore, sign tests are also performed. In the table, sign tests record the sign of dominant group and corresponding p-value.

Table A.9. Participant Demographics

	Total	Group 1	Group 2
Number of Participants	792	397	395
Completion Time (Minutes)	36.5	36.6	36.4
Number of Approvals	1993	1949	2038
Student	68	34	34
Age	39.6	39.2	40.1
Sex			
Female	397	189	208
Male	395	208	187
Race			
Asian	56	24	32
Black	85	43	42
Mixed	61	27	34
White	561	290	271
Other	16	4	12
N/A	13	9	4
Employment Status			
Full-Time	256	153	151
Starting Job Next Month	3	3	0
Not in Paid Work	56	22	34
Unemployed and Job Seeking	70	38	32
Other	28	14	14
N/A	304	153	151
Completion Time (Seconds)			

Note: Participant demographics for 792 out of 800 participants. The demographic data for the remaining 8 subjects are lost due to incorrect submissions.

Table A.10. Participant Completion Times

	Total	Group 1	Group 2
Average Completion Time (Seconds)			
Task 1	86.3	86.3	86.2
Task 2	75.3	73.2	77.3
Task 3	71.7	70.2	73.1
Task 4	73.5	72.5	74.4
Task 5	71.6	71.1	72.1
Task 6	65.0	63.2	66.9
Task 7	82.8	81.3	84.3
Task 8	76.8	73.0	80.6
Task 9	74.0	70.2	77.8
Task 10	75.6	69.3	81.9
Task 11	70.8	70.1	71.6
Task 12	73.3	71.6	75.0
Task 13	73.4	71.7	75.0
Task 14	75.9	69.3	82.6
Task 15	71.9	67.4	76.5
Task 16	75.2	71.5	78.8

Note: Participant Completion time in seconds. Due to technical issues, the completion times of 5 subjects are not recorded. Additionally, two subjects' time counter resets at the end of each round, which prevents their completion time for the last question in each task being recorded. We use their times spent on second to the last question in each round as approximations.

Appendix B

Supplement Materials for Chapter 2

B.1 Additional Elements of Theoretical Axiomatization

Our theoretical development is constructed with an assumption of differentiability (assumption 1). We now present a technical axiom that assures this differentiability (axiom 10). The technical consequences of this assumption are described in a subsequent lemma. We focus on the decision-maker's preference over sequences of lotteries with statewise converging outcome pairs. The axiom requires that the decision-maker's preferences between the options in such sequences should converge to indifference at appropriate rates. Before stating the axiom, we introduce three pieces of notation. First, let $||\cdot||$ be the Euclidean norm on $\mathbb{R}^n$. Second, given a finite sequence $\{x_i\}_{i=1}^n$ over $\mathbb{R}^n$, define the *sequence length* as $\sum_{i=1}^{n-1} ||x_{i+1} - x_i||$. Third, define $mesh(\{x_i\}_{i=1}^n) = \max_{i=1,\ldots,n-1} ||x_{i+1} - x_i||$.

Axiom 25 (Regularities at Indifference)**.** *Given a compact set K on X and a positive sequence $\{\alpha_i\}_{i=1}^n$ on $\mathbb{R}_+$ converging to 0, then for all outcome pairs $x, y \in K$ with $x \sim_u y$ and unit vector e_i, if $\{p_n\}_{n=1}^\infty$ are the probabilities such that $\{(x + \alpha_n e_i, y), 1 - p_n; (0, e_1), p_n\} \in \Pi \setminus \hat{\Pi}$ for all n, we have $\frac{p_n}{\alpha_n}$ and $\frac{\alpha_n}{p_n}$ converges uniformly for every x, y in K.*

The consequence of axiom 10 is summarized below:

Lemma 4. *A correlation sensitive preference satisfies axioms 10 if and only if $\phi(x, y)$ is uniformly differentiable at points $x \sim_u y$.*

Proof. See Appendix B.5.1 □

Our next axiom speaks to the existence of utility, $u(\cdot)$, in a smooth salience representation. Such a utility function exists if and only if there exists a utility function representing the preference over deterministic outcomes satisfying axiom 8. To formalize this observation, we construct the following "*local*" lotteries. For every outcome pair x and y, we define $L^n_{(x,y)}$ as

$$\{(x, x+\frac{1}{n}(y-x)), \frac{1}{n};\ (x+\frac{1}{n}(y-x), x+\frac{2}{n}(y-x)), \frac{1}{n}; \ldots; (x+\frac{n-1}{n}(y-x), y), \frac{1}{n}\}.$$

Intuitively, $L^n_{(x,y)}$ first divides the line segment from x to y into n segments of the same length, and in each state it moves outcome pairs from the endpoints of one segment to the endpoints of the next. Notice that a smooth salience representation assigns $L^n_{(x,y)}$ the value $\sum_{i=1}^{n} f(\sigma(x+\frac{i-1}{n}(y-x), x+\frac{i}{n}(y-x)))(u(x+\frac{i-1}{n}(y-x))-u(x+\frac{i}{n}(y-x)))$. Consequently, as n approaches infinity, the value is approximately $f(0)(u(y)-u(x))$, which eventually separates out the salience distortion from the utility difference. This argument shows that $\{(a_1,a_2),p;(b_1,b_2),1-p\} \in \hat{\Pi}_{\mathbb{E}(u)}$ if and only if $pL^n_{(a_1,a_2)} + (1-p)L^n_{(b_1,b_2)} \in \widehat{\Pi}$ for all but finitely many n. Therefore, $\Pi_{\mathbb{E}(u)}$ can be approximated by elements inside Π generated by a mixture of lotteries $L^n_{(\cdot,\cdot)}$. Our next axiom suggests that when inspecting properties of $L^n_{(\cdot,\cdot)}$, the preference approximately follows EU.

Axiom 26. *For every $x,y \in X$, let $\pi^n_{(x,y)} = \frac{1}{3}L^n_{(0,x)} + \frac{1}{3}L^n_{(x,y)} + \frac{1}{3}L^n_{(y,0)}$ and p_n be the probability s.t. $p_n\pi^n_{(x,y)} + (1-p_n)\delta_{(e_1,0)} \in \Pi \setminus \widehat{\Pi}$ or $p_n\pi^n_{(x,y)} + (1-p_n)\delta_{(0,e_1)} \in \Pi \setminus \widehat{\Pi}$, then $p_n \to 1$ as $n \to \infty$.*

Lemma 5. *Given assumption 1, a correlation sensitive preference satisfies axioms 1, 7, and 11 if and only if $\phi(x,y) = f(\sigma(x,y))(u(x)-u(y))$ where $u(\cdot)$ is continuously differentiable and has positive partial derivatives while $f \circ \sigma$ is positive, symmetric, continuous, and obtain its global minimum at all points (x,y) such that $x = y$.*

Proof. See Appendix B.5.1 □

B.2 Extensions to Context-Dependent Preferences

Following settings from the context-dependent preferences literature, we allow decision-makers to formulate flexible preferences over deterministic outcomes under different choice sets. Formally, let there be N options in the consumption set, each choice C_i is an element of $l_0(\mathbb{R})$, which is the set of sequences on real numbers with finitely many non-zero terms. We assume that there is a utility representation $u : \Pi_{i=1}^N l_0(\mathbb{R}) \to \mathbb{R}^N$. One can think of u as a collection of utility indices over N options, such that option C_i would be chosen from the choice set $\{C_k\}_{k=1}^N$ if $u_i(\{C_k\}_{k=1}^N)$, the i^{th} entry of $u(\{C_k\}_{k=1}^N)$, obtains the highest value of all $u(\{C_k\}_{k=1}^N)$. We assume u is continuous w.r.t. the product norm. Given $A \in l_0(\mathbb{R})$, denote $RA = (0, A_1, A_2, \ldots)$. Now, we impose some regularities on u:

R1. (*Absolute Monotonicity*) If $C_i \geq C_j$, $u_i(C_1, C_2, \ldots, C_N) - u_j(C_1, C_2, \ldots, C_N) \geq 0$ with strict inequality if $C_i > C_j$.

R2. (*Comparative Monotonicity*) If $C_1' \geq C_1$, then
$u_1(C_1', C_2, \ldots, C_N) - u_j(C_1', C_2, \ldots, C_N) \geq u_1(C_1, C_2, \ldots, C_N) - u_j(C_1, C_2 \ldots, C_N)$ for all j.

R3. (*Diminishing Sensitivity*) If $C_i \geq 0$ for all i, and there exists C_m such that $C_1 \geq C_m$ then for all $\xi \geq 0$ at least one of the following holds:

(a) (*Pairwise Diminishing Sensitivity*)
$|u_1(C_1 + \xi, C_2, C_3, \ldots, C_m + \xi, \ldots, C_N) - u_m(C_1 + \xi, C_2, C_3, \ldots, C_m + \xi, \ldots, C_N)|$ is decreasing in ξ.

(b) (*Uniform Diminishing Sensitivity*)
$|u_1(C_1 + \xi, C_2 + \xi, \ldots, C_N + \xi) - u_m(C_1 + \xi, C_2 + \xi, \ldots, C_N + \xi)|$ is decreasing in ξ.

R4. (*Reflexive*) For all collections $\{C_k\}_{k=1}^N, \{S_k\}_{k=1}^N$ in $l_0(\mathbb{R}_+)$ and $i, j \leq N$,
$u_i(C_1, \ldots, C_N) - u_j(C_1, \ldots, C_N) \leq u_i(S_1, \ldots, S_N) - u_j(S_1, \ldots, S_N)$ implies
$u_i(-C_1, \ldots, -C_N) - u_j(-C_1, \ldots, -C_N) \geq u_i(-S_1, \ldots, -S_N) - u_j(-S_1, \ldots, -S_N)$.

R5. (*Ordering Symmetry*) For all permutations σ over $\{1,\ldots,N\}$,

$u_i(C_1,C_2,\ldots) = u_{\sigma(i)}(C_{\sigma(1)},C_{\sigma(2)},\ldots)$

R6. (*Non-myopia*) When time is involved, as in our leading application, we impose one additional assumption: For all $\{C_i\}_1^N$, we have

$|u_1(RC_1,RC_2,\ldots)-u_j(RC_1,RC_2,\ldots)| \leq |u_1(C_1,C_2,\ldots)-u_j(C_1,C_2\ldots)|$ for all j, and there exists $\varphi \in l_0(\mathbb{R})$ such that:

$|u_1(RC_1+R\varphi,RC_2,\ldots)-u_j(RC_1+R\varphi,RC_2,\ldots)| \geq |u_1(C_1,C_2,\ldots)-u_j(C_1,C_2\ldots)|$.

We now give a brief discussion on these regularities. *Absolute monotonicity* implies each dimension of choices is an economic good since an increasing consumption in any dimension leads to a higher utility index.[1]

For a general context-dependent utility representation, an increase in one option may influence the utility assigned to others. Although we allow such possibilities, we retain a requirement that the increased option always becomes relatively more attractive. *Comparative monotonicity* guarantees that a measure of such relative attractiveness is provided by the utility differences between those options before and after the increment. In turn, *comparative monotonicity* imposes a certain degree of consistency on utility representations across consumption sets.

Diminishing sensitivity and *reflection* are analogous to the same components in definition 1. As shown in proposition 6, these two regularities lead deliver a connection between gain-loss attitudes and salience theory. Notice that when preferences are context independent, or there are only two alternatives in the consumption set, pairwise and uniform diminishing sensitivity are equivalent. Otherwise, these two are independent requirements. To the best of our knowledge,

[1]*Absolute monotonicity* may not hold under some circumstances, but one can always salvage this property by a change of measure. For example, if one is choosing a wine by comparing among different qualities and prices, one would prefer a lower price under similar quality. Nevertheless, we can change redefine the price attached to each wine as the consumer's remaining savings after corresponding purchase.

there is no consensus on a precise definition of diminishing sensitivity in general settings, so to keep the generality of the current presentation, we admit both criteria. *Ordering symmetry* is a technical assumption requiring that changes of the order assigned to options doesn't alter corresponding utility indices assigned to the options. One immediate implication is that we can always change the order of options so that the comparison between option i and j is equivalent to between option 1 and 2. Lastly, *Non-myopia* in intertemporal applications implies a weak form of time discounting – as we move all options to future, their differences become less significant, but we can always offset this discounting by changing some option's payoffs in future periods.

Models endowed with this representation include focusing (Koszegi and Szeidl, 2012), deterministic version of relative thinking (Bushong et al., 2021), a general version of salience for consumer choice (Bordalo et al., 2013b),[2] and pairwise normalization (Landry and Webb, 2021).[3]

Definition 5. *Let the consumption set be* $\mathscr{C} \equiv \{C_k\}_{k=1}^N$*, then:*

- *A preference obeys the focusing model of Koszegi and Szeidl (2012), if for each option* C_i*,* $u(C_i) = \sum_{n=1}^k \mu\left(\max_{C_i' \in \mathscr{C}} \{C_{in}'\} - \min_{C_i' \in \mathscr{C}} \{C_{in}'\}\right) \cdot C_{in}$*, where* C_{in} *is the* n^{th} *entry of option* C_i *and* $\mu(\cdot)$ *is an increasing and positive function. In addition* $\mu(x)x$ *is increasing in* x*.*

- *A preference obeys the relative thinking model of Bushong et al. (2021), if for each option* C_i*,* $u(C_i) = \sum_{n=1}^k \nu\left(\max_{C_i' \in \mathscr{C}} \{C_{in}'\} - \min_{C_i' \in \mathscr{C}} \{C_{in}'\}\right) \cdot C_{in}$*, where* C_{in} *is the* n^{th} *entry of option* C_i *and* $\nu(\cdot)$ *is an decreasing and positive function. In addition* $\nu(x)x$ *is decreasing in* x*.*

[2] The general structure is proposed in Herweg and Müller (2021). As shown in **?**, the original model in Bordalo et al. (2013b) may violate the absolute monotonicity requirement for some exogenous reference point. Furthermore, one can show that even with only two options (so that the reference point is fully endogenized), in the presence of three or more attributes, the model can also violate monotonicity, which we find less intuitive.

[3] More broadly, the categorical preference introduced by **?** doesn't fall into the current setting unless more restrictions are fulfilled. Specifically, the additional requirement is that for every $x \geq y$, regardless of their categories, we have $x \succeq_r y$ for all reference points, r.

- *A preference obeys the pairwise normalization model of Landry and Webb (2021), if for each option C_i, $u(C_i) = \sum_{n=1}^{k} \sum_{C_j \neq C_i} \frac{C_{in}}{C_{jn} + C_{in} + \beta}$ with the convention $\frac{0}{0} = 0$, where C_{in} is the n^{th} entry of option C_i, and $\beta \geq 0$. In addition, all attributes are required to be non-negative.*

We now show that the above models satisfy all regularities. Since time plays no role in these models, we can restrict our attention to options in $\mathbb{R}^k$ for some positive integer k instead of the whole $l_0(\mathbb{R})$ set. Given an option $C_i \in \mathbb{R}^k$ from some consumption set $\{C_k\}_{k=1}^N$, assuming all attributes, which are entries of C_i, are economic goods, it's clear that $R1$, *absolute monotonicity*, trivially holds. In addition, since in all models, there is some utility assigned to each choice, $R5$, *ordering symmetry*, also trivially holds. Also, by a direct check using definitions, one can show that all three models satisfy $R3$,[4] *diminishing sensitivity*, and $R4$, *reflection*.[5] Hence, the only nontrivial part is *comparative monotonicity*.

Lemma 6. *Focusing, relative thinking, and pairwise normalization all satisfy comparative monotonicity.*

Proof. See Appendix B.5.1. □

Unfortunately, the general salience theory of Bordalo et al. (2013b) doesn't satisfy comparative monotonicity. The reason is that when there are three or more nonconstant dimensions, the normalization of total salience weights makes the difference under various consumption sets incomparable. To circumvent this issue, we can use the form of salience theory proposed in Herweg and Müller (2021): given a consumption set $\mathscr{C}$, the utility of each option $u(C_i) = \sum_{n=1}^{k} f(\Delta_i^k) C_{ik}$, where Δ_i^k is the salience level of dimension k for option i, and f is an increasing function. One can easily show that this form satisfies all regularities.

In our definitions of the salience function, level, and decision weight, all evaluations are independent from the preference within each state. Therefore, for the predictions in example 1,

[4]Focusing and relative thinking satisfy uniform diminishing sensitivity while pairwise normalization satisfies pairwise diminishing sensitivity.

[5]For pairwise normalization, reflection is irrelevant.

2, and 3, so long as the preference between streams under the most salient state agrees with the discounted utility we assumed before, our predictions stay valid given the effect from salience is sufficiently large (i.e., that θ is small enough). It turns out that only *absolute monotonicity* and *non-myopic discounting* are needed for the preferences to align.

What if the independence between state salience and intrastate utility doesn't hold? For instance, small physical differences may produce enormous utility differences. If the decision-maker weights state salience according to utilities instead of physical units, will behavior be dramatically altered? To investigate this problem, we introduce the following proposition.

Proposition 11. *Let $d : \mathbb{R}_+ \to \mathbb{R}_+$ be a continuous and strictly increasing function. Suppose the context-dependent utility presentation u over the consumption set satisfies R1 to R5, then the function $\tilde{\sigma}$ defined by $\tilde{\sigma}(X,Y) = d(|u_1(X,Y) - u_2(X,Y)|)$ is a free salience function.*[6]

Proof. See Appendix B.5.1. □

Proposition 11 implies that the determination of state salience is flexible. One can replace classical utility theory with some other model that incorporates interstate salience comparisons. Since salience theory implies that the decision-maker places higher decision weight on larger differences, proposition 11 suggests that results from salience and other models mentioned above should be similar. For preferences under uncertainty, so long as outcome comparisons are made within each state, and each state's decision weight is adjusted accordingly, our results are qualitatively invariant.[7] While for future investigations, it is worthwhile to explore effects from combinations of behavioral sources, in this paper, we focused on multidimensional salience theory and keep the basic discounted utility formulation over deterministic payoff streams for subsequent empirical analysis.

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[6]Notice that $\hat{\sigma}(\cdot,\cdot)$ defined by $\hat{\sigma}(u_1(X,Y),u_2(X,Y)) = d(|u_1(X,Y) - u_2(X,Y)|)$ can also be considered as a BGS salience function.

[7]Leland and Schneider (2016); Schneider et al. (2018); Schneider and Leland (2021) investigate a relevant model in which state uncertainty is nested with attributes. In particular, they consider that under each subjective state, lottery X and Y give different monetary rewards with different objective probability in an Anscombe-Aumann act style (Anscombe et al., 1963). In their model, the state probability is subject to distortion from both probabilities of winning and magnitudes of reward.

B.3 Behavioral Deviations from DEU

1. *Salience-Based Present Bias*:

Lemma 7. *Suppose the decision-maker follows rank-based multidimensional salience theory* $\frac{U(F_2)}{U(F_1)}$ *is decreasing in p.*[8]

Proof. The utility difference ratio between these two options is:

$$\frac{U(F_2)}{U(F_1)} = \frac{p^2q\theta^2 + (1-p)pq}{p^2q\theta^2 + (1-pq)p\theta} \frac{\delta v(1+r)}{v(1)}$$

Taking derivative w.r.t. p, we have $\frac{U(F_2)}{U(F_1)}$ is decreasing w.r.t p:

$$\begin{aligned}
&\frac{\partial}{\partial p}\frac{pq\theta^2 + (1-p)q}{pq\theta^2 + (1-pq)\theta} \\
=&\frac{\left(q\theta^2 - q\right)\left[pq\theta^2 + (1-pq)\theta\right] - \left(q\theta^2 - q\theta\right)\left[pq\theta^2 + (1-p)q\right]}{(pq\theta^2 + (1-pq)\theta)^2} \\
=&\frac{q\theta^2\left[(1-pq)\theta - (1-p)q - pq + pq\theta\right] - q\left[\theta - q\theta\right]}{(pq\theta^2 + (1-pq)\theta)^2} \\
=&\frac{q\theta^2\left[\theta - q\right] - q\left[\theta - q\theta\right]}{(pq\theta^2 + (1-pq)\theta)^2} = \frac{q\theta\left[\theta(\theta - q) - (1-q)\right]}{(pq\theta^2 + (1-pq)\theta)^2} \leq 0
\end{aligned}$$

□

2. *Intertemporal Hedging*

Lemma 8. *Suppose the decision-maker follows rank-based multidimensional salience theory, then* $\frac{U(F_3)}{U(F_4)}$ *is increasing in p.*

[8]Notice that, prospect theory and other non-expected utility models based on nonlinear probability also have similar predictions. Nevertheless, multidimensional salience predicts a monotonic change in preference while others may require p, q to be within certain ranges.

Proof.

$$\frac{U(F_3)}{U(F_4)} = \frac{(p\theta^2+(1-p)\theta)\delta v(1+r)}{(p\theta^2+(1-p))v(\frac{1}{2})+(p\theta^2+(1-p)\theta)\delta v(\frac{1+r}{2})}$$
$$= \frac{\delta v(1+r)}{\frac{p\theta^2+(1-p)}{p\theta^2+(1-p)\theta}v(\frac{1}{2})+\delta v(\frac{1+r}{2})}$$

Since $\frac{p\theta^2+(1-p)}{p\theta^2+(1-p)\theta}$ is decreasing in p and $v(\frac{1}{2})>0$, $\frac{U(F_3)}{U(F_4)}$ is increasing in p. □

3. *Correlation Dependence*

Lemma 9. *Suppose the decision-maker follows rank-based multidimensional salience theory, then* $\frac{U(F_3)}{U(F_4)}$ *is increasing in* γ.

Proof.

$$\frac{U(F_3)}{U(F_4)} = \frac{((1+4\gamma)\theta^2+(1-4\gamma)\theta)\delta v(1+r)}{((1+4\gamma)\theta^2+(1-4\gamma))v(\frac{1}{2})+((1+4\gamma)\theta^2+(1-4\gamma)\theta)\delta v(\frac{1+r}{2})}$$
$$= \frac{\delta v(1+r)}{\frac{(1+4\gamma)\theta^2+(1-4\gamma)}{(1+4\gamma)\theta^2+(1-4\gamma)\theta}v(\frac{1}{2})+\delta v(\frac{1+r}{2})}$$

As $\gamma \in [-0.25, 0.25]$ $\frac{(1+4\gamma)\theta^2+(1-4\gamma)}{(1+4\gamma)\theta^2+(1-4\gamma)\theta}$ is decreasing in γ and $v(\frac{1}{2})>0$, $\frac{U(F_3)}{U(F_4)}$ is increasing in γ. □

4. *Reordering Dependence*

Lemma 10. *Suppose the decision-maker follows rank-based multidimensional salience theory, then* $U(F_3)-U(F_4) \leq U(F_3)-U(F_4')$.

Proof.

$$4(U(F_3)-U(F_4)) = (\theta^2+\theta)\delta v(1+r)-(\theta^2+1)v(\frac{1}{2})-(\theta^2+\theta)\delta v(\frac{1+r}{2})$$

$$4(U(F_3)-U(F_4'))=(\theta^3+1)\delta v(1+r)-(\theta^3+\theta^2)v(\frac{1}{2})-(\theta^3+\theta)\delta v(\frac{1+r}{2})$$

Since $\theta^3+1\geq\theta^2+\theta$, $\theta^2+1\geq\theta^3+\theta\geq\theta^3+\theta^2$, the result follows. □

5. *Precautionary Saving*. Consider the following two-period example, suppose that the decision maker receives an expected payment I in each period. In the first period, the payment is deterministic while in the second period the payment is either 0 or $2I$ with equal chances. The decision maker now chooses between whether or not to save half of the first-period payment to hedge the risk in the second period. The state space and corresponding choices are summarized in the following table.

Option	State	
	H	L
F_5	$[I,0]$	$[I,2I]$
F_6	$[\frac{I}{2},\frac{I}{2}]$	$[\frac{I}{2},\frac{5I}{2}]$

For simplicity, assume the decision maker has utility function $v(c_1)+v(c_2)$. Then $U(F_6)-U(F_5)>0$ if and only if $v(I)-v(\frac{I}{2})\leq\pi_H\cdot v(\frac{I}{2})+\pi_L\cdot(v(\frac{5I}{2})-v(2I))$, where π_H and π_L are decision weights on state H and L, respectively. Notice that under DEU, $\pi_H=\pi_L=0.5$, therefore, a sufficient condition for the inequality to hold is v has a positive third derivative. On the other hand, for multidimensional salience theory, it is possible that $\Delta_H>\Delta_L$, which implies $\pi_H>\pi_L$.[9] In the limiting case where $\pi_H=1$ and $\pi_L=0$, a sufficient condition for the aforementioned inequality to hold is v being concave. Therefore, in the presence of salience distortions, we can relax the classical restriction on the third derivative to predict prudent behaviors.

[9]Furthermore, $\pi_H>\pi_L$ has to be true if one imposes *diminishing sensitivity* on all pairs of nonnegative streams as in proposition 6 part 3.

B.4 Convex Time Budget Predictions

To provide predictions of multidimensional salience theory in the Convex Time Budget (CTB) design, we first require a mapping from stream payments, denominated in currency units, to discounted utility values. We follow the prior time preference literature on CTBs without "present" payments and assume a time separable, exponentially discounted utility function, such that the stream $[c_t, c_{t+k}]$ has value $u(c_t) + \delta^k u(c_{t+k})$. We assume $u(\cdot)$ is weakly concave, but not dramatically so.[10] This assumption is consistent with the prior time preference literature from CTBs (Andreoni and Sprenger, 2012a,b). We also assume that individuals follow the rank-based salience model of equation (2.2), applied to the stream values noted above. One subtlety of the CTB environment is that subjects face a continuum of possible menu choices in each condition, with salience evaluated for every possible stream comparison. Two technical assumptions ruling out extreme subjective time value of money and extreme diminishing sensitivity of the salience function facilitate the statement of our results. Under these assumptions, we prove analogs of behaviors in examples 1, 2, and 3 for CTB choice, c_t^j, $j \in \{MULT, SING, CERT, IND, POS, NEG\}$ with multidimensional salience.[11] Proofs are in the Appendix.

Proposition 12. Preference for Certainty and Intertemporal Hedging in CTBs:

Suppose that

a. $u(\cdot)$ *is strictly increasing and weakly concave, while the approximation* $\sigma(u(c_t) + \delta^k u(c_{t+k}), u(c'_t) + \delta^k u(c'_{t+k})) \approx \sigma(c_t + \delta^k c_{t+k}, c'_t + \delta^k c'_{t+k})$ *maintains ;*

b. individuals follow rank-based multidimensional salience theory with salience function as in equations (2.1) and (2.2) and $\beta \leq (\sqrt{2} - 1)\frac{m}{1+r}$*;*

[10] Provided $u(\cdot)$ is not dramatically concave, the perceived difference between $[c_t, c_{t+k}]$ and $[c'_t, c'_{t+k}]$ can be approximated by the perceived difference between $c_t + \delta^k c_{t+k}$ and $c'_t + \delta^k c'_{t+k}$. This approximation is used in the proof of our analog propositions.

[11] Given the budget constraint in each problem, predictions for the sooner choice, c_t^j, in these conditions also provide implicit predictions for the later choice, $c_{t+k}^j = m - (1+r)c_t^j$.

c. $0.5 \leq (1+r)\delta^k \leq 2$

Then, there exists $c^* > 0$ *such that:*

(1). If $c_t^{CERT} \geq c^*$, $c_t^{IND} \in [c^*, c_t^{CERT}]$.

(2). If $c_t^{CERT} \leq c^*$, $c_t^{IND} \in [c_t^{CERT}, c^*]$.

(3). If $c_t^{CERT} = c^*$, $c_t^{IND} = c^*$.

Similarly,

(1). If $c_t^{SING} \geq c^*$, $c_t^{MULT} \in [c^*, c_t^{SING}]$.

(2). If $c_t^{SING} \leq c^*$, $c_t^{MULT} \in [c_t^{SING}, c^*]$.

(3). If $c_t^{SING} = c^*$, $c_t^{MULT} = c^*$.

Moreover:

(4). Fixing interest rate to some r, if c_t^{CERT} *is the choice under r, and* c_t^{SING} *is the choice under* $r/0.8$*, then* $c_t^{CERT} < c_t^{SING}$.

In addition, $c^* \leq \dfrac{m}{2(1+r)}$ *if and only if* $(1+r)\delta^k \geq 1$ *and* $c^* \to \dfrac{m}{2(1+r)}$ *as* $\beta \to 0$.

Proof: See Appendix B.5.1

Proposition 12 states that there exists some threshold, c^*, related to the midpoint of the CTB budget, $\frac{m}{2(1+r)}$. If c_t^{CERT} is above this adjusted midpoint, c^*, then c_t^{IND} will lay weakly below c_t^{CERT}, while if c_t^{CERT} is below c^*, then c_t^{IND} will lay weakly above c_t^{CERT}. Given different values of $(1+r)$ across tasks, the intertemporal demand schedule of c_t^{CERT} against $1+r$ is likely to cross the adjusted midpoint, c^*. Proposition 12 implies that the intertemporal demand schedule for c_t^{IND} will cross that of c_t^{CERT} at exactly the point that c_t^{CERT} crosses the modified midpoint, c^*. This behavioral prediction of multidimensional salience differs from that of DEU, which predicts $c_t^{CERT} = c_t^{IND}$. The analysis for SING vs. MULT is similar.

The intuition of Proposition 12 follows from our introductory example of avoiding a critical state of non-payment. For example, if a decision, $c_t^{CERT} \geq c^*$, generates unequal payoffs through time, then subjecting both c_t and c_{t+k} to independent payment risk induces decision-makers to hedge towards more equal payoffs with $c^* \leq c_t^{IND} \leq c_t^{CERT}$. Such hedging allows the decision-maker to receive a better stream in the critical state where only the later payment is made. The comparison between c_t^{CERT} and c_t^{SING} is similar. However, under *SING*, c_t has a higher payment probability than c_{t+k}, so under a same interest rate, $c_t^{CERT} < c_t^{SING}$ should trivially hold. Regardless, our theory predicts that after balancing the probability difference with a higher interest rate, $c_t^{CERT} < c_t^{SING}$ should still hold. The reason is identical as before: to avoid a critical state of non-payment.

Proposition 12 also clarifies that c^* will be the budget midpoint for β near zero.[12] Bordalo et al. (2012, 2013b) indicate that β should indeed be close to zero such that the exact budget midpoint, $\frac{m}{2(1+r)}$, should be a reasonable approximation of c^*.

Proposition 13. Correlation Dependence in CTBs:

Suppose the conditions and $c^* > 0$ *are the same as in proposition 12, then:*

(1). If $c_t^{IND} \geq c^*$, $c_t^{NEG} \in [c^*, c_t^{IND}]$.

(2). If $c_t^{IND} \leq c^*$, $c_t^{NEG} \in [c_t^{IND}, c^*]$.

(3). If $c_t^{IND} = c^*$, $c_t^{NEG} = c^*$.

(4). $c_t^{CERT} = c_t^{POS}$.

Proof: See Appendix B.5.1.

Proposition 13 states that deterministic and positively correlated intertemporal risks will yield the same menu choice, $c_t^{CERT} = c_t^{POS}$. There also exists a connection between independent

[12]Intuitively, β in our salience function captures the relative strength of ordering and diminishing sensitivity. As β decreases, the relative effect of diminishing sensitivity increases. States in which streams differ by the same amount, but have different absolute levels, can thus be treated more differently.

and negatively correlated risks, c_t^{IND} and c_t^{NEG}. While c_t^{IND} will be pulled towards the modified midpoint, c^*, relative to c_t^{CERT}, c_t^{NEG} will be even further pulled towards c^*. Reasonable parameterizations of equations (2.1) and (2.2) and utility indicate that the first of these effects is quite large, leaving relatively little distance between c_t^{IND} and c^*. Hence, quite similar behavioral predictions are made for c_t^{NEG} and c_t^{IND}.[13] In contrast, DEU predicts equality for all of the values c_t^{CERT}, c_t^{IND}, c_t^{POS} and c_t^{NEG}.

The intuition of Proposition 13 also follows from our introductory examples. The correlation structure of intertemporal risks alters what states can be realized. In *POS*, either both payments or neither payment will be realized; the latter state generates no differences between options and so induces no additional distortion beyond *CERT*. In *NEG*, only one of the payments will be made, and so individuals will hedge similarly to *IND* to assure themselves positive stream values in all states.

B.5 Proofs

B.5.1 Proofs in Main Text

Proposition 6: Consider $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}_+$ such that for every $X, Y \in \mathbb{R}^n$, $\sigma(X,Y) = \tilde{\sigma}(h(X), h(Y))$ where $h : \mathbb{R}^n \mapsto \mathbb{R}$ is continuously differentiable and additive separable, and $\tilde{\sigma} : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}_+$ is a BGS salience function. Then:

1. If $\sigma(X,Y) = \tilde{\sigma}(h(X), h(Y))$ is a multidimensional salience function, then $h(X) = \sum_{i=1}^{n} g(x_i)$ where $g(\cdot)$ is some strictly monotonic function.

2. Conversely, if $g(x_i)$ is concave for $x_i > 0$, convex for $x_i < 0$, and $g(0) = 0$, $\tilde{\sigma}(\sum_{i=1}^{n} g(x_i), \sum_{i=1}^{n} g(y_i))$ is a multidimensional salience function for all BGS salience functions $\tilde{\sigma}$.

3. Furthermore, if we assume that in addition for every X, Y in $\mathbb{R}_+^n$, we have $\sigma(X,Y) \geq$

[13] As one example, if $c_t^{IND} = m/2(1+r)$, then $c_t^{IND} = c_t^{NEG}$.

$\sigma(X+\varepsilon, Y+\varepsilon)$ for all $\varepsilon > 0$. Then $\sigma(X,Y) = \tilde{\sigma}(\sum_{i=1}^{n} g(x_i), \sum_{i=1}^{n} g(x_i))$ is a multidimensional salience function if and only if $g(\cdot)$ is a non-constant linear function.

Proof. Since $h(X)$ is additively separable, $h(X) = \sum_{i=1}^{n} f_i(x_i)$ where $f_i(\cdot)$ are functions on real numbers. Define $g_i(x) = f_i(x) - f_i(0)$, then $h(X) = \sum_{i=1}^{n} g_i(x_i) + b$, where $b = \sum_{i=1}^{n} f_i(0)$. since $h(X)$ is continuously differentiable, so is $g_i(x)$.

Claim 1: for every i, $g_i(x)$ is monotonic.

Proof of claim 1: We now show $g_1(x)$ is monotonic, cases for other $g_i(x)$ can be proved similarly. Suppose not, that is there are $x, y \in \mathbb{R}$ such that $g_1'(x) > 0$ and $g_1'(y) < 0$. We now show x must be equal to y, and hence reach a contradiction and prove the claim. If $x > y$, with $g_1(\cdot)$ continuously differentiable and $g_1'(y) < 0$, there is $\varepsilon > 0$ such that $\varepsilon < x - y$, $g_1(y+\varepsilon) < g_1(y)$, and $|g_1(y+\varepsilon) - g_1(y)| < |g_1(x) - g_1(y)|$. By inclusion of salience function, $\sigma([x,0,0,\ldots,0],[y,0,0,\ldots,0]) > \sigma([x,0,0,\ldots,0],[y+\varepsilon,0,0,\ldots,0])$. Therefore, $\tilde{\sigma}(g_1(x) + b, g_1(y) + b) > \tilde{\sigma}(g_1(x) + b, g_1(y+\varepsilon) + b)$. With $g_1(y+\varepsilon) < g_1(y)$, by inclusion, it must be that $g_1(x) < g_1(y)$. But, with $g_1'(x) > 0$, there is $\delta > 0$ such that $g_1(x) < g_1(x+\delta) < g_1(y)$. Therefore, $\tilde{\sigma}(g_1(x) + b, g_1(y) + b) > \tilde{\sigma}(g_1(x+\delta) + b, g_1(y) + b)$. On the other hand, with $x > y$, $\sigma([x,0,0,\ldots,0],[y,0,0,\ldots,0]) < \sigma([x+\delta,0,0,\ldots,0],[y,0,0,\ldots,0])$ which contradicts to $\sigma(X) = \tilde{\sigma}(h(X))$. Therefore, $x \leq y$.

If $x < y$, by a similar argument, we will reach a contradiction. Hence, it must be that $x = y$. As a result, $g_1'(x) > 0$ and $g_1'(y) < 0$ cannot be true. Therefore, $g_1(\cdot)$ is monotonic.

□

Claim 2: either $g_i'(x) > 0$ for all i, or $g_i'(x) < 0$ for all i.

Proof of claim 2. Let e_i be the unit vector of the i^{th} coordinate. That is $e_1 = [1,0,\ldots,0]$, $e_2 = [0,1,0,0,\ldots,0], \ldots$ Given $x > 0$, find $\varepsilon > 0$ such that $|g_i(x)| > |g_j(\varepsilon)|$ for all $j \neq i$, by upper ordering and inclusion, $\sigma(xe_i, \vec{0}) < \sigma(xe_i + \varepsilon e_j, \vec{0})$ where $\vec{0} = [0,0,\ldots,0]$. That is, $\tilde{\sigma}(g_i(x) + b, b) < \tilde{\sigma}(g_i(x) + g_j(\varepsilon) + b, b)$. If $g_i'(x) \geq 0$, with $|g_i(x)| > |g_j(\varepsilon)|$, by inclusion it must be true that $g_j(\varepsilon) > 0$. Therefore, with claim 1, $g_j'(\cdot) \geq 0$. If $g_i'(x) \leq 0$, with $|g_i(x)| > |g_j(\varepsilon)|$, by inclusion it must be true that $g_j(\varepsilon) < 0$. Therefore, with claim 1, $g'_j(\cdot) \leq 0$. Since by ordering $g'_i(\cdot) \neq 0$, the claim follows.

From now on, we assume $g'_i(x) > 0$. By reflection of salience function, following arguments still hold if $g'_i(x) < 0$. $\square$

Claim 3: there is $g(x) : \mathbb{R} \mapsto \mathbb{R}$ s.t. $g_i(x) = g(x)$ for all i.

Proof of claim 3. for every $x \in \mathbb{R}$, by compatibility, $\sigma(xe_i, 0) = \sigma(xe_j, 0)$. That is $\tilde{\sigma}(g_i(x)+b, b) = \tilde{\sigma}(g_j(x)+b, b)$. By claim 2, $g_i(x)+b \geq b$ if and only if $g_j(x)+b \geq b$. By inclusion, $g_i(x) = g_j(x)$ for all x.

The sufficiency of concave in g for $x > 0$ and convex in g for $x < 0$ follows directly from diminishing sensitivity and reflection. Now we prove the last part of the proposition: assume for every X, Y in $\mathbb{R}^n_+$, $\sigma(X,Y) \geq \sigma(X+\varepsilon, Y+\varepsilon)$ for all $\varepsilon > 0$. $\square$

Claim 4: Given an arbitrary $a \in \mathbb{R}^+$, $g(x+a) - g(x)$ is constant for all $x \in \mathbb{R}^+$; $g(z-a) - g(z)$ is constant for all $z \in \mathbb{R}^-$.

Proof of claim 4. Let $x, y \geq 0$, consider $\sigma(xe_1+ye_2, ye_1+xe_2) = \tilde{\sigma}(g(x)+g(y)+b, g(x)+g(y)+b) = 0$. By diminishing sensitivity and $ae_1 > 0$, $\sigma((x+a)e_1+ye_2, (y+a)e_1+xe_2) \leq \sigma(xe_1+ye_2, ye_1+xe_2) = 0$. According to the BGS formulation, $g(x+a)+g(y)+b = g(y+a)+g(x)+b$. The rest of claim 4 can be proved using reflection of the salience function. $\square$

($\Rightarrow$) With claim 4, the rest of the proof is straightforward. Since $g(x)$ is differentiable, $g'(x)$ exists for all $x \in \mathbb{R}$. With $\lim\limits_{a \to 0} \dfrac{g(x+a)-g(x)}{a} = g'(x)$, $g'(x)$ is constant for all $x \in \mathbb{R}_+$. In addition, at 0, we have $g'(0) = \lim\limits_{a \to 0^+} g'(x) = \lim\limits_{a \to 0^-} g'(x)$. It follows that $g'(x)$ is constant. With upper and lower ordering, $g'(x) \neq 0$. That is, $h(X) = \sum\limits_{i=1}^{n} ax_i + b$ with $a \neq 0$. If $b \neq 0$, by diminishing sensitivity and reflection, we have $\sigma(-\dfrac{b}{3a}e_i, 0) > \sigma(-\dfrac{b}{3a}e_i, -\dfrac{2b}{3a}e_i)$. But, we also have $\tilde{\sigma}(2/3b, b) < \tilde{\sigma}(1/3b, 2/3b)$, and so we reach a contradiction. Therefore, $b = 0$.

($\Leftarrow$) If $h(X) = a\sum\limits_{i=1}^{n} x_i$, one can easily check that $\tilde{\sigma}(h(X), h(Y))$ fulfills all requirements in definition 1 and the stronger version of *diminishing sensitivity*. $\square$

Proposition 7: *A salience function $\sigma : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}_+$ satisfies strong compatibility if* *and only if* $\sigma(X,Y) = \tilde{\sigma}(\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} y_i)$ *where* $\tilde{\sigma} : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}_+$ *is a salience function.*

Proof. ($\Rightarrow$) As an example, consider outcomes in $\mathbb{R}^2$, strong compatibility means

$$\sigma([x_1,x_2],[y_1,y_2]) = \sigma([x_1,0]+[0,x_2],[y_1,0]+[0,y_2]) = \sigma([x_1+x_2,0],[y_1+y_2,0]).$$

In this way, we can combine the whole vector to its first row by summing it up entry-by-entry.

Notice for every $X \in \mathbb{R}^n$, $X = \sum_{i=1}^{n} x_i e_i$. If $\sigma()$ satisfies strong compatibility, $\sigma(X,Y) = \sigma(\sum_{i=1}^{n-1} x_i e_i + x_n e_1, \sum_{i=1}^{n-1} y_i e_i + y_n e_1) = \sigma(\sum_{i=1}^{n-2} x_i e_i + (x_{n-1}+x_n)e_1, \sum_{i=1}^{n-2} y_i e_i + (y_{n-1}+y_n)e_1)$. By induction, $\sigma(X,Y) = \sigma(e_1 \sum_{i=1}^{n} x_i, e_1 \sum_{i=1}^{n} y_i)$. The result follows from the fact that $\sigma(e_1 \sum_{i=1}^{n} x_i, e_1 \sum_{i=1}^{n} y_i)$ is equivalent to $\tilde{\sigma}(\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} y_i)$.

($\Leftarrow$) If

$$\sigma([x_1,x_2],[y_1,y_2]) = \sigma([x_1,0]+[0,x_2],[y_1,0]+[0,y_2]) = \sigma([x_1+x_2,0],[y_1+y_2,0]).$$

By the definition of strong compatibility, the result is straightforward. □

Lemma 2: Π induces a smooth correlation-sensitive presentation with monotonic ϕ if and only if completeness, strong independence, Archimedean continuity, monotonicity, and continuity are satisfied.

Proof. In Lanzani (2022), he shows axiom 1 , 2, and 3 are necessary and sufficient for the existence of skew symmetric $\phi()$ such that $\pi \in \Pi \Leftrightarrow \sum_{(x,y)} \phi(x,y)\pi(x,y) \geq 0$. Axiom 4 is a direct generalization from the same axiom in Lanzani (2022). The only difference is that when there is only a single dimension the natural ordering on real numbers is complete while it's not for higher dimensional spaces. Therefore, here we focus on the continuity axiom.

Suppose $\phi(\cdot)$ is not continuous, there is a sequence $(x_n,y_n) \to (x,y)$ such that $\phi(x_n,y_n) \nrightarrow \phi(x,y)$. This implies that there are outcomes (z,w) such that $\phi(z,w) > 0$ (picking some (x_n,y_n)

or (x,y) and reversing the coordinates if necessary). Within $\{(x_n,y_n)\}_{n=1}^{\infty}$, we can identify a subsequence (x_{n_k},y_{n_k}) such that either $\phi(x,y)-\phi(x_{n_k},y_{n_k}) \geq \delta$ or $\phi(x,y)-\phi(x_{n_k},y_{n_k}) \leq -\delta$ for some $\delta > 0$ and all k. In addition, by skew-symmetry of ϕ, we can assume $\phi(x,y) \leq 0$.

First, suppose that $\phi(x,y)-\phi(x_{n_k},y_{n_k}) \geq \delta$. Equivalently, $\phi(y_{n_k},x_{n_k})-\phi(y,x) \geq \delta$. Because $\phi(x,y) \leq 0$ and $\phi(z,w) > 0$, let $p = \dfrac{\phi(z,w)}{\phi(z,w)-\phi(x,y)+0.5\delta}$, we have $\{(y,x),p;(w,z),1-p\} \notin \Pi$ since

$$\phi(y,x)p+\phi(w,z)(1-p) = \frac{0.5\phi(w,z)\delta}{\phi(z,w)-\phi(x,y)+0.5\delta} < 0.$$

But, on the other hand,

$$\phi(y_{n_k},x_{n_k})p+\phi(w,z)(1-p) = \frac{\phi(z,w)(\phi(y_{n_k},x_{n_k})-\phi(y,x)-0.5\delta)}{\phi(z,w)-\phi(x,y)+0.5\delta} > 0.$$

In summary, if $\phi(x,y)-\phi(x_{n_k},y_{n_k}) \geq \delta$, we have

$$\{(y,x),p;(w,z),1-p\} \notin \Pi \text{ and } \{(y_{n_k},x_{n_k}),p;(w,z),1-p\} \in \Pi \ (\forall k),$$

and hence, it violates axiom 5.

Second, suppose that $\phi(x_{n_k},y_{n_k})-\phi(x,y) \geq \delta$.

On one hand, if $\phi(x,y) < 0$, let $q = \dfrac{\phi(z,w)}{\phi(z,w)-\phi(x,y)-a\delta}$ for some $a \in (0,1)$ such that $a\delta < \phi(y,x)$. We have $\{(x,y),q;(z,w),1-q\} \notin \Pi$ since

$$\phi(x,y)q+\phi(z,w)(1-q) = \frac{-a\phi(z,w)\delta}{\phi(z,w)-\phi(x,y)-a\delta} < 0.$$

But,

$$\phi(x_{n_k},y_{n_k})q+\phi(z,w)(1-q) = \frac{\phi(z,w)(\phi(x_{n_k},y_{n_k})-\phi(x,y)-a\delta)}{\phi(z,w)-\phi(x,y)-a\delta} > 0.$$

Consequently, Axiom 5 is also violated.

On the other hand, if $\phi(x,y) = 0$, let $q = \dfrac{\phi(z,w)}{\phi(z,w)+\delta}$. We have $\{(x,y),q;(w,z),1-q\} \notin \Pi$ because $\phi(w,z) < 0$. Since $\phi(x_{n_k},y_{n_k})-\phi(x,y) > \delta$, we have $\phi(x_{n_k},y_{n_k}) > \delta$. Therefore,

$\{(x_{n_k}, y_{n_k}), q; (w,z), 1-q\} \in \Pi$ because

$$q\phi(x_{n_k}, y_{n_k}) + \phi(w,z)(1-q) \geq q\delta + \phi(w,z)(1-q) = \frac{\phi(z,w)(\delta - \delta)}{\phi(z,w)+\delta} = 0.$$

Hence, Axiom 5 is still violated. Therefore, Axiom 5 implies continuity in ϕ.

Conversely, if ϕ is continuous, axiom 5 is the direct consequence of correlation-sensitive presentation. The detail is omitted here. □

Lemma 3: If Π in addition satisfies axioms 1, it induces a continuous utility u on X that is strictly increasing.

Proof. Define a binary relation $\succeq$ on outcome X by $x \succeq y$ if $\delta_{(x,y)} \in \Pi$. Then this relation is complete, transitive, and continuous. By Debreu's representation theorem, the result follows. □

Proposition 8: Under assumption 1, a correlation sensitive preference induces a smooth salience representation with utility function $u(\cdot)$, and some salience function, $\sigma(\cdot,\cdot)$, if and only if Π satisfies axioms 1 to 8.

Proof. We proceed with several claims:

Claim 1: Assumption 1 and continuity of $\phi(\cdot,\cdot)$ imply that $\phi(x,y)$ is continuously differentiable at points $x \sim_u y$.[14]

Proof of claim 1. Since $\phi(\cdot,\cdot)$ is uniformly differentiable, given $x, y \in X$, for all $\varepsilon \in \mathbb{R}_+$, we can find $n^* \in \mathbb{N}$ s.t. $\forall (x',y') \in X \times X$ with $x' \sim_u y'$ and $||(x',y') - (x,y)|| < 1$, $\left| \frac{\phi(x'+\frac{e_i}{n^*}, y')}{1/n^*} - \frac{\partial \phi(x',y')}{\partial x_i} \right| < \frac{\varepsilon}{3}$. Since $\phi(\cdot,\cdot)$ is continuous, we can find $\delta \in (0,1)$ such that $\left| \frac{\phi(x''+\frac{e_i}{n^*}, y'')}{1/n^*} - \frac{\phi(x+\frac{e_i}{n^*}, y)}{1/n^*} \right| < \frac{\varepsilon}{3}$ if $||(x'',y'') - (x,y)|| < \delta$. Since $\delta < 1$, it is also true that $\left| \frac{\phi(x''+\frac{e_i}{n^*}, y'')}{1/n^*} - \frac{\partial \phi(x'',y'')}{\partial x_i} \right| < \frac{\varepsilon}{3}$. Therefore, using triangle inequality, we arrive at $\left| \frac{\partial \phi(x,y)}{\partial x_i} - \frac{\partial \phi(x'',y'')}{\partial x_i} \right| < \varepsilon$ for all indifferent outcome pairs (x'',y'') such that $||(x'',y'') - (x,y)|| < \delta$. Since ε is arbitrary, the result follows. □

[14]Formally, our continuously differentiable means for all sequences of outcome pairs $\{(x_n, y_n)\}_{n=1}^{\infty}$ s.t. $x_n \sim_u y_n$ for all n and $(x_n, y_n) \to (x,y)$, we have $\nabla \phi(x_n, y_n) \to \nabla \phi(x,y)$ under the sup norm.

Claim 2: Given assumption 1 and utility u with continuous and positive partial derivatives, a correlation sensitive preference satisfies axioms 1, 7, and 8 if and only if $\phi(x,y) = f(\sigma(x,y))(u(x)-u(y))$ where $f\circ\sigma$ is positive, symmetric, continuous, and obtain its global minimum at all points (x,y) such that $x=y$.

Proof of claim 2. Define $f\circ\sigma$ as

$$f(\sigma(x,y)) = \begin{cases} \dfrac{\phi(x,y)}{u(x)-u(y)} & u(x)\neq u(y) \\ \\ \lim\limits_{t\downarrow 0}\dfrac{\phi(x+te_1,y)}{u(x+te_1)-u(y)} & u(x)=u(y). \end{cases}$$

When $u(x)=u(y)$, $f(\sigma(x,y))$ is well-defined due to the fact both ϕ and u have nonzero partial derivatives at indifference. Since u represents the preference over deterministic outcomes, $f\circ\sigma$ is positive.

The continuity of $f\circ\sigma$ at points (x,y) such that $u(x)\neq u(y)$ follows from the continuity of ϕ and u. For points (x,y) at which $u(x)=u(y)$, for every sequence (x_n,y_n) converging to (x,y), we can find at least one of the following type of subsequences (x_{n_k},y_{n_k}). First, $u(x_{n_k})=u(y_{n_k})$ for all but finitely many k. In this case, $f(\sigma(x_{n_k},y_{n_k}))\to f(\sigma(x,y))$ by the continuity of partial derivatives.

Second, $u(x_{n_k})>u(y_{n_k})$ for all k. In this case, using monotonicity, for all n_k, we can identify $t_{n_k}\in\mathbb{R}_+$ such that $(x_{n_k}-t_{n_k}\sum_{i=1}^n e_i)\sim_u y_{n_k}$. Let us denote $(x_{n_k}-t_{n_k}\sum_{i=1}^n e_i)$ as x'_{n_k}, we have $f(\sigma(x_{n_k},y_{n_k})) = \dfrac{\phi(x_{n_k},y_{n_k})}{u(x_{n_k})-u(y_{n_k})} = \dfrac{\phi(x_{n_k},y_{n_k})-\phi(x'_{n_k},y_{n_k})}{u(x_{n_k})-u(x'_{n_k})}$. With $(x_{n_k},y_{n_k})\to(x,y)$, $t_{n_k}\to 0$. By uniform differentiability of ϕ and continuous differentiability of u, for all $\varepsilon>0$, there is $N^*\in\mathbb{N}$, s.t. for all $n_k>N^*$, $|\sum_{i=1}^n \frac{\partial\phi(x'_{n_k},y_{n_k})}{\partial x_i} - \frac{\phi(x_{n_k},y_{n_k})-\phi(x'_{n_k},y_{n_k})}{t_{n_k}}|<\varepsilon$ and $|\sum_{i=1}^n \frac{\partial u(x'_{n_k})}{\partial x_i} - \frac{u(x_{n_k})-u(x'_{n_k})}{t_{n_k}}|<\varepsilon$.

Furthermore, since u represents the preference over deterministic outcomes, for every $x\in X$, we have $\{s\in X:\ u(s)=u(x)\}=\{s\in X:\ \phi(s,x)=0\}$. Since $u(\cdot)$ is continuously differentiable and has positive derivatives, by implicit function theorem, for every $x \in X$, if we denote it as $(x_1,\ldots,x_n)$, there is a continuously differentiable map $t : \mathbb{R}^{n-1} \mapsto \mathbb{R}$ such that $t(x_1,\ldots,x_{n-1}) = x_n$ and $u(s_1,\ldots,s_{n-1},t(s_1,\ldots,s_{n-1})) = u(x)$, for all s within some open set containing $(x_1,\ldots,x_{n-1})$. On one hand, we have $t_i(s) = -\frac{u_i(s,t(s))}{u_n(s,t(s))}$ for all $i = 1,\ldots,n-1$. On the other hand, we have

$$\phi\bigg(s_1,\ldots,s_{n-1},t(s_1,\ldots,s_{n-1}),y\bigg) = 0,$$

for every $y \in X$ such that $y \sim_u x$. Consequently, when $s = (x_1,\ldots,x_{n-1})$, using chain rule, we have

$$t_i(x_1,\ldots,x_{n-1}) = -\frac{u_i(x)}{u_n(x)} = -\frac{\partial\phi(x,y)/\partial x_i}{\partial\phi(x,y)/\partial x_n},$$

for $i = 1,\ldots,n-1$. This implies that $\frac{u_i(x)}{\partial\phi(x,y)/\partial x_i}$ is constant across all dimensions. Therefore, for n_k large enough, we have $f(\sigma(x_{n_k},y_{n_k})) \approx \frac{\sum_{i=1}^n \partial\phi(x'_{n_k},y_{n_k})/\partial x_i}{\sum_{i=1}^n \partial u(x'_{n_k})/\partial x_i} = \frac{\partial\phi(x'_{n_k},y_{n_k})/\partial x_1}{\partial u(x'_{n_k})/\partial x_1}$ which converges to $f(\sigma(x,y))$ by definition.

For the last type of subsequences, we have $u(x_{n_k}) < u(y_{n_k})$ for all k. By a similar argument, we still have $f(\sigma(x_{n_k},y_{n_k})) \to f(\sigma(x,y))$.

In summary, we just showed that every subsequence of sequence (x_n,y_n) has a further subsequence whose salience values converge to $f(\sigma(x,y))$. Therefore, $f(\sigma(x,y))$ is continuous.

For the symmetry of $f(\sigma(x,y))$, if $u(x) \neq u(y)$, the result follows trivially. If $u(x) = u(y)$, consider the sequence $\{(x+\frac{e_1}{n},y)\}_{n=1}^{\infty}$. Since $f \circ \sigma$ is continuous, we have $f(\sigma(x+\frac{e_1}{n},y)) \to f(\sigma(x,y))$ and $f(\sigma(y,x+\frac{e_1}{n})) \to f(\sigma(y,x))$. By monotonicity, for every n, $x+\frac{e_1}{n} \succ_u y$, so $f(\sigma(x+\frac{e_1}{n},y) = f(\sigma(y,x+\frac{e_1}{n}))$. Therefore, $f(\sigma(x,y)) = f(\sigma(y,x))$.

For global minimal at (x,x) for all $x \in X$, let us consider axiom 7. With $x \not\sim_u y$, it suggests that given $\varepsilon \in \mathbb{R}_+$ and $\{p_n\}_{n=1}^{\infty}$ with $\frac{|u(x)-u(y)|}{u(x)-u(x-e_1/n)}\frac{p_n}{(1-p_n)} = 1+\varepsilon$, it must be true that $\frac{|\phi(x,y)|}{\phi(x,x-e_1/n)}\frac{p_n}{1-p_n} > 1$ for all but finitely many n. Hence, $\frac{f(\sigma(x,y))}{f(\sigma(x,x-e_1/n))} > \frac{1}{1+\varepsilon}$ for all but finitely many n. Therefore, $\frac{f(\sigma(x,y))}{f(\sigma(x,x))} \geq \frac{1}{1+\varepsilon}$ since $f \circ \sigma$ is continuous. Since ε is arbitrary, $f(\sigma(x,y)) \geq f(\sigma(x,x))$. By a similar argument, we have $f(\sigma(x,y)) \geq f(\sigma(y,y))$. For points at which $x \sim_u y$, we can consider $(x+\frac{e_1}{n},y) \to (x,y)$ and achieve the same conclusion. The last thing we need to show is $f(\sigma(x,x)) = f(\sigma(y,y))$ for all $x, y \in X$. According to axiom 8, for all $x, x' \in X$, we have $\frac{\partial u(x)/\partial x_i}{\partial u(0)/\partial x_j} = \frac{\partial \phi(x,y)/\partial x_i|_{(x,y)=(x,x)}}{\partial \phi(x,y)/\partial x_j|_{(x,y)=(0,0)}}$, so $\frac{\partial u(x)/\partial x_i}{\partial \phi(x,y)/\partial x_i|_{(x,y)=(x,x)}} = \frac{\partial u(0)/\partial x_1}{\partial \phi(x,y)/\partial x_j|_{(x,y)=(0,0)}}$. Moreover, Since both $u(x)$ and $\phi(x,y)$ have positive partial derivatives over x, we can find a constant number $c > 0$ such that $c\frac{\partial u(x)}{\partial x_1}|_{x=0} = \frac{\partial \phi(x,y)}{\partial x_1}|_{(x,y)=(0,0)}$. Therefore, $f(\sigma(x,x)) = \lim_{t \downarrow 0} \frac{\phi(x+te_1,x)}{u(x+te_1)-u(x)} = \frac{\partial \phi(x,y)/\partial x_1|_{(x,y)=(x,x)}}{\partial u(x)/\partial x_1} = c$. □

Claim 3: If Axiom 2 holds, $f \circ \sigma$ satisfies upper ordering; if Axiom 3 holds, $f \circ \sigma$ satisfies lower ordering; if Axiom 4 holds, $f \circ \sigma$ satisfies inclusion.

Proof of claim 3. We can assume $u(x) > u(y)$. For if $u(x) = u(y)$, we can consider $(x+\varepsilon, y)$ for arbitrary $\varepsilon \in \mathbb{R}^n_{++}$. By continuity in ϕ and $f \circ \sigma$, we can use a limit argument in the end. As an illustration, let $x' \geq x$ such that $u(x') > u(y)$.

Consider axiom 2. If the first case is true, we can find $\alpha \in [0,1]$ such that $[u(y) - u(x')]\alpha + [u(x' \vee z) - u(y)](1-\alpha) = 0$ while $[u(y) - u(x')]f \circ \sigma(x,y)\alpha + [u(x' \vee z) - u(y)]f \circ \sigma(x' \vee z, y)(1-\alpha) \geq 0$. With $u(x') > u(y)$, α is in the interior. In this case, we have $f \circ \sigma(x', y) \leq f \circ \sigma(x' \vee z, y)$. As $x' \downarrow x$, we have $f \circ \sigma(x,y) \leq f \circ \sigma(x \vee z, y)$.

For the second case, if $u(y \vee z) = u(x')$, we can find z' arbitrarily close to z that breaks the equality. The condition implies the existence of $\alpha \in (0,1)$ such that $|u(x') - u(y)|\alpha = |u(x') - u(y \vee z')|(1-\alpha)$. And $|u(x') - u(y)|f \circ \sigma(x', y)\alpha \leq |u(x') - u(y \vee z')|f \circ \sigma(x', y \vee z')(1-\alpha)$. Therefore, with α in the interior, we have $f \circ \sigma(x', y) \leq f \circ \sigma(x', y \vee z')$. Fix x', as $z' \to z$, by continuity we have $f \circ \sigma(x', y) \leq f \circ \sigma(x', y \vee z)$. Next, as $x' \downarrow x$, we have $f \circ \sigma(x,y) \leq f \circ \sigma(x, y \vee z)$.

Therefore, $f \circ \sigma(x,y) \leq \max\{f \circ \sigma(x \vee z, y), f \circ \sigma(x, y \vee z)\}$. This is the exact definition of upper ordering.

The analysis for axiom 3 and axiom 4 are similar, they are omitted here. □

Claim 4: If Axiom 5 holds, then $f \circ \sigma$ satisfies diminishing sensitivity.

Proof of claim 4. If $\{(x,y), p; (y+\varepsilon, x+\varepsilon), 1-p\} \in \Pi_{\mathbb{E}(u)}$, we have $[u(x) - u(y)]p \geq [u(x+\varepsilon) - u(y+\varepsilon)](1-p)$. Consider the probability p that makes the equality holds. With

$x > y \geq 0$, $p \in (0,1)$. By Axiom 5, we have $[u(x)-u(y)]f(\sigma(x,y))p > [u(x+\varepsilon)-u(y+\varepsilon)]f(\sigma(x+\varepsilon,y+\varepsilon))(1-p)$, so $f(\sigma(x,y)) > f(\sigma(x+\varepsilon,y+\varepsilon))$. Therefore, $f \circ \sigma$ satisfies diminishing sensitivity. □

Claim 5: If Axiom 6 holds, then $f \circ \sigma$ satisfies reflection.

Proof of claim 5. Again, thanks to continuity and monotonicity of utility, we can assume $x \not\sim_u y$, $x' \not\sim_u y'$, $-x \not\sim_u -y$, and $-x' \not\sim_u -y'$. In this case, there exists some $\alpha, \beta \in (0,1)$, we have $|u(x)-u(y)|\alpha = |u(x')-u(y')|(1-\alpha)$, and $|u(-x)-u(-y)|\beta = |u(-x')-u(-y')|(1-\beta)$. Axiom 6 states the following equivalence

$$|\phi(x,y)|\alpha \geq |\phi(x',y')|(1-\alpha) \Leftrightarrow |\phi(-x,-y)|\beta \geq |\phi(-x',-y')|(1-\beta),$$

which is equivalent to

$$f(\sigma(x,y)) \geq f(\sigma(x',y')) \Leftrightarrow f(\sigma(-x,-y)) \geq f(\sigma(-x',-y')).$$

This is exactly the reflection condition. □

Therefore, we conclude that with axioms 1-8, $\phi(x,y) = f(\sigma(x,y))(u(x)-u(y))$ with $f(\sigma(x,y))$ satisfying all properties of a free salience function except for $f(\sigma(x,x)) = 0$ for all $x \in X$. However, we can define $\hat{\sigma}(x,y) = f(\sigma(x,y)) - f(\sigma(0,0))$ and $\hat{f}(x) = x + f(\sigma(0,0))$. With $f(\sigma(x,x))$ being constant, $\hat{\sigma}(x,x) = 0$, so $\hat{\sigma}(x,y)$ is a free salience function. Furthermore, $\hat{f}(\hat{\sigma}(x,y)) = f(\sigma(x,y))$, so $\phi(x,y)$ indeed has a smooth-salience presentation.

The only if direction to check the necessity of axiom 1-8 is straightforward given a smooth-salience presentation. The proof is omitted here. □

Corollary 1: Under assumption 1, a correlation sensitive preference induces a smooth salience representation with utility function $u(\cdot)$, and some salience function, $\sigma(\cdot,\cdot)$, if and only if Π satisfies axioms 1 to 9.

Proof. Given $x \succeq_u y$, for $r \in \mathbb{R}_+$ sufficiently small, there exits $\varepsilon \in X_+$ with $||\varepsilon|| = r$ such that

$u(x+\varepsilon) > u(y)$ and $u(x_\tau+\varepsilon_\tau) \neq u(y_\tau)$. By Axiom 13, for some $\alpha \in (0,1)$ we have

$$(u(x+\varepsilon)-u(y))\alpha = |u(y_\tau)-u(x_\tau+\varepsilon_\tau)|(1-\alpha)$$

$$\Rightarrow (u(x)-u(y))\sigma(x+\varepsilon,y)\alpha \geq |u(y_\tau)-u(x_\tau)|\sigma(x_\tau+\varepsilon_\tau,y_\tau)(1-\alpha)$$

$$\Rightarrow \sigma(x+\varepsilon,y) \geq \sigma(x_\tau+\varepsilon_\tau,y_\tau).$$

As ε goes to 0, ε_τ also goes to 0. With $\sigma(\cdot)$ being continuous, $\sigma(x,y) \geq \sigma(x_\tau,y_\tau)$. Since τ is a permutation, it is invertible. With a same argument (switch (x_τ,y_τ) to its conjugate if necessary), we have $\sigma(x_\tau,y_\tau) \geq \sigma(x_{\tau\tau^{-1}},y_{\tau\tau^{-1}}) = \sigma(x,y)$. Therefore, $\sigma(x,y) = \sigma(x_\tau,y_\tau)$.

The other direction is a direct check of the definition. The proof is omitted here. □

Proofs of Appendix B.1

Lemma 4: A correlation sensitive preference satisfies axiom 10 if and only if $\phi(x,y)$ is uniform differentiable at points $x \sim_u y$.

Proof. Consider the lottery in axiom 10: $\{(x+\alpha_n e_i,y),1-p_n;\,(0,e_1),p_n\}$. If the decision-maker is indifferent between the two options in this lottery, correlation-sensitive preference implies that $\phi(x+\alpha_n e_i,y)(1-p_n) = \phi(e_1,0)p_n$, for all n, and hence,

$$\phi(e_1,0)\liminf_{n\to\infty}\frac{p_n}{(1-p_n)\alpha_n} = \liminf_{n\to\infty}\frac{\phi(x+\alpha_n e_i,y)}{\alpha_n}.$$

Since both $\frac{\alpha_n}{p_n}$ and $\frac{p_n}{\alpha_n}$ converges, $\lim_{n\to\infty}\frac{p_n}{\alpha_n}$ is positive and finite. Therefore, $\phi(e_1,0)\lim_{n\to\infty}|\frac{p_n}{\alpha_n}| = \lim_{n\to\infty}|\frac{\phi(x+\alpha_n e_i,y)-\phi(x,y)}{\alpha_n||e_i||}|$. Therefore, by skew-symmetry of $\phi(\cdot,\cdot)$, we showed that all right partial derivatives of $\phi(x,y)$ at $x \sim_u y$ exists with $\frac{\partial_+\phi(x,y)}{x_i} > 0$ and $\frac{\partial_+\phi(x,y)}{y_i} < 0$ for all $i = 1,\ldots,n$.

Since given a compact set K, $\frac{p_n}{\alpha_n}$ converges uniformly for every $x,y \in K$ with $x \sim_u y$, using an argument similar to claim 1 from the proof of proposition 2.3, we can conclude $\phi(x,y)$ has continuous right partial derivatives if $x \sim_u y$. Furthermore, for all $\varepsilon \in \mathbb{R}_+$, we can find $n^* \in \mathbb{N}$ s.t. for all $n > n^*$ and e_i, we have $\left|\frac{\phi(x,y)-\phi(x-\frac{e_i}{n},y)}{1/n} - \frac{\partial_+\phi(x-\frac{e_i}{n},y)}{\partial x_i}\right| < \varepsilon$ for all x,y in some compact set K. In addition, with continuous right derivatives, $\frac{\partial_+\phi(x-\frac{e_i}{n},y)}{\partial x_i} \to \frac{\partial_+\phi(x,y)}{\partial x_i}$ as $n\to\infty$. Since K is compact, the right partial derivative is uniformly continuous over K. Thus, we can find $\tilde{n} > n^*$ s.t. $\forall n > \tilde{n}$, $\left|\frac{\phi(x,y)-\phi(x-\frac{e_i}{n},y)}{1/n} - \frac{\partial_+\phi(x,y)}{\partial x_i}\right| < \varepsilon$ for all $x,y\in K$ with $x\sim_u y$. Therefore, $\phi(x,y)$ is uniformly partial differentiable and has continuous partial derivatives at $x\sim_u y$.

To avoid potential confusion, for now we denote κ the number of dimensions. Let $\{e_i\}_{i=1}^{2\kappa}$ be the natural orthonormal basis for $X\times X$, for every $h\in X\times X$ with $h\neq 0$ and $||h||< 1$, we decompose $h=\sum_{i=1}^{2\kappa} a_i^h e_i$ with $||h||=\sum_{i=1}^{2\kappa}|a_i^h|\in(0,1)$. Fixing a compact set K, notice that the set $K+\{h\in X\times X : ||h||\leq 1\}$ is also compact.[15] By uniformly partial differentiability, given $\varepsilon\in\mathbb{R}_+$, we can find $n^*\in\mathbb{N}$ such that if $|a_i|<\frac{1}{n^*}$ for all $i=1,\ldots,2\kappa$. we have

$$\left|\frac{\phi((x,y)+\sum_{i=1}^{j} a_i^h e_i)-\phi((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{|a_j|} - \frac{a_j\phi_j((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{|a_j|}\right| < \frac{\varepsilon}{4\kappa},$$

where $j=1,\ldots,2\kappa$, $\sum_{i=1}^{0} a_i^h e_i = 0$, and $\phi_j(\cdot,\cdot)$ is the partial derivative of $\phi(\cdot,\cdot)$ respect to the j^{th} coordinate. Therefore, whenever $|a_i|<\frac{1}{n^*}$ for all $i=1,\ldots,2\kappa$, we have

$$\left|\frac{\phi((x,y)+h)-\phi(x,y)}{||h||}\right| = \left|\sum_{j=1}^{2\kappa}\frac{\phi((x,y)+\sum_{i=1}^{j} a_i^h e_i)-\phi((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{|a_j|}\frac{|a_j|}{||h||}\right|.$$

Therefore, for every $(x,y)\in K$ we have

$$\begin{aligned}
&\left|\frac{\phi((x,y)+h)-\phi(x,y)}{||h||} - \sum_{j=1}^{2\kappa}\frac{a_j\phi_j((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{||h||}\right| \\
=&\left|\sum_{j=1}^{2\kappa}\left(\frac{\phi((x,y)+\sum_{i=1}^{j} a_i^h e_i)-\phi((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{|a_j|} - \frac{a_j\phi_j((x,y)+\sum_{i=1}^{j-1} a_i^h e_i)}{|a_j|}\right)\frac{|a_j|}{||h||}\right| \\
\leq&\frac{\varepsilon}{2}.
\end{aligned}$$

Furthermore, since $\phi(\cdot,\cdot)$ has uniformly continuous partial derivatives over any compact

[15]Given two sets A and B of a vector space V, $A+B=\{x\in V : \exists a\in A, b\in B\, s.t.\, a+b=x\}$.

set, we can identify $\tilde{n} > n^*$ s.t. $\forall h \in X \times X$ with $||h|| \leq \frac{1}{\tilde{n}}$, we have

$$\phi_j((x,y)+h) - \phi_j(x,y) < \frac{\varepsilon}{4\kappa}.$$

Together with the above inequality, we have

$$\left| \frac{\phi((x,y)+h) - \phi(x,y)}{||h||} - \sum_{j=1}^{2\kappa} \frac{a_j \phi_j(x,y)}{||h||} \right| < \varepsilon,$$

for all $(x,y) \in K$. Therefore, $\phi(\cdot,\cdot)$ is uniformly differentiable.

The other direction is a direct check of the definition. The proof is omitted here. □

Lemma 5: Given assumption 1, a correlation sensitive preference satisfies axioms 1, 7, and 11 if and only if $\phi(x,y) = f(\sigma(x,y))(u(x) - u(y))$ where $u(\cdot)$ is continuously differentiable and has positive partial derivatives while $f \circ \sigma$ is positive, symmetric, continuous, and obtain its global minimum at all points (x,y) such that $x = y$.

Proof. In light of claim 2 in the proof of proposition 8, it suffices to construct a utility function $u(\cdot)$ that satisfies axiom 8. Since ϕ is continuously differentiable, we define

$$\psi(x) = [\phi_1(x,x), \phi_2(x,x), \ldots, \phi_n(x,x)],$$

where $\phi_i(x,x)$ is the partial derivative of $\phi(\cdot,\cdot)$ with respect to the i^{th} coordinate at point (x,x). Notice, as $n \to \infty$, the valuation of $L^n_{(y,x)}$ converges to the line integral

$$\int_0^1 \psi(x + (y-x)t) \cdot (y-x)dt,$$

which is the integration of $\psi(\cdot)$ over the straight line from x to y. Axiom 11 suggests that for any $x,y \in X$, line integrals of $\phi(\cdot)$ from 0 to x, from x to y, and from y to 0 sum up to 0. Since $X = \mathbb{R}^n$ is a path-connected set, every piecewise smooth curve can be approximated by piecewise linear curves, and $\psi(\cdot)$ is continuous, axiom 11 implies that $\psi(x)$ is a conservative vector field on X. In other words, there is a function $u : X \mapsto \mathbb{R}$ such that $\nabla u(x) = \psi(x)$.

We now show $u(\cdot)$ represents the preference over deterministic outcomes. Suppose $\phi(x,y) = 0$. From x to y, along the surface $\{s \in X : \phi(y,s) = 0\}$, we can construct a curve of finite length. Specifically, let $\gamma : [0,1] \mapsto X$ be the parametrization of the curve. Since the preference is monotonic, we can find a γ such that γ_i is strictly monotonic for $i = 1,2,\ldots,n$.[16] The resulting curve has length at most $\sum_{i=1}^{n} |x_i - y_i|$. Let us denote this length L. Let $\{s_1, s_2, \ldots, s_n\}$ be a finite partition from x to y along the curve γ, we have $\sum_{i=1}^{n-1} u(s_{i+1}) - u(s_i) = u(y) - u(x)$. Furthermore, since every such partition is contained in the cube $\prod_{i=1}^{n} [x_i \wedge z_i, x_i \vee z_i]$, for every $\varepsilon > 0$, we can find $\delta > 0$ such that if $||s_i - s_{i-1}|| < \delta$, $|u(s_i) - u(s_{i-1}) - \nabla u(s_{i-1}) \cdot (s_i - s_{i-1})| \leq \frac{\varepsilon}{2L} ||s_i - s_{i-1}||$ and $|\phi(s_i, s_{i-1}) - \nabla u(s_{i-1}) \cdot (s_i - s_{i-1})| \leq \frac{\varepsilon}{2L} ||s_i - s_{i-1}||$ for $i = 1, \ldots, n-1$ because $u(\cdot)$ is continuously differentiable and $\phi(x,y)$ is uniformly differentiable if $x \sim_u y$. In addition, since $\phi(s_i, s_{i-1}) = 0$, combining the two inequalities we have $|u(s_i) - u(s_{i-1})| \leq \frac{\varepsilon}{L} ||s_i - s_{i-1}||$. Hence $|u(y) - u(x)| \leq \varepsilon$. Since ε is arbitrary, we have $u(x) = u(y)$. To conclude, notice that if $\phi(x,y) > 0$, there is $t \in \mathbb{R}_+$ such that $\phi(x - t\sum_{i=1}^{n} e_i, y) = 0$. Since $u(\cdot)$ has positive derivatives, $u(x) > u(x - t\sum_{i=1}^{n} e_i)$. By previous argument, $u(x - t\sum_{i=1}^{n} e_i) = u(y)$. Therefore, $u(x) > u(y)$. In summary, $\phi(x,y) \geq 0 \Rightarrow u(x) \geq u(y)$. The reverse direction also holds because $\phi(\cdot,\cdot)$ is skew symmetric.

At last, $u(\cdot)$ satisfies axiom 8 follows naturally from its construction. The reverse direction is a direct check from the definitions, it is omitted here.

$\square$

Proofs for Appendix B.2

Lemma 6: Focusing, relative thinking, and pairwise normalization all satisfy comparative monotonicity.

[16] Since $\gamma_i : [0,1] \to \mathbb{R}$ are monotonic, they are differentiable almost everywhere.

Proof. We begin with two useful claims:

Claim 1: for all $x, \delta, \varepsilon > 0$ and $y \in \mathbb{R}$ s.t. $y < x, \varepsilon \leq \delta$, and $y + \delta \geq 0$, then $\mu(x+\varepsilon)(y+\delta) - \mu(x)y$ and $\nu(x+\varepsilon)(y+\delta) - \nu(x)y$ are both positive.

Proof of claim 1. Since μ is an increasing and positive function, $\mu(x+\varepsilon)(y+\delta) - \mu(x)y$ is positive. On the other hand,

$$\begin{aligned}\nu(x+\varepsilon)(y+\delta) - \nu(x)y &\geq \nu(x+\delta)(y+\delta) - \nu(x)y \\ &= \nu(x+\delta)(x+\delta) - \nu(x)x + (x-y)(\nu(x) - \nu(x+\delta)) \\ &> 0\end{aligned}$$

The last inequality holds because $\nu(x)x$ is increasing in x and $\nu(x)$ is decreasing in x. Hence the claim is valid. □

Claim 2: for all $x, \varepsilon > 0$ and $y \leq 0$ s.t. $\varepsilon \leq x$, and $x + y \geq 0$, then $\mu(x-\varepsilon)(y+\varepsilon) - \mu(x)y$ and $\nu(x-\varepsilon)(y+\varepsilon) - \nu(x)y$ are both positive.

Proof of claim 2. Since μ is increasing in x and positive while y is non-positive, $\mu(x-\varepsilon)(y+\varepsilon) - \mu(x)y = (\mu(x-\varepsilon) - \mu(x))y + \mu(x-\varepsilon)\varepsilon$ is positive. On the other hand, with $\nu(x)x$ being increasing in x and $\nu(x)$ being decreasing in x

$$\begin{aligned}\nu(x-\varepsilon)(y+\varepsilon) - \nu(x)y &= -\Big(\nu(x-\varepsilon)(-y-\varepsilon) + \nu(x)y\Big) \\ &= -\Big(\nu(x-\varepsilon)(x-\varepsilon) - \nu(x)x + (x+y)(\nu(x) - \nu(x-\varepsilon))\Big) \\ &> 0.\end{aligned}$$

□

For focusing and relative thinking, since the utility is a summation of utilities from different attributes' dimensions, we compare the change in differences in each dimension.

Specifically, it suffices to show if increasing C_i to C_i', for every attribute n and option C_j, we have

$$\mu\left(\max_{C_k'\in\mathscr{C}'}\{C_{kn}'\}-\min_{C_k'\in\mathscr{C}'}\{C_{kn}'\}\right)\cdot(C_{in}'-C_{jn})\geq\mu\left(\max_{C_k'\in\mathscr{C}}\{C_{kn}'\}-\min_{C_k'\in\mathscr{C}}\{C_{kn}'\}\right)\cdot(C_{in}-C_{jn}),$$

$$\nu\left(\max_{C_k'\in\mathscr{C}'}\{C_{kn}'\}-\min_{C_k'\in\mathscr{C}'}\{C_{kn}'\}\right)\cdot(C_{in}'-C_{jn})\geq\nu\left(\max_{C_k'\in\mathscr{C}}\{C_{kn}'\}-\min_{C_k'\in\mathscr{C}}\{C_{kn}'\}\right)\cdot(C_{in}-C_{jn}),$$

where $\mathscr{C}'$ is obtained from $\mathscr{C}$ by changing C_i to C_i'.

There are three fundamental situations in each attribute dimension. First, if increasing from C_{in} to C_{in}' doesn't alter the range of some dimension, in that dimension the difference between C_{in}' and any other C_j is indeed bigger than C_{in} comparing with C_j. Second, C_{in} is not the original minimal and increasing in C_{in} enlarges the range of that attribute (now C_{in}' is the maximal element), the increasing in C_{in} to C_{in}' is at least as large as the increasing in the maximal element of that attribute. Therefore, let $x=\max_{C_i'\in\mathscr{C}}\{C_{in}'\}-\min_{C_i'\in\mathscr{C}}\{C_{in}'\}$, $\varepsilon=C_i'-\max_{C_i'\in\mathscr{C}}\{C_{in}'\}$, $\delta=C_{in}'-C_{in}$, $y=C_{in}-C_{jn}$, using **Claim 1**, we reach the same conclusion as in the first situation. Third, if increasing from C_{in} to C_{in}' shrinks the range (C_{in} is minimal in that dimension) and C_{in}' doesn't exceed the original maximal, again let x be the original range of that dimension, $y=C_{in}-C_{jn}\leq 0$, $\varepsilon=C_{in}'-C_{in}>0$. Using **Claim 2**, we reach the same conclusion as the above two situation. The only case left is that C_{in} is originally the minimal, and C_{in}' is the new maximal. In this case, we can decompose the increment from C_{in} to C_{in}' into a linear combination of smaller increments that fit in the three fundamental cases. Therefore, comparative monotonicity holds in both focusing and relative thinking.

For pairwise normalization, notice with all attributes being non-negative, for all other options C_k, $\dfrac{C_{in}'}{C_{in}'+C_{kn}+\beta}-\dfrac{C_{in}}{C_{in}+C_{kn}+\beta}\geq 0$, and for all C_j, $\dfrac{C_{in}'-C_{jn}}{C_{in}'+C_{jn}+\beta}$ is increasing in C_{in}'. Therefore, comparative monotonicity also holds. □

Proposition 11 Let $d:\mathbb{R}_+\to\mathbb{R}_+$ be some continuous and strictly increasing function. Suppose the context-dependent utility presentation u over the consumption set satisfies R1 to R5, then the function $\tilde{\sigma}$ defined by $\tilde{\sigma}(X,Y) = d(|u_1(X,Y) - u_2(X,Y)|)$ is a *free* salience function.[17]

Proof. For ordering, given X,Y, suppose $u_1(X,Y) - u_2(X,Y) > 0$, then for any other option Z, $X \vee Z \geq X$, hence $\tilde{\sigma}(X \vee Z, Y) \geq \tilde{\sigma}(X,Y)$ by comparative monotonicity. Hence upper ordering holds. The arguments for lower ordering and inclusion are similar. Also, diminishing sensitivity and reflection holds due to the corresponding two properties of $u(\cdot)$. At last, symmetry holds by ordering symmetry. □

Proofs for Convex Time Budget Predictions

First, we derive tangency conditions under different risk structures. When there is no uncertainty, the Euler equation associated with c_t^{CERT} is

$$\frac{u'(c_t^{CERT})}{u'(m-(1+r)c_t^{CERT})} = (1+r)\delta^k. \tag{B.1}$$

Notice that since $u(\cdot)$ is weakly concave, $\dfrac{u'(c_t)}{u'(m-(1+r)c_t)}$ is decreasing in c_t.

Recall that there are four potential states: s_a, s_s, s_l, s_n. In s_a, all allocations are paid. In s_s, only the sooner allocation c_t is paid. In s_l, only the later allocation c_{t+k} is paid. In s_n, nothing is paid. Let π_s be the *objective* probability of state s. In case of each payment is paid with 50% chance, $\pi_{s_a} = \pi_{s_s} = \pi_{s_l} = \pi_{s_n} = 0.25$. And the problem becomes choosing c_t to maximize

$$(p_{s_a}^{c_t} + p_{s_s}^{c_t})u(c_t) + \delta^k(p_{s_a}^{c_t} + p_{s_l}^{c_t})u(m-(1+r)c_t).$$

Next, we establish the ordering of states. Consider the salience function of choosing c_t and comparing to some other choice c_i.

In s_a, the salience function is $\sigma(u(c_t) + \delta^k u(m-(1+r)c_t), u(c_i) + \delta^k u(m-(1+r)c_i)$;

in s_s, the salience function is $\sigma(u(c_t), u(c_i))$;

[17]Notice that $\hat{\sigma}(\cdot,\cdot)$ defined by $\hat{\sigma}(u_1(X,Y), u_2(X,Y)) = d(|u_1(X,Y) - u_2(X,Y)|)$ is "almost" a BGS salience function in the sense that it satisfy all requirements except for diminishing sensitivity.

in s_l, the salience function is $\sigma(\delta^k u(m-(1+r)c_t), \delta^k u(m-(1+r)c_i))$;

and in s_n the salience function is always $\sigma(0,0)=0$.

With $u(\cdot)$ approximately linear, and with the salience function satisfying strong compatibility, we approximate the above four values as

$$s_a\colon\ \sigma(c_t+\delta^k(m-(1+r)c_t), c_i+\delta^k(m-(1+r)c_i));$$

$$s_s\colon\ \sigma(c_t, c_i);$$

$$s_l\colon\ \sigma(\delta^k(m-(1+r)c_t), \delta^k(m-(1+r)c_i));$$

$$s_n\colon\ \sigma(0,0);$$

since when the individual chooses c_t, the salience level of each state is the average value of salience function of c_t and all possible c_i in that state. Due to the fact that there are infinitely many c_i, we assume c_i has a uniform distribution over $[0, \frac{m}{1+r}]$. The salience levels of the four states are thus:

$$\Delta_{s_a}^{c_t} = \frac{1+r}{m} \int_0^{m/(1+r)} \sigma(c_t+\delta^k(m-(1+r)c_t), c_i+\delta^k(m-(1+r)c_i))dc_i;$$

$$\Delta_{s_s}^{c_t} = \frac{1+r}{m} \int_0^{m/(1+r)} \sigma(c_t, c_i)dc_i;$$

$$\Delta_{s_l}^{c_t} = \frac{1+r}{m} \int_0^{m/(1+r)} \sigma(\delta^k(m-(1+r)c_t), \delta^k(m-(1+r)c_i))dc_i;$$

$$\Delta_{s_n}^{c_t} = 0.$$

With $0.5 \leq (1+r)\delta^k \leq 2$, we have

$$\sigma(c_t+\delta^k(m-(1+r)c_t),c_i+\delta^k(m-(1+r)c_i))$$
$$\leq \min\{\sigma(\delta^k(m-(1+r)c_t),\delta^k(m-(1+r)c_i)),\sigma(c_t,c_i)\},$$

for all c_t, c_i. The reason is that in s_a both allocations are paid, so comparing it to either s_s or s_l it is "penalized" more from diminishing sensitivity. Furthermore, the difference between payments in s_a is $(1-\delta^k(1+r))(c_t-c_i)$, the difference in s_s is c_t-c_i, and the difference in s_l is $\delta^k(1+r)(c_t-c_i)$. With $0.5 \leq (1+r)\delta^k \leq 2$, $|1-\delta^k(1+r)|$ is smaller than both 1 and $\delta^k(1+r)$. In summary, in s_a, the payments are the highest while the difference between two streams is the lowest.

Notice that for all c_t, there is a c_i which makes the inequality strict. Therefore, $\Delta_{s_a}^{c_t}$ and $\Delta_{s_n}^{c_t}$ will always be in the third and fourth place among the four levels. Using change of variables, we can rewrite

$$\Delta_{s_l}^{c_t} = \frac{1+r}{m}\int_0^{m/(1+r)} \sigma(\delta^k(1+r)c_t',\delta^k(1+r)c_i')dc_i'$$
$$\equiv \frac{1+r}{m}\int_0^{m/(1+r)} \sigma(\delta^k(1+r)c_t',\delta^k(1+r)c_i)dc_i.$$

where $c_t' = \frac{m}{1+r} - c_t$, and $c_i' = \frac{m}{1+r} - c_i$.

Now we show that there is a threshold c^* such that $\Delta_{s_s}^{c_t} \geq \Delta_{s_l}^{c_t}$ if and only if $c_t \leq c^*$.

Suppose $\beta > 0$ and $\delta^k(1+r) \geq 1$, then

$$
\begin{aligned}
&\Delta_{s_s}^{c_t} - \Delta_{s_l}^{c_t} \\
&= \int_0^{m/(1+r)} \left(\frac{|c_t - c_i|}{c_t + c_i + \beta} - \frac{|c_t' - c_i|}{c_t' + c_i + \frac{\beta}{\delta^k(1+r)}}\right) dc_i \\
&= \underbrace{\int_0^{m/(1+r)} \left(\frac{|c_t - c_i|}{c_t + c_i + \beta} - \frac{|c_t' - c_i|}{c_t' + c_i + \beta}\right) dc_i}_{\Pi(c_t)} + \underbrace{\int_0^{m/(1+r)} \left(\frac{|c_t' - c_i|}{c_t' + c_i + \beta} - \frac{|c_t' - c_i|}{c_t' + c_i + \frac{\beta}{\delta^k(1+r)}}\right) dc_i}_{e(c_t)}.
\end{aligned}
$$

where $c_t' = m/(1+r) - c_t$. We now use two lemmas to show $\Pi(c_t)$ is decreasing in c_t, and $e(c+t))$ decreasing in c_t if $c_t \leq \frac{m}{2(1+r)}$. Then, on one hand, we have $\Delta_{s_s}^{c_t} - \Delta_{s_l}^{c_t}$ is decreasing if $c_t \leq \frac{m}{2(1+r)}$. On the other hand, we have $e(c_t) < 0$ while $\Pi(\frac{m}{2(1+r)}) = 0$ and is decreasing, so $\Delta_{s_s}^{c_t} - \Delta_{s_l}^{c_t} < 0$ if $c_t \geq \frac{m}{2(1+r)}$. And hence, we prove the existence of c^*.

Lemma 11. *$\Pi(c_t)$ is decreasing in c_t.*

proof for lemma 11.

$$
\begin{aligned}
\frac{d\Pi(c_t)}{dc_t} = &\underbrace{2\ln\left(\frac{2c_t + \beta}{c_t + \beta}\right) - \frac{2c_t + \beta}{c_t + \beta} + 2\ln\left(\frac{2c_t + \beta}{c_t + \frac{m}{1+r} + \beta}\right) - \frac{2c_t + \beta}{c_t + \frac{m}{1+r} + \beta} + 2}_{A(c_t)} \\
&+ \underbrace{2\ln\left(\frac{2c_t' + \beta}{c_t' + \beta}\right) - \frac{2c_t' + \beta}{c_t' + \beta} + 2\ln\left(\frac{2c_t' + \beta}{c_t' + \frac{m}{1+r} + \beta}\right) - \frac{2c_t' + \beta}{c_t' + \frac{m}{1+r} + \beta} + 2}_{A(c_t')}.
\end{aligned}
$$

Given that $A(c_t') = A(\frac{m}{1+r} - c_t)$, notice:

a. $A(c_t)$ is concave.

b. $A(\frac{m}{2(1+r)}) \leq 0$.

For a. notice that

$$A'(c_t) = \frac{\beta^2}{(2c_t+\beta)(c_t+\beta)^2} + \frac{(\frac{2m}{1+r}+\beta)^2}{(c_t+\frac{m}{1+r}+\beta)^2(2c_t+\beta)}.$$

with $0 \le c_t \le \frac{m}{1+r}$, both terms are decreasing in c_t. Thus, $A(c_t)$ is concave.

Let $\beta' = \frac{2(1+r)\beta}{m}$, then

$$\begin{aligned} A(\frac{m}{2(1+r)}) &= 2\ln(\frac{2+\beta'}{1+\beta'}) - \frac{2+\beta'}{1+\beta'} + 2\ln(\frac{2+\beta'}{3+\beta'}) - \frac{2+\beta'}{3+\beta'} + 2 \\ &= 2\ln\frac{(2+\beta')^2}{(1+\beta')(3+\beta')} - 2\frac{(2+\beta')^2}{(1+\beta')(3+\beta')} + 2. \end{aligned}$$

with $\frac{(2+\beta')^2}{(1+\beta')(3+\beta')} > 1$ and decreasing in β', we have $A(\frac{m}{2(1+r)})$ is increasing in β' while $\lim_{\beta'\to\infty} A(\frac{m}{2(1+r)}) = 0$, so $A(\frac{m}{2(1+r)}) \le 0$. Using Jensen's inequality, we have

$$\frac{d\Pi(c_t)}{dc_t} = 2(0.5A(c_t) + 0.5A(c_t')) \le 2A(\frac{m}{2(1+r)}) \le 0.$$

□

Since we restrict our attention to $\delta^k(1+r) > 1$, we have the following result:

Lemma 12. *$e(c_t)$ is decreasing in c_t if $\beta \le (\sqrt{2}-1)\frac{m}{1+r}$ and $c_t \le \frac{m}{2(1+r)}$.*

proof of lemma 12. Notice that

$$\frac{de(c_t)}{dc_t} = -\frac{de(c_t)}{dc_t'} = -\frac{d}{dc_t'}\int_0^{m/(1+r)} \frac{|c_t'-c_i|}{c_t'+c_i+\beta}dc_i + \frac{d}{dc_t'}\int_0^{m/(1+r)} \frac{|c_t'-c_i|}{c_t'+c_i+\frac{\beta}{\delta^k(1+r)}}dc_i.$$

With $\delta^k(1+r) > 1$ and fundamental theorem of calculus, we have

$$\frac{de(c_t)}{dc_t} = -\int_{\frac{\beta}{\delta^k(1+r)}}^{\beta} \left(\frac{d^2}{dc_t' d\beta} \int\limits_0^{m/(1+r)} \frac{|c_t' - c_i|}{c_t' + c_i + \beta} dc_i \right) d\beta.$$

Therefore, it suffices to show that

$$\frac{d^2}{dc_t' d\beta} \int\limits_0^{m/(1+r)} \frac{|c_t' - c_i|}{c_t' + c_i + \beta} dc_i \geq 0.$$

Using the Newton-Leibniz formula and some algebraic manipulations, it can be shown that

$$\begin{aligned} &\frac{d^2}{dc_t' d\beta} \int\limits_0^{m/(1+r)} \frac{|c_t' - c_i|}{c_t' + c_i + \beta} dc_i \\ &= \frac{1}{2c_t' + \beta} \left(\frac{\frac{m}{1+r} - c_t'}{c_t' + \frac{m}{1+r} + \beta} + \left(\frac{\frac{m}{1+r} - c_t'}{c_t' + \frac{m}{1+r} + \beta}\right)^2 - \frac{c_t'}{c_t' + \beta} + \left(\frac{c_t'}{c_t' + \beta}\right)^2 \right). \end{aligned}$$

Notice $\frac{1}{2c_t' + \beta} > 0$, $-\frac{c_t'}{c_t' + \beta} + (\frac{c_t'}{c_t' + \beta})^2 \geq -\frac{1}{4}$ and $\frac{\frac{m}{1+r} - c_t'}{c_t' + \frac{m}{1+r} + \beta} + (\frac{\frac{m}{1+r} - c_t'}{c_t' + \frac{m}{1+r} + \beta})^2$ is decreasing in c_t'. Thus, it suffices to show the equation above is non-negative at $c_t' = \frac{m}{2(1+r)}$. With $\beta \leq (\sqrt{2} - 1)\frac{m}{1+r}$, the result follows. □

Notice that $\Pi(c_t) = -\Pi(\frac{m}{1+r} - c_t)$. By lemma 11, we have $\Pi(c_t) \geq 0$ if and only if $c_t \leq \frac{m}{2(1+r)}$. With $\delta^k(1+r) \geq 1$, $e(c_t) \leq 0$, so $\Delta_{s_s}^{c_t} - \Delta_{s_l}^{c_t} \leq 0$ if $c_t \geq \frac{m}{2(1+r)}$.

In addition, by lemma 12, we have $\Delta_{s_s}^{c_t} - \Delta_{s_l}^{c_t}$ is decreasing if $c_t \leq \frac{m}{2(1+r)}$. Under $\delta^k(1+r) \leq 2$ and $\beta \leq (\sqrt{2} - 1)\frac{m}{1+r}$, it can be shown numerically that $\Delta_{s_s}^{0} - \Delta_{s_l}^{0} > 0$. Thus, with intermediate value theorem, there is a threshold $0 < c^* < \frac{m}{2(1+r)}$ such that $\Delta_{s_s}^{c_t} = \Delta_{s_l}^{c_t}$ at c^*. Moreover, $\Delta_{s_s}^{c_t} \geq \Delta_{s_l}^{c_t}$ if and only if $c_t \leq c^*$. Therefore,

$$\begin{cases} \Delta_{s_s}^{c_t} > \Delta_{s_l}^{c_t} > \Delta_{s_a}^{c_t} > \Delta_{s_n}^{c_t} \text{ if } c_t < c^*, \\ \Delta_{s_l}^{c_t} > \Delta_{s_s}^{c_t} > \Delta_{s_a}^{c_t} > \Delta_{s_n}^{c_t} \text{ if } c_t > c^*, \\ \Delta_{s_s}^{c_t} = \Delta_{s_l}^{c_t} > \Delta_{s_a}^{c_t} > \Delta_{s_n}^{c_t} \text{ if } c_t = c^*. \end{cases} \tag{B.2}$$

These three rankings also hold if $0.5 \leq \delta^k(1+r) < 1$. Notice that $c_t = \frac{m}{1+r} - c_t'$, simplify $\frac{\beta}{\delta^k(1+r)} = \beta'$, for each c_t we have:

$$\begin{aligned} \Delta_{s_l}^{c_t'} - \Delta_{s_s}^{c_t'} &= \int_0^{m/(1+r)} \left(\frac{|c_t - c_i|}{c_t + c_i + \beta'} - \frac{|c_t' - c_i|}{c_t' + c_i + \beta}\right) dc_i \\ &= \underbrace{\int_0^{m/(1+r)} \left(\frac{|c_t - c_i|}{c_t + c_i + \beta'} - \frac{|c_t' - c_i|}{c_t' + c_i + \beta'}\right) dc_i}_{\Pi(c_t)} + \underbrace{\int_0^{m/(1+r)} \left(\frac{|c_t' - c_i|}{c_t' + c_i + \beta'} - \frac{|c_t' - c_i|}{c_t' + c_i + \beta}\right) dc_i}_{e(c_t)}. \end{aligned}$$

Therefore, by the exact argument above, the results maintain (except that now $c^* > \frac{m}{2(1+r)}$).

Proposition 12: Suppose that

a. $u(\cdot)$ is strictly increasing, and weakly concave, but the approximation $\sigma(u(c_t) + \delta^k u(c_{t+k}), u(c_t') + \delta^k u(c_{t+k}')) \approx \sigma(c_t + \delta^k c_{t+k}, c_t' + \delta^k c_{t+k}')$ maintains ;

b. individuals follow rank-based salience theory with salience function as in equations (2.1) and (2.2) and $\beta \leq (\sqrt{2} - 1)\frac{m}{1+r}$;

c. $0.5 \leq (1+r)\delta^k \leq 2$

Then, there exists $c^* > 0$ such that:

(1). If $c_t^{CERT} \geq c^*$, $c_t^{IND} \in [c^*, c_t^{CERT}]$.

(2). If $c_t^{CERT} \leq c^*$, $c_t^{IND} \in [c_t^{CERT}, c^*]$.

(3). If $c_t^{CERT} = c^*$, $c_t^{IND} = c^*$.

Similarly,

(1). If $c_t^{SING} \geq c^*$, $c_t^{MULT} \in [c^*, c_t^{SING}]$.

(2). If $c_t^{SING} \leq c^*$, $c_t^{MULT} \in [c_t^{SING}, c^*]$.

(3). If $c_t^{SING} = c^*$, $c_t^{MULT} = c^*$.

Moreover:

(4). Fixing interest rate to some r, if c_t^{CERT} is the choice under r, and c_t^{SING} is the choice under $r/0.8$, then $c_t^{CERT} < c_t^{SING}$.

In addition, $c^* \leq \dfrac{m}{2(1+r)}$ if and only if $(1+r)\delta^k \geq 1$ and $c^* \to \dfrac{m}{2(1+r)}$ as $\beta \to 0$.

Proof. Notice that the first two rankings in (B.2) hold for a certain range of c_t, so the decision weight is *constant* around c_t if $c_t \neq c^*$. Therefore, assuming $c_t \neq c^*$, the tangency condition $\dfrac{u'(c_t)}{u'(m-(1+r)c_t)} = (1+r)\delta^k \dfrac{p_{s_a}^{c_t} + p_{s_l}^{c_t}}{p_{s_a}^{c_t} + p_{s_s}^{c_t}}$ where

$$(1+r)\delta^k \frac{p_{s_a}^{c_t} + p_{s_l}^{c_t}}{p_{s_a}^{c_t} + p_{s_s}^{c_t}} = \begin{cases} (1+r)\delta^k \dfrac{1+\theta^2}{\theta+\theta^2} & \text{if } c_t > c^*, \\ \\ (1+r)\delta^k \dfrac{\theta+\theta^2}{1+\theta^2} & \text{if } c_t < c^*. \end{cases} \tag{B.3}$$

where $\theta \in (0,1]$. At the boundary point c^*, however, the tangency condition is not well-defined.

Recall $\dfrac{u'(c_t)}{u'(m-(1+r)c_t)}$ is decreasing in c_t, and c_t^{CERT} solves the decision-maker's problem under certainty while c_t^{IND} solves decision-maker's problem under uncertainty. Suppose $c_t^{CERT} > c^*$, the tangency condition under certainty is

$$\frac{u'(c_t^{CERT})}{u'(m-(1+r)c_t^{CERT})} \geq (1+r)\delta^k,$$

with equality holds whenever $c_t^{CERT} < m/(1+r)$. If $c_t^{IND} < c^*$, the tangency condition from equation B.3 says that

$$\frac{u'(c_t^{IND})}{u'(m-(1+r)c_t^{IND})} \leq (1+r)\delta^k \frac{\theta+\theta^2}{1+\theta^2},$$

with equality holds for every $c_t^{IND} > 0$. With $\theta \in (0,1]$ and monotonicity of MRS, this requires $c_t^{IND} \geq c_t^{CERT}$, which cannot be true. On the other hand, if $c_t^{IND} > c^*$, by a similar argument of decreasing MRS and structure from equation B.3, we get $c_t^{IND} \leq c_t^{CERT}$. In addition, if θ is sufficiently small, it is possible that there is no solution for either tangency condition in equation B.3. In this case, individual would allocate c^* to the sooner account.[18] A similar argument is sufficient to prove the rest of proposition 12. Notice that if $c_t^{CERT} = c^*$, c_t^{IND} can only be at c^* if $\theta < 1$. □

Proposition 13:

Suppose the conditions and $c^* > 0$ are the same as in proposition 12, then:

(1). If $c_t^{IND} \geq c^*$, $c_t^{NEG} \in [c^*, c_t^{IND}]$.

(2). If $c_t^{IND} \leq c^*$, $c_t^{NEG} \in [c_t^{IND}, c^*]$.

(3). If $c_t^{IND} = c^*$, $c_t^{NEG} = c^*$.

(4). $c_t^{CERT} = c_t^{POS}$.

Proof. The salience ranking conditions are analogous to those in proposition 12. On the other hand, correlation changes tangency conditions. Under negative-correlated risk structure, the tangency condition becomes

[18]Assume $\theta < 1$, since $\frac{2+2\theta}{2+\theta+\theta^2} > \frac{1+\theta+2\theta^2}{1+\theta+\theta^2+\theta^3}$, by continuity of utility function, allocation $(c^*, \frac{m}{1+r} - c^*)$ yields greater utility than other allocations which sufficiently close to c^*.

$$\frac{u'(c_t^{NEG})}{u'(m-(1+r)c_t^{NEG})} = (1+r)\delta^k \frac{p_{s_l}^{c_t}}{p_{s_s}^{c_t}} \text{ where}$$

$$(1+r)\delta^k \frac{p_{s_l}^{c_t}}{p_{s_s}^{c_t}} = \begin{cases} (1+r)\delta^k \frac{1}{\theta} & \text{if } c_t^{NEG} > c^* \\ \\ (1+r)\delta^k \theta & \text{if } c_t^{NEG} < c^*. \end{cases} \tag{B.4}$$

Notice that $\frac{\theta+\theta^2}{1+\theta^2} > \theta$, by a similar argument from proposition 12 we can get 1, 2, and 3.

Under positive-correlated risk structure, the tangency condition becomes

$$\frac{u'(c_t^{POS})}{u'(m-(1+r)c_t^{POS})} = (1+r)\delta^k \frac{p_{s_a}^{c_t}}{p_{s_a}^{c_t}} = (1+r)\delta^k,$$

which is the same as equation B.1. The result follows. □

B.6 Structural Estimation

In this section, we describe two structural estimators. Within the CTB design under multidimensional salience, there is no decision weight distortion when both the sooner and later payments are both riskless. On the other hand, by proposition 12, under independent risks, our theory predicts a specific behavioral pattern. Notice, in CTB design subjects modify choices due to variations in interest rates, time delays, and risk structures. Consequently, the degree to which salience distorts attention is endogenously determined. Therefore, pooling all data together may cause misidentification of utility curvature and the salience effect. To circumvent this issue, these two structural estimators can both be considered as two-stage estimators. In stage one, they both use data under riskless condition to identify utility curvature and time discounting. In the second stage, they deploy different assumptions to estimate salience effect from risky condition. In practice, we use MLE estimators to conduct these two stages simultaneously.

We begin with assuming every subject processes a CRRA utility function, that is:

$$u(c_t, c_{t+k}) = (c_t - w)^{\alpha} + \delta^k (c_{t+k} - w)^{\alpha},$$

where α is a measure for risk aversion, δ is the time discounting factor, and w is the Stone - Geary minimum consumption level.

With this structural assumption, from tangency conditions (B.1) and (B.3), we get:

$$\log(c_t^{CERT} - w) - \log(c_{t+k}^{CERT} - w) = \log \frac{c_t^{CERT} - w}{c_{t+k}^{CERT} - w} = \frac{1}{\alpha - 1} \log(1+r) + \frac{k}{\alpha - 1} \log(\delta). \quad \text{(B.5)}$$

$$\log \frac{c_t^{IND} - w}{c_{t+k}^{IND} - w} - \log \frac{c_t^{CERT} - w}{c_{t+k}^{CERT} - w} = \begin{cases} \frac{1}{\alpha - 1} \log(\frac{1+\theta^2}{\theta+\theta^2}) & \text{if } c_t^{IND} > \frac{m}{2(1+r)}, \\ \\ \frac{1}{\alpha - 1} \log(\frac{\theta+\theta^2}{1+\theta^2}) & \text{if } c_t^{IND} < \frac{m}{2(1+r)}. \end{cases} \quad \text{(B.6)}$$

For simplicity, we denote $Y_{CERT}^* = \log \frac{c_t^{CERT} - w}{c_{t+k}^{CERT} - w}$ and $Y_{IND}^* = \log \frac{c_t^{IND} - w}{c_{t+k}^{IND} - w}$. Notice, however, Y_{CERT}^* , Y_{IND}^* are not completely identified since c_t is restricted to be in $[0, \frac{m}{1+r}]$. As a result, Y_{CERT}^* and Y_{IND}^* are censored in to

$$Y_{CERT} = \max\left(\min(Y_{CERT}^*, \ln \frac{-w}{m/(1+r) - w}), \ln \frac{m/(1+r) - w}{-w}\right),$$

$$Y_{IND} = \max\left(\min(Y_{IND}^*, \ln \frac{-w}{m/(1+r) - w}), \ln \frac{m/(1+r) - w}{-w}\right).$$

At the boundary point, $c_t = \frac{m}{2(1+r)}$, the first order condition is not well-defined. For this reason, we discard these observations in subsequent structural analysis. Nevertheless, one should notice that according to proposition 12 choosing c_t at $\frac{m}{2(1+r)}$ only under risky environment

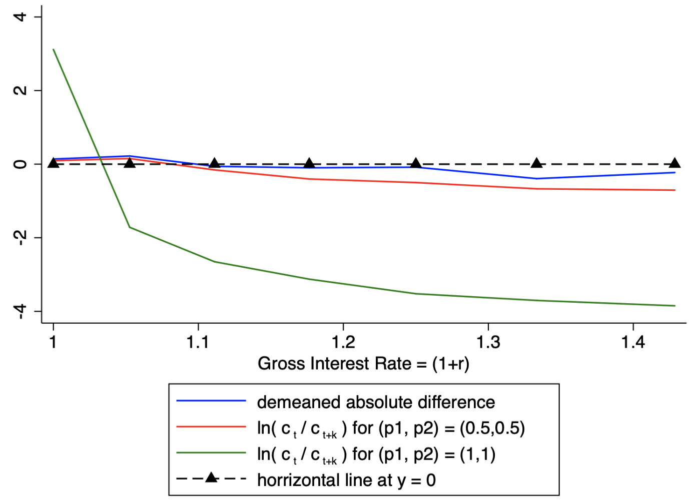

Figure A1. Average Difference of log Consumption Ratio

Notes: The Figure presents the aggregate behavior of $N = 226$ from Andreoni and Sprenger (2012b), Miao and Zhong (2015), and Cheung (2015). The averages are calculated after dropping data whose allocations are doubly censored. The red and green lines represent log values of allocation ratio between sooner and later account under risky and riskless condition. To ensure existence, we add a minimal consumption value 0.1 to both allocations in all data. The blue line gives the demeaned absolute difference between the log allocation ratio. The black dash line is a horizontal reference line.

indicates a strong salience distortion. That is, assuming subjects follow rank-based salience theory, they would choose the portfolio $(\frac{m}{2(1+r)}, \frac{m}{2})$ if the salience parameter θ is below some lower bound between 0 and 1. In this case, discarding these subjects would potentially understate the salience effect. However, the two strategies give different degrees of credibility to rank-based salience theory, so for consistency, we choose to discard those subjects. The results suggest that there is still significant salience distortion even if after discarding the observations that are most likely to be severe.

Equation B.6 implies $|Y^*_{IND} - Y^*_{CERT}|$ is a constant which equals to $\frac{1}{1-\alpha}\ln(\frac{1+\theta^2}{\theta+\theta^2})$. Figure A1 presents the aggregate behavior and difference permitting a direct assessment of $\frac{1}{1-\alpha}\ln(\frac{1+\theta^2}{\theta+\theta^2})$, which appears quite stable across values of $(1+r)$. Since the absolute differences are relatively stable, our first strategy uses a two stage method to approach this problem. In the first stage, we use the allocation ratio under the riskless condition to derive the utility curvature, α, and discounting, δ. In the second stage, we use the sample mean of the absolute difference level and estimators from the first stage to estimate the salience parameter, θ.

Two implicit assumptions are made in our initial estimation method. The first is that *all* the discrepancies between Y^*_{CERT} and Y^*_{IND} can be explained by rank-based multidimensional salience theory, but in fact only 78% of data is consistent. One can ameliorate this issue by ignoring those data. The second is that the errors in decisions made under uncertainty do not directly contribute to the error in second stage. Otherwise, the second stage estimation is biased. To circumvent these issues, we introduce the following approach. The exact form of tangency condition in equation B.6 can be inferred from c_t^{IND} and proposition 12. Therefore, we develop a different two-stage method. As in the first strategy, we use observed choices from the riskless environment to estimate curvature and discounting. Then, we use subjects' choices from risky condition to infer the exact tangency form in equation B.6 and estimate the salience parameter. However, we cannot exogenously control for the salience effect. As a consequence, one may be concerned that the tangency condition we deduct is endogenous. Therefore, we use the interest rate, $(1+r)$ as an instrumental variable for the actual rank of the states. Estimations from both approaches are provided in Table B.1. While similar estimates for θ to those previously discussed are obtained in Andreoni and Sprenger (2012b) and Cheung (2015), estimates for Miao and Zhong (2015) show some sensitivity to methodology.[19] Across all the estimates of Table B.1, we document values of the salience parameter, θ, that differ substantially and significantly from the DEU value of $\theta = 1$. To complete current discussion, we describe the estimation procedures in detail.

[19]One reason for the estimation difference is that Miao and Zhong (2015) have substantially more choices in condition IND that are exactly at the midpoint of budget constraints, $(715/1554 \approx 46\%)$, compared to Andreoni and Sprenger (2012b) $(240/1120 \approx 21\%)$ and Cheung (2015) $(247/882 \approx 28\%)$. These middle allocations are treated as censored observations in the IV approach and imply a strong salience effect.

Absolute Difference Approach

Fixing time span, k, and interest rate, $(1+r)$, for each individual we have

$$\begin{aligned} Y^*_{CERT} &= \frac{1}{\alpha-1}\ln(1+r) + \frac{k}{1-\alpha}\ln(\delta) + u, \\ |Y^*_{CERT} - Y^*_{IND}| &= \frac{1}{1-\alpha}\ln(\frac{1+\theta^2}{\theta+\theta^2}) + v, \\ u &\sim \mathcal{N}(0, \sigma_u^2), \\ v &\sim \mathcal{N}(0, \sigma_v^2). \end{aligned}$$

In the first stage, we use data collected from the riskless condition to infer curvature, α, and discount factor, δ. The likeihood function we estimate here is $\ln f(Y_{CERT}|\alpha, r, k, \delta)$, which can be expressed as:

$$\begin{aligned} &I\{Y_{CERT} = \frac{m}{1+r}\}\ln\left(\Phi\Big(\frac{\frac{1}{\alpha-1}\log(1+r) + \frac{k}{\alpha-1}\log(\delta) - \frac{m}{1+r}}{\sigma_u}\Big)\right) \\ &+I\{Y_{CERT} \in (0, \frac{m}{1+r})\}\ln\left(\frac{1}{\sigma_u}\phi\Big(\frac{\frac{1}{\alpha-1}\log(1+r) + \frac{k}{\alpha-1}\log(\delta) - Y_{CERT}}{\sigma_u}\Big)\right) \\ &+I\{Y_{CERT} = 0\}\ln\left(\Phi\Big(1 - \frac{\frac{1}{\alpha-1}\log(1+r) + \frac{k}{\alpha-1}\log(\delta)}{\sigma_u}\Big)\right). \end{aligned}$$

where I is the indicator function and Φ, ϕ are the standard normal cdf and pdf.

As mentioned above, if both Y^*_{CERT} and Y_{IND} are not identified, we are not able to infer the accurate range of absolute difference. On the other hand, if either Y^*_{CERT} or Y_{IND} is at some bound, we observe a lower bound of the absolute value. Thus, the likelihood function for the second stage is

$$I\{(Y_{CERT},Y_{IND}) \notin (0,\frac{m}{1+r})^2\}\ln\left(\Phi\Big(\frac{|Y_{IND}-Y_{CERT}|-\frac{1}{\alpha-1}\ln(\frac{1+\theta^2}{\theta+\theta^2})}{\sigma_v}\Big)\right)$$
$$+I\{(Y_{CERT},Y_{IND}) \in (0,\frac{m}{1+r})^2\}\ln\left(\frac{1}{\sigma_v}\phi\Big(\frac{|Y_{IND}-Y_{CERT}|-\frac{1}{\alpha-1}\ln(\frac{1+\theta^2}{\theta+\theta^2})}{\sigma_v}\Big)\right).$$

IV Approach

Define a variable *rank* equals to $\operatorname{sgn}\Big(c_t-\frac{m}{2(1+r)}\Big)$. That is, $rank=1$ if s_l is most salient. Notice that, we ignore the data in which $c_t=\frac{m}{2(1+r)}$. As discussed before, we use $\ln(1+r)$ as an instrument for *rank*. Since $\ln(1+r)$ is exogeneously given and has a relatively narrow range, we use a linear probability model in the first stage:

$$rank = d+\beta\ln(1+r)+e.$$

where d is a constant. Then, the structural equations are:

$$Y^*_{CERT} = \frac{1}{\alpha-1}\ln(1+r)+\frac{k}{1-\alpha}\ln(\delta)+\gamma_1 e+u,$$
$$Y^*_{IND} = \frac{1}{\alpha-1}\ln(1+r)+\frac{k}{1-\alpha}\ln(\delta)+\frac{1}{\alpha-1}\,rank\,\ln(\frac{1+\theta^2}{\theta+\theta^2})+\gamma_2 e+v,$$
$$\begin{bmatrix} u \\ v \end{bmatrix} \sim \mathcal{N}(0,\Sigma),\ \text{where } \Sigma=\begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}.$$

By design, *rank* is independent of v, and e is independent of u,v. Denote

$\Pi_1 = \frac{1}{\alpha-1}\ln(1+r)+\frac{k}{1-\alpha}\ln(\delta)+\gamma_1 e$, and

$\Pi_2 = \frac{1}{\alpha-1}\ln(1+r)+\frac{k}{1-\alpha}\ln(\delta)+\frac{1}{\alpha-1}\,rank\,\ln(\frac{1+\theta^2}{\theta+\theta^2})+\gamma_2 e.$

If both choices are uncensored, the likelihood function becomes:

$$I(riskless=1)\ln(\frac{1}{\sigma_u}\phi(\frac{Y_{CERT}-\Pi_1}{\sigma_u}))+I(riskless=0)\ln(\frac{1}{\sigma_v}\phi(\frac{Y_{IND}-\Pi_2}{\sigma_v})),$$

where $I(riskless=1)$ is the indicator function suggesting the data point is under riskless condition($I(riskless=0)$ indicates whether data is under risky condition).

Next, we consider the case in which only one of two choices from an individual is uncensored. Notice, since u,v are potentially correlated, we may reduce the standard error of one given another. Let $u=\frac{\rho\sigma_u}{\sigma_v}v+\varepsilon$ where $\varepsilon\sim\mathscr{N}(0,(1-\rho^2)\sigma_v^2)$, and $v=\frac{\rho\sigma_v}{\text{ß}_u}u+\eta$ where $\eta\sim\mathscr{N}(0,(1-\rho^2)\sigma_u^2)$. Denote the variance of ε and η as σ_ε^2 and σ_η^2 respectively.

If choices under certainty is censored, we have the likelihood function as:

$$\begin{aligned}
&I(riskless=1\cap Y_{CERT}=\bar{Y})\ln(\Phi(\frac{\Pi_1-\bar{Y}-\frac{\rho\sigma_u}{\sigma_v}(Y_{IND}-\Pi_2)}{\sigma_\varepsilon}))\\
&+I(riskless=1\cap Y_{CERT}=\underline{Y})\ln(1-\Phi(\frac{\Pi_1-\underline{Y}-\frac{\rho\sigma_u}{\sigma_v}(Y_{IND}-\Pi_2)}{\sigma_\varepsilon}))\\
&+I(riskless=0)\ln(\frac{1}{\sigma_v}\phi(\frac{Y_{IND}-\Pi_2}{\sigma_v})).
\end{aligned}$$

If choices under uncertainty is censored, we have the likelihood function as:

$$\begin{aligned}
&I(riskless=0\cap Y_{IND}=\bar{Y})\ln(\Phi(\frac{\Pi_2-\bar{Y}-\frac{\rho\sigma_v}{\sigma_u}(Y_{CERT}-\Pi_1)}{\sigma_\eta}))\\
&+I(riskless=0\cap Y_{IND}=\underline{Y})\ln(1-\Phi(\frac{\Pi_2-\underline{Y}-\frac{\rho\sigma_v}{\sigma_u}(Y_{CERT}-\Pi_1)}{\sigma_\eta}))\\
&+I(riskless=1)\ln(\frac{1}{\sigma_u}\phi(\frac{Y_{CERT}-\Pi_1}{\sigma_u})).
\end{aligned}$$

If both are censored notice that $u=Y_{CERT}^*-\Pi_1$ and $v=Y_{IND}^*-\Pi_2$. Let $F(X,Y,z)$ be the cdf of bivariate normal distribution of (X,Y) with marginal variances 1 and correlation z. The likelihood function is:

$$
\begin{aligned}
&I(Y_{CERT}=\bar{Y}\cap Y_{IND}=\bar{Y})F(-\frac{Y^*_{CERT}-\Pi_1}{\sigma_u},-\frac{Y_{IND}-\Pi_2}{\sigma_v},\rho)\\
&+I(Y_{CERT}=\underline{Y}\cap Y_{IND}=\bar{Y})F(\frac{Y^*_{CERT}-\Pi_1}{\sigma_u},-\frac{Y_{IND}-\Pi_2}{\sigma_v},-\rho)\\
&+I(Y_{CERT}=\bar{Y}\cap Y_{IND}=\underline{Y})F(-\frac{Y^*_{CERT}-\Pi_1}{\sigma_u},\frac{Y_{IND}-\Pi_2}{\sigma_v},-\rho)\\
&+I(Y_{CERT}=\underline{Y}\cap Y_{IND}=\underline{Y})F(\frac{Y^*_{CERT}-\Pi_1}{\sigma_u},\frac{Y_{IND}-\Pi_2}{\sigma_v},\rho).
\end{aligned}
$$

Table B.1 provides the full results of the two methodologies.

Table B.1. Estimation Results

	Andreoni and Sprenger (2012b)		Miao and Zhong (2015)		Cheung (2015)	
Method:	Absolute Difference	IV	Absolute Difference	IV	Absolute Difference	IV
	(1)	(2)	(3)	(4)	(5)	(6)
Parameters:						
Curvature: α	0.963	0.978	0.955	0.982	0.970	0.969
	(0.009)	(0.003)	(0.012)	(0.004)	(0.005)	(0.004)
Discounting: δ	1.003	1.001	1.010	1.002	0.999	0.999
	(0.001)	(0.000)	(0.004)	(0.000)	(0.000)	(0.000)
Salience: θ	0.648	0.614	0.556	0.339	0.754	0.730
	(0.033)	(0.011)	(0.064)	(0.009)	(0.019)	(0.012)
Variances:						
$\ln\sigma(u)$	2.074	2.192	2.185	2.500	1.935	1.823
	(0.146)	(0.116)	(0.138)	(0.145)	(0.121)	(0.086)
$\ln\sigma(v)$	1.650	1.160	1.791	0.966	1.457	0.974
	(0.091)	(0.110)	(0.076)	(0.110)	(0.090)	(0.129)
Correlations:						
γ_1		6.389		4.060		3.472
		(1.184)		(1.580)		(0.846)
γ_2		17.246		52.744		7.699
		(2.552)		(11.782)		(1.162)
ρ		0.649		0.592		0.287
		(0.074)		(0.080)		(0.100)
N	594.000	880.000	615.000	839.000	501.000	635.000
ll	-1405.616	-2461.735	-1321.036	-2395.512	-1582.251	-2186.345

Notes: Estimates of salience parameter, θ, utility curvature α, and daily discount factor δ assuming background consumption parameter $\omega=-0.1$. $\ln\sigma(u)$ and $\ln\sigma(v)$ are log standard variances. γ_1,γ_2 are the correlation between the log consumption ratio and first-stage residual. ρ represents the correlation between choices under certainty and independent risk. Columns 1, 3, and 5 use the absolute difference between the log consumption ratio of individuals decisions made under uncertainty and certainty condition. Columns 2, 4, and 6 use the IV approach. Columns 1 and 2 gives results from Andreoni and Sprenger (2012b) using data with $p1=p2=1$ or $p1=p2=0.5$. Columns 3 and 4 gives results from Miao and Zhong (2015). Columns 4 and 5 gives results from Cheung (2015) using the observation in which the first payment was paid 1 week after the experiment. Standard errors clustered at individual level in parentheses.

B.7 Experimental Hypothesis

In this section, we provide a formal analysis of our experimental hypotheses. The first hypothesis, we call *Intertemporal Hedging* , is also documented in Andreoni and Sprenger (2012b), Miao and Zhong (2015), and Cheung (2015). The second hypothesis, we call *Reordering Dependence*, is novel and can distinguish from other models positing decisions only depend on marginal distributions of options. Below, we explain the predicted behavioral patterns in detail.

Intertemporal Hedging

Recall in Choice 1, Option A gives \$18 in one week and \$2 in four weeks after the experiment while Option B gives \$10 in both one week and four weeks after the experiment. Since there is no risk in Choice 1, multidimensional salience theory predicts that decison makers' choices are governed by their intertemporal utility function: $u(x)+\delta^t u(y)$, where $u(\cdot)$ is the flow utility with $u(0)=0$ and δ is the time discounting. Denote $Diff_{AB}^{(1)}$ is the utility difference between Option A and Option B in Choice 1. That is, $Diff_{AB}^{(1)} = u(18)+\delta^t u(2) - u(10) - \delta^t u(10)$. As a result, Option A is preferred than Option B in Choice 1 if $Diff_{AB}^{(1)} \geq 0$. In addition, we define $Diff_{AB}^{(2)}$ and $Diff_{AB}^{(3)}$ analogously. On the other hand, in Choice 2, salience levels in different states are: $\Delta_{HH} = \sigma([18,2],[10,10])$ in HH, $\Delta_{HT} = \sigma([18,0],[10,0])$ in HT, $\Delta_{TH} = \sigma([0,2],[0,10])$ in TH, $\Delta_{TT} = \sigma([0,0],[0,0])$ in TT. According to definition 1, by lower ordering, we have $\sigma([18,2],[10,10]) \leq \sigma([18,2],[10,2])$. By diminishing sensitivity, we have $\Delta_{HH} \leq \Delta_{HT}$. By compatibility, we have $\Delta_{HT} \leq \Delta_{TH}$. Therefore, we have $\Delta_{TT} \leq \Delta_{HH} \leq \Delta_{HT} \leq \Delta_{TH}$. Thus, we have:

$$
\begin{aligned}
&(1+\theta+\theta^2+\theta^3)Diff_{AB}^{(2)} \\
&= (\theta+\theta^2)u(18)+(1+\theta^2)\delta^t u(2)-(\theta+\theta^2)u(10)-(1+\theta^2)\delta^t u(10) \\
&= (\theta+\theta^2)Diff_{AB}^{(1)}+(1-\theta)\delta^t\Big(u(2)-u(10)\Big) \\
&\leq (\theta+\theta^2)Diff_{AB}^{(1)}.
\end{aligned}
$$

where $\theta \in (0,1]$ is the salience parameter. As a result, we have the following prediction: if one subject chooses Option B in Choice 1, she will choose Option B in Choice 2. However, the reverse may not be true. Therefore, our theory predicts that there should be more subjects choose Option B in Choice 2 than in Choice 1.

Reordering Dependence

In Choice 3, the salience levels are:
$\Delta'_{HH} = \sigma([18,2],[0,0])$ in HH, $\Delta'_{HT} = \sigma([18,0],[10,10])$ in HT, $\Delta'_{TH} = \sigma([0,2],[0,10])$ in TH, $\Delta'_{TT} = \sigma([0,0],[10,0])$ in TT. Consequently, by a similar analysis, $\Delta'_{HT} \leq \Delta'_{TH} \leq \Delta'_{TT} \leq \Delta'_{HH}$. Then:

$$
\begin{aligned}
&(1+\theta+\theta^2+\theta^3)Diff_{AB}^{(3)} \\
&= (1+\theta^3)u(18)+(1+\theta^2)\delta^t u(2)-(\theta^2+\theta^3)u(10)-(\theta+\theta^3)\delta^t u(10) \\
&\Rightarrow (1+\theta+\theta^2+\theta^3)(Diff_{AB}^{(3)}-Diff_{AB}^{(2)}) \\
&= (1+\theta^3-\theta-\theta^2)u(18)+(\theta-\theta^3)u(10)+(1-\theta)(1+\theta^2)\delta^t u(10) \\
&= (1-\theta^2)(1-\theta)u(18)+(\theta-\theta^3)u(10)+(1-\theta)(1+\theta^2)\delta^t u(10) \geq 0.
\end{aligned}
$$

Therefore, our theory predicts that there should be more subjects choose Option B in Choice 2 compared with in Choice 3.

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