UNIVERSITY OF CALIFORNIA,

IRVINE

Modelling and Optimization of Smart Mobility Systems with Agent Envy as a Paradigm for Fairness and Behavior

DISSERTATION

submitted in partial satisfaction of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in Civil and Environmental Engineering

by

Daisik Danny Nam

Dissertation Committee:

Professor R. Jayakrishnan, Chair

Professor Jean-Daniel Saphores

Professor Will Recker

Professor Jae Hong Kim

2019

DEDICATION

To

my lovely wife, Hojin Paik

and

my beloved kids, Geunho and Dahae

and

my parents

Yunseong Nam and Pilnam Lee

Chun Gyun Paik and Jeong On Park

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

This research journey was an adventure which was possible only with the help of various individuals. I want to give gratitude to the following people for their invaluable help and support.

First and foremost, I would like to express my deep gratitude to my advisor, Professor R. Jayakrishnan. His enduring support and constant advice made this research possible. I benefited greatly from intense and invaluable discussions with him. I would like to thank my committee members, Professor Will Recker. Professor Jean-Daniel Saphores, and Professor Jae Hong Kim. They were willing to spend their time and gave me comments, encouragement, and advice. I would also like to express my gratitude to professors in ITS, Professor Wil Recker, Professor Stephen Ritchie, Professor Jean-Daniel Saphores, Professor Wenlong Jin, Professor Michael McNally and Professor Michael F. Hyland for teaching courses which have motivated my research.

I have been fortunate to have mentors who have enhanced my career path with their constant guidance, motivation, and most of all by being excellent role models role. My sincere and heartfelt gratitude goes to Dr. Hyunmyung Kim, Professor, Myongji University, and Dr. Minyoung Park, Professor, Inha University, and Dr. Inchul Yang, Korea Institute of Construction Technology (KICT), and Dr. Younshik Chung, Professor, Yeungnam University for the invaluable guidance and support throughout my Ph.D program.

I am also indebted to my colleagues Riju Lavanya, Dr. Felipe de Souza, Dr. Roger Lioret-Batlle, Sunghi An, and Bumsub Park. I would like to thank their invaluable research ideas, technical support, and writing help.

I must say that this adventure was very joyful because of my colleagues. I would like to thank Dr. Kyungsoo Jeong, Dr. Kyung Hyun, Dr. Jun Hyeong Park, Youngeun Bae, Jeonghoon Woo, Dr. Karina Hermawan, Dr.Qinglong Yan, Dr. Jianbo (Gabriel) Yu, Dr. Suman Kumar Mitra, Dr. Daniel Rodriguez Roman, Dr. Jae Hun Kim, Dr. Neda Masoud, Koti Reddy Allu, Navjyoth Sarma JS, Lu Xu, Pratiik Malik, Irene Martinez Josemaria, Khalid Abdulghanim, Xuting Wang, and Yuqiao Li not only for their enjoyable moral support, but also for all the long, deep and interesting discussions we shared. I would like to thank my research team, Dingtong Yang, Eduardo Marino, Marjan Mosslemi, Negin Shariat, and Pengyuan Sun.

I was fortunate to have supporting professors and research fellows. My sincere thanks to Dr. Young-Ihn Lee, Professor, Seoul National University, Dr. Deokgeun Yoon, KICT, and Dr. Soyoung Iris You, Korea Railway Research Institute, and Dr. Kijung Ahn, and Dr. Sungil Shin, the Seoul Institute, and Dr. Jooyoung Kim, the Korea Transportation Institute, and Dr. Wooyoung Ahn, Professor, Kongju University, and Dr. Wonho Suh, Professor, Hanyang University, Dr. Sang Gu Kim, Professor Chonnam University, Dr Soongbong Lee, Dr. Hyeonsup Lim, Bumsik Kim, DaeJin Kim for their appropriate help and encouragement.

I am thankful to the deeply committed staff in the Department of Civil Engineering and Institute of Transportation Studies for their warm and kind support: Cam Nguyet Tran, Dr. Zhe Sun, April Heath, and Lorrie L Aguirre.

I'm very grateful for my parents, brothers, sisters-in-law, nephews and nieces for their unconditional love, support, and understanding. Also, my greatest gratitude goes to my lovely wife, Hojin Paik, and my two kids, Geunho and Dahae Nam.

Finally, all praise, honor, and glory to my Lord Jesus Christ for his richest grace, mercy and guidance for the accomplishment of my Ph.D. adventure.

CURRICULUM VITAE

Daisik Danny Nam

EDUCATION

2014-Current University of California, Irvine
Ph. D. Candidate, Institute of Transportation Studies

2008-2010 Seoul National University
M.S., City Planning in Transport Studies

2002-2008 University of Seoul
B.S., Transportation Engineering

HONORS AND AWARDS

2018 Best Presentation Award, Global Top Talent Forum 2018, Hyundai Motors Group, San Diego, CA, USA

2017 Best Research Proposal, Korea Railway Research Institute Open Innovation Forum in UKC 2017, Washington DC, USA

2017 Student Essay Competition of ITS World Congress 2017 sponsored by SWRI, Montreal, Canada

2009 Best Presentation Award, Korean Society of Transportation

PROFESSIONAL AND EDITORIAL ACTIVITIES

2018-Current Organizer, Python-based Open-source Project for Transportation Studies, PyTrans (www.pytrans.github.io)

2018 Session Chair, Cooperative Traffic, Informs Annual Meeting 2018

2018-Current Reviewer**,** IEEE Access

2017-Current Reviewer**,** Artificial Intelligence and Advanced Computing Applications (ABJ70), Transportation Research Board

2017-Current Reviewer**,** KSCE Journal of Civil Engineering

PUBLICATIONS

[1] Daisik Nam, Dingtong Yang, Sunghi Ahn, Jiangbo Yu, R. Jayakrishnan, and Neda Masoud, Designing a Transit-Feeder System Using Multiple Sustainable Modes: P2P Ridesharing, Bikesharing, and Walking, *Transportation Research Record: Journal of the Transportation Research Board*, pp. 131-140, 2018. DOI: 10.1177/0361198118799031 – presented in 96 TRB annual meeting in 2018.

[2] Kim, Hyunmyung, Daisik Nam, Wonho Suh, and Seung Hoon Cheon. Origin-Destination Trip Table Estimation based on Subarea Network OD Flow and Vehicle Trajectory Data, *Transportation Planning and Technology*, Vol 41, Issue 3, pp. 265-285, 2018.

[3] Daisik Nam and Wonho Seo, Smart Transportation Systems with Artificial Intelligence, Kakao AI report, 2017 available at https://brunch.co.kr/@kakao-it/173

[4] Ko, Joonho, Kim, Daejin, Daisik Nam, Lee, Taekyung. Determining Locations of Charging Stations for Electric Taxis Using Taxi Operation Data, *Transportation Planning and Technology*, Vol 40, Issue 4, pp. 420-433, 2017.

[5] Neda Masoud, Daisik Nam, Jiangbo Yu, and R. Jayakrishnan. Promoting Peer-to-Peer Ridesharing Services as Transit System Feeders, *Transportation Research Record: Journal of Transportation Research Board*, No. 2650. pp. 74-83, 2017. DOI: 10.3141/2650-09 – presented in 96 TRB annual meeting in 2017.

[6] Roger Lloret-Batlle, Neda Masoud, and Daisik Nam. P2p Ridesharing with Ride-Back on Hov Lanes: Towards a Practical Alternative Mode for Daily Commuting, *Transportation Research Record: Journal of the Transportation Research Board*, No. 2668. pp. 21-28, 2017. DOI: 10.3141/2668-03. – presented in 96 TRB annual meeting in 2017.

[7] Daisik Nam, Kyung Kate Hyun, Hyunmyung Kim, Kijung Ahn, and R. Jayakrishnan. Analysis of Grid Cell–Based Taxi Ridership with Large-Scale GPS Data, *Transportation Research Record: Journal of the Transportation Research Board*, 2544, pp. 131–140, 2016. – presented in 95 TRB annual meeting in 2016.

[8] Tiratanapakhom, Tawin, Hyunmyung Kim, Daisik Nam, and Yongtaek Lim. Braess' Paradox in the Uncertain Demand and Congestion assumed Stochastic Transportation Network Design Problem. *KSCE Journal of Civil Engineering*, pp. 1-10., 2016.

[9] Kim. H, Daisik Nam, Cheon. S. Determination of Representative Path set from Vehicle Trajectory Samples, *Journal of Computing in Civil Engineering*, 2016, 04015052.

[10] Kim. B. S., Daisik Nam, Kim. D.Y., and Kim. K.M. The Effect of Expanding Urban Railway Infrastructure on Land Prices using DID method, *Korea Planning Association*, V49 n5, 2014.

[11] Cheon. S. H., Gwon. S. G., Daisik Nam, Lim. H. S. and Lee. Y. I. Analysis of Spatial Influential Zone for Road-sign Information using The Variable Radius Buffer Model, *Journal of Korean Society of Transportation*, v.29, no.2, 2011.

[12] Daisik Nam and Lee. Y. I. An Algorithm for Monitoring Freeway Link Travel Speeds based on the DSRC System, Master`s thesis, *Seoul National University*, 2010.

CONFERENCES

[1] Daisik Nam and R Jayakrishnan. Nash Equilibrium with User-Focused Incentives for Envy-minimized System Optimum Travel Routes. *Informs Annual Meeting*, 2018.

[2] Daisik Nam and R Jayakrishnan. Social Optimum Navigation with Incentives and Costs Schemes (SONICs) for Smart Urban Mobility. *TRARC2018*, IPAM UCLA, 2018.

[3] Daisik Nam, Riju Lavanya, Inchul Yang, Woo Hoon Jeon, and R Jayakrishnan. A Long Short-Term Memory Neural Network Approach for Traffic Density Estimation with Sensor-equipped Probe Vehicles, *ISTS-IWTDCS*, Matsuyama, Japan, 2018.

[4] Daisik Nam, Dingtong Yang, Sunghi An, Jiangbo Yu, R. Jayakrishnan, and Neda Masoud. Designing a Transit-Feeder System Using Multiple Sustainable Modes: P2P Ridesharing, Bike sharing, and Walking, *In Transportation Research Board 97th Annual Meeting*, 2018.

[5] Jiangbo Yu, Dingtong Yang, Daisik Nam, Sunghi An, and R. Jayakrishnan. An Implimentation-Ready Unsupervised Learning Approach for Multi-Van Multi-Criteria Bike Rebalancing Under Dynamic Demand, *In Transportation Research Board 97th Annual Meeting*, 2018.

[6] Daisik Nam, Riju Lavanya, Inchul Yang, Woo Hoon Jeon, and R Jayakrishnan. Traffic Density Estimation Using Radar Sensor Data from Probe Vehicles, *ITS World Congress 2017,* Montreal Canada., 2017.

[7] Daisik Nam. Traffic Sensing by Various Manners, *ITS World Congress 2017 Montreal*. Canada., 2017.

[8] Daisik Nam and R Jayakrishnan. Social Optimum Navigation with Incentives sCheme (SONIC) for Smart Urban Mobility, *Korean Society of Transportation 76th conference*, 2017.

[9] Daisik Nam, J.W Cho, H. Kim, and R. Jayakrishnan. Model Based on Deep Learning for Predicting Travel Mode Choice*, In Transportation Research Board 96th Annual Meeting*, no. 17-6512. 2017.

[10] Neda Masoud, Daisik Nam, Jiangbo Yu, and R. Jayakrishnan. Promoting Peer-to-Peer Ridesharing Services as Transit System Feeders, *In Transportation Research Board 96th Annual Meeting*, 2017.

[11] Roger Lloret-Batlle, Neda Masoud, and Daisik Nam. P2p Ridesharing with Ride-Back on Hov Lanes: Towards a Practical Alternative Mode for Daily Commuting, *In Transportation Research Board 96th Annual Meeting*, 2017.

[12] Kim, H, Daisik Nam, Yang, Inchul, Lim Yongtaek. Evacuation Time Minimization Model using Traffic Simulation and GIS Technology-Laguna Beach Tsunami scenario, *ESRI 2016 User Conference*, 2016.

[13] Daisik Nam, Kyung Kate Hyun, Hyunmyung Kim, Kijung Ahn, and R. Jayakrishnan. Grid Cell Based Taxi Ridership Analysis Using Large Scale GPS Data, *In Transportation Research Board 95th Annual Meeting*, 2016.

[14] Park. H. J, Park. D. J, Shin. S.I., Lee. C. H. and Daisik Nam. The Methodologies for Evaluating Integrated Home Delivery Systems, *Journal of Korean Society of Transportation,* 2013.

[15] Daisik Nam, Chang. H. H. and Lee. Y. I. Development of Reliability Index of Bus considering Passenger`s Waiting Time, *17th ITS world Congress*, Korea, 2010.

[16] Chang. H. H., Lee. Y. I., Han. D. H. and Daisik Nam. A Cellular Automata Model for Bidirectional Pedestrian Flows-the Dynamics of Spatial Usage, *Eastern Asia Society for Transportation Studies*, Indonesia, 2009.

[17] HAN. M. J., Daisik Nam, Kang. Y. E. and Lee. Y. I. Development of Pedestrian Simulator for Transit Center Operational Analysis, *AIT International Student Seminar*, Thailand, 2009.

[18] Cheon. S. H., Gwon. S. G., Daisik Nam, Lim, H. S. and Lee, Y. I. Analysis of Geometric Influential Zone for Road-sign Information through The Variable Radius Buffer Model Analysis, *Korean Society of Transportation*, 2009.

[19] Daisik Nam and Lee. Y. I. Surveying and Developing Algorithms of Transfer Passenger Characteristics from Subway Exit to Bus Stop, *The Korea Institute of ITS*, 2009.

PROJECTS

2018-Current	Autonomicity: Agent-based Modeling of the Impacts of Shared/Connected Mobility Modes on Transit systems and Potential Synergetic Options, SB1, USA
2018-Current	Development of a Machine-learning based Road Congestion Level Estimation Algorithm Considering Consecutive Multiple links, Korea Institute of Civil Engineering and Building Technology, Korea
2017-Current	A Smart Mobility Portfolio with an Integrated, Intermodal, Incentivized, and Shared (3-Is) Transportation System, SB1, USA
2017-2018	Analysis of Comprehensive Multi-modal Shared Travel Systems with Transit, Rideshare, Carshare and Bikeshare Options, UCConnect, USA
2017	Advanced Development of a traffic density estimation algorithm and its evaluation in a microscopic simulation framework, Korea Institute of Civil Engineering and Building Technology, Korea
2016-2017	Designing a Transit-Feeder System Using Bikesharing and Peer-to-Peer Ridesharing, UCConnect, USA
2016	Development of a Traffic Density Estimation Algorithm with Simulation and Analysis of Lane-by-Lane Traffic Characteristics, Korea Institute of Civil Engineering and Building Technology, Korea

2015-2016 Promoting Peer-to-Peer Ridesharing Services as Transit System Feeders, UCConnect, USA

WORK AND RESEARCH EXPERIENCE

2014-Current GSR, Institute of Transportation Studies, University of California, Irvine

- *Project management, assistant research scientist, and data analyst*
  - Promoting Peer-to- Peer Ridesharing Services, UC Connect & Caltrans
  - Designing a Transit-Feeder System Using Bikesharing and Peer-to-Peer Ridesharing as Transit System Feeders, UC Connect & Caltrans
  - Analysis of Comprehensive Multi-modal Shared Travel Systems with Transit, Rideshare, Carshare and Bikeshare Options, UC Connect & Caltrans
  - Developing of a Traffic Density Estimation Algorithm by using data from multiple vehicle sensors (Radar, Camera, and GPS), KICT, Korea

2013-2014 Visiting researcher, Department of Transportation System Research, The Seoul Institute, Seoul, Korea

- *Research on Seoul's transport policies based on large scale database*
  - Analyzing taxi users' and driver's behavior using digital taco gauge
  - Planning the strategies for Integrated-Home Delivering Logistics Services

2010-2012 Transportation specialist, Geo-spatial Information Team, KAKAO Co., Seoul, Korea

- *transportation data design and real-time map service applications*
  - Managed and planed portal map services associated with transportation information *(http://maps.daum.net)*
  - Developed routing guidance services for vehicle, public transportation and pedestrian and its integration
  - Introduced real time traffic information for users (Vehicle, Bus) to services (web, mobile applications)
  - Analyzed raw data regarding users` behavior and designed relevant services

2008-2010 Research Assistant, Environment Planning Institute, Seoul National University, Seoul, Korea

- *Transportation engineering and ITS*
  - Developed a 3-Dimensional microscopic pedestrian behavior simulator
  - Evaluated feasibility of Bus Priority Signal
  - Geographical analysis of road-sign information

2008 Internship, Korea Database Center, Korean Transportation Institute (KOTI), Gyeonggi-do, Korea

ABSTRACT OF THE DISSERTATION

Modelling and Optimization of Smart Mobility Systems

with Agent Envy as a Paradigm for Fairness and Behavior

By

Daisik Danny Nam

Doctor of Philosophy in Civil Engineering

University of California, Irvine, 2018

Professor R. Jayakrishnan, Chair

Smart Urban Mobility in the future demands a paradigm shift. Transportation supply needs to be designed to incorporate individual-level preferences in an era of readily-available information about other users and network performance. It is, therefore, reasonable to expect that an individual would have information to compare his/her transportation allocation with other users. For individuals having the same goal (e.g., the shortest path to the destination from the same departure location and time), the peer to peer comparison may induce 'envy' if the user perceives his/her assigned travel option to be worse than that of his/her peers.

In turn, a user may adjust his/her travel options until he/she does not feel envy. This concept is an extension of the well-known travel behavior assumption called "User Equilibrium". Existing behavior models, however, do not allow users to compare their allocations with

others on an individual basis. Furthermore, it is assumed that users have perfect information about their own alternative and all users are homogeneous. A smart mobility system of the future may also include users who are not human but machines such as logistics, an autonomous vehicle that may have programmed behavior, and thus they too can be considered "agents" in our analysis.

This dissertation is dedicated to modeling a smart mobility system which accounts for individual level of allocation. Mobility systems that include connected, autonomous, and subscribed components to various extents will all qualify as smart systems in this context. More specifically, we focus on the optimization of the allocation problem to achieve both system-wide efficiency and minimum envy among individuals. We consider envy to be an important allocation aspect in the transportation system. Maximizing the efficiency of a system necessarily brings about some level of unfairness where some users (or agents) are allocated to inferior alternatives. When agents having superior alternatives can compensate the envy of groups having inferior alternatives, an envy-free state can be achieved—which can be shown to be Pareto efficient state. Using a combination of pricing and incentives, we propose an optimization model to arrive at this new equilibrium.

This research has significant contributions in that the proposed model provides a framework to combine system-wide objectives with individual users' utility objectives. Furthermore, we consider user heterogeneity, which has not been researched in the general area of transportation assignment. The proposed optimization model can be applied to pricing strategies both for commercial and public agencies, who have real-time information about customer characteristics and system performance.

Numerical results from running our optimization on both illustrative and real networks show that the proposed model converges to both envy-free and system optimum states with appropriate allocation and pricing schemes. Our findings show that the proposed smart mobility system technically works efficiently without governmental subsidy since the budget-balance mechanism trades off credits among users. In addition, the level of user heterogeneity affects the amount of credits charged or disbursed.

Two roads diverged in a wood, and I—

I took the one less traveled by,

And that has made all the difference.

Robert Frost: The Road Not Taken

Chapter 1

1.1 Introduction

In the most basic terms, traffic can be understood as the result of the complex interplay between transportation supply (roads) and transportation demand (people's desire to travel). Transportation supply is distinctive in that it is a static service that cannot be stocked or transferred in the spatial and temporal dimensions, whereas travel demand is dynamic and takes place over space and time. In popular areas, demands are so high that infrastructure cannot service all of the demand, while in other areas, many streets and freeways lie underutilized. This imbalance occurs because travelers on the road remain isolated from their environment and are only capable of estimating their individual, short-term benefits of transportation (quite inaccurately at times, though it only degrades the system further).

Since there are physical limitations to the supply of infrastructure that can be added to the transportation system, our vision is of a smart and dynamic city mobility system that is able to respond to travel demands and more evenly distribute it over space and time. The engineering and scientific community has been building vital advancements—such as Autonomous and Connected vehicles that interact with the Internet of Things—that have been crucial in making this vision a reality.

Connected and Autonomous Vehicles (CAVs) can mitigate traffic congestion to some extent by increasing effective road capacity by virtue of lowering reaction time and other operating characteristics. However, congestion is still expected to be a pervasive problem in the future if we rely only upon improved vehicle performance. Even more exciting changes are to come that can tackle ever-increasing traffic congestion, for which we must look beyond improved vehicle performance.

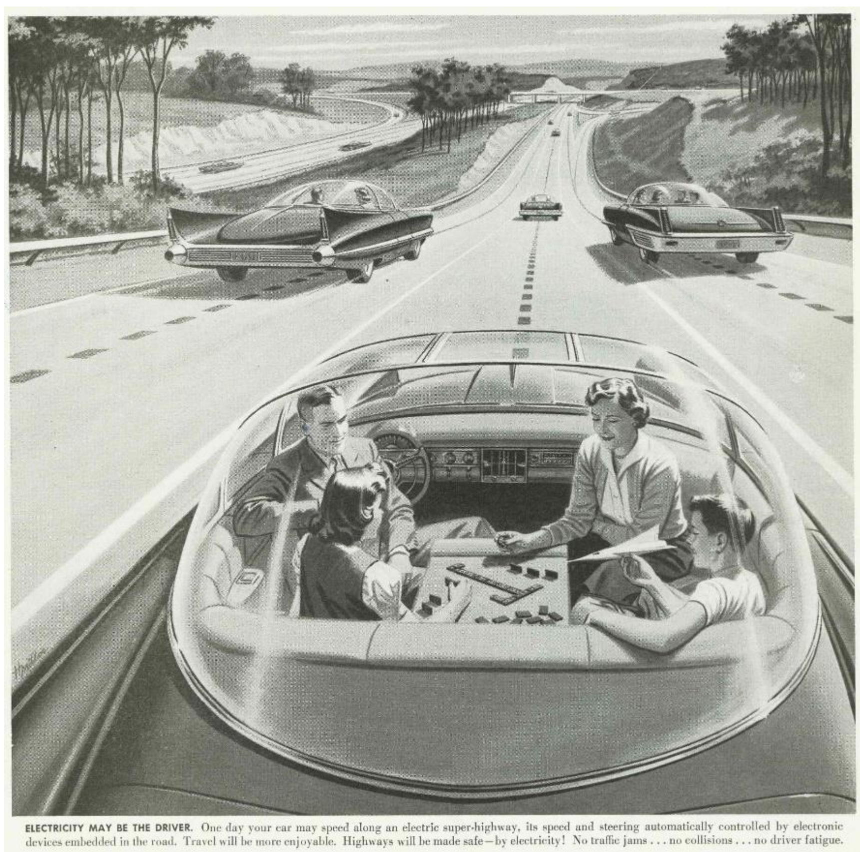


Figure 1.1 Driverless Car of the Future

Source: An advertisement for "America's Electric Light and Power Companies," Saturday Evening Post, the 1950s. Credit: The Everett Collection. Referred to: Marc Weber (2014), Computer Science Museum

We are inspired by the vintage advertisements for cars in Figure 1, that appeared in magazines in the 1950s—a period that is rightly called the Golden Age of American Futurism. This advertisement gives us a hint to solve the imbalance of traffic demand. It is not just that the family is traveling in an autonomous car; they are also engaging in leisurely activities. It suggests to us that they may not be sensitive to their travel times if other factors are in play. Perhaps some travelers could yield their shortest path to others if they are compensated adequately. The remainder of this chapter explores current efforts to manage demand, builds this idea in more depth, and envisions how future transportation systems will efficiently control demand patterns.

1.2 Transportation as a Smart Mobility

Transportation supply has the characteristic that it is shared by users. Transportation supply (i.e. roads) publicly provides services to individuals, which broadly can be regarded as shared mobility where we all have rights to utilize and responsibilities to maintain.

CAV research focuses on various strategies for such vehicle control as platooning, incident signal propagation, signal auction, and variable speed limits. The underlying assumption here is that vehicles can exchange information with each other and collaborate with each other to achieve certain goals. Here, we extend the potential of CAVs to vehicle routing strategies. By exchanging travel information within a peer-to-peer communication protocol, vehicles can decide their route and cooperate for a better transportation system (e.g., lower emissions), which we define a smart mobility as a smart usage of shared mobility.

1.3 A Collaborative Mobility System

A major cause of congestion is the 'selfish' travel behavior of individual drivers. This travel behavior has been codified in the form of Wardrop's 1st Principle, which states that drivers continue to minimize their travel time until it cannot be further improved, and thus the travel times of all used routes are eventually equal. This condition is called User Equilibrium (UE), which corresponds to the Nash Equilibrium in Game Theory. This selfish behavior aggravates traffic conditions even if additional infrastructure facilities in the form of new lanes are provided, since travelers want to use the newly developed shortest route. To complicate matters further, even if we increase capacity to accommodate growing demand, there will always be new induced demand. Therefore, to achieve the optimal conditions for the system, demand patterns should be more evenly distributed over the available supply. However, until the advent of connected and automated vehicles, the only way to do so was by means of congestion pricing, which is problematic for various reasons.

Traditionally, solving transportation problems has been regarded as a responsibility of the government. However, in the future, the private sector and individuals can also contribute to even distribution of traffic congestion. Businesses have an incentive to attract vehicles passing by their stores. Let us consider a simple situation of a traveler who has ample time to arrive at her destination. The revealed preference of the driver indicates that she tends to stop by Starbucks to grab a coffee during longer trips. When a certain part of her route starts to get congested, the incentive mechanism could possibly suggest an alternative route which may be just a little longer but also comes with a reward in the form of a Starbucks gift card or credit.

1.4 Envy: A Possible Behavior Paradigm for P2P Comparisons in the Future

Connected and Autonomous Vehicle technology is expected to transform the notion of urban transportation systems from simple concrete, steel, and asphalt into a much smarter, interactive digital information-based system (Glancy 2013). Wireless communication technologies enhance transportation connectivity and allow dynamic data exchange using a broad range of advanced systems, which enables transportation systems to be cooperative systems (Mike Pina 2012).

Connected systems with open communication will also open up new possibilities for travel. One possibility for more extensive use of real-time information is the development of peer-to-peer (P2P) communication among travelers, which introduce new opportunities of smart and collaborative consumption of transportation supply for better efficiency. Interconnected travelers can communicate their route decisions to others. Within this peer-to-peer communication framework, a real-time demand control system can encourage travelers to participate in a collaborative mobility system. Travelers with a greater sense of urgency or a high value of time will be willing to pay some amount of money for a faster travel option. Unlike current marginal cost pricing strategies, a collaborative mobility system will distribute the money obtained from faster route allocations to travelers who are willing to yield their priority to others. We can envision several other innovative applications in the context of CAV (Connected and Automated Vehicles) when travelers exchange information across the system. In the past, complex negotiations between human drivers were not possible due to safety reasons, but in a CAV environment, such ad-hoc collaborative negotiations are possible, and even necessary (for example, in disaster routing scenarios or in cases of medical emergencies). Furthermore, the autonomous vehicle can communicate with other connected vehicles on the road requesting them to yield their route.

Cities of the future necessitate a paradigm shift with the emerging and rapidly changing landscape of autonomous vehicles, subscription-based mobility services, connected vehicle technologies, and systems integration in the planning of transportation systems. System modeling is a necessary tool to study various policy alternatives, due to the complexities of such future cities. Unlike traditional top-down approaches for the transportation planning process, current and future transportation systems require more detailed and individual level of understanding of the various entities that interact with one another in ways that may or may not be collaborative. We propose to develop a comprehensive agent-based framework for system modeling that is applicable for several smart city contexts of relevance with their own associated networks, demand-generation modules, supply-side details, business models, vehicles, goods and travelers that encompass the full spectrum of mobility. Associated research projects can then study specific topics, such as for example: no rush shipping in Logistics, the effects of ride-sharing systems or autonomous vehicles on the transit systems, or the system-wide effects of user-side subscription services from Transportation Network Companies (TNCs).

This research takes a rather broader view of the word “behavior” which is in the title of the dissertation. In decades of past research in the transportation domain, behavior was intrinsically considered the response of individuals to the supply and demand conditions they experience, rather invariably in an uncontrolled way, or at least not with explicitly understood control. On the other hand, as all drivers who have been using navigation systems for a decade or more around the world know, their “behavior” in route choice has largely become subservient to what the navigation systems show. In a similar manner, in future smart mobility systems, the behavior of users may become increasingly controlled by the data availability and the dissemination of information.

It is also possible that the overabundance of data and information may make it humanly impossible to analyze and respond to, with native “behavior”. This leads to the conclusion that behavior may become increasingly dependent on what is programmed in the system than what is natively the response of the users. Of course, when the user is a machine, such as an autonomous vehicle that is moving without even a traveler controlling it, then the behavior is fully a programmed behavior. In such systems, what paradigm should be used for ensuring at least some notions of fairness and societal acceptance? This research delves into such issues and develops a framework of analysis that uses plausible concepts such as based on agent envy that are applicable to the smart mobility systems of the future.

Chapter 2

2.1 Efficient Transportation Supply Allocations and envy of users

A major cause of congestion is the 'selfish' travel behavior of individual drivers. This travel behavior has been codified in the form of Wardrop's 1st principle, which states that drivers continue to minimize their travel time until it cannot be further improved—thus the travel times of all used routes are equal. This equilibrium condition is called User Equilibrium (UE), which is also known as Nash Equilibrium.

This selfish behavior aggravates traffic conditions even if additional infrastructure facilities in the form of new lanes are provided since travelers want to use the newly developed shortest route. To achieve the system optimal condition, demands should be well distributed over the available supply. Boyce and Xiong (2004) present graphical examples of how drivers are dispersed in a system optimal (SO) state. In the SO condition, dispersing vehicles on a network minimizes total travel time of the transportation system. The SO assignment inherently lacks desirable equity property because travel time differences among SO routes. Although the SO assignment improves the overall social welfare, the travel times among the routes in the SO pattern are different. This implies that some guided drivers experience unfairness (Roughgarden 2002).

Research in engineering, economics, and even human psychology, is arriving at the conclusion that pricing schemes hold the key to change the selfish behavior of drivers and improve efficiencies of transportation system. It is likely that monetary tolls or incentives are quite effective in changing drivers' behavior. Furthermore, recent studies show that social and psychological incentives could be used as a substitute for financial rewards. This section reviews current research efforts to achieve the SO condition.

2.2 Existing Pricing Schemes for Efficient Allocation of Transportation Supplies

Various theories have been proposed in the past to achieve system optimum condition—road pricing theory being a prominent example. This theory finds the optimum condition by implementing marginal travel costs, which enables policy makers to collect congestion tolls. The amount of toll is the difference between marginal and average travel times (Beckmann, McGuire, and Winsten 1956; Small 1992; Dial 1999; L. N. Liu and McDonald 1998; Burris 2003) . Since 1995, the number of toll roads has been increasing. However, a significant proportion of newly constructed toll roads has failed to attract the expected number of drivers (Zhou et al. 2009). Furthermore, unfairness is also a primary concern since drivers stick to their 'right' to free travel on urban roads (Arnott, Palma, and Lindsey 1994), making congestion pricing policies politically difficult to implement in cities around the world (H. Yang and Zhang 2003; Ben-elia and Ettema 2009; H. Yang and Wang 2011; Y. M. Nie 2012).

Many ideas have been proposed as alternatives for congestion tolls. Yang and Wang (2011) devise and mathematically analyze a tradable credit scheme. Their scheme has the following features:

1) Some amount of credit is initially allocated to participants.

2) Then, the credit is given to participants who drive congested roads when they contribute to improving road efficiency by avoiding the (predefined) congested roads.

3) Participants can trade this credit in a competitive market.

By assuming all travelers are participants, . Yang and Wang (2011) conclude that the tradable credit scheme can guide traffic to have a social optimum pattern. Nie (2012) examines Yang and Wang's tradable credit scheme and focuses on how transaction cost affects the tradable mobility credits in both an auction market and a negotiated market. By realizing the fact that a tradable credit scheme is more controversial as an infringement of personal freedom than congestion toll, Nie (2012) examines how the government offers a proper price including transaction cost. The Braess's Paradox network shown in Figure 2.1 is used to illustrate their numerical results. The amount of total credits is calculated by measuring the travel time difference between each route. We can find that the toll of each link is determined by the difference between travel time of the longest route and the travel time of the current route as shown in Table 2.1. The paper designs two types of trading markets. The first being an auction market, where each driver purchases credits in the trading market. In contrast, the government initially distributes total credits to drivers in the negotiated market.

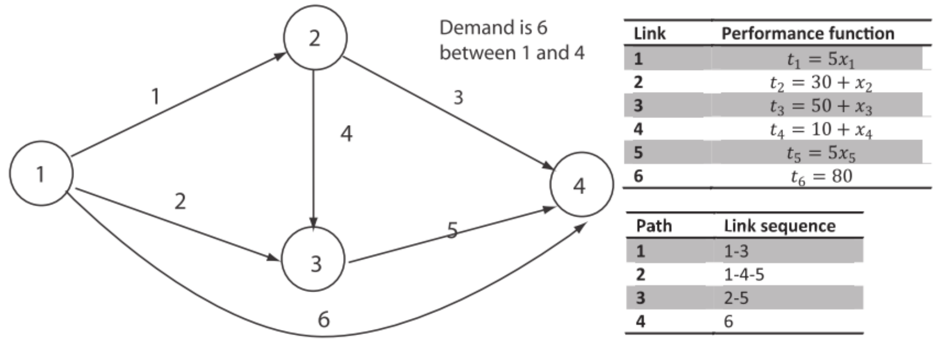


Figure 2. 1 The revised Braess network proposed by Nie (2002)

Table 2. 1 Initial allocation of credits and link tolls (Nie, 2002)

	UE solution				SO solution				
Path	Flow	Time	Total Cost	Toll	Flow	Time	Total Cost	Toll	Total Toll
1	0.00	68.57	0.00	0.00	1.79	64.50	115.56	14.50	25.98
2	3.71	62.29	231.35	0.00	0.75	44.50	33.38	34.50	25.88
3	2.29	62.29	142.37	0.00	3.46	54.50	188.48	24.50	84.73
4	0.00	80.00	0.00	0.00	0.00	80.00	0.00	0.00	0.00
Total	6		373.71		6		337.42		136.58

Note: Tolls given in the last column are first-best marginal tolls.

He et al. (2013) extend Nie (2012)'s research to the mixed user's condition where the participants are categorized into two groups: 1) individual travelers and 2) transportation firms, such as logistic companies and transit agencies. All of the participants receive an initial allocation of credits from the government. The amount of initial allocation is mathematically calculated by a mixed equilibrium assignment with system optimum approach. He et al. (2013) assume that individual drivers will follow the Wardrop's 1st principle but users in a transportation firm cooperate among themselves to minimize the firm's total cost (Cournot-Nash players). Nie and Yin (2013) and Nie (2015) apply the tradable mobility credit scheme to the departure time selection problem for managing rush hour traffic. Drivers not driving during the peak hour earn credits by contributing to congestion relief. Moreover, the participants driving during the peak hour need credits in order to drive. The participants who need more credits can purchase them in a trading market. Tradable credit scheme could be an alternative of congestion toll when the credits are reasonably allocated to the participants. However, initial allocation is another challenge (He et al. 2013). In addition, Nie (2012) pointed out that some people might be restricted to driving a credit-consuming road since they cannot purchase the credit with an affordable price. This might create another instance of unfairness, which should be carefully considered.

In the past, route guidance and traffic information services have been designed to provide shortest route information to their users. However, there are recent studies about route guidance to improve the efficiency of transportation systems. Jahn, Mohring, and Schulz (2008) proposed a system-optimal routing model by considering unfairness of the routes in the system-optimum condition. They find that the system optimum path can sometimes result in unreasonably long paths as shown in Figure 2.2. By assuming that only a few drivers are willing to sacrifice their time, they suggest the constrained system optimum path where drivers are assigned only to "acceptable paths". Their findings support an argument in Nie (2012)'s research that the amount of toll is based on the difference between the travel time of the longest path and the current path. The longest path can be unreasonably longer than other routes, thus the total amount of toll increases significantly in a large network.

Figure 2. 2 System optimum without and with restrictions on the length of a path

Source: Jahn, Mohring, and Schulz (2008)

Bosch et al. (2011) initially propose the concept of social navigation by assuming that some drivers could be altruistic. This navigation system provides route information considering both the personal preference and social cost. They assume that 40% of drivers select their route based on their expectation and 40% of drivers change their route dynamically, and 20% of people can consider the social cost. The drivers, considering the social cost, change their route according to their level of altruism. Djavadian et al. (2014) examine how many drivers will follow the recommended guidance according to various factors such as driving experience, incentive, and information. They conducted a survey and found that as drivers become more familiar with the network, they do not follow the guidelines strictly. In addition, a small number of participants drive the longer route even if they do not receive any compensation. As expected, they found that there is a high participation rate of social navigation when the drivers were well informed and well rewarded.

Pan et al. (2012) propose proactive vehicle re-routing strategies. Recognizing that the system cannot force the drivers to select the longer path, they suggest several re-routing strategies to avoid congestion. Among them, the Entropy Balanced *k* Shortest Paths (EBkSP), which balance the traffic load with multiple paths, could meaningfully improve the overall congestions. R. Liu et al. (2014) employ Pan et al. (2012)'s EBkSP to develop a participatory navigation system, which is called "Themis". Themis predicts future traffic flow and speed. This information is used to make people drive less popular routes by showing a score. The score is calculated by considering the average estimated time of arrival and popularity of a route. According to the authors, however, the intuitive meaning of the score is hard for the drivers to understand.

Many researchers conceptually give their opinion that providing incentives will contribute to changing travelers' behavior.(E. T. Verhoef, Nijkamp, and Rietveld 1996; Roughgarden 2002; Boyce and Xiong 2004; Jahn, Mohring, and Schulz 2008; Zhou et al. 2009; Ben-elia and Ettema 2009; Ben-Elia, Tillema, and Ettema 2010; Bosch et al. 2011; H. Yang and Wang 2011; Y. M. Nie and Yin 2013; Hu et al. 2014; Djavadian et al. 2014; Kumar 2015). The relationship between toll and incentive can be likened to the "Carrot or Stick" idiom. Findings in psychology literature such as Ben-elia and Ettema (2009) and Ben-Elia, Tillema, and Ettema (2010) show that an incentive scheme brings about better outcomes than punishment. They also showed that a reward scheme can be effectively used to manage travel demand from a survey test for “Spitsmijden (Dutch for peak avoidance)” in the Netherlands. They show that participants change their commuting schedule to avoid peak hour when they earn rewards. Ben-Elia, Tillema, and Ettema (2010) and Kumar (2015) analyze the behavioral changes from the “Yeti phone rewards”. Desirable behavior changes are not sustained when participants receive a “Yeti phone” which is the participants’ ultimate purpose. Thus, both studies show the impacts of incentives and insist that the incentive should be consistently provided since the desirable behavior changes from incentives do not persist when incentives are no longer provided.

Chiu(2014) proposes an active travel demand management system. This is the patent for the “Metropia” which is a company servicing a mobile app-based transportation information. The algorithm suggests several departure time windows to the users. The suggested departure time is to contribute to relieving traffic congestions by incentivizing the users to change to a non-congested travel time window. Once a user selects a desired traffic time window, the M-time-dependent minimal marginal cost path algorithm produces M routes. The incentive-offering algorithm may also leverage the user's preferences stored by the system. As their travel time decreases by choosing the suggested departure time window (i.e., the less congested times), greater incentives are rewarded to the users. Hu et al. (2014) apply Chiu’s active demand management platform, which is called “Smartrek”, to 36 commuters in Los Angeles. Their incentive scheme changes 60% of participants travel behavior. They also found that the travelers who change their departure schedule with following the suggested route save their travel time significantly by 20.12%. The travelers who only change their departure time reduce their travel time by 19.40%. The travelers who only follow the guided route save their travel time by approximately 10.0%.

Zhou et al. (2009) point out that constructing more toll roads may not necessarily contribute to reducing overall road congestion, which was the main purpose of providing a new road in the first place. To promote toll road usages and to increase system efficiency, the authors propose an incentive strategy for truck drivers to make them drive toll roads when the toll roads are not fully utilized.

The research mentioned above focuses on the effectiveness of monetary incentives. However, there are opposite findings that the financial form of incentive may not be effective to change the drivers' behavior (See review of Riggs and Obispo (2015). In other words, social and psychological forces can replace the role of financial rewards and Riggs's findings indicate that financial incentives are not always effective. Interestingly, Djavadian et al. (2014) examined the effectiveness of people's social responsibility and incentive for "Social navigation". From an experimental study, they found that various factors (altruism, familiarity with the current route, uncertainty associated with the social route, bounded rationality) affect the driver's behavior changes following the guidance.

2.3 Illustrative Examples of Current Pricing schemes

This section illustrates some existing pricing schemes and finds the possible pricing schemes that accommodate both tolls and incentives. Figure 2.3 is the hypothetical network for the different pricing cases, which is a simple transportation network with two routes connecting two nodes, each with their own travel characteristics and costs. The link performance function consists of a free-flow travel time and increased travel time according to the link flow.

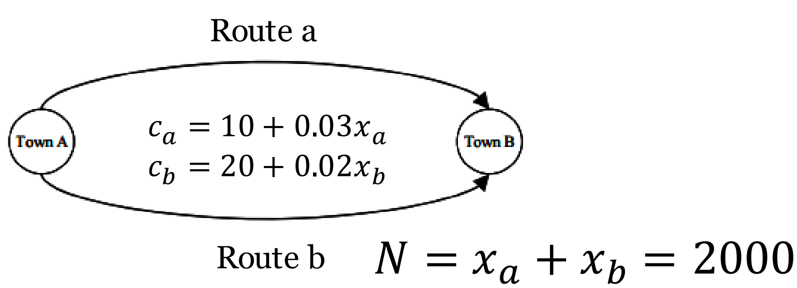


Figure 2. 3 A simple two routes network

We can simply find the UE solution that the travel times of route a (c_a) is identical to the travel times of route b (c_b). In addition, the SO solution can be found that both routes have the same marginal cost travel time ($MC_a = MC_b$).

$$\mathrm{MC_i} = c_i(x_i) + x_i c'(x_i) \tag{2.1}$$

Figure 2.4 visually solves the UE and SO solution for the network. (Note that the path volume of route b is flipped in the graph.) As can be seen, the travel time of both routes in UE condition are equilibrated at 40 minutes (c^{UE}). Its total travel time is 80,000. The total travel time of the SO solution is 79,500 minutes, which is less than that of UE. However, there is a travel time difference between two routes. In the SO condition, the optimum travel time of route a (c_a^*) is 37 minutes. However, the other route has a longer travel time (c_b^*) at 42 minutes.

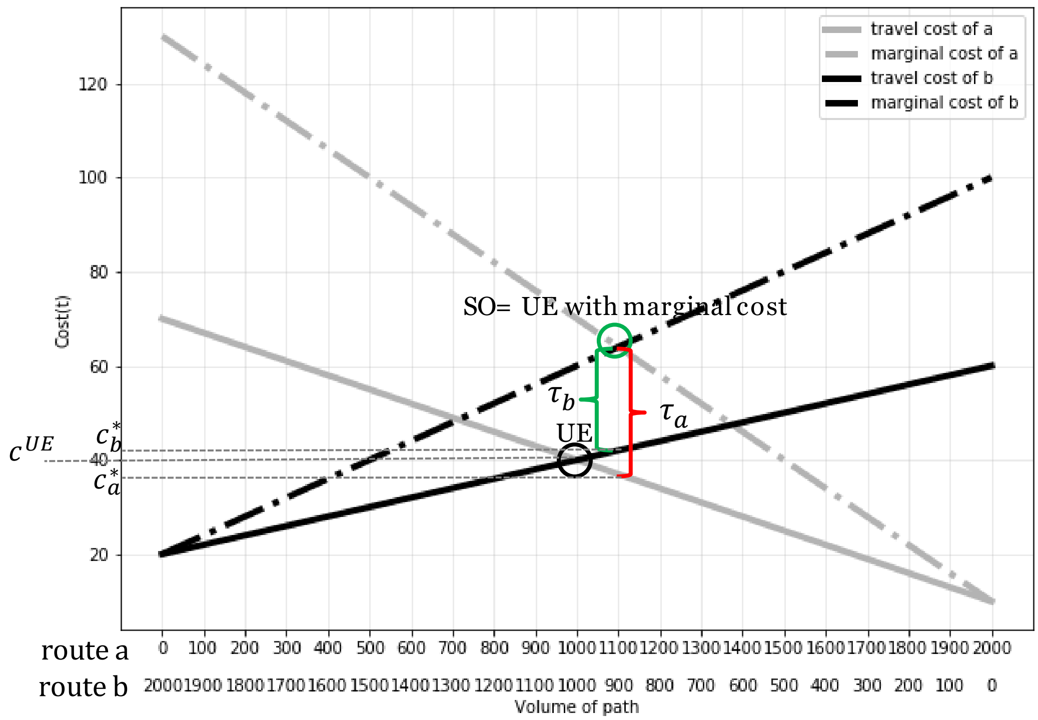


Figure 2. 4 Illustrative solutions for UE and SO

Pigou (1920) and Knight (1924) introduce a marginal cost-based toll strategy, hereafter we call it first-best toll. The amount of toll is the second term of the marginal cost (Eq 2.2)

$$\tau_i = x_i c'(x_i) \tag{2.2}$$

Efficient transportation systems with various travel options can be achieved with pricing characteristics that are different. Examples with various routes with different travel times are shown in Table 2.1. For the SO condition, The first-best toll strategy charges tolls to both routes. The toll for Route a is the equivalent amount of 27 mins and the toll of other route is 22 minutes.

Instead of charging tolls to all routes, the second-best toll strategy design a system that both tolled routes and untolled routes exist for equity reason, since it is generally acceptable to provide free routes to road users (E. T. Verhoef, Nijkamp, and Rietveld 1995; E. T. Verhoef, Nijkamp, and Rietveld 1996; Steimetz, n.d.; H. Yang and Zhang 2003; and Rouwendal and Verhoef 2004).

For the routes in Figure 2.4, the second-best toll strategy only levies to the shortest path users at the amount of 5 minutes and the longer route is free. For the Incentive cases, on the other hand, the longest users earn incentives of 5 minutes. It is noteworthy that the price difference between the two paths in the three strategies is 5 minutes. This pattern implies that pricing can be any value as long as the price difference among SO routes is equivalent to the marginal travel time difference between routes. More interestingly, the marginal travel time difference is the same with the travel time difference.

Table 2. 2 Comparisons of different pricing strategies

Type	t_a	t_b	p_a	p_b	Total Travel Time
0. No strategy (UE)	40.0	40.0	N/A	N/A	80,000
1. First best	37.0	42.0	+27.0	+22.0	79,500
2. Second-best	37.0	42.0	+5.0	Zero	79,500
3. Incentive	37.0	42.0	N/A	- 5.0	79,500
4. Toll & Incentive	37.0	42.0	any	any	79,500

2.4 Findings and Distinctive Differences from Previous Research

To increase the efficiency of a system, some people need to yield their priority, which implies that improving overall efficiency should accompany various options. For example, the path travel times of an origin and destination might vary if we want to minimize the total costs of a system. When the system forces people to give away their priority, it raises immediate concerns regarding fairness. Congestion pricing imposes a toll to the drivers who want to drive fast, but some people cannot use the toll roads because they cannot afford the toll. Tradable credit schemes try to solve the unfairness problem by initially allocating some amount of credits to individuals and allow individuals or firms to trade credits. Tradable credit schemes, however, cannot escape a fundamental criticism that they restrict the human's basic right of moving, since some people need to purchase credits to move. Furthermore, it is hard to find a concrete and fair way to distribute the credits by a government agency.

There are other efforts to change travelers' behavior through route guidance. Some research makes the unreasonable assumption that people can be altruistic in that they yield their shortest path to others. Monetary incentives can be a way to make drivers behave according to a system optimum pattern by compensating their inconvenience. Social responsibility and individual's preference also play a role in changing travelers' behavior, underscoring our contention that individual travel choice is complex and is affected by a number of factors, both qualitative and quantitative.

From previous research, we infer that various preferences, including monetary incentives, affect individuals' travel decisions. This research attempts to describe these various preferences by introducing the idea of perceived utility. We realize that there are different individual groups with various characteristics. "Social nudge" affects some people to be altruistic, some travelers consider that arriving at their destination as soon as possible is their main priority, trip preferences of some individuals rely on their trip purpose, and some might be willing to change their route when they can get rewards. The success of this strategy is dependent on individuals' participation.

Chapter 3

3.1 Agent-based Envy Model

This chapter presents the methodology used to model agent-based envy behavior for a traffic assignment problem. It includes a review of the limitation of aggregated-behavior models to highlight the importance of agent-based modelings for efficient and fair allocation of transportation supply. It also addresses methods for implementing the "Envy" concept, which is a well-known theory, often described using "a fair cake-cutting problem" in Economics. We extend the envy optimization to transportation network problems.

The cornerstone of criticism of traditional transportation problems is the idea that macroscopic modeling is based on a representative agent that simplifies the behavior of travelers. It is evident that transportation demand comprises heterogeneous users—they value their travel property (e.g., time) differently according to their travel distance, income, arrival time, and trip type. A macroscopic model, however, mainly overlooks these demand characteristics by focusing only on the supply side of massive infrastructure. In addition, agents are assumed to recognize full information of supply conditions based on their experiences. With the assumption that the behavior of system users is homogeneous, an aggregate model concludes that a system provides an equal allocation to all users (UE) or a system can restrict users' behavior for an optimum condition (SO) by imposing marginal costs. The bottom line of aggregated models is the assumption that all users are unilaterally selfish in their travel with the representative yardstick. Its equality condition can be arguable in that actual agents do not satisfy the given allocation when their preferences are not homogeneous. Some research efforts take this heterogeneity into account by introducing multiclass commodities for categorized demands (Meng, Liu, and Wang 2012; Arnott, Palma, and Lindsey 1994). Even introducing multiple groups could consider the degree of heterogeneity in the demand model, however, macroscopic modeling still assumes that an indexed traveler in a group represents all travelers in the same group (Arnott, 1994). However, transportation systems have features that are more complex. Technically, an aggregated model can also define the number of groups as much as the number of populations, it looks unfeasible in terms of computation complexity.

Agent-based models have practicality in analyzing and forecasting complex transportation systems. In the context of modeling transportation in future cities, agents refer to any autonomous entities within transportation systems. Examples are business travelers, private drivers for shopping, truck drivers, shippers, public agencies, and connected and autonomous vehicles.

Although agent-based models have been studied, inferring the behavior of agents at the micro-level has been difficult. A new paradigm can be imagined if we can collect data to calibrate a model at the individual level in the future. From Peer-to-Peer (P2P) information exchanges, a system operator might design a mechanism allowing users to compare their selected options with others'. Privacy issues could deter this mechanism from real-world applications. However, it becomes feasible when current information exchange techniques, such as connected vehicles and encrypted currencies, are available. For example, we can imagine future transportation systems wherein connected vehicles cooperate with each other by exchanging their travel option through encrypted private information—meaning that only such permitted information as destination, route, and preferred arrival time can be accessed.

In this chapter, we are interested in how individuals respond to given transportation systems when we consider travelers' different standards for their travel property. If we assume that users' travel propensities are not anonymous and a system operator routes travelers, we are also interested in how agents feel mistreated when they compare their given allocations to that of others.

3.2 Notations and Definition of Variables

This section addresses some notations and definitions. Our research scope covers optimization techniques for a network-wide transportation assignment problem and envy minimization. For a better understanding of how the proposed method works, this chapter focuses on a static version of the optimization problem. Based on this chapter, we will extend this optimization technique to the dynamic case in Chapter 6.

First, the level of detail of the problem is set to an agent. Let I be the set of agents. Each agent i traveling from origin r to destination s has path k. This differs from aggregated models in that the unit of the path of aggregated models is OD flow itself, which is equivalent to the total number of agents in an OD pair. This agent-level problem also comes with a path binary for k, $f_{i,k}^{rs}$ of an individual. Link flow x_a is aggregated by a route-link binary $\delta_{i,a,k}^{rs}$; this binary is a unit value if link a has been utilized for path k of an agent $f_{i,k}^{rs}$. A link performance function that indicates the relationship between link flow (x_a) and link travel time is monotonically increasing with respect to traffic flow.

The heterogeneity of travelers is considered using their valuation function $(\mathrm{V_i})$ of travel property (e.g., travel time, θ_i). An agent compares his/her travel option with that of others based on his/her standard and feels envious $e_{i\mathrm{j}}$ to others' if his/her travel option is worse off. A detailed explanation will be addressed in the next section.

The summary of notations employed throughout the chapter is as shown in Table 3.1.

Table 3. 1 Notations

i	an agent $i \in$ I
r, s	origin $r \in R$ and destination $s \in R$
k	a path k in a path set of an origin and destination pair
a	a link $a \in A$
θ_i	value of time of an agent i (generated from a distribution)
p_i	price term for an agent i
e_{ij}	envy function of an agent i relatively felt to trip information (e.g., time and price) of an agent j
V_{ij}	valuation function for an agent i relatively evaluate to trip information (e.g., time and price) of an agent j
$\delta_{i,a,k}^{rs}$	route-link binary (if link a is used for route k of r, s, i, then 1, otherwise 0)
$f_{i,k}^{rs}$	path k binary of an agent i of an OD pair r, s (if route k is used by i, then 1, otherwise 0)
x_a	traffic volume of link a
t_a	travel time of link a (convex and non-decreasing function of x_a, $t_a = t_a(\mathrm{x}_a)$)

3.3 Envy and Agent-level Envy Comparisons

For transportation system management, our main interest is in maximizing the efficiency of a system given the supply. On the other hand, from a social justice point of view, fair allocation of supply to demand is also a pivotal issue. Both terms conflict with each other; therefore, a solution satisfying both is unlikely to be achieved. For example, expected allocations for maximizing system efficiency indicates that the supply should be allocated unequally to different agents, but it is almost impossible to impinge upon the agents' freedom to select their travel option. Thus, a strategy for maximizing efficiency with various options is nonetheless poorly applied in an actual world because of unfair variances of alternatives. SO routing strategies in transportation assignment problems are examples. Generally, SO in moderate traffic is accompanied by unfair problems in that the quality of allocation is inequitable.

It is evident that an agent feels disgruntled if a system manager guides an agent to an allocation that is distinctively slower than that of others. The level of feeling of unfairness might be different according to the agent's valuation of an allocation. Thus, our approach considers the individual level of symmetric comparisons of agents' own yardsticks at the individual level. Recognizing the heterogeneity of individual travelers, we can minimize the system-wide envy level.

To design agent-level transportation systems, we propose to use the concept of "Envy", which is a theory from Economics, as a new paradigm for fairness, efficiency, and behavior.

With the possible technology of peer-to-peer data exchange, it is not difficult to imagine how individuals behave regarding their transportation options or how a system manager designs transportation systems. When an agent can access other agents' route information, an agent might make their best effort to minimize the equality gap with others'. A system service manager could become concerned with this user's behavior. On the other hand, a system manager might prefer to design a system maximizing efficiency or profit. The best strategy is to design a system that achieves optimum efficiency while not intruding on users' freedom of selection.

In the 1940s, Envy was initially employed for a fair division problem by economists and mathematicians (Barbanel 2005). Gamow and Stern (1958) introduce the Envy-Free (EF) theory in the context of the fair cake-cutting problem. Envy-Free implies that each agent believes that their allocation is greater than or at least the same the share of others; each player, in turn, is satisfied with an allocated piece of cake according to their preference. Thus, an EF region exists if there is a certain level of heterogeneity of preference. Varian (1974) uses Envy to design a fair allocation system. He defines fair allocation in terms of Equity, Envy, and Efficiency. Here are the definition and its relationships.

"If, in a given allocation, agent i prefers the bundle of agent j to his own, we will say i Envies j. If there are no envious agents at allocation x, we will say x is equitable. If x is both Pareto Efficient and Equitable, we will say x is Fair."

As we discussed, our research interest is to find the optimum (Efficient) allocation of transportation supply while agents believe their allocation is equitable; this interest is in line with Varian's definition of Fairness.

To implement Envy theory in the transportation domain, we define agents' behavior as follows:

Definition 3.1. Each agent has his/her own preference θ_i to travel options (e.g., value of time) and these preferences are heterogeneous.

Definition 3.2. The distribution of valuation is known to a system manager and the distribution is not necessarily based on certain well-known distributions, such as a normal distribution. However, the output of the valuation function is always non-negative according to the positive allocations.

Definition 3.3 An agent compares his/her travel option with others' only who are in the same travel property (i.e. origin and destination).

Definition 3.4. An agent($i \in I^{rs}$) feels envy when he/she finds that an option given to or selected by other travelers ($j \in I^{rs}$) provides a higher value than his/her current selection (Eq 1).

$$e_{ij} = (V_{ij}(\theta_i, t_j) - V_{ii}(\theta_i, t_i))\delta_{ij} \qquad \forall\, i\,, j\,, i \neq j,\; i\,, j \in I^{rs} \tag{3.1}$$

where δ_{ij} is 1 if "Envy" is positive and zero otherwise

This definition ensures privacy for travelers because a system is not necessary to expose other vehicles' personal information (e.g. value of time). This implies that an agent is indifferent to other agents' preferences. In other words, it does not matter for an agent to identify how rich other agents are, what their trip purposes are, or how urgent they are. An agent evaluates its envy based on its interpretation. An agent judges its envy by comparing others' options using their own criteria (Definition 3.1). In this study, the value of time is used, but it can be extended to various normalizers such as the value of emotion. We can interpret Eq (3.1) that an agent does not feel envy when its valuation is higher than that of others; thus, envy of *i* to *j* (e_{ij}) is always non-negative.

Definition 3.5. Agent's greedy behavior to the shortest path

In addition to mutual envy comparisons among agents, an agent feels envious (e_{is}) if its allocation is worse than that of the shortest path. This definition is equivalent to the basic assumption of UE. Similar to Definition 3.1 and Definition 3.4, envy is evaluated based on an agent's valuation. Interestingly, this definition is in line with the objective function for a solution of a gradient descent projection for assignment models that is used for a path-based traffic assignment algorithms (Jayakrishnan et al. 1994). This finds an equilibrium solution by comparing a route with the shortest path and adjusting traffic loads to the path.

$$e_{is} = (V_{is}(\theta_i, t_s) - V_{ii}(\theta_i, t_i))\delta_{is} \qquad \forall\, i\,, j\,, i \neq j \qquad (3.2)$$

where δ_{ij} is 1 if "Envy" is positive and zero otherwise

Definition 3.6. Agents change their travel option until they do not feel envious of others' (envy minimizer).

This is based on a different behavioral assumption than employed in traditional static or dynamic traffic assignment models. Static traffic models postulate that day-to-day travel experiences make agents have full information about traffic after certain periods that affects route choice decisions, that converge at a certain equilibrated level. This assumption is valid for the situation where agents rely on such static traffic information as paper maps, and even digital maps without real-time traffic information. With the availability of real-time traffic information (mobile applications, radio, digital maps, and Variable Message Signs) dynamic traffic assignments imply User Optimum behavior assumption, i.e., that a traveler makes a route decision based on current traffic condition. Our future vision is drawn on Connected Autonomous Vehicles (CAVs) and block-chain technology to facilitate the possibility of peer-to-peer travel information exchanges, which enable travelers to compare their option with that of others. In turn, an agent changes a travel option until it feels comfortable with the other agents' allocation.

The optimization of social welfare for maximizing efficiency and minimizing envy is our research interest; for the feasible optimization problem, we relax the definition of envy (3.1) to equation (3.3), and (3.4).

$$e_{ij} \leq My_{ij} \qquad \forall\, i, j, i \neq j \tag{3.3}$$

$$-e_{ij}+\left[V_{ij}\left(\theta_i,t_j\right)-V_{ii}\left(\theta_i,t_i\right)\right]\leq\left(1-y_{ij}\right)M \qquad \forall\, i\,,j\,,i\neq\, j \qquad (3.4)$$

where $M=$ large number;
and where $y_{ij}=0,1\,;e_{ij}\geq 0$

If $y_{ij}=0$, then
$e_{ij}\leq 0$
$e_{ij}\geq -M-\left[V_{ij}\left(\theta_i,t_j\right)-V_{ii}\left(\theta_i,t_i\right)\right]$

If $y_{ij}=1$, then
$e_{ij}\leq M$
$e_{ij}\geq\left[V_{ij}\left(\theta_i,t_j\right)-V_{ii}\left(\theta_i,t_i\right)\right]$

When we consider valuation as a multiplicative function of both a value of time (θ_i) and travel time(t_i), Eq (3.3) is simplified as Eq (3.5) and Eq (3.6). These equations deal with each agent's heterogeneity (i.e., valuation of time, θ_i) and it is worthy to note again that envy is evaluated differently by point of view. As shown in Eq (3.6), other agents' value of time or their level of envy does not matter to an agent *i*.

$$-\theta_i\mu_i+e_{ij}\ \geq\ -\theta_i\mu_j \qquad (3.5)$$

$$e_{ij}\ \geq\ -\theta_i(\mathrm{t}_j-t_i) \qquad (3.6)$$

3.4 Allocated System Efficiency with Envy Minimization (ASEEM)

Now, let us have an example. There are two roads from a node to a destination. There are two agents, and each should select one of two routes. The travel time of each route (t_a) is a function of link flow (x_a) as follows.

$$t_a = 1 + x_a \; t_a = 1 + 1.5x_a$$

Each agent has a different preference for travel time. Here, the unit of the valuation function is regarded as price, thus agent *i*'s value of time is 2.0-unit price per time, which is twice that of agent *j*.

$$V_{\mathrm{i}} = -2.0t, V_j = -1.0t$$

It is evident that both users aim to use the shortest path. However, if two agents use the same route, the used route is no longer the shortest path and total travel time becomes high. Although this condition promises envy-free allocation, we cannot say this is the desired traffic pattern. In addition, this example cannot have a UE condition since demand is a unit value. This is an important property and will be addressed by Lemma 3.1. To minimize total travel time (SO), two agents must use different routes. Here, we can imagine how we allocate the route to agents to minimize the total level of envy. Allocating route 1 to agent 1 comes with -4 unit price and he/she values that the other's option is -5. In turn, agent 1 will not feel envious of agent 2 according to Eq (3.1). But, agent 2 will envy at the amount of 0.5 unit price. On the other hand, when we distribute route 1 to agent 2 and route 2 to agent 1, the level of envy of agent 1 is 1 unit price. This case will bring twice the envy of the previous combination.

We can generalize the example by introducing a multi-objective optimization problem; we call it Allocated System Efficiency with Envy Minimization (ASEEM). Its objective function consists of three objectives: efficiency, equity, alternative availability; as Eq (3.7). Here, efficiency is considered as minimizing the total travel time of the entire system. We consider equity by minimizing the sum of envy of an agent. Here, the equity term might be overweighed according to the number of agents if we sum all the envy values of an agent's comparisons. This overweight problem can be tackled by the setting of β coefficient. However, we reduce redundancy of the envy term by only calculating maximum envy of agents among envy sets from individually symmetrical comparisons, we can reduce redundancy of the envy term. This is consistent with the Minimax approaches for envy optimizations (Dall'Aglio and Hill 2003; Fleurbaey 2008). Here, α, β, γ are weight parameters. When $\alpha = 0$, the goal of optimization becomes standard minimax envy-minimizing optimization. On the other hand, $\beta = 0 \; and \; \gamma = 0$, brings the results of SO assignment problem. Lastly, γ is used to incorporate behavior that every agent desires the fastest travel option as Definition 3.4.

$$\text{Min obj} = \alpha \sum_{a \in A} \mathrm{x}_a t_a \left(\mathrm{x}_a\right) + \beta \sum_{i \in I} \max_{i \neq j} \{e_{ij}\} + \gamma \sum_{i \in I} \mathrm{e}_{\mathrm{is}} \tag{3.7}$$

Below are the constraints for this problem. Note that this problem is a mixed integer linear programming, which is computationally complex.

The optimization is subject to:

$$\sum_{k\in K} -t_a\delta^{rs}_{i,a,k}\theta_i + e_{ij} \ \geq \ \sum_{k'\in K} -t_a\delta^{rs}_{j,a,k'}\theta_i \qquad \forall\, i,\ \mathrm{j} \in I,\ \mathrm{i} \neq j \qquad (3.7a)$$

$$\mathrm{x}_a \ = \ \sum_{rs}\sum_i\sum_k f^{rs}_{i,k}\delta^{rs}_{i,a,k} \qquad \forall\ \mathrm{rs, i}\ \in I, k\ \in K^{rs} \qquad (3.7b)$$

$$\mathrm{q}^{rs} = \sum_k f^{rs}_{i,k} \qquad \forall\ k\ \in K^{rs} \qquad (3.7c)$$

$$f^{rs}_{i,k} \geq 0 \qquad \forall\ i\ , j\ , k \qquad (3.7d)$$

$$0 \leq e_{ij} \leq e_{max} \qquad \forall\ i\ ,\ j\ ,\ i \neq\ j \qquad (3.7e)$$

Eq (3.7a) is a function for addressing both route choice behavior and envy comparisons. An OD pair (rs) has multiple paths. If function k path in the path set of OD pair is selected by agent i, a route binary $\delta^{rs}_{i,a,k}$ is a unit value, otherwise it is zero. Similarly, $\delta^{rs}_{j,a}$ is a unit value if an agent j selects k' path. Eq (3.7b) and Eq (3.7c) are similar with mathematical formulations for static traffic assignment models. We can define boundary rationality of travel time of agents by setting the allowable level of envy for the system by defining maximum allowable envy (e_{ij}) in Eq (3.7e). In other words, a traveler regards his/her travel with the best selection if travel time is within a boundary. In addition, a system manager can find the solution for the best allocation under the definition of their fairness level.

If we define e_{ij} as a very small value near zero, the optimization results in either User Equilibrium or All or Nothing assignment. With the proper value of α, β, and γ we can generalize the state of UE as Lemma 3.1.

Lemma 3.1 (An agent always wants the shortest path regardless of their valuation)

$$e_{ij} > -\theta_i\left(t_j - t_i\right) \text{ iff } \mu_j > \mu_i \text{for all } i, j \qquad (3.8)$$

Lemma 3.2. (UE Compatibility regardless of heterogeneity)

With the objective function for the minimized total travel time and fair allocations, regardless of heterogeneity of agents' valuation, the envy constraint only allows the objective function to be bounded to the solution of UE.

Proof:

$$e_{ij} = -\theta_i\left(t_j - t_i\right) \text{ for all } i, j \tag{3.9}$$

$$e_{ij} \;=\; \epsilon \quad iff \quad t_j \cong t_i \tag{3.10}$$

This solution finds the User Equilibrium condition when we define envy-freeness ($e_{ij} = 0$) or very small value (ϵ). This process can be interpreted as that a traveler changes their route until their travel time is less or equal than that of other travelers.

An interesting finding from Lemma 3.1 is that even though there are heterogeneous travelers in terms of travel time preferences (e.g. the value of time), without discrimination of pricing or other factors, all travelers act in a selfish manner which is equivalent to Wardrop's 1st Principal or Nash Equilibrium. The solution has the same total travel time no matter how the value of times by travelers are heterogeneous. In the UE condition, where all travelers' travel time becomes the same ($t_j = t_i$), envy *i* to *j* is always zero.

Lemma 3.3 The objective value of agent-based modeling is higher than continuous flow models because of unit variable characteristics.

Proof:

This formulation is based on an agent's behavior, thus it does not always guarantee UE solutions coming from continuous demand curve. In an agent-based model, it is evident that path flow cannot be divided into a decimal value. Instead, path flows are always an integer value. In this case, various travel options might strongly bind to the same allocation. Thus, it is necessary to relax the UE constraint in continuous variable models to meaningfully small positive value. Here, we denote it as Epsilon (ε).

Lemma 3.4(Group Envy-Family Envy Problem)

Agents in the same route group do not feel envious toward each other.

Proof:

Eq (3.6) can be extended to prove this lemma. As long as an agent *i* receives the same allocation (t_i) as the other agent *j* (t_i), agent *i* does not feel envy.

This can be regarded as a family cake-cutting problem with optimum distributions. There are various alternatives to the allocation of an efficient transportation system. For example, say K is the path set for OD pairs, and its number of alternatives is always lower than or at least equal to the number of participants. Furthermore, an agent belongs to one of the alternative groups. Thus, we can interpret this case as group envy.

Lemma 3.5 (Shortest path group- No Envy)

In the solution, no one in the shortest path group feels envious.

Proof:

Eq (1) indicates that $e_{ij} = 0$ if $V_{ii} \geq V_{ij}$. Since a travel time is always negative to an agent's utility, a non-shortest path travel time is always longer than the shortest path.

$$e_{ij} = max(-\theta_i(t_j - t_i), 0) \tag{3.11}$$

Lemma 3.6 (Controlling envy for CSO or SO)

We can design the system optimum status by relaxing envy.

Proof:

By allowing agents to feel envious, $0 \leq e_{max}$ a system can expect a better efficient transportation system (Constrained System Optimum, CSO) and if allowable envy is set to be high enough, we can get the system optimum (SO) solution.

$$0 \leq e_{ij} \leq e_{max} \qquad \forall\, i\,, j\,, i \neq\, j \tag{3.12}$$

Lemma 3.7 Envy is minimized in the state of allocative efficiency

Proof:

Lloret-Batlle (2017)'s definition *"A social choice function is allocative efficient if* $\forall \theta \in \Theta, k(\theta)$ $satisfies\ the\ following\ condition$: $k(\theta) \in \underset{k \in K}{argmax} \sum_{i=1}^{n} v_i(k, \theta_i)$ ". This implies that efficiently allocated resources according to individual's valuation minimizes total envy. For example, allocating shortest path to agents who have high valuation and vice versa provides a system to have minimum envy. A simple example shown in the beginning of section 3. 4 is enough to prove this condition.

3.5 Numerical Example

We examine the proposed ASEEM in the well-known Braess' paradox network with the associated Link Performance Functions (LPFs) as in Table 3.2. In this chapter, we assume that LPFs are linear functions for the solvability of the resulting Mixed Integer Linear Programming problem. We admit that linear functions cannot handle the nature of well-known non-linear functions that characterize congestion effects. Although this chapter will focus only on the characteristics of ASEEM, nonlinear programming approaches for ASEEM models will be examined in Chapter 5.

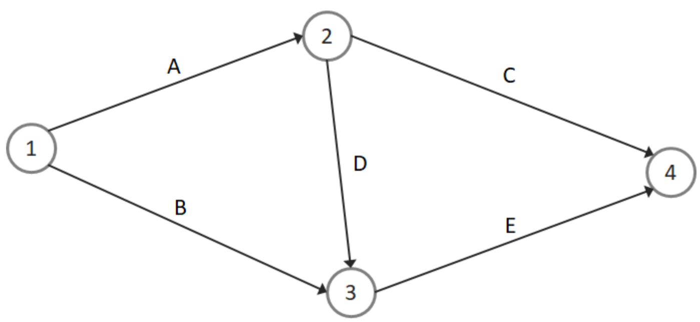


Figure 3. 1 Braess' Paradox Network

We design a hypothetical network by referring to the network used in Nie (2012). As shown in Figure 3.1, there are five links in Brass Network and their LPFs are assumed to be non-negative and increasing functions according to the flow. A continuous path flow set assumption requires demand for an OD for Nie's network necessarily to be large in order to produce integer path flows. The main reason for using small demand is because representative flow patterns can be identified with the continuous demands. The mechanism of our research, however, is based on agent-based modeling, in which demands cannot be disseminated into decimal values. Furthermore, testing characteristics of the proposed model with the small number of agents limits its analytical experiments as being delved in Lemma 3.3. We can increase the number of agents in an OD pair. However, the increased number will result in different path travel time, which confuses the comparative analysis. Thus, we modify the LPFs by introducing scale factors in LPFs.

Figure 3.1 and Table 3.2 indicate the configurations of the hypothetical network. In addition, Table 3.3 shows path sets of the network. There are three paths and the shortest path in free-flow conditions is Path #3 (B-E) which takes 30 time-units. The longest path (a-c) takes 50 time-units. But, as more vehicles select a route, longer travel time will result since travel time is a function of link volume.

Table 3. 2 Link characteristics of the Braess' Paradox Network

ID	Head	Tail	LPF
A	1	2	$\mathrm{t_a} = 5x_a/\omega$
B	1	3	$\mathrm{t_b} = 30 + x_b/\omega$
C	2	4	$\mathrm{t_c} = 50 + x_c/\omega$
D	2	3	$\mathrm{t_d} = 10 + x_d/\omega$
E	3	4	$\mathrm{t_e} = 5x_e/\omega$

Table 3. 3 Path set of the Braess' Paradox Network (from 1 to 4)

Path	Link Sequence	Free flow time
1	a-c	50
2	a-d-e	40
3	b-e	30

To examine the characteristics of the proposed method, initially, we generate agents whose a valuation function for time differs according to their travel preferences. In here, we consider that a travel time variable is the sole effective factor for agents' decisions.

We set the total number of agents to be 24 and their value of time is generated from a truncated normal distribution. The mean value and standard deviation for the normal distribution are $1/time unit and 0.3 respectively. Since the value of time cannot be negative or zero (Small 1992), we convert those generated values to a small number, say $ 0.001 /time unit.

Table 3.4 indicates the results of standard traffic assignments of both UE and SO. The total travel time of UE is 1494.9 and SO is 1349.7. Note that these continuous models produce decimal values of flow, which is incompatible with agent-based models. Our models are based on individual- level decisions; thus, optimal values of total travel times for both UE and SO are higher than that of continuous models as shown in Table 3.4. UE of agent-based models can be achieved if we interpret travel time gaps of paths are within a small range (0.8 time unit). In contrast, as shown in both Tables 3.4 and 3.5, alternatives of SO in both models have travel time gaps. There are three paths. The shortest path in Table 3.4 takes 44.5 time-units, which is 19.75 time units faster than the longest route and is faster than the second shortest route by an amount of 10.25. The gap between the second shortest path and the longest path is 9.50 time-units.

Table 3. 4 Braess Paradox Network (UE and SO, continuous models)

	UE			SO		
Path ID	Flow	Time	Total Path Cost (TPC)	Flow	Time	Total Path Cost (TPC)
1	0	-	-	7.2	64.5	462.3
2	14.9	62.3	925.4	3.0	44.5	133.5
3	9.1	62.3	569.5	13.8	54.5	753.9
Total Travel time'			1494.9			1349.7

Table 3. 5 Braess Paradox Network (UE and SO, agent-based models)

	UE			SO		
Path ID	Flow	Time	Total Path Cost (TPC)	Flow	Time	Total Path Cost (TPC)
1	0	-	-	7	64.25	449.8
2	15	62.5	937.5	3	44.5	133.5
3	9	62.3	560.7	14	54.75	766.5
Total Travel time'			1498.2			1349.8

As our objective function consists of three sub objectives, we examine how different combinations of weights affect the entire systems. We examine four cases as shown in Table 3.6. Case (A) is examines how an agent's greedy behavior aggravates the entire transportation system as defined in Definition 3.4. By decreasing γ from 1.00 to zero, we can understand that taking less account into the greed behavior of a system plays a role in reducing total travel time and inducing agent's envy. Case (B) equally divides weights into thirds. Case (C) put more weight on the total travel time objective instead of greedy behavior. Case (D) and (E) are configured by disregarding γ. Case (E) is only of the minimization of total travel time.

Table 3. 6 Cases for Analysis on the effectiveness of weights for objectives

Case	**Weights for objectives**		
	α	β	γ
A	0.00	0.00	1.00
B	0.33	0.33	0.33
C	0.50	0.25	0.25
D	0.50	0.50	0.00
E	1.00	0.00	0.00

Figure 3.2 indicates the relationship between allowable envy and total travel time. Standard deviation of agents' valuation is assumed to be 0.2 in this case. We compare several scenarios by changing the weights of the objective function (Eq 7). Recap that the total travel time of UE and SO are 1,498.2 and 1,349.8 respectively. Travel time of All or Nothing (AoN) is 1,824. Generally, larger allowable envy (e_{max}) settings, lower the total travel time. Significantly later allowable envy settings can find the SO condition. An extremely small value of e_{max} results in All or Nothing (AoN) assignments in the case of (A) and (B). This condition is based on the characteristics of an agent-based model. Note that the path travel times of the UE condition in the given network are 62.5 and 62.3 respectively. There are 0.2 travel time unit gaps.

Figure 3.2 (a) shows the effectiveness of e_{max} according to the weight configurations of the multiple components of the objective function. Interestingly, only taking the envy to the shortest time into account brings the status of UE and AoN assignments (Case (A)). The solution of the small e_{max} value brings only AON in Case (A). Greater than 0.2 of e_{max} finds the UE solution. All cases show UE solution by the certain threshold of e_{max}. As weighing less on the greedy behavior objective components (β and γ), the total travel time decreases.

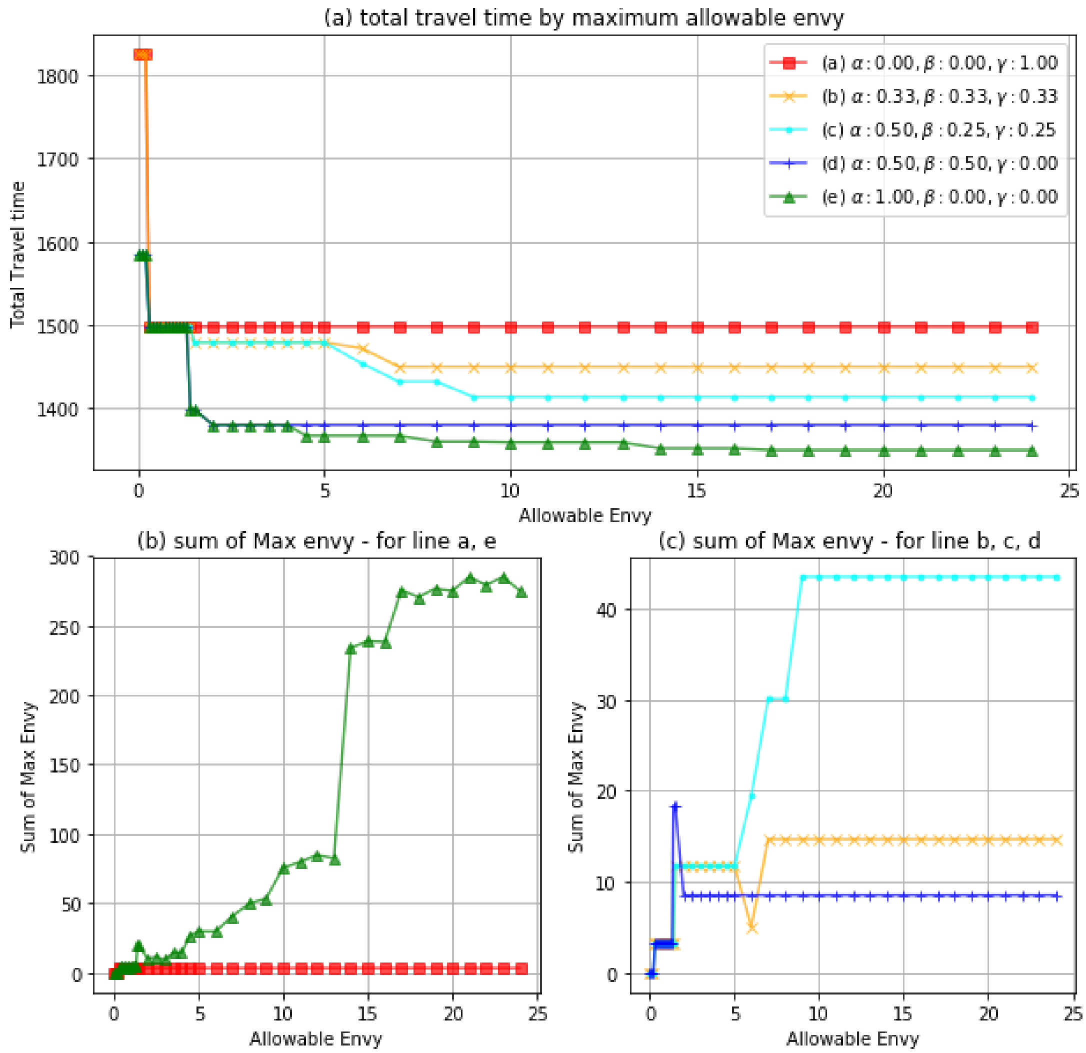


Figure 3. 2 Performance by allowable envy and objective functions

The range of the total travel time between UE and SO is regarded as the constrained system optimum (CSO). Case (B), (C), and (D) fall into this status over the certain threshold of e_{max}. However, SO condition is only guaranteed in Case E. In other words, by setting weight parameters to $\alpha = 1, \beta = 0, \text{and } \gamma = 0$, a system can find the system optimal solutions with the setting of sufficient maximum allowable envy. Here, it is noteworthy to take a look at how envy is induced among those cases. Figures 3.2 (b) and (c) indicate how efficient transportation systems lead to unfairness in terms of the sum of maximum envy. As expected, a high level of unfairness accompanies SO status from Case E. This is in line with the relaxed boundary rationality enabling an efficient system (Literature). The small amount of allowable envy achieves UE condition. If it is very small or equal to zero, it becomes equivalent to the results of AoN the assignment. UE and AoN solutions are envy free, but not efficient systems.

In contrast to Case E in Figure 3.2 (b), Figure 3.2 (c) shows that the objective functions of Cases (B), (C), and (D) have a capability of controlling both inefficiency and unfairness to certain levels but not to an optimum. From the comparisons between Cases (B), (C) and (D), we can infer that a lower weight of γ can result in better solutions for both efficiency and fairness. Furthermore, the differences among Cases (C) and (D) are related to β and γ. Case D puts more importance on envy between agents by disregarding envy for the shortest path. From our experience, we arrive at the best combination, for the hypothetical network, that considering only total travel time and envy among agent (Case (D)) results in the condition that total travel time is optimal and envy is minimized.

In Case (D), we can observe Allocation Efficiency (Lemma 3.7). Table 3.5 is an example of the case in which e_{max} =20, which is relatively large. The sum of maximum envy of this setting is 8.50 and Total travel time is at 1349.8. The shortest path (Path ID: 2) is assigned to agents having higher valuations (2.0 unit-time to 1.5) and the longest path (Path ID:1) is allocated to the lowest VOT group. Other cases, which put less weight on envy among agents, do not come with minimized envy and their results do not show Allocation Efficiency.

Table 3.7 indicates the results of Case D.

Table 3. 7 Examination of ASEEM

Path ID	Flow	Time	Total Path Cost (TPC)	VOT range
1	7	64.25	449.8	0.5-0.001
2	4	44.5	133.5	2.0-1.5
3	13	54.75	766.5	1.5-0.5
			1349.8	

3.6 Discussion

In this chapter, we designed an agent-based transportation model with an envy minimization scheme. Furthermore, we propose a new paradigm of traveler's behavior by incorporating characteristics of CAVs and P2P data exchanges with the future block-chain implementations. We employed envy mechanisms to develop a network optimization model. The proposed model has another contribution in that heterogeneity of travelers are significantly modeled, which traditional models assume it to be homogeneous.

We assume that a system manager finds the best path allocation of OD demand when she/he knows the distribution of valuation of time. The objective function for a system is designed to maximize efficiency and minimize envy among agents. This multi-objective goal is tested by various weights of each component of the objective function.

Results show that if a system manager does not consider equity in the system, it may induce significant envy. A system manager might design his/her objective function to minimize envy by setting β and γ to positive values; then, it will deter a system from the system optimum status and such systems cannot be a solution for envy-free.

From the experiments, we also find that envy exists, even though a system manager sets the objective for envy minimization. This suggests implementing pricing schemes to design a system where system efficiency is maximized while no agent feels unfairness. Chapter 4 will address a model with a pricing scheme.

Chapter 4

4.1 Allocated System Efficiency with Envy Minimized Price Matching (ASEEM-PM)

As we have analyzed in Chapter 3, an efficient system intrinsically accommodates envy. Although ASEEM minimizes the sum of maximum envy of agents, it is almost impossible to achieve Envy-Free allocations; this signifies the importance of introducing a pricing scheme. In other words, pricing or incentivizing plays a significant role in attracting agents to follow alternative traffic patterns that minimize envy. To accommodate pricing, we introduce pricing terms to Eq (3.2) as displayed in Eqs (4.1)-(4.3) . The pricing term can take any value (toll, incentive, or free), but the system needs to find its appropriate amount. Eq (4.2) can also be represented using the resources, a value of time, and pricing terms.

$$e_{ij} = (V_{ij}(\theta_i, t_j, p_j) - V_i(\theta_i, t_i, p_i))\delta_{ij} \qquad \forall\, i\,, j\,, i \neq j \qquad (4.1)$$

where δ_{ij} is 1 if “envy” is positive and zero otherwise

As the same with Eq (3.4) and Eq (3.5), we relax Eq (4.1) to Eqs (4.2) – (4.3)

$$\theta_i t_i + e_{ij} - p_i \ \geq\ \theta_i t_j - p_j \qquad (4.2)$$

$$e_{ij} \ \geq\ \theta_i(\mathrm{t}_j - t_i) + p_i - p_j \qquad (4.3)$$

Note that this solution cannot be solved without a budget constraint; i.e., Eq (4.4). This sets a criterion of how pricing terms are formulated. Setting budget equal to zero means budget-balanced where the money collected from shorter allocation users is distributed to other agents.

$$\sum_{i \in I^{rs}} p_i = B^{rs} \tag{4.4}$$

Since a budget constraint is related to pricing terms, budget B should be carefully defined. The solution might be unfeasible if a system manager sets a budget B to be positive and price terms to be an incentive (negative p_i). If p_i is positive, agent i should pay for their allocation, which might be considered as a toll. In contrast, negative pricing term implies that agent i will receive incentives for their allocation.

We can also prevent abusive behavior where an agent travels only for incentives as Eq (4.5)

$$\theta_i \mu_i \geq p_i \qquad \forall \boldsymbol{i} \in \boldsymbol{I} \tag{4.5}$$

By restricting the toll and incentive amount as a policy, a system manager can control maximum payments for allocations as Eq (4.6). If a manager wants to implement a sole incentive scheme, she/he can set p_{max} to be negative. Similar, for a congestion toll scheme, a positive p_{min} find the optimal congestion pricing with the consideration of user's heterogeneity and of minimization of their envy.

$$p_{min} \leq p_i \leq p_{max} \qquad \forall \boldsymbol{i} \in \boldsymbol{I} \tag{4.6}$$

Let us assume that a system manager can provide a subsidy for this system, then, a budget constraint becomes a negative value. Enough amount of funding allows a system to provide an incentive to agents who are willing to yield their fast option. This implies free-taxi or transit services that are currently under spotlight as shared-mobility. However, an incentive strategy does not always have pros since induced demands and abusive behavior will be caused by free cakes. This will be addressed in section 4.2. In addition, this objective function can find the solution for the second-best strategy where roads without a toll exist.

Finally, the purpose of the objective function is to find the optimal pricing scheme to achieve both UE and efficient transportation system under the constraints of budget and maximum allowable envy.

$$\text{Min obj} = \alpha \sum_{a\in A} \mathrm{x}_a t_a\,(\mathrm{x}_a) + \beta \sum_{i\in I} \max_{i\neq j}\{e_{ij}\} + \gamma \sum_{i\in I} \mathrm{e}_{\mathrm{is}} \tag{4.7}$$

subject to:

$$\sum_{k\in K} -t_a \delta^{rs}_{i,a,k}\theta_i + e_{ij} - p_i \geq \sum_{k'\in K} -t_a \delta^{rs}_{j,a,k'}\theta_i - p_j \qquad \forall\, i,\, \mathrm{j} \in I,\ \mathrm{i} \neq j \tag{4.7a}$$

$$\mathrm{x}_a = \sum_{rs} \sum_i \sum_k f^{rs}_{i,k} \delta^{rs}_{i,a,k} \qquad \mathrm{i} \in I, k \in K^{rs} \tag{4.7b}$$

$$q^{rs} = \sum_{\mathrm{i} \in I} \sum_k f^{rs}_{i,k} \qquad \forall\, \mathrm{rs}, \forall\, k \in K^{rs} \tag{4.7c}$$

$$f^{rs}_{i,k} \geq 0 \qquad \forall\, i, k \tag{4.7d}$$

$$\sum_{k\in \mathrm{K}} f^{rs}_{i,k} = 1 \qquad \forall\, i, \forall\, \mathrm{rs} \tag{4.7e}$$

$$0 \leq e_{ij} \leq e_{max} \qquad \forall\, i, j, i \neq j \tag{4.7f}$$

$$p_{min} \leq p_i \leq p_{max} \qquad \forall\, i \tag{4.7g}$$

$$\sum_{i\in I^{rs}} p_i = B^{rs} \qquad \forall\, \mathrm{rs} \tag{4.7h}$$

$$\theta_i \mu_i \geq p_i \qquad \forall\, i \in I \tag{4.7i}$$

4.2 Characteristics of ASEEM-PM

This section addresses the characteristics of ASEEM-PM.

Lemma 4.1 (Compatibility to traffic assignment strategies)

We can design the system optimum status by relaxing envy and pricing.

Proof:

Table 4.1 shows the relationship between control variables and the status of traffic assignment assumptions. With very small envy in the amount of ε, the model converges to the results of All or Nothing (AoN) or User Equilibrium assignment. Pricing policy can promote a more efficient system while envy is restricted to a small amount. Finally, allowing maximum pricing to be large-enough can achieve the status of system optimum and no one is likely to complain about his/her travel option, which is the status of Envy-Free. It is also equivalent to the Perceived UE (PUE) that is UE with respect to agent' goal of achieving minimum of both travel time and toll.

Table 4. 1 Compatibility to traffic assignment strategies

e_{max}	p_{max}=0	$\lvert p_{max}\rvert$ is small	$\lvert p_{max}\rvert$ is large enough
ε	AoN, UE	CSO, Envy	SO & PUE, Envy-free
Small	CSO, Envy	CSO, Envy	-
Large enough	SO, Envy	SO, Envy	-

This implies that a pricing scheme can play a pivotal role in the system design for both efficiency and equity. From the assumption of bounded rationality, we can allow a system to have better efficiency (CSO) but it could not be optimal (SO). Furthermore, bounded rationality admits that agents feel envious. If maximum allowable envy is set too high, the designed system could lose confidence from participants since induced envy will frustrate agents who follow the given guidelines. If a system forces people to unfair options without enough compensation, an equity problem will be significantly raised, which cannot be a sustainable scheme. Thus, we can conclude that the pricing scheme is an important feature of the proposed system. Now, we need to know how to define proper pricing.

Lemma 4.2 (Pricing strategy depends on the budget and pricing range)

The proposed model finds the best toll and incentive amounts based on the optimization. The pricing policy depends on the setting of the budget and allowable maximum envy. As can be seen in Table 4.2, the proposed model can be designed to incorporate situations where a government wants either to distribute subsidy or to charge operational costs for the system. Table 4.2 indicates how the budget affects configurations of the pricing scheme. First, both tolls and incentives will be features of a system in the budget-balanced case since the collected tolls are transferred as compensation to the other agents.

From this characteristic, we can also infer that excessive funds will result in a situation where every agent gets payments from his/her travel, which will be unlikely to happen or undesirable in terms of abusive behavior. A significant portion of agents will unnecessarily travel under this condition, which induces extra demand from abusive behaviors.

A proper amount of funding will help the system to differentiate payments: some will pay and others will get payments. There exists a condition that only detoured agents get incentives under the funding condition. In contrast, if a system manager sets his/her model for excessive profit, all agents will be required to pay for their travel, which is similar to the marginal cost toll strategy. A system manager can even design a pricing scheme similar to the second-best toll where only some portion of routes are charged—others being free.

Table 4. 2 Relationship between budget and pricing policy

Budget, B	$\boldsymbol{p \geq 0}$	$\boldsymbol{p \in \mathrm{IR}}$	$\boldsymbol{p \leq 0}$
Subsidy (--)	N/A	Incentives	Incentives
Subsidy (-)	N/A	Toll, Incentives	Incentives
Balanced (0)	N/A	Toll, Incentives	N/A
Profit (+)	Toll	Toll, Incentives	N/A
Profit (++)	Toll	Toll	N/A

Figure 4.1 visualizes the details of the relationships among variables addressed in Table 4.2. If we assume there are two agents and two routes in the system and they must select one of

the routes; say agent *1* selects a faster route then agent 2 is forced to select a slower route. The x-axis represents the budget. Each agent's price is determined by budget as shown in Figure 4.1. In other words, this diagram shows how an individual gets payments or is charged under the budget setting by a system manager. Positive price means toll and negative is for incentives. For a simple explanation, this diagram is drawn with the assumption that agents' valuation is the same, thus the price gap among agents is equal to the gap of allocations.

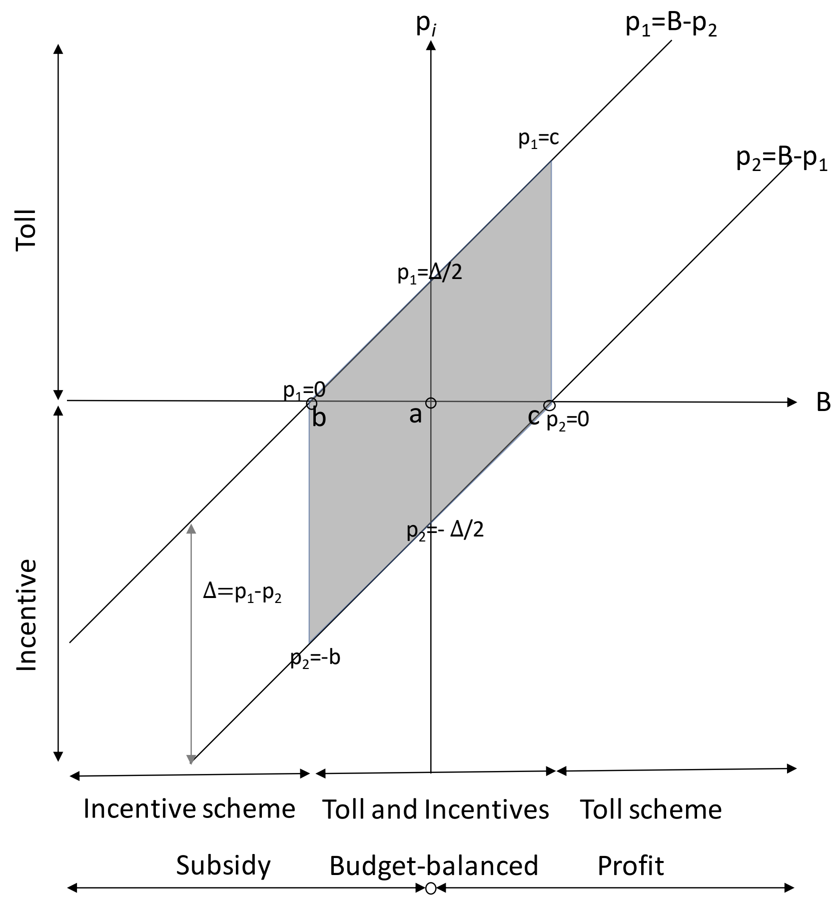


Figure 4. 1 Relationship between pricing and budget

A feasible budget range for ASEEM-PM that both payments and tolls exist is shown as a shaded rectangle in Figure 4.1. In the budget-balanced case (a), the sum of prices equals to zero; agent *1* who selects a faster option would pay a toll that transfers to agent *2* in compensation for her/his inconvenience. There is no brokerage or system operational cost or profit. A system can manage those costs by increasing the budget. At point (c), agent *1* will be charged the amount of (c). Agent *2* will still be free of charge, which is to the Second-best toll (Rouwendal and Verhoef 2004; E. T. Verhoef 2002; E. T. Verhoef, Nijkamp, and Rietveld 1996; E. T. Verhoef 2004; Steimetz, n.d.; E. Verhoef, Nijkamp, and Rietveld 1995; H. Yang and Zhang 2003). Both agents should pay for their travel if budget is set to be greater than (c). On the other hand, if a subsidy is available from either government or commercial sector, the amount of payment for agent 1 will be smaller. At point (b), agent *1* will not pay for the shortest path, but agent *2* will get an incentive for their longer route. Excessive budget lower than (b) would make a situation that every agent gets incentives for their journey.

Lemma 4.3 (Allocation Efficiency with pricing)

When agents' valuation is ordered decreasingly, allocation efficiency with pricing is envy-free if and only if:

High VOT – assigned to the shortest path and pay tolls

Low VOT – assigned to the longest path - pay less tolls or receive rewards

Proof:

Proof of this lemma is nothing but the proof for the allocation efficiency theorem in Lloret-Batlle and Jayakrishnan (2016), showing that envy-freeness is achieved if and only if the allocation is monotonously decreasing and the price is a function of valuation of individual and allocation difference. Our problem reinterprets the monotonously decreasing allocations into increasing travel time allocation. And an agent's valuation, $\theta = (\theta_{1,} \cdots, \theta_{n,})$, is decreasing order

a) Travel time is monotonous increasing

$$t_1 \leq \cdots \leq t_n \tag{4.7}$$

b) The prices follow from this expression:

$$\forall i \in I, p_i = \sum_{k \leq i} \alpha_i, \alpha_i \in \Re, \alpha_k \in [-\theta_{k-1}(t_k - t_{k-1}), -\theta_k(t_k - t_{k-1})] \; \forall k > 1 \tag{4.8}$$

Proof. Condition a): From the envy comparison constraints Eq (4.2), we can derive Eq (4.9c'). According to the valuation order (i < j), θ_i is always greater or equal than θ_j. Thus, t_j is always greater or equal than t_i.

$EF \Rightarrow$ Eq (4.7):		
$-\theta_i t_i - p_i \geq -\theta_i t_j - p_j$	(4.9a)	Envy comparisons from P2P communications
$-\theta_j t_j - p_j \geq -\theta_j t_i - p_i$	(4.9b)	
$-\theta_i t_i - p_i + \theta_i t_j + p_j \geq 0$	(4.9a')	Reorder of Eq 4.9a
$-\theta_j t_j - p_j + \theta_j t_i + p_i \geq 0$	(4.9b')	Reorder of Eq 4.9b
$-(\theta_i - \theta_j)\, t_i + (\theta_i - \theta_j)\, t_j \geq 0$	(4.9c)	4.9a'+4.9b'
$-(\theta_i - \theta_j)(t_i - t_j) \geq 0$	(4.9c')	Reorder of Eq 4.9c

Proof. Condition b)

$EF \Rightarrow$ Eq (4.8):		
$-\theta_j(\mathrm{t}_j - \mathrm{t}_i) + p_i \geq p_j$	(4.9d)	Reorder of Eq 4.9b’
Set $p_1 = \alpha_1$	(4.9e)	
$-\theta_2(\mathrm{t}_2 - \mathrm{t}_1) + \alpha_1 \geq p_2$	(4.9f)	
$-\theta_3(\mathrm{t}_3 - \mathrm{t}_2) + \alpha_2 \geq p_3$	(4.9g)	
$-\theta_n(\mathrm{t}_n - \mathrm{t}_{n-1}) + \alpha_{n-1} \geq p_n$	(4.9h)	Generalization of (f, g)
$\alpha_{n-1} \geq p_n + \theta_n(\mathrm{t}_n - \mathrm{t}_{n-1})$	(4.9i)	Since $\mathrm{t}_n - \mathrm{t}_{n-1} \geq 0$, then $\alpha_{n-1} \geq p_n$

EF ⇒*decreasing pricing order*: as shown in Eq (4.9i), $\alpha_{n-1}$is always equal to or greater than p_n since $\mathrm{t_n - t_{n-1}}$, $\theta_{\mathrm{n-1}}$ are positive values. This property reserves that the pricing is also a decreasing order. It is easy to infer that the higher valuation groups pay more with the faster route. Interestingly, in the budget balanced case, the pricing scheme where a higher valuation group pays more, and a lower valuation group is assigned to the longer paths and receives incentive.

Furthermore, from Eq (4.8), we can note that an agent *n-1* can pay the price difference as large as the perceived amount of travel time difference of the next agent k, $-\theta_{\mathrm{n-1}}(\mathrm{t_n - t_{n-1}})$. And agent n is acceptable the minimum price gap to the upper agent, $-\theta_{\mathrm{n}}(\mathrm{t}_n - \mathrm{t_{n-1}})$. This is the subsequent procedure; we can find the price pattern as Eq (4.10):

$\mathrm{i < j}$ and $\mathrm{t_i < t_j}$: $-\theta_{\mathrm{i}}(\mathrm{t_i - t_j}) + \sum_{\mathrm{k \leq i}} \alpha_{\mathrm{k}} > \mathrm{p_j} > -\theta_{\mathrm{j}}(\mathrm{t_i - t_j}) + \sum_{\mathrm{k \leq i}} \alpha_{\mathrm{k}}$	(4.10)

Lemma 4.4 (No price discrimination in the same path group)

Agents in the same group pay tolls or get incentives at the same amount in the minimized envy condition. In other words, if tolls differ by agents in the same path group, the envy increases, thus each group becomes to have the same price in the envy minimization.

Proof) Similar to Lemma 3.3. Agents in the same path group have identical travel time; the envy-free condition is achieved if and only if prices of agents are the same.

Lemma 4.5 Pareto efficiency with the budget balanced constraint

Proof) When we define a budget-balanced condition that the sum of toll and incentives is zero, it can be interpreted as that an agent who does not feel envy for allocations compensates agents who feel envy to others'. This definition can be regarded as "Pareto Efficiency"

4.3 Numerical experiments

In this section, we evaluate our proposed pricing scheme (ASEEM-PM) and check the properties that we have mathematically examined. There are various factors that affect the proposed model such as the distribution of valuation, a configuration of weights of an objective function, maximum allowable envy, and maximum willingness to pay. We found that considering both total travel time and the sum of maximum envy brings the best result while excluding the weight for greedy for the shortest path. Thus, overall, we set weights as Case (D) in Section 3. 5($\alpha = 0.5, \beta = 0.5$, and $\gamma = 0.0$). Similarly, the total number of agents is 24 and Braess's Paradox Network has been implemented.

4.3.1 Effectiveness of $\mathbf{e}_{max}$ and $\mathbf{p}_{max}$

First, we allow the ASEEM-PM to find the level of envy and price with p_{max} set to infinity. Agents' valuation parameter is randomly drawn from a normal distribution with mean is equal 1.0 and standard deviation 0.3. As shown in Table 4.3, ASEEM-PM leads to the best scenario that minimizes Total Travel Time (TTT) and Total Max Envy (TME). As pricing scheme controls unfairness coming from the efficient system, it achieves Envy-Free condition. Since SO assignment does not deal with heterogeneity, we assume that an agent's valuation is monotone as 1 time-unit. Although the ASEEM model devises the optimal solution by allocating longer travel time paths to agents whose valuation are low, it comes with envy. We also assume that a system manager does not want the system to be profitable or to be funded, meaning budget-balanced to zero.

Table 4. 3 Results of ASSEM-PM and its comparisons with other models

Model	Total Travel Time	Total Maximum Envy
UE	1494.9	-
SO without toll	1349.7	282.0
Congestion pricing	1349.8	21.16
Second-Best Toll	1349.8	11.48
ASEEM	1380.0	8.5
ASEEM-PM	1349.8	0.0

Table 4.4 indicates comparisons of allocated path and credits. There are only two paths in the results of UE assignment and its travel time is equilibrated at 62.3. The SO model has three paths and its travel time gaps to the shortest path are 20 for path1 and 10 for path 3: there are equivalent gaps to maximum envy relative to the shortest path. Path flows of both SO and UE are continuous variables, whereas the ASEEM models are agent-based models, restricts path flows of both models to integer variables.

Table 4. 4 Path-level comparisons among models

Type	Travel time (Vol)			Pricing			Max (Envy)		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
UE	62.3 (14.9)	62.3 (9.1)	-	-	-	-	0	0	0
SO	64. 5 (7.2)	44.5 (3.0)	54.5 (13.8)	-	-	-	20	0	10
Congestion Pricing	64.25 (7)	44.5 (3)	54.75 (14)	14.25	34.50	24.75	0.0	0.0	2.58
Second- Best Toll	64.25 (7)	44.5 (3)	54.75 (14)	0	19.75	9.5	0.0	0.0	1.58
ASEEM	59.0 (6)	-	57.0 (18)	-	-	-	1.50	-	0.0
ASEEM-PM	64.25 (7)	44.5 (3)	54.75 (14)	-7.73	14.91	0.67	0.0	0.0	0.0

ASEEM does not find the minimized travel time solution and has two paths separated by time-unit gaps of 2. However, the maximum envy felt by agents in the longer path group (path 1) is 1.5. This is because the maximum envy is bounded to the value of an extreme agent in the path group. This property will be utilized in Chapter 5 for reducing the complexity of the proposed algorithms. Finally, the pattern of path flows of ASEEM-PM is close to the SO solution. The slight differences come from the characteristics of an agent-based model.

As can be seen in Table 4.4, ASEEM-PM does not induce envy. Varying prices or incentives, according to allocation and travel time gap and heterogeneity, control the system to be Envy-Free. The shortest path group pays tolls at the amount of 14.91 and the second path groups also pay tolls but small at the amount of 0.67. Those collected tolls are used to compensate the length of path groups' inconvenience, which compromises their envy. Interestingly, the allocated efficiency is also observed in the ASEEM-PM (Lemma 4.2). The shorter paths are allocated by categorization of valuation of time. This makes sense in that agents who have higher valuation of time pay for their shortest travel time and agents who are not sensitive to travel time are willing to yield their shortest option to others with the compensation.

With the assumption that the valuation of agents is randomly sampled from $\sim N\ (1, 0.3)$, we examine all possible combinations of the setting of allowable envy and maximum willingness to pay. The maximum travel gap among the path of the hypothetical network is 20 time-units. Thus, we set the range of both e_{max} and p_{max} from zero to 20. From our prior tests, we found that objectives are significantly sensitive to $e_{max}$and p_{max} adjacent zero: so we use a more dense-sampling in this range. We vary the interval due to the slow convergence of a solution. The variables are discretized with 0.1 interval from 0 to 1.5, 0.5 interval from 1.5 to 5.0, and 1.0 interval by 20.

Figure 4.2 visualizes the effectiveness of e_{max} and p_{max}. Extreme small value of envy without pricing comes with the result of AoN. This is also an Envy-Free solution but the worst usage of transportation supply. By relaxing e_{max} and p_{max} , ASEEM-PM is able to find the equivalent solution to UE condition. We can also observe a transitional region between UE and SO. It is also known as Constrained System Optimal (CSO). But it comes with envy if pricing is not introduced. It is interesting to find waves in Envy objectives (right). These waves happen when improved travel time is not controlled by enough pricing. At the same total travel time, increasing p_{max} lowers the total maximum envy. Finally, combinations with enough e_{max} and p_{max} allow ASEEM-PM to achieve our goal-maximized efficiency and fairness. Although we set e_{max} and p_{max} to be very large, say infinity, pricing is bounded to a certain value. We will discuss this property in Section 4.3.2.

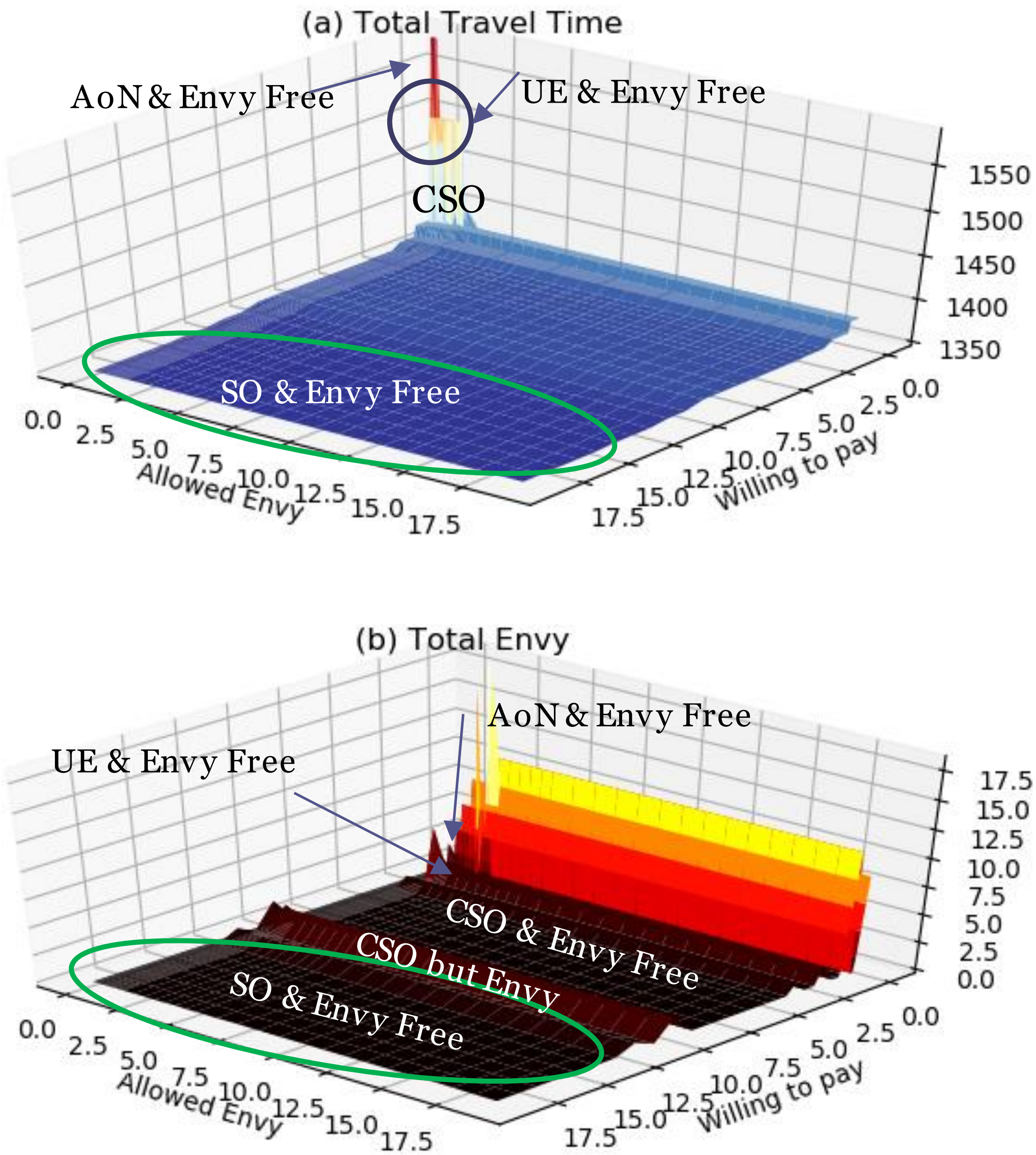


Figure 4. 2 Pattern of ASEEM-PM under different combination of e_{max} andp_{max}

4.3.2 Relationship between maximum willingness to pay and heterogeneity

The heterogeneity of agents is of critical importance in the mechanism of the proposed method. Figure 4.3 shows the histograms of generated individuals for various standard deviations. Note that we can observe the increasing number of the smallest bin as being generated by negative values replaced by the smallest value as defined in Definition 3.2.

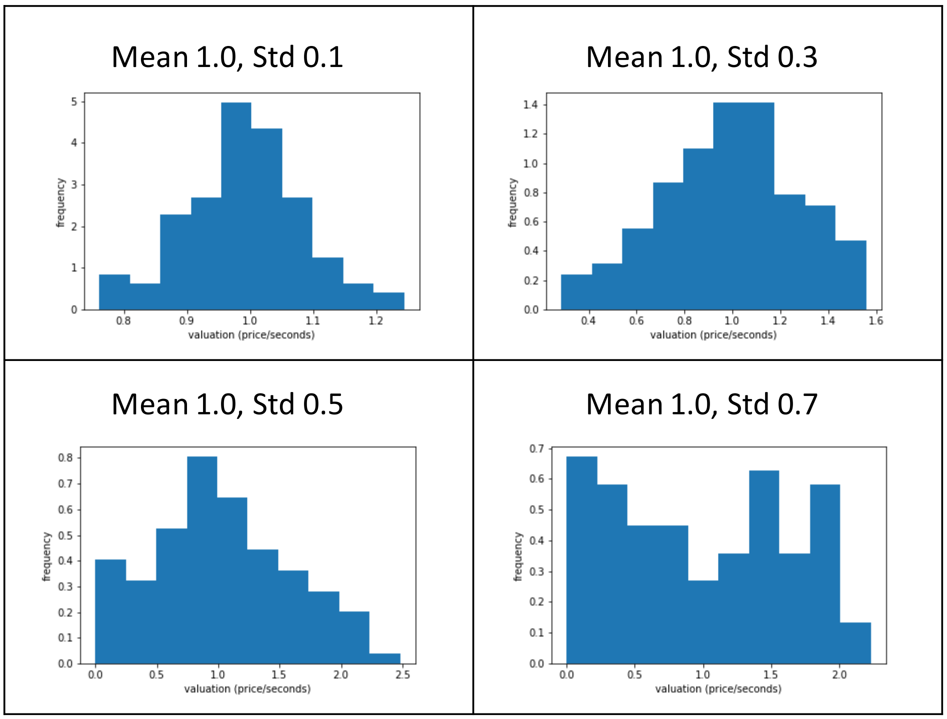


Figure 4. 3 Histograms of the value of time for generated agents

Figure 4.4 shows a comprehensive set of results on the effectiveness on p_{max} variable while e_{max} is fixed to be a large number. Pricing is related to the heterogeneity of agents, so this figure analyzes the effectiveness of p_{max} and heterogeneity together. We categorize the heterogeneity of agents' valuation into two groups. Left figures (a and c) visualize the scenarios having a relatively small standard deviation. Right (b and d) are for the relatively large standard deviation groups.

As can be seen in Figure 4.4 (a) and (b) provided enough value of p_{max} we can find the optimal solution for both the minimum total travel time and minimum total max envy. Specifically, small p_{max} settings result in significant envy regardless of the level of heterogeneity. Furthermore, it does not find the optimal solution for the minimized total travel time. This justifies the role of pricing schemes that charges enough tolls and distribute collected tolls to the other agents.

Another general pattern is that greater heterogeneity can bring higher efficiency in a lower maximum payment setting as we compare graph (a) and (b). As higher heterogeneity in Figure 4.3 (a), small amounts of maximum willing to pay find better efficiency. Figure 4 4 (b) also shows the same pattern. Sheffi (1985) shows that higher perception error for travel time could lead travelers to not use the shortest path and a more substantial variance of perception could distribute travel along paths that travelers regard as their best. Although we do not consider agents' perception error, our solution shows similar results with the heterogeneity of valuation of time for agents.

Maximum willing to pay over 12 time-units bound to 11.9 time-unit for $\sigma = 0.0$. Other heterogeneity cases, i.e., $\sigma = 0.1, 0.3, \text{and } 0.5$ are bounded 12.79, 14.91, and 16.97, respectively. This pattern implies that heterogeneity contributes efficiency with the lower maximum payments. However, the optimization problem with the high heterogeneity (σ = 0.7) does not find the solution where total travel time is lower than the user equilibrium condition, (we will address this condition in more detail in Section 4.3.3.).

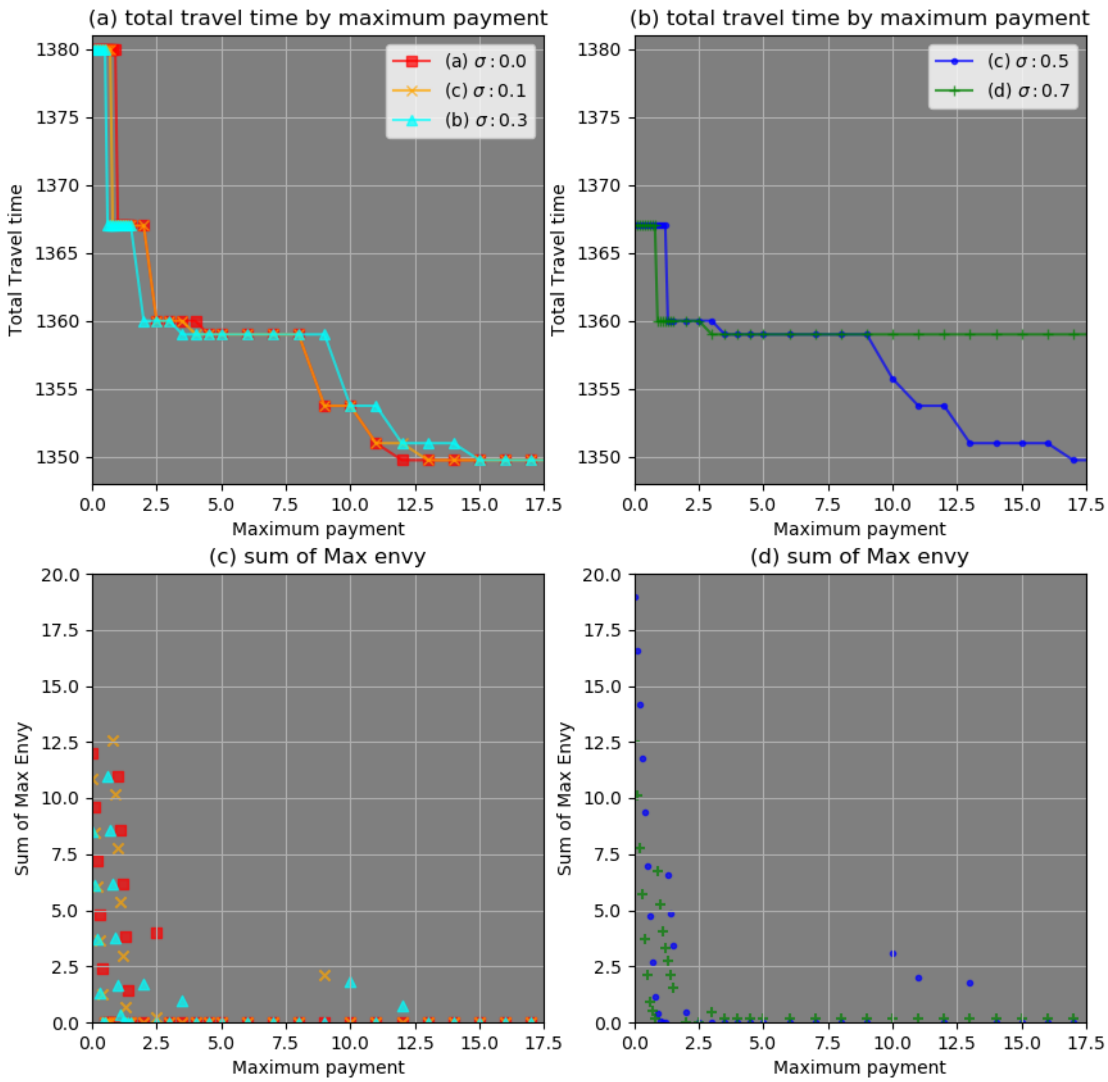


Figure 4. 4 Performance with respect to maximum payment and heterogeneity

4.3.3 Relationship between heterogeneity and budget

This section analyzes heterogeneity in more detail. This examination relaxes the p_{max} and e_{max} to be large enough, that p_i and e_{ij} control the results of optimizations. Figure 4.5 visualizes the relationship between heterogeneity and objective values.

From the solutions, we can infer that while budget constraints do not affect the minimization of total travel time, heterogeneity is an effective factor. Both the travel time and sum of envy is effectively regulated under moderate heterogeneity. However, results for heterogeneities over 0.5 standard deviations seem unstable. It is highly likely that the optimization might not find the optimum solution in both total travel time and sum of envy. We expect that large heterogeneity make the optimization problem too complex to solve in the existing solvers. It is worth restating that the proposed method is a mixed integer linear programming.

The numerical example, Braess's Paradox Network, has three binary variables relating to paths. In addition, there are individual-level envy comparisons, that associates $N \times N - 1$ constraints. Pricing terms also increase complexity. We employ Gurobi solver that solves MILPs heuristically. There are different modes for exploring the MILP search tree and we can increase the number of iterations for optimization. These settings help to find the better solution; however, our experiments could find the optimum solution that meets our expectation for the objective value. Chapter 5 will address this problem by reducing the problem sizes while not losing of generality of the proposed scheme (ASEEM-PM).

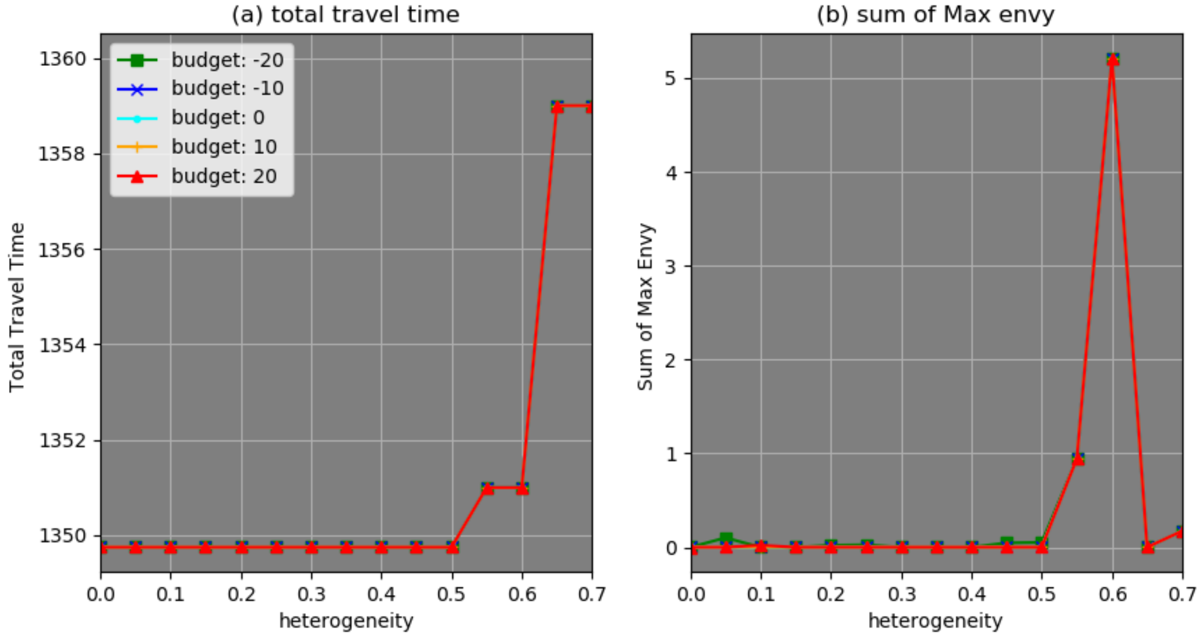


Figure 4. 5 Performance with respect to budget and heterogeneity

In addition, pricing distribution among paths might be of critical importance to a system manager and users. Moreover, price variations according to the heterogeneity are also matters of interest. Table 4.5 shows total transaction amounts according to both budget and heterogeneity. As addressed in Figure 4.5. all heterogeneity cases, except for the case of that standard deviation of valuation is higher than 0.7, have the same traffic pattern. R2 is the shortest path and R1 is the longest path. As addressed in Chapter 2.3, the pricing differences among paths for homogeneous valuation are the same to travel time gaps regardless of budget. Higher heterogeneity is related to increase the pricing gaps.

Generally, the fastest route path group (R2) tends to increase their willingness to pay as heterogeneity increases. It is also observed that incentives for the slowest route path group

(R1) get lower by heterogeneity. We can infer that the mean value of valuation of the shortest group increases by heterogeneity, which implies that agents in this group put more weight on travel time and are more willing to pay for their shortest time. Similarly, the mean value of valuation for agents in the longest group might be lower, which are less sensitive to travel time and follow the longest route by smaller incentives. Interestingly, the second shortest group (R2) might pay for their shorter travel time, but not always.

Subsidizing (budget) also affects groups pricing and total transaction cost. Total transaction cost (TTC) is computed as Eq (22).

$$TTC = \sum_{rs} \sum_{i \in I} |p_i^{rs}| \tag{22}$$

When a manager operates this system for his/her profit, the shortest path group will pay more, and the slowest group will get less paid than the budget balanced case ($B = 0$). This also affects the amount of total transaction cost.

Intuitively, a subsidy helps decrease the total amount of transacted price. Our experiments, however, find that high subsidy can also increase total transaction cost under high heterogeneity conditions. The hypothetical network has this case and mostly happens in the transitions of pricing. For example, in budget-balanced case (B=10), total transaction cost decreases as heterogeneity grows until 0.3 but then increases again. We guess that this condition occurs because of the second group (R3) where pricing changes from tolls to incentives in the subsidy cases (B>0).

Table 4. 5 Path-level pricing comparisons by heterogeneity and budget

Budget	Heterogeneity	pricing			Total Transaction
		R1	R2	R3	
-20	0	-7.18	12.57	2.32	120.48
	0.1	-6.94	13.58	1.99	117.22
	0.3	-6.47	15.56	1.33	110.62
	0.5	-6.53	17.78	0.89	111.47
	0.7	-4.32		3.92	97.67
-10	0	-7.59	12.16	1.91	116.31
	0.1	-7.50	13.22	1.63	115.05
	0.3	-7.32	15.32	1.09	112.46
	0.5	-6.42	17.14	0.25	99.89
	0.7	-4.85		3.56	97.06
0	0	-8.01	11.74	1.49	112.15
	0.1	-7.77	12.74	1.16	108.85
	0.3	-7.74	14.93	0.67	108.35
	0.5	-6.84	16.73	-0.17	100.45
	0.7	-5.09		3.15	93.06
10	0	-8.43	11.32	1.07	107.98
	0.1	-8.19	12.32	0.74	104.70
	0.3	-8.15	14.49	0.26	104.12
	0.5	-7.26	16.35	-0.59	108.111
	0.7	-5.68		2.73	92.07
20	0	-8.84	10.91	0.66	103.81
	0.1	-8.75	11.96	0.38	102.53
	0.3	-8.57	14.07	-0.16	104.44
	0.5	-7.68	15.94	-1.01	115.62
	0.7	-6.10		2.31	89.56

4.3.4 Insensitivity of solution optimality on the chosen random distribution

The symmetry characteristics of the normal distribution might be a reason for the optimality of solution. To test the impact of the symmetry characteristic on the final solution, we also evaluate the proposed method with another random probability distribution. However, it is noteworthy that the chosen distribution of valuation of time in the previous case might also be asymmetric since we manually converted some generated negative values to a small positive value. This happens when the standard deviation setting is high.

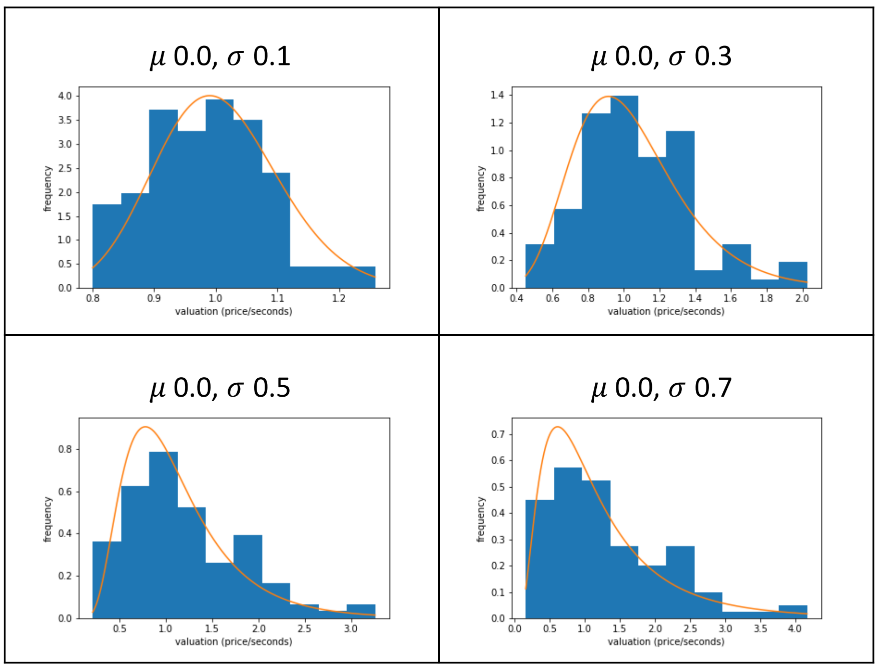


Figure 4. 6 Lognormal distribution with different sigma

Here, we examine a lognormal distribution for agent's valuation of time with different settings. A lognormal distribution is also a continuous random distribution where the random variable is lognormally distributed. Thus, the lognormal distribution only takes positive values which is reasonable assumption for valuation of time. In addition, the parameters for the lognormal distribution are μ and σ, which are the mean of logarithmic value and standard deviation of logarithmic value, respectively. As can be seen in Figure 4.6, a higher value of σ of the lognormal distribution results in an asymmetric distribution.

Figure 4.7 shows the result for the case where agents are randomly sampled from this distribution. The results show similar patterns as in the normal distribution case. An appropriate value of the maximum willingness to pay (p_{max}) setting can find the optimal solution for both efficiency of total travel time and fairness in terms of envy.

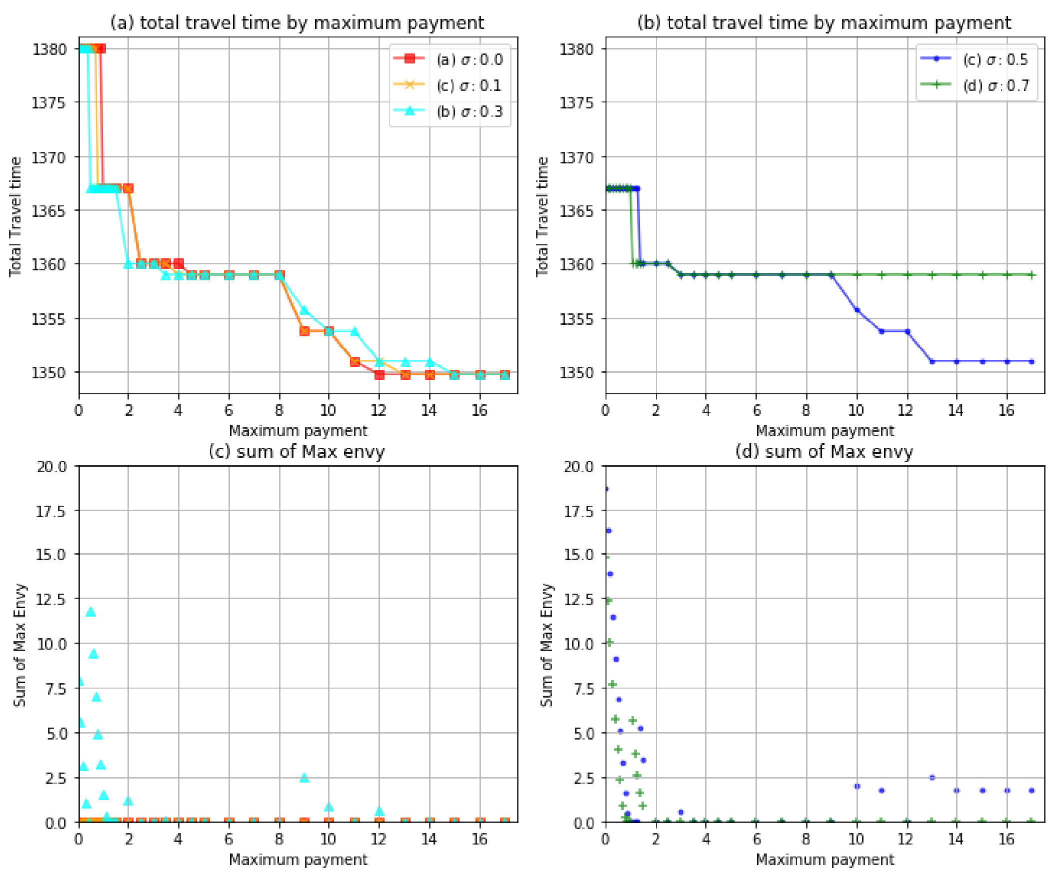


Figure 4. 7 Maximum payment and heterogeneity (lognormal distribution)

4.3.5 Limitation on computational complexity

As discussed, our model is based on a mixed integer linear programming which is known to be computationally complex. It is evident that our optimization has a significant number of constraints. In addition to constraints with respect to traffic assignment-such as origin-destination demand, path volumes, and path travel time, Peer to Peer alternative comparisons- add complexity since n agents compare the matched option of n-1 agents, which consist of n x (n-1) envy comparison variables. An objective for total max envy accompanies with the Min-Max optimization technique, meaning that the constraints include finding an individual level of maximum envy. There are also Budget constraints. Furthermore, binary variables for route choices of agents add to complexity.

The complexity of the proposed method is tested with the ASEEM-PM and the configuration of weights in the objective function is $\alpha = 0.5, \beta = 0.5, \gamma = 0.0$. We generate various number of agents (from 6 to 35) in the hypothetical network (Braess's Paradox network). Due to the demand scale parameter (ω), as defined in Chapter 3, travel time of all links remains the same regardless of demand. There are three routes as Table 4.4. the value of p_{max} and e_{max} are relaxed to very large number. As can be seen in Figure 4.6, computation time exponentially increases by the number of agents.

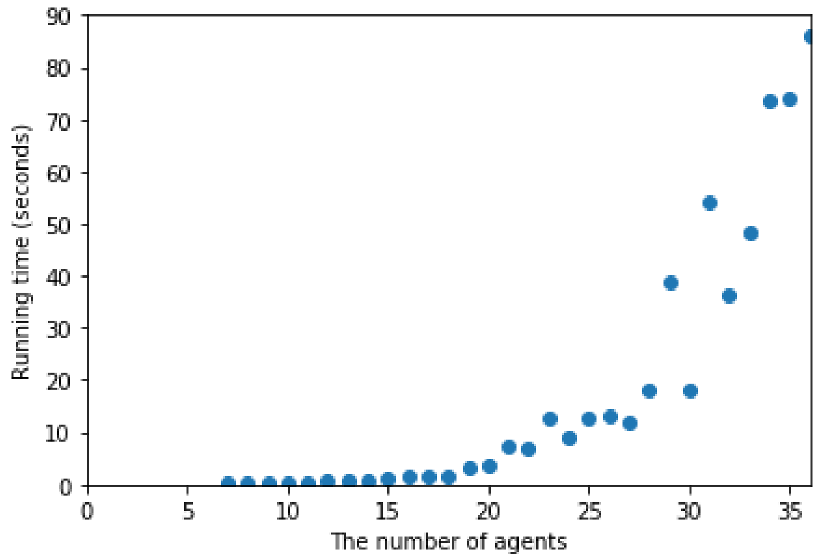


Figure 4. 7 Computing complexity of ASSEM-PM

4.4 Discussion

This chapter introduces a pricing scheme, ASEEM-PM, as an extended version of ASEEM so that agents in the optimized system do not feel envious to others. The shortest path group in the optimized system can travel faster than the UE condition and even faster than other path groups. Their convenience actually comes from the other group's inconvenience, say longer travel time. The travel time of the inconvenient group can be either longer or shorter than UE. No matter what an agent compares to his/her travel time of UE, it will be true that an agent feels unfair when she/he compares his/her resource to the shortest group. This condition justifies that the agents of the shortest path group pay and the other agents get rewarded. When money collected from the shortest path group compensates to the non-shortest path group, they might feel less unfair. However, when the shortest path group pays too much than they think reasonable, they might feel the system unreasonable. In turn, they will not select the recommended alternative that a system finds as most efficient.

We formulate ASEEM-PM's mathematical objective function and constraints, considering heterogeneity in valuation for time and optimized transportation systems. The solution finds the optimal price where nobody feels envious of others' while achieving efficient transportation condition. From this solution, we can regard an optimized system to be a fair and efficient transportation system. Although we have contributions in that the proposed method can deal with heterogeneity of agents, finds the ideal condition in both the efficiency and fairness, and computes the optimal prices for each group, one pivotal limitation lies on the scalability.

The individual level of alternative comparisons requires almost polynomial number of constraints, $n \times (n-1)$. The mathematical formulation even includes binary variables for paths. Our numerical experiences indicate the processing time increases exponentially by the number of agents. More computational time is required by larger number of route alternatives. This limitation deters the proposed method to implement in real problems.

From the numerical examples and examinations of the characteristics of the ASEEM-PM, we have come up with ideas to simplify the complexity, which motivates us to develop a decomposing the problem sets and reduce the number of comparisons among agents. Lemma 3.4 is an example in that we can remove the envy comparison constraints for agents in the same path group because agent in the same group do not feel envious toward each other.

Chapter 5

5.1 Decomposition Method for ASEEM-PM: Overview

This chapter outlines a heuristic method to apply the proposed ASEEM-PM model for actual real-world applications, where the network size, the number of agents, and heterogeneity are large enough. Designing a service focusing on travelers' behavior at an individual level requires complex computations. As discussed in Chapter 4, ASEEM-PM is unlikely to be a candidate framework for real-world applications, since it arrives at the optimal solution in reasonable time only if the number of agents in the system are small. Solving a problem where the system contains 50 agents takes more than half an hour on a sufficiently powerful desktop computer. One of the main reasons for this limitation is that Mixed Integer Linear Programming methods are used to solve the problem. Furthermore, the problem is also restricted to a linear objective function and linear constraints, an assumption not valid for realistic transportation scenarios that have non-linear characteristics

To solve this problem, we develop a heuristic method using properties of ASEEM and ASEEM-PM. The first approach is to decompose the Mixed Integer Linear Programming into a set of a known traffic assignment problem and a linear programming problem. Furthermore, we reduce the problem size by eliminating constraints. We introduce the heuristic algorithm in this section and evaluate its performance on two hypothetical networks.

5.2 Heuristic algorithm for ASEEM-PM

The mathematical formulation for ASEEM-PM consists of three main objectives: total travel time, the sum of maximum envy of agents, and the sum of envy to the shortest path. The optimal condition of the solution would be that all objectives no longer improve. If the sum of maximum envy and the sum of envy to the shortest path are minimized to optimum no matter what the given traffic condition, both objective functions can be optimal. The envy-free pricing (p) in ASEEM-PM calculates optimal pricing to find the envy-free condition. Lemma 4.1 and the experiments in Chapter 4 show that relaxing the maximum envy ($\boldsymbol{e_{max}}$) and the maximum pricing setting ($\boldsymbol{p_{max}}$) can ensure the system optimum and Envy-free conditions. This characteristic motivates us to decompose the multi-objectives into two pieces and to solve them sequentially. In other words, we can redesign the ASEEM-PM to have two steps: 1) identifying the desirable traffic condition for a given demand and supply, 2) calculating the optimum pricing to warrant envy-free condition.

This decomposition simplifies the ASEEM-PM significantly. The solution of the first objective does not need to consider the agent's heterogeneity for the value of time. This implies that the system-wide goal in this step is to identify the minimized total travel time that is not affected by agents' different preference to travel time, as shown in Lemma 5.1.

Lemma 5.1 (Agents' heterogeneity on valuation of time does not affect the minimization of total travel time)

Proof) The first term of the objective function (Eq 4.7) only takes into account the sum of the total travel time of each link, which is identical to the total path travel time of agents.

Secondly, Step 2 finds the envy-free condition for P2P envy comparisons. It is unnecessary for this step to be iterative since the pricing scheme finds the condition that the second objective is equal to zero based on the results from Step 1. The static version of the heuristic algorithm eliminates the third objective (the envy to the shortest path) from the scope since the results from aggregate a traffic assignment model in Step 1 do not deliver a unique path flow. Unique path flows can be reorganized by considering OD paths and agent's heterogeneity. However, the reorganization requires dynamic movements of each OD pair. Instead of designing the property in this algorithm, this chapter focuses on the potential of the heuristic approach to solve the ASEEM-PM problem. The unique path property will be addressed in the dynamic solution in Chapter 6.

5.3.1 Desirable Travel Pattern

The first objective of ASEEM-PM is formulated as an agent level problem, as shown in Eq (4.7) We can select formulations (4.7(b)-(e)) that only affect traffic conditions for Step1. Variables in the selected equations are independent of the agent's different preferences of travel time (θ_i), which allows an aggregate traffic assignment model to be the best candidate.

Equation (5.1) and its constraints represent the mathematical formulations of an aggregate traffic assignment model for the efficiency of urban transportation systems. This is maximized according to Wardrop's 2nd principle, also as known as "system optimal" or "social Wardrop equilibrium", where traffic is well distributed over the network.

$$\text{Min obj} = \sum_{a \in A} \mathrm{x}_a t_a\,(\mathrm{x}_a) = \sum_{a \in A} \int_0^{x_\mathrm{a}} mc_a(v_a)\mathrm{dv} \qquad (5.1)$$

subject to:

$$\mathrm{x}_a \;=\; \sum_{rs} \sum_k f_k^{rs} \delta_{a,k}^{rs} \qquad k \;\in K^{rs} \qquad (5.1a)$$

$$q^{rs} = \sum_k f_k^{rs} \qquad \forall\ \mathrm{rs}, \forall\ k \;\in K^{rs} \qquad (5.1b)$$

$$f_k^{rs} \geq 0 \qquad \forall\;\; k \in K^{rs},\; \forall\ \mathrm{rs} \qquad (5.1c)$$

There are possible assignment models to solve for System Optimum state, such as Mathematical programming (MP), traditional traffic assignment models (such as the Frank-Wolfe algorithm), and path-based assignment model (Gradient Projection or the disaggregate simplicial decomposition (DSD) algorithm).

The choice of a proper model is of critical importance because Step 2 entails knowledge of path-level flows and travel time information, which will be utilized in the agent-level envy comparisons. MP and Frank-Wolfe algorithms remove the path-level flow variables from the objective functions and constraints; consequently, the tradition models do not make use of path flow and travel time. Thus, we select the path-based traffic assignment model in our approach.

Two algorithms have received significant attention in path-based traffic assignment literature: the disaggregate simplicial decomposition (DSD) algorithm and the gradient projection (GP) algorithm. The GP algorithm outperforms DSD in terms of the faster convergence to an optimal solution since GP avoids expensive line searches (Chen, Lee, and Jayakrishnan 2002). Thus we will utilize the GP algorithm (Jayakrishnan et al. 1994), to solve the problem in Step 1.

The GP algorithm iteratively solves the assignment problem by enumerating and eliminating paths. The path information also accompanies with flow and travel time information. In contrast to solving the mathematical problem by using the Frank-Wolfe algorithm, the GP algorithm redefines the problem by focusing on the demand constraints (Eq 5.1b). Eq 5.2 reformulates the constraints to include the path flow variables. The path flows of the shortest path are the results of the operation of the gradient projection in the non-shortest path flows. Accordingly, the demand constraint is eliminated in the constraint set.

$$\mathrm{f}_{\bar{k}}^{rs} = q_{rs} - \sum_{k \in K_{rs}, k \neq \bar{k}} f_k^{rs} \tag{5.2}$$

The objective function is also reorganized to include the demand constraints and path flows (Eq 5.3). Note that the reformed formulation is a convex program with only the non-negativity constraints of the set of non-shortest path flows for all OD pairs (Eq 5.3a).

$$\min \tilde{Z}(\tilde{f}) \qquad (5.3)$$

subject to:

$$\tilde{f}_k^{rs} \geq 0 \qquad \forall \mathrm{k} \in \mathrm{K}, \mathrm{k} \neq \bar{\mathrm{k}}, \forall \mathrm{rs} \qquad (5.3\ \mathrm{a})$$

where $\tilde{f}$ is the non-shortest path flow.

The GP algorithm finds a solution with successive moves in the descent direction as a Newton approximation of the objective function. The path flows are updated according to Eq 5.4.

$$\mathrm{f}_\mathrm{k}^\mathrm{rs}(n+1) = \max\left(0, \mathrm{f}_\mathrm{k}^\mathrm{rs}(n) - \frac{\alpha}{s_k^{rs}(n)}(d_k^{rs}(n) - d_{\bar{k}}^{rs}(n)\right) \qquad (5.4)$$

where α is the step size, and $s_k^{rs}(n)$ is a scaling factor based on the second derivative Hessian of the objective function. Note that, $s_k^{rs}(n)$ is calculated by removing overlapped links between the non-shortest path and shortest path (marginal cost-based travel time). In addition, the objective function is a convex and non-decreasing function, so the scaling is diagonal and positive. $d_k^{rs,n}$ and $d_k^{rs,n}$ are the first derivative of the objective function with respective to a path volume of the non-shortest path ($f_k^{rs,n}$) and of the shortest path ($f_{\bar{k}}^{rs,n}$) as shown in Eq (5.5). This expression is equivalent to the marginal path travel time.

$$d_\tau^{rsk,n} = \frac{\partial Z}{\partial h_\tau^{rsk,n}} = \sum_t \sum_a mc_a^n \cdot \delta_{\tau\mathrm{ta}}^{\mathrm{rsk}} \qquad \forall\, rsk, \forall\, \tau \in T \qquad (5.5)$$

As GP process eliminates the path violating non-negativity constraint Eq (5.3 a), the max function is defined as Eq (5.4), which is theoretically the orthogonal projection to positive orthant in Gradient Projection algorithm

5.3.2 Agent-based Envy Minimization for a Given SO Traffic Pattern

Once the previous step has identified the desired travel pattern, the next step finds the optimum pricing that minimizes agent's envy. The second objective focuses on envy, and the constraints sets consist of OD path travel time and its price at the agent level. An agent measures envy by comparing the allocations with others' if and only if the agents belong to the same origin and their destinations group (Definition 3.3). The proposed heuristic utilizes this local comparison property, making the local objective set for each OD pair independent. In this manner, the decomposition method solves the complete problem efficiently. Furthermore, the proposed heuristic does not include the agent-level of path-link binary variables ($\delta_{i,a,k}^{rs}$), accordingly, the problem is no longer a mixed integer programing problem. With the given path travel time and volume from Step 1, this problem becomes a simpler linear optimization problem. We choose constraints (Eq 4.7 (a), (f)-(i)) that are related to the envy minimization for an OD pair, as Eq (5.6).

$$\text{Min obj} = \sum_{i \in I} \max_{i \neq j}\{e_{ij}\} \tag{5.6}$$

subject to:

$$-t_{k,i}^{rs}\theta_i + e_{ij} - p_i^{rs} \geq -t_{k,j}^{rs}\theta_i - p_j^{rs} \qquad \forall\, i,\ \mathrm{j} \in I^{rs},\ \mathrm{i} \neq j, \forall\ \mathrm{rs} \tag{5.6a}$$

$$e_{ij} \geq 0 \qquad \forall\, i \in I, , \forall\ \mathrm{rs} \tag{5.6b}$$

$$p_{min}^{rs} \leq \mathrm{p}_i^{rs} \leq p_{max}^{rs} \qquad \forall\, i \in I, , \forall\ \mathrm{rs} \tag{5.6c}$$

$$\sum_{i \in I^{rs}} p_i = B^{rs} \qquad \forall\ \mathrm{rs} \tag{5.6d}$$

$$\mathrm{t}_{rs}^* \in \max_{t_{rs}} \sum_i -t_{k,i}^{rs}\theta_i \qquad \forall\ \mathrm{rs} \tag{5.6e}$$

where, $t_{k,i}^{rs}$ is path travel time of path k for Origin (r) and Destination (r) for agent i. $t_{k,j}^{rs}$ in Eq (5.6a) is the other agent j who agent i compares their allocation with. Similarly, p_i^{rs} is the pricing for agent i and p_j^{rs} is for the comparison target (agent j). The system manager can regulate the minimum pricing p_{min}^{rs} and the maximum pricing p_{max}^{rs}. This setting affects the total maximum envy but the mathematical programming procedure solves the problem to minimize the total maximum envy. Additionally, we relax p_{min}^{rs} and p_{max}^{rs} to negative infinity and positive infinity to make the problem envy-free.

Here, the second step requires a procedure that converts the aggregated path flow pattern to the agent level. The path information from the prior step only contains travel time and its path flow over total OD flow. For this conversion, we utilize Allocation Efficiency (Lemma 3.7 and Lemma 4.3) that is the result of a complete problem. It is proven that AE minimizes the total maximum envy. To consider this property, the generated agents, based on the known distribution, are allocated according to the sorted order of AE (Eq 5.6e). In other words, OD paths enumerated by travel time are allocated to agents in order of valuation, meaning that the shortest path is assigned to the agents having a high valuation in the simulation.

The final results of the entire problem will come with the travel options following the system optimum pattern, which contain information of travel time and its price (either toll or incentive). In an actual application, an agent has all route options and finds an alternative that does not bring envy, which is essentially the same traffic pattern found in Step 1.

5.3 Reducing problem size by the characteristics of ASEEM-PM

Section 5.2 addressed the decomposition method for ASEEM-PM. Although it strives to simplify the original complex problem, the method of Step 2 finds the objective by comparing N to N-1 envies. Therefore, it is still computationally complex, and we need to design methods to find more efficient methods without loss of generality. This section introduces the method to reduce the problem size using the characteristics of ASEEM and ASEEM-PM.

First of all, Lemma 3.4 (Group Envy-Family Envy problem) helps eliminate a significant number of comparison constraints. Although the SO status has more paths than UE, its number is definitely less than OD demand, meaning that each path flow ($\mathrm{n_k}$) is more than unit value. In practice, there is a limited number of paths in an OD pair, say less than five. Furthermore, ASEEM-PM does not have a price difference in the same path group (Lemma 4.4). These two properties make the p2p envy comparisons in the same path group redundant. Accordingly, we can remove $\mathrm{n_k} \times (n_k - 1)$ constraints.

More interestingly, Lemma 5.2 states that the amount of pricing is determined by extreme valuation in each group and budget allows the p2p comparisons to be path group comparisons.

Lemma 5.2 (Amount of Credit determined by extreme values in each group and budget)

Proof)

Let i, j are paths of an OD path group ($i, j \in K^{rs}$) and agent a belongs to one of the paths and path travel time follows as $t_i < t_j$. θ_i^a represents the valuation of agent a in path i, θ_i^- is the agent having the lowest valuation in path i. θ_i^- and $\overline{\theta_i}$ are for the highest valuation and the representative valuation (such as mean, 85 percentiles, and etc.), respectively. We generalize envy in Eq (4.1) to the path level comparison as Eq (5.7), where e_{ij}^a is the envy that agent a in path i feel to path j.

$$e_{ij}^a \geq \max\left(-\theta_i^a\left(t_j - t_i\right) - \left(p_j - p_i\right), 0\right) \tag{5.7}$$

The possible envy-free pricing (e_{ij}^a=0) with the $\overline{\theta}_i$ is Eq (5.8)

$$p_i = \overline{\theta_i}(t_j - t_i) + p_j \tag{5.8}$$

We replace p_i in Eq (5.8) with Eq (5.6), then, we can find Eq (5.9)

$$e_{ij}^a \geq \max\left(\left(-\theta_i^a + \overline{\theta_i}\right)\left(t_j - t_i\right), 0\right) \tag{5.9}$$

For the shortest path group (i), agent a feels envy to j (e_{ij}^a) if agent a' valuation θ_i^a is higher than $\overline{\theta_i}$. Thus, the envy-free condition is achieved only if the representative valuation for a price for path j is set to be the minimum valuation (θ_i^-) among agents in path group i. This finding is in line with Mayet and Hansen (2000). The pricing for the shortest path where every agent in path i does not feel envy is Eq (5.10). Eq (5.11) also implies that the shortest path group is willing to pay more than the other group at the amount of the perceived time difference of the representative agent (θ_i^-). When the pricing is higher than this value, some agents in the group feel envious. Eq (5.11)

$$p_i \leq \theta_i^-(t_j - t_i) + \mathrm{p_j} \tag{5.10}$$

$$p_i - \mathrm{p_j} \leq \theta_i^-(t_j - t_i) \tag{5.11}$$

For agent b in the longest path group (j), we reformulate Eq (5.9) to the agent b's point of view as Eq (5.12). Since $t_i - t_j$ is negative, an envy free condition is achieved only if θ_j^b is equal to or higher than $\overline{\theta}_j$. Similarly, θ_j^b should be the maximum valuation in the path group j, which implies that price for path j is set based on the agent who has the highest valuation, Eq (5.13).

$$e_{ji}^b \geq \max\left(\left(-\theta_j^b + \overline{\theta}_j\right)\left(t_i - t_j\right), 0\right) \tag{5.12}$$

$$p_j \geq \theta_j^+(t_j - t_i) + \mathrm{p_i} \tag{5.13}$$

We can also extend this to multiple paths case.

Let i, j, k $\in \mathrm{K}^{rs}$ and a $\in \mathrm{I_i}^{rs}$, b$\in \mathrm{I}_j^{rs}$, and c $\in \mathrm{I_k}^{rs}$ and $\mathrm{t_i} \leq t_j \leq t_k$

A pricing range for envy-free exists for each path group is as shown in Eq (5.14) to Eq (5.16)

$$\mathrm{p_i} \geq \theta_i^-\left(t_j - t_i\right) + \mathrm{p_j} \geq \theta_i^-(t_k - t_i) + \mathrm{p_k} \tag{5.14}$$

$$\theta_j^+(t_j - t_i) + \mathrm{p_i} \leq p_j \leq \theta_j^-(t_k - t_i) + \mathrm{p_k} \tag{5.15}$$

$$\mathrm{p_k} \leq \theta_k^+(t_k - t_j) + \mathrm{p_j} \leq \theta_k^+(t_k - t_i) + \mathrm{p}_i \tag{5.16}$$

The position of maximum valuation or minimum valuation is determined by the portion of the group among total OD volume. Furthermore, the portion of groups is identified by system optimum assignments (Step 1). From Lemma 5.2, we can infer that an agent in a paying group makes an effort to minimize their payment and an agent in an incentivized group does his/her best to maximize incentives. This means that the system estimates the optimum price by considering compensation to agents assigned to an inferior alternative and the optimization finds a solution to be bounded to the first agent (having θ_i^+) in the alternatives group.

From Lemma 5.2, we prove that the pricing for path k is determined by the extreme valuations in the group. For the mathematical problem, we generalize the equations from Eq (5.134) to Eq (5.16) into the two constraints (5.17 to 5.18). Here, $e_{kk'}^-$,is the envy that agent having the lowest valuation in k path group feels to the path group k'. $e_{kk'}^+$ is the envy for the agents having the highest valuation. Thus, if those agents located at the extreme valuation in a path group do not feel envy, every agent in the same path group also does not feel envy.

$$e_{kk'}^- \geq \; -\theta_k^-(t_{k'}^{rs} - t_k^{rs}) - \;(p_{k'}^{rs} - \; p_k^{rs}) \qquad \forall\, \mathrm{k}, \mathrm{k}' \in K^{rs}\,, \mathrm{k} \neq \mathrm{k}' \qquad (5.17)$$

$$e_{kk'}^+ \geq \; -\theta_k^+(t_{k'}^{rs} - t_k^{rs}) - \;(p_{k'}^{rs} - \; p_k^{rs}) \qquad \forall\, \mathrm{k}, \mathrm{k}' \in K^{rs}\,, \mathrm{k} \neq \mathrm{k}' \qquad (5.18)$$

We also set the budget constraints for the mathematical formulation. The given proportion from Step 1 fixes the number of agents $n_k$in a path group k, thus budget constraint for OD pair rs is as Eq (5.19)

$$\sum_{k \in K^{rs}} n_k p_k = B^{rs} \tag{5.19}$$

Eq (5.20) shows the entire mathematical formulation for Step 2. The peer to peer envy comparisons, which requires $\mathrm{N} \times (\mathrm{N}-1)$ computations, is simplified to the path-level comparisons. If there are k existing paths in an OD pair, its computation complexity becomes $2 \times \mathrm{K} \times (\mathrm{K}-1)$. Usually, the number of OD paths is indeed not high in real-world; consequently, the decomposed heuristic method can be computationally efficient.

$$\text{Min obj} = \sum_{\mathrm{k} \in K^{rs}} \max_{i \neq j} \{e_{kk'}\} \tag{5.19}$$

subject to:

$$-t_k^{rs}\theta_k^- + e_{kk'}^- - p_k^{rs} \geq -t_{k'}^{rs}\theta_k^+ - p_j^{rs} \qquad \forall\, \mathrm{k}, \mathrm{k}' \in K^{rs}, \mathrm{k} \neq \mathrm{k}' \tag{5.19a}$$

$$-t_k^{rs}\theta_k^+ + e_{kk'}^+ - p_k^{rs} \geq -t_{k'}^{rs}\theta_k^+ - p_j^{rs} \qquad \forall\, \mathrm{k}, \mathrm{k}' \in K^{rs}, \mathrm{k} \neq \mathrm{k}' \tag{5.19b}$$

$$e_{kk'}^- \geq 0,\ e_{kk'}^+ \geq 0 \qquad \forall\, \mathrm{k}, \mathrm{k}' \in K^{rs}, \mathrm{k} \neq \mathrm{k}' \tag{5.19c}$$

$$p_{min}^{rs} \leq \mathrm{p}_k^{rs} \leq p_{max}^{rs} \qquad \forall\, i \in I \tag{5.19d}$$

$$\sum_{k \in K^{rs}} n_k p_k = B^{rs} \tag{5.19e}$$

$$\mathrm{t}_{rs}^* \in \max_{t_{rs}} \sum_i -t_{k,i}^{rs}\theta_i \qquad \forall\, \mathrm{rs} \tag{5.19f}$$

5.4 Numerical Example

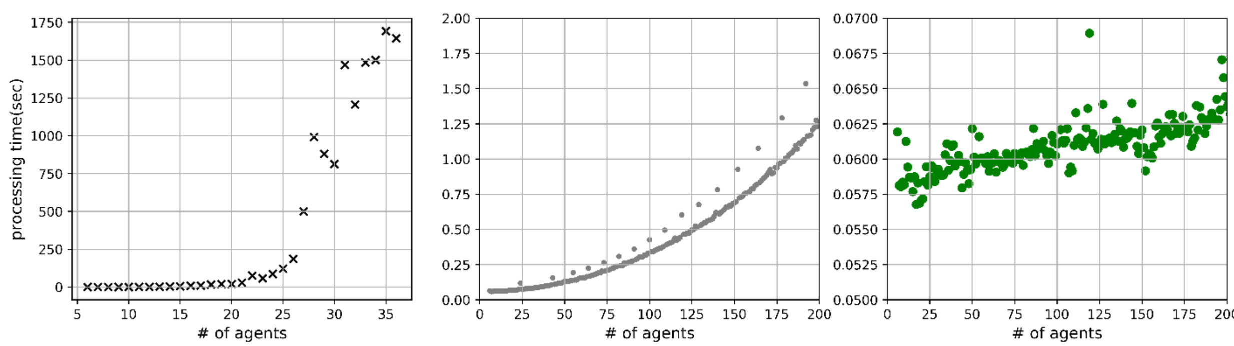
5.4.1 Evaluation Environment

In this section, we compare the proposed decomposition method with a standard mathematical programming technique. For the base case of the mathematical programming problem, we set the weights of the objective function as Case D in Section 3.5 ($\alpha = 0.5, \beta = 0.5$, and $\gamma = 0.0$). The number of agents is a significant factor affecting the mathematical problem (Eq 4.7). The individual level of route-link binary incident matrix adds complexity ($\delta_{i,a,k}^{rs}$). The proposed method devises to reduce the computation complexity, as explained in the previous section. The first approach (GP+Peer to Peer Envy comparisons) is to eliminate $\delta_{i,a,k}^{rs}$ by decomposing the full problem into two steps: 1) GP for identifying SO traffic pattern, and 2) finding the agent-level envy-free pricing solution. The first approach still remains a NP-hard problem which requires a non-polynomial time for the P2P envy comparisons. The second approach that we propose simplifies the NP hard problem by leveraging the characteristics of ASEEM-PM.

For the evaluation, we use Python 3.56 (64bit) with Gurobi solver 8.10. The computing processor is Intel® Core™ i7-6800k @ 3.40Ghz with 32.0 GB memory. Since the given solver (Gurobi) can only solve mixed-integer linear programming problems in which the objective function and its constraints consist of linear and non-decreasing equations, we test the performance on the Braess's Paradox Network.

5.4.2 Processing time

Figure 5.1 indicates the comparisons of the processing time. The mathematical programming (MILP) spends about 1600 seconds for 36 agents, which is unlikely to be a candidate application. For Figure 5.2 (a), the first decomposed method (GP+P2P Envy comparisons) reduces the processing time significantly. It only takes about 0.20 seconds for the 36 agents case. However, the processing time still exponentially increases the number of agents increases, typically $O(n^2)$. The proposed heuristic enables the problem to be $O(n)$, and only takes 0.057 seconds for 36 agents and 0.06 seconds for 200 agents.



(a) Full problem (b) GP+P2P Envy comparisons (c) The proposed heuristic

Figure 5. 1 Comparisons of processing time with respect to the number of agents

5.4.3 Performance on solving the optimum

We evaluate whether the proposed solution finds the exact solution for ASEEM-PM. Table 5.2 indicates the results of the total travel time and total max envy with respect to the various number of agents. For the six agent cases, all methods find the exact solution where both the total travel time and total maximum envy are minimum. However, with the increased number of agents, MILP does not find the optimum solution. For example, the 10 agents case shows that the total travel time is 564.80, which is higher than 562.60 for the SO travel time. Also, for all cases with more than 20 agents, envy-free state is not achieved, and even the travel time is not an optimum. MILP is generally solved using the Branch and Bound algorithm. A significant number of integer variables make the optimization problem difficult to solve since memory and solution time exponentially increase. This is due to the fact that integer values for the variables result in many combinations needed to be checked. Although Gurobi solver can handle a large number of integer variables by heuristically presolving, cutting planes, and parallel processing, it still cannot find the optimal solution. Even when our method finds an optimum solution, there are possible cases where the total travel time is not an optimum. In the process of conversion from an aggregate model to agents-based model, our method rounds up the continuous value. Sometimes, the rounded values are not the solution to the optimum. There are some green dots and cyan "x" markers higher than the optimum travel time in Figure 5.2. It implies that a simple rounded function can have not an optimum solution. However, the deviation from optimum is not significant and this deviation rarely occurs when the number of agents is higher. Therefore, we do not develop further steps to find the optimum solution in those cases.

Table 5. 1 Comparisons of objective values for models

Total Agents	Total Travel Time			Total Max Envy		
	MILP	GP+P2P	Proposed	MILP	GP+P2P	Proposed
6	339.00	339.00	339.00	0.00	0.00	0.00
10	564.80	562.60	562.60	0.00	0.00	0.00
15	845.60	844.40	844.40	0.00	0.00	0.00
20	1127.60	1125.20	1205.70	0.36	0.00	0.00
25	1409.92	1406.56	1406.56	0.03	0.00	0.00
30	1691.20	1687.20	1687.20	0.63	0.00	0.00
35	1973.03	1968.63	1968.63	1.43	0.00	0.00
40	–	2249.45	2249.45	–	0.00	0.00
50	–	2811.84	2811.84	–	0.00	0.00
60	–	3374.30	3374.30	–	0.00	0.00
70	–	3936.60	3936.60	–	0.00	0.00
80	–	4498.90	4498.90	–	0.00	0.00
90	–	5061.27	5061.27	–	0.00	0.00
100	–	5702.98	5702.98	–	0.00	0.00
150	–	8435.48	8435.48	–	0.00	0.00
200	–	11247.25	11247.25	–	0.00	0.00

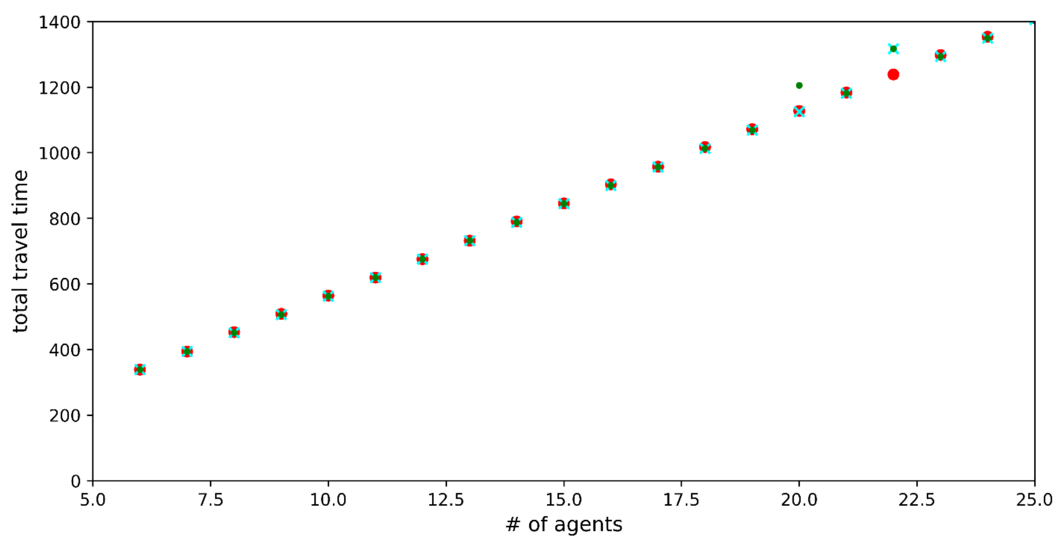


Figure 5. 2 Comparisons of objective values with the respect to the number of agents

5.4.2 Application

One significant advantage of the proposed method is that the objective function and its constraint set can accommodate a non-linear function if the function is convex and non-decreasing. This property is essential since the proposed method can depict congestion effects where the travel time exponentially increases according to the traffic density or flow of a link. This section examines the proposed method on the well-known Sioux-Falls network (LeBlanc, Morlok, and Pierskalla 1975), which is not realistic but has been used in various network design problems and is available at the Open-source based developer platform (https://github.com/bstabler/TransportationNetworks/tree/master/SiouxFalls).

The hypothetical network consists of 24 zones, 24 nodes, and 76 links. Its total demand is 360,600 trips from 576 OD pairs. The link performance function follows the traditional BPR (Bureau of Public Roads). As can be seen in Eq (5.20), the link performance function in this hypothetical network is an increasing and non-negative function.

$$\mathrm{t_a}(x_a) = t_0(1 + \alpha \left(\frac{x_a}{c_a}\right)^{\beta}) \tag{5.19}$$

Table 5.2 shows the comparisons of results. The Gradient Projection is applied for the continuous model, and its output is categorized into UE and SO. The proposed heuristic is used for an agent-based model. UE-A is nothing but the converted version of UE. SO has the optimum total travel time at 7,194,256 seconds that represents an improvement of 3.82%. ASEEM-PM also has a similar ratio of travel time reductions. However, total travel miles increase for SO and ASEEM-PM. This increase is because the SO assignment distributes vehicles over the space. As can be seen in Table 5.2, SO and ASEEM-PM have more path counts than the equilibrated traffic condition.

Table 5. 2 Results of Sioux-Falls Network application

Index	Continuous Model			Agent-based Model		
	UE	SO	Diff (%)	UE-A	ASEEM-PM	Diff (%)
Total Travel Time (minutes)	7,480,189	7,194,256	-3.82	7,480,566	7,194,806	-3.82
Total Travel Distances(miles)	569,853	595,170	4.44	569,861	595,185	4.44
Path count	682	746	9.38	682	746	9.38

Table 5.3 shows results obtained from ASEEM-PM. Total maximum is minimized to zero (Envy free) in ASEEM-PM, which is 137.36 for the UE-A case. Even when UE-A is the equilibrated condition in the agent-based model, it has a small amount of travel time gap among paths. Without pricing, even efficient allocation of the given allocation for the system optimum accompanies envy at the amount of 102,708,01. We calculate the total transaction cost by summing up the absolute value of pricings (toll and incentive) of agents. To achieve the SO traffic pattern with the given valuation distribution ($N(1, 0.3)$, the price that is equivalent to 52,412.61 minutes is transacted in this system. Because we assume the budget-balanced condition in this case study, half of the transaction cost is regarded as tolls for faster route users, which compensate for slower route users (incentives)

Table 5. 3 Total Maximum Envy and Transaction cost

Measurement	UE-A	ASEEM (w/o PM)	ASEEM-PM
Total Travel Time (minutes)	7,480,566	7,194,806	7,194,806
Total Maximum Envy (minutes)	137.36	102.708.01	0.00
Total Transaction cost (minutes)	0.00		52412.61
Max Toll	-		9.08
Min Incentive	-		-5.18
Transaction cost per agent	-		0.15

We also examine the proposed method by various demand levels. The demand is scaled from 36,060 to 721,200 trips. Figure 5.3 indicates that the total travel time improves in all cases. In contrast, total travel miles increase. Interestingly, improvements have an upper limit. This pattern implies that the SO routing strategy is not able to help a system improve efficiency in a low or extremely high demand level case. Figure 5.4 visualizes the speed profile, which is calculated by dividing the total traveled miles over to the total traveled hours. As can be seen, there is no distinctive travel time difference in the low demand (less 100,000) and the high demand (over 400,000).

Figure 5.4 shows the trend of transaction cost per agent by the demand level. The more demand, the more congestion, the more transacted amount per agent, which implies that there is a valid demand range for the system where the supply of a system is not underutilized, or demands are not too excessive.

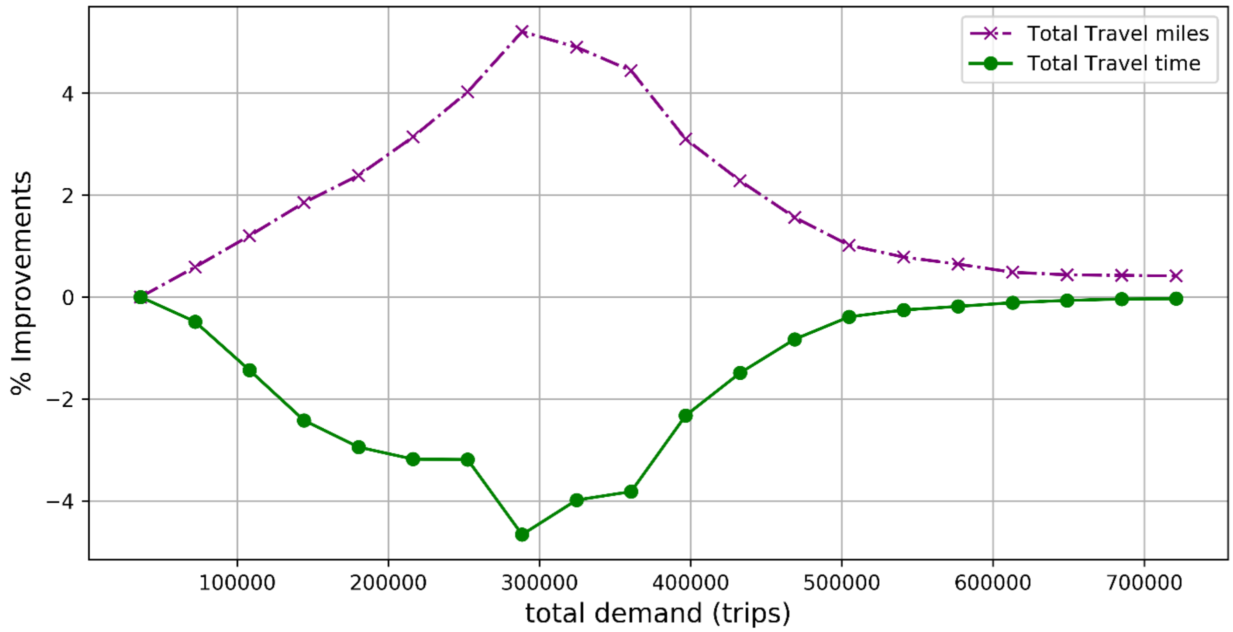


Figure 5. 3 Improvements in Travel time and miles with respect demand

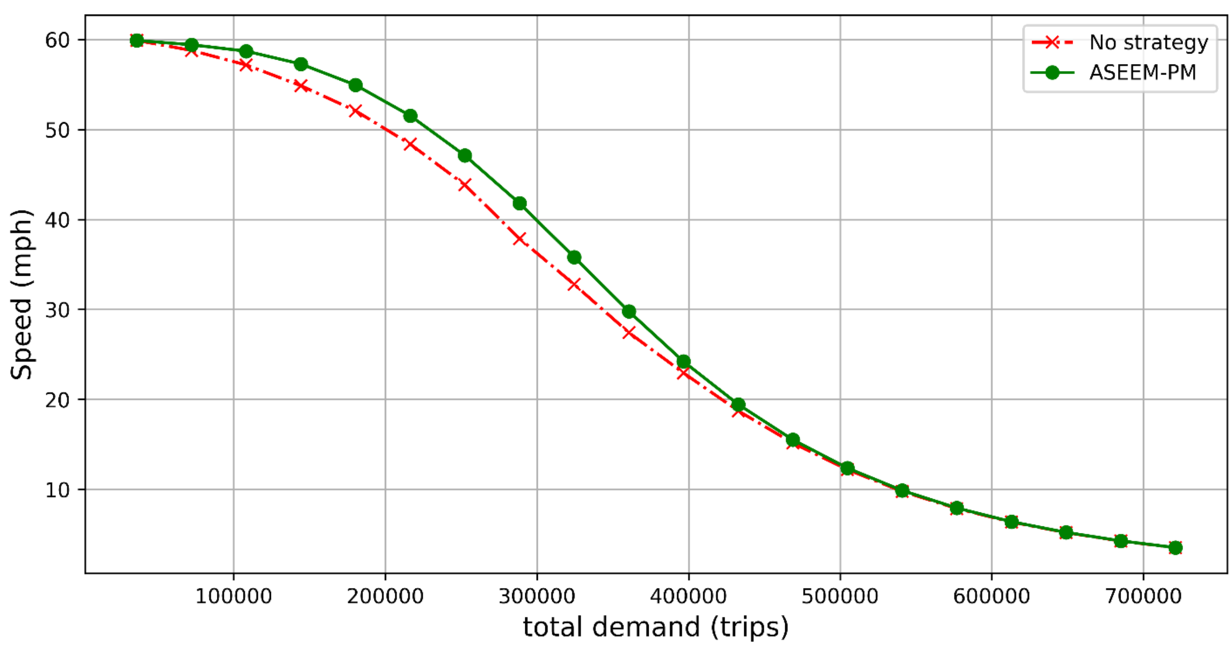


Figure 5. 4 Space Mean Speed with respect to demand

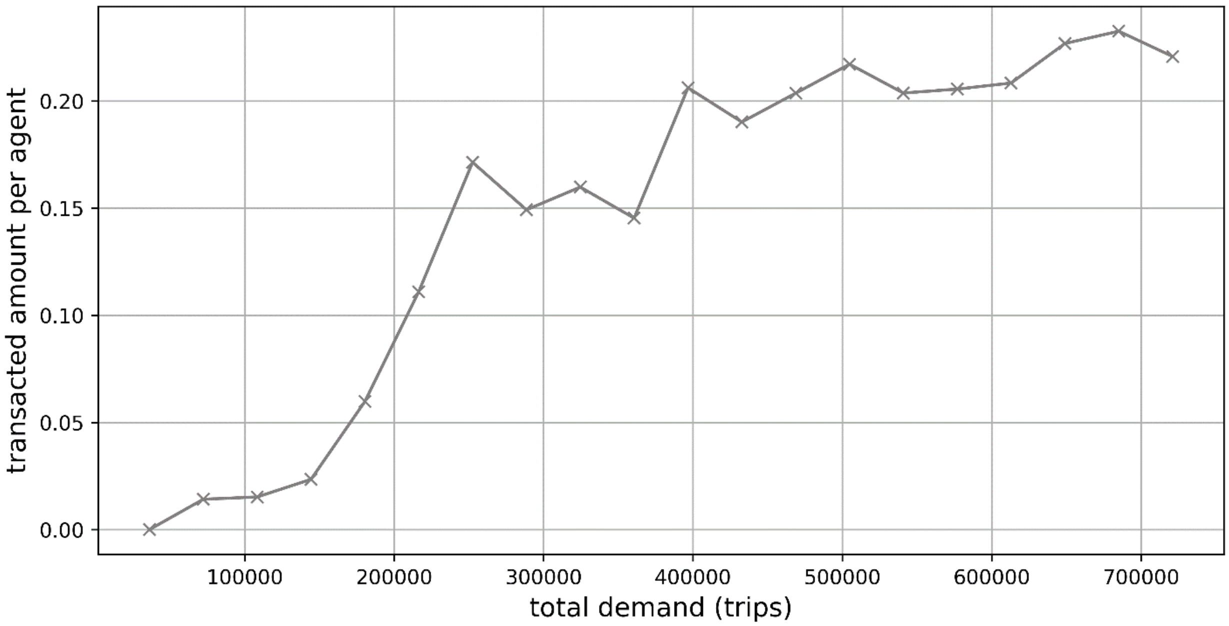


Figure 5. 5 Travel time and miles with respect to demand

Chapter 6

6.1 Social Optimum Navigation with Incentives and Costs Schemes (SONICs)

In the preceding chapters, we detailed the static version of our proposed optimization problem. However, for such a system to be implemented in the real world, it would have to consider time and space dynamics. Our dynamic routing system is kept flexible by design in order to accommodate innovative applications in the future. Our goal is to find socially desired solutions in terms of efficiency and fairness. With that in mind, we name this framework Social Optimum Navigation with Incentives and Costs Schemes (SONICs).

The details are as follows. First, a user provides their personal profile and basic preferences. Then, just like in conventional car navigation systems, the user enters their destination in the form of an address or as a place name that could be a selectable Point of Interest (POI). Based on the provided information and the estimated distribution of the valuation function, the central server finds possible alternatives that include route and pricing information. Note that those alternatives are likely to be the system optimum route patterns from transportation models and already stored on the server. The server provides the user with a list of alternatives. Users are assumed to select the most attractive option that induces minimum envy under bounded rational recognition. After driving on the selected route, the driver might earn rewards or pay tolls, depending on their choice.

To achieve the overall vision of SONICs, this chapter extends the static version of ASEEM-PM to dynamic cases. Traffic varies by time of day. Furthermore, as agents travel on their route, their envy may differ according to time dynamics (such as traffic conditions). Any pricing scheme, therefore, needs to account for this variance. In other words, the actual application of the proposed model needs to implement time-space dynamics. Thus, we reformulate the proposed ASSEEM-PM model.

We first review basic behavioral assumptions of traffic assignment models and show our expectation of future behavior under possible technologies such as P2P communication, Block-chain, and agent-level marketing. Static traffic assignment models rely on the assumption that all travelers have complete information about their route, and they change their route based on their experience. This assumption is likely to be sound when travelers only depend on printed maps or their experience. Now that real-time information has become readily available, dynamic traffic assignment models assume an optimal traveler's behavior where travelers select the fastest travel option based on current traffic conditions (greedy assumption), known as the Dynamic User Optimum (DUO). This assumption is valid when we only consider the availability of real-time traffic information. However, we can postulate that this is not a system-optimal solution since greedy behavior does not improve the efficiency of a system. As for Definition 5.3, we expect that agents in the future will exchange their travel information with others as long as the system permits it. Our goals regarding dynamic cases are the same as before: maximizing system efficiency while minimizing agents' envy. This chapter will address the following questions: 1) how to find the best alternatives under dynamic traffic demands and conditions, 2) how unfairness is evoked by efficient transportation, 3) how to obtain the best pricing policy for minimizing envy.

6.2 Formulation and Overview of Solution

In this section, we present a dynamically allocated system efficiency with envy minimization-price match (DASEEM-PM), which is a temporal generalization of the multi-objective optimization of ASEEM. The formulation of the DASEEM-PM problem is as follows

$$\min Z = \alpha \sum_{t\in T} \sum_{a\in A} x_{a,t} t_{a,t}\left(x_{a,t}\right) + \beta \sum_{i\in I} \max_{i\neq j}\{e_{ij}\} \tag{6-1a}$$

subject to:

$$q_{\tau}^{rs} = \sum_{i\in I} \sum_{k\in K_{\tau}^{rs}} h_{i,\tau}^{rsk} \qquad \forall\, r,s \in R, \tau \in T^{D} \tag{6-1b}$$

$$h_{i,\tau}^{rsk} = \begin{bmatrix} 1 & if\ path\ k\ is\ used \\ 0 & otherwise \end{bmatrix} \qquad \forall\, i\,,\tau \tag{6-1c}$$

$$\sum_{i} h_{i,\tau}^{rsk} = 1 \qquad \forall\, i\,, k \tag{6-1d}$$

$$x_{ta} = \sum_{i\,\in I} \sum_{r,s,k,\tau} h_{\tau}^{rsk} \delta_{it\tau a}^{rsk} \qquad \forall\, r,s \in R, i\ \in I, k\ \in K^{rs} \tag{6-1e}$$

$$\delta_{it\tau a}^{rsk} = \Psi\left[h_{i,\tau}^{rsk},\ \forall i,r,s,k,\tau\right] \qquad \forall\, r,s\,,q\ \in q^{rs}, k\ \in K_{\tau}^{rs}, a\ \in A \tag{6-1f}$$

$$\sum_{r,s,k,\tau} -t_a \delta_{it\tau a}^{rsk} \theta_i + e_{ij} - p_{i\tau}^{rs} \geq \sum_{r,s,k',\tau} -t_a \delta_{jt\tau a}^{rsk'} \theta_i - p_{j\tau}^{rs} \qquad \forall\, i,\, j \in I,\, i \neq j \tag{6-1g}$$

$$0 \leq e_{ij} \leq e_{max} \qquad \forall\, i\,,\, j\,,\, i \neq\ j \tag{6-1h}$$

$$\sum_{i\in I} p_{i}^{rs\tau} = B^{rs\tau} \qquad \forall\, r,s,\tau \tag{6-1i}$$

Here, $x_{a,t}$ and $t_{a,t}(x_{a,t})$ are the traffic load of link a at time t and its travel time respectively, both of which are aggregated-values of the individual's path-link incidence matrix. This implies that the first objective function is of importance for systemwide travel time minimization problem. The second objective function consists of envy comparisons at an individual level. In other words, our objective has two components: 1) total travel time and 2) minimization of the sum of the maximum envy of individual, which allow us to solve two objectives in two steps.

Constraints (6-1b) to (6-1f) are reformulated from an aggregated traffic assignment model to incorporate a version of dynamic traffic assignment model at the agent level. Instead of regarding flow as aggregated behavior, these constraints are formulated to consider each agent. For example, Eq (6-1b) is formulated for OD demand conservation which is the sum of an agent's path flow binary $h_{i,\tau}^{rsk}$. The value of $h_{i,\tau}^{rsk}$ is 1 if path k for rs is used for agent i departing at time τ as in Eq (6-1c). Consequently, the sum of $h_{i,\tau}^{rsk}$ for agent i is a unit value, as in Eq (6-1d). The term $\delta_{it\tau a}^{rsk}$ is a path-link variable of an individual i whose origin and destination are r and s, respectively, her path is k, departure time is τ, and time to arrive a link a at time t, Eq (6-1f). The term Ψ represents the calculation for enumerating traveled links of a path k for agent i, then converting it to the path to path link incidence matrix ($\delta_{it\tau a}^{rsk}$). Link flow at time t, $x_{a,t}$, is the sum of agents passing link a at time t, which is represented in Eq (6-1e).

Peer to Peer envy comparison constraints are formulated as from Eq (*6-1g)* to Eq (*6-1i).* Eq (6-1g) addresses envy comparison. It indicates that if agent i pays more than agent j ($p_i \geq p_j$) for the same path travel time, agent i feel envious (e_{ij}) at the amount of the price difference. Similarly, if the path travel time of agent j is shorter than that of agent i without price difference ($p_i = p_j$), agent i feels envious (e_{ij}) to agent j. It is noteworthy that the feeling of envy is only measured by agent i's valuation θ_i. In Eq (8g), $\sum_{r,s,k,\tau} -t_a \delta_{it\tau a}^{rsk}$ is the path travel time of path k for agent i. Thus, envy in the latter case, can be interpreted as a degree of travel time difference as perceived only by agent i.

Note that the agent-based model with the proposed formulation brings with it a significant number of constraints, including the individual level of path binary variables, making the problem intractable. Furthermore, this problem has temporal-dynamics, which increases its complexity. Similar to the decomposition method discussed in Chapter 5, the problem set of the DASEEM-PM model can also be discretized into two steps. First, we find a system optimum traffic pattern from a continuous traffic assignment model, i.e., dynamic traffic model. Then, we find the appropriate amount of pricing that allows agents not to feel envious to others. From this two-step process, we can compute a solution that minimizes both objectives.

6.3 Identification of systemwide-efficient routes

6.3.1 Dynamic Assignment for a Dynamic System Optimum Condition

For a real-world implementation application of the proposed method, it is necessary to understand the spatiotemporal dynamics of traffic conditions on the network. Dynamic System Optimum Assignment can be a candidate solution to find desirable dynamic traffic patterns considering the time constraints of route guidance. Jayakrishnan, Tsai, and Chen (1995) develop a methodology to calculate traffic patterns at the link level, showing the number of assigned vehicles on links in each time step for DUE or DSO conditions. The requirements for our solutions are path-based flow patterns, which will be used in pricing schemes. Based on the computed patterns, we can solve the efficient route allocation problem with the consideration of pricing trade-off among agents in the same origin-destination pair. Thus, we develop a path-based system optimum solution by applying a Gradient Descent Projection method and a time-dependent network structure.

The fundamental idea that undergirds this research is that the SO/DSO conditions have more diverse routes compositions than the corresponding UE condition ((Boyce and Xiong 2004; I. Yang 2011). In some cases, we might arrive at the same traffic pattern with greedy routing. For SO routes mostly during peak hours, however, the number of routes of an OD pair tend to be higher. Furthermore, travel times of some routes are longer than the shortest path's travel time, which are likely not preferred by travelers. Although previous studies on SO route guidance assume that drivers are so altruistic that they can yield their shortest route for improving social benefits, we still subscribe to the more realistic assumption that travelers are greedy with respect to their travel cost (time and money). Guiding travelers to the longer path could be problematic for traditional route guidance systems.

As shown in Chapter 5, an aggregated model can be chosen as a solution for an agent-based model. Eq (6.2a)-(6.2e) depict formulations for finding appropriate travel patterns at an aggregated level. Dynamic System Optimum can be implemented to find the right amount of incentives. The objective of the system is explicit: minimization of total travel time. Eq (6.2) is the same formulation as in the UE case except that the link cost function comes not from an actual link cost function ($f_a(w)$) but a marginal cost ($mc_a(w)$) function. For the system optimum pattern, defining $f_a(w)$ in Eq (6.2a) to be a marginal link cost function implies minimization of increased total travel time (marginal cost time) by an additional agent on a link. We can reinterpret the SO traffic pattern in which all marginal costs among paths are equilibrated, so that the system cannot be better off by an agent's route switch. Eq (6.2b) and (6.2c) show Origin-Destination demand and link flow, respectively. Eq (6.2d) is the OD-path link incidence matrix.

$$\min Z = \sum_{t \in T} \sum_{a \in A} \int_0^{x_{ta}} mc_a(w)\, dw \qquad (6.2a)$$

subject to:

$$\mathrm{q}_\tau^{\mathrm{rs}} = \sum_{k \in K_\tau^{rs}} h_\tau^{rsk} \qquad \forall\, \mathrm{r,s} \in \mathrm{R}, \tau \in \mathrm{T^D} \qquad (6.2b)$$

$$x_{ta} = \sum_{o,d,k,\tau} h_\tau^{rsk} \delta_{\tau t a}^{rsk} \qquad \forall\, \mathrm{t} \in \mathrm{T}, \mathrm{a} \in \mathrm{A} \qquad (6.2c)$$

$$\delta_{\tau\mathrm{ta}}^{\mathrm{rsk}} = \Psi[h_\tau^{rsk}, \forall r, s, k, \tau] \qquad \forall\, \mathrm{r,s} \in \mathrm{R}, \tau \in \mathrm{T^D}, k \in k_\tau^{rs}, \mathrm{a} \in \mathrm{A} \qquad (6.2d)$$

$$h_\tau^{rsk} \geq 0 \qquad \forall\, \mathrm{r,s} \in \mathrm{R}, \tau \in \mathrm{T^D}, k \in k_\tau^{rs} \qquad (6.2e)$$

Here, Eq 6.2b addresses demand conservation constraint of an OD demand. Intuitively, the OD demand (q_{τ}^{rs}) is the total sum of path flow (h_{τ}^{rsk}). The term x_{ta} in Eq (6.2c) is traffic load (vehicle/mile) which is the number of vehicles existing on a link a time. The term $h_{i,\tau}^{rsk}$ in Eq 6-2c and 6-2d is a binary variable for a used path k meaning that if a path k is used, $h_{i,\tau}^{rsk}$ is 1; otherwise it is 0. $\delta_{\tau ta}^{rsk}$ in Eq 6.2d is a value in the time-dependent path-link incidence matrix.

6.3.2 Link Performance Functions

It is noteworthy that traffic load, x_{ta}, differs from traffic flow (vehicle/hour) that is used for the static version of the assignment problem. Jayakrishnan, Tsai, and Chen (1995) underscore the importance of using Traffic load in Dynamic Traffic Assignment models. We employ a modified Greenshields equation, as used by them, for the link performance function. As shown in Eq (6.3), link travel time (f_a^t) is a function of "the number of vehicles in a unit length(density)" existing on a link instead of using link flow in a static traffic assignment.

$$f_a^t(k_a^t) = \frac{L_a k_j}{u_{min} k_j + (u_{max} - u_{min})\left(k_j - k_a^t\right)} \qquad \forall\, a \in A, \mathit{Link\ Cost\ Function\ for}\ k_a^t \leq k_j \qquad (6.3)$$

Where u_{min} *= minimum speed at jam density (ft/sec)*

u_{max} *= free flow speed (ft/sec)*

k_a^t *= density on the link a in time step t (veh/mile)*

$k_{j,a}$ *= jam density of link a(veh/mile)*

L_a *= length of link a (ft)*

$f_a^t(k_a^t)$ *= travel time of link a at time step t (sec)*

The original Greenshields model can account for the situation in which density is greater than the jammed density. The traffic assignment model that we employ is a dynamic version of Gradient Projection which finds an equilibrated traffic condition by implementing an iterative traffic assignment procedure. Its initial step is to assign all OD traffic onto the shortest path, such as All or Nothing assignment. Intuitively, assigning all demands to one route will end up with jammed traffic on that link. Thus, Greenshields model cannot be used to represent a realistic traffic condition. To prevent this situation, Jayakrishnan, Tsai, and Chen (1995) utilize the modified Greenshields speed-density relationship as a link performance function with the minimum speed (8.8 ft/sec, or 6 mph). The choice of their link performance function may appear arbitrary at first glance, but its properties are amenable for optimization in that this density-based link cost function is monotonically increasing, convex, and is twice differentiable, which is sufficient to show that Hessian of the resulting objective function is positive definite, i.e. monotonically increasing with respect to density.

The link cost function (6.3) is written as $\mathrm{f}_a^t(L_a, k_a^t)$, hereafter written as $f_a^t(\, k_a^t)$, since the assignment variable is link load, x_a^t(set of vehicles for origin-destination pairs), which is related to density directly as shown in Eq (6.4)

$$k_a^t = \frac{x_a^t}{n_a L_a} \qquad \forall\, a \in A, \qquad link\ density \qquad (6.4)$$

where n_a, = number of lanes on link a

Again, our research interest is to find the system optimized-traffic pattern. As is well-known, the formulation of Dynamic System Optimum solution can be found by applying the marginal cost function as the link cost function. The marginal cost function is the derivative of total cost $xf(k)$, vehicles x time, as in Eq (6.5). We can generalize it to the density-cost relationship. Because density k is the number of vehicles (x) on a unit length of the link (L), we can convert x into L x k, similarly, we can also find that $\frac{dk}{dx} = 1/L$ (Eq 6.6).

$$\boldsymbol{mc} = \frac{\boldsymbol{dx \cdot f(k)}}{\boldsymbol{dx}} \tag{6.5}$$

$$= \frac{\boldsymbol{dk}}{\boldsymbol{dx}} \cdot \frac{\boldsymbol{dx \cdot f(k)}}{\boldsymbol{dk}}$$

(6.6)

Here, $x_a = n_a L_a k_a$, let $n_a L_a = L$, and $\frac{dk}{dx} = 1/L$

From Eq (6.5) and (6.6), we can infer that the marginal cost is the sum of link travel time and the marginal congestion effects ($kf'(k)$.)

$$mc = \frac{1}{L} \cdot \frac{dL \cdot k \cdot f(k)}{dk} \tag{6.7}$$

$$= \frac{dk \cdot f(k)}{dk}$$

$$= f(k) + kf'(k)$$

Note that the marginal cost function includes the first derivative of a link cost function. MC can be interpreted as the added cost when an additional vehicle entering a link contributes to the total travel time. The term $f'(k)$ is the additional travel time on a link when unit density increases and computed as Eq (6.8).

$$f'(k) = \frac{k_j \cdot L(u_{max} - u_{min})}{\left(k_j \cdot u_{max} + k \cdot (u_{max} - u_{min})\right)^2} \tag{6.8}$$

6.3.3 Time-dependent networks

A dynamic network can be regarded as a set of multiple static networks interconnected in time and space. Jayakrishnan, Tsai, and Chen (1995) propose the terms *s_link* and *d_link* to represent dynamic characteristics of the traffic. As a vehicle drives on a *s_link* a (a static network), it traverses multiple dynamic links whose traffic conditions differ in timesteps according to the traffic load at time step t. Link travel time of link a for a traveler entering the link at t_{enter} is the sum of travel time of d_link at t (Eq 6-5). Here, exit time (t_{exit}) for the traveler is the cumulative time of link traversal times. Figure 6.1 illustrates a vehicle entering link a at timestep t and exiting that link at timestep t+4.

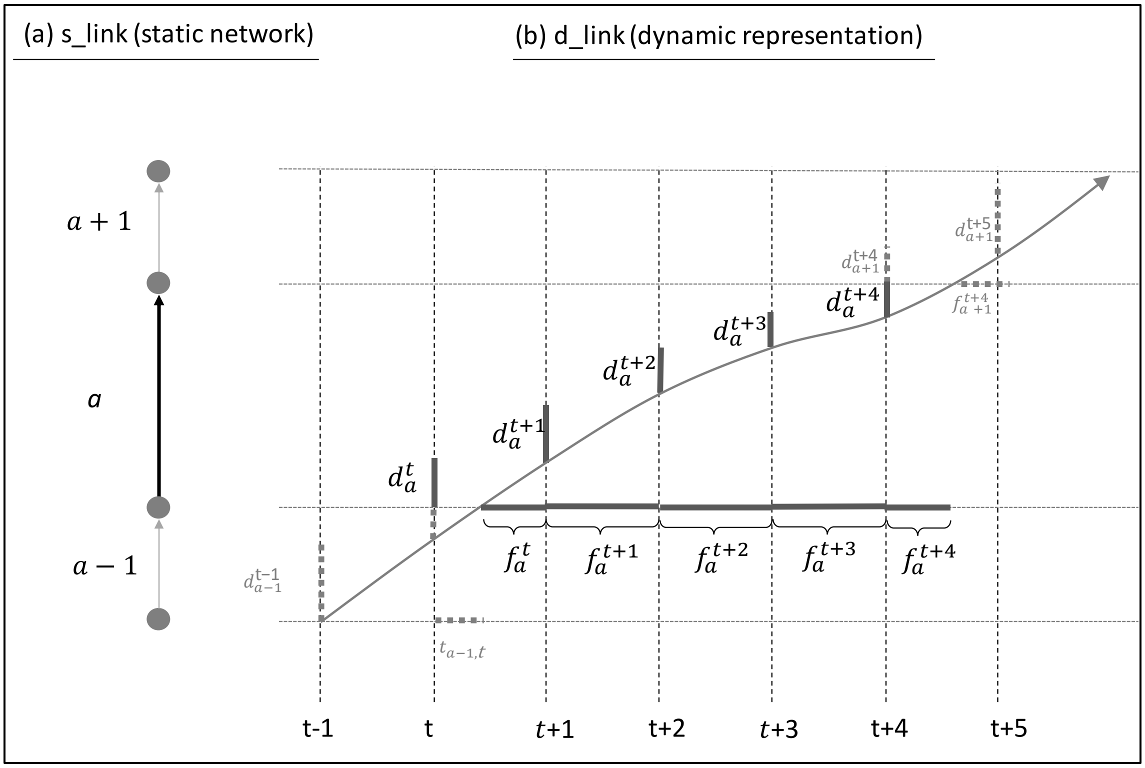


Figure 6. 1 The relationship between s_link and its dynamic link (d_link)

Link exit time can be calculated by iteratively checking the current location of a vehicle (set of vehicles) entering at time *t*. Once, the vehicle's location exceeds the length of a link at time step *t+n*, *n* can be regarded as a traveled time of a vehicle. This property determines the link exit time t_{exit} in calculating total the travel time spent in link a. Here, we can also define an important property of the minimum link length.

Definition 6.1. The minimum link length requirement

The length of link *a*, L_a, must be longer than the travel distance of a free flow condition (Eq 6.9).

$$L_a \geq u^{max} \cdot \Delta \qquad \forall\, a \in A \qquad (6.9)$$

If the link length is shorter than the minimum distance, a vehicle can skip the following dynamic link, therefore travel time at time t of link a cannot be calculated.

Definition 6.2. The total travel distance of link a with dynamic link

Once a vehicle finishes traveling on link a, it is evident that the sum of total travel distance equals the total link length (Eq (6.10)). However, we need to carefully consider travel time and its travel distance in System Optimum assignments, which will be detailed in Definition 6.5 and Lemma 6.1.

$$\sum_{t=t_{enter}}^{t_{exit}} d_a^t = L_a \qquad \forall\, a \in A \qquad (6.10)$$

where,

$$d_a^t = u_n^t \cdot f_a^t$$

Definition 6.3. travel time (s_d link) and travel distance at time t

Here, travel time at t, f_a^t, is less than or equal to the time interval (Eq (6.11)) since the maximum travel time of a vehicle in a time interval t is the length of the time interval. Once the link travel time is larger than time interval, the traffic condition follows the next time step. This condition of less travel time might only be applied when a vehicle enters or exits a link; otherwise, it is equal to the time interval.

$$f_a^t = \min(f_a^t(x_a^t), \Delta) \qquad \forall\, a \in A \qquad (6.11)$$

Definition 6.4. Total travel time of a link with various traffic conditions

It is noteworthy that a vehicle experiences different traffic conditions in different time steps under congestion. This time dynamics can be incorporated into our model if we discretize traffic conditions in terms of time steps. In Figure 6.1, a vehicle experiences different traffic conditions in 5 timesteps (from t to t+4). Its total travel time can be calculated as eq (6.12).

$$f_a = \sum_{t=t_{enter}}^{t_{exit}} f_a^t(k_a^t) \qquad \forall\, a \in A \qquad (6.12)$$

It is possible for a vehicle to enter a link in the middle of a time slot as $f_a^{t=t_{enter}}$ in Figure 6.1. In the same manner, exit time points might also lie in the middle of a time step. Definition 6.4 addresses time dynamics of travel time and travel distance for a vehicle traversing a link.

Table 6. 1 An example of the s_d link table

	f_a^t							Travel time	Clock$_{out}$
	time interval t								
$Clock_{in}$	ts2	ts3	ts4	ts5	ts6	ts7	ts8		
7	3.0	10.0	10.0	2.6				25.6	32.6
15		5.0	10.0	10.0	5.2			30.2	45.2
23			7.0	10.0	10.0	6.8		33.8	56.8
25			5.0	10.0	10.0	9.2		34.2	59.2
35					5.0	10.0	10.0	31.0	66.0
42						8.0	10.0	28.0	70.0

Lemma 6.1. The discrepancy between link travel time and marginal travel time

Though previous research has shown that SO assignments can be done by employing a marginal cost function as a link cost function, it is theoretically reasonable only for a static assignment because this does not consider time dynamics (e.g., a link entering time and exit time). Even when previous research studies have considered this time dynamics, the details are often overlooked. This section recapitulates the importance of the discrepancy and describes a methodology to handle the time dynamics of SO. In the regime of non-free flow traffic condition, a marginal link cost (time) is always higher than or equal to the actual link travel time. It is unnecessary for UE assignments to consider this discrepancy. Using a marginal cost function for a link exit time, however, might bring unreasonable results when an actual arrival time of a vehicle differs from the estimated arrival time, resulting in a vehicle being assigned to an incorrect dynamic link. As addressed in Definition 6.3, the travel time of s_link consists of a set of dynamic link travel time, and the s_d link matrix stores the actual travel time of the s_link in a dynamic form.

Definition 6. 5. Static-dynamic-time specification table (*s_d_t* link table)

As a vehicle enters a link at time t and travels along with the link during some time periods, we can calculate the proportion of each time slot during the journey of a link as Eq (6.13). For example, a vehicle entering a link at the beginning of time slot t and exiting a link at end of time slot $t + 5$ has its travel evenly distributed during t to $t + 5$. If a vehicle arrives at link a the middle of time slot t, the first time slot (t) takes a lower proportion of the travel. We define the link and time-spent proportion as *s_d_t* link table (ϕ_l^t, Eq (6.13)), This table stores information about how *d_link* is connected to the *s_link* in addition to exit time step. Table 6.1 shows an example of *s_d_t* link table. We can easily expect that the summation of each row is a unit value. Let us assume that a time interval is 10 seconds. A vehicle approaching a link at 7 seconds (*Clock_in*)in a simulation clock that is time step 1 (t_{in}), exits link at 37 seconds (*Clock_out*). It travels a link for 30 seconds, but it takes 4 simulation time steps from 1(t_{in}) to 4 (t_{out}). The Table 6.1 also indicates First-In-First-Out (FIFO) property of a link supply and demand.

$$\phi_l^t = \frac{f_a^t}{\sum_{t' \in [t_{in}, t_{out}]} f_a^{t'}} \qquad \forall\, a \in A, \forall\, t \in T \qquad (6.13)$$

Table 6. 2 An example of a dynamic ϕ table

	ϕ_l^t							
	time step							
Clock_in	ts2	ts3	ts4	ts5	ts6	ts7	ts8	Sum
7	0.1	0.4	0.4	0.1				1.0
15		0.2	0.3	0.3	0.2			1.0
23			0.2	0.3	0.3	0.2		1.0
25			0.1	0.3	0.3	0.3		1.0
35					0.2	0.3	0.3	1.0
42						0.3	0.4	1.0

Definition 6.6. Redefinition of Dynamic Marginal cost

From the *s_d_t* link table, we can synchronize the actual link exit time. Marginal cost can be computed by considering how a vehicle contributes to congestion at each time step. It is the summation of the product of marginal cost of a time slot and its respective proportion as shown in Eq (6.9). From this equation, we can preserve the conservation of link flow without loss of generality of actual travel time. This marginal cost of path information is used in the lower problem in the proposed GP-based Bi-Level programming for DSO.

$$\mathrm{Mc}_a^t = \sum_{t \in [t_{in}, t_{out}]} \phi_l^t \cdot \mathrm{mc}_a^t \qquad \forall\, a \in A, \forall\, t \in T \qquad (6.13)$$

6.3.4 GP-based Bi-Level programming formulations for DSO

This section shows the proposed algorithm to solve the Dynamic System Optimum problem by using a Dynamic Traffic Assignment model. Jayakrishnan, Tsai, and Chen (1995) propose a bi-level programming approach for the efficient solution of DTA problem. Yang (2011) introduces a simulation-based DTA problem and employ the Gradient Projection (GP) algorithm. We develop a method to find the pattern of system optimum assignment by integrating the above two methods, as shown in Figure 6.2.

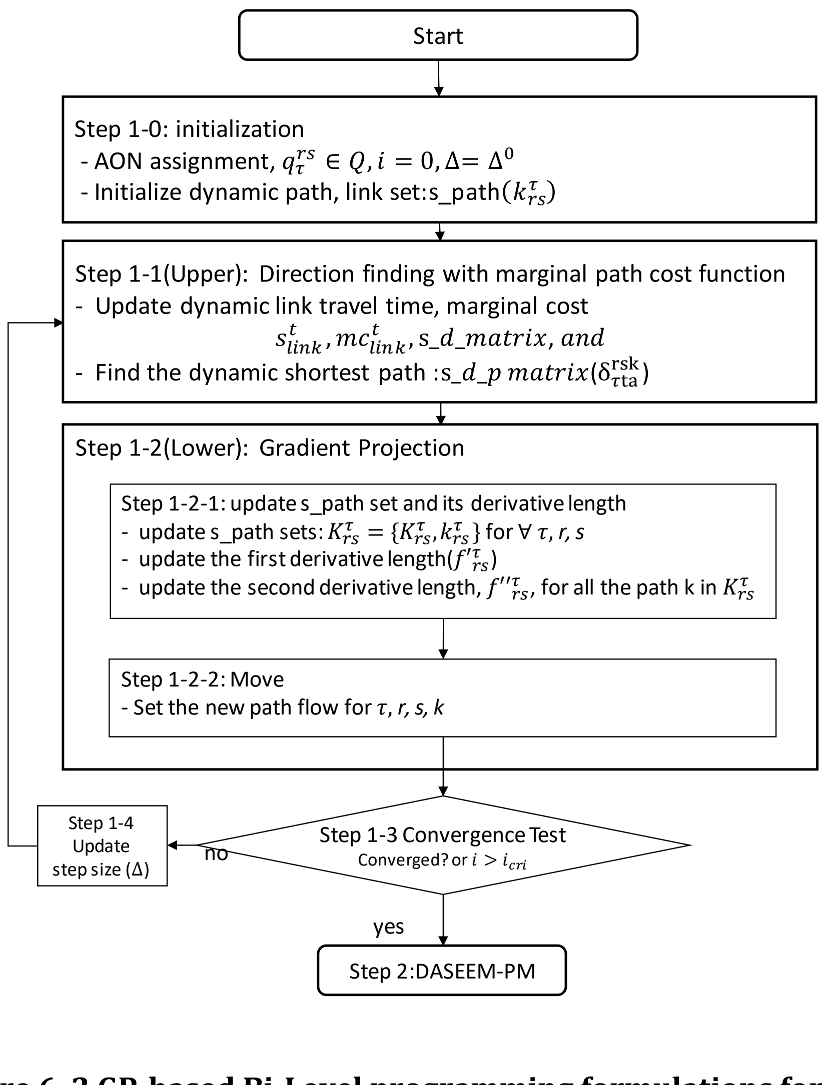


Figure 6. 2 GP-based Bi-Level programming formulations for DSO

Before introducing the bi-level programming framework, we reformulate the DSO problem for solving the problem using GP algorithm. A static version of GP algorithm is proposed in Jayakrishnan et al. (1994). They introduce a new objective function (6.14), consisting of only non-shortest path flows instead of link flow. Recall that we use the term "load" instead of "flow". The problem constraints are focused only on the non-negativity of path loads.

$$\text{Min } \tilde{Z}(\tilde{h}) \tag{6.14}$$

Subject to:

$$\tilde{h} \geq 0$$

The objective function includes demand constraints (Eq 6.1b). These are reordered for the algorithm, as shown in Eq (6.15). It considers the least marginal cost path flow ($\tilde{h}_{i,\tau}^{rs\bar{k}}$) that is the rest of total flow ($\tilde{h}_{i,\tau}^{rsk}$) with respect to the non-least marginal cost path.

$$\tilde{h}_{\tau}^{rs\bar{k}} = q_{rs}^{\tau} - \sum_{\substack{k \in K_{rs}^{\tau} \\ k \neq \bar{k}_{rs}^{\tau}}} \tilde{h}_{\tau}^{rsk} \qquad \forall \text{ rs}\tau \tag{6.15}$$

where K_{rs}^{τ} is the set of non-least marginal cost path and $\bar{k}_{rs}^{\tau}$ s the least marginal cost path of an OD pair (rs) departing at timestep τ.

With the consideration of time-dynamics and its dynamic links, we divide the GP algorithm into a bi-level programming problem. The upper problem (Step 1-1) finds a time-dependent shortest path of OD pairs in each iteration. This process generates the relationship between a static link (*s_link*) and its associated dynamic links (*d_link*), which connects the upper problem to lower problem. The shortest path tree is associated with *s_d_t* table of links, which will define the s_d_p incidence value ($\delta_{\tau \mathrm{ta}}^{\mathrm{rsk}}$) for the lower problem. Eq (6.16) indicates the objective function and its constraints for the DSO problem. Note that this is for an origin, a destination, and a departure time. As seen in Eq (6.16), a path for each OD is the minimum marginal cost path, which is consistent with SO routing literature (Jayakrishnan, Tsai, and Chen 1995; Bosch et al. 2011; I. Yang 2011; Djavadian et al. 2014).

$$\text{Min} \sum_{t} \sum_{(i,j=a)\in L} mc_a^t \, \delta_{\tau \mathrm{ta}}^{\mathrm{rsk}} \qquad \forall\, rs\tau \qquad (6.16)$$

subject to:

$$\sum_{t<t^i} \sum_{j\in N} \delta_{ji}^{\tau} - \sum_{t>t^i} \sum_{j\in N} \delta_{ij}^{\tau} = \begin{cases} 1 & i = s \\ 0 & i \neq r, s \\ -1 & i = r \end{cases} \qquad \forall\, i,\ \mathrm{j} \in I,\ \mathrm{i} \neq j \qquad (6.16a)$$

$$\tau^i = \tau^r + \sum_{\tau} \sum_{(i,j)\in L} f_{ij}^{\tau} \, \delta_{\tau \mathrm{ta}}^{\mathrm{rsk}} \qquad \forall\, \mathrm{rs}, \mathrm{i} \in I, k \in K^{rs} \qquad (6.16.b)$$

In the second step (Step 1-2), from the identified least marginal cost path, the GP algorithm updates the path-sets (Step 1-2-1) and makes successive moves (1-2-2) in every iteration in the gradient direction that is the minimum of the Newton approximation of objective function, Eq (6.17). The direction for any path flow can be obtained from the gradient difference between non-shortest path and shortest path, and the move size is determined by the second derivative of the objective function with respect to path flows of a non-shortest path. The non-negative constraints form the feasible space of path loads of dynamic OD pairs. Once a move is infeasible, it bounds to the constraint boundary that is zero.

$$\tilde{h}_{\tau}^{rsk,n+1} = \max\left(0, \tilde{h}_{\tau}^{rsk,n} - \frac{\alpha^n}{s_{\tau}^{rsk,n}}\left(d_{\tau}^{rsk,n} - d_{\tau}^{rs\bar{k},n}\right)\right) \qquad \forall\, rsk, \forall\, \tau \in T \qquad (6.17)$$

where

$$s_{\tau}^{rsk,n} = \sum_{t}\sum_{a} \frac{\partial mc_a^n}{\partial x_a^n} \delta_{\tau ta}^{\mathrm{rsk}} - 2\sum_{t}\sum_{a} \frac{\partial mc_a^n}{\partial x_a^n} \cdot \delta_{\tau ta}^{\mathrm{rsk}} \cdot \delta_{\tau ta}^{\mathrm{rs\bar{k}}} \qquad \forall\, rsk, \forall\, \tau \in T \qquad (6.17a)$$

A brief explanation of the equations follows. More details can be found in Yang (2011). The term $d_{\tau}^{rsk,n}$ is the first derivative of the objective function with respect to a path flow variable ($h_{\tau}^{rsk,n}$) as shown in Eq (6.18). This expression is equivalent to the marginal path travel time.

$$d_{\tau}^{rsk,n} = \frac{\partial Z}{\partial h_{\tau}^{rsk,n}} = \sum_{t}\sum_{a} mc_a^n \cdot \delta_{\tau ta}^{\mathrm{rsk}} \qquad \forall\, rsk, \forall\, \tau \in T \qquad (6.18)$$

Likewise, $s_{\tau}^{rs\bar{k},n}$ is the second derivative length of the objective function that can be computed by differentiating Eq (6.18) with respect to a path flow variable. Since it consists of marginal costs, the second derivative length is the sum of derivatives of marginal costs on a path in a Beckmann-transformed objective function with integrals. Furthermore, $s_{\tau}^{rs\bar{k},n}$ is a diagonal positive-definite Hessian matrix since the marginal cost function is non-negative and increasing convex function. Note that the second derivative of objective function takes overlapped links between the non-shortest path and shortest path into account by eliminating those links that are physically connected.

(Step 1-3 Convergence test)

The iterative bi-level framework bases its convergence on statistical measurements. The measurements are associated with the nodal arrival intervals in the Upper and Lower Problems. Root Mean Squared Error (RMSE) can be a candidate, as shown in Eq (6.19). However, it is vulnerable with respect to the scale of the entire travel time. Hence, we employ relative root mean squared error (R_RMSE) as shown in Eq (6.20).

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{\tau}\sum_{r}\sum_{i\in N}(\alpha_{\tau}^{ri} - \beta_{\tau}^{ri})^{2}}{\Gamma}} \qquad \forall\, rsk, \forall\, \tau \in T \qquad (6.19)$$

where

α_{τ}^{ri} is the nodal arrival time of the UP

$\forall\, rsk, \forall\, \tau \in T$ (6.19a)

β_{τ}^{ri} is the nodal arrival time of the LP

$\Gamma$is the total number of the nodal arrival estivates (Eq 6.19 b)

$$\Gamma = R \times T \times (N - 1) \qquad (6.19b)$$

$$\mathrm{R}_{\mathrm{RMSE}} = \frac{\mathrm{RMSE}}{\left(\frac{\sum_{\tau}\sum_{r}\sum_{i\in N}\alpha_{\tau}^{ri}}{\Gamma}\right)} \qquad (6.19b)$$

Since our interest is in the path-level optimization problem in which all path costs are equilibrated, we also consider the other criterion that is based on the path level definition of equilibrium. Equilibrium in routing means that every route in the same origin and destination and departure time has the same travel cost. In system optimum, marginal costs of all routes are within a reasonable range. We adopt the relaxed duality gap (RDG, Eq (6.20)), as employed by Yang (2011).

$$\mathrm{RDG} = \frac{\sum_{\tau}\sum_{r}\sum_{s}\sum_{k\in K_{\tau}^{rs}} h_{\tau}^{rsk,n} \cdot \max\left(0, mc_{\tau}^{rs} - \overline{mc}_{\tau}^{rs} \cdot (1+\omega)\right)}{\sum_{\tau}\sum_{r}\sum_{s} q_{\tau}^{rs} \cdot - \overline{mc}_{\tau}^{rs}} \tag{6.20}$$

Here, ω is the boundary rationality factor ($0 \leq \omega < 1$), which relaxes travel time differences. This factor assumes that travelers tolerate some travel time differences. Similarly, a system can be indifferent to some level of marginal cost differences among paths. If ω is set to be zero and the convergence criterion of RDG is almost zero, the proposed algorithm attempts to find the perfect system optimized solution. Vice versa, a large value of ω relaxes the problem in which higher total travel time is achieved than in the SO condition. In our study, ω is set to be zero.

(Step 1-4: update step-size)

GP algorithm is an iterative solution that is an approximate version of the Newton method, which has a positive step-size modifier (hereafter, step-size). Taking a proper step size affects the convergence to the optimum and the convergence rate. The step-size can be a constant or an adaptive value. Jayakrishnan, Tsai, and Chen (1995) define α as a constant value of 1 from experience. Chen, Zhou, and Xu (2012) introduce a self-adaptive function determined in each iteration by comparing path flows and costs of the previous step. Both approaches are for a static case. Yang (2011) empirically choose the value of α value between 0 to 1. Gentile (2015) who adopts an adaptive step-size function, notes: *"This is the nonsummable diminishing step-size, and proved to be effective in their numerical tests, because it makes possible to reach a null value of probability for a given alternative in a finite number of iterations, which is impossible when adopting the classical MSA approaches".*

We refer to Gentile's step-size function as shown in Eq (6.21)

$$\alpha^n = \alpha^0 \left(\frac{\eta}{\eta + n_{worse}} \right) \tag{6.21}$$

Here, n_{worse} is an integer cumulative function, Eq (6.21), that is added when the objective function of the current step is worse than that of the previous step. A larger n_{worse} drives α to be scaled down, meaning that the move size decreases (or slower convergence) so that it prevents jumping to another worse solution. n_{worse} is a Boolean function in its input x. If x is true, it becomes 1 otherwise its value is 0.

$$n^i{}_{worse} = \sum_{i=2}^{i-1} Bool\left(\gamma^i - \gamma^{i-1} < 0\right) \tag{6.22}$$

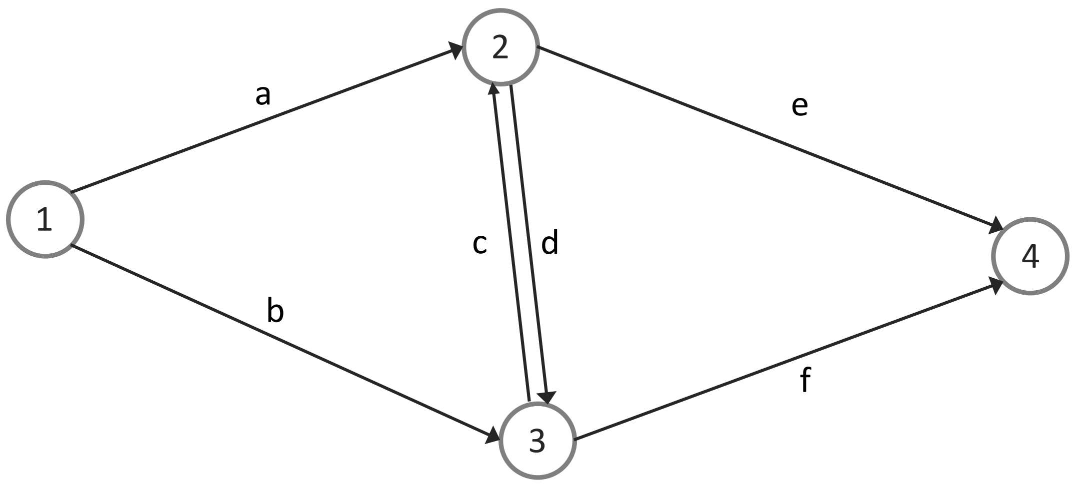
$$where\ \gamma: sum\ of\ node\ arrival\ time$$

6.4 Dynamic ASEEM-PM (DASEEM-PM)

Once we identify the desired traffic pattern in a system-wide aggregated manner, the next step is to find the best pricing scheme where individuals follow the assigned alternative without envy. Our assumption of traveler's behavior is that an agent selects the best alternative that has minimum envy. From the identified options from the previous step, the system manager arrives at the estimation of envy-free pricing for each OD- τ pair based on the valuation distribution. If there is a single route for an OD- τ pair, agents in the same pair may not compare their route with others since there is no difference in allocation and price among members in the same path group (Lemma 4.4).

Here, we can argue the existence of alternatives although the prior step (Step 1) concludes there is only one path for an efficient traffic pattern. It is noteworthy that the output from Step 1 is not a unique solution since an aggregated traffic model only provides a unique solution for an objective function. In other words, the solution from Step 1 guarantees the unique solution exclusively for the objective function and its link flows. However, the computed path flows can be various, which means that the computed path flow solution cannot be a unique solution to our problem.

Figure 6.3 is an example of the non-unique path solution. We can see that an OD pair (from 3 to 4) can have multiple paths (path 5 (f), and path 6 (c-e)) although a solution finds there is only one path (path 5). This limitation might induce user dissatisfaction if the system only provides a single option. It will be more feasible when agents passing at node 3 have the same alternatives that are path 3 and path 4. In this manner, we define a Node-to-Destination array for Definitions 6.7 and 6.8. Agents who pass by a node r' at time interval t compare their envy with others who are passing at the same time to the same destination (s).



OD	Path ID	Links	Nodes	OD path volume scenarios				
				Case1	Case2	Case3	Case4	Case5
1->4	1	a, e	1, 2, 3					
	2	a, d, f	1, 2, 3, 4					
	3	b, f	1, 3, 4	6	5	4	3	2
	4	b, c, e	1, 3, 2, 4	4	5	6	7	8
3->4	5	f	3, 4		1	2	3	4
	6	c, e	3, 2, 4	4	3	2	1	0

Figure 6. 3 An example of non-unique path flows for OD pairs (Homogeneous agent)

Definition 6.7 Dynamic Envy from Node (decision point) to Destination (DEND)

$$e^{r's}_{ij,\tau,r'} = (V_{ij}(\theta_{i\tau}, \mu^{r's}{}_{j}) - V_{i}(\theta_{i\tau}, \mu^{r's}{}_{i}) - (p^{r's}_{i\tau} - p^{r's}_{j\tau}))\delta_{ij} \qquad \forall\, i\,, j, r\,, i \neq j \qquad (6.22)$$

where δ_{ij} is 1 if “Envy” is positive and zero otherwise

Definition 6.8 Cumulated Dynamic Envy at each Node (Decision Point) (CDEeN)

$$\mathrm{e}_{ij} = \sum_{i,t} e^{rs}_{i,t,r'} \tag{6.23}$$

where (6.23a)

$$e^{rs}_{i,\tau,r'} = e^{r's}_{ij\tau r'} + e^{rr'}_{itr'-1}$$

Let us assume that there are two agents, *i* and *j*. Both agents have the same destination (s) and they are currently passing node r' in the same time step (τ). The term $e^{r's}_{i,\tau,r'}$ represents DEND for agent *i*. The term CDEeN ($e^{rs}_{i,\tau,r'}$) is an additive function which sums the envy from the previous node ($r' - 1$) on the route and current envy at node r'($e^{r's}_{ij,\tau,r'}$). This represents envy that agent *i* feels regarding agent *j.* Here, agent *i* only compares resource of other agents whose destination (s) and current node (r') are the same. In other words, when agent *i* arrives at node $r'$and time step (τ), the agent will select a route until the resource of the selected route cannot be better than agent *j's* (Eq 6.2).

Lemma 6.2 Unique path from DASEEM-PM

The allocation efficiency with the pricing (Lemma 4.3) reconstructs non unique path flows to unique path flow if each agent is dynamically reallocated to one of the possible alternatives at every decision point.

Proof) The proposed DEND function reorganizes the results from the previous step to have a unique path flow pattern. Figure 6.4 indicates the organized path flow pattern. There are two used paths for Figure 6.3. Node 3 is a common decision point for Path 3, 4 of OD pair (1 to 4) and Path 5,6 of OD pair (3 to4). Vehicles, arriving at node 3 from node 1, meet other vehicles departing from node 3. There are two routes from node 3 to a destination (node 4). The shortest path to the destination is by using link *f*, which takes 130 seconds. The detouring path via node 2 takes 150 seconds. At the decision point (node 3), the minimized-envy path pattern consists of six agents to link *e* and eight agents to link *i* according to the allocation efficiency (Lemma 4.3). From the pattern, the envy to the shortest path is also minimized. For example, let us assume that system provides the information only based on Path ID 3 in Figure 6.3. If agents whose origin is node 3 are assigned to the longest path even though they can travel the shortest path and are willing to pay, this given guidance will induce envy to them.

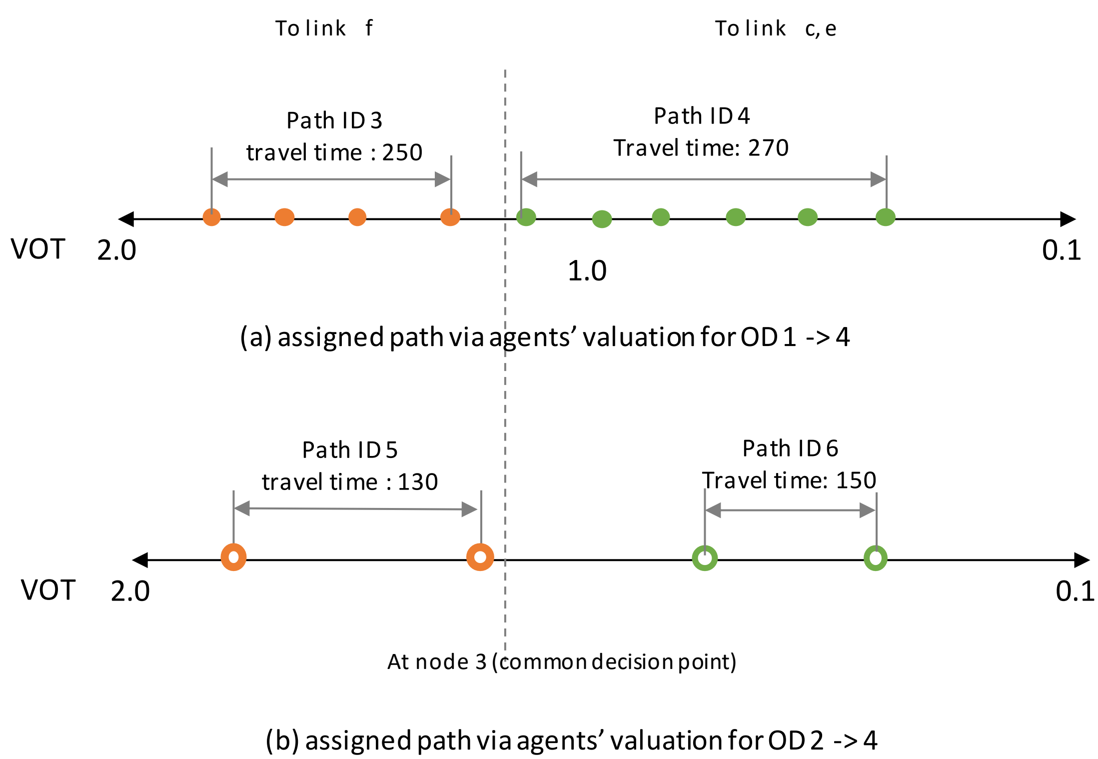


Figure 6. 4 Unique path flow with agent heterogeneity

To identify agents passing a node (decision point) at time step t, it is necessary to carefully track the movement of agents in each time step. The node to destination array for existing agents in a network is updated in Step 2-4 by updating agents' position in every time step. Agent's position of the next time step is calculated as shown in Figure 6.5.

* Update Agent Position at each time step	
Function UAP ($\mathbf{pos}_{\mathrm{i,t-1}}, \boldsymbol{t}, \boldsymbol{u_a}, \boldsymbol{u_b}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{n2d\_array}$)	
current_link = a	current link of an agent at t
$\mathrm{d_{t,a}} = \mathrm{u_a} \cdot \Delta$	travel distance at t for Δ
$\mathrm{pos_{i,t}} = \mathrm{pos_{i,t-1}} + \mathrm{d_{t,a}}$	next position
if $\mathrm{pos_{i,t,a}} \geq \mathrm{l_a}$ then:	check agent passes a node at t+1
$\mathrm{tt}_{b,\mathrm{t}} = \Delta - \frac{\mathrm{l_a}}{\mathrm{u_a}}$	travel time for link b at t
current_link = b	update agent's current link
$\mathrm{pos_{i,t}} = \mathrm{u_b} \cdot \mathrm{tt_{b,t}}$	update agent's position on the next link
$n2d\_array$.append(self.id)	add the agent to a node-to-destination array
end if	
return current_link, $\mathrm{pos_{i,t}}$, n2d_array	return agent's position, and updated node to destination array.

Figure 6. 5 Pseudo-code for updating an agent position in each time step

Once each agent is generated from an origin node at time t, the agent will reach their destination by traversing multiple links and nodes. The agent's position depends on the link's traffic condition (e.g., speed of the current link, u_a). Note that it is possible for the agent to travel two consecutive links in one time step. In this case, the speed of the next link (u_a) determines the position of agent in the next link. The node passing at time t is the decision point for the agent which selects the best route minimizing envy among alternative. Thus, this function appends this agent to the node to destination array for steps 2-3 in the next time interval.

Figure 6.6 presents the overall process of DASEEM-PM.

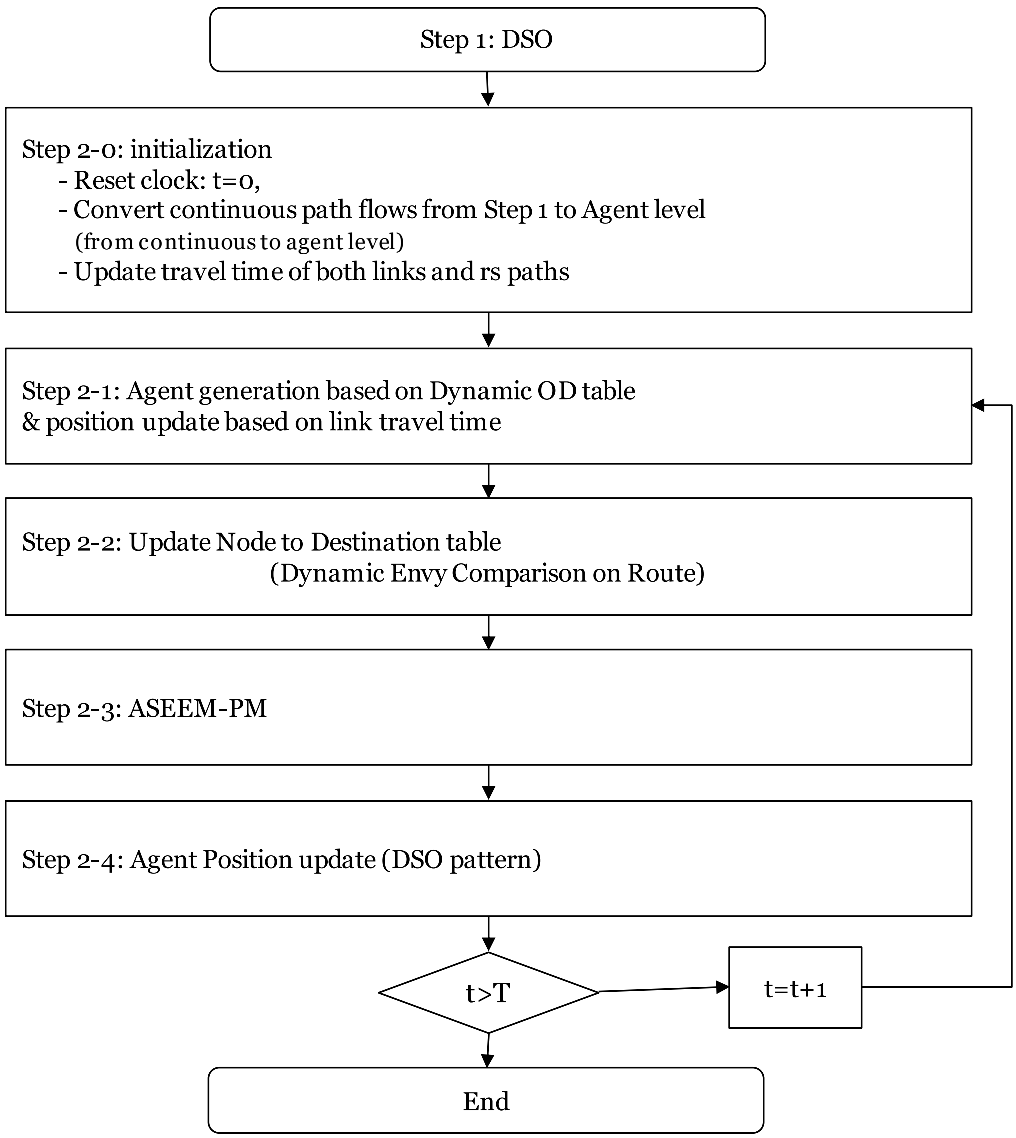


Figure 6. 6 the overall process of the DASEEM-PM

6.5 Numerical Examples

6.5.1 Simulation Environment Settings

We examine the proposed method on a hypothetical network (a dynamic version of Braess Paradox Network). Initially, a single origin-destination case shows an example of how routes vary under the SO condition and the proposed pricing scheme plays a role in minimizing envy induced by allocations for SO. Also, two OD pair cases indicate how "node-to-destination array updates" function provides fair and reasonable allocations for users having different origins but coincidently heading to the same destination.

Table 6. 3 Link characteristics of Braess's Paradox network

Link ID	Head node	Tail node	Jam density (veh/mile)	# of lanes	Length(ft)	Speed(ft/sec)
a	1	2	160.0	6	6000.0	51.33
b	1	3	160.0	6	3000.0	51.33
c	2	3	160.0	6	900.0	51.33
d	3	2	160.0	6	900.0	51.33
e	2	4	160.0	6	6000.0	51.33
f	3	4	160.0	6	6000.0	51.33

Table 6.3 and Figure 6.7 provide more details of the hypothetical network and its link performance function parameters. As discussed, a modified Greenshields model is applied to all links. As shown in Table 6.4, there are four route alternatives for an OD (1 to 4) and two routes for an OD (2 to 4). One of the routes in both od pairs share a link (c) and the other

route for both has the common links (d and e). When agents departing from node 1 and driving to node 4 pass node 2, they will meet other agents with the same destination, but whose origin is node 2. We will utilize this condition in Step 2 to identify unique path flow patterns under the assumption that our system knows the distribution of the valuation function.

Table 6. 4 Possible path combinations for hypothesis network's OD pairs

OD	Path ID	Links	Nodes	Travel time (free flow, seconds)
1->4	1	a, e	1, 2, 3	233.78
	2	a, d, f	1, 2, 3, 4	251.32
	3	b, f	1, 3, 4	175.34
	4	b, c, e	1, 3, 2, 4	192.87
2->4	5	e	2, 4	116.89
	6	d, f	2, 3, 4	134.42

Figure 6.7 depicts the time-varying demand characteristics of all origin and destinations. The proposed assignment module distributes agents in the network for an hour. Each time interval chosen for the simulation is 15 seconds. Consequently, a simulation lasting 1 hour consists of a total of 240-time intervals. The peak interval group has the highest demand ratio, with approximately 1 percent of all time intervals, which is five times that of the lowest demand interval group. The base demand scenario is assumed to have 5952 agents for a single OD pair. We also study various traffic conditions by varying the demand scale from 0.33 to 1.33, which represents a variation of demand from 1,984 to 7,936 agents for a single OD scenario and from 2,504 to 10,016 agents for a multiple ODs scenario. The agents'

valuation is randomly from a lognormal distribution with the values of μ and σ to be 0 and 0.5, respectively.

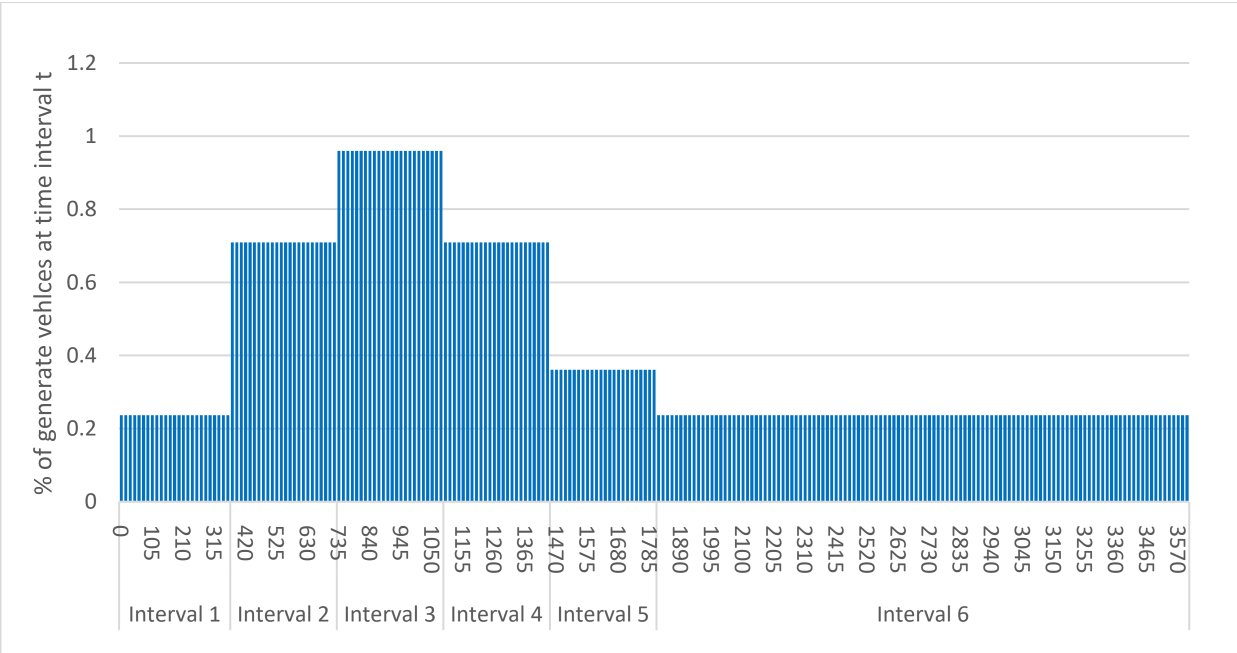


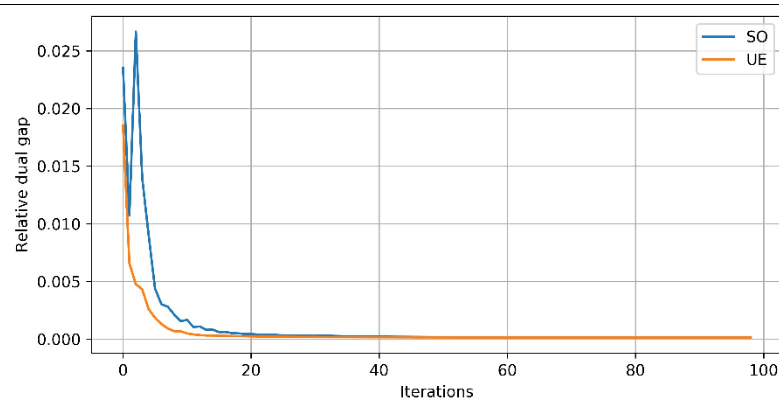
Figure 6. 7 Demand profile for all OD pairs

Table 6. 5 Base demand profile and the number of generated agents (per timestep)

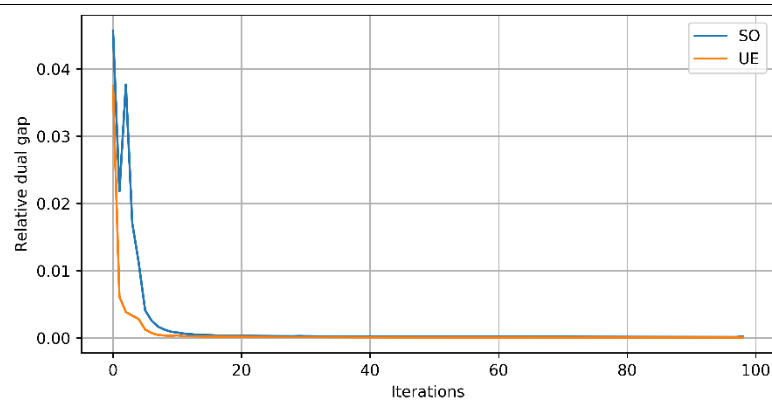
OD	Time Interval group						Total agents
	1	2	3	4	5	6	
Periods (seconds)	(0~360)	(361~720)	(721~1080)	(1081~1440)	(1441~1800)	(1801~3600)	
1->4	14.0	43.0	57.0	43.0	21.0	14.0	5,952
3->4	4.0	11.0	14.0	11.0	5.0	4.0	1,560
Total	18.0	54.0	71.0	54.0	26.0	18.0	7,512

6.5.2 Results of Dynamic System Optimum Assignment

Figure 6.8 shows the results of the convergence for Dynamic SO Assignment. DUE results have been provided for comparison. Relative Dual Gap has been utilized for the test and the travel cost tolerance factor (ω) is set to be zero. If the relative gap is worse than that in the previous step, the step size decreases by increasing $n^{i}{}_{worse}$. Both tests for single OD and two ODs scenarios indicate that RDG converges rapidly in terms of the number of iterations. From the experiments, we set the convergence criteria to RDG = 10^{-4} and the maximum iteration to 100.



(a) Convergence pattern for a single OD pair



(b) Multiple OD pairs

Figure 6. 8 Convergence Pattern of the proposed DTA module

Overall, the total travel time of DSO pattern for a single OD pair scenario is 1,316,308 seconds, which shows an improvement of 1.48% over the total travel time of DUE condition (1,336,084 seconds). Figure 6.9 shows the dynamic pattern of the average speed of both DSO (Green line) and DUE (Red dashed line). Average travel speed is calculated by using Space Mean Speed, which is total vehicle miles per total vehicle travel hours at each time step. In the initial simulation-steps, when traffic is not fully loaded, the average travel speed is similar to that in the free-flow condition. Travel speed drops significantly as demand increases in both DUE and DSO. However, we can see that the variance in the travel speed is lower in the DSO than in the DUE condition.

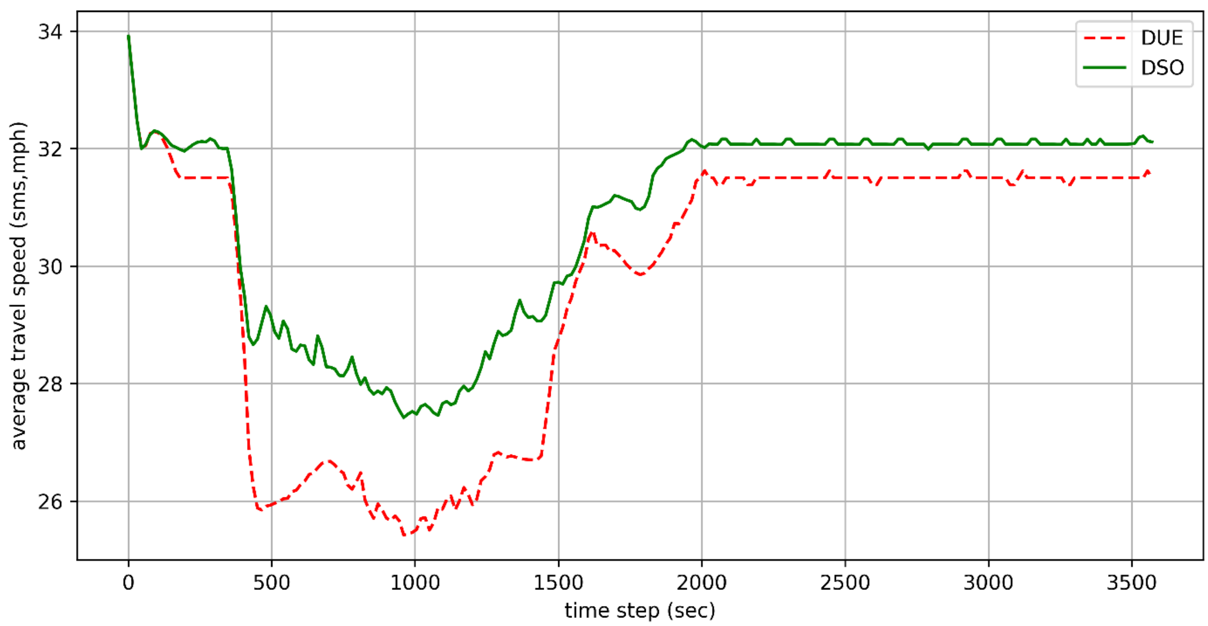
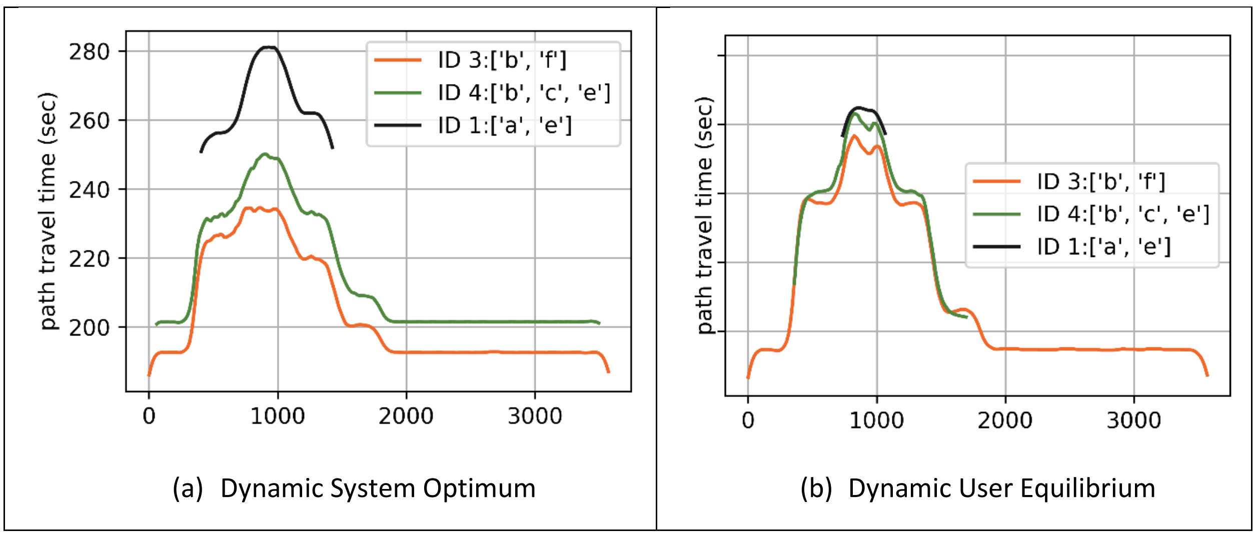


Figure 6. 9 Average travel speed of DSO and DUE (single OD)

Figure 6.10 shows the details of the path travel time results from Step 1 (DTA) for the single OD scenario. There are three used paths in DSO assignment in which the total travel time is minimum. Each path has a different travel time, as shown in Figure 6.10 (a). During the peak hour (Interval 3), Path ID 3 (b, f) has the highest travel time at 280 seconds and the shortest path is ID 1. We compare the path travel time with the results from DUE. As can be seen in Figure 6.10 (b), the status of Dynamic User Equilibrium is achieved from our proposed dynamic traffic assignment module in that the path travel time of all used paths becomes identical within a small tolerance.



(a) Dynamic System Optimum

(b) Dynamic User Equilibrium

Figure 6. 10 Identified Path Travel times of DSO and DUE with a single OD pair

Figure 6.11 depicts path flow patterns of both DSO and DUE. It should be noted that the path flow results are smoothed by averaging two consecutive points. It is commonly known that dynamic path flows fluctuate by time since the traffic conditions in the previous time step affect conditions in the following step. Accordingly, the route of the following step might differ from the preceding step in congested conditions.

Overall, as expected, traffic of DSO condition has a more even spatial distribution that in the DUE condition. The DSO condition utilizes Path ID 1 and 3 earlier than the DUE condition, and DSO assigns more vehicles to Path ID 1 that is the longest path. Furthermore, path ID 3 is generally less utilized in the DSO. These patterns imply that better traffic conditions can be fulfilled only if some agents are willing to yield the shortest path. Furthermore, when we consider the path travel time variances of DSO together, we can reconfirm that optimized traffic condition comes with travel time gaps among alternatives. We also can find that optimized system pattern has spatially dispersed traffic by time, which efficiently utilizes the given transportation supply.

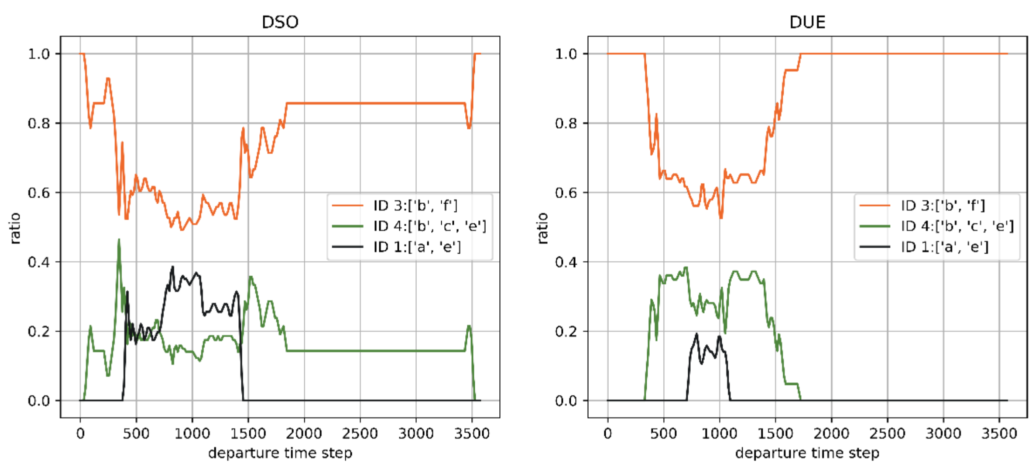


Figure 6. 11 Dynamic path usage of DSO and DUE with a single OD pair

The proposed DTA method is also capable of finding the optimized traffic pattern for the multiple ODs scenario. Figure 6.12 depicts the comparison of travel speed over the entire network and time for both DSO and DUE. Similar to the single OD scenario, the average travel speed of DSO is higher than DUE over time. Figure 6.13 shows that the multiple ODs case also has travel time differences between alternatives in the DSO pattern. The maximum difference is about 48.5 seconds in peak interval for OD pair 1(1->4) and is approximately 17.5 seconds for OD pair 2 (3->4). Whereas the maximum differences of UE for both ODs are within 10 seconds. These gaps correlate to the congestion level. DUE also has several path alternatives but its travel time is equilibrated in all OD pairs.

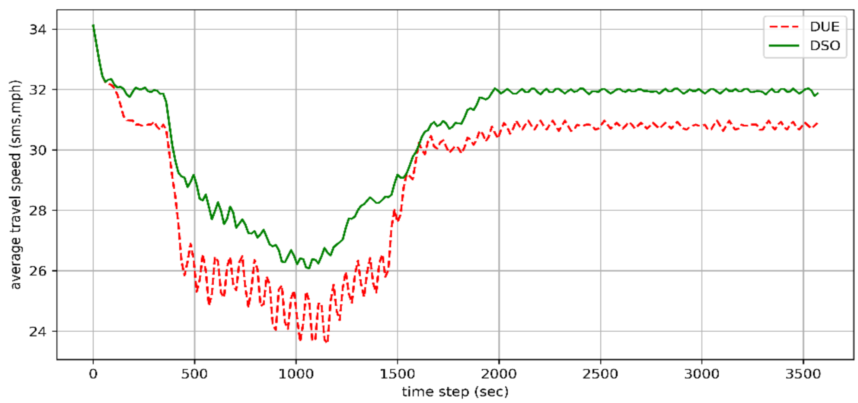
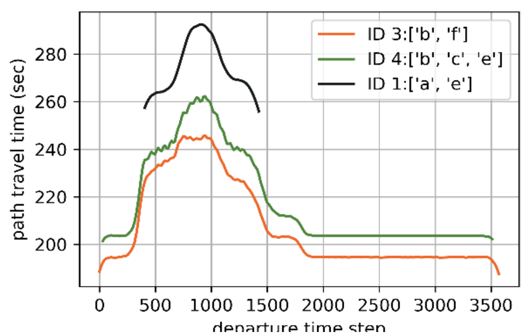
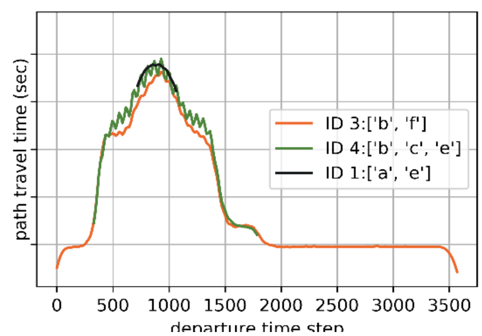


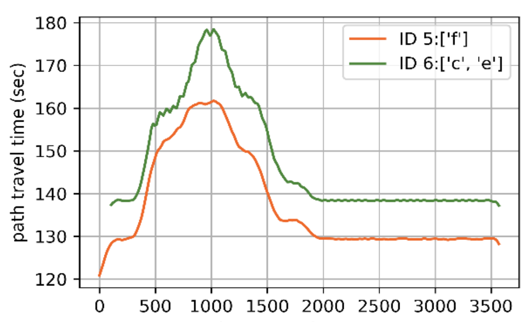
Figure 6. 12 Average travel speed of DSO and DUE (Multiple ODs)



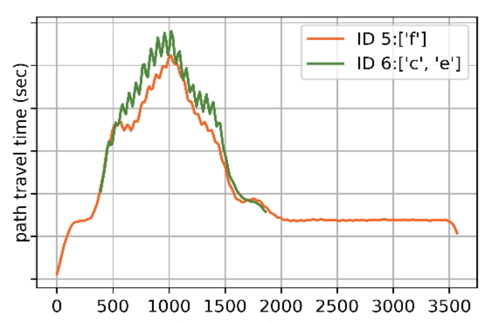
(a) Dynamic System Optimum
(origin: 1, destination: 4)



(b) Dynamic User Equilibrium
(origin: 1, destination: 4)



(c) Dynamic System Optimum
(origin: 3, destination: 4)



(d) Dynamic User Equilibrium
(origin: 3, destination: 4)

Figure 6. 13 Identified Path Travel times of DSO and DUE with multiple OD pairs

Figure 6.14 provides the link travel speed of each link for the multiple ODs scenario. Our experiments show that the travel speed of each link for DSO is faster than for DUE except for link a and e. However, the travel speed under DSO is not worse than under DUE. Figure 6.15 depicts how the detour routes improve the overall network efficiency. In DUE, most agents travel along Path ID 3 and 4, which induces congestion on link b, c and f. In addition, agents departing from node 3 add congestion on link e and f. However, the DSO pattern recommends that more agents in OD pair (1 to 4) drive on path 1 by selecting link a instead of link b to reduce congestion on link b, c, and f. Although we can identify the general patterns of path usage, it is hard to draw conclusions on the identified path flow pattern since it is not unique. However, it is enough to utilize this finding in the next step (Step 2) to compute the unique solution, which will be addressed in the following section.

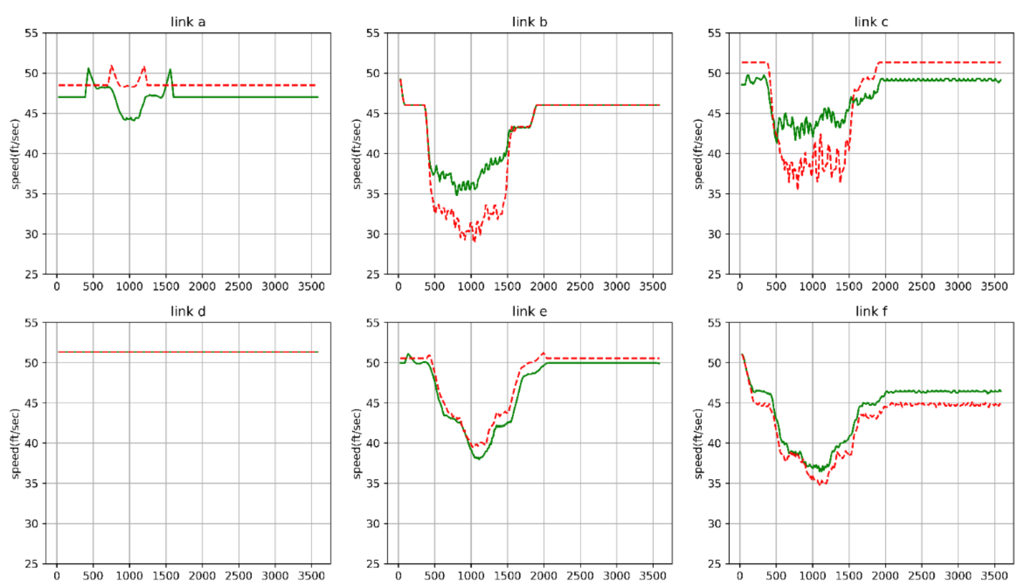


Figure 6. 14 Comparisons of Link travel speed profile (Multiple ODs scenario)

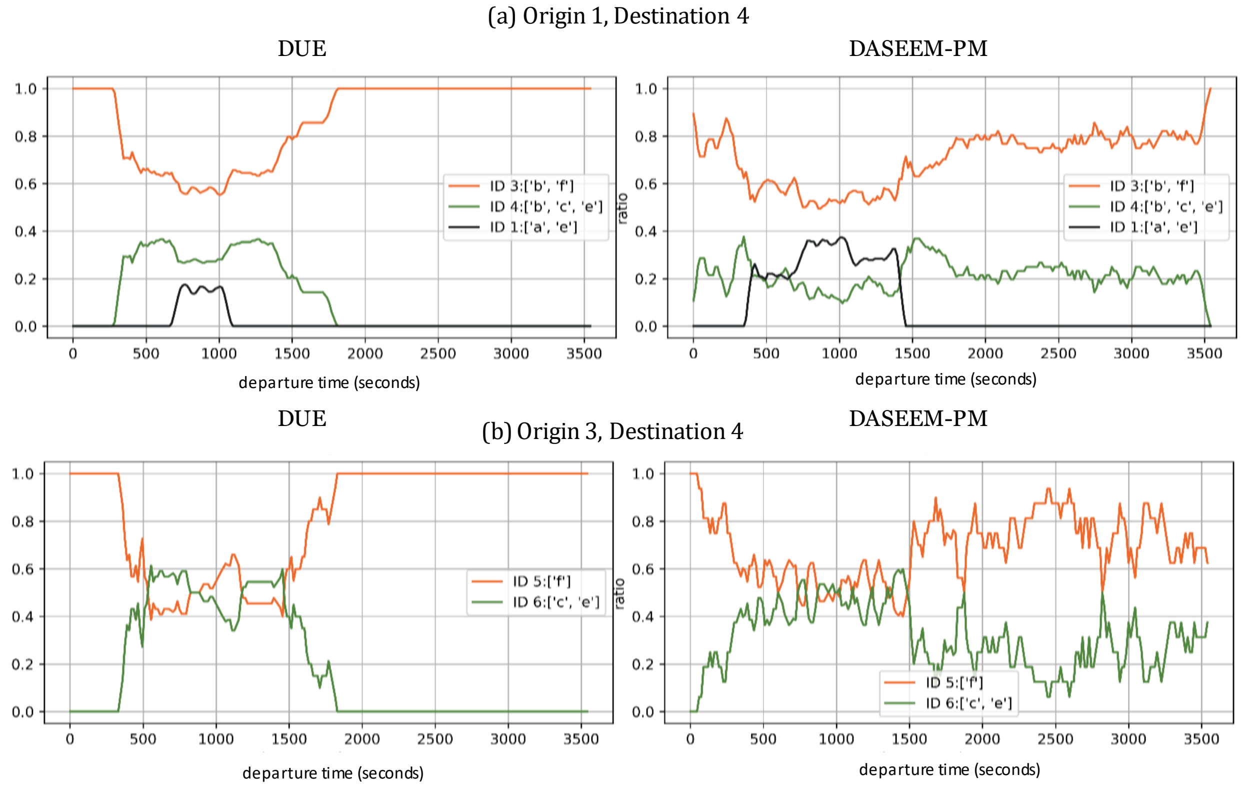


Figure 6. 15 Dynamic path usage of DSO and DUE with multiple OD pairs

6.5.3 Results of DASEEM-PM

Based on the results from DSO (Step 1) for the multiple ODs scenario, this section examines the proposed DASEEM-PM (Step 2). It is noteworthy that Step 1 finds the system optimum traffic pattern by assuming that OD demand has a homogenous agent, but Step 2 generates agents given agents' heterogeneity and allocates those agents based on the distribution of agents' valuation and the estimated path flow pattern.

We evaluate the proposed method by comparing several indices with other models. DUE is for the base scenario when no strategy is applied. DASEEM is implemented for the case when the system forces an agent to the given path from Step 1. DASEEM-PM estimates optimal pricing for each SO path that minimizes envy among agents who have the same destination at the same decision point at a certain time. Then, an agent selects the best option among estimated options that system provides. In addition, we also introduce monetary benefit as a performance index. It is noteworthy that the envy function counts valuation only if other's selected option is better than his/her selected option (Eq. 4.1). The monetary benefit function calculates how an agent feels benefit when his/her selected option is better than others', as shown in Eq (6.23). Similar to the sum of the maximum envy objective in Eq (4.7), we calculate the sum of maximum benefit (Eq 6.24).

$$mb_{ij} = \max\left(\theta_i(\mathrm{t_j} - \mathrm{t}_i) + (\mathrm{p}_j - p_i), 0\right) \tag{6.23}$$

$$\sum_{i \in I} \max_{i \neq j}\{mb_{ij}\} \tag{6.24}$$

Table 6.6 provides a numerical summary of the performance of each scenario. Total maximum envy is minimized to zero in DASEEM-PM, which supposed to be 40,581units in DASEEM. This implies that without pricing scheme, assigning agents to SO routes induces significant envy. It is interesting that DUE also generate envy since DUE in agent-based modeling results in large travel time gaps in OD routes. Among agents who have the same origin and destination and the same departure time, some agents might feel satisfaction when they arrive at their destination earlier than others'. DUE also has this satisfaction, whose value is 15,549.35 units. Agents who matched to the fastest path group in DASEEM will might feel more benefit than agents in DUE since the travel time gaps of DSO between routes tend to be substantial. Whereas agents matched to longer paths feel envy, agents belonging to the shortest path might feel benefit. Note that the Allocation Efficiency function in DASEEM matches the paths with respect to the agents' valuations. Consequently, DASEEM minimizes total envy while maximizing total monetary benefit. However, DASEEM could still not be considered as a feasible application because of unfairness (envy). However, the pricing scheme in DASEEM-PM minimizes envy to 0 and maximizes monetary benefit that agents perceive from P2P travel option comparisons.

Table 6. 6 Overall performance of DASEEM-PM

Index	DUE	DASEEM	DASEEM-PM
Total max Envy	5,681.96	41,348.64	0.00
Total max Monetary Benefit	15,549.35	143,906.52	92,865.00
Total Transaction	-	-	64.367.10

Figure 6.16 depicts envy and monetary benefit by time. Agents in DASEEM are more likely to be dissatisfied by the system because of high level of envy during peak time. Some agents might feel more benefits from the DASEEM system because agents having higher valuation receive shorter paths without pricing. DUE has insignificant amounts of envy and monetary value over the time. Note that agents in DASEEM-PM do not feel envy. Instead, they feel benefited from the system although some pay and some detour because of the pricing estimated by DASEEM-PM.

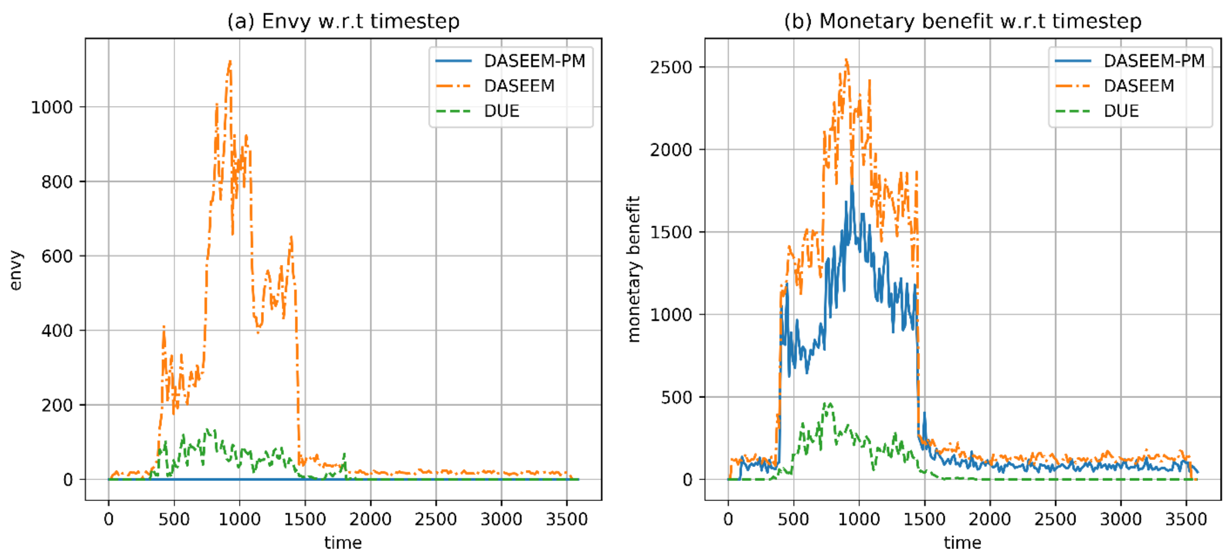


Figure 6. 16 Dynamic profile of Envy and Monetary benefit

DASEEM-PM controls agent's envy coming from the differences in travel time for each alternative by estimating prices to minimize envy for agents under budget balanced condition. Figure 6.17 shows the network-wide dynamic characteristics of tolls and incentives. The red dashed color and green color lines depict tolls and incentives, respectively. These two lines are symmetric since the total amount of collected tolls are used to compensate for the other path groups. The amount of incentives and tolls is associated with traffic conditions in which the gap between alternatives becomes higher during significant congestion

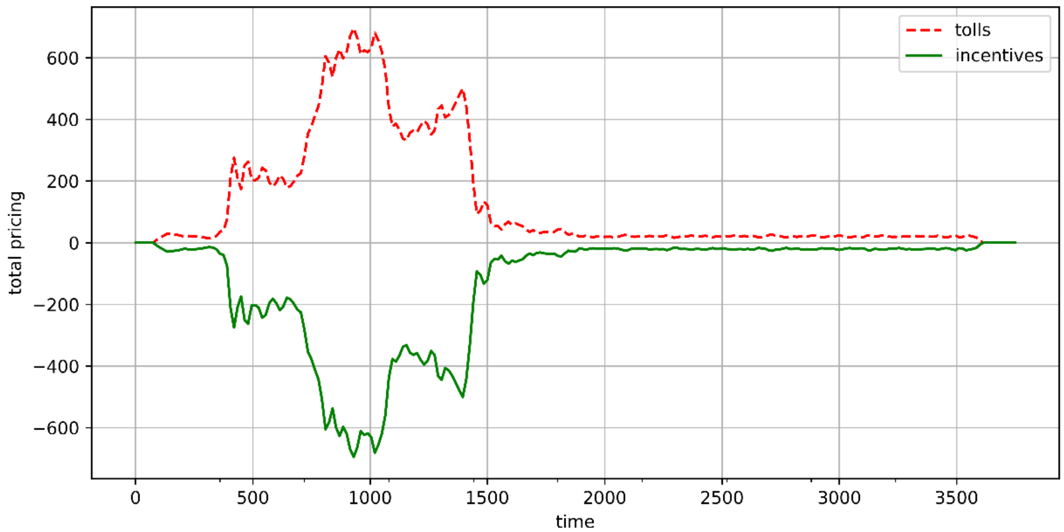
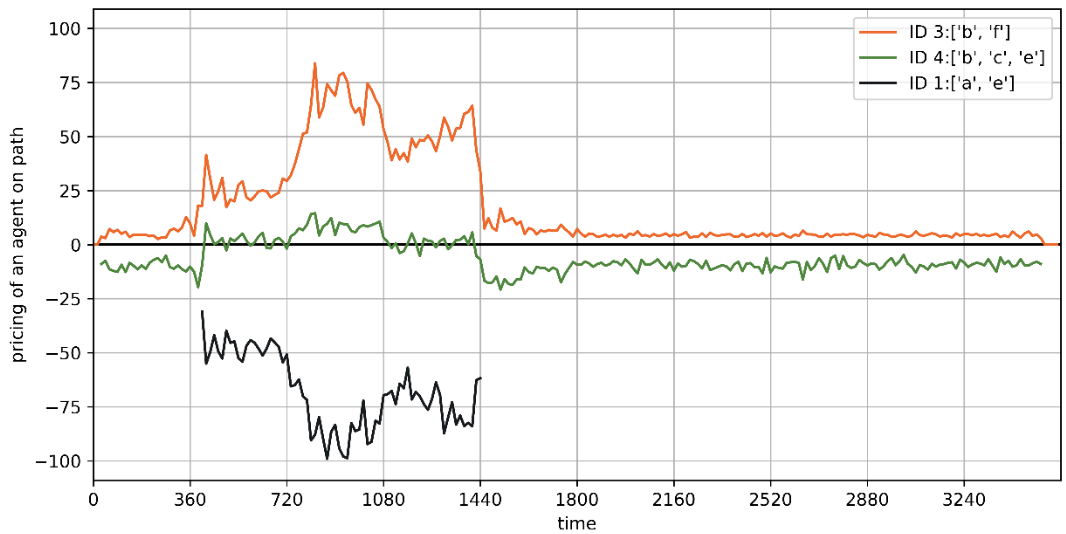
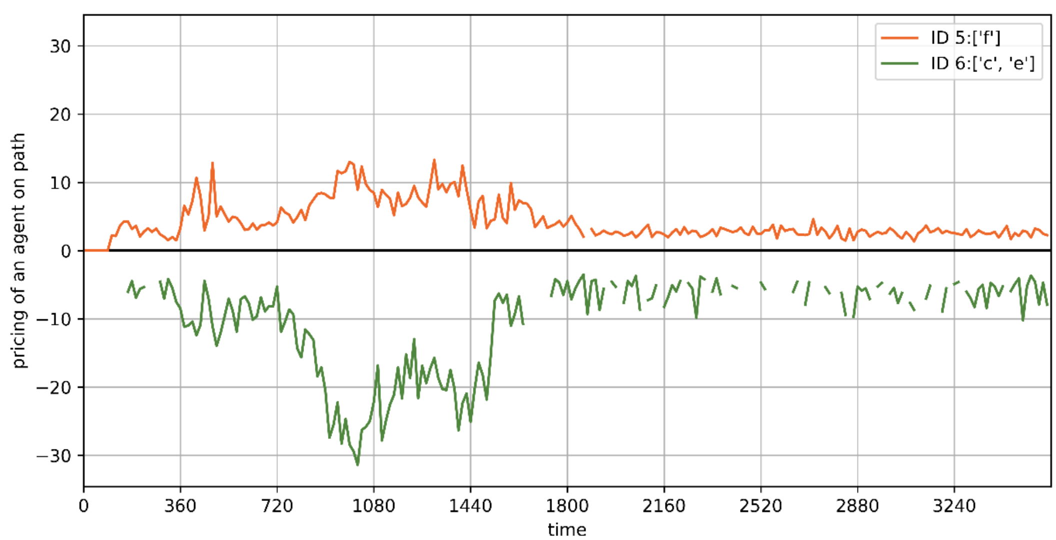


Figure 6. 17 Dynamic pattern of Total cost and incentives

Figure 6.18 indicates dynamic pricing by time for each alternative. For the three path alternatives case (origin 1 and destination 4, Figure 6.18 (a)), the shortest path always pays a toll. The price of the second option varies (ID 4). In low traffic, the pricing of the second fastest option is incentives. However, it becomes a toll after a certain point (from 390 seconds). The third option that is the longest path (ID 1) always receives incentives. The other OD group departs from node 3 and its destination is 4 (Figure 6.18 (b)). Remarkably, some periods (from 1800 seconds) have only tolls instead of an incentive option. This is because of DEND (Definition 6.7), which updates paths and prices at each decision point. In the scenario, agents passing node 3 consist of two OD pair groups. First group includes agents departed from node 1 and pass node 3 for node 4 at time *t*. At the same time, the other group departs from 3 to node 4. DEND function updates at time *t* and node 3 for agents in Node-to-destination table and Price matching module in DASEEM-PM estimates the best prices for agents in the table.



(a) origin 1, destination 4



(b) origin 3, destination 4

Figure 6. 18 Dynamic pricing for each path alternative

Figure 6.19 confirms Lemma 6.2 that Step 2 reallocates path flows to a unique travel path pattern. The profile of generated agents at t of both OD pairs follows the dynamic demand profile shown in Figure 6.7. The number of agents of OD pair 1(node 1 to 4) is proportional. In other words, the demands of OD pair 1 at t is always four times larger than the demand of OD pair 2 at t.

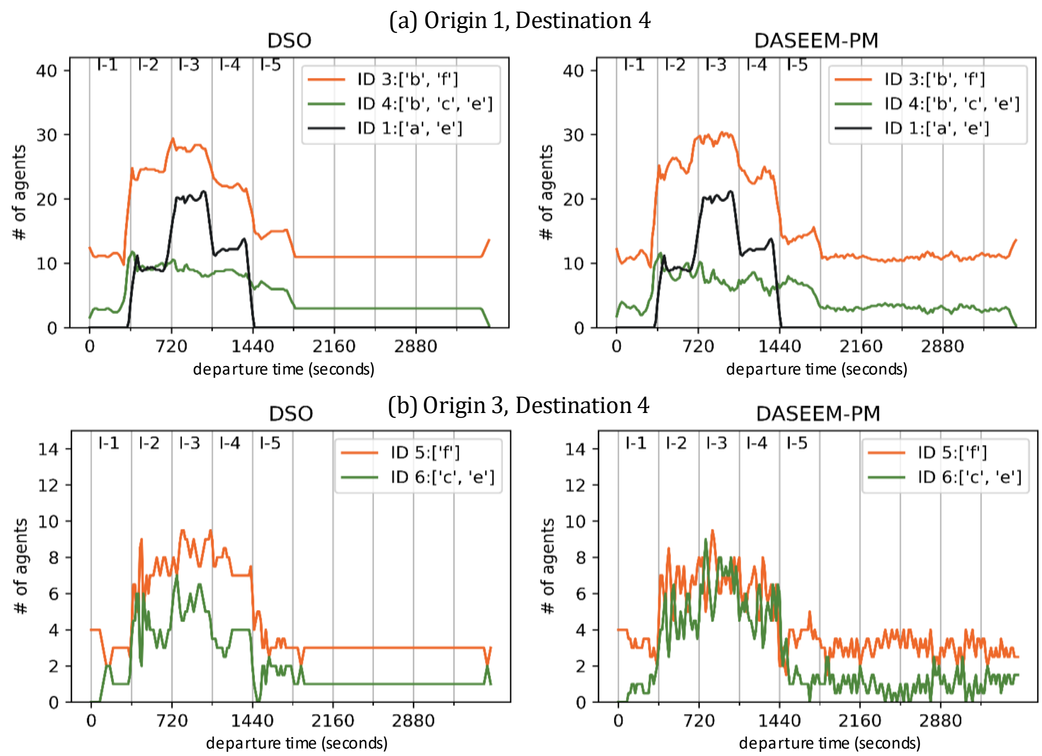


Figure 6. 19 Organized-unique path flow pattern

The left figure of Figure 6.19 is a dynamic path flow pattern for DSO identified by Step 1, which is non-unique. Specifically, some of the path flows for Path ID 3 and 4 can vary with path flow of Path ID 5 and 6 of OD pair 2 (node 3 to 4). Those paths contain the same decision point (node 3). At node 3, agents heading to node 4, no matter their origin, have the same path alternatives. One of the alternatives is to go to node 2 (link c) or to node 4 (link f).

Let us recall that our simulation in Step 2 generates agents having heterogeneous valuation function and assigns those agents to the network based on the estimated allocated efficiency from step 1. This property enables us to track the distribution of valuation of agents at each node, which we use to reallocate agents to the given turning ratio to the next node.

Figure 6.20 helps understand how the DEND function reorganizes the path flow. While the Path 1' path flow remains the same, the path flow ratio Path 2 and 3 have changed. Fewer agents travel on Path 3 than the identified DSO pattern. Those agents initially assigned to link f (Path 3) change their next link to link c (Path 1) at node 3. Since the results of Step 1 have the unique link flow pattern, link flows and turning ratios should be identical. Consequently, the amount of changes of Path 4 affects the path flow of OD pair 2. Interestingly, it is easy to identify that agents have shifted to path 6 since the DSO pattern of interval 3 (I-3) and 4 (I-5) indicates lower path usage of Path 5. In addition, agents who are supposed to travel along Path 5 in OD pair 2 also change their route to Path 6. This reallocation is dependent on the distribution of agents' valuation.

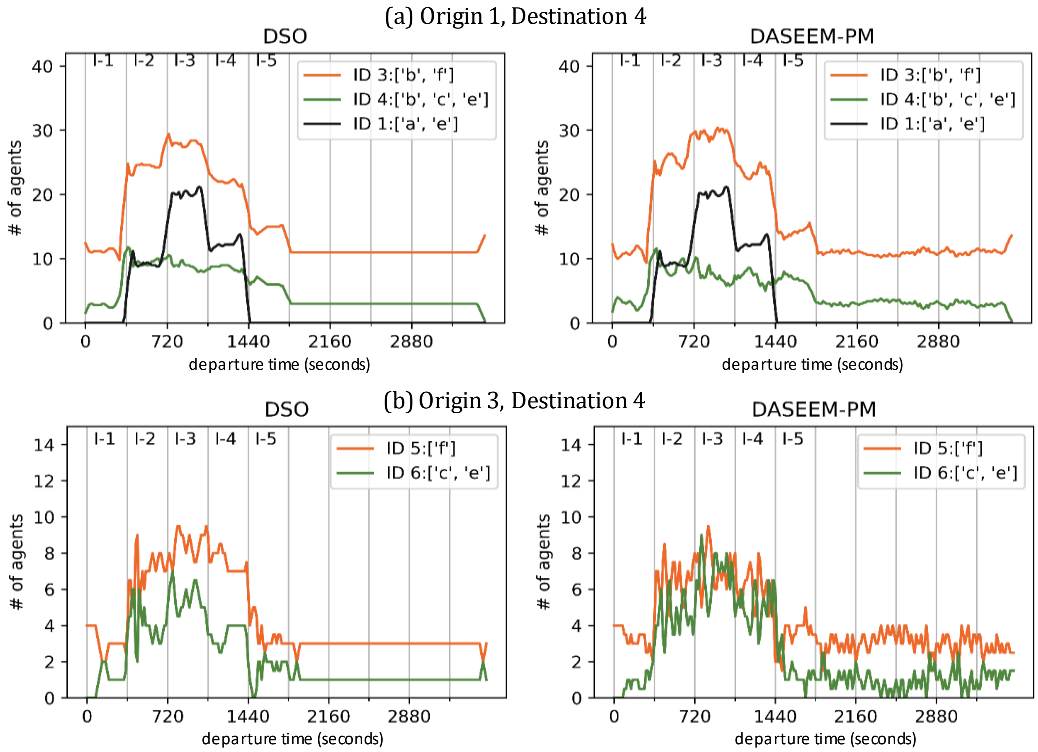


Figure 6. 20 Organized-unique path ratio pattern

Chapter 7

7.1 Numerical Study

In this chapter, we examine the proposed DASEEM-PM method and its solution algorithm using an actual network. We select the Irvine Triangular network as our study area whose network and demand information is available (Park 2009). Their network has been coded for a microscopic simulation (PARAMICS). From their model, we can obtain dynamic OD demands. Also, we estimate the parameter of the Link Performance Function (LPF), which is a modified Greenshields' model, by analyzing the relationship between link density and link travel time from the microscopic model. We calibrate the LPF by comparing the basic geometry for the network with the SCAG travel demand model such as capacity, the number of lanes, link type, and free-flow speed. Note that that our current model does not consider signal controls that were presented in Park (2009). We assume that the calibrated link performance functions can serve as a proxy for signalized intersections, with reduced capacity due to signal phases.

7.1.1 Irvine Triangular Network and Its Dynamic Demand

A total of 27,484 agents are generated in the simulation. We relaxed the maximum allowable envy and the maximum pricing; thus, the pricing is determined by travel time differences in the node to a destination pair, the distribution of valuation, and budget. We set the budget constraint to be budget balanced, meaning that the total amount of collected tolls are used for the total incentives.

Figure 7. 1 Irvine Triangular Network

The study network includes various road types and multiple OD pairs, as shown in Figure 7.1. There are 231 nodes and 312 links. The time scope of the simulation is 7 am for a one-hour period in the morning weekday peak. The simulation time interval is 15 seconds. There are 270 OD pairs and 21,068 dynamic OD pairs. The total number of agents in this case study is 27,484.

Dynamic demands are loaded to the network during an hour. Valuation of agents from dynamic demands is drawn from a lognormal distribution with the values of μ and σ to be 1 and 0.5, respectively, as shown in Figure 7.2. The simulation ends when all agents arrive at their destination. On first glance, it may seem from the cumulative curve of the departure pattern in Figure 7.3 that agents are likely to be generated from a uniform distribution.

However, Figure 7.4, indicating the dynamic demand patterns of top four OD pairs having high demands, confirm that the OD demand referred to Park (2009), contains time-varying dynamics.

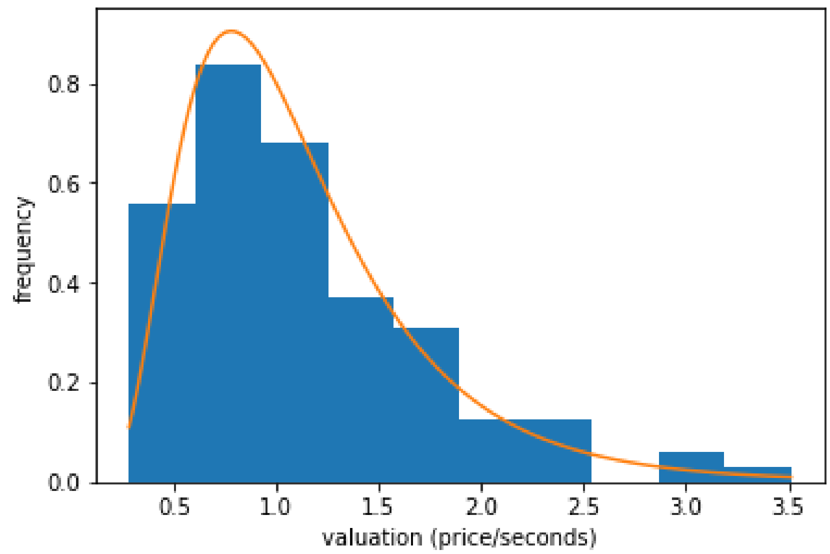


Figure 7. 2 An example of gents' valuation distribution

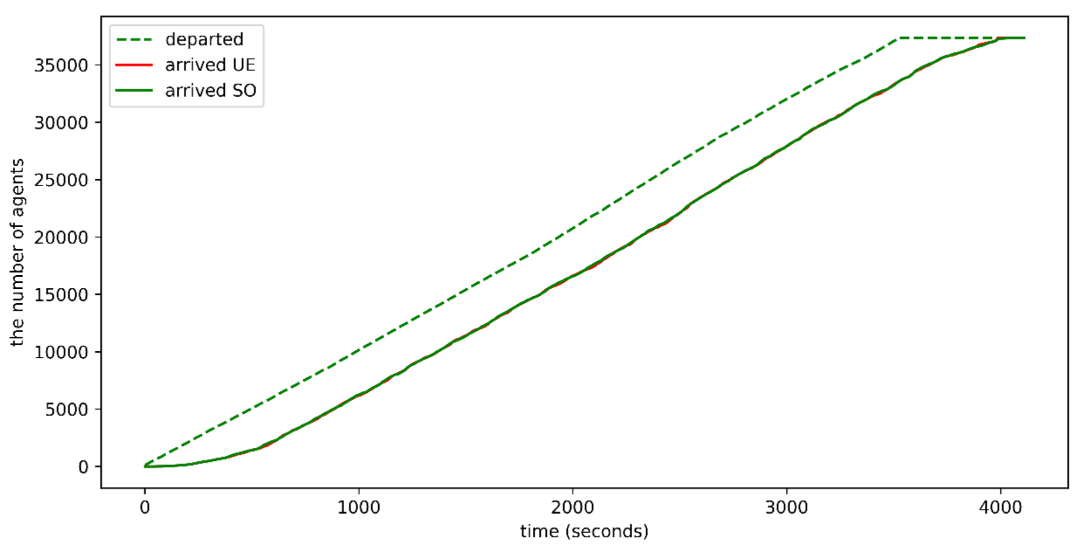


Figure 7. 3 Cumulative curve for agents' departure and arrivals

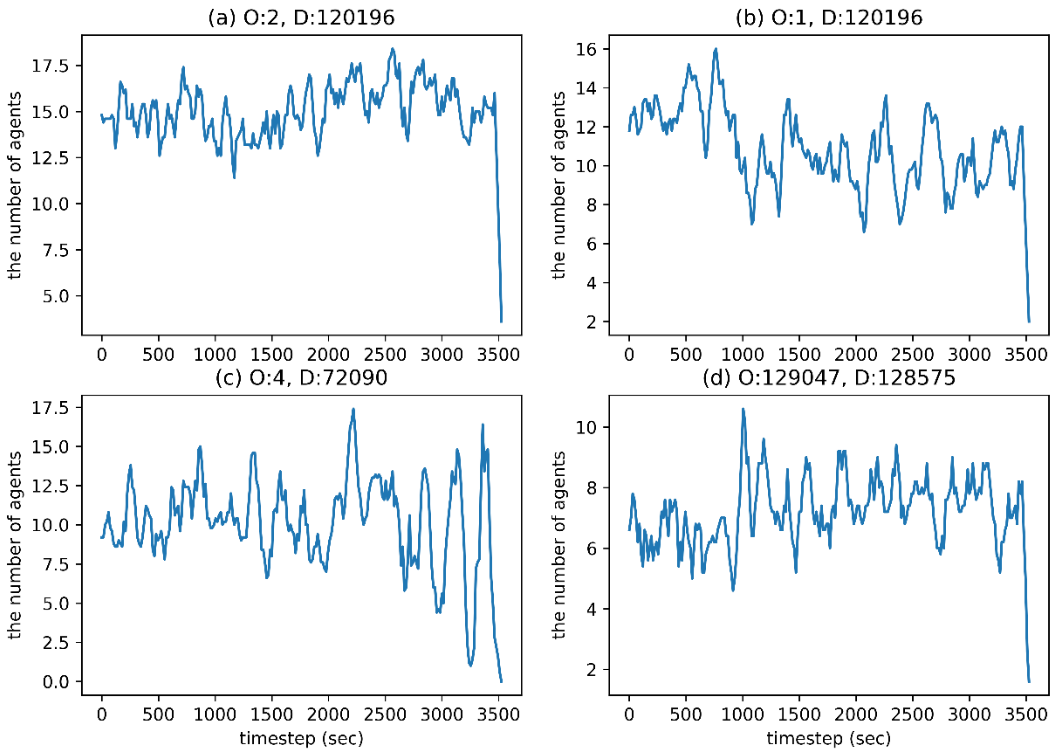


Figure 7. 4 Dynamic demand pattern of top 4 OD pairs

7.1.2 Scenarios for Comparative Studies

We verify our proposed model by comparing the results with the DUE model. There are 4 cases for the comparative analysis, as shown in Table 7.1. Case1 is the base scenario in which no strategy is applied (DUE). Agents in Case 1 are assumed to seek the shortest path and to change their path until their travel cannot be better off. In the equilibrated condition, all agents have the same travel time, but this greedy behavior makes the entire system inefficient. Case 2 is for the best condition in terms of efficiency. The solutions of both Case 1 and Case 2 are computed based on the assumption that agents have the same preference on travel time (homogeneous). There is no pricing scheme in Case 1 and Case 2. Case 3 and Case 4 takes heterogeneity and pricing into account in the model for fairness and efficiency. The difference between Case 3 and 4 is the DEND function, according to which the system manager of Case 4 finds agents approaching a node and updates the node to the destination demand table at every time step, which is used to process dynamic pricing to minimize envy. The system recommends travel options for the agents in the node to destination table if better options are available to the agent.

Table 7. 1 Definition of cases and its component

Case	Model	Pricing	Heterogeneity	DEND (Node update)
Case 1	DUE	x	x	x
Case 2	DASEEM	x	✓	x
Case 3	DASEEM-PM	x	✓	✓
Case 4	DASEEM-PM	Toll & Incentive	✓	✓

7.2 Results

Table 7.2 summarizes the basic comparisons of all the cases. First, the proposed model improves the efficiency of the given supply. The total travel time of the proposed model is 14,399,160 (seconds), which represents an improvement of 0.44% over Case 1(DUE). This improvement can be achieved due to the spatially distributed traffic. The total number of paths of Case 2 is 565, which is 196 more than the number of paths of Case1. Case 4 has the increased number of paths of 603 because of the previously defined DEND function. The total travel time of Case 2, 3 and 4 is higher than in Case1 because the optimized system distributes traffic over the space using a larger number of paths. Note that Case 4 has the highest number of paths since the DEND function reorganizes paths for agents at each node while traffic patterns remains the same. Although there are total travel distances increases for Case 2, 3, 4, the average travel speed, which can be calculated by both total travel time and total travel distances, improves from 42.95 mph to 43.36 mph.

Table 7. 2 Results of comparative analysis

Index	Case 1	Case 2	Case 3	Case 4
Total Travel Time (seconds)	14,467,252	14,399,160	14,399,160	14,399,160
Total Travel Distances (ft)	911,395,789	915,674,204	915,674,204	915,674,204
Average Speed (SMS,mph)	42.95	43.36	43.36	43.36
Total number of paths	369	565	565	603
Total max Envy	100,290	136,917	0	0
Total Transaction	0	0	138,305	140,790
Total Max envy to the shortest path	30,951	151,691	57,466	0
Total Max Monetary Benefit	79,165	362,047	212,842	234,037

Figure 7.5 shows a detailed analysis of the dynamic case in terms of the average travel speed. Overall, DASEEM-PM tends to have the lower total travel times (green line) than DUE (red dashed line) in the entire time period, meaning that the proposed method affects the overall system efficiency. Although the proposed system has the higher total travel distance, Figure

7.5 shows that DASEEM-PM has better speed in the entire system. Dynamic pricing plays role in making people select the DSO pattern as a green line in Figure 7.5

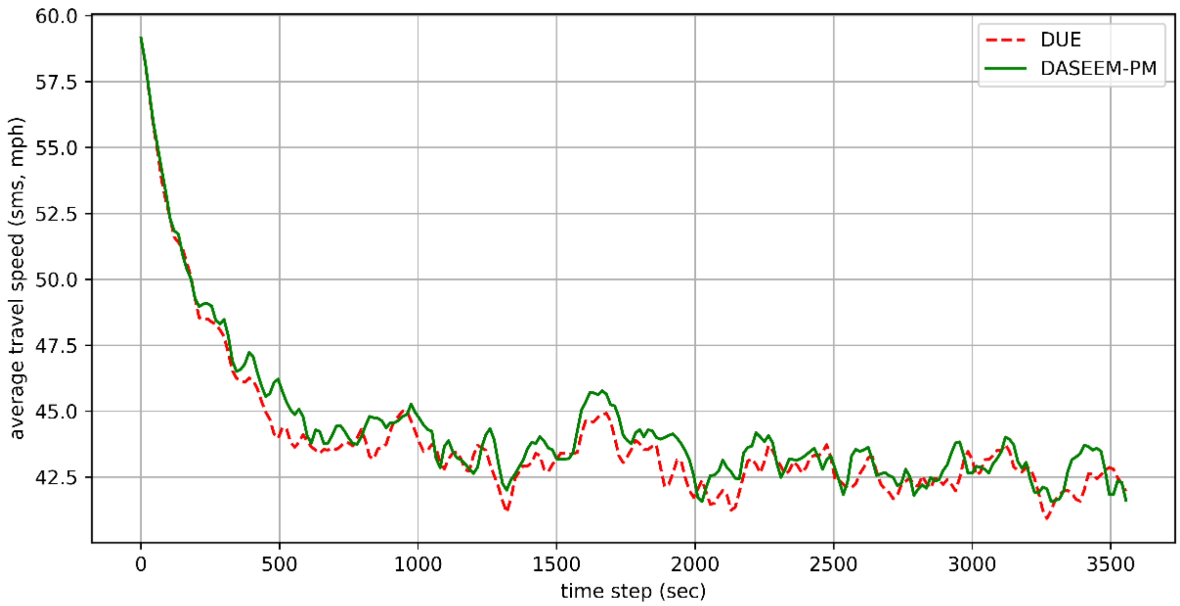


Figure 7. 5 Comparisons of a trend of average speed over the entire simulation time

Figure 7.6 also visualizes the total pricing (tolls and incentives) over the simulation time. As more traffic exists in the network and causes higher marginal effects on congestion, transaction costs increase. Before 07:08:15 am (485 seconds), the pricing gaps between tolls and incentives are small due to less severe traffic congestion and fewer number of agents in the network.

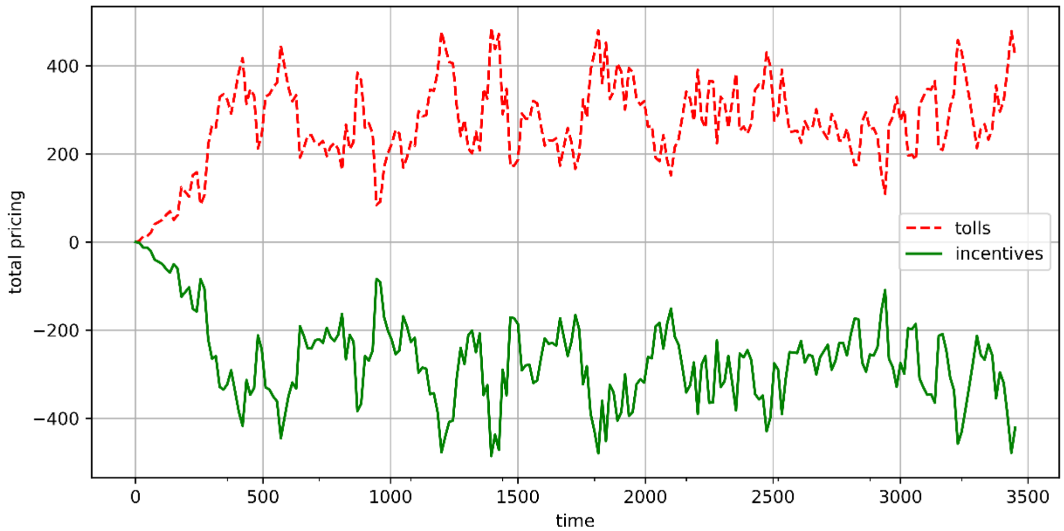


Figure 7. 6 Estimated total tolls and incentives for the Irvine Triangular network

Figure 7.7 depicts the envy of agents in each case. While Case 1 (red dotted line) and 2 (green hashed line) have envy, Case 3 (orange hashed line) and 4 (blue line) do not have envy due to price matching. Similar to the Braess' Paradox Network scenario in Chapter 6, Case 2 (Green line), which is DASEEM with no pricing, has the highest envy. However, agents in Case 3 are likely to feel envious when they realize that no shortest path is available to them as an option. Figure 7 (b) shows how the agents feel envy to the shortest path. Case 3, which does not include the DEND function, has significant level of envy to the shortest path although the level of envy is lower than in Case 3. DASEEM-PM of Case 4 updates the node-to-destination table at each time step, then travel options from a node to destination are processed with pricing according to agents set in the table. Thus, agents always receive travel options where one of them is the shortest path.

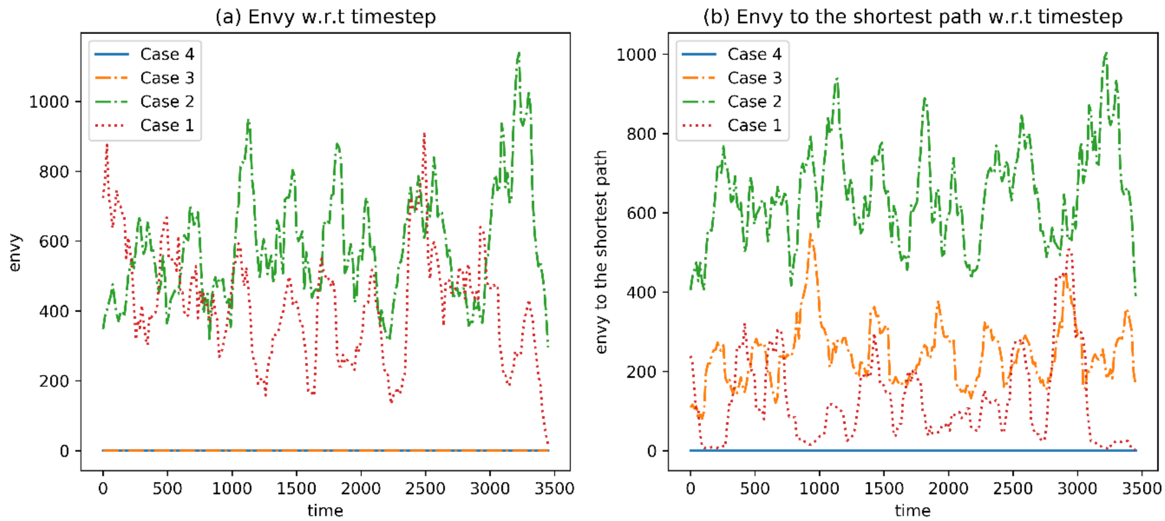


Figure 7. 7 Dynamic profile of envy and envy to the shortest path (Irvine)

Figure 7.8 shows the dynamic trend of monetary benefit that agents receive because their best option is better than other agents' travel time and cost. Since travel time gaps of Case 1 are insignificant, its monetary benefit is small. The monetary benefit of CASE 2 is the highest over the same time period. However, it is noteworthy that envy and envy to the shortest path of Case 2 are also highest, which has a negative effect on the fairness of the system.

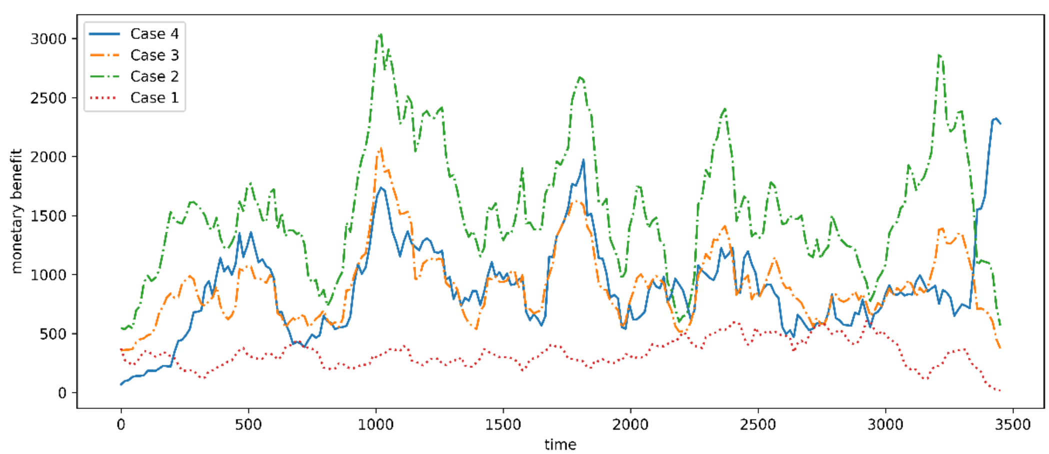


Figure 7. 8 Dynamic profile of Monetary benefit (Irvine)

We further analyze the travel time and pricing for the selected OD pairs. The OD paths of DUE have a similar travel time as the definition of UE. Moreover, the number of paths is significantly limited. For example, most OD time instances have one or two paths. The maximum number of paths is 4. On the other hand, DSO and the proposed method (both Case 2 and 3) have more paths. Figures from 7.9 to 7.11 indicate examples of the various paths. Figure 7.9 shows all paths for Interstate 5 (I-5) which is a major interstate highway in California. In the Irvine Triangular network, its origin and destination are the entry and exit points of Interstate 5 (I-5), respectively. DSO has five identified travel paths in the time step, whereas DUE has only one path.

Figure 7. 9 Paths for an origin 2 to a destination 4 (I-5 southbound)

Figure 7.10 also visualizes the five paths from Interstate 5 to Interstate 405 (Northbound). There are five paths in DASEEM-PM over the simulation periods, whereas only one path is used in DUE. The thickness of each path in Figure 7.8 indicates the relative path flow. It is evident that most of the flow is assigned to Path 1. DASEEM-PM finds the optimal pattern that minimizes travel time by distributing travel demands over the possible paths. Figure 7.9 confirms this pattern.

DUE pattern has two paths (Path ID 1 and 2). Upon closer inspection of the overlapped paths and actual road geometry, we can regard that the two paths are the same. However, DASEEM-PM generates total 8 paths. Most agents still select Path ID 1 and 2, which are shorter paths. Some agents selecting the detoured routes contributes to minimizing total travel time (efficiency).

Table 7.3 indicates the number of used paths for DASEEM-PM and DUE over the entire simulation periods. The DUE patterns have a limited number of paths. The I-5 Southbound case has 4 paths. I-405 Northbound and Woodbridge to I-405 case have 1 and 2 paths, respectively. In contrast, DASEEM-PM efficiently utilize road supply by distributing flows, which is important because even small fraction of distributed traffic could play role in improving efficiency. As can be seen in Table 7.3, the paths of DASEEM-PM not used in DUE have a relatively small number of path flows. For example, only 62 agents among 3,502 in the I-405 Northbound direction are able to influence the system to make it efficient. In the same manner, the other paths for I-5 Southbound than DUE have 31 agents over total 5,298 agents.

Figure 7. 10 Paths for an origin 4 to a destination 1 (I-405 northbound)

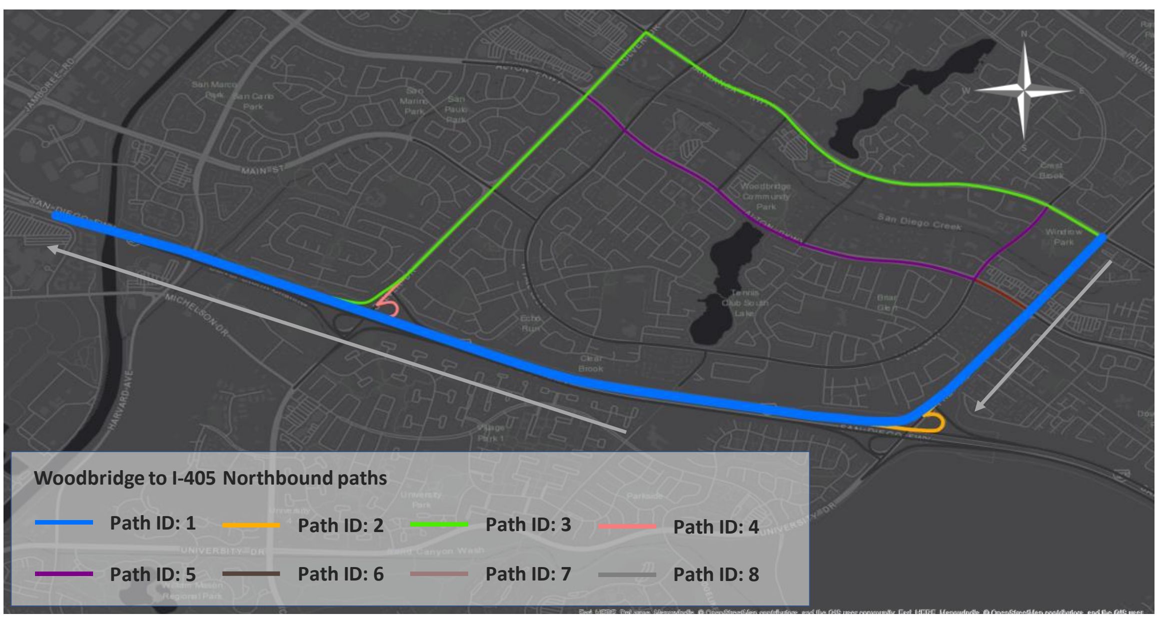
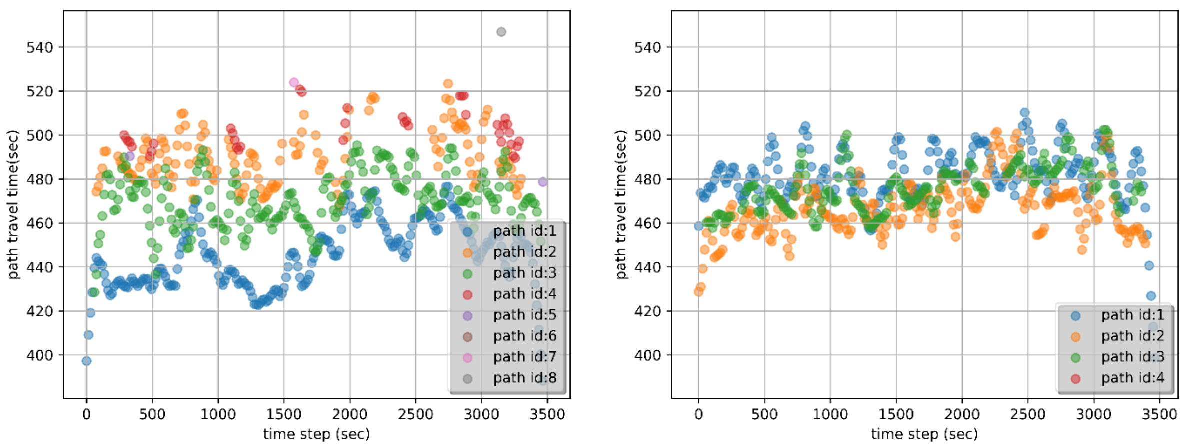


Figure 7. 11 Paths from Woodbridge to I-405 northbound

Table 7. 3 Path flows of the selected OD pairs

Direction	Path ID	Path Volume	
		DASEEM-PM	DUE
I-5 Southbound	1	2,933	3,389
	2	1,027	1,018
	3	976	889
	4	331	2
	5	18	
	6	11	
	7	1	
	8	1	
Total		5,298	5,298
I-405 Northbound	1	3,440	3,502
	2	54	
	3	4	
	4	3	
	5	1	
Total		3,502	3,502
Woodbridge to I-405 Northbound	1	344	540
	2	214	92
	3	32	
	4	18	
	5	11	
	6	5	
	7	4	
	8	4	
Total		632	632

Figure 7.12 compares the path travel times by simulation time between DASEEM-PM and DUE for the OD pair of I-5 southbound. In addition, Figure 7.13 shows the pricing according to the paths. As for the definition of DUE, travel time among paths are almost the same. However, DASEEM-PM achieves minimized travel time from different travel times among paths as shown in Figure 7.12 (a). Although there are 8 paths in DASEEM-PM, only four paths are mostly used over the timesteps.



(a) Path travel times of DASEEM-PM

(b) Path travel times of DUE

Figure 7. 12 Comparisons of path travel time between DSO and DUE (I-5 Southbound)

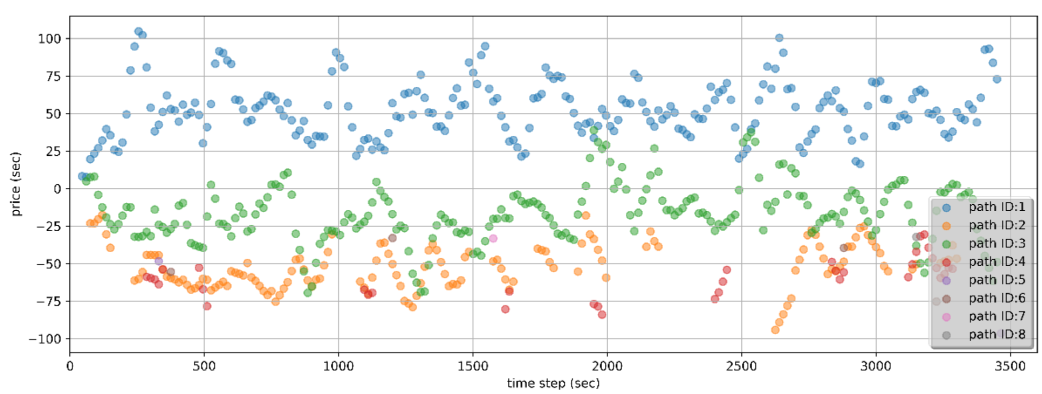
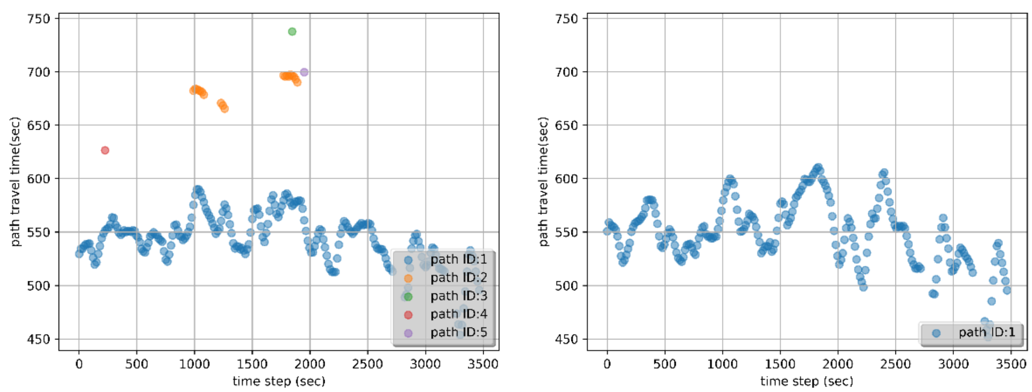


Figure 7. 13 Tolls and Incentives of paths (I-5 Southbound)

The incentives and tolls are related to the path travel time gaps as shown in Figure 7.12 (a) and Figure 7.13. The pricing estimated by DASEEM-PM is shown in Figure 7.13. The shortest path (path 1, blue line) has the highest tolls ranging from 0 to 110 time-units). In this OD pair case, the tolls collected from path 1 are distributed to the other paths. These patterns are similarly observed in the other OD paths. Figure 7.14 to 7.18 further exemplify these patterns.



(a) Path travel times of DASEEM-PM (b) Path travel times of DUE

Figure 7. 14 Comparisons of path travel time (I-405 North)

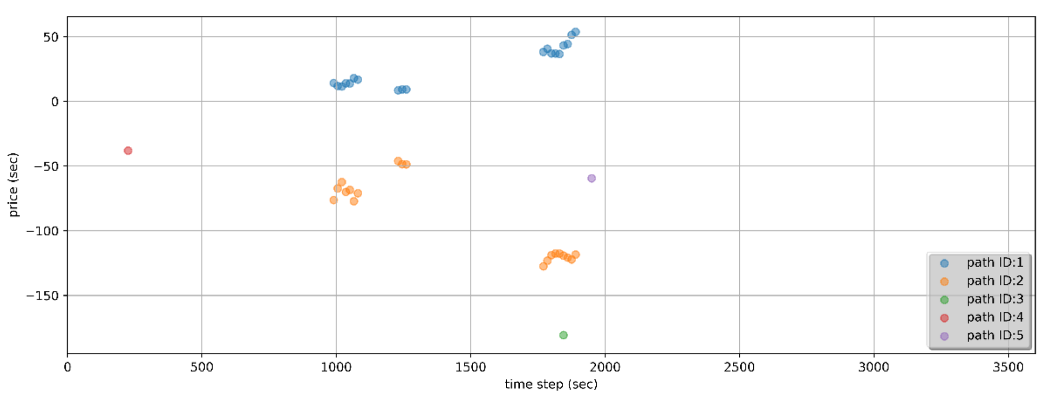
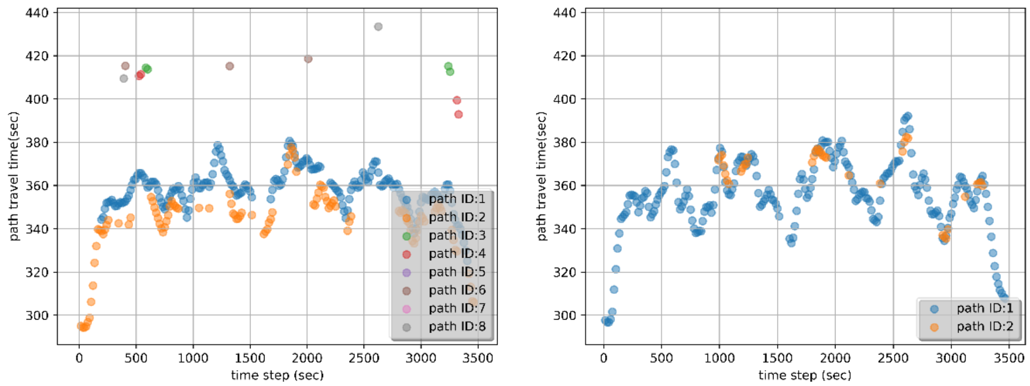


Figure 7. 15 Tolls and Incentives of paths (I-405 Northbound)



(a) Path travel times of DASEEM-PM (c) Path travel times of DUE

Figure 7. 16 Comparisons of path travel time (Woodbridge to I-405 Northbound)

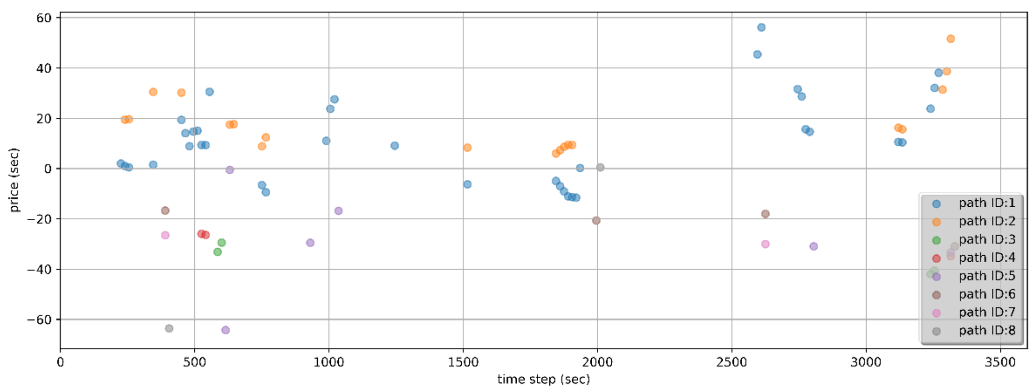


Figure 7. 17 Tolls and Incentives of paths (Woodbridge to I-405 Northbound)

Chapter 8

Future technologies will require smart management of urban infrastructure systems. Our primary research interest is to develop smart urban mobility platforms for achieving both fairness and efficiency, goals that often compete with each other. We focus on the design of a transportation planning platform which maximizes system efficiency while minimizing user 'envy'. Traditional planning methods limit the demand characteristics as an aggregated behavior, meaning that all travelers have identical behavior. Even efforts to consider the heterogeneity of travelers only focus on broad categorical travelers who are assumed to have the same behavior within their group. The transportation design resulting from the assumption of homogeneous behavior results in traveler dissatisfaction since not all travelers can be satisfied with their assigned supply. To solve this problem, we have developed extensions to the envy-free allocation theory, an idea from Economics. Our solution arrives at a pricing scheme, achieving both Pareto efficiency and fair allocations by considering an individual-level preference and efficient allocation of supply.

This dissertation introduces a pricing scheme for a transportation system optimized with respect to envy. The pricing scheme incentivizes agents to follow the system optimum traffic pattern, without feeling envious. The shortest path group in the optimized system can travel faster than in the UE condition and even faster than other path groups. Their added convenience comes from the other group's inconvenience in the form of longer travel times, who are appropriately compensated. The travel time of the inconvenient group can be either longer or shorter than the travel time under UE conditions. It will be true that in any efficient system, an agent will feel unfairness when she/he compares their allocations with the shortest path group. Therefore, financial incentives are an obvious way to compensate people who are willing to sacrifice their travel time to others. At the same time, when the shortest path group pays too much than they deem reasonable, they might not opt into such a system, and not comply with the recommended alternative that the system finds as most efficient.

Firstly, we formulate ASEEM-PM's mathematical objective function and constraints, considering the heterogeneity of valuation for time and optimized transportation systems. The solution finds the optimal price where nobody feels envious to others while achieving an efficient transportation state. From this solution, we can regard the optimized system to be both fair and efficient. The proposed method can address agent heterogeneity and optimality in both the efficiency and fairness and computes the optimal prices for each group. However, its current limitation is scalability.

The individual level of alternative comparisons requires a polynomial number of constraints, $n \times (n-1)$. The mathematical formulation includes binary variables for paths. Our numerical experiences indicate the processing time increases exponentially by the number of agents. More computational resources are required as the number of route alternatives increases. This limitation prevents the ASEEM-PM as currently envisioned to be implemented in a real-world scenario, which were also addressed in subsequent chapters.

By analyzing the characteristics of the mathematical formulations of ASEEM-PM, we reduce the problem complexity. The proposed decomposition heuristic simplifies the original mixed-integer linear programming problem, which decreases the computation complexity from NP-complete to that of linearly increasing processing time with respect to the number of agents. The experiments on the Braess' Paradox network indicate that the proposed heuristic finds the optimum solution that MILP could not guarantee. Furthermore, the proposed heuristic enables us to solve non-linear programs in our framework within a feasible time period. We extended our examination to a much larger network that has a non-linear link cost function (Sioux Falls network).

The proposed ASEEM-PM and its heuristic solution are extended to the dynamic transportation problem for real-world implementation, which considers the time and space-dependent nature of transportation systems. For identifying the dynamic system optimum condition in the given dynamic demand and supply, we develop a dynamic traffic assignment model for SO (SODTA) by integrating a bi-level programming version of dynamic traffic assignment model (Jayakrishnan et al. 1995) with Yang et. al. (2011) Gradient Projection approach. The dynamic version of ASEEM-PM (DASEEM-PM) utilizes the results from the SODTA model. The DASEEM-PM considers time-space movements of agents and agents' behavior, which is defined as Dynamic Envy to a Destination (DEND) at each node. This property is sufficient to prove that the solution from the DASEEM-PM guarantees a unique path travel pattern. We implemented DASEEM-PM on both Braess's Paradox Network and the actual experimental network in Irvine, CA, USA (Irvine Triangular Network). The results indicate that the proposed method maximizes the efficiency of a system and minimizes envy of agents.

This research has applications in the public and private sector. An immediate candidate application would be pricing policies for road systems. Furthermore, with the rapid adoption of cloud-based mobile platform services, our research contributions can enhance public transportation, multimodal route planning services, and transit feeder systems, to name just a few applications. This research can also play a vital role in improving current tolling strategies such as tradable credit schemes, by introducing dynamic tolling and incentivizing.

In the private sector, our research has implications for No-rush shipping, a common shipping tactic employed by Amazon and other retailers. These logistic companies must determine the value of the incentives offered to customers who choose to have their items shipped slower. Our research can also be applied to companies in the Mobility as a Service (MaaS) market, to design flexible options for customers and price these options to induce certain desirable travel and consumption patterns.

References

Arnott, S., A. Palma, and D. Lindsey. 1994. "The Welfare Effects of Congestion Tolls with Heterogeneous Commuters." *Journal of Transport Economics and Policy* 28 (2): 139–61.

Barbanel, J.B. 2005. *The Geometry of Efficient Fair Division*. Cambridge University Press.

Beckmann, M., C. B. McGuire, and C. B. Winsten. 1956. *Studies in the Economics of Transportation*. Yale University Press: New Haven.

Ben-Elia, E, T Tillema, and D Ettema. 2010. "Road Pricing vs. Peak-Avoidance Rewards: A Comparison of Two Dutch Studies." *12th World Conference for Transportation Research*. http://www.civil.ist.utl.pt/wctr12_lisboa/proceedings.htm.

Ben-elia, Eran, and Dick Ettema. 2009. "Carrots versus Sticks : Rewarding Commuters for Avoiding the Rush-Hour — a Study of Willingness to Participate Faculty of Geosciences Key Words : Abstract This Paper Deals with the Potential Participation in a Reward Scheme to Avoid" 16: 68–76.

Bosch, Alje Van Den, Bart Van Arem, Mohamed Mahmod, and James Misener. 2011. "Reducing Time Delays on Congested Road Networks Using Social Navigation." *2011 IEEE Forum on Integrated and Sustainable Transportation Systems*, 26–31. https://doi.org/10.1109/FISTS.2011.5973596.

Boyce, David, and Qian Xiong. 2004. "User-Optimal and System-Optimal Route Choices for a Large Road Network." *Review of Network Economics* 3 (4): 371–80. https://doi.org/10.2202/1446-9022.1058.

Burris, Mark W. 2003. "The Toll-Price Component of Travel Demand Elasticity." *International Journal of Transport Economics*.

Chen, Anthony, Der Horng Lee, and R. Jayakrishnan. 2002. "Computational Study of State-of-the-Art Path-Based Traffic Assignment Algorithms." *Mathematics and Computers in Simulation*. https://doi.org/10.1016/S0378-4754(01)00437-2.

Chen, Anthony, Zhong Zhou, and Xiangdong Xu. 2012. "A Self-Adaptive Gradient Projection Algorithm for the Nonadditive Traffic Equilibrium Problem." *Computers and Operations Research* 39 (2): 127–38. https://doi.org/10.1016/j.cor.2011.02.018.

Chiu, Yi-chang. 2014. Active Traffic and Demand Management System, issued 2014.

Dall'Aglio, Marco, and Theodore P. Hill. 2003. "Maximin Share and Minimax Envy in Fair-Division Problems." *Journal of Mathematical Analysis and Applications* 281 (1): 346–61. https://doi.org/10.1016/S0022-247X(03)00107-0.

Dial, Robert B. 1999. "Network-Optimized Road Pricing: Part II: Algorithms and Examples." *Operations Research* 47 (2): 327–36. https://doi.org/10.1287/opre.47.2.327.

Djavadian, Shadi, Raymond G. Hoogendoorn, Bart van Arem, and Joseph Y. J. Chow. 2014. "Empirical E Valuation of Drivers ' Route Choice Behavioral Responses to Social Navigation." *Transportation Research Board 93rd Annual Meeting. January 12-16, Washington, D.C.* https://doi.org/10.3141/2423-07.

Fleurbaey, M. 2008. "To Envy or To Be Envied? Refinements of the Envy Test for the Compensation Problem. Rational Choice and Social Welfare: Theory and Applications Essays in Honor of Kotaro Suzumura." *Springer Berlin Heidelberg*, 95–118.

Gamow, George, and Marvin Stern. 1958. *Puzzle-Math*. Viking Press.

Gentile, Guido. 2015. "Solving a Dynamic User Equilibrium Model Based on Splitting Rates with Gradient Projection Algorithms." *Transportation Research Part B: Methodological* 0: 1–28. https://doi.org/10.1016/j.trb.2016.02.005.

Glancy, Dorothy J. 2013. "Sharing the Road: Smart Transportation Infrastructure." *Fordham Urban Law Journal* 41 (5).

He, Fang, Yafeng Yin, Nima Shirmohammadi, and Yu Marco Nie. 2013. "Tradable Credit Schemes on Networks with Mixed Equilibrium Behaviors." *Transportation Research Part B: Methodological* 57: 47–65. https://doi.org/10.1016/j.trb.2013.08.016.

Hu, Xianbiao, Yi-chang Chiu, Steve Delgado, Lei Zhu, Ruijun Luo, Paul Hoffer, and Sumin Byeon. 2014. "Behavior Insights for an Incentive-Based Active Demand Management Platform." *Transportation Research Board 93rd Annual Meeting Compendium of Papers*, 1–17.

Jahn, Olaf, Rolf H Mohring, and Andreas S Schulz. 2008. "SYSTEM-OPTIMAL ROUTING OF TRAFFIC FLOWS WITH USER CONSTRAINTS IN NETWORKS WITH CONGESTION." *Operations Research* 53 (4): 600–616.

Jayakrishnan, R., Wei K. Tsai, Joseph N Prashker, and Subodh Rajadhyaksha. 1994. "A Faster Path-Based Algorithm for Traffic Assignment." *Transportation Research Record, Journal of TRB* 1443: 75-83.

Jayakrishnan, R, Wei K Tsai, and Anthony Chen. 1995. "A DYNAMIC TRAFFIC ASSIGNMENT MODEL WITH TRAFFIC-FLOW RELATIONSHIPS." *Trmpn. Res.-C*. Vol. 3.

Knight, F.H. 1924. "Some Fallacies in the Interpretation of Social Cost." *Q. J. Economics* 38: 582–606.

Kumar, Vivek. 2015. "The Impacts of an Incentive-Based Intervention on Peak Period Traffic : Experience from the Netherlands." The University of Texas at Austin.

LeBlanc, Larry J., Edward K. Morlok, and William P. Pierskalla. 1975. "An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem." *Transportation Research* 9 (5): 309–18. https://doi.org/10.1016/0041-1647(75)90030-1.

Liu, Louie Nan, and John F. McDonald. 1998. "Efficient Congestion Tolls in the Presence of Unpriced Congestion: A Peak and Off-Peak Simulation Model." *Journal of Urban Economics* 44 (3): 352–66. https://doi.org/10.1006/juec.1997.2073.

Liu, Ruilin, Hongzhang Liu, Daehan Kwak, Yong Xiang, Cristian Borcea, Badri Nath, and Liviu Iftode. 2014. "Themis : A Participatory Navigation System for Balanced Traffic Routing." *Vehicular Networking Conference (VNC), 2014 IEEE*, 159–66. https://doi.org/10.1109/VNC.2014.7013335.

Lloret-Batlle, Roger. 2017. "Peer-to-Peer and Collaborative Consumption of Supply in Transportation Systems." University of California, Irvine.

Lloret-Batlle, Roger, and R. Jayakrishnan. 2016. "Envy-Minimizing Pareto Efficient Intersection Control with Brokered Utility Exchanges under User Heterogeneity." *Transportation Research Part B: Methodological* 94: 22–42. https://doi.org/10.1016/j.trb.2016.08.014.

Mayet, J., and M. Hansen. 2000. "Congestion Pricing with Continuously Distributed Values of Time." *Journal of Transport Economics and Policy* 34 (3): 359–69.

Meng, Qiang, Zhiyuan Liu, and Shuaian Wang. 2012. "Optimal Distance Tolls under Congestion Pricing and Continuously Distributed Value of Time." *Transportation Research Part E: Logistics and Transportation Review* 48 (5): 937–57. https://doi.org/10.1016/j.tre.2012.04.004.

Mike Pina. 2012. "ITS Strategic Research Plan, 2010 – 2014 Progress Update 2012," 1–195. http://www.its.dot.gov/strategicplan/.

Nie, Yu (Marco). 2015. "A New Tradable Credit Scheme for the Morning Commute Problem." *Networks and Spatial Economics* 15 (3): 719–41. https://doi.org/10.1007/s11067-013-9192-8.

Nie, Yu Marco. 2012. "Transaction Costs and Tradable Mobility Credits." *Transportation Research Part B: Methodological* 46 (1): 189–203. https://doi.org/10.1016/j.trb.2011.10.002.

Nie, Yu Marco, and Yafeng Yin. 2013. "Managing Rush Hour Travel Choices with Tradable Credit Scheme." *Transportation Research Part B: Methodological* 50: 1–19. https://doi.org/10.1016/j.trb.2013.01.004.

Pan, Juan, Mohammad A. Khan, Iulian Sandu Popa, Karine Zeitouni, and Cristian Borcea. 2012. "Proactive Vehicle Re-Routing Strategies for Congestion Avoidance." *Proceedings* -

IEEE International Conference on Distributed Computing in Sensor Systems, DCOSS 2012, 265–72. https://doi.org/10.1109/DCOSS.2012.29.

Park, Ji Young. 2009. “Network-Wide Signal Control with Distributed Real-Time Travel Data.” University of California, Irvine.

Pigou. 1920. *The Economics of Welfare*. New York: MacMillan,.

Riggs, William, and San Luis Obispo. 2015. “Testing Social Norms As an Incentive to Active Transportation Behavior,” 1–17.

Roughgarden, Tim. 2002. “How Unfair Is Optimal Routing?” *Proceedings of the Thirteenth Annual ACM-SIAM Symposium on Discrete Algorithms*, 203–4. http://portal.acm.org/citation.cfm?id=545381.545406.

Rouwendal, Jan, and Erik T. Verhoef. 2004. “Second-Best Pricing for Imperfect Substitutes in Urban Networks.” *Research in Transportation Economics* 9: 27–60. https://doi.org/10.1016/S0739-8859(04)09002-X.

Sheffi, Yoseff. 1985. *Urban Transportation Networks*. *PRENTICE-HALL, Inc.* https://doi.org/10.1016/0191-2607(86)90023-3.

Small, K. A. 1992. *Urban Transportation Economics*. Edited by Chur. Switzerland: Harwood Academic Publishers.

Steimetz, Seiji. n.d. “Volume 35 , Issue 4 A Two-Route Ramsey Pricing Problem : Second-Best Congestion Pricing with an Untolled Alternative and a Revenue Constraint” 35 (4): 2535–42.

Varian, Hal R. 1974. “Equity, Envy, and Efficiency.” *Journal of Economic Theory* 9 (1): 63–91. https://doi.org/https://doi.org/10.1016/0022-0531(74)90075-1.

Verhoef, Erik, Peter Nijkamp, and Piet Rietveld. 1995. “Second-Best Regulation of Road Transport Externalities.” *Journal of Transport Economics and Policy* 29 (2): 147–67. https://doi.org/10.2307/20053070.

Verhoef, Erik T. 2002. “Second-Best Congestion Pricing in General Static Transportation Networks with Elastic Demands.” *Regional Science and Urban Economics* 32 (3): 281–310. https://doi.org/10.1016/S0166-0462(00)00064-8.

Verhoef, Erik T., Peter Nijkamp, and Piet Rietveld. 1996. “Second-Best Congestion Pricing: The Case of an Untolled Alternative.” *Journal of Urban Economics* 40: 279–302. https://doi.org/10.1006/juec.1996.0033.

Yang, Hai, and Xiaolei Wang. 2011. “Managing Network Mobility with Tradable Credits.” *Transportation Research Part B: Methodological* 45 (3): 580–94. https://doi.org/10.1016/j.trb.2010.10.002.

Yang, Hai, and Xiaoning Zhang. 2003. “Optimal Toll Design in Second-Best Link-Based Congestion Pricing.” *Transportation Research Record* 1857 (1): 85–92. https://doi.org/10.3141/1857-10.

Yang, Inchul. 2011. “The Interplay of Urban Traffic Route Guidance , Network Control and Driver Response : A Convergent Algorithmic and Model-Based Framework.” University of California, Irvine.

Zhou, L., M. Burris, R. Baker, and T. Geiselbrecht. 2009. “Impact of Incentives on Toll Road Use by Trucks.” Transportation Research Record: Journal of the Transportation Research Board.