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IN ASYMPTOTIC LIGHT FIELDS

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Simple Formulas for the Volume Absorption Coefficient in Asymptotic Light Fields[*]

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INTRODUCTION

It is now a well-established fact that the properties of the light field in optically deep homogeneous stratified media (such as deep cloud layers, oceans, lakes, etc) become extremely regular and predictable at great depths.[1] The conceptual and practical consequences of this fact, however, are only in the first stages of exploration.[2,3] In this note we derive some further consequences from this regularity. In particular, we use the established regularity of the light field to derive several simple, exact, formulas relating the volume absorption coefficient to the common limiting value k_∞ of the K-functions[4] for irradiance. In this way we supplement the exact formula:[5]

$$\alpha(z) = \frac{1}{h(z)} \frac{d\bar{H}(z,+)}{dz} \tag{1}$$

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for the volume absorption function   with several alternate
formulas which are especially suitable for engineering calculations
and as handy rules of thumb relating a and k_ω . These
formulas are as follows:

$$(I) \qquad a = k_\omega \frac{\overline{H}(z,-)}{h(z)} \quad , \quad z \geqq z_0 \,,$$

$$(II) \qquad a = k_\omega \frac{(1-R_\omega)}{D(-) + R_\omega D(+)} \,,$$

$$(III) \qquad a = \frac{3}{4} k_\omega \,,$$

where:

k_ω is the common limit, as $z \to \infty$, of the
K -functions $K(z,\pm)$, $k(z)$ for
irradiance and scalar irradiance,
respectively.

$D(\pm)$ are the limits, as $z \to \infty$, of the distribution functions $D(z, \pm)$.

R_∞ is the limit, as $z \to \infty$, of the reflectance function $R(z, -) = H(z, +)/H(z, -)$.

$\overline{H}(z, -) = H(z, -) - H(z, +)$, the net downward irradiance at any depth $z \geq z_0$, where z_0 is the depth below which the light field has essentially attained its asymptotic structure.

The details of the derivation of (I), (II), and (III) will now be given.

DERIVATION OF (I)

Short Derivation

The short derivation starts with (1), and the fact that there exists a depth z_0 below which the logarithmic derivatives of $H(z, -)$, $H(z, +)$, and $h(z)$ are constant and equal to a common value k_∞ (reference 3, equations (22), (24), and (35)). Therefore

$$\frac{d\bar{H}(z,+)}{dz} = \frac{dH(z,+)}{dz} - \frac{dH(z,-)}{dz}$$

$$= -k_\omega H(z,+) + k_\alpha H(z,-) = k_\omega \bar{H}(z,-), \qquad (2)$$

for all $z \geqq z_0$. Hence:

$$a = k_\omega \frac{\bar{H}(z,-)}{h(z)} . \qquad (I)$$

Long Derivation

The long derivation is essentially an exercise in the use of the integrated form of the divergence relation for the light field vector:[5]

$$\bar{P}(z,-) = a \, v \, \dot{U}(M) , \qquad (3)$$

where M is <u>any</u> region of the optical medium, S is its boundary, and $\overline{P}(S,-)$ is the net inward flux across S into M. $U(M)$ is the radiant energy content of M, v is the speed of light in M, and α is the required value of the volume absorption coefficient.

It is interesting to observe that (3) yields a value of α in <u>any</u> homogeneous medium, regardless of the structure of the light field:

$$\alpha = \frac{\overline{P}(S,-)}{v\,U(M)} \, . \qquad (4)$$

The numerator of (4) can be obtained by traversing the boundary of M with flat plate collectors. The denominator is obtained by probing the interior of M with a spherical collector (to find $h(p)$ at each point p) and integrating the values over the volume of M.

In the present case, the extreme regularity of the asymptotic light field allows one to estimate $U(M)$ <u>knowing only one value</u> of the scalar irradiance at a boundary point of M. This fact holds also for $P(S,-)$. Specifically, consider a region M in the form of a vertical column of unit cross section, and bounded by two parallel planes at depth z_1, and z_2, such that $z_0 \leq z_1 \leq z_2$.

The medium is homogeneous and stratified; hence

$$\overline{P}(s,-) = \overline{H}(z_1,-) + \overline{H}(z_2,+).$$ (5)

By hypothesis, we have:

$$H(z_2,\pm) = H(z_1,\pm)\, e^{-k_\omega(z_2-z_1)},$$ (6)

so that

$$\overline{P}(s,-) = \widetilde{H}(z_1,-)\left[1 - e^{-k_\omega(z_2-z_1)}\right].$$ (7)

Furthermore,

$$U(M) = \int_{z_1}^{z_2} h(z)\, dz$$

$$= h(z_1) \int_{z_1}^{z_2} e^{-k_\omega(z-z_1)}\, dz$$

$$= \frac{h(z_1)}{k_\omega}\left[1 - e^{-k_\omega(z_2-z_1)}\right].$$ (8)

Inserting (7) and (8) into the general formula (4), we have
the desired result.

$$a = k_\omega \frac{\overline{H}(z_1, -)}{h(z_1)} \, , \quad z_1 \geqq z_0 \, . \tag{I}$$

DERIVATION OF (II)

The formula (II) can be obtained directly from (I) by
writing

$$\overline{H}(z, -) = H(z, -) - H(z, +) \, ,$$
$$h(z) = h(z, -) + h(z, +) \, ,$$

and invoking the definitions of $R(z, -)$, and $D(z, \pm)$.
That is, in general:

$$\overline{H}(z, -) = H(z, -) - R(z, -) H(z, -) = H(z, -) \left[1 - R(z, -) \right]$$

and

$$h(z) = D(z, -) H(z, -) + D(z, +) H(z, +) \, .$$

So that when $z \geqq z_0$, we have

$$a = k_\omega \frac{(1 - R_\omega)}{D(-) + R_\omega D(+)} \, , \tag{II}$$

which is the desired alternate formula. We observe in passing
that (II) is a limiting form of the exact formula:

$$a(z) = \frac{K(z,-) - R(z,-)\, K(z,+)}{D(z,-) + R(z,-)\, D(z,+)} \; , \qquad (9)$$

which follows from equation (25) of reference 4. Clearly, as
$z \to \pm\infty$, equation (9) takes the limiting form (II). Further-
more if we assume $D(\pm) = 2$, as is done in the classical
Schuster two-flow theory of the light field, then (II) reduces
to the classical relation:

$$a = \frac{k_\infty}{2}\; \frac{1 - R_\infty}{1 + R_\infty} \; . \qquad (10)$$

APPLIED NUMEROLOGY: A RULE OF THUMB

Formula III is to be taken as a convenient rule of thumb,
and as such, is subject to possible revision whenever specific
optical media are under study. Formula III arises from the
following observed regularities in the values of R_∞ and $D(\pm)$ _in natural waters_:
R_∞ is usually found to be in the neighborhood of 0.02 ,
give or take 0.01 . Furthermore $D(\pm)$ appears to be such

that the sum $D(+) + D(-)$ is usually very nearly equal to 4; and the ratio $D(+)/D(-)$ is usually very nearly equal to 2, over great ranges of depths and in many media. Solving these two simultaneous equations yields, to two significant figures:

$$D(-) = 1.3 \, ,$$
$$D(+) = 2.7 \, ,$$

$$\text{(11)}$$

which agrees very well with experimental results. It follows that, to the nearest rational number with small integers for numerator and denominator, we have from (II):

$$a = \frac{3}{4} k_\omega \, ,$$

or $$\text{(III)}$$

$$\frac{4}{3} a = k_\omega \, .$$

Any similarity between the appearance of the fraction 4/3 in (III), and the index of refraction of water is, unfortunately, coincidental.

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