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Beam Bunching in a Final Storage Ring

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In this note, we consider the possibility of carrying out the final bunching of a particle beam for heavy ion fusion in a storage ring or synchrotron using parameters as follows:

Total kinetic energy	$Q = 1$ Megajoule
Kinetic energy per particle	$= 35$ GeV
Burst duration on target	$\Delta t = 6$ nanosec[a]
Particle mass (Bismuth)	$Am_p = 209\ M_p = 196\ \mathrm{GeV}/c^2$
Particle charge	$qe = 1\ e$
Ring radius	$R = 400$ meters

From these we calculate for later use

$\beta = v/c = 0.529$

$\gamma = (1-\beta^2)^{-1/2} = 1.18$

$N = 1.78 \times 10^{14}$ total particles

For the bunching calculation, we shall use the approach of D. Judd[1] in which the length L of a beam of particles acted on by an electric field with linear gradient $dE/d\ell = 2E_{max}/L$ evolves according to the equation [MKSA units]

$$\frac{d^2L}{dt^2} + \frac{2qe}{Am_p\gamma^3}\left[E_{max} - \frac{6gqeN}{(4\pi\varepsilon_0)\gamma^2L^2}\right] - \left(\frac{4c\varepsilon_\ell}{\gamma^3}\right)^2 \frac{1}{L^3} = 0 \qquad (1)$$

where the geometrical factor $g \approx 1.5$,

(a) Beam power during the six nanosecond portion of the pulse should be 100 terawatts. The use here of 1 megajoule, or 170 terawatts may be regarded as an over-specification affording some degree of safety factor.

$$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \text{ meter/farad},$$

and $\pi\varepsilon_\ell$ = longitudinal emittance area in $(z, \Delta\beta\gamma)$ space.
This equation may be adapted to motion in a ring under the action of a sinusoidal voltage $V \sin h\theta$ per turn with N/h particles per bunch by setting

$$E_{max} = \frac{hVL}{4\pi R^2}, \text{ (with } \frac{2hL}{\pi R} < 1 \text{ for linearity)},$$

and replacing γ^{-3} by $(-\eta/\gamma) = (\gamma^{-2} - \gamma_{tr}^{-2})\gamma^{-1}$,

N by N/h, and ε_ℓ by ε_ℓ/h.

Following Judd's definition of dimensionless variables, the equation becomes

$$\frac{d^2\rho}{d\tau^2} + \frac{1}{2}\left(\rho - P\rho^{-2} - \frac{1}{2}S\rho^{-3}\right) = 0 \qquad (2)$$

in which

$$\rho = L/L_o$$

$$\tau = t/T$$

$$L_o = L \text{ at } t = o$$

$$P = \frac{6\, gq\, eN\, 4\pi R^2}{(4\pi\varepsilon_o)\gamma^2 h^2 V\, L_o^3}$$

$$S = \frac{16\, Am_p c^2(-\eta)\varepsilon_\ell^2 4\pi R^2}{qe\gamma h^3\, V\, L_o^4}$$

$$T^2 = \frac{Am_p\gamma\;\; 4\pi R^2}{4qe(-\eta)hV}$$

For our case, assume $S \ll P$ to allow dropping the emittance term and let $d\rho/d\tau = 0$ at $T = 0$. A first integral is then

$$\left(\frac{d\rho}{d\tau}\right)^2 = (1-\rho)\left(\frac{1+\rho}{2} - \frac{P}{\rho}\right). \qquad (3)$$

If most of the bunching takes place while circulating in the ring and a large compression is obtained, as needed, then $P \ll 1$ and the final minimum bunch length is approximately (as found by Judd)

$$\rho_{min} \approx 2P. \qquad (4)$$

From this result and the definitions of P and ρ we can calculate the voltage per turn, V, needed to produce a 6 nanosecond bunch of length

$$L_{min} = \beta c\Delta t = (0.529)(3 \times 10^8)(6 \times 10^{-9}) = 0.9525 \text{ meter.}$$

The beam must initially have a bunching factor B_f of about 1/4 to assure linearity of the bunching field used. This initial length is

$$L_o = \frac{2\pi RB_f}{h} \quad \text{with} \quad B_f \simeq 1/4.$$

Substituting in equation 4 we obtain

$$V = \frac{12\, g\, q\, e\, N}{(4\pi\varepsilon_o)\gamma^2 L_{min}\, \pi(B_f)^2}$$

$$= \frac{(12)(1.5)(1.6 \times 15^{-19})(1.783 \times 10^{14})(9 \times 10^9)}{(1.39)(0.9525)\, \pi(1/16)}$$

$$= 17.78 \text{ MV/turn.}$$

To supply this bunching voltage with r.f. cavities, assume that 20% of the circumference can be filled with cavities that could provide an average voltage gradient at frequency f of $50(f/10^6)^{1/2}$ kV/m. In a full turn, then, the bunching voltage would be

$$V = (0.2)(2\pi R)(5 \times 10^4)(h\beta c/2\pi R\ 10^6)^{1/2}$$

$$= 6.31\ h^{1/2} \text{ MV/turn}$$

In this case, $h = 8$ would be needed to give 17.85 MV/turn at a frequency of 0.505 MHz.

Before adopting a harmonic number, we must examine the transverse space charge conditions during the bunching in the ring. We shall propose that some final bunching shall occur after leaving the ring in $\pi 200 = 628$ meters of transport to the target. While still in the ring, one could hope to bunch to the point where the space charge fields shift ν^2 to one-half its normal value, i.e. $\Delta\nu^2/\nu^2 = -0.5$. This limit, if permissible, would allow a maximum bunching factor while in the ring given by: *

$$\frac{1}{B_f} \approx -(\pi\beta\gamma\varepsilon)\ \frac{A\beta\gamma^2\nu}{q^2 N r_p}\ \frac{\Delta\nu^2}{\nu^2} \qquad (5)$$

Use $\beta\gamma\varepsilon = 10^{-5}$ radian meters emittance

$\nu \approx 10$

$r_p = 1.53 \times 10^{-18}$ meter

$$\text{then } \frac{1}{B_f} = \pi 10^{-5}\ \frac{(209)(0.529)(1.39)(10)}{(1.78 \times 10^{14})(1.53 \times 10^{-18})(2)} = 88.64$$

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* This formula neglects the substantial change in ν when $\Delta\nu^2$ is large. Hence it underestimates the allowed limit by a factor $\left(1 + \frac{\Delta\nu^2}{\nu_o^2}\right)^{-1/2} = (1-0.5)^{-1/2} \doteq \sqrt{2}$. Alternatively, one could say that the numerical result above corresponds more correctly to a change of ν from 10 to 7.81.

(For comparison, the value for $\Delta\nu = 1/4$ is $1/B_f = 8.86$, smaller by just the factor $\nu = 10$). At the target we must have a final bunching that would correspond to

$$\frac{1}{B_f} = \frac{2\pi R}{hL_{min}} = \frac{800\ \pi}{0.9525\ h} = \frac{2639}{h}$$

A detailed and correct treatment of the bunching process will not be attempted here, but it will be useful to note that over the major part of the process in the ring, the bunch length is given by the approximate integral of equation (3)

$$\rho \approx \cos \tau / \sqrt{2}. \qquad (6)$$

This expression gives the approximate bunching time as $\tau \approx \pi/\sqrt{2} = 2.22$ or $t \simeq 2.22\ T$. An evaluation of T gives

$$T = \frac{317}{h} \times 10^{-6} \text{ sec.}$$

Hence the bunching time is about

$$t = (2.22)\ \frac{317}{h} \times 10^{-6} = \frac{704}{h} \text{ microseconds.}$$

In terms of turns in the ring this is

$$\frac{\beta ct}{800\,\pi} = \frac{44.47}{h} \text{ turns .}$$

A drift distance to the target equal to one quarter turn occurs outside the ring and we may ask what value of h would permit the bunching to be less than 88.6 at extraction. Using the approximate equation (6), we find that the harmonic number must be greater than 5.1. The correct bunching formula will require a larger number, so the space charge condition as well as the need to raise the r.f. cavity frequency indicate a value of $h \simeq 8$, which we shall adopt.

The parameters of the bunching process are then:

$h = 8$, frequency = 0.5053 MHz

$V = 17.85$ MV/turn, e.g. 503 m (20% of circumference) of cavities at 35.5 kV/m

Bunching at start	= 4.0	
at extraction	= 56.6	
at target	= 329.9	
Bunching period in ring	= 5.53 turns	
Distance to target	= 0.25 turn	= 628 meters

The foregoing has explored only bunching by r.f. cavities in the ring. Stronger fields could be generated by pulsed cavities, but no examination of the use of pulsed cavities in a storage ring has been made. Pulsed cavities could, of course, be used in the external transport lines to augment and complete the bunching initiated in a ring.

Ref. 1 - D. Judd, paper in this publication, p.

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