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Abstract

An explicit expression for the multiplicity distribution $P(n)$ of secondaries from high energy collisions is derived (in an easily justifiable approximation) for the case of a partially coherent source with a finite autocorrelation length in the chaotic component. Until now analytical expressions were available only for the cumulants of this distribution.

Recently, much interest has been centered around large fluctuations in rapidity density in narrow rapidity windows ("spikes"). Such spikes have been investigated (for a review, see [1]) in the context of intermittent emission patterns, possibly connected with fractal properties of the emitting process, as well as in the hope of observing signals for quark-gluon-plasma formation. Natural choices for *global* characteristics of such spikes in large collections of events are factorial moments and/or cumulants. As was shown in ref.[2], the behavior of the factorial moments allows interpretations within the framework of quantum statistics (QS) as well.

It would be of interest to have the ability of judging *individual* events like, e.g., the famous spike reported by the NA22 Collaboration [3], in terms of the simple probability for the number of tracks observed in a given rapidity window to exceed some reasonable limit. This endeavor was until now frustrated by the fact that the QS formalism for sources with finite correlation lengths, as formulated in optics by Jaiswal and Mehta [4] and applied to particle physics by Fowler and Weiner [5,6], provided closed expressions only for the factorial cumulants μ_k.

However, an explicit expression for the multiplicity distribution $P(n)$ itself is needed, as the desired measure for the improbability of a spike of multiplicity $n > n_c$ is given by

$$F(n_c) \equiv \sum_{n=n_c+1}^{\infty} P(n). \tag{1}$$

A great effort to solve this problem numerically was undertaken by Shih [7]. However, that approach involves tremendous computational resources, and since no analytic expression is obtained, the entire calculation has to be repeated any time one wants to try a new set of parameters.

We present here a simpler approach based on the observation that a certain (very involved) function appearing in the factorial cumulants reveals, upon numerical inspection, a trivial (exponential) behavior.

Given the generating function of the factorial moments, $G(s)$, the desired distribution $P(n)$ is obtained from

$$P(n) = \frac{(-1)^n}{n!} \frac{d^n}{ds^n} G(s)\bigg|_{s=1} \tag{2}$$

In the following we derive an expression for $G(s)$ and use it to obtain $P(n)$ by means of symbolic manipulation programs (Macsyma).

The Jaiswal Mehta formalism [4] describes the very general case of a multiplicity distribution for partially coherent fields, where the correlations in the chaotic component are characterized by a finite correlation length ξ,

$$\begin{aligned} \gamma(y_1, y_2) &\equiv \; < \pi(y_1)\, \pi(y_2) > \\ &= \; < n >_{chaotic} \exp\left(-\frac{|y_1 - y_2|}{\xi}\right) \end{aligned} \tag{3}$$

where $< n >_{chaotic}$ is the mean multiplicity of chaotically produced particles. Thus, the multiplicity distribution $P(n)$ for a rapidity interval of width ΔY is characterised by its mean value $< n >$, the chaoticity parameter p,

$$p \equiv \frac{< n >_{chaotic}}{< n >}, \tag{4}$$

the ratio

$$\beta \equiv \frac{\Delta Y}{\xi} \tag{5}$$

and the number of independent particle species considered, ν.

For $\xi \rightarrow 0$, $P(n)$ is explicitly given by the Glauber-Lachs distribution [8], which has as limiting cases the Poisson and the negative binomial distribution (at p=0 and p=1, respectively).

For finite $\xi \neq 0$ and $p \neq 1$, no analytic expression for $P(n)$ can be derived. However, one may analytically calculate the factorial cumulants μ_k, which are defined in terms of the generating functions

$$\begin{aligned} G(s) &= \sum_{n=0}^{\infty} (1-s)^n \, P(n) \\ &= \exp[H(s)] \end{aligned} \tag{6}$$

and

$$H(s) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} s^k \mu_k \tag{7}$$

One finds [4]

$$\mu_k = \frac{< n >^k}{\nu^{k-1}} [(k-1)! p^k B_k(\beta) + k! p^{k-1} (1-p) \bar{B}_k(\beta)] \tag{8}$$

where the constants B_k, $\bar{B}_k$ are given by

$$\begin{aligned} B_k &= \frac{1}{(\Delta Y)^k} \int_0^{\Delta Y} dy_1 \cdots \int_0^{\Delta Y} dy_k \; \gamma(y_1 - y_2) \, \gamma(y_2 - y_3) \cdots \gamma(y_k - y_1) \\ \bar{B}_k &= \frac{1}{(\Delta Y)^k} \int_0^{\Delta Y} dy_1 \cdots \int_0^{\Delta Y} dy_k \; \gamma(y_1 - y_2) \, \gamma(y_2 - y_3) \cdots \gamma(y_{k-1} - y_k) \end{aligned} \tag{9}$$

Substitution of eq(8) into eq(7) allows us to express $H(s)$ as

$$H(s) = H_1(s) + H_2(s) \tag{10}$$

where

$$H_1(s) = \nu \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \left(\frac{p<n>s}{\nu}\right)^k B_k(\beta) \tag{11}$$

is formally identical with the generating function for a completely chaotic system with mean multiplicity $<n>_1 = p<n>$; for the latter case, an analytic expression has been found [4] which we may use here to obtain

$$H_1(s) = \nu\,[\beta - \ln\{\cosh(z) + \frac{1}{2}(\frac{z}{\beta} + \frac{\beta}{z})\sinh(z)\}] \tag{12}$$

with

$$z \equiv \sqrt{\beta^2 + \frac{2\beta p<n>s}{\nu}} \tag{13}$$

To further evaluate the second term on the rhs of eq(10),

$$H_2(s) = \nu\,\frac{(1-p)}{p} \sum_{k=1}^{\infty}(-1)^k \left(\frac{p<n>s}{\nu}\right)^k \bar{B}_k(\beta) \tag{14}$$

we note that the k-dependence of the constants $\bar{B}_k$ can to a good approximation be described by an exponential fit,as shown in Fig.1.

$$\bar{B}_k(\beta) = a(\beta)\,\exp[-b(\beta)\,k] \tag{15}$$

With such an ansatz, the sum in eq(14) is reduced to a geometric series, and, after collection of all the different terms, the complete generating function for $P(n)$ takes the form

$$G(s) = \left(\frac{\exp[\beta + b\,\frac{1-p}{p}\,(w-1)]}{\cosh(z) + \frac{1}{2}(\frac{z}{\beta} + \frac{\beta}{z})\sinh(z)}\right)^{\nu} \tag{16}$$

where $z = z(s)$ is defined in eq(13), and

$$w(s) = \left(1 + exp(-b)\,\frac{p<n>s}{\nu}\right)^{-1} \tag{17}$$

Successive expressions, albeit increasingly involved, for $P(n)$ can be derived and stored once and for all by symbolic computation. Evaluation with required sets of parameters then becomes trivial.

Fig.2 shows an example of $P(n)$ for various values of β. The quantity of interest for the study of spikes, viz. $F(n_c)$, is shown for the case $n_c = 8$ in Fig.3 as a function of the chaoticity p. As an example we consider the specific case of the NA22 event, where $< n >\simeq 0.6$; the probaility for such a fluctuation is seen to be very low indeed as long as the emitting source is largely coherent (small p); at large p the probability increases and becomes also quite dependent (within a factor of 2) on the autocorrelation length ξ.

The authors would be glad to provide the symbolic programs used here to any interested reader.

We are indebted to C.C.Shih for useful discussions and for the use of his routines allowing computation of $\bar{B}_k$ beyond the published expresssions ($k \leq 5$).

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Figure Captions

Fig.1 The functions B_k (solid lines) and $\bar{B}_k$ (dotted lines) plotted against k, for β =0.1, 0.2, 0.5, 1.0, 2.0 and 5.0 (from top to bottom).

Fig.2 The multiplicity distribution $P(n)$, for a mean multiplicity $< n >= 4$ and a chaoticity parameter value $p = 0.8$. The curves are for β =0.1 (solid), 0.2 (dotted), 0.5 (dashed) and 1.0 (dashdotted).

Fig.3 The probability to count more than 8 particles as a function of the chaoticity p, for $\beta = 0.1$ (solid lines) and $\beta = 1.0$ (dotted lines). The two upper curves correspond to a mean multiplicity $< n >= 3$, the two lower curves to $< n >= 1$.

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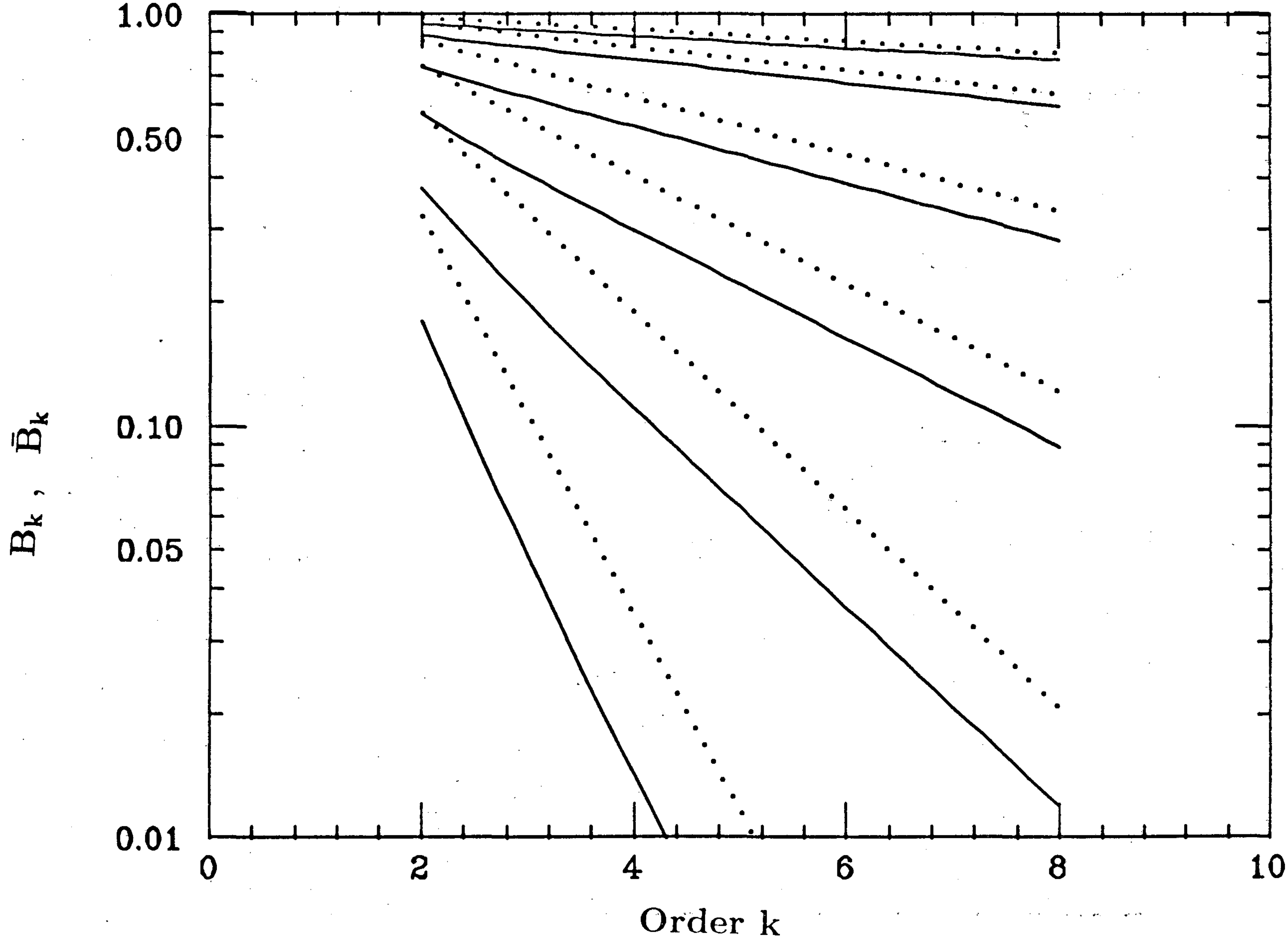

Fig. 1

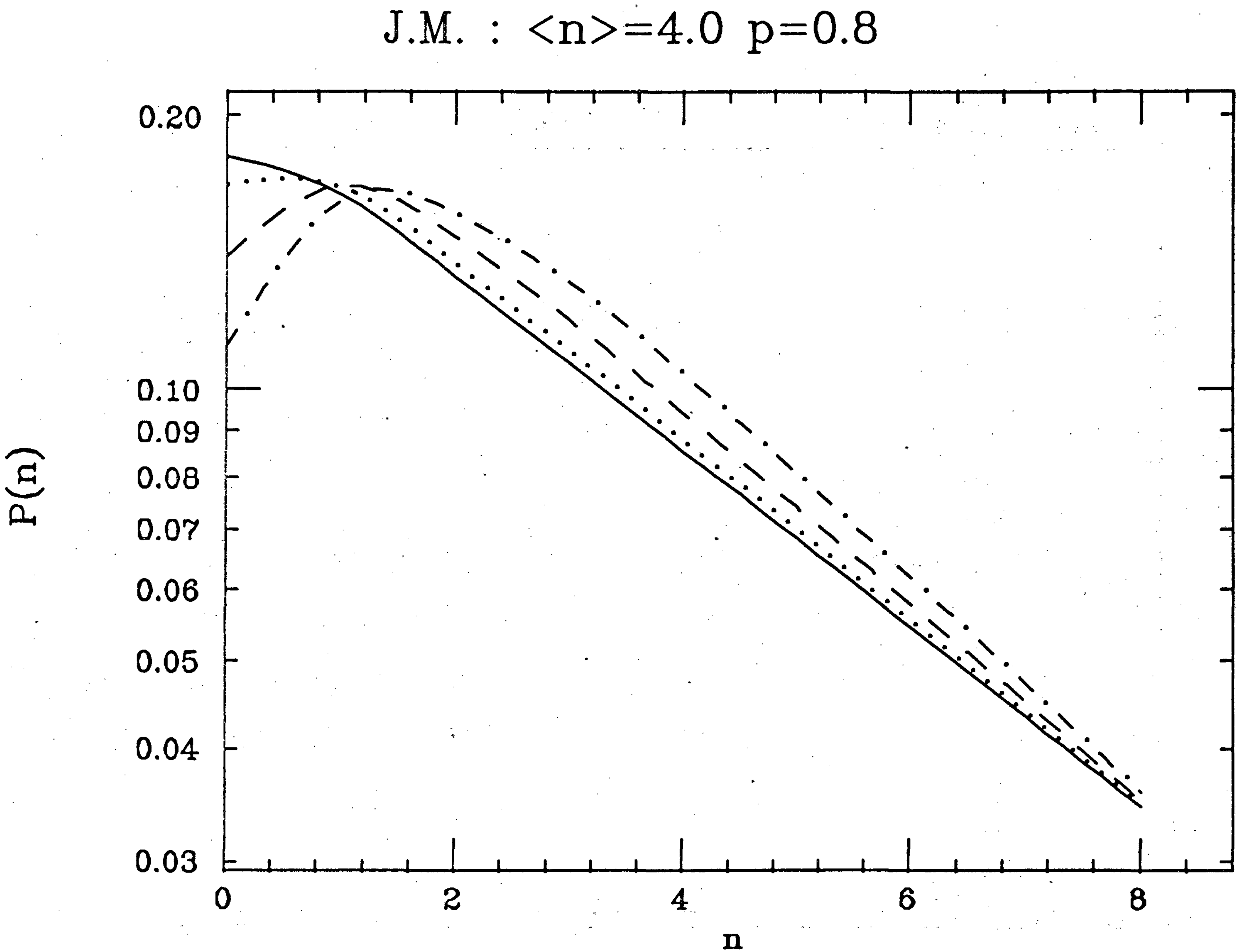

Fig. 2

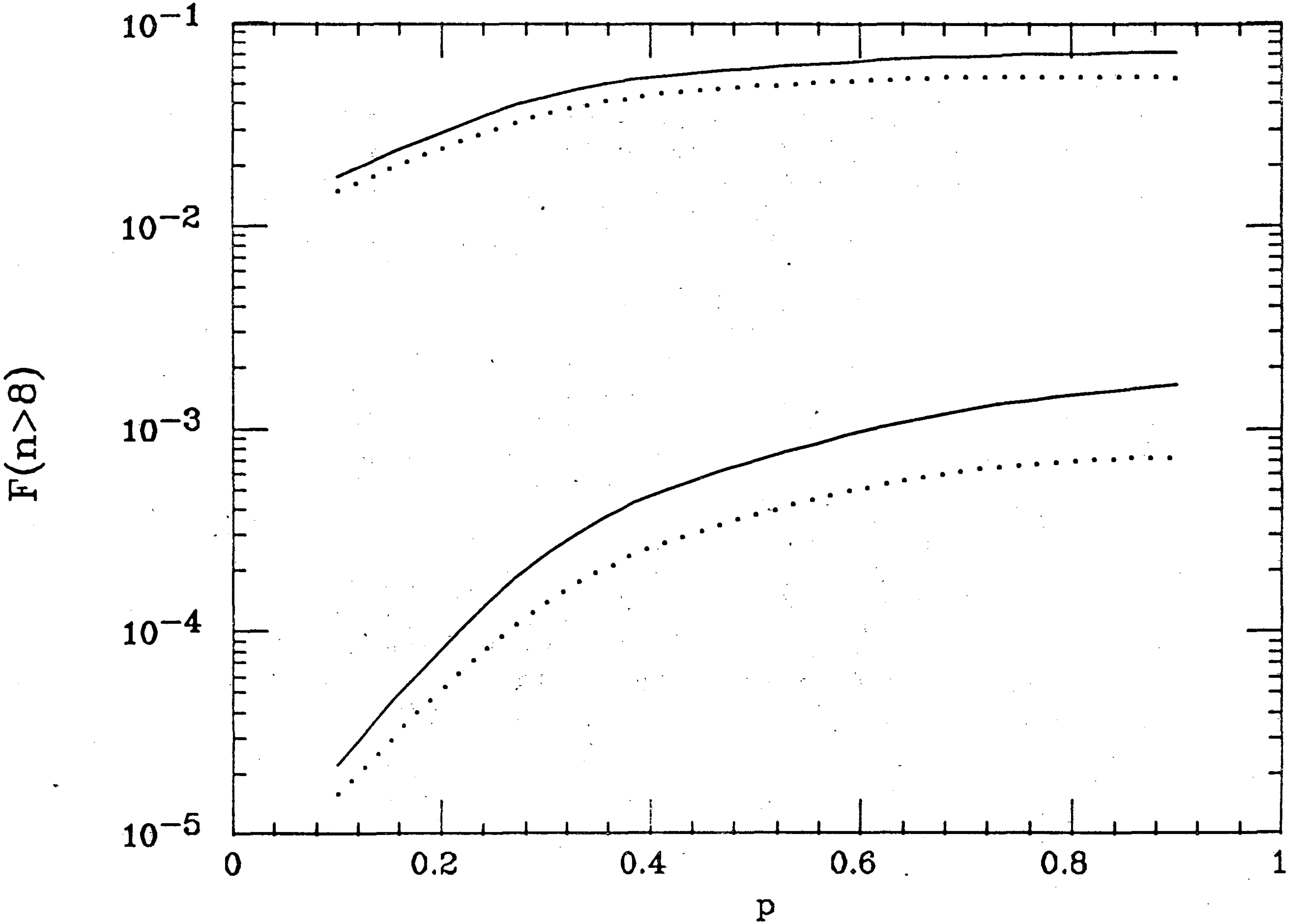

Fig. 3