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ATCMIC BEAM STUDIES: HYPERFINE-STRUCTURE SEPARATIONS IN POTASSIUM-42, RUBIDIUM-84, AND CESIUM-129; MAGNETIC MOMENT AND HYPERFINE-STRUCTURE ANOMALY IN POTASSIUM-42

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MAGNETIC MOMENT AND HYPERFINE-STRUCTURE ANOMALY
IN POTASSIUM-42

Jhan M. Khan

(Ph. D. Thesis)

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ATOMIC BEAM STUDIES:

Hyperfine-Structure Separations in K^{42} , Rb^{84} , and Cs^{129} ;
Magnetic Moment and Hyperfine-Structure Anomaly in K^{42}

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IN POTASSIUM-42, RUBIDIUM-84, AND CESIUM-129;
MAGNETIC MOMENT AND HYPERFINE-STRUCTURE ANOMALY
IN POTASSIUM-42

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ABSTRACT

The atomic beam flop-in magnetic resonance method has been used to measure the hyperfine-structure splitting ($\Delta \nu$) and magnetic moment (μ_I), in one case, of four radioactive alkali isotopes. The electronic ground state is $^2S_{1/2}$. The results obtained are:

Isotope	Half life	Nuclear spin	$\Delta \nu (\frac{\text{Mc}}{\text{sec}})$	$\mu_I (\text{nm})$
K^{42}	12.5 hr	2	1258.877(4)	-1.1395(20)
Rb^{84}	33 d	2	3083.159(4)	
Cs^{127}	6 hr	1/2	8900(150)	
Cs^{129}	31 hr	1/2	9229(30)	

From the $\Delta \nu$ and μ_I of K^{42} it is possible to determine the hyperfine-structure anomaly,

$$^1\Delta^2 = \left[\left(\frac{\Delta \nu_1}{\Delta \nu_2} \frac{\mu_{I_2}}{\mu_{I_1}} \frac{I_1}{I_2} \frac{2I_2+1}{2I_1+1} \right) - 1 \right] \times 100\%,$$

with respect to K^{39} , K^{40} , and K^{41} . By convention "1" refers to the lighter isotope. These are:

$$^{39}_{\Delta}^{42} = + 0.20(25)\%$$

$$^{40}_{\Delta}^{42} = - 0.25(25)\%$$

$$^{41}_{\Delta}^{42} = + 0.42(25)\%$$

The three anomalies are compared with the predictions of two models for the distribution of nuclear magnetism.

The 12.5-hr K^{42} isotope is reactor-produced by an (n, γ) reaction from the 6.9% K^{41} isotope. The target material was KCl (99.9%) and was encapsulated in quartz. The 33-day Rb^{84} isotope was cyclotron-produced by a (p, n) reaction from the 59.9% Kr^{84} isotope. The krypton was bombarded in gaseous form, by 12-Mev protons. The 31-hr Cs^{129} isotope was cyclotron-produced by an $(\alpha, 2n)$ reaction from the 100% I^{127} isotope. The target material was BaI_2 in powder form. An $(\alpha, 4n)$ reaction on the same target material produced Cs^{127} .

I. INTRODUCTION

The properties of nuclei have been studied by many methods since the nucleus emerged as an entity in atomic structure. Since the nucleus was first observed as a scattering center possessing the property of mass and charge, many of its characteristics have been recognized. Among these are spin angular momentum, magnetic and electric multipole moments, and manifestations of a finite extent.^{1, 2} These are frequently referred to as the static properties of nuclei, in that their measurement generally does not produce internal changes in the nucleus. Nuclear models have been proposed to describe these properties. By use of nuclear energy-level separations and construction of a set of nuclear wave functions consistent with these spacings, detailed theoretical models for nuclear structure have been produced.³⁻⁵ To test these models, nuclear spectral data have been collected, and remarkable correlations have been made. By way of example, the books Elementary Theory of Nuclear Shell Structure by Mayer and Jensen and Theories of Nuclear Moments by Blin-Stoyle show the degree of qualitative--and sometimes quantitative--agreement between theory and data in the study of nuclear level structure.^{6, 3} In the experiments presented here, two properties have been explored. First is the magnetic dipole moment; next is the interaction of this nuclear moment with the electronic magnetic field. The latter interaction, in the case of s-state electronic configuration, can be used to probe the nuclear volume and the distribution of the nuclear magnetism. This has been summarized in a paper by Eisinger and Jaccarino⁷ in which are presented the measurements and theory to the date of their work. A very exacting test of a theory of nuclear structure would be its ability to predict both the magnetic moment and the distribution of nuclear magnetism. The number of experimentally determined moments is presently quite large, covering most nuclei. However, the measurement of the effects of the distribution of nuclear magnetism employing the hyperfine-structure interaction has been limited to just over a dozen.⁸

There are numerous methods capable of investigating the static properties of nuclei. Of these, the atomic-beam magnetic-resonance method enjoys a distinct advantage. It is capable of studying ground-state and some excited-state atoms with stable or radioactive nuclei while the atoms are free from interactions with other atoms. Therefore, the hyperfine-interaction Hamiltonian describing the system contains only the nuclear and electronic terms of the atom. As the field of atomic beams has been reviewed by a number of authors,^{9, 10, 11, 12, 13} a detailed description of the general techniques and methods need not be given here. However, in a following section we present the physical principles of the method. For the present we consider the atomic system as being composed of an electronic magnetic moment coupled to a nuclear magnetic moment, both of which interact with an external magnetic field. The vector magnetic moments are proportional to the corresponding angular momentum vectors. The interactions of the magnetic moments with each other and with the external field are expressed in terms of vector products. The matrix elements of the interactions are therefore proportional to the matrix elements of the angular-momentum vector products involved.¹⁴ With these matrix elements the complete interaction matrix can be constructed and diagonalized, yielding the eigenvalues for the energy levels of the atomic system in the homogeneous magnetic field used in the experiments. This calculation is the basis for the theory of atomic hyperfine structure.¹¹

The experimental method employs three regions of interaction through which the neutral atoms must pass. The first region establishes the orientations for the electronic magnetic moment. This condition is referred to as spatial quantization and is produced through the quantum-mechanical interaction of the electron magnetic moment with the magnetic field. The second region also contains a magnetic field and contains the radio-frequency magnetic field perturbation. This perturbation can cause transitions from one orientational state to another, given the correct value of magnetic field and radio frequency satisfying the Bohr

condition. The third region serves to separate, spatially, those atoms which make electronic orientational transitions from those which do not.

In the experimental apparatus these interaction regions are in fact three magnets. Two inhomogeneous magnetic fields are separated by a region of homogeneous field. All three fields have the same direction in space, in common with the gradients of the two inhomogeneous fields. The fields exist within a vacuum chamber. At one end of the chamber is a source of neutral atoms. This source is a stainless steel oven containing the sample material. The oven is heated to 500 to 700°C for the alkali atom beams considered here. The atoms pass out of the oven through a small slit and are collimated by a second set of slits and pass down the axis of the vacuum chamber and magnet system. As the atoms enter the first inhomogeneous magnetic field, the interaction of the electronic magnetic moment with this field produces an acceleration of the atom transverse to the beam axis. If the orientation of this magnetic moment does not reverse, the displacement is augmented in the third magnetic field, which is also inhomogeneous. However, if a transition takes place that reverses the projection of the electronic magnetic moment refocusing occurs. This is termed the refocusing or "flop-in" condition, and the refocused atom returns to the axis of the system, where a suitable detector is located. The detailed trajectories are determined by the design characteristics of the apparatus.

The specific apparatus used in these experiments was constructed by Dr. Gilbert O. Brink, Norman Braslau, and the author from the vacuum chamber and magnet system of an older apparatus. The original structure was described by Bemski¹⁵ and the modifications appear in the thesis of Norman Braslau.¹⁶ The purpose was to introduce such characteristics as to allow precision measurements of the hyperfine-structure splitting and nuclear magnetic moments of radioactive alkali atoms. Consistent with this purpose, the system design and detectors

(hot-wire surface ionization) were specified. The study of radioactive alkali atoms was made possible by the introduction of a deposition system which allowed the monitoring of the decay rate of refocused radioactive beams by scintillation and proportional-flow β counters.

The first studies carried out were on rubidium isotopes. The hyperfine-structure separation and magnetic moment of Rb^{85} were remeasured to check the apparatus.¹⁶ These results were consistent with previous determinations. Measurements of these properties for Rb^{86} have been completed; ^{16, 17, 18} reported here are the results for K^{42} , and in progress by J. D. Faust is the study of the isotopes Rb^{81} and Rb^{82} . Also under construction in the laboratory is a first-order focusing mass spectrometer capable of operating at pressures less than 10^{-8} mm Hg. This device was designed by the author and constructed by the Physics Department machine shop. It is hoped that reduction of gas-scattered background ions may permit direct electrical observation of long-lived radioactive atoms.

Included, in addition to the hfs separation and magnetic moment of K^{42} , are the hfs separations of Rb^{84} , Cs^{127} , and Cs^{129} . In the following section (Section II) is a review of hyperfine-structure theory and a development of the quantum-mechanical formalism which gives the energy levels for the $J = 1/2$ atoms. Section III presents the nuclear models applicable in interpreting the hfs anomaly. In Section IV is a description of the apparatus used, followed in Section V by the experimental procedure. The experimental results are presented in Section VI, and in Section VII, a discussion of the hfs anomaly of K^{42} .

II. THEORY OF HYPERFINE STRUCTURE

A. Review

The hyperfine interaction was first recognized in medium- and high-resolution optical spectroscopy early in the 1920's. Detailed study of this interaction had to wait for several years until G. F. Uhlenbeck and S. Goudsmit proposed the electron intrinsic spin to explain fine structure. Pauli suggested the existence of a nuclear spin and associated magnetic dipole moment to explain the hyperfine-structure splitting observed.¹⁹ This nuclear magnetic moment, when incorporated into the vector coupling model of the atomic electrons, gave a consistent explanation of the hyperfine structure and its field dependences. Thus armed, optical spectroscopists have been able to establish the nuclear spins of many of the stable isotopes, and have been able to establish the level ordering and to measure the magnitude of the hyperfine-structure separation to within 10% in some cases. Refinements in the method came, which reduced the Doppler broadening of the optical lines by liquid air cooling of sources, and sharpened the optical resolving power by employing the Fabry-Perot etalons.

Further extensions of theory for single electron spectra came with the theoretical evaluation of the electron magnetic field at the nucleus by Fermi and Segrè.²⁰ This specification of field combined with hyperfine-structure separations in alkali atoms gave values of the nuclear magnetic moments which had an ultimate accuracy of a few percent. Breit and Rabi in 1931 evaluated the energy-level dependence upon field, using quantum-mechanical methods.²¹ It is this formalism which serves as the basis for the work presented here. Many others studied nuclear spins and magnetic moments, employing the field dependence of the effective magnetic moments of the orientational states of the atomic electron-nucleus system in combination with the Stern-Gerlach type of atomic beam deflection experiment. Later Rabi introduced the radio-frequency resonance method which increased the sensitivity and accuracy of the method and rendered the older methods more or less obsolete.²²

It is therefore the purpose of this section to present the Breit-Rabi and Fermi-Segrè formulas and show the manner in which these are applied to atomic-beam magnetic resonance method.

B. Theory of the Hyperfine-Structure Interaction

To define clearly the nature of the hyperfine interaction and the energy levels of the system in low, high, and intermediate magnetic fields, we must specify the properties of the atomic electron-nucleus system. (At an early point in the development, the single-valence-electron restriction will be introduced which will limit the number and type of the interactions of the nucleus with its electronic environment).^{10,11}

1. The nucleus, to first approximation, is regarded as a point charge of magnitude Ze , possessing sufficient mass to render nuclear motion negligible.
2. The electrical interaction of the electrons with the nucleus is assumed to be coulombic.
3. The electron possesses an intrinsic spin angular momentum $\hbar/2$.
4. Associated with the spin angular momentum of the electron is a magnetic moment $\vec{\mu}_s = \gamma_s \hbar \vec{s} = g_s \mu_0 \vec{s}$, where γ = gyromagnetic ratio, g_s = electronic spin g value μ_0 = Bohr magneton $= \frac{e\hbar}{2mc}$.
5. Associated with the electron orbital motion is an angular momentum $\vec{l}$, with magnetic moment

$$\vec{\mu}_l = g_l \mu_0 \vec{l}$$

6. The total electronic magnetic moment is the vector sum of these two, $\vec{\mu}_J = \vec{\mu}_s + \vec{\mu}_l$. The total electronic magnetic moment can be expressed

$$\vec{\mu}_J = g_J \mu_0 \vec{J}$$

where

$$\vec{J} = \vec{s} + \vec{l}$$

7. The nucleus possesses an intrinsic spin angular momentum

$\hbar I$ and a magnetic moment $\vec{\mu}_I = g_I \mu_0 \vec{I}$,

where $g_I \approx \frac{m}{M} g_J$,

m = electronic mass,

M = proton mass.

To specify the hyperfine interaction of the nuclear magnetic moment with the electronic field we state the classical form

$$\mathcal{H} = - \vec{\mu}_I \cdot \vec{H}_J, \quad (\text{II-1})$$

where $\vec{H}_J$ is the vector magnetic field of the atomic electrons at the site of the nucleus.

For the case of interest here, the single valence electron in the s-state, the interaction has been calculated by Fermi and Segrè, assuming a nuclear dipole of zero extent. The magnitude of the field is given by

$$H_J = \frac{8\pi}{3} \mu_J |\psi(0)|^2, \quad (\text{II-2})$$

where $\psi(0)$ is the value of the electron wave function at the site of the nucleus.

The magnetic field $\vec{H}_J$ at the nucleus transforms under rotation of coordinate system like $\vec{J}$, and hence has matrix elements proportional to $\vec{J}$, for representations in which $\vec{J}$ is diagonal. By combining expressions for the nuclear magnetic moment and the field, one obtains

$$\mathcal{H} = ha \vec{I} \cdot \vec{J}, \quad (\text{II-3})$$

where

$$ha = - \frac{8\pi}{3} \mu_I \mu_J |\psi(0)|^2. \quad (\text{II-4})$$

In the absence of external fields this term gives the hyperfine-structure separation. However, with the addition of external magnetic field two additional terms appear: the interactions of the electronic and nuclear magnetic moments with $\vec{H}_C$, the external "C" field, giving

$$\mathcal{H} = ha \vec{I} \cdot \vec{J} - g_I \mu_0 \vec{I} \cdot \vec{H}_c - g_J \mu_0 \vec{J} \cdot \vec{H}_c. \quad (\text{II-5})$$

This Hamiltonian may be solved in either of two representations. These representations are characterized by the degree of coupling of $\vec{I}$ and $\vec{J}$. In the low-field case, the dominant term is the hyperfine interaction proportional to $\vec{I} \cdot \vec{J}$, with $\vec{I} \cdot \vec{H}_c$ and $\vec{J} \cdot \vec{H}_c$ regarded as perturbations, and $\vec{F} = \vec{I} + \vec{J}$, with F, M_F (the projection of F on the field axis) being constants of motion. For high magnetic fields $\vec{I}$ and $\vec{J}$ are strongly coupled to the external field, and the hyperfine interaction is then a perturbation. The two extreme coupling cases may be seen in Fig. 1. The development of these two cases is not of general usefulness here; it can be found in the literature.

The Hamiltonian may be solved exactly for its eigenvalues in either system, giving the Breit-Rabi formula.²¹ The secular determinant for a given $M = M_I + M_J$ can be readily solved for $J = 1/2$, and results in

$$W(F, M) = -\frac{\Delta W}{2(2I+1)} - \frac{\mu_I}{I} H_c m \pm \frac{\Delta W}{2} \left[1 + \frac{4M}{2I+1} x + x^2 \right]^{1/2} \quad (\text{II-6})$$

where

$$\Delta W = h \Delta \nu = ha \left(\frac{2I+1}{2} \right),$$

$$x = \frac{\left(-\frac{\mu_J}{J} + \frac{\mu_I}{I} \right) H_c}{\Delta W},$$

and the $+$ sign refers to $F = I + 1/2$, and the $-$ sign refers to $F = I - 1/2$, provided the square root is factored first if possible. The above equation, known as the Breit-Rabi formula, is applicable to atoms for which either I or J is $1/2$. The form remains the same but J and I interchange roles for $I = 1/2$. (Plots of $W(F, m)$ for $J = 1/2$, $I = 1/2$ and 2 , respectively will be seen in Figs. 3 and 4).

In the cases in which I and J are greater than $1/2$, the form of the Hamiltonian becomes more complicated. Additional terms appear by virtue of the interaction of higher multipole moments of the nuclear charge and current distribution with higher derivatives of the electronic magnetic and electric fields, as allowed by the symmetry properties of each. The restriction upon the observable nuclear multipole moments states that owing to the parity of the multipole operators and the nuclear wave function only even electric moments and odd magnetic moments will be nonvanishing. Therefore, a nuclear spin of 1 has only an observable magnetic dipole moment and electric quadrupole moment, spin $3/2$ has a magnetic dipole and octupole moment and an electric quadrupole. At present, only dipole, quadrupole, and octupole moments have been observed, although it is wholly possible that higher moments exist and may be discovered with the advent of greater precision.

C. Observable Transitions and the "Flop-In" Technique

A schematic representation of the apparatus appears in Fig. 2. A and B magnets supply the inhomogeneous fields and the C magnet supplies the one homogeneous field. The transverse forces used in the deflections of the atoms can be expressed as

$$\vec{F} = -\vec{\nabla}W = -\frac{\partial W}{\partial \vec{H}} \vec{\nabla} \vec{H} = \mu_{\text{eff}} \vec{\nabla} \vec{H}.$$

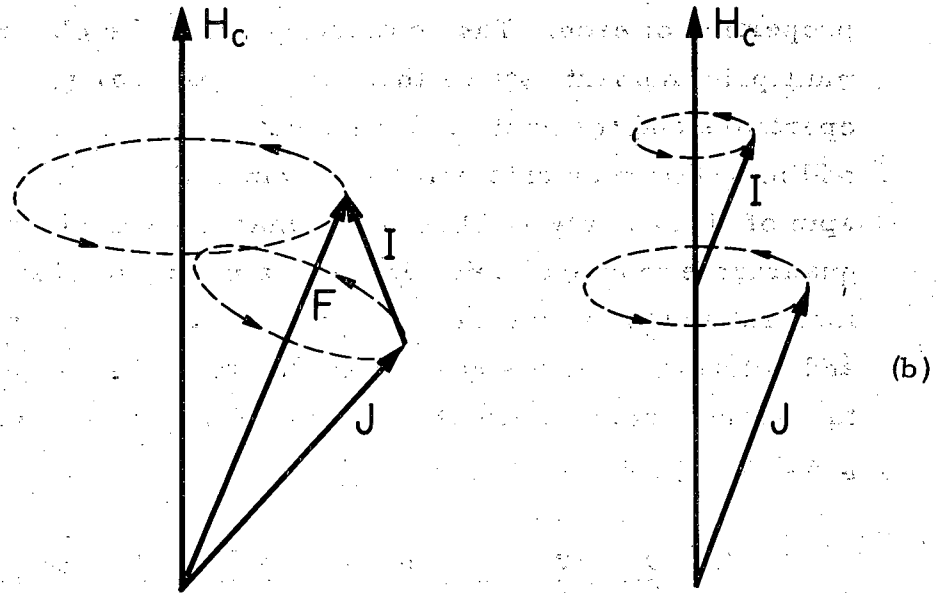
The effective magnetic moment of the atom in the various energy levels can be obtained directly from the Breit-Rabi formula; it is

$$\mu_{\text{eff}} = -\frac{\partial W}{\partial H} = mg_I \mu_0 + (g_I - g_J) \mu_0 \frac{\left(\frac{2m}{2I+1} + x \right)}{2 \left(1 + \frac{4m}{2I+1} x + x^2 \right)^{1/2}} \quad (\text{II-7})$$

where

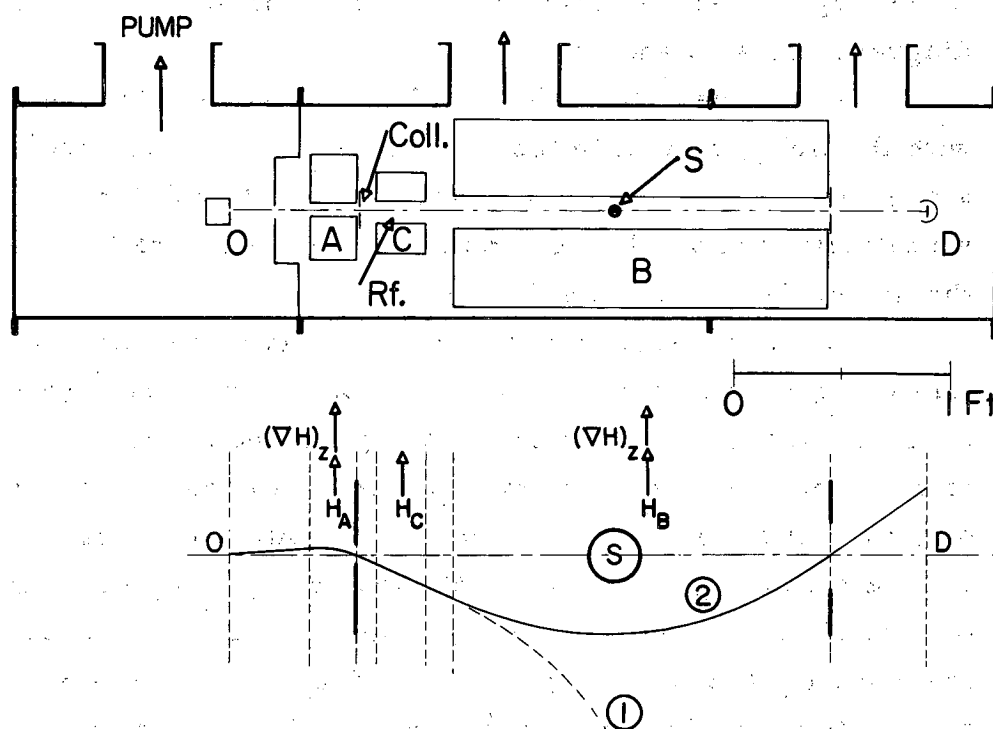
$$g_I = \frac{\mu_I}{I}$$

$$g_J = \frac{\mu_J}{J}$$



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Fig. 1. $\vec{I} + \vec{J}$ in strong and weak fields.



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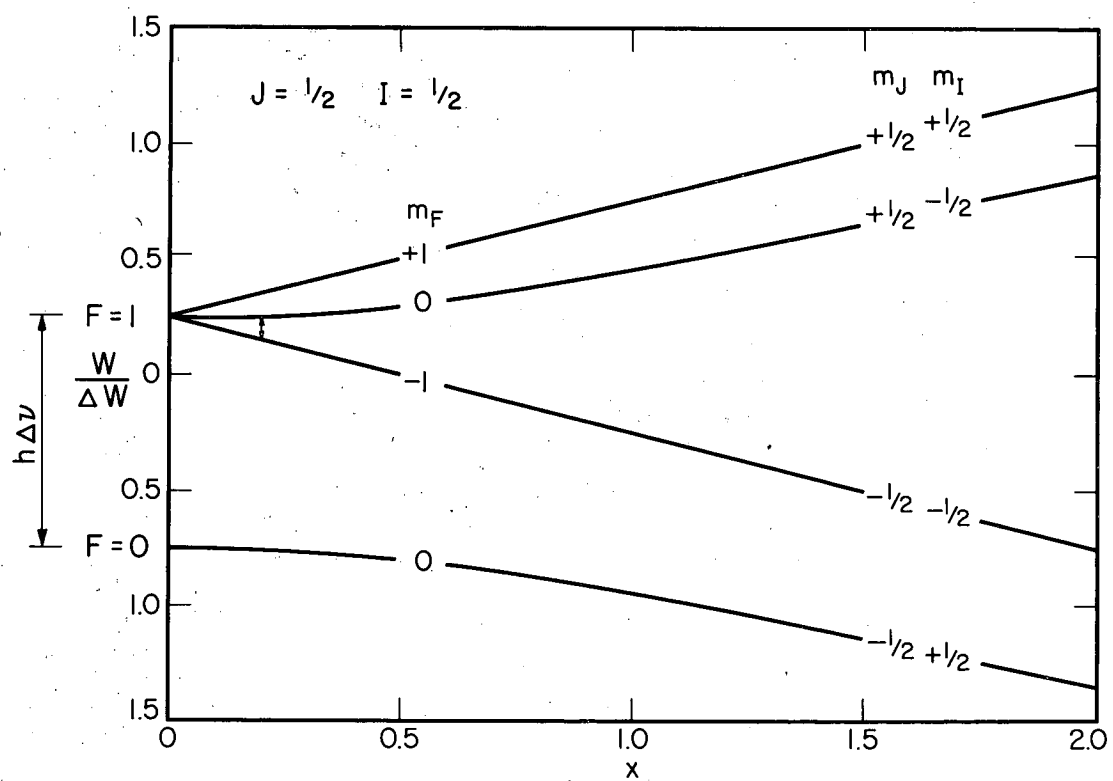
Fig. 2. Atomic beam apparatus (schematic).

as expressed in F, M_F representation, with the $\mp$ signs corresponding to $F = I + 1/2$ and $I - 1/2$ respectively. As the value of (x) increases, the effective magnetic moment approaches $\pm \mu_0$, since $|g_J| > |g_I|$. This can also be seen by observing that the effective magnetic moment is the negative slope of the energy curves drawn in the Breit-Rabi diagram, Figs. 3 and 4.

For values of $x \geq 2$ the effective magnetic moment that interacts with the field gradient in the A and B magnets is essentially $\pm g_J \mu_0 m_J$. This value of x serves as a criterion for the field value, while deflection amplitudes and geometry considerations determine the usable field gradients.

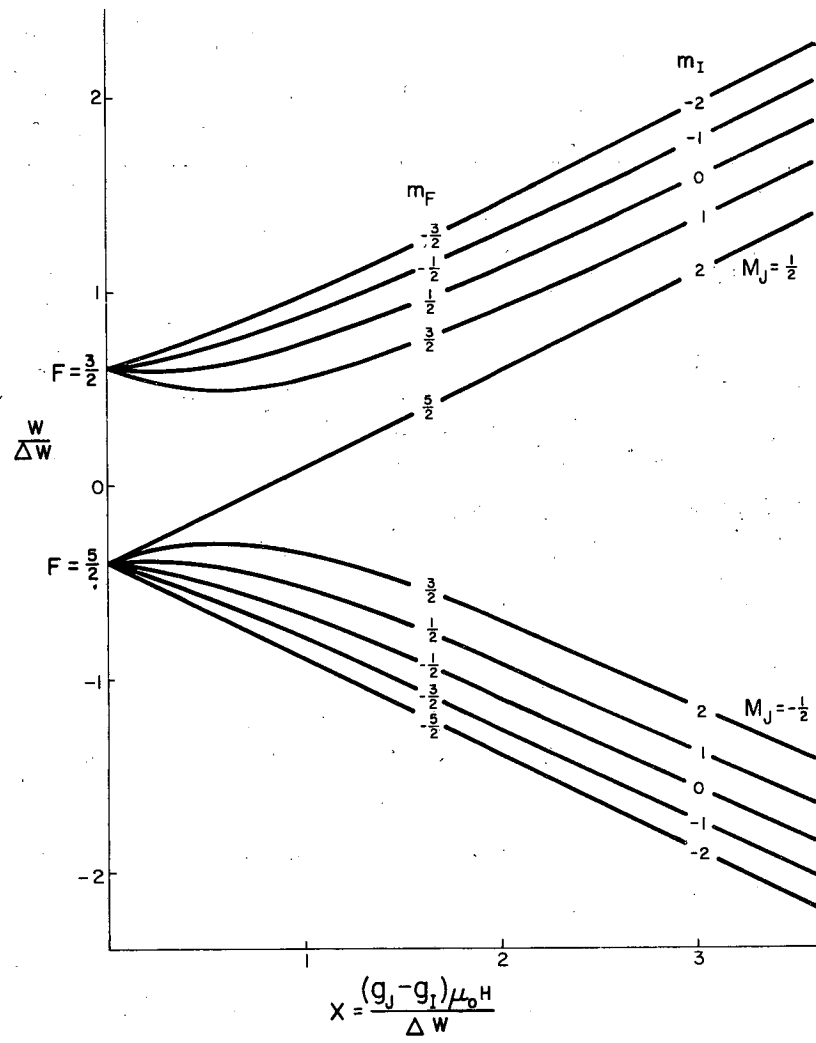
Given the above x value in the A and B magnets, it is easy to follow the atomic orbits through the apparatus. The direction of the magnetic gradients is the same in both magnets. If, therefore, the effective moment of the atom maintains the same orientation throughout, the deflection in the A magnet is continued in the B magnet, and Path 1 of Fig. 2 describes the trajectory. However, if a perturbation is introduced which reverses the direction of the effective moment, the transverse force in the B magnet is reversed and the atom returns to the axis, as shown in Path 2 of Fig. 2.

The two classes of perturbation found in experimental practice are spatially varying magnetic fields and radio-frequency magnetic fields. The static-field perturbations can arise from any rapid change in field along the trajectory which has a large Fourier component at the resonance frequency. Generally this perturbation is too small by several orders of magnitude to be observed in properly designed apparatus. One may also set the C field to zero, which then randomizes the orientations. This is generally found experimentally and establishes the zero-field condition. Radiofrequency transitions may be excited by a magnetic field rotating at the resonance frequency. In practice, however, oscillatory fields replace the rotating fields. The effect of employing the oscillatory field is to add a counterrotating magnetic



MU-13310

Fig. 3. Breit-Rabi diagram; $I = 1/2$, $J = 1/2$.



MU-16436

Fig. 4. Breit-Rabi diagram; $I = 2$, $J = 1/2$,
 $g_I < 0$.

field vector of the same frequency and of phase such that the vector sum lies in a given plane. The effect upon the resonance frequency has been estimated by Bloch and Siegert. They showed that there is a shift in frequency of the resonance, given by

$$\omega_0 - \omega_r = \frac{(2b_1)^2}{\omega_0 - \omega_1}, \text{ where } \omega_1 = -\omega_0. \quad (\text{II-8})$$

Typical value of b_1 is 10^4 cps and of ω_0 is 10^8 cps, thus

$$\omega_0 - \omega_r \approx 1 \text{ cps} \ll \text{experimental uncertainty.}$$

The plane of the oscillatory rf magnetic field with respect to the fixed C field establishes the selection rules for the resonance transitions.

For an rf magnetic field perpendicular to the fixed field, π or $\Delta m = \pm 1$ transitions may take place. For the field component parallel to the fixed field, σ or $\Delta M = 0$ transitions are allowed. The F-state selection rule states $\Delta F = 0, \pm 1$. By noting that in the Breit-Rabi diagram of Fig. 4 for the case $I = 2$, $g_I < 0$ (as for Rb^{84} and K^{42}), one sees that the refocusing transitions at large x correspond to $\Delta M_J = \pm 1$ in the high-field or Paschen-Bach region. Following these curves backward, we see that for $\Delta F = 0$ there is only one refocusing transition: $F = I + 1/2, M = -I - 1/2 \longleftrightarrow M = -I + 1/2$. For $\Delta F = \pm 1$ all transitions are refocusing except

$$\begin{pmatrix} F = I + 1/2 \\ M = I + 1/2 \end{pmatrix} \longleftrightarrow \begin{pmatrix} F = I - 1/2 \\ M = I - 1/2 \end{pmatrix}$$

The expressions for the frequencies of the transitions are given by the Breit-Rabi formula (Eq. II-6). In the following sections we state these explicitly in application to evaluating the hyperfine structure and nuclear magnetic moment.

D. Determination of $\Delta \nu$

One of the quantities of interest is the zero-field hyperfine separation. This cannot be evaluated directly, as there would be no reference axes at zero field with which to reorient. The functional dependence of the Breit-Rabi formula upon $\Delta \nu$ can be used to determine its value from resonances taken at finite values of the static magnetic field.

For preliminary determination the refocused $\Delta F = 0$, $\Delta M = \pm 1$ transition can be used. The exact expression for this transition frequency can be expanded in the low-field approximation to obtain, to second order in H ,

$$\nu = \frac{2.80}{2I+1} H_c + \left(\frac{2.80}{2I+1} H_c \right)^2 \frac{2I}{\Delta \nu} \left(\text{in (Mc/sec)} \right). \quad (\text{II-9})$$

As the field increases, one must use the exact expression, solving for $\Delta \nu$:

$$\Delta \nu = \frac{\left(\nu + \frac{g_I \mu_0 H_c}{h} \right) \left(- \frac{g_J \mu_0 H_c}{h} - \nu \right)}{\left(\nu + \frac{g_J \mu_0 H_c}{(2I+1)h} + \frac{2I}{2I+1} \frac{g_I \mu_0 H_c}{h} \right)} \quad (\text{II-10})$$

The g_J is assumed to be known; the only quantity that has not been evaluated is g_I . By the following method of successive approximations one may obtain $\Delta \nu$ to within a few megacycles. First, one neglects the g_I term and obtains a preliminary value of $\Delta \nu$. According to the Fermi-Segrè formula the ratio of the splitting factors of two isotopes of the same element becomes

$$\frac{a_1}{a_2} \approx \frac{g_{I_1}}{g_{I_2}} \quad (\text{II-11})$$

The a is related to the hyperfine-structure splitting by

$$h \Delta \nu = \frac{ha(2I+1)}{2} \quad (II-12)$$

The approximate result obtained is

$$\frac{\Delta \nu_1}{|g_{I_1}|(2I_1+1)} \approx \frac{\Delta \nu_2}{|g_{I_2}|(2I_2+1)} \quad (II-13)$$

By use of this relation, the g_I can be obtained, with ultimate accuracy of a few percent from an isotope whose g_I is known. Inserting this into Eq. (II-10), one obtains a corrected $\Delta \nu$, and the process can be repeated to converge upon the value limited by the experimental error of the measurement or by the Fermi-Segrè formula.

The final step employs excitation of a $\Delta F = \pm 1$ transition at low field where all frequencies converge to $\Delta \nu$ as H_c approaches zero. The experimental radio-frequency hairpin used to introduce the oscillatory field is designed to allow only $\Delta M = \pm 1$ transitions with large transition probability. For $I + J$ equal to an integer, there exist only $\Delta M = 0$ transitions for which the frequencies are field-independent to first order in H . However, for I and J equal to a half-integer, one may obtain the value of $\Delta \nu$ which is limited only by the uncertainty width resulting from the radio-frequency loop that excites the transition.

The general expression for the frequency of such a transition is

$$\nu = g_I \mu_0 H_c (m_+ - m_-) + \frac{\Delta \nu}{2} \left[\left(1 + \frac{4m_+}{2I+1} x + x^2 \right)^{1/2} + \left(1 + \frac{4m_-}{2I+1} x + x^2 \right)^{1/2} \right] \quad (II-14)$$

If one expands this to second order in x , neglecting terms in g_I , one obtains

$$\nu = \frac{\Delta \nu}{2} \left[2 + \frac{2x}{2I+1} (m_+ - m_-) + x^2 \left(1 - \frac{2}{(2I+1)^2} [m_+^2 + m_-^2] \right) \right]. \quad (\text{II-15})$$

This equation indicates that for $m_+ = \pm 1/2$ and $m_- = \mp 1/2$ the term linear in x vanishes. For the particular application in these experiments, for $I = 2$, this becomes

$$\nu = \Delta \nu \left[1 + \frac{12}{25} x^2 \right]. \quad (\text{II-16})$$

For typical low-field measurements H_c is about 2 gauss.

$$x^2 \approx \left(\frac{2.8 H_c}{\Delta \nu} \right)^2 \approx \left(\frac{5.6}{3 \times 10^3} \right)^2 \approx (2 \times 10^{-3})^2 \approx 4 \times 10^{-6}$$

For:

$$H_c = 2 \text{ gauss}$$

$$\Delta \nu \approx 3,000 \text{ Mc,}$$

which allows $\Delta \nu$ to be determined, for example, to within a few kilocycles out of 3000 Mc. It is possible to use the Breit-Rabi formula without approximations to obtain an even better value.

E. Determination of g_I

From the evaluation of the hyperfine-structure separation, the nuclear g factor can be obtained to a few percent from comparison with another isotope, using the Fermi-Segre formula. For greater accuracy, it is necessary to obtain a g value which is not dependent upon the Fermi-Segre formulas explicitly. This can be done by observing that the second term of equation (II-14), involving $\Delta \nu$, has stationary values. We define this second term

$$\bar{\nu} = \frac{\Delta \nu}{2} \left[\left(1 + \frac{4m_+}{2I+1} x + x^2 \right)^{1/2} + \left(1 + \frac{4m_-}{2I+1} x + x^2 \right)^{1/2} \right], \quad (\text{II-17})$$

and the derivative with respect to x can be made zero for certain values of m_+ and m_- and x , which can be visually estimated from the Breit-Rabi diagram.

For the case in question (for Rb^{84} and K^{42}) the value of m_+ and m_- are $+1/2, +3/2$ and $+3/2, +1/2$, respectively. Under the conditions of $\frac{\partial \bar{\nu}}{\partial x} = 0$, the difference in frequency between the two resonance transitions gives a value for the nuclear g factor,

$$\frac{\partial \bar{\nu}}{\partial x} = 0 = \frac{\Delta \nu}{2} \left[\frac{1}{2} \frac{\frac{4m_+}{2I+1} + 2x}{\left(1 + \frac{4m_+}{2I+1} x + x^2 \right)^{1/2}} + \frac{1}{2} \frac{\frac{4m_-}{2I+1} + 2x}{\left(1 + \frac{4m_-}{2I+1} x + x^2 \right)^{1/2}} \right].$$

For $I = 2$, $m_+ = 1/2, 3/2$, $m_- = 3/2, 1/2$

$$0 = \frac{2x - 6x/4}{(1 - 6/5x + x^2)^{1/2}} + \frac{2x - 2x/5}{(1 - 2/5x + x^2)^{1/2}};$$

the value of x for which $\frac{\partial \bar{\nu}}{\partial x} = 0$ is

$$x = \frac{14 - 4\sqrt{6}}{10} = 0.420 \ 204 \ 103. \quad (\text{II-18})$$

For the value of H_c corresponding to this x , the frequencies of the transition $\left(\begin{smallmatrix} F = 5/2 \\ m = 3/2 \end{smallmatrix} \right) \longleftrightarrow \left(\begin{smallmatrix} F = 3/2 \\ m = 1/2 \end{smallmatrix} \right)$ or ν_+ and the transition

$\left(\begin{smallmatrix} F = 5/2 \\ m = 1/2 \end{smallmatrix} \right) \longleftrightarrow \left(\begin{smallmatrix} F = 3/2 \\ m = 3/2 \end{smallmatrix} \right)$ or ν_- are given by

$$\nu_{\pm} = \bar{\nu} \pm g_I \mu_0 H_c,$$

and the difference frequency is given by

$$\nu_+ - \nu_- = 2 g_I \mu_0 H_c. \quad (\text{II-19})$$

Thus the accuracy with which g_I may be obtained is determined by the accuracy of $\nu_+ - \nu_-$ and μ_0, H_c .

The final considerations concern any additional effects that might invalidate the conclusions put forth. Some perturbing effects are: (a) electronic-state admixing due to Coulomb interaction of the electrons, (b) second-order contributions to the hyperfine energy from near-lying fine-structure states or excited states, and (c) changes in the electron wave functions in the region of a finite nucleus.

The state admixing in alkali atoms has been considered by Lamb, who concludes that this might explain the change in g_I . This effect cannot alter the g_I as determined by the above method. Schwartz has considered the second-order effects in the hyperfine energy and proposes them to be of the order $(\Delta \nu)^2 / \delta$, where δ is the energy increase necessary to reach the next interacting state.²³ For K^{42} this is less than $10^{-4}\%$.

The last effect, which is of great importance in optical spectroscopy, appears in the form of a change in scale when the g_I value is deduced from the $\Delta \nu$, and does not appear at all when the g_I is evaluated from the separation of two resonance lines whose frequencies are measured at field-independent points. This effect must be considered in evaluating the anomaly, and introduces a correction factor of the order of $10^{-4}\%$.

III. THEORY OF ANOMALOUS HYPERFINE STRUCTURE

A. Introduction

Given, by some experimental method, the hyperfine-structure separation and magnetic moment for two isotopes of the same element, one may ask how these may be related. For the case of s electrons (whose spatial distribution is spherically symmetric) the delta function or contact term of the electron-nucleus magnetic dipole interaction gives rise to the Fermi Segrè formula, which is based on the assumption of a nucleus with zero spatial extent. This states^{7, 9, 24}

$$\frac{\Delta \nu_1}{\Delta \nu_2} = \frac{a_1}{a_2} \frac{2I_1 + 1}{2I_2 + 1} = \frac{2I_1 + 1}{2I_2 + 1} \frac{|g_{I_1}|}{|g_{I_2}|} \frac{|\psi_1(0)|^2}{|\psi_2(0)|^2} \quad (\text{III-1})$$

To a first approximation the squares of the wave functions cancel, as they belong to the same electron configuration. However, the ratio of the $\Delta \nu$'s fails to satisfy the above relation, by as much as a percent. This leads to the definition of the hyperfine-structure anomaly. For each isotope the value of $\Delta \nu$ differs from the Fermi-Segrè value by a factor $1 + \epsilon$. Since the individual ϵ , the fractional hyperfine-interaction reduction, may not be measured directly, owing to inadequate knowledge of the electron wave function, we must evaluate its change from isotope to isotope of a given element. For small values of ϵ , the following ratio is used to define the hyperfine structure anomaly:

$$\frac{1 + \epsilon_1}{1 + \epsilon_2} \approx 1 + \epsilon_1 - \epsilon_2 = 1 + {}^1\Delta^2, \quad (\text{III-2})$$

where, by convention, the smaller superscript refers to the lighter nucleus. Therefore, the hfs anomaly is defined as

$${}^1\Delta^2 = \left[\left(\frac{\Delta \nu_1}{\Delta \nu_2} - \frac{|g_{I_2}|}{|g_{I_1}|} \frac{2I_2 + 1}{2I_1 + 1} \right) - 1 \right] \times 100\% \quad (\text{III-3})$$

Up to this point we have made no statement as to the source of these deviations. It is now necessary to give the nucleus further properties. The nuclei whose properties are considered in the following discussions are of medium and heavy atoms. The nucleons contained within these nuclei are attracted by extremely short-range forces, range of the order of 10^{-13} cm. These forces drop off as $1/r^n$, where n is near 6. Free nucleons have the properties of finite spatial extent, spin angular momentum, magnetic dipole moments, and--in the case of protons--charge. In the aggregate state these properties are combined in complex ways. There exists, for the whole nucleus, the property of a nonzero volume which reflects an incompressibility of the nucleons. Between the individual nucleons of one type (protons or neutrons) there are forces which depend upon the relative orientation of their angular momentum vectors.

Taking the nucleus as a whole, it has been found that a central-force-field shell model can be applied even though the existing forces are essentially interparticle.³ This shell theory has qualitatively predicted the nuclear energy-level scheme, including the magic-number shells. The success of this approach suggests further analogies with atomic electrons. The property of use here is that of a nucleon orbit within the nucleus. This property is utilized by Bohr and Weisskopf to introduce the orbital magnetic moment of unpaired nucleons to predict nuclear magnetic moments. The essential features of this approach have been accepted.

The shell theory predicts other nuclear properties. Of these, the electric quadrupole moment prediction has proven inaccurate in certain cases. The large values observed in experiment suggest that many protons are contributing, rather than the single unpaired proton. This suggests the possibility of a collective motion of (the closed shell or core) nucleons. The core may have associated with it volume and surface oscillations, as well as an effective volume angular momentum. This was considered in detail by Bohr and Mottelson, and proved consistent with nuclear energy level data. This, then, represents an

additional term that may appear in the expression for the nuclear magnetic moment prediction.

Another ramification of the finite size of the nucleus is a modification of the electronic wave function at the nucleus.^{15, 26} Therefore any interaction which depends upon this will likewise be altered. This occurs in the hyperfine interaction and must be considered in the prediction of the hyperfine-structure anomaly.

To summarize, we see that the evaluation of any corrections to the Fermi-Segrè formula must take into account the orbital and spin states of the unpaired nucleons as well as any collective motion of a core composed of all the nucleon pairs. In addition, changes in the electron wave function must be included.

To further complicate the picture, the odd-odd nucleus introduces the problem of coupling of the unpaired proton and neutron to each other and to any core motion.

The two tests that must be applied to the models available are the prediction of the nuclear moment and the hyperfine-structure anomaly.^{6, 27} Three models are currently applied to the problem and employ the Bohr-Weisskopf theory. The first two differ in the weighting of nuclear ground-state single-particle and core contributions. The last introduces configuration mixing. This has the effect of admixing into the ground-state single-particle wave function small amounts of the wave functions of excited states of the nucleons. As applied to odd-even and even-odd nuclei the prediction of magnetic moments has been quite successful. However, its application to odd-odd nucleons remains as a future development.²⁸

B. General

The hyperfine-structure anomaly has been observed in atoms in the s and $p_{1/2}$ electronic state in which the electron has a non-negligible probability to being in the nucleus. The $p_{1/2}$ -state, in relativistic theory, has an s -state wave function component in regions near the nucleus, of the order $Z\alpha$ times the pure s -state value. We shall here limit ourselves to s -state atoms.

The Fermi-Segrè formula was calculated by assuming the nucleus to be a point dipole magnetic moment. The inadequacy of this assumption was first suspected by Kopfermann. The first isotopes experimentally studied were Rb^{85} and Rb^{87} .²⁹ A measured anomaly of 0.35% was obtained.³⁰ As time passed additional measurements were made and theoretical investigation proceeded. Bohr and Weisskopf in a series of three articles proposed a model of the hyperfine interaction which combined the effects of changes in the electron wave function, separate spin and orbital nucleon contributions to the nuclear magnetism, and a collective mode of closed shell motion in the nucleus which could couple with the unpaired nucleons.^{31, 32, 33} They applied these theories to the anomaly prediction for Rb^{85} and Rb^{87} with relative success.

Further developments have refined this approach. They include more exact evaluation of the electron wave function and the incorporation of the results of the collective model. Another approach has been the configuration mixing model to explain the magnetic moment deviations from the Schmidt limits.

C. The Bohr-Weisskopf Theory

With the failure of the simple model employing a uniform distribution of nuclear magnetism whose radius varied as $A^{1/3}$, it became necessary to separate the nucleon magnetic moment into two intrinsically different parts--the spin and orbital components. To first approximation for odd-even and even odd-nuclei, all nucleon spins except the "valence" or unpaired nucleon spin couple to zero. Therefore, the hyperfine interaction reduces to the interaction of a single electron with a single nucleon moving in the nuclear volume, using a relativistic wave function for the electron.

Using the hyperfine interaction constant defined by Eq. (II-3), we separate this into

$$a = a_s + a_\ell, \quad (\text{III-4})$$

which are to be compared with the point dipole hyperfine interaction constant,

$$a_{pd} = - \frac{8\pi}{3} g_I g_J |\psi(0)|^2.$$

For the values of a_s and a_ℓ :

$$a_s = - \frac{8\pi}{3} g_s g_J \int_N s_z \omega(\vec{R}) d\tau_n \left[\int_R^\infty FG - \zeta^*(s_z, \theta, \phi) \int_0^R \frac{r^3}{R^3} FG dr \right], \quad (\text{III-5})$$

$$a_\ell = - \frac{8\pi}{3} g_\ell g_J \int_N |\bar{\phi}(\vec{R})|^2 \ell_z d\tau_n \left[\int_R^\infty FG dr + \int_0^R \frac{r^3}{R^3} FG dr \right], \quad (\text{III-6})$$

where $\omega(\vec{R})$ = distribution of nuclear spin magnetization,

$\bar{\phi}(\vec{R})$ = nuclear wave function,

ζ^* = spin asymmetric distribution function.

As $R \rightarrow 0$ the expression for $a_s \rightarrow a_{pd}$ and $a_\ell \rightarrow 0$. If we define

$$\kappa_s = \frac{\int_0^R \left(1 + \zeta^* \frac{r^3}{R^3} \right) FG dr}{\int_0^\infty FG dr}, \quad (\text{III-7})$$

$$\kappa_l = \frac{\int_0^R (1 - \frac{r^3}{R^3}) FG dr}{\int_0^\infty FG dr}, \quad (III-8)$$

and $\bar{\kappa}_s$ and $\bar{\kappa}_l$, the average values over the nucleus, we obtain

$$\bar{\kappa}_s = \frac{a_{pd} - a_s}{a_{pd}}, \quad (III-9)$$

$$\bar{\kappa}_l = \frac{a_{pd} - a_l}{a_{pd}},$$

which represent the fraction reduction due to the finite extent of the orbital and spin nuclear magnetization.

To evaluate the expressions for the κ 's, information about the electron wave functions is necessary, as well as the angular asymmetry of the spin distribution expressed by ζ^* which has been calculated by Bohr³¹⁻³³ for various single-particle states.

For a given isotope, the relative contribution of spin and orbital magnetization to the magnetic moment must be established. For the vector coupling $\vec{I} = \vec{l} + \vec{s}$ for a single unpaired nucleon and $\vec{\mu}_I = \vec{\mu}_s + \vec{\mu}_l$, using the g-factor notation, one has

$$a_s = \frac{g_s}{g_I} \frac{g_I - g_l}{g_s - g_l}$$

$$a_l = \frac{g_l}{g_I} \frac{g_s - g_I}{g_s - g_l} \quad (III-10)$$

For the total fractional hyperfine, interaction reduction, we have

$$\epsilon = - (a_s \bar{\kappa}_s + a_l \bar{\kappa}_l). \quad (III-11)$$

The electronic wave function is calculated for the region within the nucleus, assuming the nuclear potential of a uniform charge distribution

$$V = \left(\frac{3}{2} - \frac{1}{2} x^2 \right) \frac{Ze}{R} ; \quad \text{for } x = \frac{r}{R} \leq 1 ,$$

where R = radius of the nuclear charge distribution. With these wave functions, $\bar{\kappa}_s$ and $\bar{\kappa}_l$ are evaluated in the form

$$\bar{\kappa}_s = \left[b_{s2}(S_{1/2}) + (b'_{s2}(S_{1/2})) \right] \left\langle \frac{R^2}{R_e^2} \right\rangle - \left[b_{s4}(S_{1/2}) + (b'_{s4}(S_{1/2})) \right] \left\langle \frac{R^4}{R_e^4} \right\rangle$$

$$\bar{\kappa}_l = b_{l2}(S_{1/2}) \left\langle \frac{R^2}{R_e^2} \right\rangle - b_{l4}(S_{1/2}) \left\langle \frac{R^4}{R_e^4} \right\rangle ,$$

where the coefficients b are tabulated for the above expressions as well as the radial averages.

In an extension of the above method Stroke³⁴ extends the series solution for the κ 's and tabulates new coefficients. This represents a slight increase of precision.

D. Collective Model

When quadrupole moments were measured and found to be many times that associated with a single nucleon, it became necessary to propose a nonspherical nuclear core. The simplest form of this asymmetry was the ellipsoid, with a symmetry axis associated with it. Next, surface oscillations were proposed, which might contribute to the magnetic moment. As a consequence, an additional term could be added, yielding

$$\vec{\mu}_I = \frac{e}{2mc} (g_s \vec{S} + g_l \vec{l} + g_R \vec{R}) , \quad (\text{III-13})$$

where $\bar{R}$ is the angular momentum of the core, $g_R \approx Z/A$, the g value of a uniformly charged core.

The regions in which the collective model should be applicable lie far from the spherically symmetric magic-number configuration, and in regions of large A (A being the total nucleon number).

E. Configuration Mixing Model

Blin-Stoyle and Arima and Horie have developed a configuration-mixing theory which predicts many magnetic moments for odd-even and even-odd nuclei.^{6,35,36,37} In application to the hfs anomaly of Rb^{85} and Rb^{87} the value predicted is 0.34%, compared to 0.35% observed.

The wave function employed has the form

$$\bar{\Psi}_i = \psi_i + \sum_j a_j \phi_j,$$

where the ϕ_j 's can differ only by one single-particle state from ψ_i with $\Delta l = 0$.

The smallness of a_j (< 1) allows one to limit the contribution to terms linear in a_j in the expression for the expectation value of the magnetic moment:

$$\bar{\mu} = \langle \bar{\Psi} | (\sum_n g_s^n \vec{S} + g_l^n \vec{L}) | \bar{\Psi} \rangle_{I,I}. \quad (III-14)$$

F. Conclusion

The basic validity of the Bohr-Weisskopf approach is well established and serves as the basis for the current hfs anomaly predictions. The greatest ambiguity that exists at present is associated with collective modes, possible interconfiguration mixing, and--for odd-odd nuclei--the coupling schemes.

(Stroke and Blin-Stoyle have recently extended the Bohr-Weisskopf formalism to include explicitly detailed information of the nucleon configurations of odd-A nuclei.³⁸) As a consequence, only the modified single-particle-model anomaly predictions will be compared with the experimental results. Two cases will be considered in which the intrinsic and orbital nuclear g values respectively, are changed, to allow fitting of the magnetic moment to the observed value. These changes are then reflected in the predicted hyperfine-structure anomaly.

IV. EXPERIMENTAL APPARATUS

A. General Characteristics

The apparatus used in these experiments has been described in detail in the thesis of Norman Braslau.¹⁶ The vacuum tank and magnets were built by Bemski¹⁵ for research on alkali halide molecules. The modification and reconstruction for application to atomic beam studies was carried out by Dr. Gilbert O. Brink, Norman Braslau, and the author. Many additions have been made since the summer of 1958 when the first alkali beam was observed in the reconstructed apparatus. The first problem to be solved at that time was monitoring the stable carrier beam, to allow normalization of the radioactive beam. The placement of a surface ionization detector in the path of the thrown-out beam allowed relative normalization to within $\pm 5\%$. The generation of radio-frequency signals required no unusual efforts, and comparison with a primary standard produced frequencies stable to $1/10^8$ over the course of an experiment. The most formidable problem lay with stabilization of the homogeneous C field, which varied because of changes in the C-magnet current and in the strength of fringe fields from adjacent magnets. This was solved by continuously monitoring a refocused transition in the stable alkali carrier, and using a frequency modulator and phase-sensitive detector to correct for the field drifts. A brief discussion of the apparatus components, below, concludes with a summary of the radio-frequency generation methods.

B. Machine Geometry

The physical structure of the machine was dictated by its earlier application to the study of molecules in the sigma-electronic state. Under these conditions Bemski required long deflecting fields and employed large gradients to produce usable deflections using the nuclear moment interaction with the field gradient. The over-all length of the vacuum chamber, from source to detector end, is 78-5/8 inches. The deflection region occupied 36.82 inches of this distance. The changes involved reduction of the field gradients consistent with the refocusing conditions as developed by R. J. Sunderland.³⁹ Thus, Fig. 5 shows the relative placement and location of the source, buffer chamber, magnets, collimator, stop wire, and detectors. The essential geometric characteristic of the machine is the symmetry about the collimating slit, located at the center of the C magnet.

C. The Vacuum Characteristics

The chambers shown in the machine layout are employed to allow differential pumping along the path length of the beam, hence eliminating spurious gas by-products of the oven chemistry from the interaction region of the main chamber. A pressure differential 10^2 can be obtained by pumping the oven, buffer, and main chamber independently, with apertures between, which are large enough only for the beam.

The external vacuum wall is a stainless steel cylinder of 12 in. o.d. There are ports for entrance of connections for water cooling, electrical lead-ins, etc. Figure 6 shows the external appearance of the apparatus.

The system has a full complement of ionization gauges, thermocouple gauges and appropriate controls, and associated pressure interlocks for fail-safe operation.

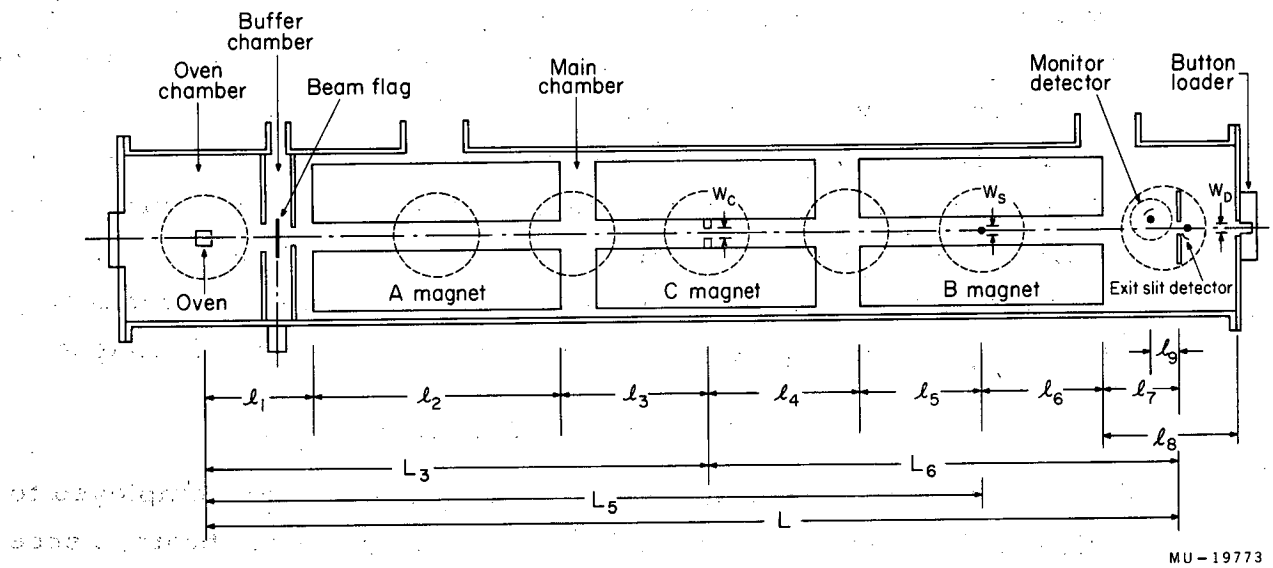
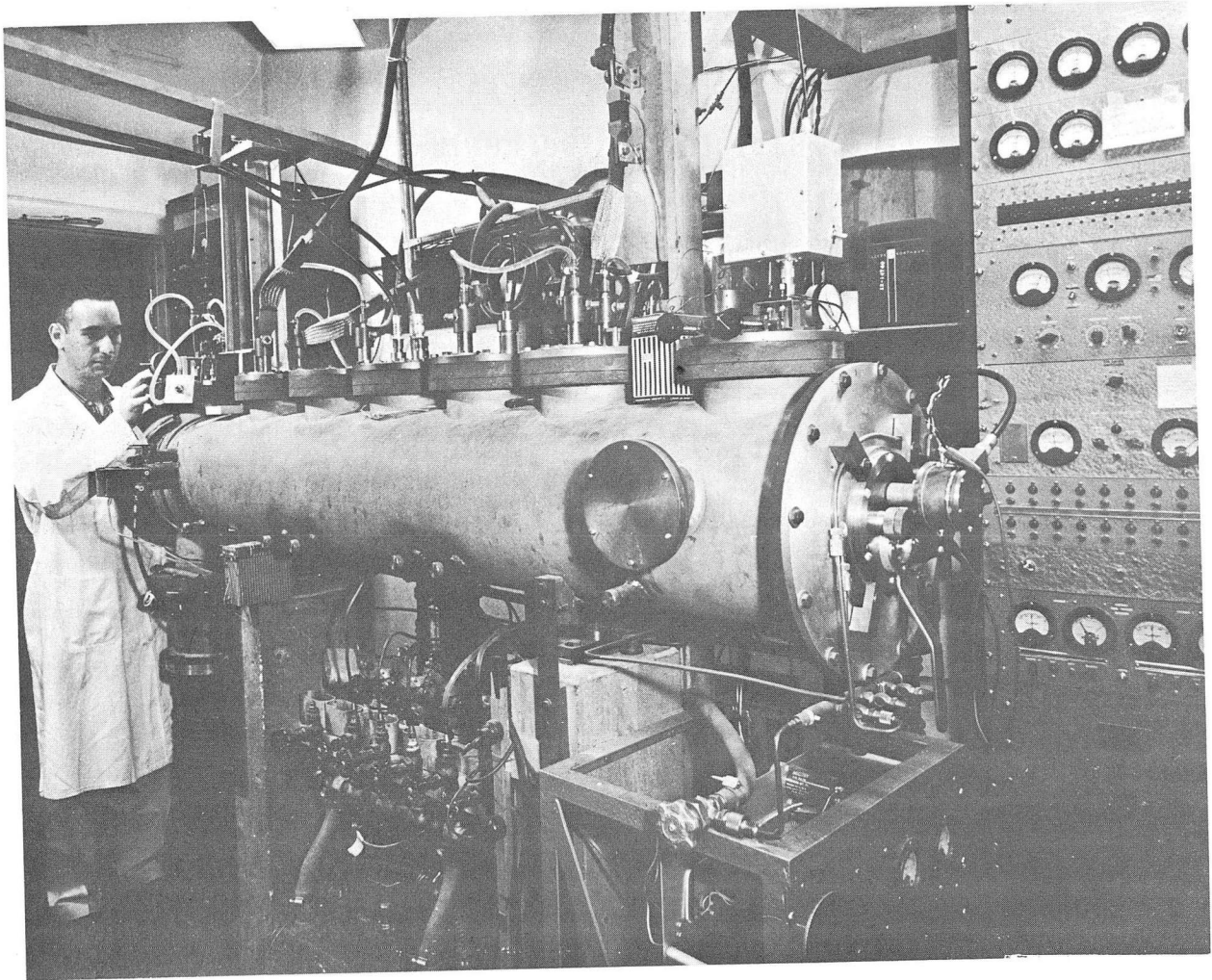


Fig. 5. Layout of vacuum tank and magnet.



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Fig. 6. Atomic beam apparatus.

The machine suspension system employs the familiar three-point method, using ball bearing expansion compensators. The machine is supported by two concrete pillars attached to the floor.

D. The Magnet System

The two deflecting magnets of the Rabi type and the homogeneous field magnet are of the low-impedance type. Figure 7 shows pole tip design. The A and B magnets are 17.5 in. long. A current of 350 amp at 4 volts is passed through the flattened copper tubing of the two magnets connected in series. The magnitude of the C field, in which the transitions occur, is determined by the requirements of the particular experiment. The C field is $15\frac{3}{8}$ in. long, with a gap width $\frac{1}{4}$ in. and a height of 2 in.

The total resistance of the 20 turns of flattened copper tubing is 0.0071 ohms, which at 350 amp can produce a field of 8000 gauss. In these experiments fields no larger than 500 gauss were employed, requiring only of the order of 35 amp. In the A and B magnets the only field requirement lies in the decoupling of the $\vec{I}$ and $\vec{J}$, such that the Paschen-Bach field dependences of the energy levels are approached. This condition is satisfied for all the alkali elements considered here.

A later section deals specifically with the stable resonances employed, using a frequency-modulated locking signal and a phase-sensitive detector, to stabilize the C field.

E. The Oven and Oven Loader

The ovens employed in the production of beams of neutral alkali atoms are shown in Fig. 8. The two ovens on the left show the latest models. With press-fit tops, the escape of the sample through the thread gaps of the right-hand oven is eliminated. The ovens are $7/8 \times 1 \times 1$ in. in dimension with two mutually perpendicular V notches in the bottoms to allow accurate location on three support pins. The slit width for the experiments was 0.005 in. with a jaw thickness of $1/8$ in. These ovens were mounted in the oven loader and lowered into the vacuum.

The oven loader is employed to introduce the oven, containing the radioactive and stable materials, into the oven chamber through a series of two intermediate vacuum regions. As the lifetime of certain radioactive samples is short (several hours), it is necessary to expedite all procedures.

The oven loader is shown in exterior view and in cross section in Figs. 9 and 10. The loader moves the oven past three O rings defining the boundaries of the rough (forepump) vacuum of a few microns, the fine (backing DP) vacuum, and the oven chamber. The process requires about 5 minutes. Generally, an additional 10 to 15 minutes is needed to allow the oven chamber to reach an operating pressure of 3×10^{-6} mm Hg.

The oven is positioned by screw adjustment with dial-indicator position monitoring. The two motions are translational (perpendicular to the beam axis) and rotational. The oven position is reproducible, upon removal and replacement, to within a few thousandths of an inch.

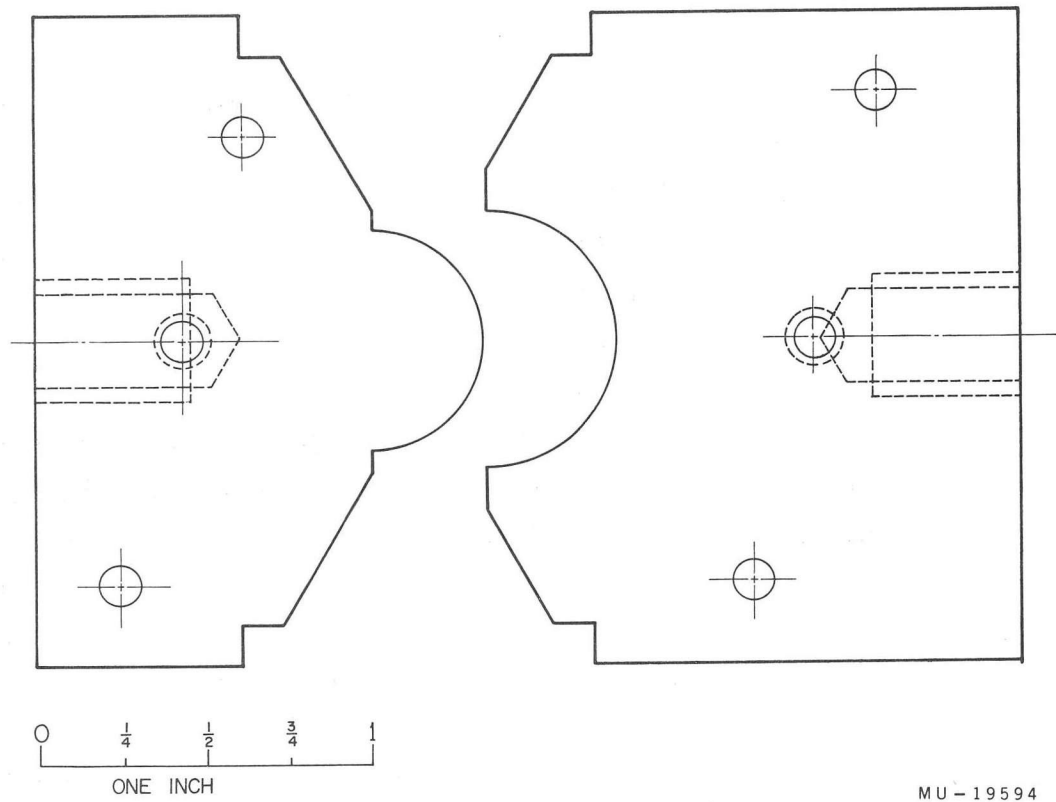
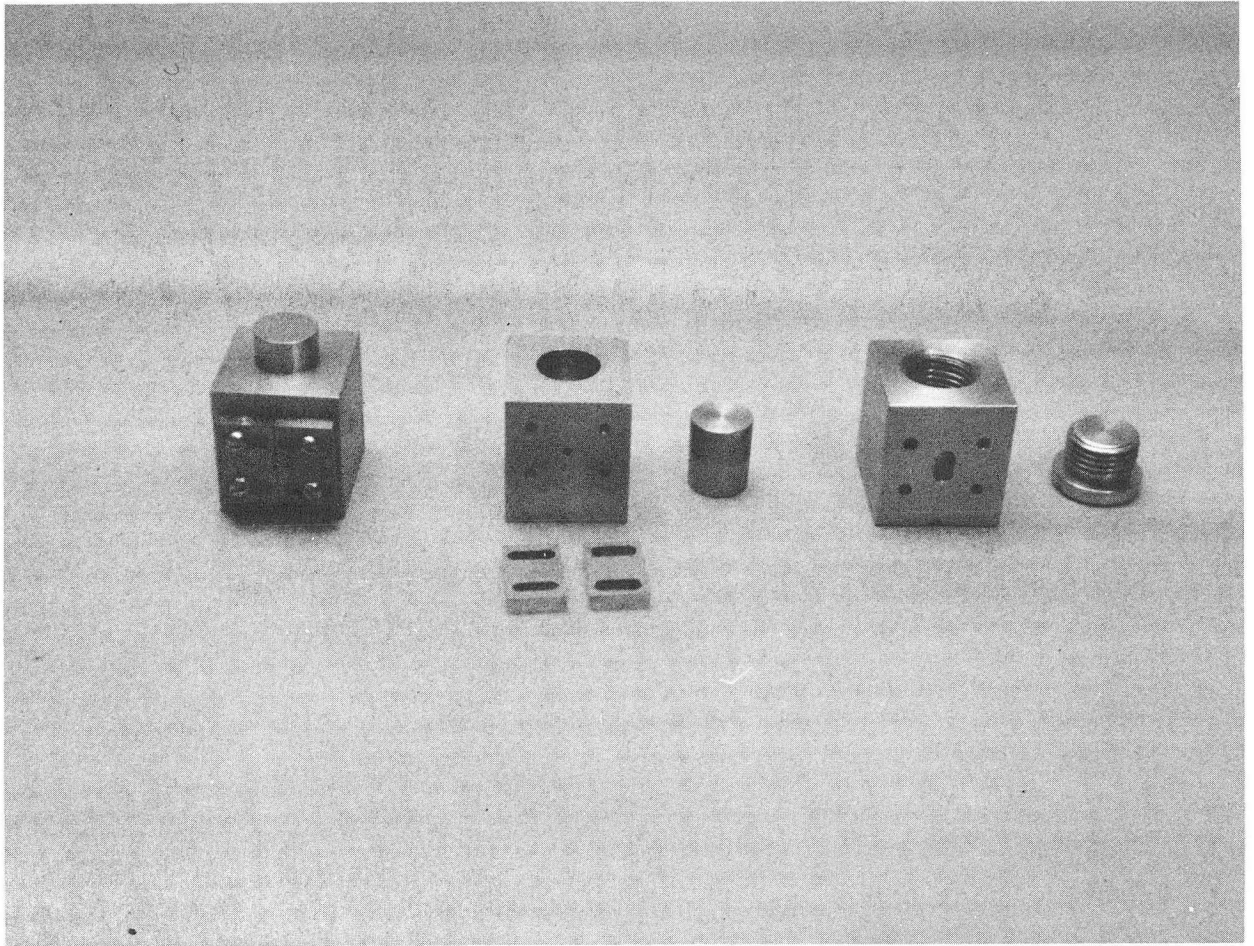
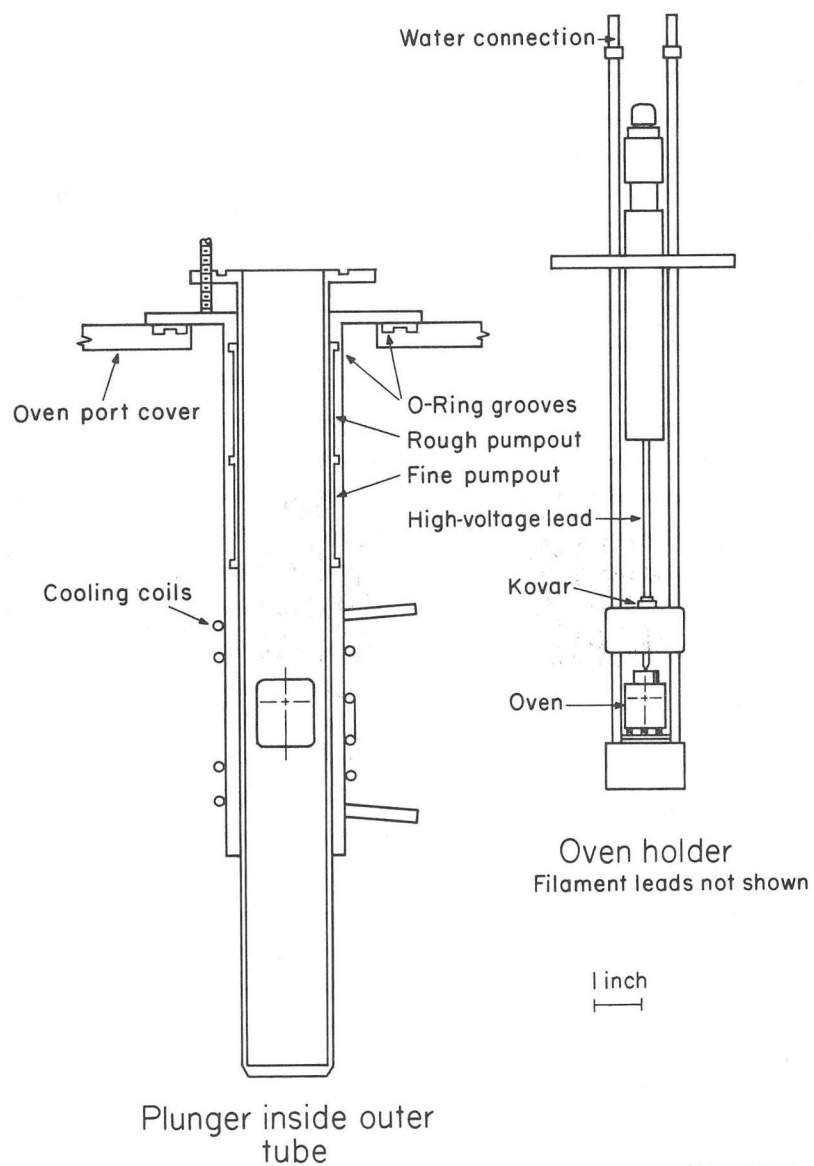


Fig. 7. Deflecting magnet pole tips.



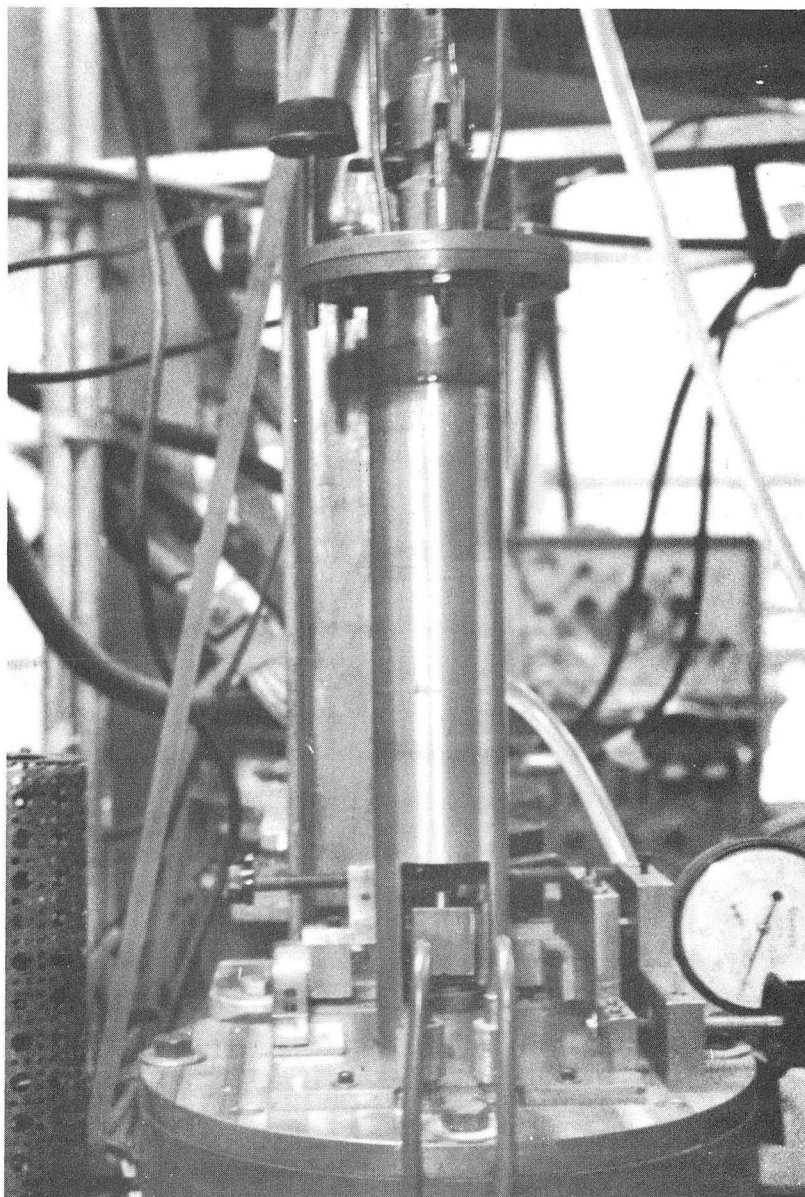
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Fig. 8. Stainless steel ovens.



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Fig. 9. Cutaway view of oven loader.



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Fig. 10. Oven loader .

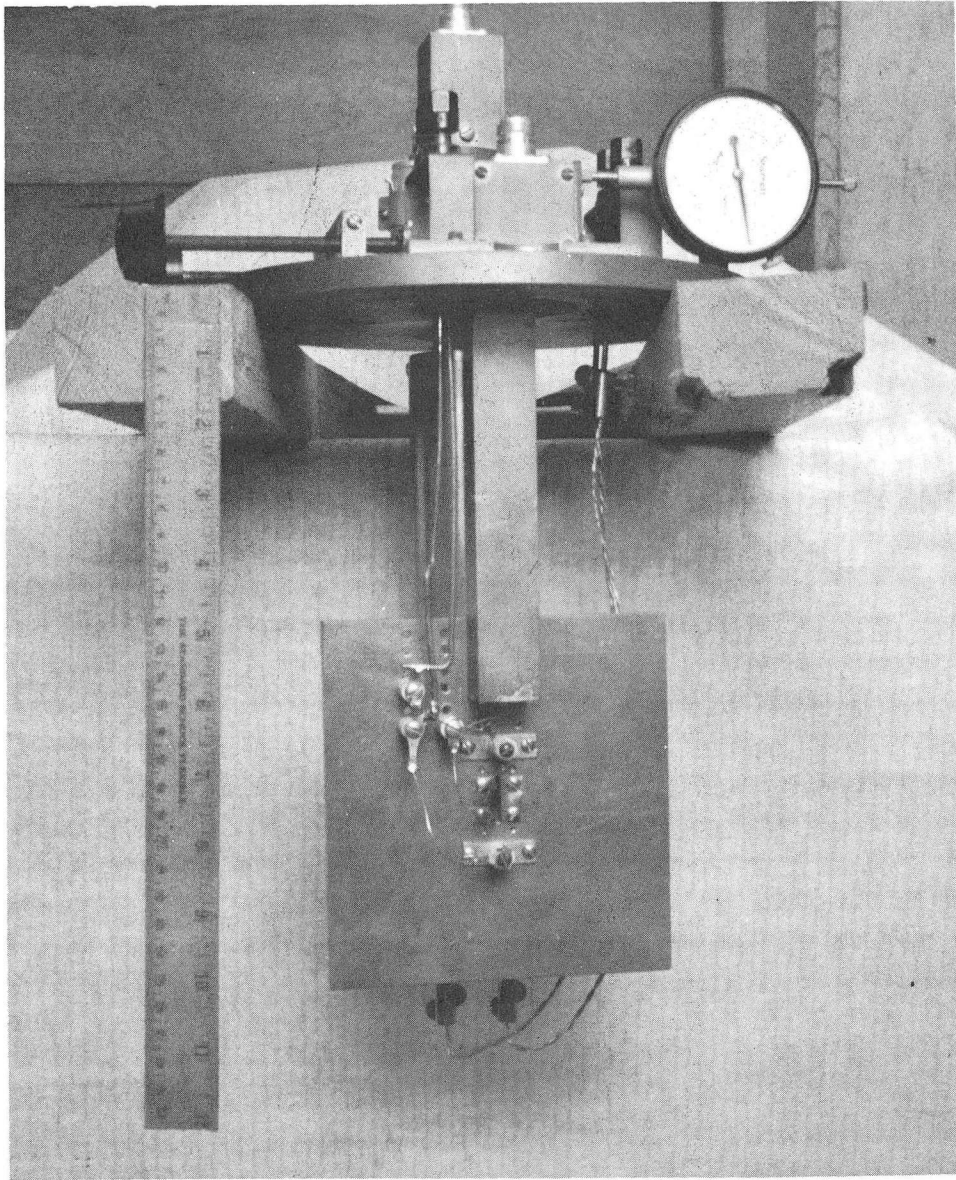
F. The Detector Assembly

In the experiments described here it is necessary to have two surface ionization detectors in operation during the full period of the experiment. In addition, the beam line-up procedure requires a third detector which can be placed at the focal point of the beam, normally occupied by the sulfur-coated brass buttons employed to collect the radioactive atoms refocused at resonance condition.

The first and second detectors are shown in Figs. 11 and 12. They are independently movable and are placed appropriately to their function. The first, closest to the B magnet, is called the monitor detector. This surface ionization detector intercepts the thrownout beam and has shown itself capable of monitoring the full beam level to within 10%.

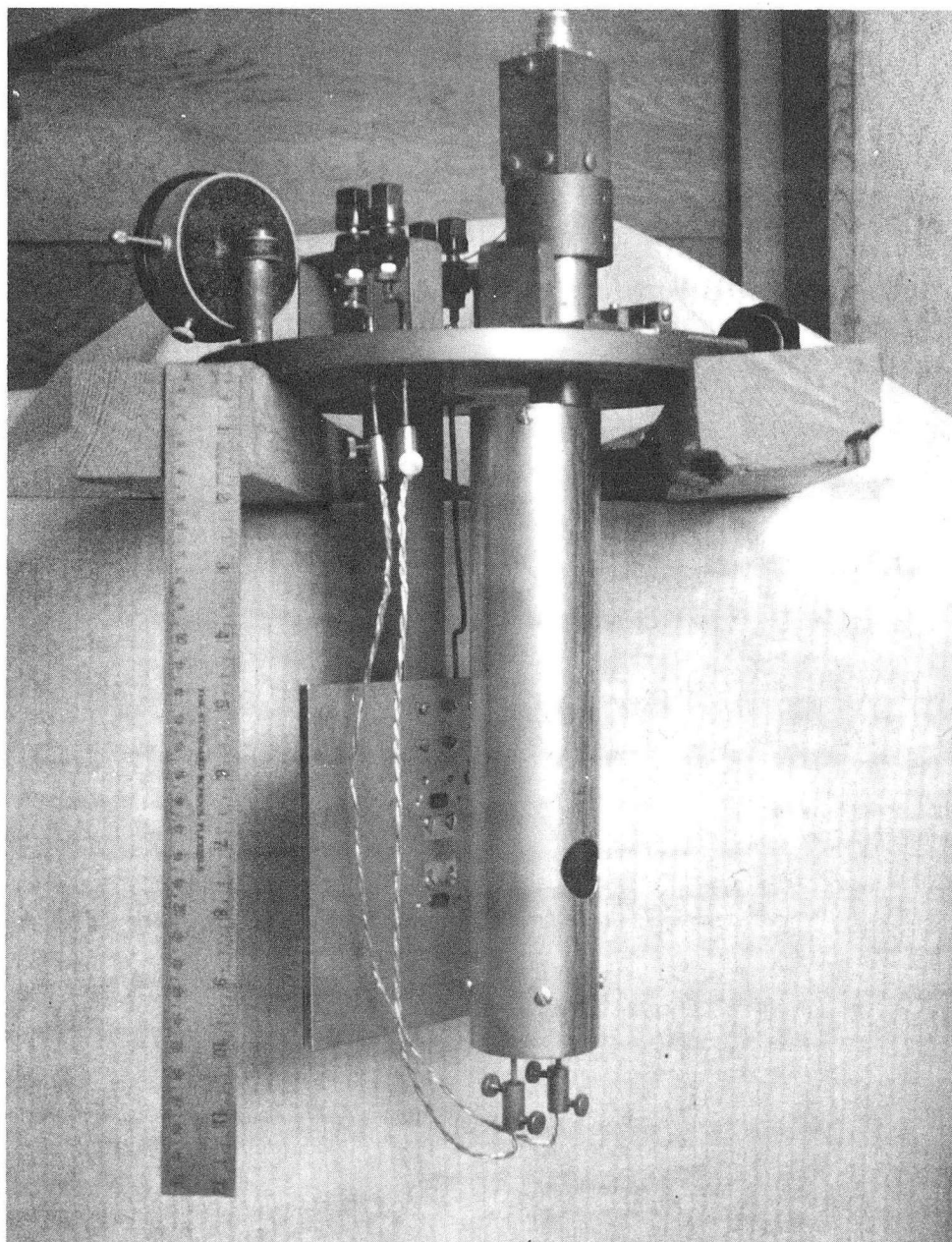
The second detector, called the locking detector, is mounted in the exit which defines the refocused beam. This detector is capable of observing refocused beams of resonance amplitude 1×10^{-12} amp in the stable isotopes. These resonances are employed in stabilizing the C field.

The third detector is located in the rotating button loader shown at the end of the atomic beam machine in Fig. 6. When in position, it allows alignment of the undeflected beam and assures that the refocused atoms will fall on the surface of the sulfur-coated buttons. The mechanism that houses the button loader ionization detector contains two button ports which allow the sulfur-coated brass buttons to be introduced into the beam path. The button passes through two intermediate vacuum chambers before entering the detector chamber. This allows minimum button-changing time, generally of the order of 2 minutes. In addition to the two button ports and the ionization detector there is a glass-windowed observation port which is used to check machine alignment.



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Fig. 11. Locking detector.



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Fig. 12. Monitor detector.

G. Radioactive Detection

It has been found that sulfur coatings are capable of retaining the alkali atoms deposited upon them in the evacuated detection or main chamber of the machine. (These buttons will be seen in Fig. 21). The buttons, upon removal after exposure, are placed in radiation detectors.

H. The Radio-Frequency Apparatus

Three functions must be performed. To establish a C field constant in time it is necessary to supply signals to the locking hairpin up to 5000 Mc/sec with a frequency-modulation band width up to a megacycle, which is stable to several parts in 10^5 . Next, the measurement of the magnetic field at the search hairpin, which is located in another portion of the C magnet, requires frequency generation up to several thousand megacycles. This is called the calibrating resonance measurement.

The resonance search requires frequencies up to 5000 Mc/sec with stability of one part in 10^8 . This system may also be used for the calibration when change-over time is only a few minutes.

The frequency generation has been accomplished by means of frequency multiplication and amplification. The oscillators used for the precision measurements were the Gertsch AM-1, 1 to 2 and 20 to 40 Mc; the Gertsch FM3, 500 to 1000 Mc, which could be locked to a James Knight Co. JKFS 1100 secondary frequency standard (the primary standard employed was a National Co. "Atomichron"). Other oscillators used for calibration purposes were the Tektronix 190A, 0.5 to 50 Mc, and the Hewlett-Packard 608A, 10 to 500 Mc.

By use of the above oscillators to produce the fundamental frequencies, arbitrarily high frequencies could be produced which were limited only by the transmission-line characteristics and amplifier ranges. These caused no difficulties in the range through 4000 Mc.

For amplification in the range 0.2 to 200 Mc the Hewlett-Packard 460A and 460B amplifiers and the Instruments for Industries model 500 amplifier were used. Above this frequency, traveling-wave tube amplifiers were employed, which were built around Sylvania and Huggins traveling-wave tubes.

The array of radio-frequency equipment used for each experiment is shown in Figs. 13, 14, 15, 16, and 17.

I. Hairpins

At medium and high frequencies the transmission-line termination, which provides the exciting oscillatory magnetic field, must be considered an integral part of the frequency-generating system. This comes about for frequencies whose wave length is comparable to the transmission-line length. Impedance matching, proper coupling, and reflection minimizing (from discontinuities near the termination) must all be considered carefully in producing the optimum radio-frequency magnetic field at the position of the atomic beam.

Two types of terminations or hairpins are shown in Figs. 18, 19 and 20. These are the strap, or single-oscillatory-field hairpin, and the Ramsey, or separated-oscillatory-field hairpin. Each of these has a characteristic shape of atomic beam resonance associated with it. The strap hairpin produces a single peaked resonance whose half width can be related to the thermal velocity of the beam atoms by the relation (at optimum oscillatory field strength).

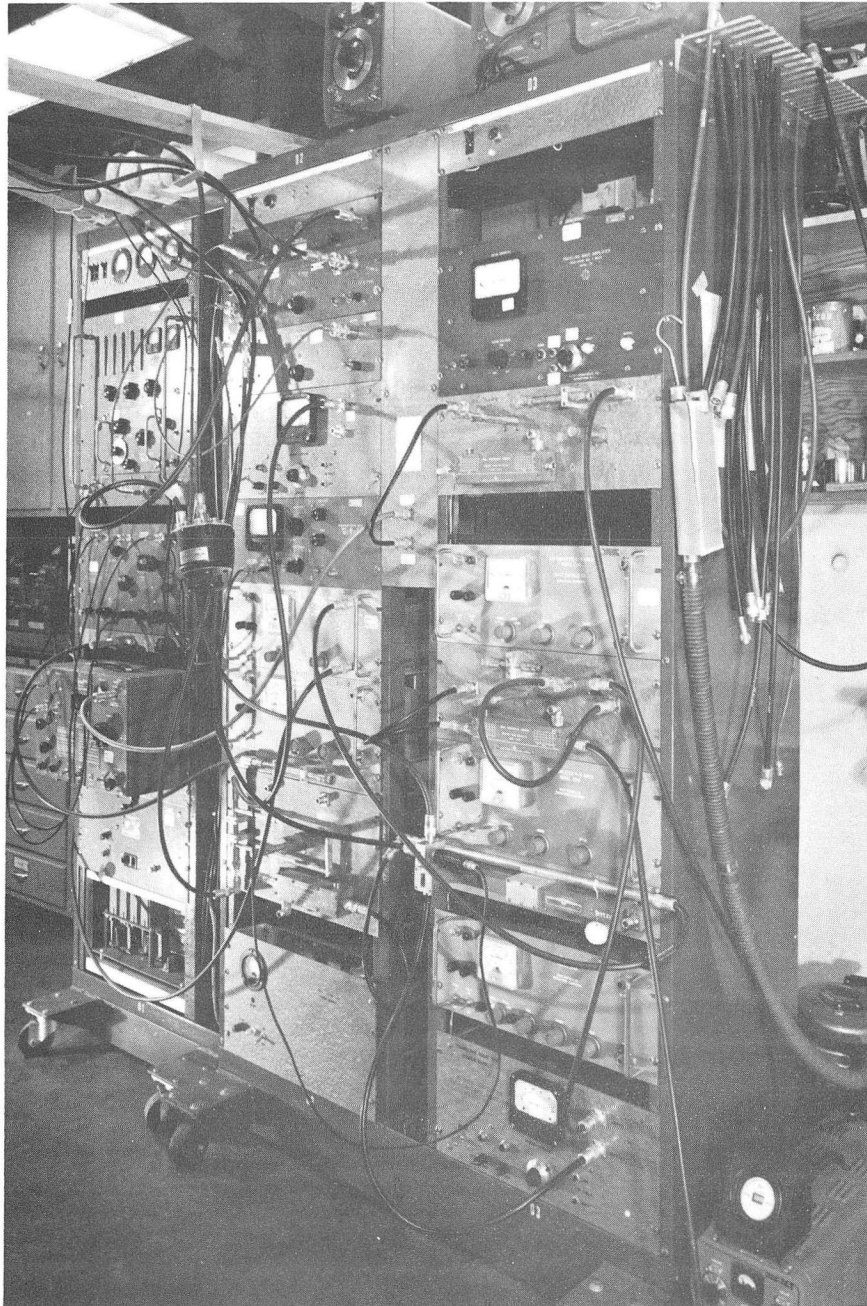
$$\text{Full width at half resonance} = 1.072 \, a/\ell ,$$

where

$$a = \sqrt{2kT/M} ,$$

$$\ell = \text{length of oscillatory field.}$$

The Ramsey hairpin has associated with it a broad resonance pedestal upon which occurs an interference-type pattern. If the radio-frequency signals of the two oscillatory regions are in phase (0 deg phase difference), there occurs a three-peak interference pattern whose central



ZN-2382

Fig. 13. Radio-frequency relay rack.

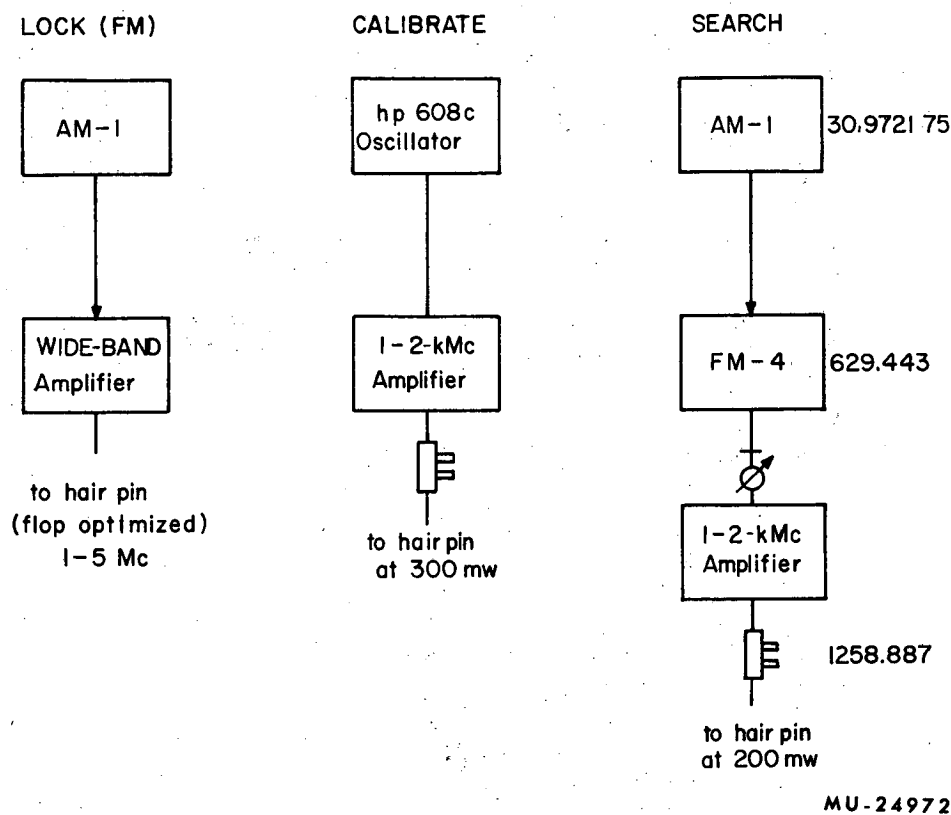


Fig. 14. Frequency generation for $\Delta \nu$ search, K^{42} ; K^{42} : $H \approx 1.4$ gauss. $\Delta \nu$ search.

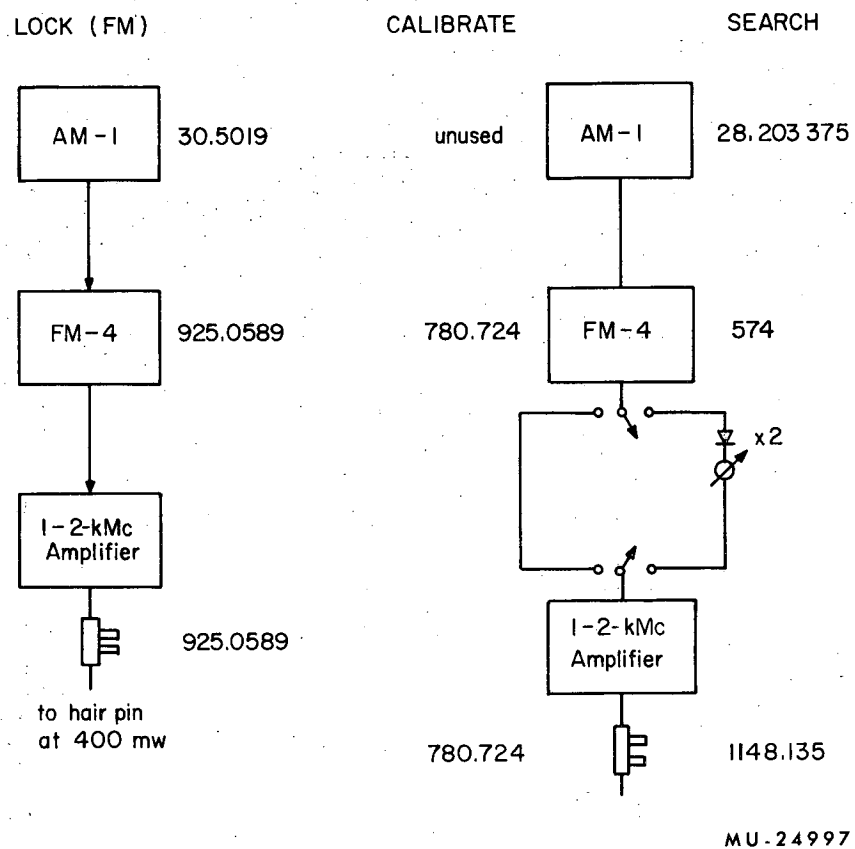


Fig. 15. Frequency generation, ν_+ and ν_- for K^{42} : $H = 188.9$ gauss Moment search.

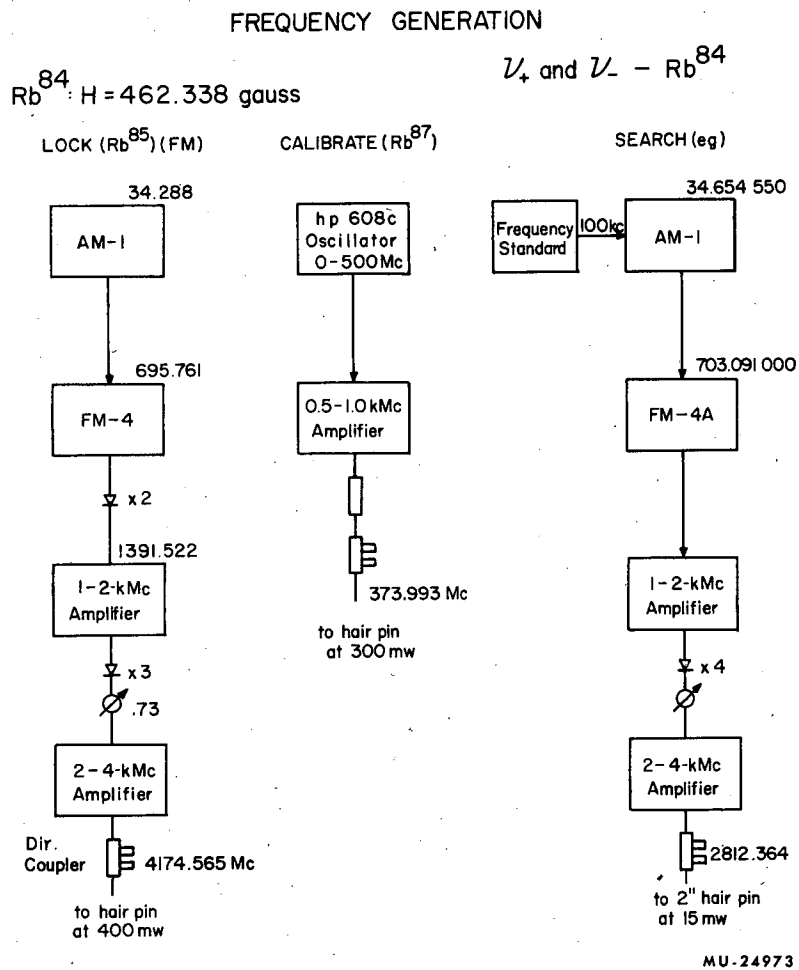
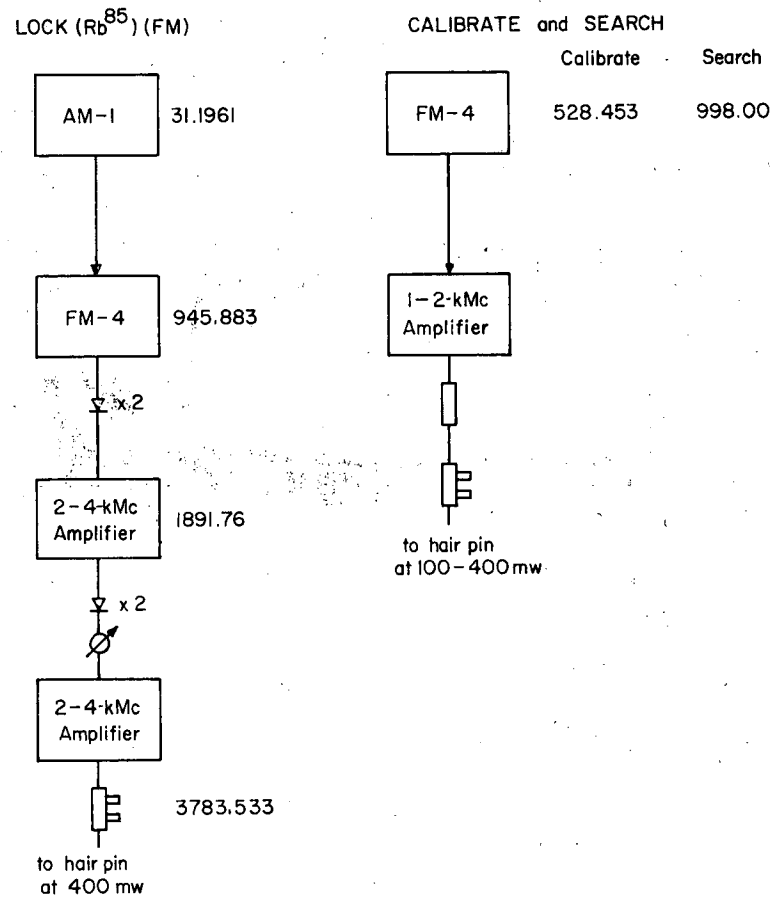
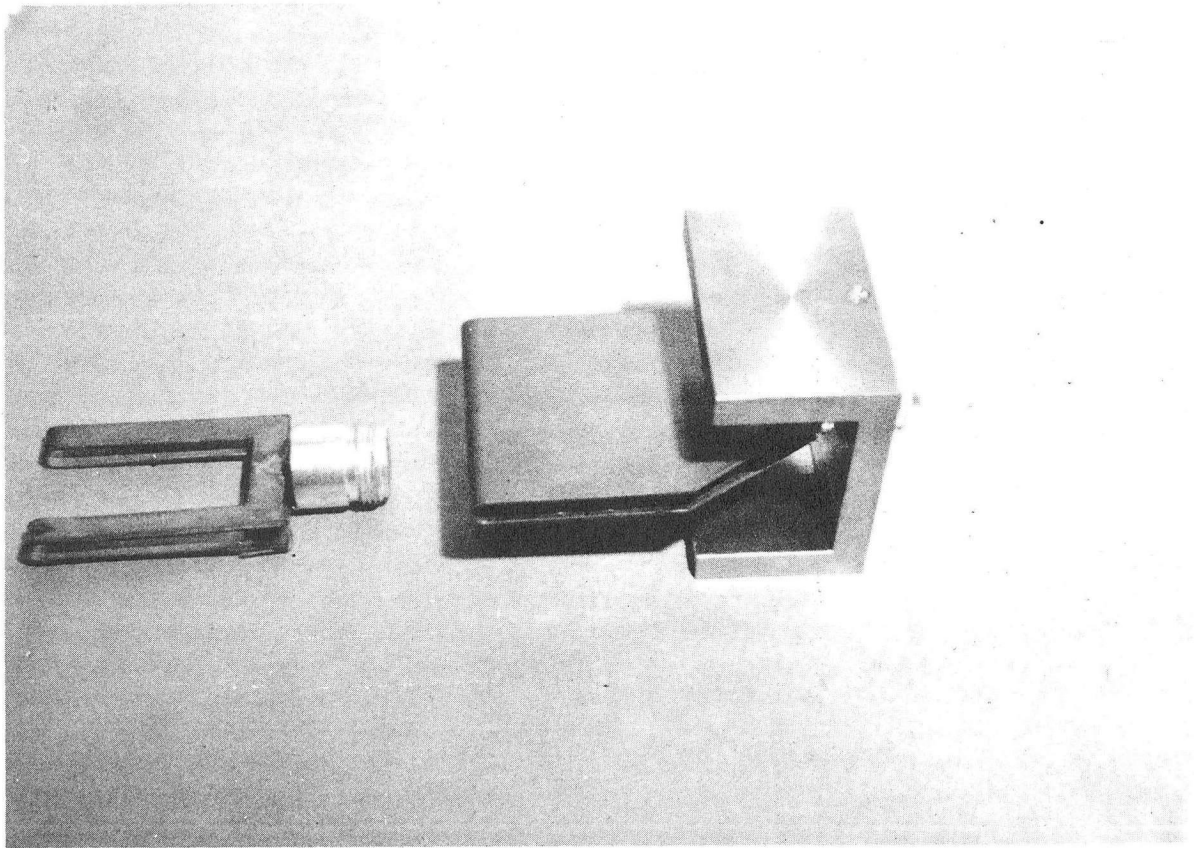


Fig. 16. Frequency generation, ν_+ and ν_- for $Rb^{84}: H = 462.338 \text{ gauss}$.



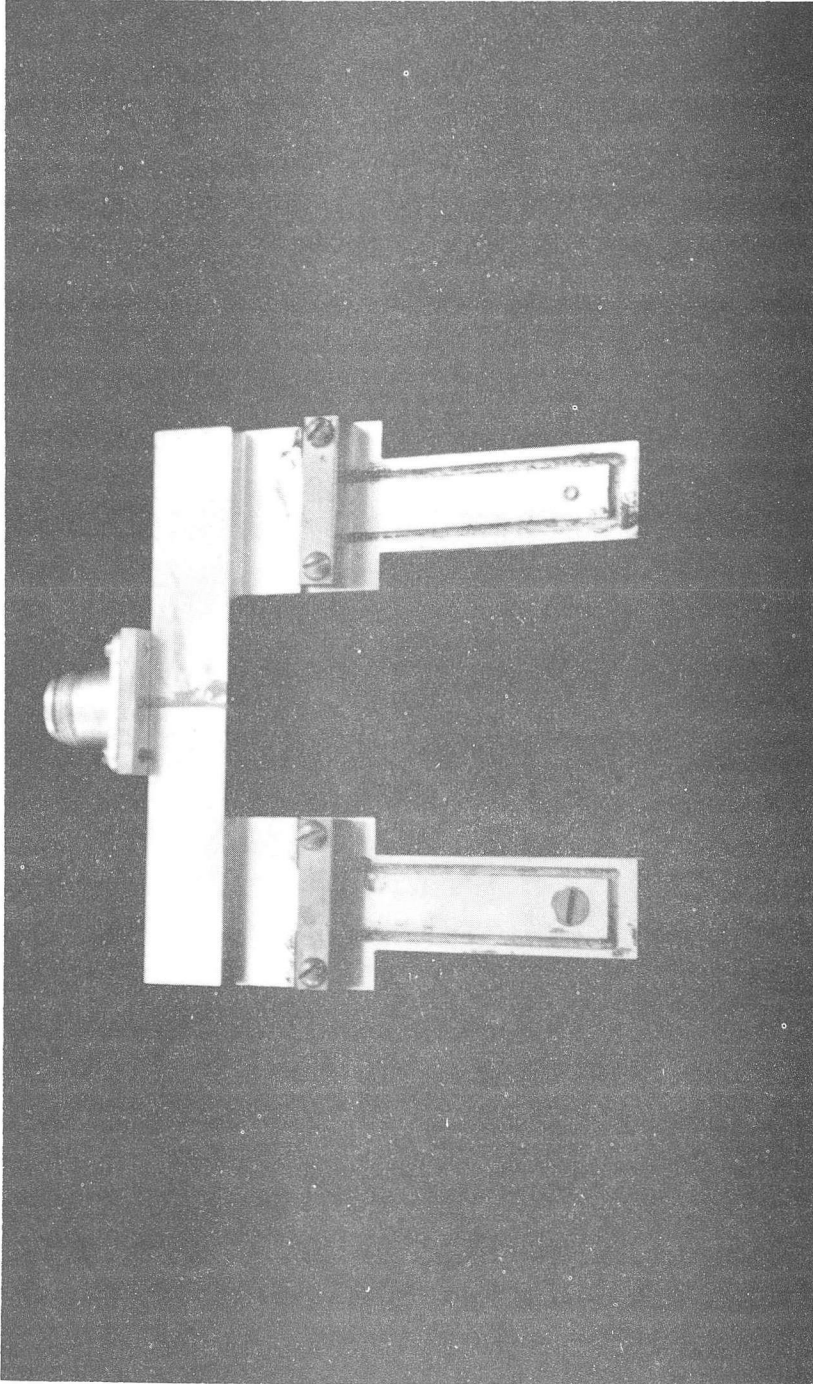
MU-24974

Fig. 17. Frequency generation for
Cs¹²⁹: H = 649.515.



ZN-2386

Fig. 18. Two-inch strap hairpin and early Ramsey hairpin.



ZN-2389

Fig. 19. Ramsey hairpin.

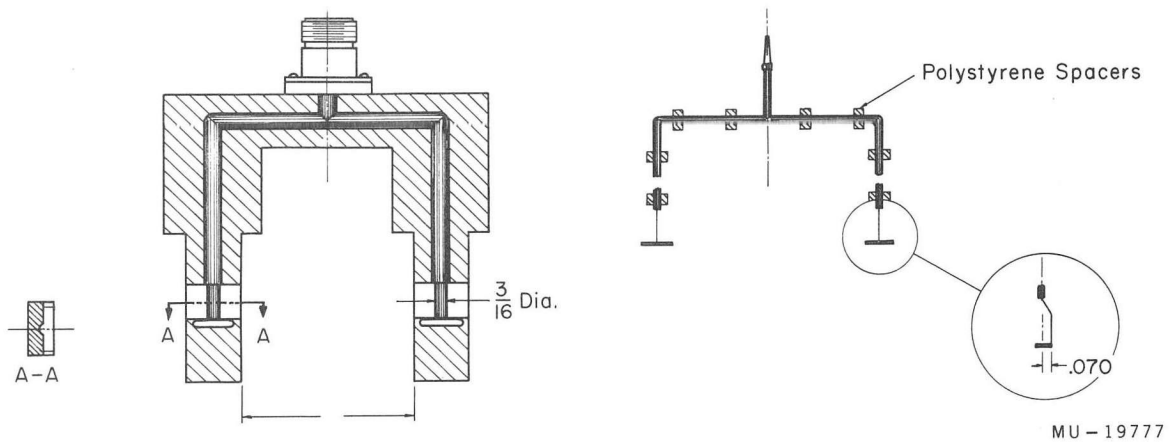


Fig. 20. Ramsey hairpin-details.

peak is larger than the two secondary peaks. The width of the pedestal is related to the length of the individual oscillatory field regions, and the central peak width is related to the separation of the oscillatory field regions (at optimum field strength) by

$$\text{Full width at half resonance} = 0.65 a/L .$$

J. Stabilization System for the C Field

This part of the experimental apparatus was designed and perfected by Dr. Gilbert O. Brink and Norman Braslau.⁴⁰ The signal impressed upon the locking hairpin was of the form

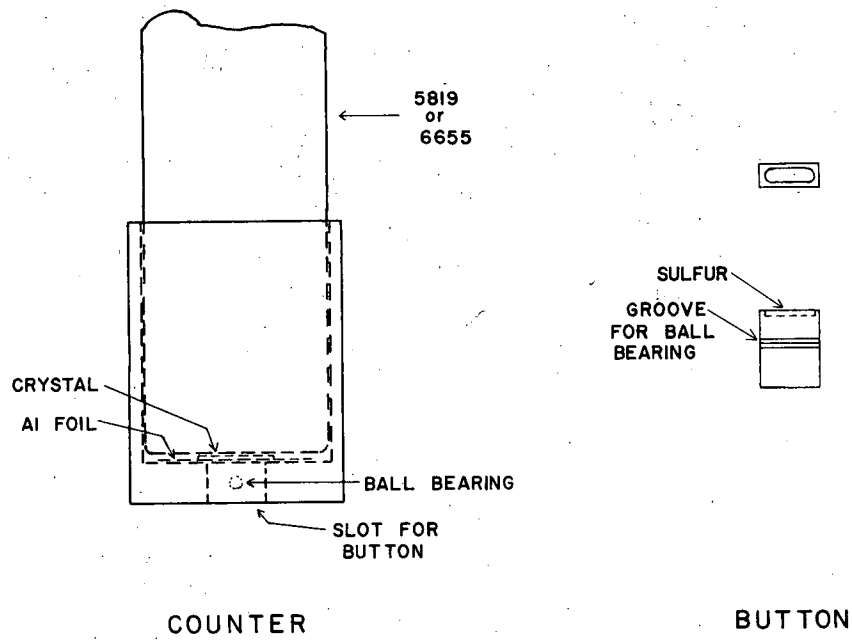
$$\nu = \nu_0 + \nu_1 \cos (2\pi\nu_2 t) .$$

Here ν_0 might be any value from 1 to 5000 Mc; ν_1 was optimized for the particular resonance but was on the order of the line width, being between 0.1 and 1 Mc. The modulation frequency, ν_2 , was 100 cps.

The signal from the locking ionization detector is fed into a phase-sensitive detector which compares the signal with the 100-cps reference. The error signal from the phase-sensitive detector is then fed to the C magnet current regulator containing two Delco 2N277 transistors which add an incremental current to that from the main 4-volt submarine batteries. By this means, the C fields can be kept constant to several parts in 10^5 .

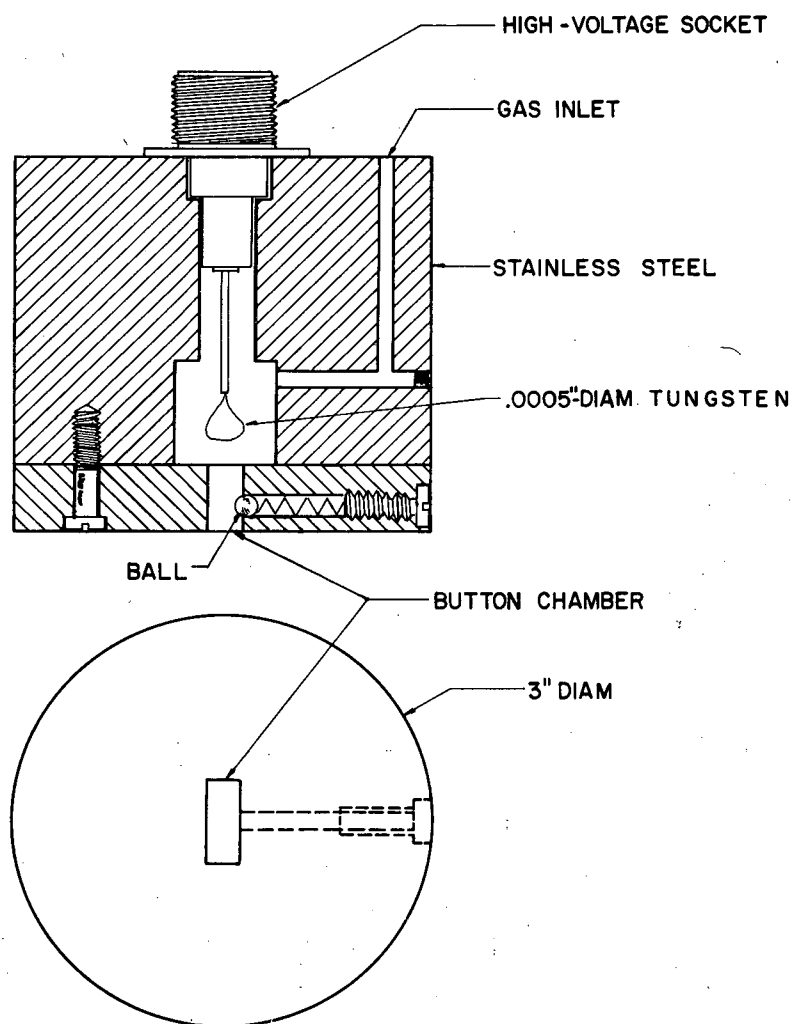
K. Radiation Detection Systems

The determination of the resonance condition for radioactive beams requires that the refocused atoms deposited on the buttons be counted by assaying the decay radiation. Two sets of detectors have been used in these experiments. For γ -radiation detection, thallium-activated NaI crystals combined with photomultiplier, pulse-height discriminator, and scaler were used. They can be seen in Fig. 21. For β -particle detection, proportional-flow counters were used. These are shown in Fig. 22.



MU-12676

Fig. 21. Scintillation counter and button.



MU-17401

Fig. 22. β -counter head.

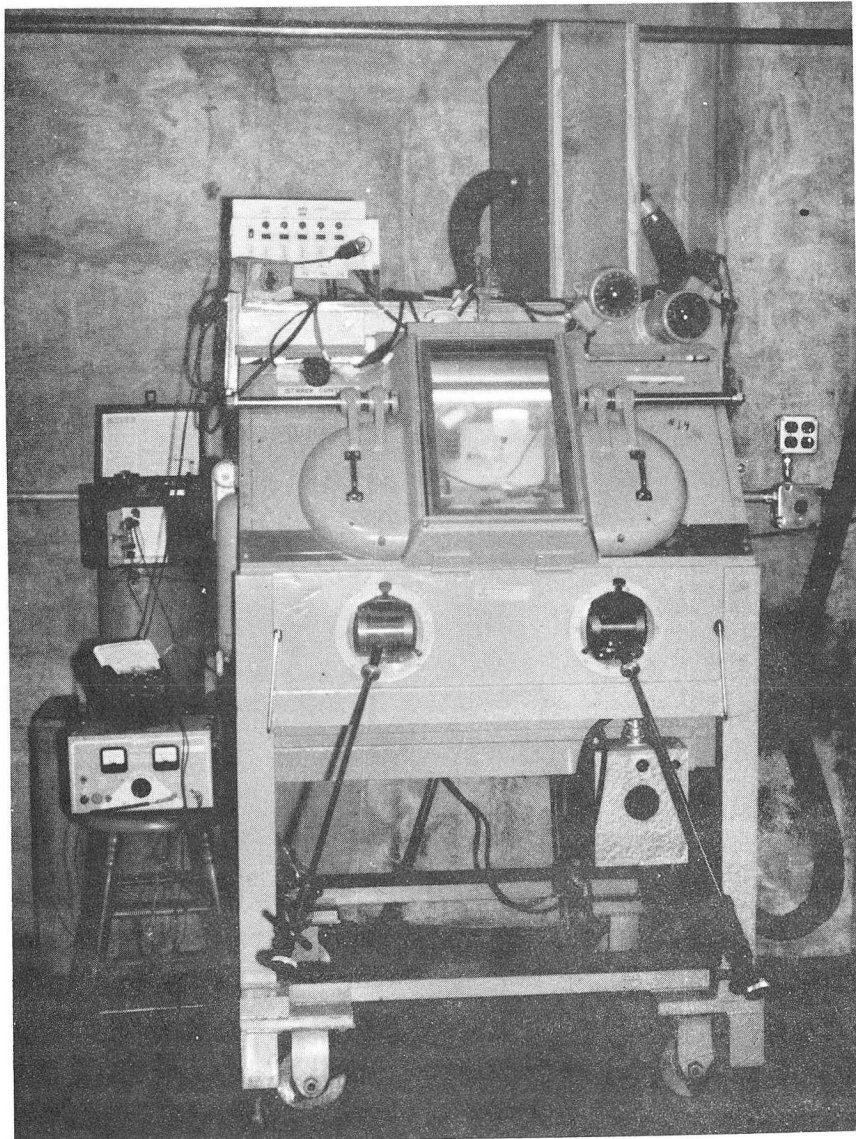
V. EXPERIMENTAL PROCEDURE

The experimental procedure (excluding source preparation) can be broken up into steps, as follows.

1. Transfer of reactant into atomic beam oven.
2. Transfer of oven into atomic beam machine.
3. Production of the beam.
4. Check of oven alignment.
5. Measurement of specific activity of beam
(cpm/(min. of exposure)(stable beam level)).
6. Degaussing of C magnet.
7. Observation of the Majorana transition (zero-field refocusing)
8. Changing magnetic field to desired value.
9. Observation of locking transition.
10. Adjustment of lock-in system for optimum error signal.
11. Calibration of magnetic field (altering locking frequency to obtain proper field if necessary).
12. Exposure of sulfur-coated brass buttons at zero rf and at search frequencies.
13. Transporting of buttons to counting room.
14. Counting of activity on buttons.
15. Feedback of counting-rate information to experimenter.
16. Recount of buttons after experiment, if necessary, for improvement of statistics or to establish decay constant.

The chemistry of radioactive isotopes is carried out in shielded boxes (shown in Fig. 23) known as "caves." With the aid of a Health Chemistry monitor, the oven is next transferred to the atomic beam machine.

The oven is placed upon three supporting pins in the cylindrical oven loader shown in Fig. 10. The oven is then lowered into the main vacuum chamber through two intermediate-pressure chambers pumped to 80μ and 10μ respectively. The oven is then ready for outgassing and the final heating for beam production.



ZN-2366

Fig. 23. Cave for handling radioactive materials.

In the production of alkali beams low heating power is required. A 0.010-in. thoriated tungsten filament at 1/8 in. from the front face of the oven is heated with a current of 12 amp at 10 volts. Generally it is necessary to place a potential of some 100 volts between the filament and the oven and electron-bombard the oven. The electron bombardment power is generally from 3 to 25 watts under normal circumstances. The heating process requires from 20 minutes to 1 hour owing to the rapid outgassing of the oven and walls of the oven loader.

When a beam is observed initially, the deflecting magnets are off. The stop wire and beam flag are also withdrawn so that there is an unobstructed path for the atoms from the oven to the detector end of the atomic beam machine. In order to align the oven to be collinear with the axis of the atomic beam machine, the stop wire (whose position is known to be on the axis) is lowered into the path of the beam. Next the oven position is adjusted so that the image of the stop wire lies in the center of the beam pattern. The beam pattern is defined by the oven (line source) and the fixed collimating slit located at the center of the C magnet.

After the oven is aligned, one then exposes a sulfur-coated button to the unobstructed beam for 5 minutes. This button is subsequently assayed and an indication of the specific activity is given which can be used to predict the amplitude of the resonance signal being sought and to correlate successive experiments.

The next procedure is to cycle the C magnet through hysteresis loops of successively smaller area until one obtains the zero-field condition. There is a fringe field impressed by virtue of the placement of the A and B magnets. To obtain a state of zero field, one must impose a current upon the windings of the C magnet such that H_c is opposed to the direction of the fringe fields from the A and B magnets. The indicator of the zero-field state is the so-called Majorana flop, which is due to a random repopulation of the states in the C-field region when the reference axis of quantization is removed.

One can then use the calibration of field vs C magnet current (assuming the "degaussed" magnet history) to place the magnetic field value very close to the desired field ($\pm 5\%$).

The magnetic field is fine-tuned to observe the locking transition. When any ambiguities in transition identity are removed, one closes the servo loop and uses the output of the phase-sensitive detector to lock the field control to the resonance peak. The locking characteristics are optimized by varying the width of the resonance (by power change), by changing the time constant and noise associated with the hot-wire surface ionization detector (by filament current change), by changing relative phases of input vs reference and band width of frequency modulation signal fed to hairpin.⁴⁰

For the typical experiment two radio-frequency hairpins are used simultaneously. One of these, usually a strap, feeds in the modulated locking signal. The second serves to introduce the calibration and search frequencies successively. The calibration frequency (the frequency of an observable resonance in a stable isotope whose field-vs-frequency relation is known) is introduced into the same hairpin as the search frequency in order to establish the field at that location. (The radio-frequency-generating systems for each of the isotopes is given at the end of the section on experimental procedure.)

The procedure for detection of refocused radioactive atoms employs a sulfur-coated brass button which is exposed to the beam. The first button exposed during the run observes only background atoms, as the rf is off. This establishes a base line, which may be repeated throughout the run at the experimenter's discretion. Successive buttons are exposed at fixed frequencies for a time determined to give a reasonable compromise dictated by resonance counting rate, apparatus stability, and total predicted length of experiment. Typical exposure periods lie between 5 and 15 minutes.

The exposed buttons are removed from the apparatus and delivered, by means of pneumatic tube, to a counting room physically located some 400 feet away from the atomic beam machine. The

radiation from the atoms deposited upon the sulfur is detected by either scintillation or proportional-flow β counters, depending upon the nature of the radiation. The information of counting rate and its uncertainty are then communicated from the counting room to the experimenter by the intercommunication system.

VI. RESULTS

A. General

The results of the experiment upon four radioactive alkali atoms are as follows

Cs^{127}	(6 hr)	$\Delta \nu = 8900(150) \text{ Mc}$
Cs^{129}	(31 hr)	$\Delta \nu = 9229(30) \text{ Mc}$
Rb^{84}	(33d)	$\Delta \nu = 3083.159(4) \text{ Mc}$
K^{42}	(12.5 hr)	$\Delta \nu = 1258.877(4) \text{ Mc}$

For each isotope there is a brief discussion in which the observed resonances are presented together with the experimental conditions. In the following, the criteria of analysis are presented and the experimental errors considered in turn.

In order to estimate the validity of the results of any experiment it is necessary to know the characteristics of the apparatus, the procedure employed, and the systematic errors. The final tests of the complete measurement process are whether its results are consistent with measurements made previously by other experimenters, and whether its results are internally consistent.

The apparatus includes the following units: (a) vacuum system with source, deflecting magnets, and detectors; (b) frequency-generating equipment; (c) current supply for deflecting and transition-region magnets; (d) electrometers for electrical detection system; (e) a feedback system for maintaining a constant magnetic field in the transition region, including oscillators, hairpin, phase-sensitive detector, and current-correcting attachment to transition-region magnet current supply.

These individual components have been described earlier in this presentation and are described in detail in a previous article.¹⁶ Their characteristics may be summarized as follows.

- The frequency generation is stable to $1/10^8$.

- The deflecting-magnet supply (submarine batteries) has a drift of about $1/10$, but does not affect the transition-region magnetic field except through fringe effects, which can be determined and allowed for by periodic calibration of the uniform field.

- The homogeneous magnet lock-in system has a maximum variation of $5/10^5$ in field over the period of the experiment.

- The continuous electrical monitoring of stable carrier beam gives an amplitude normalization that is accurate to within 5%--which is less than the statistical uncertainty of the amplitudes as determined by radioactive detection.

Two of the quantities (frequency and field) mentioned above are directly measurable and are unambiguously known.

The sources of error to be considered next are less definable, but we can evaluate their magnitudes by noting their characteristics and applying various tests. Five known effects associated with the hairpins and fields which can produce displaced or distorted resonances are: Bloch-Siegert effect, Millman effect, Doppler effect, non-uniformity of field, and phase-shift effect.

At this point it is useful to consider the two types of hairpins used in the experiments presented here. The first type is the single-strap hairpin. This was used to investigate the rubidium and cesium isotopes. For potassium, in order to increase the precision of the measurement, a separated-field hairpin was used. This consists of two strap hairpins $1/8$ inch long separated by 2 inches. The first four error-producing effects are present in both types of hairpin. The phase-shift effect is significant only in the separated-field hairpin.

1. Bloch-Siegert Effect

The general solution of the time-dependent Schroedinger equation is obtained by assuming that the perturbing magnetic field is rotating about an axis in space with a frequency ω .

If one replaces the rotating field with one oscillating in a plane perpendicular to the axis of rotation, the time-dependent Schroedinger equation must be solved with two rotating components of frequency, ω and $-\omega$. The result, given in Eq. (II-8), is

$$\omega_r - \omega_0 \approx 1 \text{ cps,}$$

which is seen to be no more than 10^{-3} of the line width of the resonances considered.

2. Millman Effect

This effect is due to any changes in direction of the oscillatory rf magnetic field along the particle path. This can produce an explicit rotating component which will add to one or the other of the rotating components of the oscillatory field. The effect can cause an incremental frequency shift of $\nu(\pi/2)\Delta\ell$, where $\Delta\ell$ is the distance over which the change of direction occurs.

In the experiments presented here this effect has been greatly reduced by use of a strap hairpin, and is not observed.

3. Doppler Effect

If there is a standing-wave pattern in the hairpin, it can be resolved into two running waves traveling in opposite directions. Each of these can cause transitions. The Doppler effect gives the apparent frequency of each of these components as seen by a moving atom. The velocities of the running waves are $+c$ and $-c$, where c is the velocity of light.

The general expression for the two apparent frequencies is

$$\nu = \nu_0 (1 \pm v \cos \theta) / c (1 - v^2/c^2)^{1/2},$$

where ν_0 is the frequency impressed, v is the velocity of the atoms in the beam, and θ is the angle between the beam path and the standing-wave plane. The denominator gives the effect of time dilation if the particle is moving with a velocity comparable to the speed of light. This term is smaller by several orders of magnitude than the accuracy required, i. e., 10^{-9} .

To estimate the maximum shift we note that the atomic velocity is approximately 5×10^5 cm/sec:

$$v/c = 1.5 \times 10^{-5},$$

$$\theta = 0 \text{ deg},$$

$$\nu = \nu_0 \pm 20 \text{ kc for } \nu_0 = 3.000 \text{ Mc.}$$

If both running-wave components are equal in amplitude, the observed resonance is broadened symmetrically. However, if one has greater amplitude than the other, the effective resonance peak is shifted from the Bohr frequency.

This effect is significant only when standing-wave patterns are possible, i. e. when the hairpin length is comparable to the wave length of the input signal.

4. Nonuniformity of Field

The effect of a nonuniformity of field is to allow the Bohr condition to be satisfied by several frequencies. With a single-strap hairpin this produces an observed resonance that is in general unsymmetrical. For a separated-field hairpin, this effect may not be apparent. This hairpin requires only that the average of the magnetic field be equal to that of the fields at the straps. Again, this may be detected by asymmetries in the resonance.

5. Phase-Shift Effect

This effect produces peculiarly shaped resonances. In the single-strap hairpin, this is observed only in long hairpins--that is, when the hairpin length is comparable to the wave length of the radio-frequency signal. For the separated-field hairpin the effect is more complex. The signals at the two straps bear a definite relationship. When the phase in the second leads or lags behind the phase of the first by either 0 deg or 180 deg, the Bohr frequency corresponds to a maximum or minimum respectively, but at 90 deg corresponds to the position of the center of a dispersion curve. Thus, at phases other than 0 deg (for which the hairpin used here was redesigned), the resonance peak is shifted. The shift has been estimated to be $\omega(\theta/2\pi)$, where ω is the width of the resonance, and θ is the phase difference (assumed small).

To summarize, for the single-strap hairpin, we can eliminate all the sources of error except the nonuniformity of field. The Bloch-Siegert effect is at most a few cycles, compared with a line width greater than 5 kc. The Millman effect is eliminated by virtue of the strap hairpin design. The Doppler effect is negligible, as the wave length is greater than 30 cm with a hairpin length of only 1 cm or so. Likewise, the phase shift over the length of the hairpin can only be very small. To estimate the effect of field nonuniformity, the width of the line and its shape must be used as the only criterion aside from comparison of the results with those obtained from other machines.

For the separated-field hairpin, effects 1 to 3 are small, by previous arguments. The nonuniformity of field will be manifested by an unsymmetric resonance. The last effect, which can be serious under some circumstances, is the phase difference between the signals at the two strap.

The error effects in the separated-field hairpin can be subjected to another test. This is a reversal of orientation of the hairpin. Those frequency shifts which are a function of the geometry of the hairpin (Bloch-Siegert, Millman phase-shift, and Doppler effects) will be reversed upon reversal of the hairpin. This test has been applied to the resonance data of potassium-42; the resulting conclusion is that these shifts must be less than the experimental error.

B. Computer Aids to Experiments and Analysis

The two IBM computers that were available were the 650 and 704, both digital computers. In applications to atomic-beam experiments, programs have been written by members of the Berkeley atomic beam group to shorten the lengthy processes of energy-level calculation and data processing. The programs employed in this work are as follows.

1. OMNIBUS Routine

This routine, written by Professor H. B. Silsbee, contains these operations:

- a. Decomposition of a decay curve into components of chosen half lives, and extrapolation of population number back to a common time origin.
- b. Fitting the points of a resonance to a bell-shaped curve, giving the central frequency and the χ^2 of the fit.
- c. Calculation of the hyperfine-structure separation, when given a resonance peak frequency for a transition, the appropriate constants for the isotope, the field calibration frequency for a known isotope, and the uncertainties in peak position and calibration frequency.

2. F-TABLE Routine

This routine was written by Dr. W. B. Ewbank. It calculates the Breit-Rabi formula solution for the energy of any given level and the frequency corresponding to the difference energy between any two levels, when given the hyperfine structure separation, nuclear spin, nuclear magnetic moment, and electronic g factor.

C. Cesium Experiments

1. Identification

Two cesium isotopes were present in comparable abundance in the samples studied. These are Cs^{127} and Cs^{129} . Care must be taken to identify these, as their nuclear spins are the same ($1/2$) and their hyperfine-structure separations are very nearly the same.

In a preliminary experiment refocusing transitions were observed in both isotopes, at approximately the same frequency. To distinguish between these two, their different decay rates proved useful. A second assay of the counting rate of each sulfur-coated button was made 24 hours after the first. The results are shown in Fig. 24, which is consistent with the identification of Cs^{129} , as having the larger hyperfine-structure separation of the two, and a 31-hour half life. Another resonance in Cs^{129} is shown in Fig. 25.

2. Transition and Hairpin

The $\Delta F = 0$ refocusing transition between M levels 0 and -1 of the $F = 1$ group is observed. The approximate expression for the frequency of this transition is given by

$$\nu = 1.40 H_c + (1.40 H_c)^2 / \Delta \nu ,$$

which can be used to evaluate the $\Delta \nu$ approximately.

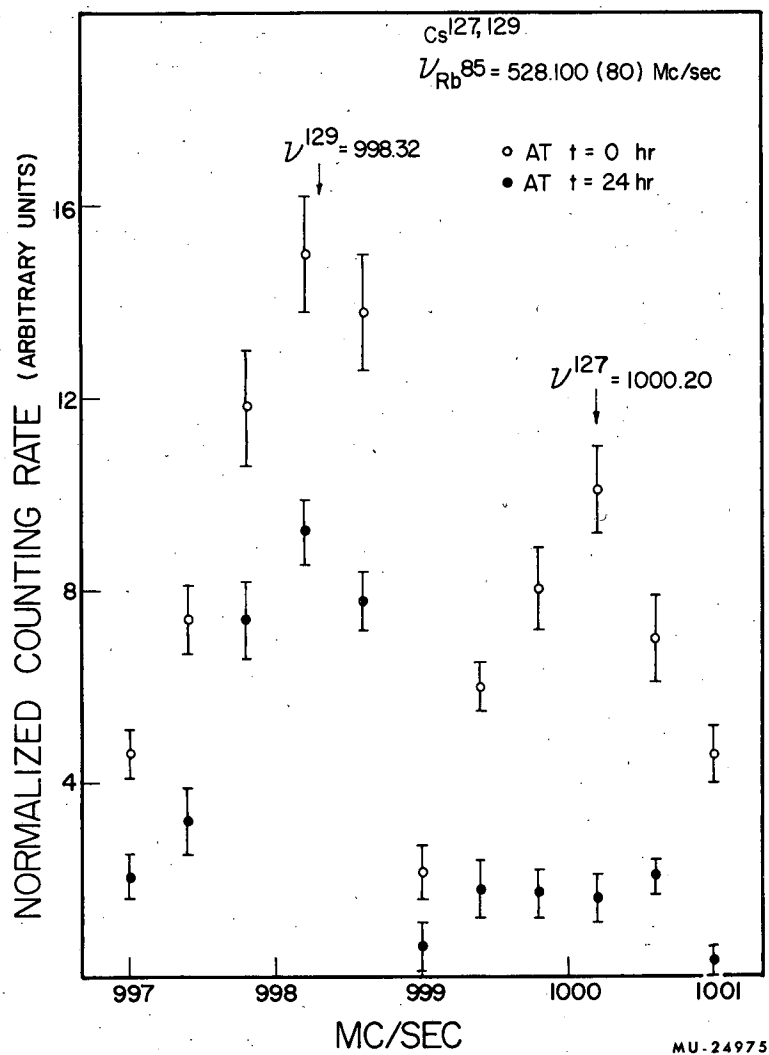
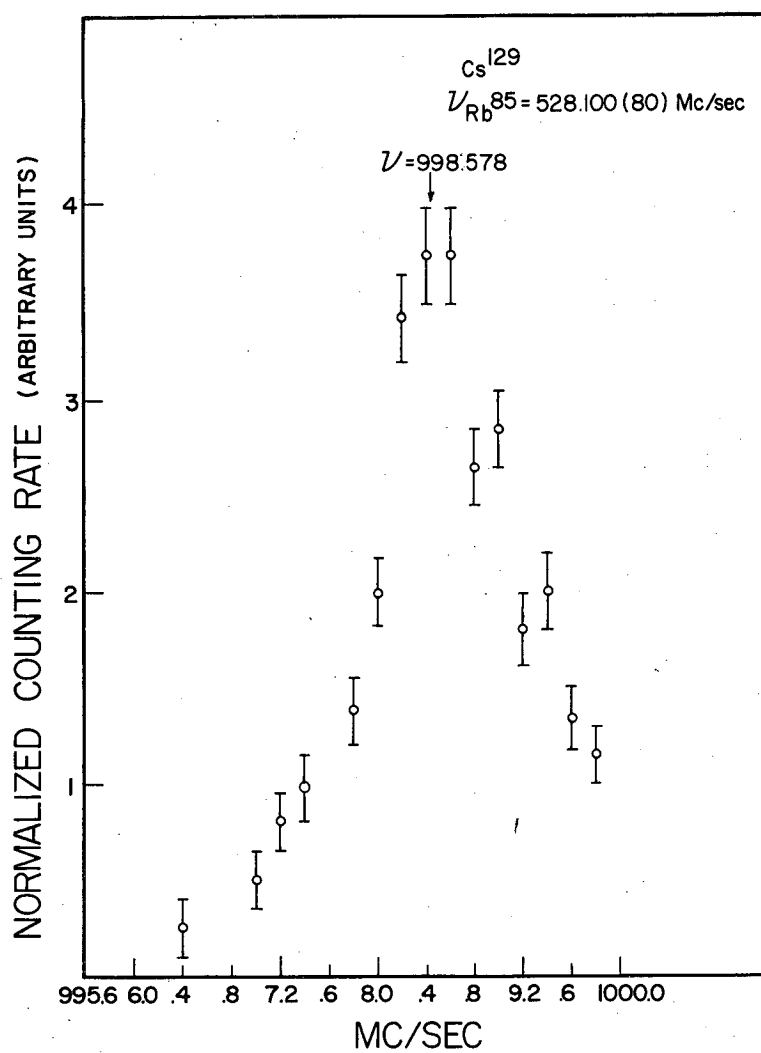


Fig. 24. Resolved resonances of Cs^{127} and Cs^{129} .
 $\nu_{\text{Rb}^{85}} = 528.100(80) \text{ Mc}$.



MU-24976

Fig. 25. A Cs^{129} resonance. $\nu_{\text{Rb}}^{85} = 528.100(80)\text{Mc}$.

In this experiment, however, the OMNIBUS routine was used to fit the resonance to a bell-shaped curve and calculate the $\Delta \nu$ by direct solution of the Briet-Rabi equation.

In this experiment a 3/4-inch strap hairpin of conventional design (Fig. 18) was used to excite the transitions.

3. Results

The experimental conditions and findings were as follows.

	<u>Run 115</u>	<u>Run 152</u>
Bombardment time (hr)	10	6
Amount of RbCl carrier (mg)	200	105
Duration of atomic beam (hr)	9	5
Calibration, Rb ⁸⁵ (Mc)	528.100(80)	528.100(80)
Magnetic field (gauss)	649.518(63)	649.518(63)
Observed resonance (Mc)	1000.20(60) ^{7*} 998.32(30) ⁹	998.578(27) ⁹
Isotope and half life	Cs ¹²⁷ (6 hr) Cs ¹²⁹ (31 hr)	Cs ¹²⁹ (31 hr)
$\Delta \nu$ (Mc)	8900(150) ⁷ 9180(100) ⁹	9229(30) ⁹

*7 = Cs¹²⁷ 9 = Cs¹²⁹

4. Conclusion

From the results of run 152, and by calculation directly from the exact solution of the Briet-Rabi equation, the value obtained was

$$\Delta \nu^{129} = 9229(30) \text{ Mc/sec.}$$

This value is in agreement, within the experimental error, with the measurement by Howard A. Shugart.⁴¹

D. Rubidium Experiments

1. General

The isotope rubidium-84, produced in the reaction $\text{Kr}^{84}(\text{p}, \text{n})$ Rb^{84} , and having a half life of 33 days, was investigated with the goal of measurement of the hyperfine-structure separation and the magnetic moment. Figures 26 and 27 show typical resonances in this isotope. At the time of the termination of the experiments on the isotope, the $\Delta \nu$ had been measured, and a value of the g_I obtained. The demonstrated distortions of the resonances produced, with the use of a simple strap hairpin, a value of the g_I which is no more accurate than the value predicted by the Fermi-Segrè value.

2. Transitions and Hairpin

The transitions observed at 1.5 gauss are $(5/2, \pm 1/2) \longleftrightarrow (3/2, \mp 1/2)$. At low field these are unresolved. The intermediate-field transitions observed were $(5/2, 3/2) \longleftrightarrow (3/2, 1/2)$ and $(5/2, 1/2) \longleftrightarrow (3/2, 3/2)$. Their frequency separation yields g_I and then average gives $\Delta \nu$. A 2-inch strap hairpin was used to excite the transitions.

3. Results

The experimental conditions and findings were as follows:
Target material: krypton gas (2000 cc at 1 atmosphere pressure)
Bombardment: In 60-inch cyclotron, Berkeley, at
 proton energy of 12 Mev;
 duration, 40 hr, yielding 800 $\mu\text{a-hr}$.
Carrier: 100 mg RbCl .
Duration of atomic beam: 13 hr.

	<u>Run 76a</u>	<u>Run 76b</u>
Calibration, Rb^{87} (Mc)	373.993(100)	2.236 (15)
Magnetic field (gauss)	462.338(130)	1.41 (1)
Observed resonance (Mc)	2812.368(2) ν^+ 2811.904(2) ν^-	3083.171 (4)
$\Delta \nu$ (Mc)	3083.159(4) (ν^+) 3083.159(4) (ν^-)	3083.159 (4)

Average $\Delta \nu$ (Mc) 3083.159(3)

4. Conclusion

The direct measurement of the $\Delta \nu$ at low fields and the average value of the resolved doublet at intermediate fields give a value of $\Delta \nu$ which is

$$\Delta \nu = 3083.159(4) \text{ Mc/sec.}$$

E. Potassium Experiments

1. General

The purpose of the experiment was to measure the hyperfine-structure separation and the magnetic moment of potassium-42, half life 12.5 hr. This has been accomplished and the results of four runs are presented here. Used in the experiment was a 2-inch Ramsey separated-field hairpin.

The transitions $(5/2, \pm 1/2) \longleftrightarrow (3/2, \mp 1/2)$, which appear as an unresolved doublet, are observed at 1.4 gauss, in Run 159. For measurement of the moment, the transitions $(5/2, +1/2) \longleftrightarrow (3/2, +3/2)$ $(5/2, +3/2) \longleftrightarrow (3/2, +1/2)$ are observed at an intermediate field of 189 gauss, where the frequency difference between the two resonances is field-independent to second order in the electronic term. At this intermediate field the frequency difference, which is independent of $\Delta \nu$, is used to obtain the nuclear magnetic moment. Such differences are observed in Runs 141, 147, 154, 160.

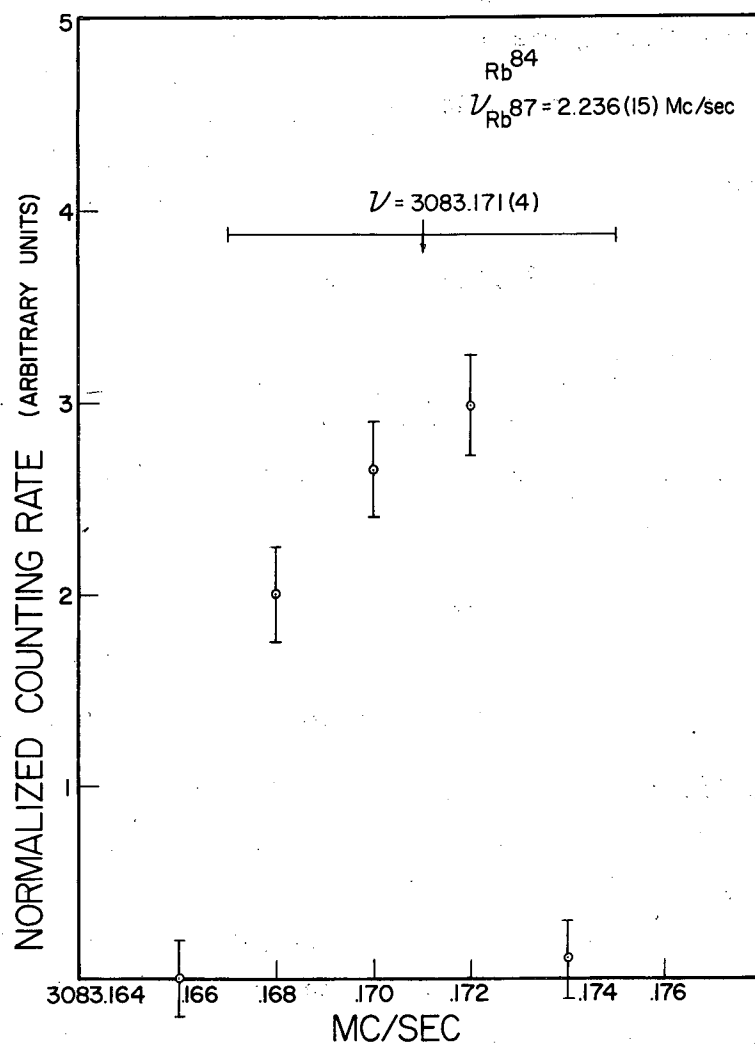


Fig. 26. A direct transition in Rb^{84} at approximately 2 gauss. $\nu_{\text{Rb}^{87}} = 2.236(15) \text{ Mc}$.

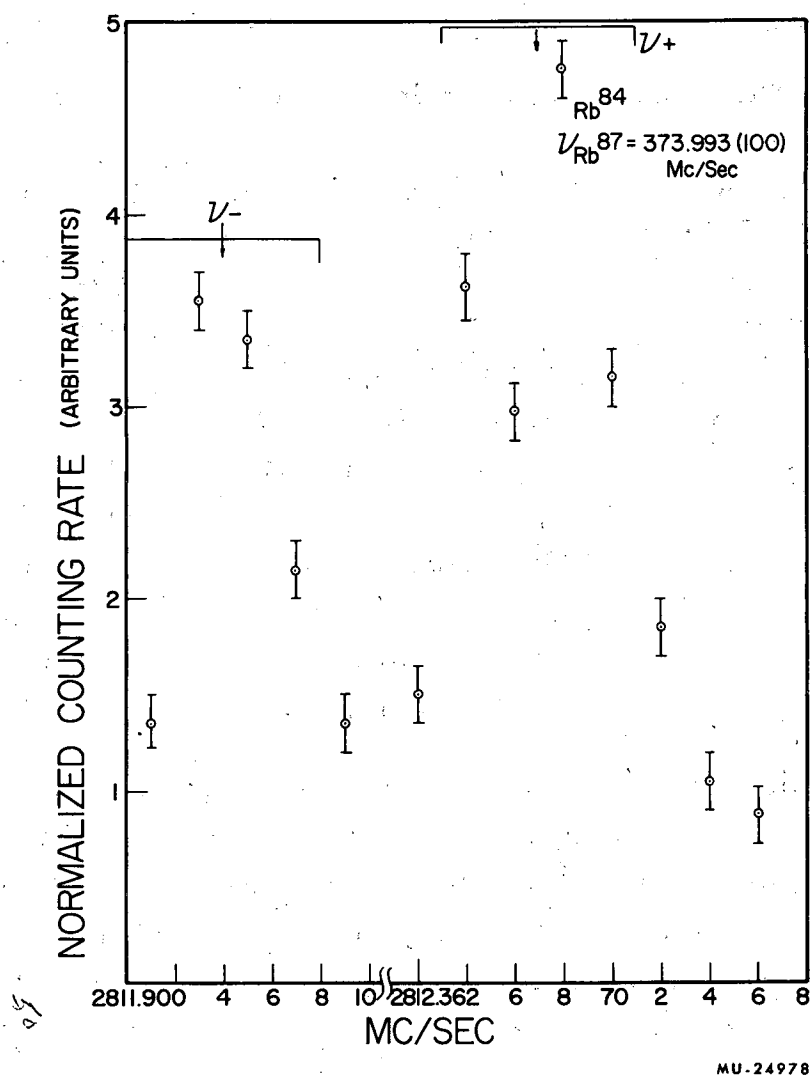


Fig. 27. The $(5/2, 3/2 \leftrightarrow 3/2, 1/2)$,
 $(5/2, 1/2 \leftrightarrow 3/2, 3/2)$ doublet in Rb^{84} .
 $\nu_{\text{Rb}^{87}} = 373.993(100) \text{ Mc}$.

Two orientations of the Ramsey hairpin were used; they are designated by + and - signs in the table of results.

2. Results

The experimental conditions and findings were as follows.

Run no.	Hairpin orientation	Calibration, K^{39} (Mc)	Magnetic field (gauss)	<u>Resonances</u> ν (visual) (Mc)	ν (computer) (Mc)
141	...	780.500(100)	188.80(40)	<u>line shapes</u>	
1471	+	780.508(100)	188.806(40)	1148.1340(2)	1148.13395(7)
1472	+	780.508(100)	188.806(40)	1148.2980(2)	1148.29805(7)
1541	+	780.750(300)	188.801(86)	1148.1341(2)	1148.13425(7)
1542	+	780.750(300)	188.801(86)	1148.2980(2)	1148.29795(7)
159	-	461.988(50)	1.40(5)	1258.883(4)	1258.8834(7)
1601	-	780.250(100)	188.665(50)	1148.1340(2)	1148.13405(7)
1602	-	780.250(100)	188.665(50)	1148.2980(2)	1148.29815(7)

In view of the nonshifted resonance value upon reversal of the orientation of the hairpin and the consistent moment measurements for Rb^{85} (g_I measured = $2.938(2) \times 10^{-4}$ compared with 2.93704×10^{-4} of Yasaitis and Smaller⁴²), an uncertainty of $\pm 1/20$ the full width at half maximum is placed on peak frequencies. A summary of the magnetic moment calculations is as follows:

Run	ν_+ (Mc)	ν_- (Mc)	$\nu_+ - \nu_-$ (Mc)	H (gauss)	μ_I nm
147(+)	1148.29805(25)	1148.13395(25)	1148.16410(35)	188.806(40)	-1.1402(25)
154(+)	1148.29795(25)	1148.13425(25)	1148.16370(35)	188.801(40)	-1.1374(25)
160(-)	1148.29815(25)	1148.13405(25)	1148.16410(35)	188.665(50)	-1.1395(25)
average				$\langle \mu_I \rangle$ = -1.1395(30)	
				$\langle g_I \rangle$ = -0.5697(15)	

3. Error Analysis

In the general discussion of the results, five sources of systematic error are described, with the conclusion that all but two were negligible. These two are the field-inhomogeneity error and the phase-shift error. These must now be considered in the light of the data available.

Nonuniformities in field are observable as an asymmetry in the resonance lines. Associated with this may be a shift in the position of the resonance peak. It is suspected that this effect appears in Fig. 28 as the difference in amplitude of the minima. This effect is difficult to isolate, as it requires a change in magnet geometry or hairpin position. This, therefore, represents the major source of error.

The second source, the phase-shift error, may be tested directly. This shift is not observed, which leads to the conclusion that the magnitude of this error must be smaller than the experimental uncertainties of the measurement.

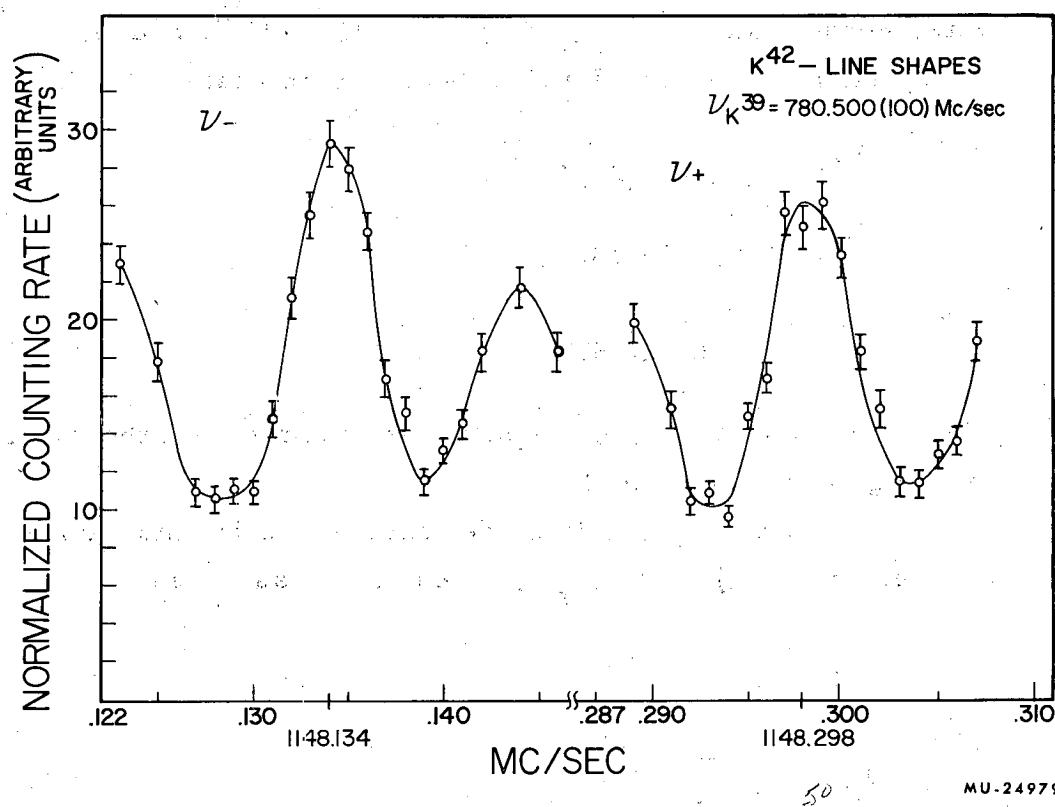


Fig. 28. K^{42} Ramsey hairpin line-shape curves.
 ν for $K^{39} = 780.500(100) \text{ Mc}$.

VII. HYPERFINE STRUCTURE ANOMALY IN POTASSIUM-42

The anomaly, defined in Section III, is given by

$$a_{\Delta}^{\beta} = \left[\left(\frac{\Delta \nu_a}{\Delta \nu_{\beta}} \frac{g_{I_{\beta}}}{g_{I_a}} \frac{2I_{\beta}+1}{2I_a+1} \right) - 1 \right] \times 100 \% , \quad (\text{VII-1})$$

with known values for $\Delta \nu$, I , g_I for K^{39} , K^{40} , and K^{41} .⁴³ This expression can be calculated for each, with respect to K^{42} :

$${}^{39}_{\Delta}{}^{42} = + 0.20(25)\% ,$$

$${}^{40}_{\Delta}{}^{42} = - 0.25(25)\% ,$$

$${}^{41}_{\Delta}{}^{42} = + 0.42(25)\% .$$

These values of the anomaly will be compared with the predictions of the single-particle model.

The single-particle model expression for the nuclear g value is derived from $\vec{\mu}_I = \vec{\mu}_p + \vec{\mu}_n$, where one assumes

$$\vec{I} = \vec{j}_p + \vec{j}_n ,$$

$$\vec{j}_p = \vec{s}_p + \vec{\ell}_p , \quad (\text{VII-2})$$

$$\vec{j}_n = \vec{s}_n + \vec{\ell}_n ,$$

and

$$g_I \mu_0 \vec{I} \cdot \vec{I} = (g_p \mu_0 \vec{j}_p + g_n \mu_0 \vec{j}_n) \cdot \vec{I} ,$$

$$g_p \mu_0 \vec{j}_p \cdot \vec{j}_p = (g_s^p \mu_0 \vec{s}_p + g_{\ell}^p \mu_0 \vec{\ell}_p) \cdot \vec{j}_p ,$$

$$g_n \mu_0 \vec{j}_n \cdot \vec{j}_n = (g_s^n \mu_0 \vec{s}_n + g_{\ell}^n \mu_0 \vec{\ell}_n) \cdot \vec{j}_n .$$

The g -values can be related, by employing the vector coupling assumed above:

$$g_I = \frac{\vec{I} \cdot \vec{j}_p}{\vec{I} \cdot \vec{I}} g_p + \frac{\vec{I} \cdot \vec{j}_n}{\vec{I} \cdot \vec{I}} g_n \quad (\text{VII-3})$$

By similar operations the g_p and g_n can be re-expressed in terms of the orbital and intrinsic-spin g values.

The specific coupling scheme for each of the potassium isotopes is given by⁴⁴

	Proton unpaired	Neutron unpaired	I	j_p	j_n
K^{39}	$d_{3/2}$	—	$3/2$	$3/2$	—
K^{40}	$d_{3/2}$	$f_{7/2}$	4	$3/2$	$7/2$
K^{41}	$d_{3/2}$	—	$3/2$	$3/2$	—
K^{42}	$d_{3/2}$	$f_{7/2}$	2	$3/2$	$7/2$

When these quantum numbers are substituted into the expression for the nuclear g value and the free-nucleon g values are employed, the result deviates in all cases from the observed value.^{27, 45}

g_I (single-particle model)	g_I (observed)
K^{39} +0.08	+0.2607
K^{40} -0.41	-0.32412
K^{41} +0.08	+0.14311
K^{42} -0.87	-0.5697

To explain these deviations, and maintain the feature of the shell model, it has been suggested that changes in the spin or orbital g values might occur in the strong nuclear fields.⁴⁶ For two unpaired nucleons in the odd-odd nucleus, it was suggested by Schwartz that the g values of neighboring odd-even or even-odd isotopes whose unpaired nucleon is in the same state might be a successful approximation.

In the two cases presented for comparison, neighboring isotope proton g values are employed. The procedure is as follows. Use the expression

$$1. \quad g_I = \frac{\vec{I} \cdot \vec{J}_p}{\vec{I} \cdot \vec{I}} \left[\frac{\vec{J}_p \cdot \vec{s}_p}{\vec{J}_p \cdot \vec{J}_p} g_s^p + \frac{\vec{J}_p \cdot \vec{l}_p}{\vec{J}_p \cdot \vec{J}_p} g_l^p \right] \quad (\text{VII-4})$$

2. For g_I substitute the observed g value of the odd-even isotope.
3. a. Substitute $g_l^p = 1$ and solve for g_s^p , yielding an effective g_s^p ,
 b. Substitute $g_s^p = \text{free-nucleon value}$, and solve for an effective g_l^p .
4. a. Substitute $g_I = g_I(\text{obs})$, $g_l^p = 1$, $g_s^p = \text{effective } g_s^p$ and $g_s^n = \text{free-nucleon } g_s^n$ into Eq. (VII-3) and solve for $g_l^n = \text{effective } g_l^n$.
5. a. Use g values from 4a to obtain a_p and a_n (proton and neutron weighting factors).
 b. Use g values from 4b to obtain a_p and a_n .
6. a. Employ a_p, a_n from 5a to obtain $-\epsilon(\text{I})$
 b. Employ a_p, a_n from 5b to obtain $-\epsilon(\text{II})$.
7. Evaluate factors b and $\left\langle \frac{R^2}{R_e^2} \right\rangle$, $\left\langle \frac{R^4}{R_e^4} \right\rangle$ from Eisinger and Jaccarino, and substitute into $-\epsilon(\text{I})$ and $-\epsilon(\text{II})$.⁷

The results of this process are:

Model I	Model II
Proton: $g_{\ell}^p = 1$	Proton: $g_s^p = 5.59$
$g_s^p = g_I^{41} = 0.14311$	$g_{\ell}^p = \frac{5}{6} \left[g_I^{41} + \frac{1}{5} g_s^p \right]$
Neutron: $g_{\ell}^n = 0$	Neutron: $g_s^n = -3.83$
$g_s^n = \frac{14}{3} \left[g_I^{42} + \frac{1}{2} g_I^{41} \right]$	$g_{\ell}^n = \frac{6}{7} \left[\frac{2}{3} \left(g_I^{42} + 1/2 g_I^{41} \right) - \frac{1}{7} g_s^n \right]$
$a_s^p = -0.927$	$a_{\ell}^p = -0.630$
$a_s^n = +0.875$	$a_{\ell}^n = -1.317$
$- \epsilon = 0.075$	$- \epsilon = 0.084$

When the same procedure is applied for K^{39} , K^{40} , K^{41} , the following numbers result: ⁴³

$a_{\Delta}^{\beta} = \epsilon_{\alpha} - \epsilon_{\beta}$					
K^A	$\epsilon(I)$	$\epsilon(II)$	$A_{\Delta}^{42}(I)$	$A_{\Delta}^{42}(II)$	$A_{\Delta}^{42}(\text{observed})$
K^{39}	+0.266	+0.331	+0.341	+0.415	+0.20(25)
K^{40}	+0.254	-0.280	-0.179	-0.196	-0.25(25)
K^{41}	+0.626	+0.666	+0.701	+0.750	+0.42(25)
K^{42}	-0.075	-0.084			

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