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BLENDE-TYPE SEMICONDUCTORS

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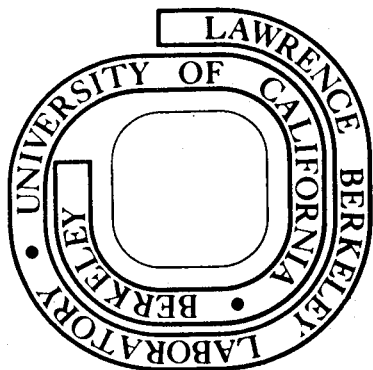
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TEMPERATURE DEPENDENCE OF THE FUNDAMENTAL EDGE OF  
GERMANIUM AND ZINC BLENDE-TYPE SEMICONDUCTORS

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## ABSTRACT

We calculate the theoretical temperature dependence of the minimum of the conduction band at  $k=0$  for Ge, GaAs, GaSb, InSb, GaP, CdTe, ZnTe, ZnSe and ZnS. We find results strongly dependent of the parameters describing the symmetrical part of the pseudopotential and we show that the use of slightly different sets of pseudopotential form factors give rise to very different values for the temperature coefficient of the fundamental edge. A comparison with experimental results (piezo-reflectance measurements) permits to discuss the choice of these parameters.

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## INTRODUCTION

Since the pioneering work of Fan<sup>1</sup> and Yu-Brooks,<sup>2</sup> a large number of calculations<sup>3-6</sup> have been carried out to theoretically investigate the temperature dependence of the band structure of cubic semiconductors. Generally speaking most of these calculations achieve qualitative agreement with experiment i.e. give a correct order of magnitude for the temperature coefficient of the main optical transitions but they never fit closely the experimental data. No attempt to improve the calculations was made because of the dispersion of the experimental results available for a given compound<sup>6</sup> and because of the lack of reliable experimental Debye-Waller factors.

However, a recent calculation of the temperature dependence of the band structure of germanium, silicon and GaAs<sup>7</sup> which uses the new pseudopotential parameters of Chelikoswky et al. including nonlocality of the  $V_0$  pseudopotential<sup>8</sup> concluded that the Yu-Brooks theory accounts well for the experimentally observed rigidity with temperature of the top valence band of these compounds. Indeed increasing the temperature of a crystal, one increases first the lattice parameter and second the electron-phonon interaction. The effect of lattice expansion can be obtained from pressure experiments. For example for the valence band of Germanium in the X direction, it corresponds to a decrease in width with a rate:  $-5.9 \cdot 10^{-5} \text{ eV/}^\circ\text{K}$ . The Yu-Brooks theory permits to calculate the second contribution and give in this case a value:  $d(\Gamma_{25} - X_4)/dT = 7.9 \cdot 10^{-5} \text{ eV/}^\circ\text{K}$  which is positive. This gives, corresponding to the effect of electron phonon interaction with

increasing temperature, an increase in width which nearly cancels the decrease in width corresponding to the effect of lattice expansion. This corresponds well to the rigidity experimentally observed.<sup>7</sup>

In this paper we use the same method to calculate the theoretical temperature dependence of the minimum of the conduction band at  $k=0$ . It is well known that slightly different pseudopotential curves can give identical band structures even if the Fourier transform of the crystal potential has different  $V_{(G^2)}$  components. We show that the use of these slightly different sets of band structure parameters give rise to very different values for the temperature coefficient of the fundamental edge. In this case the calculated values of the temperature coefficients mostly depend of the symmetrical part of the pseudopotential through the two form factors  $V_{(G^2=8)}$  and  $V_{(G^2=11)}$ , and we show that this calculation in conjunction with both:

- a reliable set of experimental Debye-Waller factors.
- well established experimental temperature coefficients may constitute a separate check of these specific pseudopotential form factors.

In order to perform this comparison we give experimental results (Piezo-reflectivity measurements) on Ge, GaAs, GaSb, InSb, CdTe, ZnSe, ZnTe and ZnS. As most of the dispersion observed in the previously published data comes from the room temperature measurements, we focus our attention on the accuracy obtained at room temperature. In that way we achieve a set of accurate data which permit a system comparison with our calculated values.

## I. THEORETICAL BACKGROUND

In this first part, we perform a direct calculation of the temperature dependence of the fundamental edge of Ge, GaAs, GaSb, InSb, CdTe, ZnSe, ZnTe and ZnS. For completeness, we also include GaP which is indirect (the conduction band minimum is in the X direction) and for which reliable data exist in the literature.<sup>9,10</sup> The electron-phonon interaction (Debye-Waller effect) is obtained in the pseudopotential formalism using the standard basis of 15 symmetrized combinations of plane-waves.<sup>11,12</sup> We first obtain the unperturbed wave functions of the two states  $\Gamma_1^c$  and  $\Gamma_{15}^v$  ( $T=0^\circ\text{K}$ ) and then calculate in first order the effect of perturbation ( $T\neq 0^\circ\text{K}$ ). In order to obtain a set of systematically reliable results, we take all the values of the mean square displacement of atoms  $\langle u^2 \rangle$  for zinc-blende materials from the work of Vetelino and Col.<sup>13</sup> In the same way we always include<sup>7</sup> the effect of lattice expansion from the experimental work of Paul and coworkers.<sup>14</sup> Lastly our calculation completely neglects the electron self energy effects. A recent estimate for III-V compounds<sup>15</sup> has shown this correction to be of the order of 5 to 20% of the Debye-Waller contribution in GaP, GaAs, and InP.

### A. Results

The 15 symmetrized combinations of [000], [111] and [200] plane waves used in the calculation are listed in reference 7 with the corresponding secular equation for a covalent crystal. Solving this equation, one obtains a set of 8 different wave functions:

$\Gamma_1^\ell$ ,  $\Gamma_1^u$ ,  $\Gamma_2^\ell$ ,  $\Gamma_2^u$ ,  $\Gamma_{25}^\ell$ ,  $\Gamma_{25}^u$ ,  $\Gamma_{15}$  and  $\Gamma_{12}$  which are:

- a pure  $\{111\}_{\Gamma_{15}}$  combination
- a pure  $\{200\}_{\Gamma_{12}}$  combination
- Two admixtures of the properly symmetrized  $\{000\}_{\Gamma_1}$  and  $\{111\}_{\Gamma_1}$

combinations which we note:

$$|\Gamma_1^\ell\rangle = \alpha_1^\ell \{000\}_{\Gamma_1} + \beta_1^\ell \{111\}_{\Gamma_1} \quad (1)$$

$$|\Gamma_1^u\rangle = \alpha_1^u \{000\}_{\Gamma_1} + \beta_1^u \{111\}_{\Gamma_1}$$

- and four admixtures:  $|\Gamma_2^\ell\rangle$ ,  $|\Gamma_2^u\rangle$ ,  $|\Gamma_{25}^\ell\rangle$  and  $|\Gamma_{25}^u\rangle$  of the properly symmetrized  $\{111\}$  and  $\{200\}$  combinations which we note:

$$|\Gamma_2^\ell\rangle = \beta_2^\ell \{111\}_{\Gamma_2} + \gamma_2^\ell \{200\}_{\Gamma_2} \quad (2)$$

with 3 other equivalent expressions.

We work under the standard assumption that the antisymmetric part of the pseudopotential in noncentrosymmetric crystals acts as a perturbation on the lone symmetric part of the pseudopotential which determine the pure  $|\Gamma_2^\ell\rangle$  and  $|\Gamma_{25}^\ell\rangle$  wave functions. This results in an admixture of  $|\Gamma_1\rangle$  and  $|\Gamma_{15}\rangle$  states with  $|\Gamma_2\rangle$  and  $|\Gamma_{25}\rangle$  respectively. One can write:

$$|\Gamma_1^c\rangle = \alpha_1 |\Gamma_2^\ell\rangle - j\beta_1 |\Gamma_1^\ell\rangle - j\gamma_1 |\Gamma_1^u\rangle \quad (3)$$

$$|\Gamma_{15}^v\rangle = \alpha_2 |\Gamma_{25}^\ell\rangle - j\beta_2 |\Gamma_{15}\rangle$$



where the factor  $j$  comes from the definition of the pseudopotential:

$$V = V_S + jV_A$$

In order to determine the numerical values of these coefficients  $\alpha, \beta, \gamma$  we must calculate the 3 different antisymmetric matrix elements labelled in the notation of Higginbotham et al:<sup>16</sup>  $V_1, V_2, V_3$ . Each corresponds to a closed combination of the 3 classical antisymmetric form factors  $v_3^-, v_4^-$  and  $v_{11}^-$  given by:

$$\begin{aligned}
 v_1 &= \langle \Gamma_{25}^\ell | H^A | \Gamma_{15}^u \rangle = -\beta_{25}^\ell v_4^- - \sqrt{2} \gamma_{25}^\ell (v_3^- - v_{11}^-) \\
 v_2 &= \langle \Gamma_2^\ell | H^A | \Gamma_1^u \rangle = v_3^- [2\alpha_1^u \beta_2^\ell - \sqrt{6} \beta_1^u \gamma_2^\ell] \\
 &\quad + v_4^- [\sqrt{6} \alpha_1^u \gamma_2^\ell - 3\beta_1^u \beta_2^\ell] - v_{11}^- [\sqrt{6} \beta_1^u \gamma_2^\ell]
 \end{aligned}
 \tag{4}$$

with a similar expression for  $V_3$ . In that way we obtain the different wave functions of the conduction and valence bands at  $k=0$ .

Now, since we know the different contributions (admixture coefficients) for a specific eigenstate, we can perform the temperature dependent calculation once we have determined the variation versus temperature of each component. As an example, for the minimum of the conduction band we can write:

$$\begin{aligned}
 \frac{d\Gamma_1^c}{dT} &= \alpha_1^2 \frac{d\Gamma_2^\ell}{dT} + \beta_1^2 \frac{d\Gamma_1^\ell}{dT} + \gamma_1^2 \frac{d\Gamma_1^u}{dT} + 2\alpha_1 \beta_1 \frac{dv_3}{dT} \\
 &\quad + 2\alpha_1 \gamma_1 \frac{dv_2}{dT} + 2\beta_1 \gamma_1 \frac{d\langle \Gamma_1^\ell | H | \Gamma_1^u \rangle}{dT}
 \end{aligned}
 \tag{5}$$

with a similar expression for the valence band. The weighting coefficients  $\alpha, \beta, \gamma$  depend on the semiconductor studied through the intensity of the antisymmetric coupling. For instance with the recent pseudopotential form factors of Chelikowsky et al.<sup>8</sup> we obtain for the conduction band in the horizontal sequence Ge-GaAs-ZnSe the contributions listed in Table I. As expected the  $|\Gamma_{2'}\rangle$  contribution decreases with increasing ionicity from about 80% in GaAs to 50% in ZnSe as the  $|\Gamma_1^l\rangle$  and  $|\Gamma_1^u\rangle$  character increases. The same kind of result is found for the valence band.

Finally we need to calculate the temperature dependence of the different matrix elements in Eq. (5). In order to perform the calculation, we work under the standard assumption that because of the effect of the thermal vibrations of the lattice, each energy state experiences a shift which is, (relative to its unperturbed value at 0°K), simply obtained by multiplying each pseudopotential form factor by the corresponding Debye-Waller factor<sup>2-7</sup> amplitude.

$$V_m \rightarrow V_m \exp\left(-\frac{m\Omega}{\sigma} \langle u^2 \rangle\right) \quad (6)$$

where

$m = 3, 8, 11$  for the 3 symmetric form factors.

$m = 3, 4, 11$  for the 3 antisymmetric form factors

$\Omega = (2\pi/a_0)^2$  where  $a_0$  is the lattice constant, in Bohr units,

and  $\langle u^2 \rangle$  is the mean-square average displacement, also in Bohr units.

This results in an attenuation of the potential seen by the electrons and for example in the simple picture of pure bonding and

antibonding states of covalent crystals (see Fig. 1) one expect a positive variation of the valence band (bonding P state) and a negative variation of the conduction band (antibonding S state) with this decrease in potential. In order to calculate directly the numerical value  $\langle \alpha \rangle$  of the corresponding temperature coefficients in the range of the linear variation, between 100 and 300°K, we develop the perturbation and write:

$$V_m \rightarrow V_m - \frac{m\Omega}{6} V_m [\Delta \langle u^2 \rangle]_{100^\circ K}^{300^\circ K} \quad (7)$$

where the first term corresponds to the unperturbed hamiltonian and the second gives the temperature dependence of the matrix elements. With our explicit expressions of the wave functions (Eq. 3), we limit the calculation to first order and obtain the numerical values given in the two first columns of Table II. In the last column, we give the references for the pseudopotential form factors used and list the corresponding parameters in Table III. In order to achieve a reliable set of results, we tried to refer all along the computation to the same work.<sup>8</sup> This has not been possible for GaSb,<sup>17</sup> ZnS and ZnTe.<sup>18</sup> However a comparison between the calculations of references 8 and 18 is possible through the use of the computations for CdTe and ZnSe with both series of parameters. In the same way all values of  $[\Delta \langle u^2 \rangle]_{100}^{300}$  are taken from the work of reference 13, except germanium (not given in ref. 13) for which we use the recent experimental data of Jex.<sup>19</sup>

## B. Discussion of Results

As expected, the Debye-Waller temperature coefficient of the valence band is positive and this is true for the entire family: the valence band goes up with increasing temperature and give the most important contribution to the experimental value of the temperature coefficient of the band gap. Roughly this contribution is of the order of  $3 \cdot 10^{-4}$  eV/°K. The exact value depends essentially of the symmetrical part of the pseudopotential through the  $v_{11}$  form factor. In the isoelectronic sequence: Ge-GaAs-ZnSe computed with the parameters of ref. 8, the temperature coefficient given for germanium is greater than the corresponding value for GaAs. This result arises from the different origin of the mean-square displacement  $\langle u^2 \rangle$  for germanium and explains our choice of a reliable, unique, source of data for all the rest of the calculation.

The temperature coefficient of the conduction band is less important. As expected, for germanium it is negative but very small:  $-5 \cdot 10^{-6}$  eV/°K. Generally speaking, the calculated value increases through the isoelectronic sequence: Ge-GaAs-ZnSe with increasing strength for the antisymmetric coupling, but remains negative. The numerical values obtained strongly depends on the semiconductor (Ge:  $-5 \cdot 10^{-6}$  eV/°K, GaAs:  $-7 \cdot 10^{-5}$  eV/°K) and on the set of parameters used. Consider for example the strongly varying values obtained for ZnSe ( $-2.5 \cdot 10^{-4}$  eV/° with the parameters of Ref. 8 and  $-9 \cdot 10^{-5}$  eV/°

with those of Ref. 18). An even more interesting result is obtained in the isoelectronic sequence InSb-CdTe. For InSb, the calculated value is very low and positive. This results in a small Debye-Waller contribution to the temperature coefficient  $\alpha_0$  of the band gap which correlates well with the small experimental value of  $\alpha_0$  obtained on this compound. For CdTe, we performed the calculation with 3 different sets of parameters. The values obtained range between:  $-1,6 \cdot 10^{-4}$  eV/° and  $+5 \cdot 10^{-5}$  eV/° which also gives a positive shift for the conduction band versus temperature and a small Debye-Waller contribution. As previously observed for InSb, this also correlates well with our experimental results on CdTe: The two compounds exhibit nearly identical temperature coefficients which are the lowest for the whole germanium-zincblende family. As the most important difference between the parameters of references 8, 18 and 20 is in the successive decrease of the  $v_{11}$  symmetric form factor, this confirms that the calculated values mainly depend on the symmetrical part of the pseudopotential (through  $v_8(s)$  and  $v_{11}(s)$  which should vary little through the horizontal sequence).

The third column of Table II gives the total Debye-Waller contribution, obtained from difference of columns 1 and 2. In order to perform the calculation, we must add to this contribution the effect of lattice expansion given in column 4. We can calculate this contribution from the experimental pressure and compressibility coefficients listed by Paul and coworkers<sup>14</sup> through the standard expression:

$$\frac{dE_o}{dT} = -3\alpha/\chi \frac{dE_o}{dP}$$

The linear dilatation coefficients are taken in the work of Novikova<sup>21</sup> and one obtains the values given in column 4. This volume or dilatation contribution is always smaller than the Debye-Waller contribution but is also negative and hence contributes to increase the temperature coefficients to the final values listed in column 5.

### C. Minimum X of Indirect Materials

For the sake of completeness it is interesting to calculate the temperature dependence of the X indirect gap of GaP. The corresponding  $|X_1^C\rangle$  wave function is an admixture of the two symmetrized combinations of plane waves  $\{100\}_{X_1}$  and  $\{011\}_{X_1}$ , given in reference 11, through the effect of both symmetric and antisymmetric coupling.

$$\begin{array}{l} \{100\}_{X_1} \\ \{011\}_{X_1} \end{array} \left| \begin{array}{cc} \{100\}_{X_1} & \{011\}_{X_1} \\ \Omega - v_4^- & \sqrt{2}(v_3 - v_3^-) \\ \sqrt{2}(v_3 - v_3^-) & 2\Omega + v_8 - 2v_4^- \end{array} \right|$$

with the parameters of reference 8, a straightforward calculation gives for the temperature coefficient of the  $|X_1^C\rangle$  minimum:

$dX_1^C/dT = -1,25 \cdot 10^{-4}$  eV/°. Taking account of the two values calculated in Table II for  $d\Gamma_1^C/dT$  and  $d\Gamma_{15}^V/dT$ , we have the 3 coefficients needed to account for the experimental temperature dependence of both the direct and the indirect gap in this material. Adding the effect of

the two strongly different corrections for lattice expansion, one obtains the results given in Table IV in excellent agreement with the experimental values.

This shows that:

- a) the Debye-Waller coefficients which we independently associate with each extremum of the band structure are correct.
- b) our calculation of the lattice expansion effect from experimental hydrostatic pressure coefficients is also sufficiently accurate to permit a fine comparison with experiment. This is the goal of our next section.

## II. EXPERIMENTS

The experimental apparatus has been previously described<sup>22,23</sup> and enable us to work in a large range of experimental conditions, independent of the spectral range investigated and of the doping level of the sample. All samples studied are mechanically polished and chemically etched by standard techniques to a final thickness of 100 $\mu$  and then glued on piezo-electric transducers in order to perform a derivative spectroscopy. With standard values for the experimental conditions ( $E \sim 3-5 \cdot 10^3$  V/cm), this corresponds to the application of an alternative stress  $\Delta T \sim 50$  kg/cm<sup>2</sup> on the sample and gives a modulation of the fundamental edge:

$$\Delta E_g \approx \frac{dE_g}{dP} \Delta T \sim 5 \cdot 10^{-4} \text{ eV} .$$

In order to obtain an accurate determination of a temperature coefficient, it is most important to determine precisely the experimental value of the transition in the highest temperature range of data. Consider for example GaAs, which is a well known semiconductor: in the recent literature one can find values of the band gap at room temperature ranging from 1,380 eV<sup>24</sup> to 1,435 eV<sup>25</sup> which constitutes simply a dispersion in crystals quality, broadening effects, interpretation of data, etc. On the other hand most of the results converge at helium temperature, resulting in a dispersion of 2 or 3  $\cdot 10^{-4}$  eV/ $^\circ$  which



constitute 50% of the true coefficient. The situation is worse for II-VI compounds in which the range of uncertainty at room temperature is of the order of 100 meV.<sup>23</sup>

A typical example of the resolution obtained at room temperature is given in Fig. 2.A. The line shape is characteristic of the first derivative of a 2 dimensional Mo critical point and permits an accurate determination of the absorption edge.<sup>26</sup> In this case, for intrinsic germanium, the broadening parameter  $\Gamma$  which corresponds to half the energy separation between the two extrema is less than 3 meV and the accuracy of the data compete with the best results obtained in low field electro-reflectance<sup>27</sup> (See Fig. 2.B) The same kind of spectrum is obtained on high purity GaAs and GaSb. In contrast, InSb always contains an important concentration of free carriers and is characterized by a 3 dimensional (non excitonic) Mo critical point.<sup>22</sup>

The temperature dependence of the band gap in these compounds is given in Fig. 3. In Fig. 4 we give the temperature dependence observed in the II-VI compounds. As expected from the results of our calculations, we see that the semiconductors of the same isoelectronic sequence are characterized by nearly the same experimental value (see Fig. 5). This experimental result confirms the predominant role played by the symmetrical part of the pseudopotential in the calculation of the temperature coefficients. The numerical values of the absorption edge at 0° and 300°K are given in Table V together with the experimental and calculated values of the temperature coefficients in the range 100-300°K. A comparison of these two series of results

show that for Ge, GaAs, GaSb and InSb the agreement obtained between the experimental values and the result of our calculation is fairly good.

It is interesting to verify how a small change of  $v_{11}^S$  is able to greatly modify this result. In Table VI, we compare the results obtained for InSb with only a slight adjustment of  $v_{11}^S$ , changing from the value 0.044 Rydberg<sup>8</sup> to the value 0.040 Rydberg.<sup>28</sup> This result in an improvement of the calculation which give now:  $-2,9 \cdot 10^{-4}$  eV/°K in much better agreement with the experimental value of  $-2,75 \cdot 10^{-4}$  eV/°K.

For the II-VI compounds, the comparison of both series of results (experiment and theory) show a systematic improvement when a set of pseudopotential form factors are used which give Fourier components of the symmetrical part of the pseudopotential lower in a II-VI compound as compared with the corresponding III-V. This result is absolutely general and is expected to provide a probe to check one or two specific form factors.<sup>29</sup> For example, we give in Table 7 a systematic comparison between our experimental values, the results of this calculation and the results obtained by TSAY et al.<sup>5-6</sup>, with the same values of the Debye-Waller factors but the pseudopotential form factors of Cohen and Bergstresser.<sup>28</sup> The improvement is general and reflects well the progress achieved in band structure calculation during the last 10 years.

## CONCLUSION

We have performed a direct calculation of the temperature dependence of the fundamental edge in germanium and zincblende type semiconductors. This calculation shows that the most sensitive level to temperature is the maximum of the valence band  $\Gamma_{15}^V$ . As the temperature increases, the valence band goes up toward the conduction band with a rate of  $\sim 3.10^{-4}$  eV/ $^{\circ}$  and gives the most important contribution to the temperature coefficient of the band gap. In contrast, the temperature coefficient of the conduction band at  $k=0$  is very small. Lastly, in both cases, the calculated values are mostly sensitive to the symmetrical pair of the pseudopotential. This effect is supported by experimental observations on semiconductors of the two most important isoelectronic sequences which exhibit experimental temperature coefficients nearly independent of ionicity i.e. roughly independent of the effect of antisymmetric couplings.

In this way, a careful comparison of the experimental values with the results of the calculation for different sets of parameters show a systematic dependence on the value of the  $v_{11}^S$ . We expect this result to give a new probe to check the results of different band structure calculations. A more detailed comparison could be made if a complete set of experimental Debye-Waller factors for the germanium-zincblende family become available.

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$$\left(\frac{d\langle u^2 \rangle}{dT}\right) = 1.98 \times 10^{-4} \text{ Bohr}^2 / ^\circ\text{K, value given in ref. 7}.$$
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29. In that way, one should mention that the form factors of ref. 8 were fit with emphasis on the valence band density of states with only a rough fit to the optical gap (J. Chelikowsky, private communication.) This may help to understand why some of the "older" form factors carefully fit to the optical data, give better value for  $V(G^2 = 11)$ .

Table I. Change in admixture coefficients for the conduction and the valence band at  $k=0$  through the horizontal sequence: Ge-GaAs-ZnSe.

	$ \Gamma_{2'}^{\ell}\rangle$	$ \Gamma_1^{\ell}\rangle$	$ \Gamma_1^u\rangle$	$ \Gamma_{25'}^{\ell}\rangle$	$ \Gamma_{15'}\rangle$
Ge	1	0	0	1	0
GaAs	0.904	-0.271j	-0.33j	0.878	-0.479j
ZnSe	0.716	-0.4j	-0.567j	0.820	-0.570j

Table II. Details of the contributions which add to give the temperature coefficients of the fundamental gap in some cubic semiconductors.

Compound	$\left(\frac{d \Gamma_1^c}{dT}\right) \text{eV}/^\circ\text{K}$	$\left(\frac{d \Gamma_{15}^v}{dT}\right) \text{eV}/^\circ\text{K}$	$\left(\frac{d E_o}{dT}\right)_{\text{D.W.}}$ ( $10^4 \text{ eV}/^\circ\text{K}$ )	$\left(\frac{d E_o}{dT}\right)_{\text{Vol.}}$ ( $10^4 \text{ eV}/^\circ\text{K}$ )	$\left(\frac{d E_o}{dT}\right)_{\text{Cal.}}$ ( $10^4 \text{ eV}/^\circ\text{K}$ )	Reference ( $V_m$ )
Ge	$-5.10^{-6}$	$2.7 \times 10^{-4}$	-2.75	-1.65	-4.4	8
GaAs	$-7.10^{-5}$	$2.45 \times 10^{-4}$	-3.15	-1.3	-4.45	8
GaSb	$-4.10^{-5}$	$2.10^{-4}$	-2.4	-1.35	-3.75	17
InSb	$1.10^{-5}$	$2.5 \times 10^{-4}$	-2.4	-0.9	-3.3	8
GaP	$-1.10^{-4}$	$2.10^{-4}$	-3	-1.5	-4.5	8
CdTe	$-1.6 \times 10^{-4}$	$5.5 \times 10^{-4}$	-7.1	-0.4	-7.5	8
	$-4 \times 10^{-5}$	$3.5 \times 10^{-4}$	-3.9	-0.4	-4.3	18
	$5 \times 10^{-5}$	$3.4 \times 10^{-4}$	-2.9	-0.4	-3.3	20
ZnSe	$-2.5 \times 10^{-4}$	$5.2 \times 10^{-4}$	-7.7	-0.7	-8.4	8
	$-9 \times 10^{-5}$	$3.54 \times 10^{-4}$	-4.44	-0.7	-5.1	18
ZnTe	$-1.1 \times 10^{-4}$	$3.8 \times 10^{-4}$	-4.9	-0.6	-5.5	18
ZnS	$-2.6 \times 10^{-4}$	$3.9 \times 10^{-4}$	-6.5	-0.6	-7.1	18



Table III. Pseudopotential form factors and lattice constant  $a_0$  (Å) used in the calculation.

Compound	$a_0$ (Å)	$v_3$ (s)	$v_8$ (s)	$v_{11}$ (s)	$v_3^-$ (a)	$v_4^-$ (a)	$v_{11}^-$ (a)	Reference
Ge	5.65	-0.269	0.038	0.035				8
GaAs	5.64	-0.252	0.000	0.080	0.068	0.066	0.012	8
GaSb	6.12	-0.21	0.015	0.04	0.09	0.03	0.01	17
InSb	6.47	-0.25	0.010	0.044	0.049	0.038	0.01	8
GaP	5.45	-0.249	0.017	0.083	0.081	0.055	0.003	8
CdTe	6.48	-0.245	-0.015	0.073	0.089	0.084	0.006	8
		-0.207	0.006	0.032	0.145	0.0715	0.026	18
		-0.20	0.012	0.027	0.168	0.075	0.028	20
ZnSe	5.65	-0.261	0.011	0.113	0.151	0.130	0.016	8
		-0.239	0.014	0.0529	0.2807	0.0978	0.0525	18
ZnTe	6.07	-0.2256	0.0022	0.035	0.087	0.1257	0.035	18
ZnS	5.41	-0.2472	0.0412	0.0526	0.301	0.1215	0.0475	18

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Table IV. Comparison of both calculated and experimental values for the temperature dependence of the direct and indirect gaps of GaP.

	$\frac{dE_{(\Gamma-X)}}{dT}$	$\frac{dE_o}{dT}$
Debye-Waller contribution	$-3.25 \times 10^{-4}$	$-3 \times 10^{-4}$
Volume contribution	$0.15 \times 10^{-4}$	$-1.5 \times 10^{-4}$
Calculated value	$-3.1 \times 10^{-4}$	$-4.5 \times 10^{-4}$
Experimental value (100°K-300°K)	$-3.1 \times 10^{-4}$ (a)	$-4.1 \times 10^{-4}$ (b)

(a) Experimental result obtained from the relation

$$E_g = 2.388 - 6.2 \times 10^{-4} T^2 / (T+460).$$

[M. B. Panish and H. C. Casey, J. Appl. Phys. 40, 163 (1969)]

(b) Value obtained from the experimental results of Dean et al.,  
J. Appl. Phys. 38, 3551 (1967).

Table V. Variation of the band gap versus temperature in cubic compounds. Experimental values for Ge, GaAs, GaSb and InSb do not include the binding energy of excitons. Values for CdTe, ZnTe, ZnSe and ZnS include excitonic effect according to the work of ref. 23.

$E_o$ (eV)	Ge	InSb	GaSb	GaAs	CdTe	ZnTe	ZnSe	ZnS
300°K	0.7985	0.172	0.722	1.424	1.529	2.300	2.724	3.731
0°K	0.888	0.235	0.810	1.515	1.606	2.395	2.823	3.833
$\left(\frac{dE_o}{dT}\right)_{\text{exp.}}$ $\times 10^4$ °K/eV	-4±0.2	-2.75±0.20	-3.7±0.2	-3.9±0.2	-2.9±0.5	-4.1±0.5	-4.5±0.5	-4.7±0.5
$\left(\frac{dE_o}{dT}\right)_{\text{cal.}}$ $\times 10^4$ °K/eV	-4.4	-3.3	-3.75	-4.45	-7.5 -4.3 -3.3	-5.5	-8.4 -5.1	-7.1

000042074674

Table VI. Example of adjustment of  $V(G^2=11)$  performed on InSb. (a) See ref. 8,  $V^S(G^2=11) = 0.044$  Rydberg (b) All pseudopotential form factors have the values of ref. 8, except  $V^S(G^2=11) = 0.04$  Rydberg (see ref. 28).

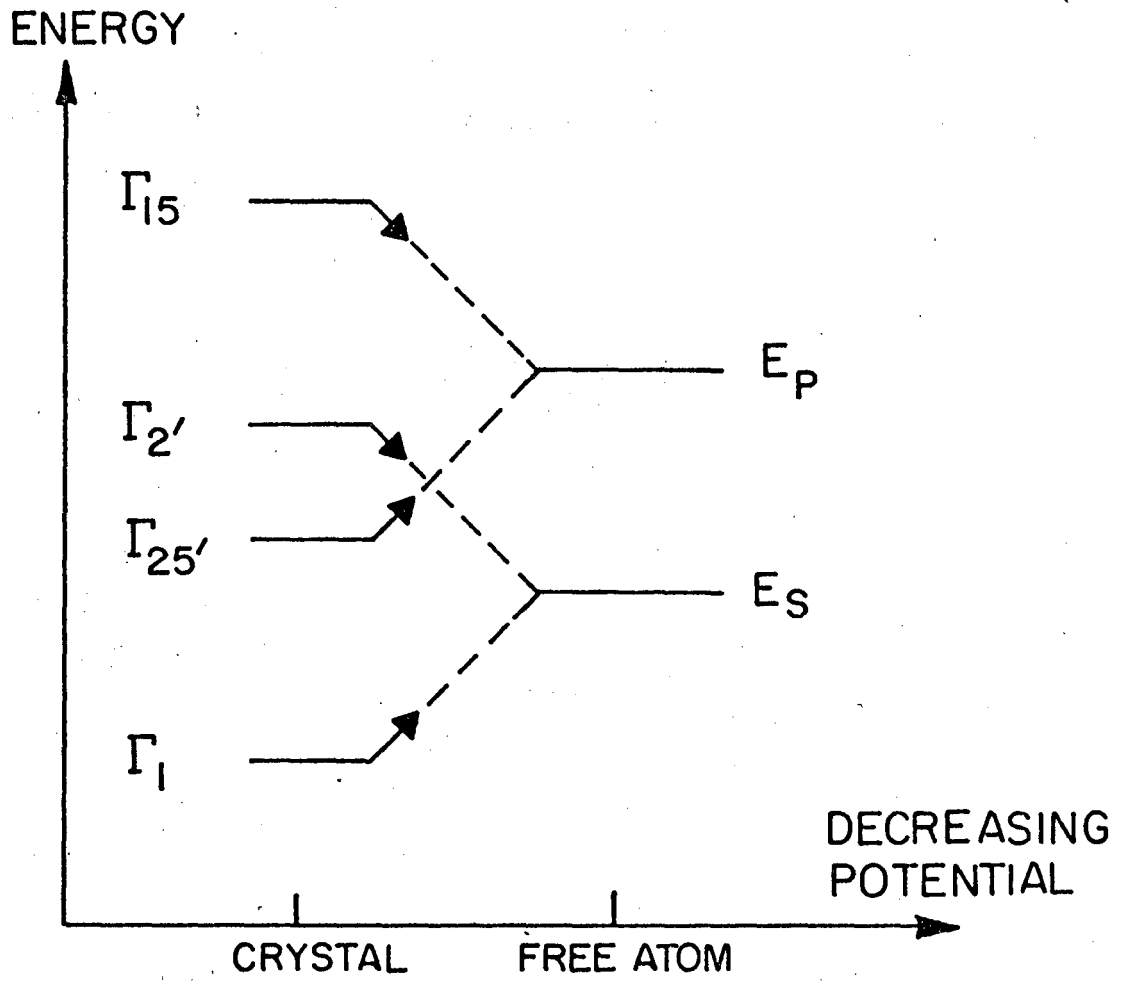
Pseudopotential form factors	$\frac{d \Gamma_1^c}{dT}$	$\frac{d \Gamma_{15}^v}{dT}$	$\left(\frac{d E_o}{dT}\right)$ Theory	$\left(\frac{d E_o}{dT}\right)$ Exp.
(a)	$1 \times 10^{-5}$	$2.5 \times 10^{-4}$	$-3.3 \times 10^{-4}$	$-2.75 \times 10^{-4}$
(b)	$4 \times 10^{-5}$	$2.4 \times 10^{-4}$	$-2.9 \times 10^{-4}$	$-2.75 \times 10^{-4}$

Table VII. Comparison of experimental results and calculated values, first in this work, second in the works of Tsay et al. (refs. 5 & 6) with a systematic use of the pseudopotential form factors of ref. 28.

	Experimental value ( $10^{-4}$ eV/°K)	Calculated value (this work) ( $10^{-4}$ eV/°K)	Reference of the pseudopotential form factors (this work)	Other works (refs. 5-6 and 28)
Ge	-4.0	-4.4	8	
GaAs	-3.9	-4.45	8	-6.3
GaSb	-3.7	-3.75	17	-4.0
InSb	-2.75	-2.9	8 with $v_{11}=0.04$	-3.5
GaP( $\Gamma_1^c - \Gamma_{15}^v$ )	-4.1	-4.5	8	-4.3
GaP( $\chi_1^c - \Gamma_{15}^v$ )	-3.1	-3.1	8	-2.75
CdTe	-2.9	-3.3	20	-6.2
ZnSe	-4.5	-5.1	18	-6.5
ZnTe	-4.1	-5.5	18	-6.7
ZnS	-4.7	-7.1	18	-6.7

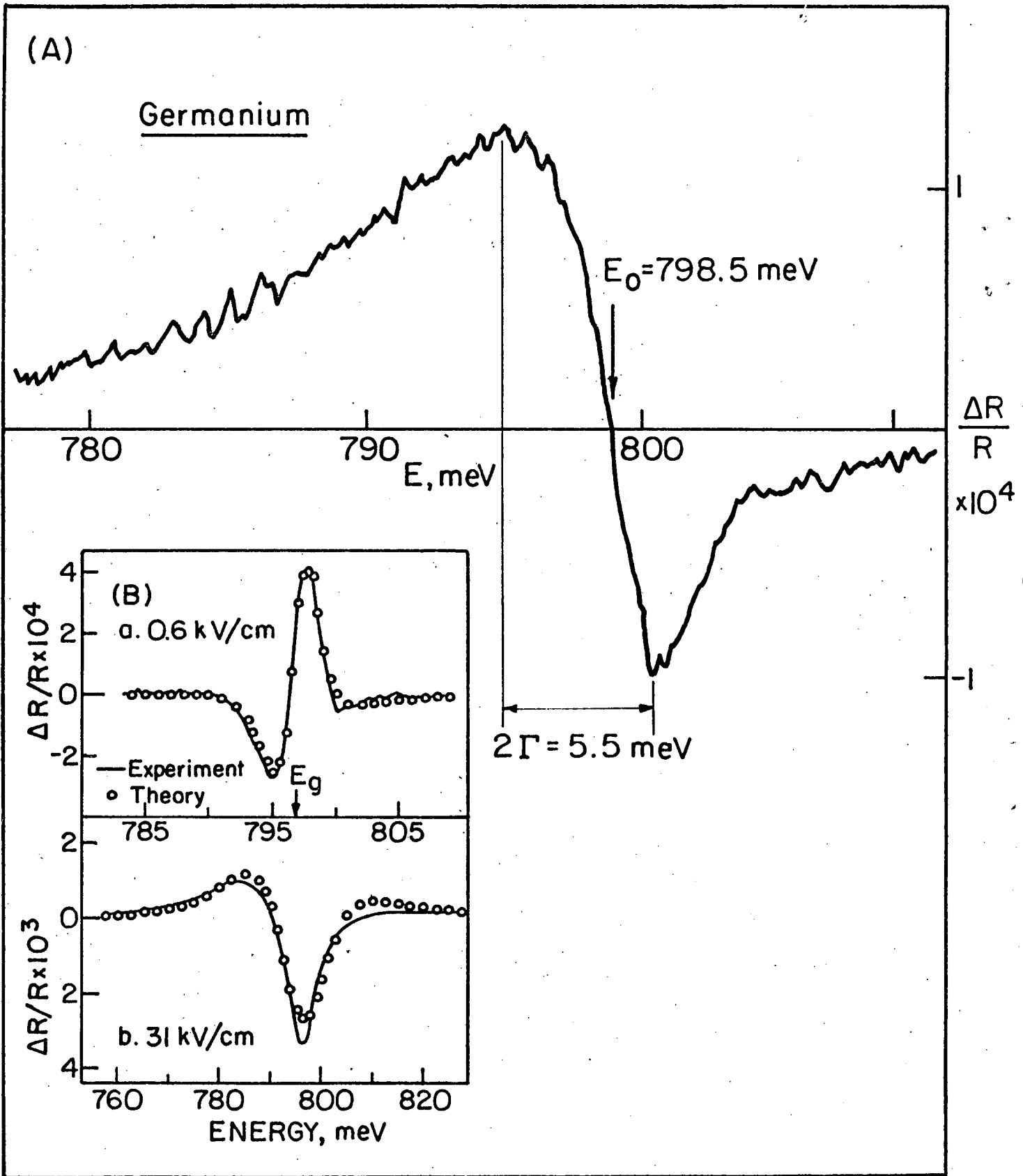
## FIGURE CAPTIONS

- Fig. 1. Correspondence between atomic energies and crystal energies at  $k=0$  for a covalent semiconductor. A decrease in potential like the effect of lattice expansion or thermal vibration will give a positive temperature coefficient for the two bonding states.  $\Gamma_1$  and  $\Gamma_{25'}$ . An increase in potential like hydrostatic pressure would give a negative pressure coefficient for these two states. In the same way one expects for the two antibonding states  $\Gamma_2^{\ell}$  and  $\Gamma_{15}$  a negative temperature coefficient and a positive pressure dependence.
- Fig. 2. Differential reflectivity spectrum of the fundamental edge of Germanium ( $300^\circ\text{K}$ )
- A) Piezoreflectivity (this work)
- $$E_0 = 798.5 \pm 1 \text{ meV}$$
- B) Electroreflectivity (a: low field  $E = 600 \text{ V/cm}$ , b: high field  $E = 3.1 \text{ kV/cm}$ , after Ref. 27). In both cases
- $$E_0 = 797 \pm 2 \text{ meV.}$$
- Fig. 3. Relative shift of the band gap in Ge, GaAs, GaSb and InSb.
- Fig. 4. Shift of the band gap in the II-VI compounds, CdTe, ZnTe, ZnSe and ZnS.
- Fig. 5. Temperature dependence of the fundamental edge versus ionicity for the two isoelectronic sequences Ge-GaAs-ZnSe and InSb-CdTe. The estimated error is  $+0.2 \cdot 10^{-4} \text{ eV}/^\circ\text{K}$  for Ge and the III-V compounds and  $\pm 0.4 \cdot 10^{-4} \text{ eV}/^\circ\text{K}$  for the II-VI compounds. We did not find in the recent literature any value for the temperature coefficient of the inverted bandgap of  $\alpha$ -Sn. According to the present scheme it should be positive  $dE_g/dT \sim 3 \cdot 10^{-4} \text{ eV}/^\circ\text{K}$ .



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Fig. 1



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Fig. 2



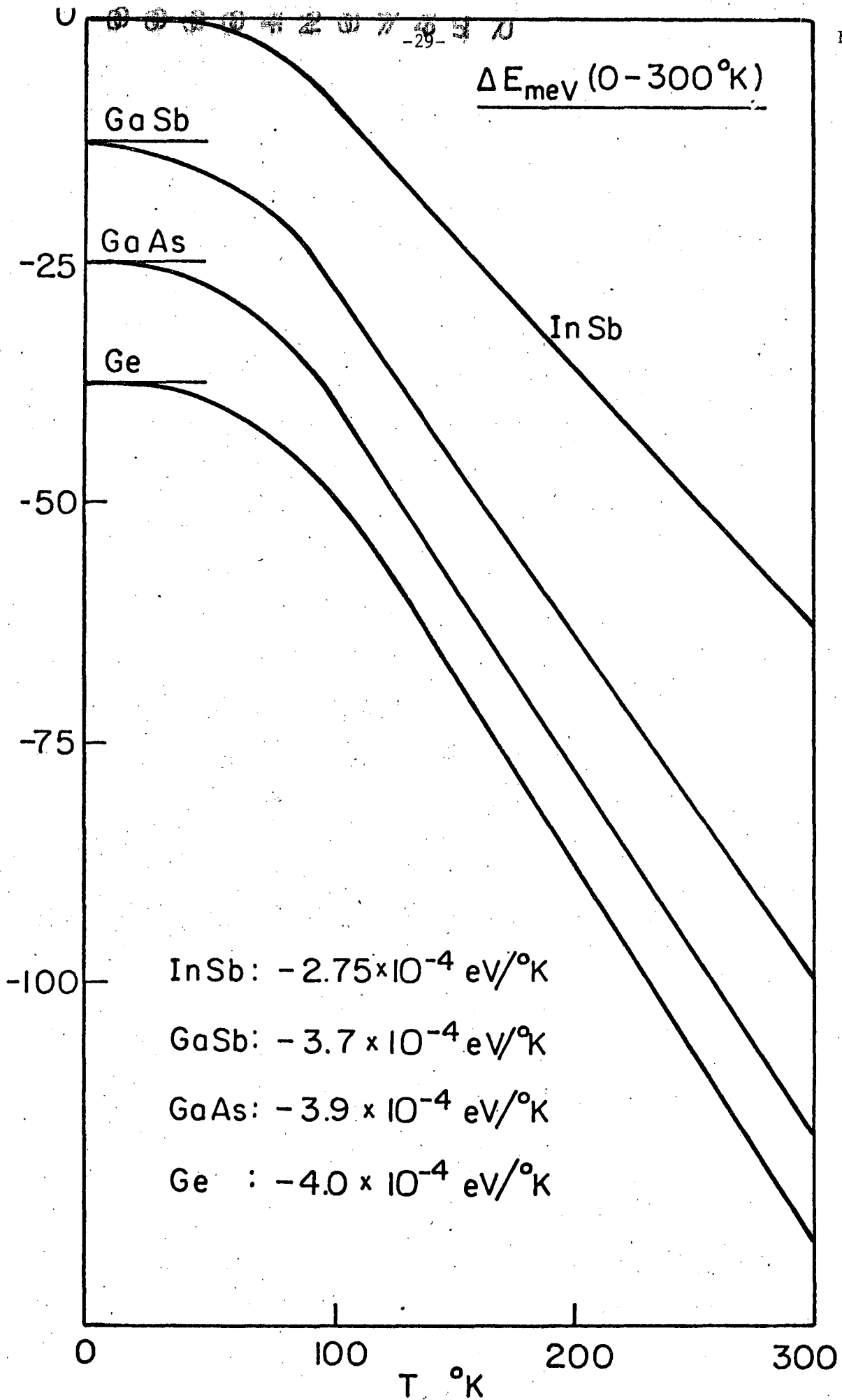


Fig. 3

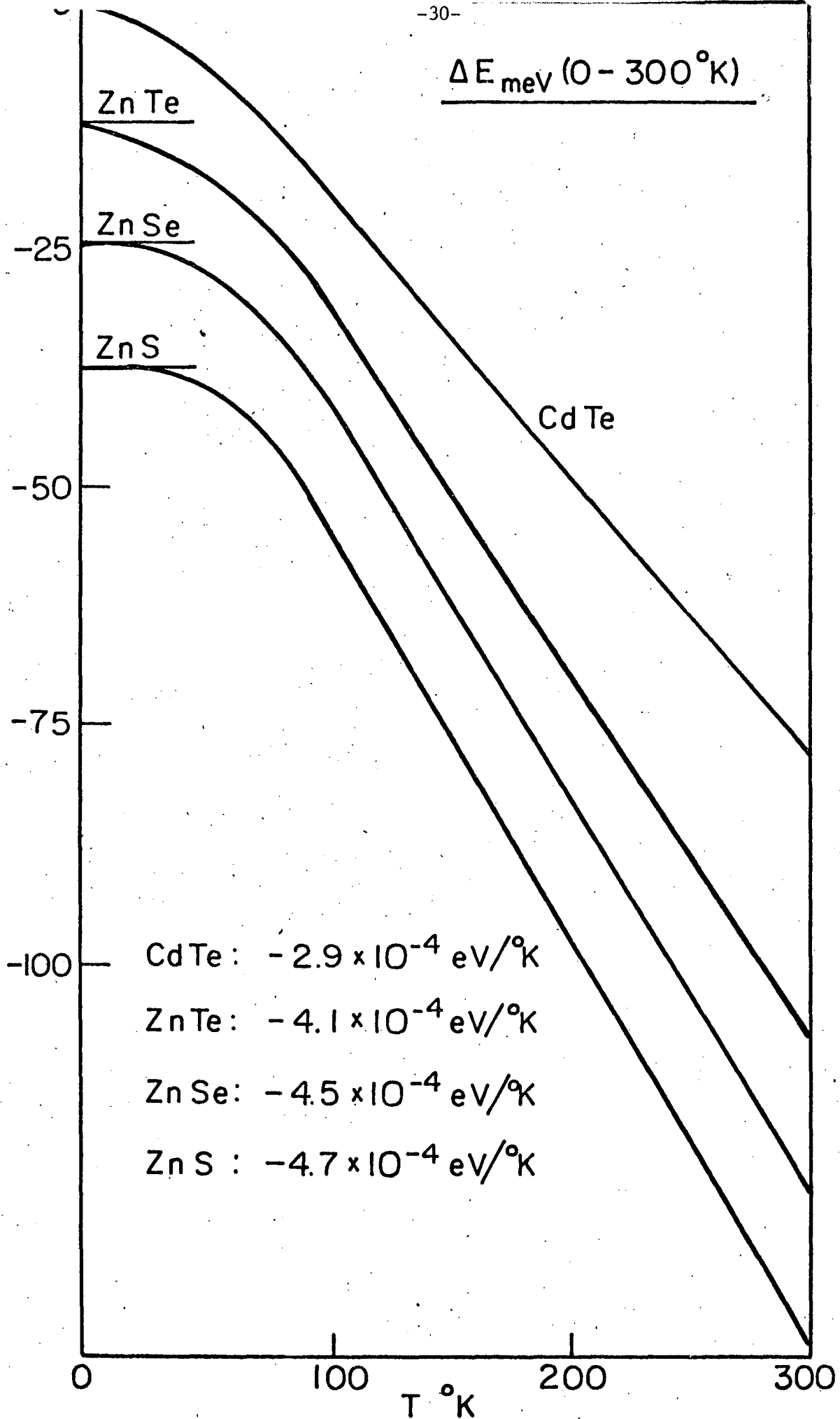
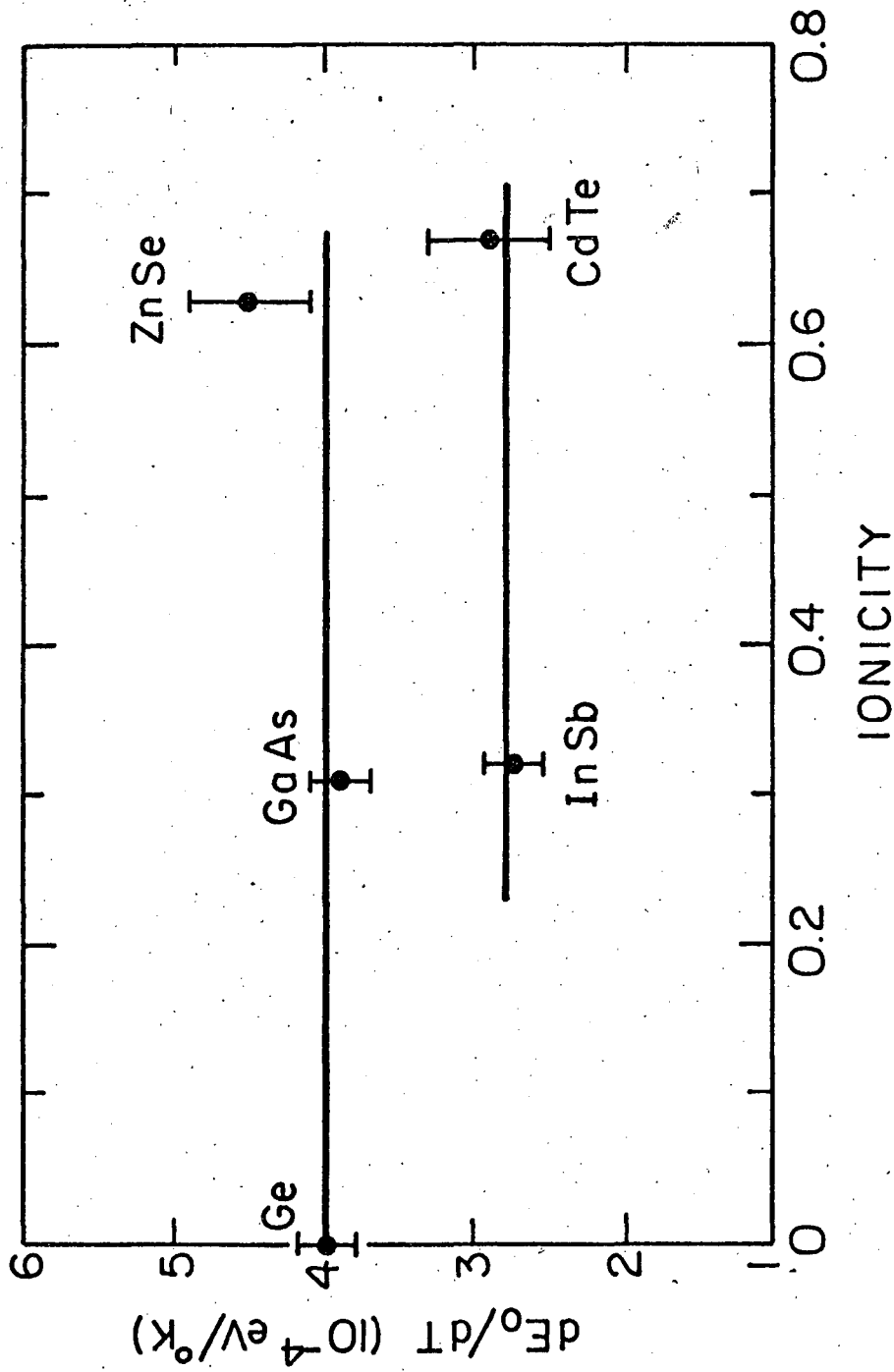


Fig. 4



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Fig. 5

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