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APPLICATION OF GENERAL THEORETICAL PRINCIPLES TO EXPERIMENTS - LECTURE 9

ASYMMETRY OF STRANGE PARTICLE DECAYS,

on the Assumption of Conservation of Parity

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If a system of two particles is in an eigenstate of parity, its internal wave function satisfies $\Psi(-\vec{x}) = \Psi(\vec{x})$ and therefore $|\Psi(-\vec{x})|^2 = |\Psi(\vec{x})|^2$, i.e., the angular distribution of the state is inversion-symmetric. Thus lack of inversion symmetry, ("asymmetry" for short) can result only from a state that is not a parity eigenstate. Such a state is not in general an eigenstate of energy, because in general energy is nondegenerate with respect to parity. Thus the state is a superposition of energy eigenstates, i.e., $\Psi = \alpha\Psi_o + \beta\Psi_e$ where $\Psi_{o,e}$ are eigenstates of both parity and energy. The phase between the two states rotates with a frequency $E_o - E_e (\hbar = 1)$, thus the asymmetry averages to zero over a time long compared to $(E_o - E_e)^{-1}$; a condition then on the observability of an asymmetry is that the energy difference between the parity eigenstates be small compared with the (time of observation)$^{-1}$ (i.e., for a decay, the decay width.)

Lee and Yang have suggested, as a solution to the $\tau - \theta$ difficulty, that "the K meson" (and therefore all particles with strangeness ± 1) is really a parity doublet, i.e., it consists of two states with opposite parity, which are degenerate with respect to the strong and electromagnetic interactions. They are, however, split by the weak decay interaction: the two parity-states decay into different states of ordinary particles, and these different states, when serving as virtual intermediate states, produce different self-energy shifts. One expects, then, that the energy difference between the parity states is of the same order as their lifetime; thus it is possible that asymmetric decay from strange particles would be observable. (Below [illegible] see q[illegible] only the effect on the asymmetry of the ratio of energ[illegible] investigate other factors necessary for observable asymmetry, let us look in detail at a particular process: $K^- + p \longrightarrow \pi^{\mp}\Sigma^{\pm}$.

Choose the axis of quantization, the "z-axis," to be along the line of flight of the Σ , then we have $m_\ell = 0$ (for if $m_\ell \neq 0$, the Σ could not be going along the z-axis: $P_\ell^{m_\ell \neq 0}(\cos\theta = \pm 1) = 0$). Therefore $m_j = m_s = \pm\frac{1}{2}$, if for simplicity we take $s_\Sigma = \frac{1}{2}$. Write, when $m_j = \frac{1}{2}$, the Σ state function as $\psi_\Sigma^{\frac{1}{2}} = h_o \psi_{\Sigma_o}^{\frac{1}{2}} + h_e \psi_{\Sigma_e}^{\frac{1}{2}}$ ($|h_o|^2 + |h_e|^2 = 1$), i.e., $h_{o(e)}$ is the amplitude for a $\Sigma_{o(e)}$ to be going along the z axis. To conserve parity, the Σ_o decays to πN in an S state, with wave function $\delta_o \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ where δ_o is proportional to the matrix element for decay (this is discussed in more detail below); the Σ_e decays to πN in a P state, with wave function

$$\delta_e \begin{pmatrix} \cos\theta \\ \sin\theta\, e^{i\phi} \end{pmatrix} .$$

Thus the state $\psi_\Sigma^{\frac{1}{2}}$ decays into

$$\psi_{\pi N}^{\frac{1}{2}} = h_o \delta_o \begin{pmatrix} 1 \\ 0 \end{pmatrix} + h_e \delta_e \begin{pmatrix} \cos\theta \\ \sin\theta\, e^{i\phi} \end{pmatrix} = \begin{pmatrix} \omega_o + \omega_e \cos\theta \\ \omega_e \sin\theta_e e^{i\phi} \end{pmatrix} ,$$

where $\omega = h\delta$. The angular distribution therefore is

$$\left|\psi_{\pi N}^{\frac{1}{2}}\right|^2 = |\omega_o|^2 + |\omega_e|^2 + 2\,\mathrm{Re}(\omega_o^* \omega_e)\cos\theta ,$$

which exhibits the asymmetry. The question now is, will this asymmetry persist when we average over initial spin $m_j = \pm\frac{1}{2}$? To answer this, we must find $\psi^{-\frac{1}{2}}$, if $\psi^{+\frac{1}{2}}$ is given. Lee and Yang constructed $\psi^{-\frac{1}{2}}$ by a reflection, say $Y \rightarrow -Y$. This reverses J_z (e.g.,

$$L_z = xP_Y - YP_x \longrightarrow x(-P_Y) - (-Y)P_x = -L_z)$$

and thus turns $m_j = \frac{1}{2}$ into $m_j = -\frac{1}{2}$, but leaves θ and therefore the angular distribution $|\psi|^2$ unchanged.

However, we have made an implicit assumption, that the amplitudes $\eta_{o,e}$ did not change under the reflection, i.e., that they transform like scalars; a pseudoscalar would change sign under the reflection (which is inversion times a proper rotation). As an example of the latter situation, consider the following model for the production of the Σ: The K-capture reaction goes through a single orbital channel, so that Σ_o comes only from the capture of a K_o, and a Σ_e comes from a K_e (or vice versa). Now, under reflection, the wave functions of the initial states change sign relative to each other, since they are of opposite parity; thus the wave functions of the $\psi_{\Sigma_{o,e}}$ states must change sign relative to one another, i.e., the coefficients $\eta_{o,e}$. Thus in this case the asymmetry reverses under the reflection, so that averaging over initial spin destroys the asymmetry. In general, interference between states of different parity does not yield an asymmetric Σ - decay.

Thus we consider the $\Sigma_{o,e}$ to come from the same state (or at least from states of the same parity); this of course requires that the Σ_o and Σ_e come out in different orbital states, to conserve parity.

<u>Note:</u> this implies that the scheme of coupling the K mesons to other particles is not the most economical possible. The latter would be the coupling of K to baryon in a single orbital state; i.e., (taking as above $s_\Sigma = \frac{1}{2}$, $s_K = 0$)

$$\begin{pmatrix} N \longrightarrow \Sigma_o K_o, \\ \text{or} \\ N \longrightarrow \Sigma_e K_e \end{pmatrix} \text{ in an S state } \underline{\text{or}} \quad \begin{pmatrix} N \longrightarrow \Sigma_o K_e \\ \text{or} \\ N \longrightarrow \Sigma_e K_o \end{pmatrix} \text{ in a P state,}$$

but not both. As we see, an asymmetry in the Σ decay would imply the existence of both couplings (and therefore of a $K_o K_e \pi$ coupling, as recommended by Schwinger.)

For example, if the reaction $K^- P \longrightarrow \pi \Sigma$ goes through a j^{+} state where $j - \frac{1}{2}$ is even the $\Sigma_{e(o)}$ must be in an odd (even) orbital state, i.e., [illegible] We can avoid questions of parity here by deriving [illegible] from $\psi^{+\frac{1}{2}}$, not by reflection, but by application of the standard lowering operator $J_x - iJ_y$. The essential point is that the coefficients $\eta_{o(e)}$ are proportional to the Clebsch-Gordan coefficients $C_{j\overline{(+)}\frac{1}{2},\frac{1}{2}}(j,\frac{1}{2};0,\frac{1}{2})$; under the lowering operation, the $\eta_{o,e}$ become $\eta'_{o,e}$, proportional to

$$C_{j\overline{(+)}\frac{1}{2},\frac{1}{2}}(j,-\tfrac{1}{2};0,-\tfrac{1}{2}) = (-)^{\ell\overline{(+)}\frac{1}{2}+j-\frac{1}{2}}\, C_{j\overline{(+)}\frac{1}{2},\frac{1}{2}}(j\;\tfrac{1}{2};0\;\tfrac{1}{2})$$

so that the relative sign of the $\eta'_{o,e}$ is opposite to the relative sign of the $\eta_{o,e}$. Since under the lowering operation we have

$$\psi_{\pi N}^{\frac{1}{2}} = \begin{pmatrix} \eta_o + \eta_e \cos\theta \\ \eta_e \sin\theta e^{i\phi} \end{pmatrix} \longrightarrow \psi_{\pi N}^{-\frac{1}{2}} = \begin{pmatrix} \eta'_e \sin\theta\, e^{-i\phi} \\ \eta'_o - \eta'_e \cos\theta \end{pmatrix},$$

the asymmetry is the same for both initial spin states.

As we said above, $\delta_{o,e}$ is proportional to the matrix element for decay; in the somewhat symbolic way we have been writing things, we have

$$\delta_o = e^{i\zeta_o}\sqrt{\lambda_o}\; e^{-\frac{\lambda_o t}{2} + im_o t} \quad \text{(similarly for e), so that}$$

$$|\psi_{\pi N}|^2 = \lambda_o e^{-\lambda_o t} \text{ is the \underline{flux} of decay states: } \int |\psi|^2\, dt = 1.$$

The matrix element for decay is real, except for the phase shift due to final-state interaction between the π and N, just as in photoproduction (see Takeda, Phys. Rev. 101, 1547 (1956)); we here ignore the phases $\zeta_{o,e}$ considering them to be lumped in with the $\eta_{o,e}$, which are unknown anyway. The time dependence of the $\delta_{o,e}$ would be irrelevant if the lifetimes and masses of the Σ_o and Σ_e were equal. The qualitative effect of their inequality is clear: if one decays quickly and the other slowly, there won't be much interference between the two; if the mass difference is large compared with the decay width, the phase between the Σ_o and Σ_e states averages to zero, as discussed above.

If the angular distribution is written as $A + 2B\cos\theta$, the asymmetry, defined by

$$a \equiv \frac{(\text{no. fore}) - (\text{no. aft})}{(\text{no. fore})\quad(\text{no. aft})}\text{, equals } \frac{B}{A}\,.$$

We have

$$A(t) = |\omega_o|^2 + |\omega_e|^2 = |\eta_o|^2 \lambda_o e^{-\lambda_o t} + |\eta_e|^2 \lambda_e e^{-\lambda_e t}\,,$$

$$B(t) = \mathrm{Re}(\omega_o^* \omega_e) = \mathrm{Re}(\eta_o^* \eta_e e^{i\Delta t})\sqrt{\lambda_o \lambda_e}\; e^{-\bar{\lambda} t}$$

where

$$\Delta = E_o - E_e \approx m_{\Sigma_o} - m_{\Sigma_e}$$

$$\bar{\lambda} = \frac{\lambda_o + \lambda_e}{2}$$

Writing

$\phi = \arg \eta_e - \arg \eta_o$, we have

$$B(t) = |\eta_o \eta_e| \cos(\phi + \Delta t)\sqrt{\lambda_o \lambda_e}\; e^{-\bar{\lambda} t}\,.$$

These are fluxes; integrating over time gives us

$$A = |\eta_o|^2 + |\eta_e|^2 \equiv 1,$$

$$B = |\eta_o \eta_e| \frac{\sqrt{\lambda_o \lambda_e}\,(\bar{\lambda}\cos\phi - \Delta\sin\phi)}{\bar{\lambda}^2 + \Delta^2} = a\,.$$

~~Here~~ a is a maximum when $\eta_o = \eta_e$, $\lambda_o = \lambda_e$, and $\Delta = 0$: $a \leq \frac{1}{2}$; this is consistent with the intuitive discussion above, and with the fact that the asymmetry results from the interference between the reaction going through the two channels.

The Σ^- decays into $\pi^- n$; if we take the angle between the lines of flight of the π^- and the Σ to define the asymmetry, the question arises what choice of the angle for $\pi^+ n$ and $\pi^0 p$

will yield the same asymmetry? This question amounts to asking for the relative signs of the decay matrix elements; in terms of i-spin eigenstates we have

$$\psi_{\Sigma^-} \longrightarrow \delta_3 \, \psi_{\pi^- n}$$

$$\psi_{\Sigma^+} \longrightarrow \frac{\sqrt{2}\,\delta_3 - \delta_1}{3}\,\psi_{\pi^0 p} + \frac{\delta_3 + \sqrt{2}\,\delta_1}{3}\,\psi_{\pi^+ n} \, .$$

If the spin of the Σ is not $\frac{1}{2}$, the analysis is the same as above, except that in addition to the average over $m_j = \pm |m_j|$ (which, as above, is trivial: $|\psi^{m_j}|^2 = |\psi^{-m_j}|^2$) we must average over $|m_j| = \frac{1}{2}, \ldots, j$. For example, if $s_\Sigma = 3/2$, then we have

$\Sigma_{o(e)} \longrightarrow (\pi N)_{D}$ state
(P state).

For

$$|m_j| = 3/2 : \quad \psi^{3/2} = \sqrt{\frac{3}{2}}\,\omega_o^3 \begin{pmatrix} \sin\theta\cos\theta\, e^{i\phi} \\ \sin^2\theta\, e^{2i\phi} \end{pmatrix} + \sqrt{\frac{3}{2}}\,\omega_e^3 \begin{pmatrix} -\sin\theta\, e^{i\phi} \\ 0 \end{pmatrix}$$

$$|\psi^{3/2}|^2 = \frac{3}{2}\sin^2\theta \left[|\omega_o^3|^2 + |\omega_e^3|^2 - 2\,\mathrm{Re}(\omega_o^{3*}\,\omega_e^3)\cos\theta \right] .$$

For

$$|m_j| = \tfrac{1}{2} : \quad \psi^{\frac{1}{2}} = -\frac{3}{\sqrt{2}}\,\omega_o' \begin{pmatrix} \cos^2\theta - \frac{1}{3} \\ \sin\theta\cos\theta\, e^{i\phi} \end{pmatrix} + \sqrt{\frac{3}{2}}\,\omega_e' \begin{pmatrix} \cos\theta \\ \frac{-\sin\theta}{2}\, e^{i\phi} \end{pmatrix}$$

$$|\psi^{\frac{1}{2}}|^2 = \frac{3}{2}\Big[\big(|\omega_o'|^2 + \big|\sqrt{\tfrac{3}{4}}\,\omega_e'\big|^2 \big)(\cos^2\theta + \tfrac{1}{3})$$

$$+ 2\,\mathrm{Re}\big(\omega_o'^{*}\sqrt{\tfrac{3}{4}}\,\omega_e'\big)\big(\tfrac{5}{3} - 2\cos^2\theta\big)\cos\theta \Big] .$$