Title
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Publication Date
1967-04-17

UCRL-17513 ✓ errata

University of California

Ernest O. Lawrence Radiation Laboratory

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AEC Contract No. W-7405-eng-48

May 16, 1967

ERRATA

TO: All recipients of UCRL-17513

FROM: Technical Information Division

Subject: UCRL-17513, "Non-Existence of Self-Conjugate Particles with Half-Integral Isospin," Huan Lee, April 17, 1967.

Page 7 Replace with the attached done

Page 13 Change in reference 7. Should read:

7. See e. g., H. P. Stapp . . . .

. . . . . Laboratory Report

UCRL-16816 . . . . done

Submitted to Phys. Rev. Letters

UCRL-17513
Preprint

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Lawrence Radiation Laboratory
Berkeley, California

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ABSTRACT

It is proved that in the usual local field theory of particles, one cannot, for any value of ordinary spin, construct the fields corresponding to the particles of a self-conjugate multiplet of half-integral isospin (SMHI). An analogous result is obtained in the analytic S-matrix framework, where the requirement of isospin invariance and crossing symmetry precludes the existence of SMHI particles of any spin. Interesting physical implications and generalizations of this result to higher internal symmetry are discussed.

Recently it has been shown by Carruthers[1] that the field operators corresponding to spinless bosons of a self-conjugate isomultiplet with half-integral isospin are non-local: if locality is demanded, then one cannot construct such fields. We shall generalize this result to the particles, with any spin, of a self-conjugate multiplet of half-integral isospin (SMHI). Furthermore we shall prove in the S-matrix framework that isospin symmetry plus the usual crossing property entail that all scattering amplitudes involving any particles of an SMHI must vanish. Interesting physical implications of this result, and generalization to higher symmetry are also discussed.

Throughout this paper, by a self-conjugate multiplet we mean an irreducible multiplet that contains the antiparticle of each particle contained in the multiplet. Consider a self-conjugate isomultiplet of spin j and isospin I. Let $a_\alpha^+(\underset{\sim}{p},\sigma)$ and $a_\alpha(\underset{\sim}{p},\sigma)$ be the creation and annihilation operators of the free particle multiplet with momentum $\underset{\sim}{p}$ and spin component σ, where α denotes the I_z-component. Isospin symmetry is expressed by

$$U^{-1}(u)\ a_\alpha(\underset{\sim}{p},\sigma)\ U(u) = \sum_{\alpha'} D^{(I)}_{\alpha\alpha'}(u)\ a_{\alpha'}(\underset{\sim}{p},\sigma)\ , \qquad (1)$$

where $U(u)$ is the unitary operator in Hilbert space that represents the $SU(2)$ transformation u, and $D^{(I)}(u)$ is the standard irreducible representation matrix[2] with dimension $(2I+1)$.

The adjoint of (1) is

$$U^{-1}(u)\, a^{\dagger}_{\alpha}(\underset{\sim}{p},\sigma)\, U(u) = \sum_{\alpha'} D^{(I)*}_{\alpha\alpha'}(u)\, a^{\dagger}_{\alpha'}(\underset{\sim}{p},\sigma) \ . \tag{2}$$

Now we construct the (2j+1)-component field in the usual way[3]

$$\phi^{\alpha}_{\sigma}(x) = (2\pi)^{-3/2} \int \frac{d^3\underset{\sim}{p}}{(2p_0)^{1/2}} \sum_{\sigma'} \Big[D^{(j)}_{\sigma\sigma'}[L(\underset{\sim}{p})]\, a_{\alpha}(\underset{\sim}{p},\sigma')\, e^{ip\cdot x}$$

$$+ \left\{ D^{(j)}\, [L(\underset{\sim}{p})] C^{-1} \right\}_{\sigma\sigma'} \left\{ H a^{\dagger}(\underset{\sim}{p},\sigma') \right\}_{\alpha} e^{-ip\cdot x} \Big] , \tag{3}$$

where $D^{(j)}[\Lambda]$ is the (j,0) or (0,j) irreducible representation of homogeneous Lorentz group, and $L(\underset{\sim}{p})$ is a boost which takes a particle of mass m from rest to momentum $\underset{\sim}{p}$; and C is a (2j+1) x (2j+1) matrix with the properties

$$C^* C = (-1)^{2j}, \quad C^{\dagger} C = I \ , \tag{4}$$

such that for any rotation R and boost $L(\underset{\sim}{p})$ we have

$$D^{(j)*}[R] = C D^{(j)}[R]\, C^{-1} \tag{5}$$

and

$$D^{(j)*}[L(\underset{\sim}{p})] = C D^{(j)}[L(-\underset{\sim}{p})]\, C^{-1} \ ; \tag{6}$$

$[Ha^{\dagger}(\underset{\sim}{p},\sigma)]_{\alpha}$ stands for $\sum_{\alpha'} H_{\alpha\alpha'} a^{\dagger}_{\alpha'}(\underset{\sim}{p},\sigma)$, where H is some matrix to be determined by the locality condition. We assume now, on the basis of their particle interpretation, that the a's and $a^{\dagger}$'s satisfy either the usual commutation $([\ ]_+)$ or anticommutation $([\ ]_-)$ rules:

$$[a_{\alpha}(\underset{\sim}{p},\sigma),\ a^{\dagger}_{\alpha'}(\underset{\sim}{p}',\sigma')]_{\pm} = \delta_{\alpha\alpha'}\ \delta_{\sigma\sigma'}\ \delta(\underset{\sim}{p} - \underset{\sim}{p}') \ , \tag{7}$$

with all other vanishing. It is then easy to work out the commutators or anticommutators for the field defined by (3):

$$[\phi^{\alpha}_{\sigma}(x),\ \phi^{\alpha'}_{\sigma'}(y)]_{\pm} = (2\pi)^{-3}\int \frac{d^3\underset{\sim}{p}}{2p_0}$$

$$\times \sum_{\lambda}\Big\{D_{\sigma\lambda}(DC^{-1})_{\sigma'\lambda}\ H_{\alpha'\alpha}\ \exp[ip\cdot(x-y)] \mp (DC^{-1})_{\sigma\lambda}\ D_{\sigma'\lambda}$$

$$\times\ H_{\alpha\alpha'}\ \exp[-ip\cdot(x-y)]\Big\}$$

$$= (2\pi)^{-3}\ C_{\sigma\sigma'}\int \frac{d^3\underset{\sim}{p}}{2p_0}\Big\{(-1)^{2j}\ H_{\alpha'\alpha}\ \exp[ip\cdot(x-y)]$$

$$\mp\ H_{\alpha\alpha'}\ \exp[-ip\cdot(x-y)]\Big\} \ . \tag{8}$$

It is well known[4] that such an integral will vanish for space-like (x-y) if, and only if, the coefficients of the two exponentials are equal and opposite, i.e.,

$$H_{\alpha\alpha'} = \pm (-1)^{2j} H_{\alpha'\alpha} ,$$

or in matrix form

$$H = \pm (-1)^{2j} \tilde{H} . \tag{9}$$

In deriving (8) we have used (4), (6), and $D^{(j)}[L(\underset{\sim}{p})] = D^{(j)\dagger}[L(\underset{\sim}{p})]$. Similarly we obtain

$$[\phi_\sigma^\alpha(x), \phi_{\sigma'}^{\alpha'\dagger}(y)]_\pm = (2\pi)^{-3} \int \frac{d^3\underset{\sim}{p}}{2p_0}$$

$$\times \sum_\lambda \left\{ D_{\sigma\lambda} D^*_{\sigma'\lambda} \exp[ip\cdot(x-y)] \mp (DC^{-1})_{\sigma\lambda} (DC^{-1})^*_{\sigma'\lambda} \right.$$

$$\left. \times \left(\sum_\beta H_{\alpha\beta} H^*_{\alpha'\beta} \right) \exp[-ip\cdot(x-y)] \right\}$$

$$= (2\pi)^{-3} \int \frac{d^3\underset{\sim}{p}}{2p_0} \left\{ D^{(j)} [L(\underset{\sim}{p})] \right\}^2_{\sigma\sigma'} \left\{ \exp[ip\cdot(x-y)] \right.$$

$$\left. \mp (HH^\dagger)_{\alpha\alpha'} \exp[-ip\cdot(x-y)] \right\} . \tag{10}$$

It is shown in Ref. 3 that $\left\{ D^{(j)} [L(\underset{\sim}{p})] \right\}^2_{\sigma\sigma'}$ is a homogeneous polynomial of order $2j$ in the momentum components p_μ, which we shall denote by $F_{\sigma\sigma'}(p_\mu)$. With this fact we can rewrite (10) as

$$[\phi^{\alpha}_{\sigma}(x),\ \phi^{\alpha'\dagger}_{\sigma'}(y)]_{\pm} = (2\pi)^{-3}\ F_{\sigma\sigma'}\left(-i\frac{\partial}{\partial x'^{\mu}}\right)$$

$$\times\int\frac{d^3\underset{\sim}{p}}{2p_0}\left\{\exp[ip\cdot(x-y)]\ \mp\ (-1)^{2j}\ (HH^{\dagger})_{\alpha\alpha'}\ \exp[-ip\cdot(x-y)]\right\}. \tag{11}$$

In order that (11) vanish outside the light-cone we must have

$$HH^{\dagger} = \pm\,(-1)^{2j}\ I\ . \tag{12}$$

Since $HH^{\dagger} = -I$ is absurd, (12) tells us we must take the normal connection between spin and statistics. Hence we can express (9) and (12) together as

$$HH^{*} = HH^{\dagger} = I\ . \tag{13}$$

On the other hand, if we use (1) and (2) in (3) and require that $\phi^{\alpha}_{\sigma}(x)$ transform under SU(2) irreducibly,

$$U^{-1}(u)\ \phi^{\alpha}_{\sigma}(x)\ U(u) = \sum_{\alpha'}\ D^{(I)}_{\alpha\alpha'}(u)\ \phi^{\alpha'}_{\sigma}(x),$$

we obtain

$$HD^{(I)*}(u)\ H^{-1} = D^{(I)}(u)\ . \tag{14}$$

The most general H satisfying (14) is of the form[5]

$$H_{\alpha\alpha'} = \eta(-1)^{I+\alpha}\,\delta_{\alpha,-\alpha'}\,, \tag{15}$$

where η is an arbitrary complex number. But (15) implies

$$(-1)^{2I}\,HH^* = HH^\dagger = |\eta|^2,$$

which is incompatible with (13) if 2I is an odd integer. The proof for 2(2j+1)-component fields and other general reducible (in spin space) spin fields follow in the same manner. Hence we conclude that, in the framework of the usual local field theory of particles, one cannot construct the free fields corresponding to the particles of any SMHI.

Jin[6] has recently attempted to generalize Carruther's result to higher spin. He starts with a field which satisfies

$$\phi^\dagger_\alpha(x) = \eta^{T*}_\alpha\,\phi_{-\alpha}(x)\,, \tag{16}$$

where $\eta^T_\alpha = \xi(-1)^{T+\alpha}$, and ξ is independent of α and satisfies $|\xi| = 1$. But (18) immediately implies $\phi_\alpha(x) \equiv 0$ for half-integral T. For, taking the adjoint of (18), we get

$$\phi_\alpha = \eta_\alpha^T \phi_{-\alpha}^\dagger ,$$

hence

$$\phi_{-\alpha}^\dagger = \eta_\alpha^{T*} \phi_\alpha ,$$

and hence

$$\phi_\alpha^\dagger = \eta_{-\alpha}^{T*} \phi_{-\alpha} . \tag{17}$$

Comparing (16) and (17) we conclude that $\phi_\alpha^\dagger(x) \equiv 0$ for

$$\eta_{-\alpha}^{T*} = \xi^*(-1)^{T-\alpha} = -\xi^*(-1)^{T+\alpha} = -\eta_\alpha^{T*} .$$

Since $\phi_\alpha^\dagger(x)$ is identically zero, the arguments of Jin involving the Källen-Lehmann representation are superfluous.

The foregoing discussion is based on the general framework of local field theory and on the usual connections between particles and fields. The question thus arises whether a physically more direct conclusion can be made in the S-matrix framework, which specifically avoids problems concerning the connection between particles and fields. We shall show next that if the S-matrix possesses isospin symmetry and the usual crossing property, then particles of SMHI cannot exist in such a theory.

The crossing property of the S-matrix, which can be derived[7] from the assumptions of (1) superposition principle,

(2) unitarity, (3) connectedness structure, (4) Lorentz invariance, and (5) analyticity on mass shell, can be expressed as[8]

$$\langle K_1|M|K_2;p,a_\alpha\rangle = \lambda\ \langle K_1;-p,\bar{a}_{-\alpha}|M|K_2\rangle\ , \tag{18}$$

$$\langle K_1'|M|K_2';-p,\bar{\bar{a}}_{-\alpha}\rangle = \lambda\ \langle K_1';p,a_\alpha|M|K_2'\rangle\ . \tag{19}$$

Here $\langle\cdot\cdot|M|\cdot\cdot\rangle$ represents a scattering amplitude, the K's denote arbitrary sets of particles which are not crossed, the symbol a_α designates a particle of type a in an isomultiplet with $I = a$ and $I_z = \alpha$, and $\bar{a}_{-\alpha}$ designates its antiparticle. The vector $p(-p)$ is the momentum of $a_\alpha(\bar{a}_{-\alpha})$; and λ is a phase factor that is independent of p but depends on[8] a and α. The dependence of the crossing phase on spin is irrelevant here, and is suppressed. If we write $\lambda = \lambda(a,\alpha)$ in (18) and $\lambda = \lambda(\bar{\bar{a}},-\alpha)$ in (19), thus exhibiting the dependence on the crossed particle in a definite way, then the equality of the factors λ in (18) and (19) is the statement

$$\lambda(a,\alpha) = \lambda(\bar{\bar{a}},-\alpha)\ . \tag{20}$$

Isospin symmetry is expressed as[9]

$$\langle a_\alpha \cdots b_\beta | M | c_\gamma \cdots d_\delta \rangle = \sum_{\alpha' \cdots \delta'} \langle a_{\alpha'} \cdots b_{\beta'} | M | c_{\gamma'} \cdots d_{\delta'} \rangle$$

$$\times D^{(a)*}_{\alpha'\alpha}(u) \cdots D^{(b)*}_{\beta'\beta}(u) \, D^{(c)}_{\gamma'\gamma}(u) \cdots D^{(d)}_{\delta'\delta}(u) \; . \qquad (21)$$

It has been shown[8] that the most general form of $\lambda(a,\alpha)$ compatible with both (18) and (21) is

$$\lambda(a,\alpha) = \lambda_a(-1)^\alpha , \qquad (22)$$

where λ_a is independent of α and can be selected arbitrarily for a given multiplet a.

Up to this point Eqs. (18) through (22) are completely general; now if we let a be an SMHI, then the multiplet labels a and $\bar{a}$ are identical in these equations, and in particular we can write (20) as

$$\lambda(a,\alpha) = \lambda(a,-\alpha) \; . \qquad (23)$$

But (23) is incompatible with (22) since the latter implies

$$\lambda(a,\alpha) = \lambda_a(-1)^\alpha = (-1)^{2\alpha} \lambda_a(-1)^{-\alpha}$$

$$= -\lambda(a,-\alpha) \; . \qquad (24)$$

Thus we shall have a contradiction unless all the scattering amplitudes involving particles of SMHI vanish identically. This result is, however, physically equivalent to the statement that particles of SMHI do not exist in the theory since we can never establish a set of non-interacting particles as an isomultiplet.

In conclusion we make the following remarks:

(a) Although our S-matrix proof is based on exact symmetry, it can be used as an explanation why self-conjugate mesons with half-integral isospin[10] are absent from the family of the strongly interacting particles. The usual belief is that strongly interacting particles will obey the exact SU(2) symmetry if the electromagnetic interaction is "turned off". Now by our result above, mesons of any SMHI do not interact through pure strong interaction at all but only through electromagnetic interaction;[11] since the latter does not obey isospin symmetry even in the approximate sense, we would never recognize these mesons as members in an isomultiplet.

(b) The method presented here can be applied to higher internal symmetry as well. Whether an irreducible self-conjugate multiplet is allowed or not depends on the existence of a matrix H satisfying (13) and

$$\sum_{\beta,\gamma} H_{\alpha\beta}\, D^{(\nu)*}_{\beta\gamma}(g)\, H^{-1}_{\gamma\delta} = D^{(\nu)}_{\alpha\delta}(g) \,, \qquad (25)$$

where $D^{(\nu)}(g)$ is the irreducible representation characterized by ν and g in any member of the symmetry group in question; now α is a set of quantum numbers which specifies a state. For example in the case of SU(3), it is easy to show that whenever D and D^* are equivalent irreducible representations, there always exist a conjugation matrix that satisfies[12] both (13) and (25). In contrast to the case of SU(2), here all the possible irreducible self-conjugate multiplets (i.e., those with a regular hexagonal weight diagram) are allowed.

(c) Notice that in the proofs presented above, parity does not enter. In a theory where parity is conserved, we have only self-conjugate bosons to consider; fermion and antifermion must have opposite parity, hence they cannot belong to the same irreducible multiplet.

ACKNOWLEDGMENTS

I am grateful to Professor G. F. Chew, Professor P. Carruthers, and Professor S. Mandelstam for helpful discussions. I would like to thank Dr. H. P. Stapp and Professor E. H. Wichmann for many useful comments and discussions, and for their critical reading of the manuscript.

FOOTNOTES AND REFERENCES

* Work done under the auspices of the U. S. Atomic Energy Commission.

1. P. Carruthers, Phys. Rev. Letters 18, 353 (1967).

2. A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton Univ. Press, 1960).

3. See e.g., S. Weinberg, Phys. Rev. 133, B1318 (1964).

4. W. Pauli, Phys. Rev. 58, 716 (1940).

5. That H takes this form is because we use the standard phase convention. In general $H = \eta D^{(I)}\,(e^{i\frac{\pi}{2}\sigma_2})$, but the relation $HH^{\dagger} = (-1)^{2I}\, HH^{*} = |\eta|^2$ is independent of convention (see Ref. 2).

6. Y. S. Jin, NYO-2262 TA-148, Preprint, 1967.

7. See e.g., H. P. Stapp, Crossing, Hermitian Analyticity and the Connection Between Spin and Statistics, Lawrence Radiation Laboratory Report, UCRL-~~18616~~ 16816, 1966. (submitted to J. Math. Phys.) Reference of earlier works on this subject can be found in the paper.

8. For the derivation of these assertions, we refer to J. R. Taylor, J. Math. Phys. 7, 181 (1966).

9. Although it is only for convenience that we require all the indices in the bra(ket) transform according to D^*(D) representation, this is possible only when D and D^* are equivalent, which is true for SU(2).

10. By charge conservation, these mesons would have to carry half-integral charges *if* we assume they could interact strongly with the existing hadrons.

11. It is tacitly assumed that the "pure" strong interaction S-matrix possesses the crossing properties (18) and (19).

12. J. J. De Swart, Rev. Mod. Phys. 35, 916 (1963).