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- [4] H. Chen, T. K. Sarkar, S. A. Dianat, and J. Brule, "Adaptive spectral estimation utilizing the conjugate gradient method," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 272-284, Apr. 1986.
- [5] T. K. Sarkar and X. Yang, "Accurate and efficient solution of Hankel matrix systems by FFT and conjugate gradient methods," in *Proc. 1987 Conf. Acoust., Speech, Signal Processing*, Dallas, TX, pp. 1835-1838.
- [6] T. K. Sarkar and E. Arvas, "On a class of finite step iterative methods (conjugate directions) for the solution of an operator equation arising in electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. AP-33, no. 10, Oct. 1985.

A Discussion of *E*-Pulse Method and Prony's Method for Radar Target Resonance Retrieval from Scattered Field

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Abstract—An efficient version of the *E*-pulse method is discussed and generalized in the context of radar target pole retrieval from scattered field. Its relation to the generalized Prony's method is investigated. It is shown that the *E*-pulse method can be efficiently implemented via a smoothing and sampling procedure and a Prony's approach. The trade-off between noise insensitivity and computational burden influenced by various parameters is pointed out.

I. INTRODUCTION

Estimation of resonant frequencies or poles of radar target from its transient response has been under active investigation since the work by Kennaugh and Moffatt [4]. Because of their aspect independence, the resonant frequencies of a conducting object can be used as a signature of that object to discriminate it from others for the purpose of target identification (e.g., see [5]). Therefore, accurate (exact in noiseless case), robust (insensitive to noise) and fast techniques for estimating resonant frequencies from transient scattered field that contains the resonant nature of a conducting object are important. We should point out that the aspect dependent (complex) resonant amplitudes (including the phases) can be easily found by a linear least square approach (e.g., see [6]) after the resonant poles are obtained so that the transient response can be reconstructed.

Many methods have been found for the pole retrieval problem. The most well-known method is Prony's method which was applied first by Van Blaricum and Mittra [6], [7] to the electromagnetic transient problem. This method has also received a great attention in signal processing community. Study of this method has resulted in its improved and generalized versions [14] and understanding of their noise perturbation properties [12], [13]. Another well-known method for pole extraction is the pencil-of functions method (e.g., see [8] and [10]) whose application to radar transient waveform identification was presented initially by Sarkar *et al.* [8]. Recent applications of the concept of the pencil-of-functions method have been found in improved and generalized forms [9] and [11]. Following the above

two methods is the newly developed *E*-pulse approach [1]–[3] by Rothwell *et al.* Similar to the pencil-of-functions method as in [8], the *E*-pulse method is based on continuous signals, or in other words, the sampling process is automatically incorporated in the *E*-pulse method. However, Prony's method is based on sinusoidal sequence or uniformly sampled sinusoidal functions.

In this communication an efficient version (most efficient version, as we believe) of the *E*-pulse method is presented in a general and refined fashion and related to a generalized Prony's method. For convenience, this version will be referred to as the *E*-pulse method. It will be observed that the *E*-pulse method simply consists of a smoothing and sampling procedure and a generalized Prony's method.

II. *E*-PULSE METHOD AND PRONY'S METHOD

Without loss of generality the available scattered or late-time transient field of a target can be represented by the following sum of exponential functions:

$$y(t) = \sum_{i=1, M} b_i \exp(s_i t), \quad 0 \leq t \leq T_y \quad (1)$$

where s_i , $i = 1, 2, \dots, M$, are the distinct resonant poles of the target which are aspect independent. The real part of s_i is the damping factor σ_i . The imaginary part of s_i is the resonant frequency ω_i . b_i , $i = 1, 2, \dots, M$, are the aspect dependent complex residues or amplitudes whose phases depend on the origin reference ($t = 0$) of the measurement. M is the number of dominant resonances induced by an exciting field. One should note that in practice there are very weak or nondominant resonances existing in the excited field $y(t)$, and these components may be considered as part of noise added to $y(t)$. It should also be noted that s_i and b_i 's are in complex conjugate pairs since the field $y(t)$ is real. The simple pole nature as in (1) is a commonly used hypothesis (e.g., see [16]).

The *E*-pulse approach [1] to finding the poles s_i from $y(t)$ is basically to use a structured extinction-pulse (*E*-pulse) of finite duration T_E to match the measured scattered field $y(t)$ in the following way:

$$c(t) \equiv e(t) * y(t) = 0, \quad \text{for } T_E \leq t \leq T_y \quad (2)$$

where the asterisk denotes convolution. The structure of $e(t)$ should be such that the poles s_i can easily be retrieved from the parameters that determine $e(t)$. As is shown in [1], a necessary and sufficient condition for (2) to be true is that the Laplace transform $E(s) \equiv \mathcal{L}\{e(t)\}$ has zeros identical to the poles s_i . Without imposing any constraint on $e(t)$ or $E(s)$, the poles have to be located by an expensive search procedure. Although one may expect that there would exist an infinite number of structures of $e(t)$ such that the above requirements can be met, we shall consider a general class of structures of $e(t)$. We assume that

$$\begin{aligned} e(t) &= \sum_{n=0, N} e_n h(t - nT) \\ &= \sum_{n=0, N} e_n \delta(t - nT) * h(t) \end{aligned} \quad (3)$$

where $\delta(t)$ is the delta function, $h(t) = 0$ for $t < 0$, and $t > T_h$, and $NT + T_h = T_E$. The structure as in (3) is a linear structure and uniquely determined by the parameters e_n provided that N , T , and $h(t)$ for $0 \leq t \leq T_h$ are fixed. If $N = M$ and $T_h = T$, (3) is the "forced *E*-pulse" [3] version of (7) and (8) in [1]. Substituting (1) and (3) into (2) and performing some simple manipulation yields

$$c(t) = \sum_{i=1, M} b_i H(s_i) \left\{ \sum_{n=0, N} e_n \exp(-s_i nT) \right\} \exp(s_i t) \quad (4)$$

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where

$$H(s_i) = \int_{0 \leq t \leq T_h} h(t) \exp(-s_i t) dt. \quad (5)$$

Now one can see that if $H(s_i)$ is not zero for all i , which can be satisfied with probability one if $h(t)$ is randomly chosen, then $c(t) = 0$ for $T_E \leq t \leq T_y$ if and only if

$$\sum_{n=0, N} e_n \exp(-s_i T_n) = 0 \quad (6)$$

for $i = 1, 2, \dots, M$. As one may see, (6) for $N = M$ is the "forced E -pulse" version of (10) in [1]. It is a well-known property of polynomials that (6) can be satisfied by e_n if and only if $N \geq M$ (provided that $s_i, i = 1, \dots, M$, are distinct). It should be noted that $\max \{\text{Im}(s_i T)\}$ should be less than π to avoid ambiguity or aliasing, which sets a practical limitation for T .

Now we have shown that the structure of the E -pulse as in (3) is sufficient to have (2) satisfied. The next questions are, "how do we find e_n which determine the E -pulse $e(t)$ from the scattered field $Y(t)$ which is assumed to be the only information available?" and "how do we retrieve the resonant poles $z_i = \exp(s_i T)$ from e_n ?" We consider the second question first assuming that (6) is satisfied. It is obvious that if $N = M$ then the poles z_i are uniquely determined by the M roots of the polynomial:

$$P_1(z) \equiv \sum_{n=0, N} e_n z^{-n}. \quad (7)$$

However, if $N > M$, can we detect the M desired roots z_i from N roots of $P_1(z)$? In general it is not possible without introducing any further information or constraint on e_n . A sufficient constraint on e_n for detecting the M desired poles from e_n 's is to minimize the norm $|e_0|^2 + |e_1|^2 + \dots + |e_{N-1}|^2$ so that the following "backward" polynomial:

$$P_2(z) \equiv e_N + \sum_{n=1, N} e_{N-n} z^{-n} \quad (8)$$

has the M desired roots z_i^{-1} and $N - M$ extraneous roots that have magnitudes less than one (i.e., inside the unit circle). The proof of this property of the minimum norm polynomial is available in either [14] or [15]. Note that in most cases s_i has a negative real part so that z_i^{-1} is outside the unit circle and, therefore, separable from $N - M$ extraneous roots of $P_2(z)$.

To compute e_n with incorporation of the minimum norm property and the condition as in (6) which is equivalent to $c(t) = 0$ for $T_E \leq t \leq T_y$, we start by substituting (3) into (2) which yields

$$c(t) = \sum_{n=0, N} e_n y_n(t) = 0 \quad (9)$$

where $T_E \leq t \leq T_y$ and

$$y_n(t) = \delta(t - nT) * [h(t) * y(t)] \quad (10)$$

which is illustrated by Fig. 1. One can see that because of the uniform tapping of the filtered signal $x(t) \equiv h(t) * y(t)$ we had the polynomial equation (6) which is exactly the nature exploited by Prony's method from uniformly sampled data sequence. Equation (9) can be equivalently written as

$$[y_0(t) \ y_1(t) \ \dots \ y_{N-1}(t)] \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_{N-1} \end{bmatrix} = -e_N y_N(t) \quad (11)$$

Fig. 1. Equivalent sampling procedure coherent in E -pulse method. Note that $NT + T_h \leq T_0 \leq t_1 \leq \dots \leq t_{L-1} \leq T_y$.

where $T_E \leq t \leq T_y$. Note that e_N can be any nonzero value. The solution, or minimum norm solution for $N > M$, to (11) may be obtained in several ways. One way is to form the following normal equation from (11)

$$Ge = -e_N g \quad (12)$$

where

$$G = \begin{bmatrix} g_{0,0} & g_{0,1} & \dots & g_{0,N-1} \\ g_{1,0} & g_{1,1} & \dots & g_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N-1,0} & g_{N-1,1} & \dots & g_{N-1,N-1} \end{bmatrix} \quad (13)$$

$$g = [g_{0,N} \ g_{1,N} \ \dots \ g_{N,N}]^T \quad (14)$$

$$g_{i,j} = \langle y_i(t); y_j(t) \rangle = \int_{T_E \leq t \leq T_y} y_i(t) y_j(t) dt \quad (15)$$

$$e = [e_0 \ e_1 \ \dots \ e_{N-1}]^T. \quad (16)$$

Note that the rank of G is M as long as $N \geq M$. This property can be used to detect the number of resonant poles in $y(t)$. The minimum norm solution (i.e., $\|e\|^2 = |e_0|^2 + |e_1|^2 + \dots + |e_{N-1}|^2$ is minimized) to the linear equation (12) can be readily obtained with reasonable size ($N \times N$) of G (e.g., using IMSL software package).

Another way of solving (11) is to use sampled values of $y_n(t)$. Note that $y_n(t)$ is already a smoothed version of $y(t)$, and the inner products required in (13) and (14), if performed digitally, cause many more computations than the following procedure. Assume that $y_n(t)$, $T_E \leq t \leq T_y$, is sampled at $t = t_i, i = 0, 1, \dots, L-1$, where $L \geq M$. Then we have from (11)

$$Ye = -e_N y \quad (17)$$

where

$$Y = \begin{bmatrix} y_0(t_0) & y_1(t_0) & \dots & y_{N-1}(t_0) \\ y_0(t_1) & y_1(t_1) & \dots & y_{N-1}(t_1) \\ \vdots & \vdots & \ddots & \vdots \\ y_0(t_{L-1}) & y_1(t_{L-1}) & \dots & y_{N-1}(t_{L-1}) \end{bmatrix} \quad (18)$$

$$y = [y_N(t_0) \ y_N(t_1) \ \dots \ y_N(t_{L-1})]^T. \quad (19)$$

It can be shown that if and only if Y has rank M , or equivalently $N \geq M$ and $L \geq M$, (17) and (11) have identical minimum norm solution. For proof, see the Appendix. The minimum norm solution to (17) can also be readily obtained. Note that the singular value decomposition (SVD) [13] which can be used for minimum norm solution serves to reduce the noise effect which would increase the rank of Y . One would expect that the larger L is, the less sensitive to noise is the solution to (17). In fact, if L is very large (with $t_0 = T_E$ and $t_{L-1} = T_y$) then

$$Y^T Y \gamma \approx G$$

$$Y^T y \gamma \approx g$$

where γ (different from T) is a sampling interval as illustrated in Fig. 1, so that (17) becomes equivalent to (12).

Referring to (17)–(19) and Fig. 1, one can see that if $T = \gamma$ then the minimum norm solution to (17) together with the rooting of the “backward” polynomial $P_2(z)$ is identical to the generalized Prony’s method (see [14] and [17]) applied to the data obtained by uniformly sampling the smoothed signal $x(t)$. The generalized Prony’s method incorporates the backward linear prediction and lengthened polynomial. Detailed noise perturbation analysis of this method is available in [17]. It is found that the optimum order N (in the sense of least sensitiveness to noise) of the polynomial is between one third and one half of the total number of sampled data.

Finally one should note that the choice of $h(t)$, $0 \leq t \leq T_h$, should be such that the noise component in $x(t)$ is minimum as long as the computation is tolerated. However, large T_h which may reduce the noise component in $x(t)$ results in small N (provided that T and T_r are fixed) which may increase the sensitiveness of the estimated poles to the noise in $x(t)$. In [1], $h(t)$ was chosen to be a rectangular window of width equal to T . A reviewer brought [18] to the authors’ attention, where an attempt was made to choose T_E (with $N = M$) in a sense of least squares curve fitting.

III. CONCLUSION

An efficient version of the newly proposed E -pulse method by Rothwell *et al.* [1]–[3] for extracting resonance poles from scattered field is presented in a general and refined fashion. Its relation to Prony’s method is investigated. It is shown that this efficient version incorporates a smoothing and sampling process and the concept of Prony’s method. So some well understood properties of Prony’s method should help us understand some natures of the E -pulse method. The trade-offs between noise insensitiveness and computations influenced by various parameters (e.g., $h(t)$, T_h , N , γ and L) should be determined by applications.

APPENDIX

Using the decomposition of $[Y, y]$ to be given, one can easily show that

- 1) Y has rank M if and only if $N \geq M$ and $L \geq M$; and
- 2) for any given t ($T_E \leq t \leq T_r$), $[y_0(t), y_1(t), \dots, y_{N-1}(t), y_N(t)]$ is a linear combination of any M rows of $[Y, y]$ so that (11) and (17) have the same solution.

One can verify the following:

$$[Y, y] = E_1 B [E_2, e_3] \quad (20)$$

where

$$E_1 = \begin{bmatrix} \exp(s_1 t_0) & \exp(s_2 t_0) & \cdots & \exp(s_M t_0) \\ \exp(s_1 t_1) & \exp(s_2 t_1) & \cdots & \exp(s_M t_1) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(s_1 t_{L-1}) & \exp(s_2 t_{L-1}) & \cdots & \exp(s_M t_{L-1}) \end{bmatrix} \quad (21)$$

$$B = \text{diag}[b_1, b_2, \dots, b_M] \quad (22)$$

$$E_2 = \begin{bmatrix} 1 \exp(-s_1 T) & \cdots & \exp(-s_1(N-1)T) \\ 1 \exp(-s_2 T) & \cdots & \exp(-s_2(N-1)T) \\ \vdots & & \vdots \\ 1 \exp(-s_M T) & \cdots & \exp(-s_M(N-1)T) \end{bmatrix} \quad (23)$$

$$e_3 = [\exp(-s_1 NT) \exp(-s_2 NT) \cdots \exp(-s_M NT)]^T. \quad (24)$$

REFERENCES

- [1] E. J. Rothwell, K. M. Chen, and D. P. Nyquist, “Extraction of the natural frequencies of a radar target from a measured response using E -pulse techniques,” *IEEE Trans. Antennas Propagat.*, vol. AP-35, no. 6, pp. 715–720, June 1987.
- [2] E. J. Rothwell, K. M. Chen, D. P. Nyquist, and W. Sun, “Frequency domain E -pulse synthesis and target discrimination,” *IEEE Trans. Antennas Propagat.*, vol. AP-35, no. 4, pp. 416–434, April 1987.
- [3] E. Rothwell, D. P. Nyquist, K. M. Chen, and B. Drachman, “Radar target discrimination using the extinction-pulse technique,” *IEEE Trans. Antennas Propagat.*, vol. AP-33, no. 9, pp. 929–936, Sept. 1985.
- [4] E. M. Kennaugh and D. L. Moffatt, “Transient and impulse response approximations,” *Proc. IEEE*, vol. 53, pp. 893–901, Aug. 1965.
- [5] A. J. Perni, “Target identification by natural resonant estimation,” *IEEE Trans. Aerospace Electron. Syst.*, vol. AES-11, no. 2, pp. 147–154, Mar. 1975.
- [6] M. Van Blaricum and R. Mittra, “A technique for extracting the poles and residues of a system directly from its transient response,” *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 777–781, Nov. 1975.
- [7] A. J. Poggio, M. L. Van Blaricum, E. K. Miller, and R. Mittra, “Evaluation of a processing technique for transient data,” *IEEE Trans. Antennas Propagat.*, vol. AP-26, no. 1, pp. 165–173, Jan. 1978.
- [8] T. K. Sarkar *et al.*, “Suboptimal approximation/identification of transient waveform from electromagnetic systems by pencil-of-functions method,” *IEEE Trans. Antennas Propagat.*, vol. AP-28, no. 6, pp. 928–932, Nov. 1980.
- [9] A. J. Mackay and A. McCowen, “An improved pencil-of-functions method and comparisons with traditional methods of pole extraction,” *IEEE Trans. Antennas Propagat.*, vol. AP-35, no. 4, pp. 435–441, Apr. 1987.
- [10] V. K. Jain, T. K. Sarkar, and D. D. Weiner, “Rational modeling by the pencil-of-function method,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 564–573, June 1983.
- [11] Y. Hua and T. K. Sarkar, “Generalized pencil-of-functions method for extracting the poles of an electromagnetic system from its transient response,” *IEEE Trans. Antennas Propagat.*, vol. 37, no. 2, pp. 229–234, Feb. 1989.
- [12] —, “Further analysis of three modern techniques for poles retrieval from data sequence,” in *Proc. 30th Midwest Symp. Circuits Syst.*, Aug. 1987, Syracuse, NY.
- [13] —, “Perturbation analysis of TK method for harmonic retrieval problems,” *IEEE Trans. Acoust., Speech Signal Processing*, vol. ASSP-36, no. 2, pp. 228–240, Feb. 1988.
- [14] R. Kumaresan, “Estimating the parameters of exponentially damped or undamped sinusoidal signals in noise,” Ph.D. dissertation, Univ. Rhode Island, 1982.
- [15] S. W. Lang and J. H. McLellan, “A simple proof of stability for all-pole linear prediction models,” *Proc. IEEE*, vol. 67, no. 15, pp. 860–861, May 1979.
- [16] L. Marin, “Natural-mode representation of transient scattered fields,” *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 809–817, Nov. 1973.
- [17] Y. Hua, “On techniques for estimating parameters of exponentially damped/undamped sinusoids in noise,” Ph.D. dissertation, Syracuse Univ., 1988.
- [18] E. J. Rothwell and K. M. Chen, “A hybrid E -pulse/least squares technique for natural resonance extraction,” *Proc. IEEE*, vol. 76, no. 3, pp. 296–298, Mar. 1988.

An Improved Approximation to the Fresnel Integral

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Abstract—The accuracy of a Fresnel-integral approximation is improved by adding an exponential term. Errors in the amplitude compo-

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