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Empirical Threshold of Representation

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An operational method using data from previous elections is proposed for determining the vote share a small party needs to have a fifty-fifty chance of winning its first seat. The resulting median value for 23 electoral systems is 1.0 per cent of the nation-wide vote, with a range from 0.1 to 8 per cent. This empirical threshold of representation is affected by assembly size, legal representation threshold (if any exists), and geographical concentration of small party votes. In turn, this threshold affects the number of seat-winning parties and the effective number of parties in the system. Empirical thresholds can also be calculated on the district level. They can then be compared with theoretical thresholds of representation, and unanticipated discrepancies occur, because apparently minor aspects of electoral rules can alter the outcome.

How large a share of the votes does a small party typically need to win its first parliamentary seat? The answer depends on the specific country and electoral system, and it has political consequences. A low representation threshold makes it easy for small parties to form and maintain themselves, while a high threshold deprives them of parliamentary exposure and thus makes their survival difficult. Instead of a clear cut-off, there is usually a broad range of vote shares where parties sometimes do and sometimes do not obtain a seat. Therefore, an operational definition of representation threshold is not self-evident. In this article such an empirical representation threshold will be defined and used. Before doing so, two other types of thresholds should be briefly discussed: the legal and the theoretical.

A number of countries have legal thresholds of representation, such as 5 per cent nation-wide votes in West Germany or 5 per cent district-wide votes in France 1986. Often there are additional loopholes (such as winning 3 direct seats in West Germany) or barriers, so that the actual threshold can be somewhat hazy. In most countries legal thresholds are not stipulated, but the nature of electoral districts and rules used introduces implicit thresholds.

On the district level, such implicit thresholds can be theoretically calculated (at least for simple electoral rules), based on district magnitude (number of seats per district), the number of parties competing, and the seat allocation formula. The theoretical inclusion (or representation) threshold is the minimum share of votes a party needs to win its first seat under the most favourable circumstances; exclusion threshold is the maximum share at which the party could still fail to win a seat under the most unfavourable circumstances (Rokkan, 1968; Rae, Hanby and Loosemore, 1971). Thresholds for other than first seat also have been calculated (Lijphart and Gibberd,

* I would like to thank Arend Lijphart and Matthew Shugart for valuable comments.

1977; Laakso, 1979) but need not concern us here. When one proceeds beyond a single district which uses a standard allocation formula (such as d'Hondt), theoretical threshold calculations bog down. Thresholds in terms of nation-wide vote shares depend on local concentrations of these votes and cannot be calculated, unless one introduces knowledge about such geographical distribution of votes. Therefore, theoretical threshold formulas up to now have been restricted to a single district.

Thus both legal and theoretical thresholds have limitations. In the case of most national assemblies neither can tell us at which votes share a party is likely to win its first seat and thus become politically credible.

A third kind of threshold of representation is proposed here, based on empirical data from previous elections. This empirical threshold can be calculated both on district- and nation-wide levels, and is a measure of ease or difficulty of entering parliamentary politics with a new party.

Empirical threshold values can be compared with those of legal and theoretical thresholds in cases where the latter can be determined unambiguously, and some useful connections emerge. These in turn may guide the determination of theoretical thresholds nation-wide.

Operational Definition of Empirical Threshold of Representation

The method for determining the empirical threshold of representation is the following. For a number of elections carried out under the same rules, find the vote shares for all those cases where a party obtained one seat but no more. Rank these votes by increasing size. Also find the vote shares for cases where parties with non-negligible vote shares failed to win a single seat, and rank these shares by decreasing size. The empirical representation threshold (T) is defined as the vote share (v) such that the number of cases where a party fails to get a seat with $v > T$ equals the number of cases where a party with $v < T$ does win a seat.

Two actual examples will clarify the procedure. In Finnish parliamentary elections of 1907–1979 (all carried out with essentially the same rules), the nation-wide vote shares resulting in one seat or zero seats, respectively, in the *Eduskunta* lined up as shown in Table 1, where the vote shares are in per cent of the nation-wide vote. The letter T in the table indicates the point where the terms in the increasing series ($S = 1$) surpass the terms in the decreasing series ($S = 0$). Any number $1.4 < T < 1.6$ satisfies our requirement for the empirical threshold, and it makes sense to pick the midpoint of this range: $T = 1.5\%$. We may not know the vote shares of the smallest parties with no seats at the right end of the $S = 0$ series, but we do not need them. Thus the definition of T is operational unambiguously, for these particular data.

TABLE 1. Percent vote shares in Finnish elections 1907–1979 that led to one or zero seats, respectively, in Parliament

$S=1:$	0.5	1.0	1.1	T	1.6	1.7	1.8	1.8	2.0	2.1	2.2	2.7
$S=0:$	2.2	1.8	1.6	T	1.4	1.2	1.2	1.1	1.0	0.8	...	

The $S=1$ series yields $1.1 < T < 1.6$; the $S=0$ series yields $1.4 < T < 1.6$. Hence $1.4 < T < 1.6$, and the centre of the possible range is $T = 1.5\%$.

Data source: Mackie and Rose 1982, pp. 104–14.

TABLE 2. Percent vote shares in Finnish electoral districts in 1983 that led to one or zero seats in the district, respectively

S=1:	2.1	2.5	3.3	3.9	4.0	T	4.8	5.1	5.3	6.1	6.4	
S=0:	8.4	7.6	7.0	5.0	4.5	T	4.1	4.0	3.5	3.4	3.0	...

Data source: *Statistical Yearbook of Finland 1983*, pp. 404–5.

Of course, the more data we have, the more we have confidence that the value of T thus determined reflects the basic properties of the given system. This system includes both the electoral rules and geographical concentrations of parties, which may change over time. Further elections may shift T . In Finland 1983, a party won a seat with an unprecedentedly low vote share of 0.4 per cent (*Statistical Yearbook of Finland 1983*: 404–5) because of a favourable electoral alliance, and hence the empirical threshold shifted to $1.2 < T < 1.4$ or $T = 1.3\%$ on the average. Note that the shift is rather small and could be reversed if a party with more than 1.6 per cent votes should fail to obtain a seat in the future.

The second example (Table 2) shows analogous data for districts in one single election. In the Finnish parliamentary elections of 1983, the *district-level* vote shares resulting in one seat or zero seats, respectively, in that particular district lined up as shown. The empirical threshold is located at $4.1 < T < 4.5$, or $T = 4.3\%$ on the average. This is much above the nation-wide T . The basic reason is that one seat nation-wide amounts to 0.5 per cent of all seats, while in the average district (magnitude $M = 14$) one seat represents 7 per cent of all seats in the district. Actually, the empirical threshold for districts is largely determined by the magnitude of a few very large ($20 < M < 30$) and very small ($6 < M < 8$) districts.¹ In the largest districts one seat represents about 4 per cent of all seats in that district, and this is where some parties at times obtain a seat with only 2 to 3 per cent of the district votes. In the smallest districts one seat represents about 20 per cent of all district seats, and this is where parties with 5 to 9 per cent votes can fail to win a seat. In fact, such failures would be more numerous than they are, if parties did not desist from running in districts where they expect to fail.

Once more, we can see that the empirical threshold depends on more than electoral rules and their average effect on major parties. It depends on geographical concentration, the existence of some very small and very large districts, the ability of parties to form local electoral alliances, and their willingness to run candidates under highly unpromising conditions. In other words, the empirical threshold depends, besides electoral rules, on electoral geography and political practices.

The procedure used implies that at $v = T$ a party has a fifty-fifty chance of winning a seat. For $v > T$ the probability of getting a seat increases beyond 50 per cent, and for $v < T$ it decreases below 50 per cent. The decrease in probability of winning a seat when v decreases in the neighbourhood of T can be gradual or steep (especially in the case of a legal threshold); the value of T does not say anything about it.

Nation-wide Empirical Thresholds of Representation

Table 3 shows all countries and periods with different electoral rules for which the data in Mackie and Rose (1982) enables one to determine the empirical representation

TABLE 3. Nation-wide empirical thresholds of representation (in per cent of nation-wide vote)

Country and period	Empirical threshold (T , in %)	Assembly size S	Threshold advantage ratio A_T	Seat-winning parties p	Effective parties N_V
Italy 1946–1979	0.1	600	0.83	10.3	3.5
Germany 1871–1912	0.2	400	0.63	13.3	6.2
Germany 1920–1933	0.2	500	0.50	12.3	6.0
United Kingdom 1918–1979	0.3	620	0.27	6.2	2.5
Spain 1977–1979	0.45	350	0.32	12.5	4.3
Switzerland 1919–1979	0.6	190	0.44	9.8	5.3
Netherlands 1956–1981	0.67 ^a	150	0.48	10.9	5.4
Netherlands 1918–1933	0.75 ^a	100	0.71	12.2	5.8
Japan 1928–1980	0.75	470	0.14	6.4+ ^d	3.3
Denmark 1920–1950	0.8	150	0.42	6.5	3.8
Belgium 1919–1981	0.85	200	0.29	6.2	3.5
Israel 1949–1981	1.0 ^a	120	0.40	11.4	4.5
Norway 1921–1949	1.4	150	0.25	6.6	3.8
Ireland 1922–1981	1.4	150	0.24	5.4+ ^d	3.1
Portugal 1975–1980	1.4	250	0.14	7.2	3.3
Finland 1907–1979	1.5	200	0.17	6.7	5.4
Denmark 1953–1981	2.0 ^{a,b}	175	0.50 ^c	7.6	4.4
France 1958–1981	2.3 ^b	470	0.09 ^c	7.4	3.8
Luxembourg 1919–1979	2.3	50	0.43	5.0	3.5
Norway 1953–1981	2.7 ^b	150	0.25 ^c	6.1	3.8
Iceland 1916–1933	3.3	36	0.42	4.5	3.2
West Germany 1961–1980	5.0 ^{a,b}	500	0.50 ^c	3.0	2.9
New Zealand 1880–1981	8.0	80	0.08	2.9+ ^d	2.4

Based on data in Mackie and Rose (1982) and, for most N_V values, Taagepera and Shugart (1989). Assembly sizes are given approximately, since they vary over time.

a. Legal threshold, confirmed empirically

b. See Note 2

c. See Note 4

d. An appreciable number of seats went to the 'Other' category in Mackie and Rose (1982) and are counted as a single party. Thus the actual p could be appreciably higher than shown here.

threshold T .² The 23 electoral systems are listed in the order of increasing values of T , which are found to range from 0.1 per cent of nation-wide votes for Italy to 8 per cent for New Zealand. The median T is 1.0 per cent. Also shown in this table are the size of the national assembly (S), the threshold advantage ratio (A_T), the number of seat-winning parties (p), and the effective number of parties (N_V), which will be discussed later.

We will first consider the inputs, that is, the factors which affect the value of T . Subsequently the output will be discussed, that is, the effect of T on the number of parties.

Assemblies vary in size, and this size imposes a lower limit on the threshold of representation. In an assembly with S members, perfect PR would entitle a party to exactly one representative when it reaches $100\%/S$ of nation-wide votes. When fractional PR shares of more than 0.5 representatives are rounded off to 1, the lower limit of representation is $50\%/S$, unless there is marked overrepresentation of certain

regions or population groups. In Figure 1 the empirical threshold is graphed against the assembly size (on logarithmic scale), and $T = 50\%/S$ is indeed the lower limit at any S . No upper limit on T is visible, since the outcome depends on electoral rules, especially existence of legal thresholds of representation, the magnitude of electoral districts, and the geographical distribution of party votes. The median relationship between T and S is close to

$$T = 180\%/S \text{ or } ST = 180\%. \tag{1}$$

Legal thresholds are at times set barely above the $50\%/S$ level (as indicated in Figure 1 for Israel and the Netherlands) and hence rarely eliminate any party. They can also be 6 times (Denmark since 1953) or even 50 times (West Germany) above the $50\%/S$ level and thus have a marked effect. But legal thresholds are relatively rare. More frequently, an empirical threshold of representation higher than that imposed by assembly size is determined by the smaller parties' ability to pack their votes into a single allocation unit, namely, their regionality.³

The cost of the first seat, in terms of votes, is of interest. Consider the advantage ratio $A = (\% \text{ seats})/(\% \text{ votes})$, which measures by how much a party is under- or over-represented compared to perfect PR (Taagepera and Laakso, 1980). The value $A = 1.0$ indicates that a party receives exactly its proportional due of seats. Now consider the advantage ratio A_T of a party located just at the empirical threshold of representation T . At $v = T$, a party has a fifty-fifty chance of winning a seat. If it does, it wins a per cent share $100\%/S$ of the S seats in the assembly, and $A = (100\%/S)/T$. If it

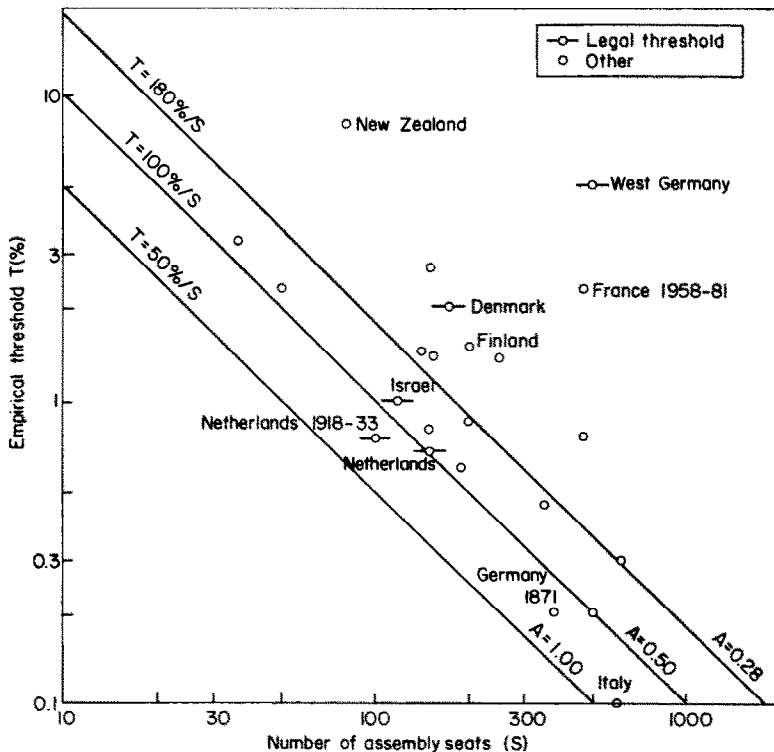


FIG 1. Empirical threshold versus size of national assembly. Data from Table 3.

fails to win a seat, $A = 0$. Thus the average advantage ratio at $v = T$ is

$$A_T = 50\%/ST. \quad (2)$$

The lines shown in Figure 1 now acquire a further meaning. The line $T = 50\%/S$ corresponds to $A_T = 1$, and the line $T = 100\%/S$ corresponds to $A_T = 0.5$. The best fit line $T = 180\%/S$ corresponds to $A_T = 0.28$. The implications are best explained by discussing two extreme cases: Italy and New Zealand.⁴

In Italy 1946–1976 parties which stand a fifty-fifty chance of obtaining a seat have an average advantage ratio of 0.83. If they do obtain a seat, they are ‘overpaid’ heavily: $A = 100\%/ST = 1.7$; in the other half of the cases they do not get anything: $A = 0$. On the average, these marginal parties come close to full proportional representation ($A = 1$); hence the enticement to remain in the electoral game is high. Italy’s very low value of T (0.1%) does not just reflect its large assembly size; the electoral system is unusually hospitable to tiny parties with some regional basis.⁵

In contrast, New Zealand’s high T (8%) reflects not only a low assembly size but also political surroundings unusually adverse to small parties. The latter reach the fifty-fifty point for winning a seat at such a high vote share that A_T is only 0.08. Even if they do win the seat, they are still grossly underpaid: $2A_T = 0.16$, meaning that their seat share is still only one-sixth of their vote share. Plurality in single-seat districts explains only part of this penalty, since the UK has a value $A_T = 0.27$ close to the median. The other part of the explanation is the smaller assembly size and the unusual homogeneity of the New Zealand electorate, which denies regional strongholds to small parties.

Imperial Germany (1871–1912) and France 1958–1981 offer another contrast to show that T and A_T are quite independent of the electoral system used. Two-round majority in single-seat districts was used, and the assembly was equally large in both cases. Yet in France the empirical threshold of representation was very high (2.3%) and came with severe underrepresentation. In contrast, in Germany T was very low (0.2%) and the threshold parties almost broke even in terms of seat-vote ratio: $A_T = 0.63$. A glance at lists of parties reveals a number of openly regional small parties in Germany, but even small parties with a non-regional label were often successful, reflecting the existence of local strongholds or favourable deals with larger parties.

The broad conclusion is that the empirical threshold, in contrast to the aforementioned legal and theoretical thresholds, does not reflect electoral laws alone but also the impact of these laws in specific political surroundings. However, the empirical threshold sometimes tells us about the impact of a change in electoral rules, within the same surroundings. When Denmark switched from d’Hondt (1920–1950) to modified Sainte-Laguë plus a legal threshold (1953–1981), T increased markedly (from 0.8 to 2.0%), although the general deviation from proportionality ($D = 0.5 \sum |v(i) - s(i)|$) changed little (from 2.2 to 2.9 per cent, on the average). Hence the empirical threshold T complements the information given by D .

We now turn to the effect of T on fractionalization of the party system. One simple measure of it is the number (p) of parties that obtain representation in the assembly by winning at least one seat. Figure 2 shows p graphed against T on log-log paper. As one might expect, p decreases as T increases. The average pattern is close to

$$p^2 T = 60\% \text{ or } p = \sqrt{60\%/T}. \quad (3)$$

This suggests that a pure two-party system ($p = 2$) would result from $T = 15\%$, which could be obtained by establishing a 15 per cent legal threshold.

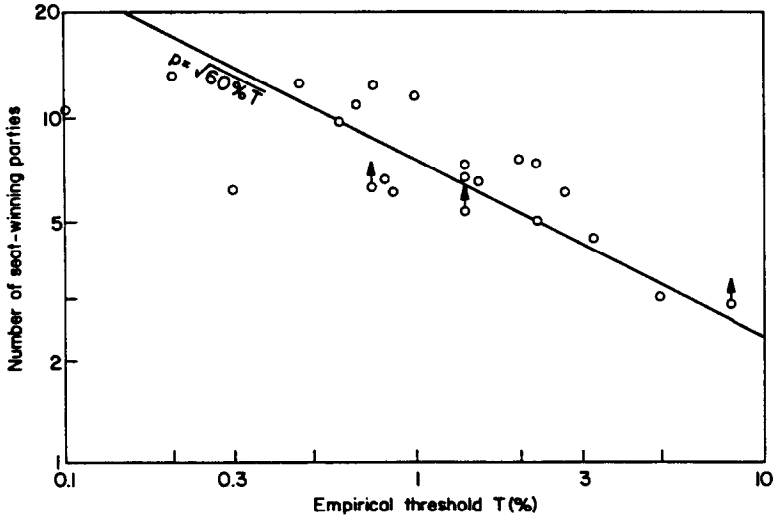


FIG. 2. Number of seat-winning parties versus empirical threshold of representation. Data from Table 3.

Since the threshold advantage ratio is in most cases inversely proportional to ST (Equation 2, with exceptions discussed in Note 4), it makes sense to graph also p against ST , as is done in Figure 3. Again an approximately linear relationship appears on the log-log scale: p tends to decrease as ST increases:

$$p = (10,000\%/ST)^{0.5}. \tag{4}$$

It should be noted that Equations 3 and 4 are mutually inconsistent for values of S different from 167. Both equations represent imperfect fits of empirical data. It remains to be seen whether it makes better theoretical sense for the number of

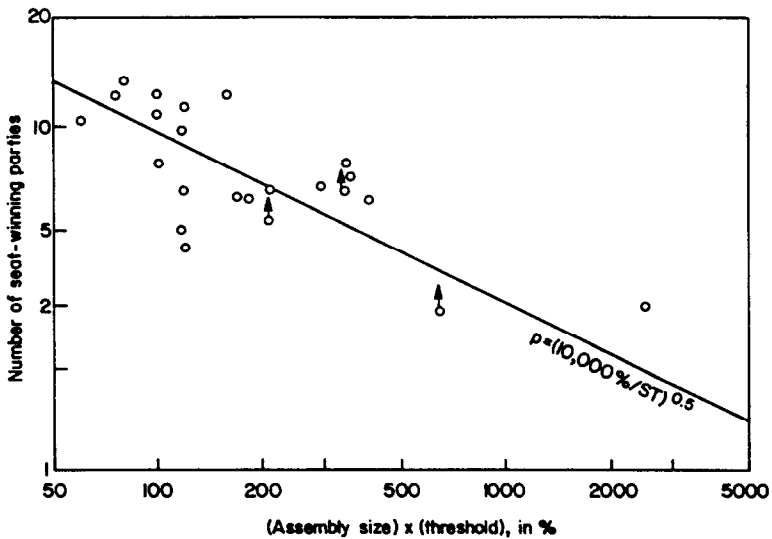


FIG. 3. Number of seat-winning parties versus the product of empirical threshold and assembly size. Data from Table 3.

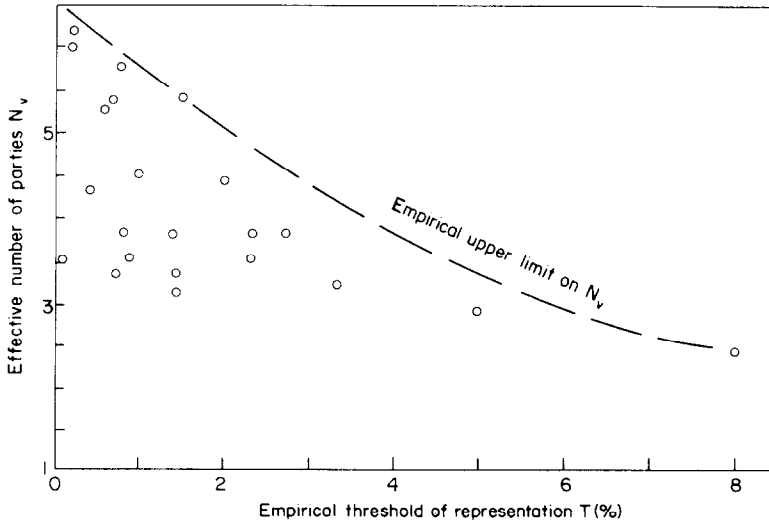


FIG. 4. Effective number of parties versus empirical threshold of representation. Data from Table 3.

seat-winning parties to depend on the threshold or the product of threshold and assembly size.

The number of seat-winning parties has its limits as a measure of overall fractionalization of the party system, since the constellation 20-20-20-20-20 is clearly more fractionalized than 49-48-1-1-1, although $p = 5$ in both cases. The effective number of parties is a more suitable measure; it yields 5.0 in the first case but only 2.1 in the second.⁶ Figure 4 shows votes-level effective number of parties (N_V) graphed versus T . It can be seen that a high empirical threshold of representation exerts a restraining influence on the effective number of parties, but a low T does not automatically lead to a high N_V . Taken together, Figures 2 and 4 indicate that a low T tends to engender numerous very small parties but has little effect on the size of the major parties. However, a high T not only eliminates tiny parties but also exerts pressure on medium-size parties, thus reducing the effective number of parties, too.

The location of threshold of representation is obviously important for the morale and hence the existence of very small parties which, together, can have an effect on the ease of forming government coalitions.⁷ The psychological cutoff for encouragement of small parties might be expected to be around $A_T = 0.50$. This is the point where the marginal parties, if they win a seat, immediately break even in terms of their vote shares so that image of 'wasted votes' is avoided. However, Table 3 indicates that in one-half the cases some small parties survive with A_T as low as 0.28. This apparent discrepancy can be explained by focusing on the district level.

District-Level Thresholds of Representation

In most cases, seats are not allocated nation-wide but in smaller districts. Before one could hope to understand the mechanisms which determine the nation-wide representation thresholds, one has to understand what happens in the districts. Considerable theoretical work on district-level inclusion and exclusion thresholds

exists, as mentioned in the introduction. Hence, once an empirical threshold is defined, the question immediately arises: how does it relate to theoretical thresholds?

This question will be addressed here only in a limited way, using data from a single country. Finland was chosen as an example because of the durability and apparent simplicity of its electoral system. Finland is the only country where multiseat districts of the same average magnitude have been used for eighty years, with the same allocation formula (d'Hondt).

Omitting the one seat assigned to the autonomous Åland Islands with its own regional party, Finland has steadily had 199 seats divided among 14 districts so that the average M is 14.2. However, individual district magnitudes ranged from 9 to 20 in 1962 and from 7 to 27 in 1983. Furthermore, parties are allowed to form district-level electoral alliances. This explains how a well-allied party in a large- M district could obtain a seat with less than 3 per cent votes (cf. Table 2) while an isolated party in a small- M district could fail to win a seat even with more than 8 per cent of votes. It will be seen that the apparently minor feature of having alliances plays havoc with the theoretical inclusion and exclusion thresholds.

For the d'Hondt allocation rule the exclusion threshold depends on district magnitude only:

$$T_E = 1/(M+1), \quad (5)$$

while the inclusion threshold depends also on the number of parties (n) competing for votes:

$$T_I = 1/(M+n-1). \quad (6)$$

The number of parties competing in a district is not always well defined. Since the inclusion threshold involves a party narrowly gaining a seat by denying it to another party, n will be approximated by the number of parties obtaining seats (p) plus one. Hence

$$T_I = 1/(M+p), \quad (7)$$

approximately. By definition, the probability of winning a seat with $v < T_I$ is 0, and the probability of winning at least one seat with $v > T_E$ is 1. Hence, since the empirical threshold reflects a fifty-fifty probability, one would expect $T_E > T > T_I$, with T approximately halfway between T_E and T_I .

As a test, Finnish districts in 1962 and 1983 were divided into three groups by magnitude, and T as well as theoretical inclusion and exclusion thresholds were calculated.

Table 4 indicates that the number of seat-winning parties tends to increase as M increases, while theoretical thresholds decrease (in line with the equations above). The empirical threshold also tends to decrease with increasing M (with a minor deviation in 1962). However, the empirical threshold is not halfway between the exclusion and inclusion thresholds but tends to be at or even below the inclusion threshold. This is an effect of electoral alliances.⁸ I see no easy way to include alliance formation in a corrected theoretical formula, and the uncorrected formula substantially overestimates the actual threshold of representation in Finland.

If theoretical thresholds are off the mark in such an apparently straightforward system as Finland's, then checking against empirical data would be even more important in the case of more complex electoral rules. This is why an empirical

TABLE 4. Theoretical and empirical thresholds of representation in Finland's districts, in 1962 and 1983

Districts	Average magnitude (M)	Seat winners (p)	Theoret. thresholds		Empirical threshold T (%)	Threshold adv. ratio (A_T)
			Exclusion T_E (%)	Inclusion T_I (%)		
<i>1962</i>						
Largest 5	18.2	5.8	5.2	4.5	4.8	0.57
Median 5	13.4	4.6	6.9	5.6	4.4	0.85
Smallest 4	10.2	4.8	8.9	6.7	6.2	0.78
All 14	14.2	5.1	6.6	5.2	4.8	0.72
Nation-wide	199	8	—	—	<3	>0.09
<i>1983</i>						
Largest 5	20.0	6.6	4.8	3.8	2.7	0.92
Median 4	13.8	5.2	6.8	5.3	4.4	0.82
Smallest 5	8.8	4.6	10.2	7.5	8	0.72
All 14	14.2	5.5	6.6	5.1	4.3	0.82
Nation-wide	199	10	—	—	<0.4	>0.63

Calculated from data in *Statistical Yearbook of Finland*, 1962, pp. 358–60 and 1983, pp. 404–5, omitting the autonomous Åland Islands and its one seat.

Seat winners = number of parties winning at least one seat in the given district.

measure such as T is needed on the district level even when theoretical threshold calculations seem possible.

We can now address the apparent discrepancy mentioned at the end of the previous section. Finland's nation-wide A_T is so low (0.17) that one might think that it should discourage marginal parties from existing. However, small parties do not run nation-wide but only in a few most favourable districts. Table 4 shows that the threshold advantage ratio in the districts is about 0.75 (± 10.20), independently of district magnitude.⁹ Thus the odds are quite favourable to small parties, since A_T is close to 1.00. The district-level threshold advantage ratio may be similarly high in all countries, but this question will not be settled here.

Taking all districts at any M together leads to threshold values close to those of the group with median M . This is a welcome result, because it tells us that we can treat a country with a fairly wide range of district magnitudes as if all districts had a medium M . Such a simple outcome could not be taken for granted because, conceivably, the average T could be determined mainly by the districts with largest M where even very small parties could land a seat.

Finally, it should be pointed out that, for all rows in Table 4, there is a relationship between district magnitude (M) and the number (p) of parties winning at least one seat in that district:

$$p/\sqrt{M} = 1.4 \pm 0.15. \quad (8)$$

In other words, the number of seat-winning parties in a district grows as the square root of district magnitude. The broader implications of this empirical observation will be discussed in a separate article.

Conclusions

An empirical threshold of representation has been defined and measured for various countries and, in one country, on district level. Comparisons with legal and theoretically calculated thresholds have been made. The empirical threshold tends to decrease as assembly size increases so that their product remains constant at 180 per cent (when threshold is in per cent), corresponding to a threshold advantage ratio of 0.28. Deviations from this average depend on geographical concentration of minor parties; no type of electoral rule is conducive to especially high or low values. The number of seat-winning parties, in turn, decreases as the threshold of representation increases. The effective number of parties, which depends mainly on the largest parties, can at low thresholds range from high to low, but is restricted to low values only when the threshold of representation becomes high.

For applications, we are mostly interested in nation-wide results such as those above. For the theory of conversion of votes into seats, the district-level results are of most interest, because this is where theoretical explanations can begin. At this simpler level theoretical thresholds of representation have been calculated. By calculating the empirical thresholds for two elections in one country (Finland) we have only scratched the surface here, but we have already raised a number of questions. Discrepancies between theoretical and empirical threshold values arise because of a minor rule on freedom to form district-level alliances. This indicates the need for comparing theoretical and empirical results in other countries. In Finland, district-level thresholds are considerably higher than the nation-wide thresholds, but at the same time the threshold advantage ratios are higher. This again needs checking with other countries, so as to have a starting base for theoretical work to connect the district and national levels. The same applies to the number of seat-winning parties. In sum, the empirical threshold of representation is a valuable means for checking the considerable earlier theoretical work by scholars like Rokkan, Rae, and Lijphart and for guiding its future direction.

Notes

1. Finland also has a one-seat district in the autonomous Åland Islands, and its single seat is regularly won overwhelmingly by a local party affiliated with the nation-wide Swedish People's Party. If we counted the Åland Party as a separate party (as is sometimes done), Table 2 would be marginally affected, but the $S = 1$ series in Table 1 would start with 28 entries (one for each election) of about 0.3 per cent, and thus the value of T would appear to be down to 0.3 per cent. This value would not tell us anything about Finnish electoral politics outside the population share of the Åland Islands. When encountering extremely low T in other countries we should check whether it is not one single regional party that determines the outcome.
2. The main requirement is that there be a sufficient number of parties that do obtain one seat or no seat, respectively, and that the vote and seat shares of such parties be listed separately rather than being lumped in the 'Other' category. In general, at least 3 cases with one seat won were required. This eliminated, for instance, *Canada* 1878–1980 where only 2 such cases occurred, leading to $1.8 < T < 2.1$. Problems arise when parties find it easier to win several seats rather than one. This is so in *West Germany*: parties with 4.3 and 4.6 per cent votes have failed to get seats because of the 5 per cent legal threshold, but parties with barely over 5 per cent of nation-wide votes obtain their proportional share, that is, about 25 seats out of a total of about 500. In such cases we take T as the votes share where $S = 0$ shifts to $S > 1$ (rather than $S = 1$). *Norway* 1953–1981 has only two cases with $S = 1$, each with 4.3 per cent of votes, but also four cases of 2 seats won with fewer votes (2.3 to 3.2 per cent). These were included along with $S = 1$ cases, to determine T . In *France* 1958–1981, parties close to $v = T$ tended to win 2

- seats, if they won any at all. In *Denmark 1953–1981* the legal threshold (2 per cent) has escape clauses, and the regional Schleswig Party won a seat with only 0.4 per cent votes in three elections, while the other parties jumped from 0 to 3 or even 4 seats once they surpassed the 2 per cent threshold. It is not clear why in 3 cases parties failed to obtain seats even with more than 2 per cent votes (Independents Party 1953 and 1957, and Justice Party 1960, with 2.2 to 2.7 per cent votes, according to Mackie and Rose, 1982: 92–9).
3. The meaningful allocation unit can be the electoral district, if this is where the allocation of seats ends, or a wider unit (possibly the entire nation), if this is where compensatory seats are allocated and parties without district seats can participate. Legal thresholds are introduced usually to counteract the effect of nation-wide compensatory seats. Once the meaningful allocation unit is established, the specific seat allocation rules used play a relatively minor role in determining T . See Taagepera and Shugart (1989) for a comprehensive discussion of these complex issues.
 4. For countries where T is basically determined by the legal threshold the outcome may be different. In West Germany, A jumps from 0 for $v < 5\%$ to 1.0 (near-perfect proportional representation) for $v > 5\%$. Hence $A_T = 0.5$ rather than the value 0.02 resulting from the general formula. The same is broadly the case also in Denmark 1953–1981. In France 1958–1981 and Norway 1953–1981 the shift at $v = T$ tends to be from 0 to 2 seats (cf. Note 2), and hence the formula used was $A_T = 100\%/ST$.
 5. Out of the 6 cases of parties winning a single seat. 5 refer to local parties: Val d'Aosta (3 times), Trieste, Sardinia. The exception is Community Front in 1958. The situation has some elements of the Åland dilemma discussed in Note 1.
 6. The effective number of parties on the votes level is defined as $N_V = (\sum v_i^2)^{-1}$, where v_i is the votes share of the i -th party and the summation is over all parties (Laakso and Taagepera 1979). N can also be calculated on the basis of seat shares, and the results then tend to be lower by about 0.4.
 7. Parties with 5 seats or less amounted on the average to 12 per cent of all seats in the Netherlands 1918–1933, 7.4 per cent in the Netherlands 1956–1981, and to 5.8 per cent in Switzerland 1919–1979. This represents a share of seats which cannot be overlooked in coalition building. In most cases with low T , parties with 5 or less seats average 1 to 2 per cent of all seats; this applies, in particular, to Italy, Imperial and Weimar Germany, and to the United Kingdom.
 8. The effect can be observed in individual districts. In the Helsinki district ($M = 20$) the Constitutional Party won a seat in 1983 with 2.5 per cent of the district votes, with only 10 parties running so that $T_I = 3.4\%$. In order to reduce T_I to 2.5 per cent, the number of parties competing would have to be 21!
 9. The threshold advantage ratio is defined as previously, simply replacing S by M : $A_T = 50\%/MT$.

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