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Alpha Decay Theoretical Calculations for
125-Neutron Nuclei

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ABSTRACT

We have applied the new normalized and an older shell-model alpha decay theory to the calculation of decay rates of three 125-neutron isotones ^{209}Po , ^{211}Rn , and ^{213}Ra . We also have applied the theory to the 2.1-ms isomer of ^{213}Ra and are able thus to select a $17/2^-$ assignment as preferable to the $13/2^+$ alternative.

Introduction

During our experimental work on the 2.1-ms $^{213\text{m}}\text{Ra}$ isomer, we found it desirable to make a rough calculation of possible alpha decay branching. Previously, shell model alpha decay theory for odd-mass spherical nuclei has been generally applied [1] only to nuclei close to the doubly-closed system ^{208}Pb , such as ^{211}Po and ^{211}Bi . These cases are simplest to treat, since the shell model states in parent and daughter are relatively pure. Almost as simple are the cases here treated of alpha decay of 125-neutron isotones to 123-neutron daughters, since 124 neutrons is a subshell before the filling of the $P_{1/2}$ neutrons.

Recently the shell model alpha decay theory was carefully re-examined and a revised version derived [2] with a renormalization of the alpha projection probability amplitude $G(R)$, designated in the revised theory $G_N(R)$. The revised theory [3] systematically enhances the absolute decay rates but does not much affect the relative rates in ^{211}Po and ^{211}Bi .

There is experimental alpha decay information from ground states of the 125-neutron isotones ^{209}Po , ^{211}Rn , and ^{213}Ra . Furthermore, the alpha branching of $^{213\text{m}}\text{Ra}$ is reported in the preceding article [4], and indeed these new results were a stimulus to make the alpha theoretical calculations here described.

Decay Rate Calculation from the 1/2- Ground States

The neutron configuration of each parent 125-neutron isotone will be assumed as a $p_{1/2}^{-1}$ hole in the 126 neutron shell. In the 123-neutron daughter states there will be appreciable configuration mixing, and it is essential that the mixing be taken into account. We use the ^{205}Pb wave functions of Arvieu et al[5] in this study. As they point out, the main mixing is of pairing type among seniority-1 states. The neutron-hole pair, coupled to zero, is distributed among the neutron orbitals of the 82-126 shell with amplitudes given in their Table 3.

Consider first the decay to the 5/2- ground state, the dominant configuration of which is $p_{1/2}^{-2}f_{5/2}^{-1}$. The $L=2$ alpha-decay matrix element connects with this configuration but not with any of the others, which have a filled $p_{1/2}^2$ orbital. Thus, the ground-to-ground matrix element is to be multiplied by 0.866, the amplitude for the connecting configuration.

Likewise, decay to the 3/2- second excited state proceeds only through its dominant $p_{1/2}^{-2}p_{3/2}^{-1}$ configuration with amplitude 0.828.

Decay to the 1/2- first excited state is a favored type of decay and consists of a coherent sum to all the seniority-1 components of the final wave function.

The intrinsic alpha-decay matrix elements were calculated with the computer code described in ref. [3]; both the older theory $G(R)$ values and the revised theory $G_N(R)$ values were computed. The renormalization by the revised theory requires specifying a closed core configuration, which we have taken as an approximation to be the ^{208}Pb configuration. The relative error of this approximation is estimated to be less than 15%.

Of alternative nearly equivalent reduced width formulas in ref.[3], we have chosen here to use eqs. (2.8) and (3.1).

$$\lambda = \delta^2 P/h$$

$$\delta_N^2 = 5 \text{ MeV} \left[R_m G_N(R_m) \right]$$

where P is the WKB barrier penetration factor and R_m is the value of the radial distance at which G_N attains its outermost maximum.

In this formula the value of $RG_N(R)$ in $\text{fm}^{-1/2}$ has to be inserted. If several shell model configurations contribute we have to use a superposition $G_N(R) = \sum A_j G_N^j(R)$. Here A_j is the amplitude of configuration admixture and $G_N^j(R)$ the reduced α -amplitude for the corresponding pure configuration.

The Coulomb barrier WKB penetration factors were calculated as in reference [3] by numerical integration between inner and outer turning points with Coulomb plus Woods-Saxon nuclear potential. This procedure is the same as that used much earlier [6] except that here we include the L -dependent centrifugal-barrier term. We have taken the same Woods-Saxon parameters as in ref.[3], $V_0 = -58.8 \text{ MeV}$, $r_0 = 1.454 \text{ fm}$ and $a = 0.560 \text{ fm}$, of one of the potentials given by McFadden and Satchler[7]. This potential yields penetrabilities which are in our cases 5 to 20% higher than

the ones calculated with the Woods-Saxon potential ($V_0 = -96.44$ MeV, $r_0 = 1.367$ fm and $a = 0.625$ fm) derived by Barnett and Lilley[8] in a more recent analysis. The range of the uncertainty in the resulting penetrability can be demonstrated by two extreme potentials: Igo's exponential potential[9] gives values which are larger by about a factor 2, and Igo's best fit Woods-Saxon potential[9], values which are smaller by about a factor 5.

Table 1 lists the various factors in the ground-state decay calculations, and the final decay rate constants are plotted in Fig.1. Only the revised theory values are used to calculate decay constants in Tables 1,2, and 3. The old theory gives smaller reduced amplitudes shown in square brackets beneath the new theory values. The notation is that numbers in parenthesis are the power of 10 multiplier for the preceding number.

We note rather good agreement for the relative rates of the unfavoured groups, with experimental decay constants varying over nine orders of magnitude. The overall prediction is, however, fast by a factor 10. The favoured decay to the $1/2^-$ final state is overestimated by an additional factor 4 or 5. This additional factor might well be attributed to the configuration mixing. It is well-known that a δ -interaction, as used by Arvieu, favors the higher angular momentum single particle states coupled to zero spin. In Arvieu's calculation the ratio of the amplitudes $A((f_{5/2})_0^2)$ is 1: 0.47 for ^{205}Pb and 1: 0.59 for ^{206}Pb . This is not necessarily so: A Gaussian interaction[10] gives 1: 1.22 for ^{206}Pb , and the pure G-matrix derived from the Hamada-Johnston potential[11] gives 1: 1.39. Therefore, a calculation with another interaction might well yield a dominant $(p_{1/2})^{-1}(p_{3/2})_0^{-2}$ configuration for ^{205}Pb .

Taking this pure configuration would decrease our rates by a factor 9.5. Therefore, we do not consider our favoured and unfavoured rates to be in serious relative disagreement.

There remains an overall factor of about 10 which is too large to be explained by the uncertainty in the penetrability discussed above. It is likely that this discrepancy is due to our primitive formula $\delta^2 = 5\text{MeV} (R_m G_N(R_m))^2$ for the reduced width using a channel radius simply at the outer maximum R_m of the reduced amplitude. The absolute rates for ^{211}Po and ^{211}Bi given by this formula were found[3] in good agreement with experiment. The difference to the present calculation is that here the radii R_m are typically 0.2 to 0.4 fm smaller than for ^{211}Po and ^{211}Bi . A better reaction theory could well be sensitive also to the position of these maxima. We have so far in these calculations neglected a correction factor for radon and radium of the square of a fractional parentage coefficient (c.f.p.) for the $(h_{9/2})_0^n \rightarrow (h_{9/2})_0^{n-2}$ proton configurations. This factor, which in pairing theory terms is simply 5 times the product of the fullness probability of $h_{9/2}$ in the parent times the emptiness probability in the daughter, is $5 \cdot \frac{n}{10} \cdot \frac{[10-(n-2)]}{10}$. That is, the $_{86}\text{Rn}$ rates should be multiplied by 1.6 and the $_{88}\text{Ra}$ rates should be multiplied by 1.8. The scatter of relative rates is not much affected by inclusion of the proton c.f.p., though the average absolute rate discrepancy is increased somewhat. Further complications may be introduced by considering proton pair configuration mixing with $f_{7/2}$ and $i_{13/2}$ orbitals besides $h_{9/2}$. Arvieu et al[5] in their Table 4 give a ^{210}Po wave function of

$0.833|h_{9/2}^2\rangle + 0.347|f_{7/2}^2\rangle - 0.317|i_{13/2}^2\rangle$, so proton configuration mixing may be quite significant. We have not wished to take the computer time to explore the complications of proton configuration mixing, since we believe the relative transition probabilities here will not be substantially altered.

Alpha Decay Rate Calculation from the ^{213m}Ra Isomer

For the 2.1-msec isomeric state of ^{213m}Ra , the energy is known within a few kilovolts but the spin and parity could not be uniquely determined from the decay data. The possible assignments are as the $\nu i_{13/2}^{-1}$ state, or as a $17/2^-$ state formed by the $(h_{9/2})_8^6$ protons coupled to the $p_{1/2}^{-1}$ neutron hole. We have therefore proceeded to calculate the alpha decay rates under each alternative assignment.

In this case, there is no longer a unique L value, but three values to the $5/2^-$ ground state, one to the $1/2^-$ state, and two to the $3/2^-$ state.

For the assumed $17/2^-$ state the calculation is closely analogous to the ground state decay, since the neutron configurations are the same. That is, the decay to $5/2^-$ and $3/2^-$ states connects with the dominant configuration in the 123-neutron daughter, and the decay to the $1/2^-$ state involves a coherent sum over the four final states with the hole pair in different orbitals. Table 2 gives the numerical results.

For the assumed $13/2^+$ state assignment with a single $i_{13/2}$ neutron hole, the alpha decay matrix element to the dominant configuration in the daughter vanishes and the decay proceeds via a smaller component in Arvieu's wave function, namely, the component

with a hole pair in the $i_{13/2}$ orbital. Table 3 gives the numerical results, and Fig. 1 plots the final decay rate constants.

It is evident from Fig. 1, despite the large uncertainty in the data, that the $17/2^-$ assignment is indicated for the isomer. The $13/2^+$ assignment would put a relatively large intensity to the $3/2^-$ second excited state, contrary to experiment. For the $17/2^-$ assignment the absolute theoretical rates exceed experiment, but this behavior is similar to that for the ground state decay of all the three 125-neutron isotones examined.

The illustration of the "electric transition" rule[12] for hindered alpha transitions can be seen in the alpha matrix element column of Table 3. For the $f_{5/2}$ final state the alpha wave function is projected from a coupling of $i_{13/2}$ and $f_{5/2}$, a $j=l+1/2$ and an $l-1/2$ orbital, respectively. The rule, arising from the singlet spin nature of the alpha particle, states that in such cases of unlike type the higher L couplings are preferred over lower L couplings, and this is seen to be the case by an increase of the reduced width of a factor of 12.5 in going from $L=5$ to 9. For these large L values the centrifugal barrier effect outweighs the intrinsic matrix element trend, and $L=9$ is predicted (last column of Table 3) to be an order of magnitude weaker than the $L=5$ component. For the decay to $p_{3/2}$ the orbitals are of like type, and the intrinsic matrix elements prefer the lower L coupling. The net effect of the "electric transition" rule is that hindrance factors will be higher for unlike orbital combinations. Thus, for the $i_{13/2}$ assignment the predicted intensity ratio of $3/2^-:5/2^-$ groups approaches

unity despite the energy difference. It is not justified to apply the rule to $j=1/2$ orbitals. The $17/2^-$ assignment gives a strong preference for ground state $5/2^-$ decay mainly because a lower L value (6) is allowed than for other groups.

It is of some theoretical interest to compare the relative reduced transition probabilities predicted by new and old theories with the predictions of the simplified delta-function alpha model [12]. In the delta function model the relative reduced rates are proportional simply to a ratio of squares of Clebsch-Gordan coefficients and the radial nuclear wave functions R_j (taken here from Blomqvist and Wahlborn, at a radius of 9 fm). So that these ratios will be small numbers greater than unity, the comparison rate is the $13/2^+ \rightarrow 5/2^-$ ($L=5$), the topmost entry in Table 3. Table 4 shows the relative reduced rates from the three forms of shell-model rate theory. For the finite-size alpha theories the entries are merely squares of ratios of matrix elements from column 5 of Tables 1 and 3. For the delta-function alpha theory, the entries are the ratios

$$R_{\text{RTP}} = \left[\frac{(2j_n+1)^{1/2} (L \ j_n \ 0 \ -1/2 | \ j_n' \ -1/2) R_n R_{n'}}{14^{1/2} (5 \ 13/2 \ 0 \ -1/2 | \ 5/2 \ -1/2) R_{i13/2} R_{f5/2}} \right]^2 \frac{(2L+1) \cdot 14^2}{11 (2j+L)^2}$$

The second factor on the r.h.s. comes from the integration over the daughter nucleus coordinates which results in this case in a factor $\frac{(2(2L+1))^{1/2}}{2j+1}$ in the reduced amplitude. Here j is $1/2$ or $13/2$, respectively.

From Table 4 we note first of all that there may be as much as an order of magnitude difference between relative rate predictions of the new theory and the delta function theory, with the old

finite-size theory usually predicting an intermediate value. The most striking difference is exhibited in the last line, where the new theory gives almost the same rate as for the first line and the delta theory gives a ratio an order of magnitude higher. However, it may be noted that similar predictions on the relative L mix within a given alpha group are made by the various theories.

Despite uncertainties associated with even the new normalized alpha decay rate theory, it was possible to use alpha decay branching to decide between alternative structural assignments for the radium-213 isomer.

Further extension of alpha rate theory beyond nearest neighbors of doubly magic ^{208}Pb should be done. To do so there is a need for shell-model calculations including configuration mixture of unpaired protons and neutrons.

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Table 1. Alpha Decay Rate Factors for 1/2- State Decay

Final State	Daughter Hole pair orbital j	Angular Momentum L	Max. Radius(fm) R_m [R_m] ^a	Outer Max. α Amplitude $R_m G_{\alpha}(R_m)$ [$R_m G_{\alpha}(R_m)$] ^a in fm ^{-1/2}	Amplitude of Configuration Admixture a_j	Z_i	Q_{α} including e ⁻ screening (MeV)	Barrier penetration factor P	Theo. Decay Constant λ (sec) ⁻¹	Exp. Decay Constant λ	Ratio Theo. : Exp.
5/2-	P _{1/2}	2	6.6 [6.9]	-3.407(-2) [-3.932(-3)]	0.856	84 86 88	4.978 6.000 6.893	3.188(-29) 9.101(-25) 7.319(-22)	5.321(-10) 1.519(-5) 1.222(-2)	b 1.12(-6) 1.89(-3)	-- 13.6 6.5
3/2-	P _{1/2}	2	6.8 [7.0]	3.649(-2) [2.659(-3)]	0.828	84 86 88	4.716 5.762 6.679	6.898(-31) 6.452(-25) 1.044(-22)	4.789(-12) 4.492(-7) 7.256(-4)	1.28(-12) 6.67(-8) 2.52(-4)	3.7 6.7 2.9
1/2-	f _{5/2}	0	6.4 [6.8]	1.077(-1) [4.270(-3)]	0.807	84 86 88	4.975 5.931 6.764	5.499(-29) 7.468(-25) 4.707(-22)	1.538(-8) 2.095(-4) 1.317(-1)	2.12(-10) 2.15(-6) 2.06(-3)	72.6 97.4 63.9
	P _{3/2}	0	6.8 [7.1]	5.667(-2) [4.053(-3)]	0.379						
	i _{13/2}	0	5.9 [6.7]	-1.484(-1) [-2.548(-3)]	-0.392						
	f _{7/2}	0	6.4 [6.8]	1.244(-1) [4.930(-3)]	0.202						
$\sum_j a_j \{ R_m G_{\alpha}(R_m) \}_j = 1.917(-1)$						84 86 88	4.975 5.931 6.764	5.499(-29) 7.468(-25) 4.707(-22)	1.538(-8) 2.095(-4) 1.317(-1)	2.12(-10) 2.15(-6) 2.06(-3)	72.6 97.4 63.9

a. Lower bracketed values are from older shell-model theory. See text.

b. Final states in ²⁰⁸Pb unresolved $\lambda(1/2-) + \lambda(5/2-) = 2.12 \times 10^{-10}$. With theory as guide we attribute full intensity to the 1/2- favored transition.

Table 2. Alpha Decay Rate Factors for assumed 17/2- state as ^{213}mRa

Final State $I\pi$	Daughter Hole pair orbital j	Angular Momentum L	R_m [R_m']	$R_m G_N(R_m)$ [$R_m' G(R_m')$]	A_j	Q_α	P	Theo. λ (Sec) $^{-1}$
5/2-	$P_{1/2}$	6	6.6 [6.9]	-1.406(-2) [-8.590(-4)]	0.866	8.666	1.161(-17)	1.308(1)
		8	6.6 [6.9]	1.089(-2) [7.220(-4)]	"	"	8.881(-19)	6.001(-1)
		10	6.6 [6.9]	-9.873(-3) [-6.938(-4)]	"	"	3.571(-20)	1.983(-2)
								$\sum_L \lambda_L = 1.370(1)$
3/2-	$P_{1/2}$	8	6.8 [7.0]	6.360(-3) [5.500(-4)]	0.828	8.442	2.087(-19)	4.397(-2)
		10	6.8 [6.9]	-1.364(-2) [-1.308(-3)]	"	"	8.199(-21)	7.946(-3)
1/2-	$\left\{ \begin{array}{l} f_{5/2} \\ P_{3/2} \\ i_{13/2} \\ f_{7/2} \end{array} \right.$	8	6.4 [6.8]	2.649(-2) [1.281(-3)]	0.807			$\sum_L \lambda_L = 5.192(-2)$
		8	6.8 [7.0]	1.386(-2) [1.199(-3)]	0.379			
		8	5.9 [6.6]	-3.687(-2) [-8.089(-4)]	-0.392			
		8	6.4 [6.8]	3.059(-2) [1.479(-3)]	0.202			
Configuration mixing			$6.4 \left\{ \sum_j A_j (R G_N(R)) \right\}_{R=R_m} = 4.288(-2)$					
			8.554					
			4.343(-19)					

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Table 3. Alpha Decay Rate Factors for assumed $i_{13/2}$ -hole state as ^{213m}Ra

Final State I^π	Daughter Hole pair orbital	Angular Momentum L	R_m [R_m]	$R_m G_N(R_m)$ [$R_m G(R_m)$]	A_j	Q_α	P	Theo. λ (sec) ⁻¹
5/2-	$i_{13/2}$	5	6.2 [6.7]	1.113(-2) [3.360(-4)]	0.249	8.666	3.277(-17)	1.912(0)
		7	6.2 [6.7]	2.002(-2) [6.565(-4)]			3.501(-18)	6.610(-1)
		9	6.2 [6.7]	3.936(-2) [1.439(-3)]			1.929(-19)	1.408(-1)
							$\sum_L \lambda_L =$	2.714(0)
3/2-	$i_{13/2}$	5	6.5 [6.9]	1.945(-2) [8.738(-4)]	0.244	8.442	7.837(-18)	1.341(0)
		7	6.5 [6.8]	1.385(-2) [6.729(-4)]			8.246(-19)	7.155(-2)
							$\sum_L \lambda_L =$	1.413(0)
1/2-	$i_{13/2}$	7	6.5 [6.8]	1.766(-2) [8.576(-4)]	0.392	8.554	1.708(-13)	6.219(-1)

Table 4. Ratios of Reduced Transition Rates
from Three Forms of Shell Model Rate Theory

Final State $I\pi$	Daughter Hole pair orbital	Angular Momentum	Relative Reduced Transition Probability		
			New Finite-size Theory	Old Finite-size Theory	Delta function Theory
5/2-	$i_{13/2}$	5	1	1	1
		7	3.24	3.82	2.82
		9	12.49	18.34	9.17
3/2-	$i_{13/2}$	5	3.05	6.76	10.88
		7	1.55	4.01	4.84
1/2-	$i_{13/2}$	7	2.52	6.51	7.85
5/2-	$p_{1/2}$	2	59.00	199.6	266.4
3/2-	$p_{1/2}$	2	26.88	157.7	321.4

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Figure Legends

Fig. 1. Comparison of experimental absolute alpha decay rate constants with the new normalized rate theory. The rightmost panel compares $^{213\text{m}}\text{Ra}$ isomer data (lower bold line) with theory (upper thin lines) for alternative assignments as $13/2^+$ or $17/2^-$. The left three panels give the comparisons for $1/2^-$ ground state decay of three 125-neutron isotones. The $5/2^-$ and $1/2^-$ states in the ^{205}Pb daughter of ^{209}Po are too close in energy to be resolved, so the dashed lines in the leftmost panel indicate that only the total intensity of these groups is known.

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