

Lawrence Berkeley National Laboratory

Lawrence Berkeley National Laboratory

Title

BETA STABLE NEUTRON STARS IN NON-LINEAR RELATIVISTIC MEAN FIELD MODELS

Permalink

<https://escholarship.org/uc/item/3n51p51x>

Author

Boguta, J.

Publication Date

1981-11-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Submitted to Physics Letters

BETA STABLE NEUTRON STARS IN NON-LINEAR
RELATIVISTIC MEAN FIELD MODELS

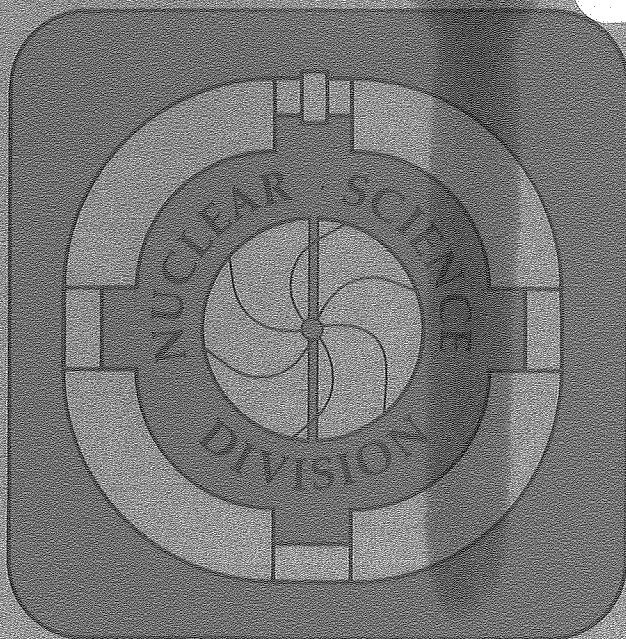
J. Boguta

November 1981

RECEIVED
LAWRENCE
BERKELEY LABORATORY
DEC 1 1981
LIBRARY AND
DOCUMENTS SECTION

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782



LBL-12584
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Beta Stable Neutron Stars in Non-Linear
Relativistic Mean Field Models*

J. Boguta
Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

Abstract

Properties of beta stable neutron stars are investigated in relativistic non-linear mean field models that correctly describe known bulk properties of symmetric nuclear matter. The requirement of beta stability can significantly affect the neutron matter equation of state and in turn the static properties of neutron stars.

*This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

A number of calculations predicting the static properties of cold neutron stars have been done using a variety of neutron matter equations of state.¹ Neutron matter density at the center of the neutron star is believed to reach several times nuclear densities encountered in normal nuclei. To compute the properties of neutron stars using the Tolman-Oppenheimer-Volkov equations the neutron matter equation of state must be known at these large densities. This equation of state is usually calculated in models that violate the most elemental requirements, such as relativity. An exception to this criticism is the work of Chin and Walecka,² who used a particular relativistic field theory to obtain the equation of state at high densities.

The idea of a neutron star was originally conceived by Landau³ immediately after learning of neutron's discovery. He imagined a heavenly body made of neutrons and held together by gravity. Strictly speaking, this idea cannot be correct, because of neutron's beta decay. A neutron star must have a certain percentage of protons and electrons. The requirement of beta stability in neutron stars has been neglected in the belief that its fulfillment would lead to a small and negligible proton concentration and only trivial modification of neutron matter equation of state. That this assumption can be incorrect for proton concentration in beta stable neutron matter was shown by the author.⁴ It was shown that about twice nuclear density [$k_F(n) \approx 400$ MeV] beta stable neutron matter can be 15% protons. At neutron star densities, this percentage can be much larger. The presence of a large percentage of protons in neutron matter (electrons will keep the system electrically neutral) will affect the equation of state. The equation of state of beta stable neutron matter and the corresponding modifications of the static properties of

neutron stars has never been investigated in detail. We find that in some models, the modification of the equation of state due to beta stability is substantial.

A useful method to study the properties of dense nuclear matter is relativistic quantum field theory in the mean field approximation. The Lagrangian describing the nucleons ψ interacting with isoscalar field σ , isoscalar vector field ω_μ and isovector field $\hat{R}_\mu$ is taken to be

$$\begin{aligned} \mathcal{L} = & -\bar{\psi}(\gamma_\mu \partial_\mu + m_N)\psi - \frac{1}{2} (\partial_\mu \sigma)^2 - U(\sigma) \\ & - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}_{\mu\nu} - \frac{1}{2} m_V^2 \omega_\lambda \omega_\lambda - \frac{1}{2} m_V^2 \hat{R}_\lambda \cdot \hat{R}_\lambda \\ & + ig_V \bar{\psi} \gamma_\lambda \psi \omega_\lambda + ig_r \bar{\psi} \gamma_\mu \vec{\tau} \cdot \hat{R}_\mu - g_S \bar{\psi} \psi \sigma \end{aligned} \quad (1)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \\ \vec{G}_{\mu\nu} &= \partial_\mu \hat{R}_\nu - \partial_\nu \hat{R}_\mu \end{aligned} \quad (2)$$

and ψ is an isospin doublet of protons and neutrons,

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad (3)$$

In the above Lagrangian m_N , m_V , m_σ correspond to the nucleon, omega and rho meson and sigma mesons respectively, while g_S , g_V , and g_r are the corresponding coupling constants. The potential function $U(\sigma)$ is taken to be a general quartic polynomial of the form

$$U(\sigma) = \frac{m_S^2}{2} \sigma^2 + \frac{b}{3} \sigma^3 + \frac{c\sigma^4}{4} . \quad (4)$$

Such a model of nuclear matter was first considered by Schiff,⁵ who showed that it can lead to nuclear matter saturation. The properties of nuclear matter for a specific choice of b and c were considered by Boguta and Bodmer⁶ and Boguta and Rafelski⁷ (called the BB-model). The parameters were determined by fitting nuclear matter compressibility and surface energy together with binding energy, saturation density and symmetry energy. The choice of $b = c$ leads to a model considered by Walecka.⁸ Recently, the author has considered a model⁹ (called the B-model) in which the values of b and c were fitted to reproduce the energy and density behavior of the single particle potential in nuclear matter predicted by the variational calculations of Pandharipande and Friedman.¹⁰ This model describes closed shell nuclei very well.¹¹ In this work we investigate the effects of beta stability in the BB-model and the B-model. These models are of interest because both saturate symmetric nuclear matter at $\rho_0 = 0.1625/\text{fm}^3$ with binding energy of -15.75 MeV/particle and their low density equations of state are almost identical. The bulk properties of nuclear matter are equally well described by both models.

For infinite nuclear matter, assuming translational and rotational invariance for the ground state, the equations of motion become

$$m_s^2 \sigma + b\sigma^2 + c\sigma^3 + g_s[\rho_s(p) + \rho_s(n)] = 0 \quad (5a)$$

$$m_v^2 \omega_0 - g_v[\rho_v(p) + \rho_v(n)] = 0 \quad (5b)$$

$$m_v^2 R_0^{(0)} - g_r[\rho_v(n) - \rho_v(p)] = 0 \quad (5c)$$

where ω_0 is the time component of the omega field and $R_0^{(0)}$ is the time component of the neutral rho field. The energy density ϵ is given by

$$\epsilon = \frac{1}{2} \left(\frac{g_v}{m_v} \right)^2 [\rho_v(n) + \rho_v(p)]^2 + \frac{1}{2} \left(\frac{g_r}{m_v} \right)^2 [\rho_v(n) - \rho_v(p)]^2 + U(\sigma) + 2 \int^{k_F(n)} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + m^{*2}} + (n \leftrightarrow p) \quad (6)$$

$$m^* = m_N + g_s \sigma \quad (7)$$

The pressure P at zero temperature is given by

$$P = \rho_B^2 \frac{d(\epsilon/\rho_B)}{d\rho_B} \quad (8a)$$

with

$$\rho_B = \rho_v(n) + \rho_v(p) \quad (8b)$$

The Fermi energies of the neutrons and protons are

$$E_F(n) = g_v \omega_0 + g_r R_0^{(0)} + \sqrt{k_F(n)^2 + m^{*2}} \quad (9a)$$

$$E_F(p) = g_v \omega_0 - g_r R_0^{(0)} + \sqrt{k_F(p)^2 + m^{*2}} \quad (9b)$$

The requirement of beta stability implies that the reaction $n \rightarrow p + e^- + \bar{\nu}_e$ has reached an equilibrium. That is

$$E_F(n) = E_F(p) + k_F(e) \quad (10)$$

where we assume that electrons are relativistic ($k_F(e) \gg m_e$). Charge neutrality requires that

$$k_F(p) = k_F(e) \quad (11)$$

Beta stability then becomes

$$2 \left(\frac{g_r}{m_v} \right)^2 [\rho_v(n) - \rho_v(p)] + \sqrt{k_F(n)^2 + m^{*2}} - \sqrt{k_F(p)^2 + m^{*2}} = k_F(p) \quad (12)$$

We have implicitly assumed that the neutrons and protons have the same effective mass m^* . This means that we neglect the effects of the 0^{++} isovector field. This approximation is justified because the scalar meson nonet is taken to be $Q^2\bar{Q}^2$ quark configuration, and the isovector member has two strange quarks.¹²

For a nonrotating neutron star the equation of hydrostatic equilibrium is given by the Tolman-Oppenheimer-Volkov equations. They are

$$\frac{dP(r)}{dr} = - \frac{G[d(r) + P(r)/c^2] [m(r) + 4\pi r^3 P(r)/c^2]}{[r - 2Gm(r)/c^2]r} \quad (13a)$$

$$\frac{dm(r)}{dr} = 4\pi r^2 d(r) \quad (13b)$$

where $P(r)$ is the pressure and $m(r)$ is the amount of mass the spherically symmetric neutron star has up to a distance r away from the center of the star. Here $d(r)$ is the energy-mass density. The relationship between the pressure and the particle density, or the energy-mass density, is given by the equation of state. Given a central energy-mass density d_c one computes the neutron star mass $m(R)$ and the radius R by integrating the Tolman-Oppenheimer-Volkov equations to the point $P(R) = 0$.

The pressure P as a function of energy-matter density (d_c) is obtained by solving the field equations Eq. (5a-5c) together with the beta stability equation Eq. (12) for given values of $C_s = (g_s/m_s)/m_N$, $C_v = (g_v/m_v)m_N$, $C_r = (g_r/m_r)m_N$, $b/g_s^3 = \bar{b}$ and $c/g_s^4 = \bar{c}$. For the BB-model the values are $C_s = 8$, $C_v = 3$, $C_r = 5.8$, $\bar{b} = 1.76$, $\bar{c} = 6.82$. For the B-model they are $C_s = 15.6$, $C_v = 12.5$, $C_r = 4.4$, $\bar{b} = -5.2 \times 10^{-3}$, $\bar{c} = 2.46 \times 10^{-3}$. Both models saturate infinite symmetric nuclear matter at a density of $\rho = 0.1625/\text{fm}^3$ with a binding energy of -15.75 MeV/particle with a symmetry

energy coefficient of 35 MeV. The BB-model has nuclear compressibility of $K = 200$ MeV and the B-model has $K = 280$ MeV. The two models derive nuclear saturation by different mechanisms. While in the BB-model the attraction and repulsion come mainly from the scalar field, that is, the nuclear force is highly non-Yukawa, in the B-model repulsion comes mainly from the ω -field with a small admixture of many body effects (σ^3 and σ^4 contributions). The implication of this for beta stability is dramatic.

In Fig. 1 we show the pressure as a function of baryon density for the BB-model and B-model for pure neutron matter [$\rho_v(p) = 0$] and for beta stable neutron matter. In the BB-model the difference in the equation of state for beta stable and pure neutron matter is very significant. For the B-model the difference in the equations of state is relatively small. The proton concentration in beta stable neutron matter at about $4\rho_0$ in both models is 30%.

The neutron star mass, as a function of energy-mass density is obtained by solving the Tolman-Oppenheimer-Volkov equations, Eq. (13a-13b), given the equation of state and the central density of the star. In Fig. 2 we show the results for B-model and BB-model for pure neutron matter and beta stable neutron matter. As expected, the mass of the star as a function of the central mass density in the B-model changes by about 20%, in beta stable stars. The central density at the maximum mass increases by about 10% since the equation of state is softer for beta stable neutron matter. The characteristics of neutron stars in the BB-model are considerably modified when the beta stability requirement is imposed.

The B-model and the BB-model were constructed to reproduce known bulk properties of symmetric nuclear matter at saturation. At, or below, nuclear saturation density, the two equations of state are almost identical. Thus we can conclude that the knowledge of bulk properties of nuclear matter at or below saturation is by far insufficient to determine the properties of beta stable neutron stars. Furthermore, the requirement of beta stability can rule out the BB-model as a reasonable model, which could not have been done just from the knowledge of the bulk properties of matter alone. The importance of beta stability cannot be prejudged beforehand but must be verified in each model to see whether it does not dramatically alter the predicted neutron star properties.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

References

- 1) For a review of neutron stars and a detailed list of references see V.R. Pandharipande, D. Pines and R.A. Smith, Ap. J. 208, 550 (1976)
G. Baym and C. Pethick, Ann. Rev. Nucl. Sci. 25, 27 (1975)
W.D. Arnett and R.L. Bowers, Ap. J. Supp. Series 33, 415 (1977)
- 2) S.A. Chin and J.D. Walecka, Phys. Lett. 52B, 24 (1974)
- 3) L. Rosenfeld, in Astrophysics and Gravitation, Proc. Solvay Conference on Physics, 16th. Brussels: Editions de l'Universite de Bruxelles, p. 174

- 4) J. Boguta, Remarks on the Beta Stability in Neutron Stars, LBL-11465
(to be published)
- 5) L.I. Schiff, Phys. Rev. 84, 1 (1951); 84, 10 (1951)
- 6) J. Boguta and A.R. Bodmer, Nucl. Phys. A292, 413 (1977)
- 7) J. Boguta and J. Rafelski, Phys. Lett. 71B, 22 (1977)
- 8) J.D. Walecka, Ann. Phys. 83, 491 (1974)
- 9) J. Boguta, Density Dependence of the Single Particle Potential in
Nuclear Matter, LBL-12628
- 10) B. Friedman and V.R. Pandharipande, Phys. Lett. 100B, 205 (1981)
- 11) J. Boguta, Finite Nuclei in Non-Linear Relativistic Field Theory of
Nuclear Matter, LBL-13166
- 12) R.L. Jaffe, Phys. Rev. 15D, 267 (1976)

Figure Captions

- Fig. 1. Pressure as a function of baryon density in various relativistic field models.
- Fig. 2. Neutron star mass as a function of mass density in various relativistic field models.

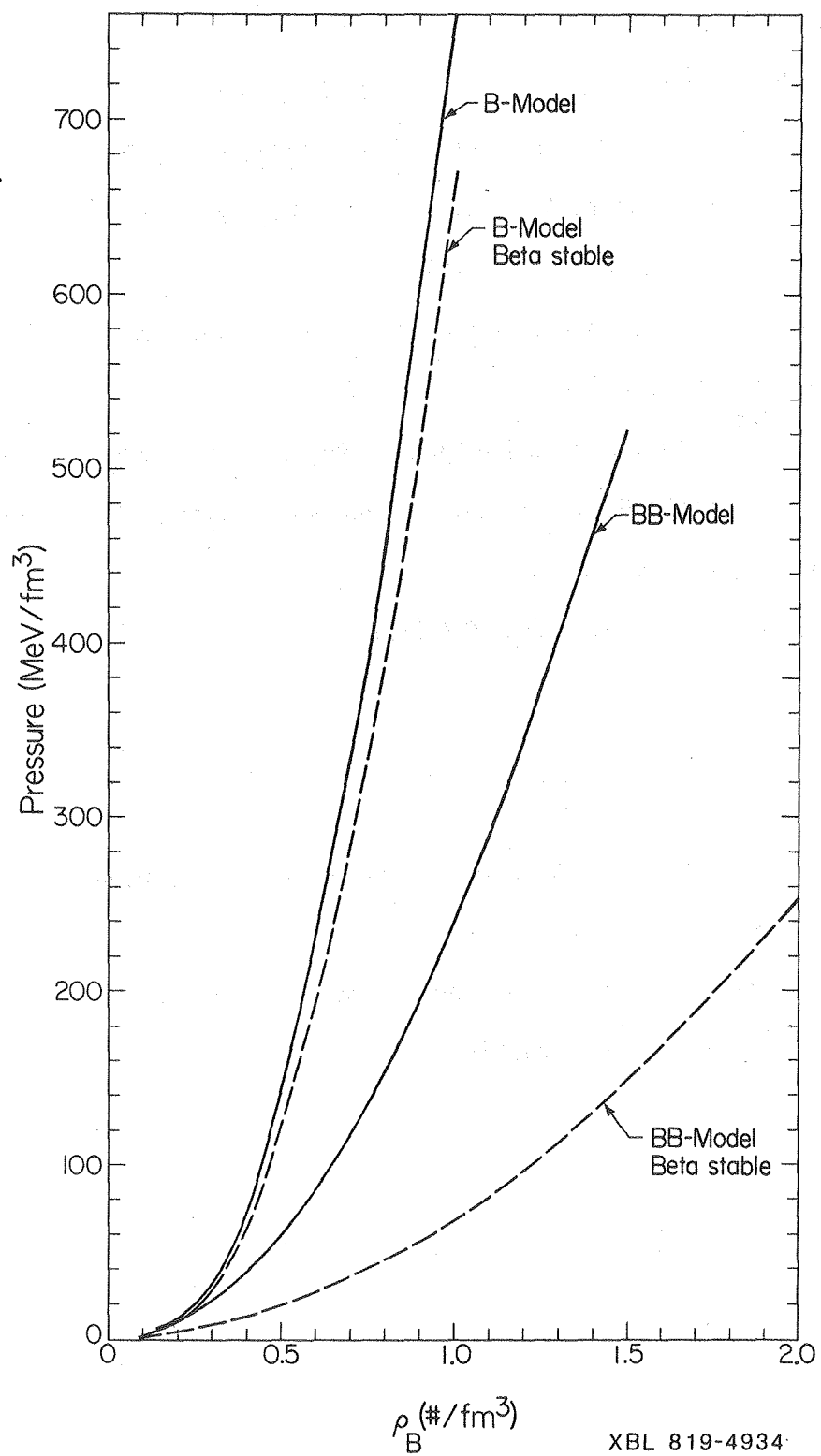


Fig. 1

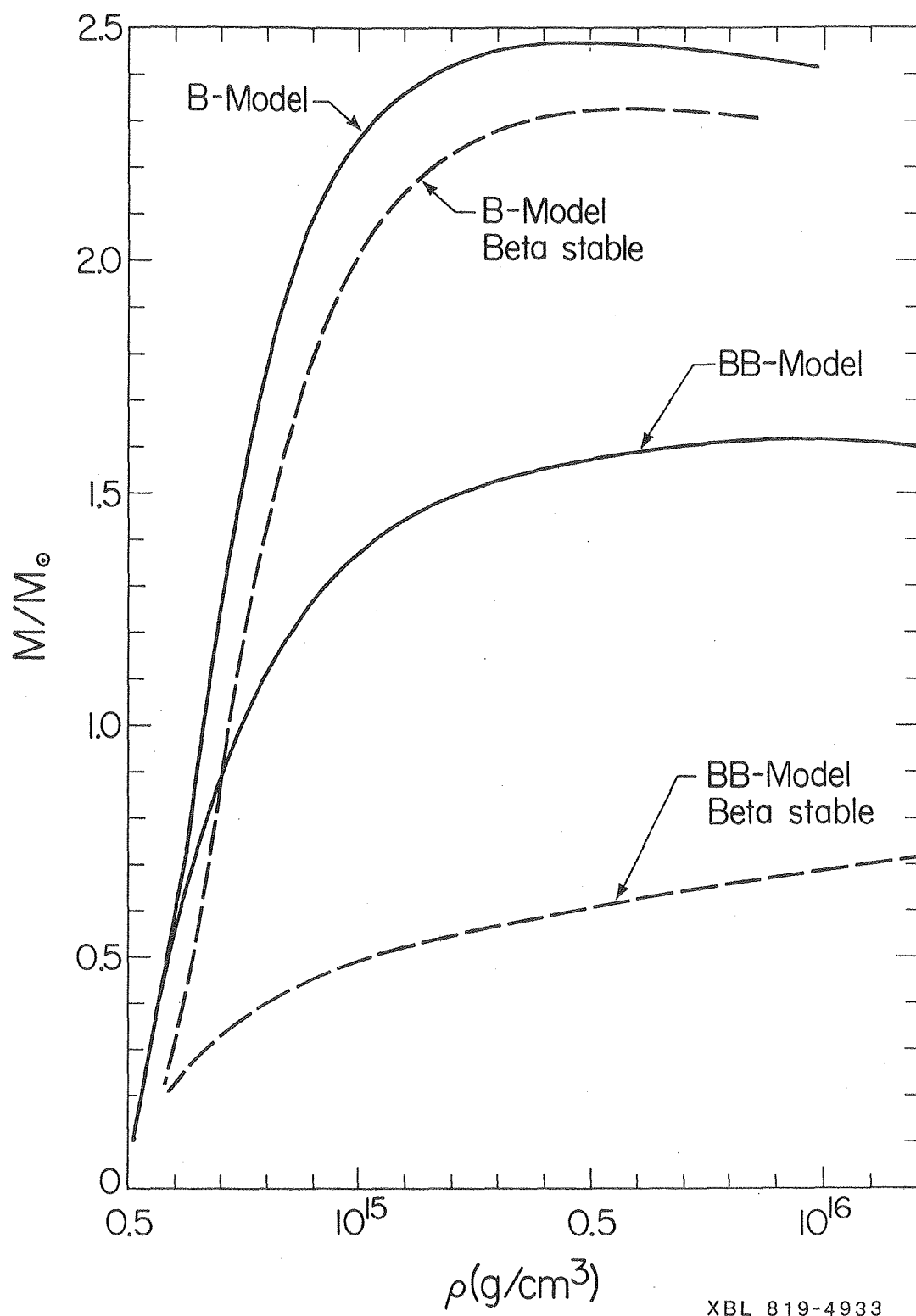


Fig. 2

