REFERENCE

1. H. Rossi, *Vector fields on analytic spaces*, Ann. of Math. **78** (1963), 455–467.

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ON PIECEWISE LINEAR IMMERSIONS

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The purpose of this note is to prove an existence theorem for immersions of piecewise linear manifolds in Euclidean space. A more comprehensive theory of piecewise linear immersions has been worked out by Haefliger and Poenaru [1].

All maps, manifolds, microbundles, etc. are piecewise linear unless the contrary is explicitly indicated.

Let M be a manifold without boundary, of dimension n. Denote the tangent microbundle of M by τ_M, and the trivial microbundle over M of (fibre) dimension k by ϵ^k. Let

$$\nu: M \xrightarrow{i} E \xrightarrow{j} M$$

be a microbundle of dimension k such that E is a manifold. An *immersion* of M in R^{n+k} is a locally one-one map $f: M \to R^{n+k}$.

I say f has a *normal bundle of type* ν if there is an immersion $g: E \to R^{n+k}$ such that $gi=f$. (It is unknown whether f necessarily has a normal bundle, or whether all normal bundles of f are of the same type.)

The converse of the following theorem is trivial.

THEOREM. *Assume that if $k=0$, then M has no compact component. There exists an immersion of M in R^{n+k} having a normal bundle of type ν if there exists an isomorphism*

$$\phi: \tau_m \oplus \nu \to \epsilon^{n+k}$$

PROOF. We may assume that $i(M)$ is a deformation retract of the total space E of ν. By Milnor [3], $\tau_E \,|\, i(M)$ is isomorphic to $\tau_M \oplus \nu$; it follows from the existence of ϕ that τ_E is trivial. According to [3]

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there is a parallelizable differential structure α on E compatible with the piecewise linear structure. Let $h: E_\alpha \to R^{n+k}$ be a differentiable immersion, which exists by Hirsch [2] or Poenaru [4]. (If $k=0$, the assumption that M has no compact component is used here.) Approximate h by a piecewise linear immersion $g: E \to R^{n+k}$, using the theory of C^1 complexes of Whitehead [5]. Clearly $gi: M \to R^{n+k}$ is an immersion having a normal bundle of type ν.

REMARKS. (1) The assumption that M is unbounded is unnecessary, since a bounded manifold can be embedded in its interior. However, τ_M must be redefined if M has a boundary.

(2) It is not hard to define the concepts of "immersion plus normal bundle"—essentially an immersion of E—and of a "regular homotopy" of these; one can then prove a uniqueness theorem.

BIBLIOGRAPHY

1. A. Haefliger and V. Poenaru, *La classification des immersions combinatoires*, Publ. Math. I.H.E.S., no. 23, pp. 75–91.

2. M. W. Hirsch, *On imbedding differentiable manifolds in euclidean space*, Ann. of Math. **73** (1961), 566–571.

3. J. Milnor, *Microbundles and differentiable structures*, (mimeographed). Princeton Univ., Princeton, N. J., 1961.

4. V. Poenaru, *Sur la théorie d'immersions*, Topology **1** (1962), 81–100.

5. J. H. C. Whitehead, *On C^1 complexes*, Ann. of Math. **41** (1940), 809–824.

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