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Impact of Void Ratio and State Parameters on the Small Strain Shear Modulus of Unsaturated Soils

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ABSTRACT

The unsaturated small strain shear modulus, G_{max}, is a key reference value in predicting relationships between dynamic shear modulus and shear strain amplitude and is thus a key quantity to properly model the behavior of dynamically-loaded geotechnical systems such as pavements, rail beds, and machine foundations. From the interpretation of the experimental G_{max} results for unsaturated soils, different definitions of trends between G_{max} and the stress state of the unsaturated soils and material properties are proposed. However, in most of trends, the relationship between the stress state and void ratio is considered and the effect of void ratio on the unsaturated small strain shear modulus is not fully investigated. In the study presented herein, G_{max} data published in the technical literature for two different types of unsaturated soils are critically reviewed with the goal of identifying trends with path-dependent stress state and void ratio. The literature data is also used to evaluate the reliability of an existing approach in predicting the small strains shear modulus of unsaturated soils under different loading conditions.

Keywords: Small strain shear modulus, unsaturated soil, void ratio, mean effective stress, double hardening, hydraulic hysteresis

1. INTRODUCTION

The progression of stress waves through the soil with time in the case of earthquake ground shaking or machine foundations can be predicted using solutions to the wave equations (Kramer 1996), and the corresponding strains in the soil can be estimated using constitutive modelling. In either case, the analysis depends on some representative material properties, which are the dynamic properties of soils (e.g. shear modulus and damping ratio) and the Poisson's ratio. The dynamic properties of soils have been studied theoretically and experimentally for several years under both saturated (Hardin and Black 1968, 1969; Hardin and Drnevich 1972; Hardin 1978; Iwasaki et al. 1978; Stokoe et al. 1999) and unsaturated conditions (Cabarkapa et al. 1999; Mancuso et al. 2002; Inci et al. 2003; Marinho et al. 1995; Vassallo et al. 2007; Sawangsuriya et al. 2009, Khosravi and McCartney 2009; Ng et al. 2009; Khosravi et al. 2010; Khosravi and McCartney 2011; Khosravi and McCartney 2012). Of particular interest has been to understand the shear modulus of unsaturated soils at shear strain amplitudes less than 10^{-6} (elastic range of strain) which is defined as the small strain shear modulus, G_{max}. Based on the results presented in these studies, G_{max} of unsaturated soils is dependent on different variables, such as state of stress, void ratio, soil grain characteristics (shape, size, mineralogy), and degree of saturation S_r. Based on the definition of stress state considered for analysis and from the interpretation of G_{max} results, two general forms of predictive relationships for G_{max} have been used in literature. In one form of equations (Inci et al. 2003; Sawangsuriya et al. 2009; Khosravi and McCartney 2009; Khosravi et al. 2010), the single-value mean effective stress definition proposed by Bishop (1959) for unsaturated soils is incorporated into the expression of G_{max} proposed by Hardin and Black (1968) and a relationship for the small strain shear modulus of unsaturated soils was developed as follows:

$$G_{max} = A\, f(e) p'^{n} \tag{1}$$

where *f*(e) is the void ratio function, A and n are fitting parameters that can be defined by fitting Eq. (1) to a set of G_{max} and p′ is the unsaturated mean effective stress defined as:

$$p' = p_n + \chi\psi \tag{2}$$

where p_n is the mean net confining stress defined as the difference between total mean stress and pore air pressure ($p_n = p - u_a$), and ψ is the suction. For low suction magnitudes (less than 300 kPa), ψ is equal to the matric suction, which is the difference between the pore air pressure and pore water pressure ($\psi = u_a - u_w$). For higher suction magnitudes, the total suction should be considered in this equation to incorporate the effects of osmotic suction. The term χ is the effective stress parameter, which ranges from 0 (for dry soils) to 1 (for saturated soils) and has been defined using different approaches proposed in literature (Khalili and Khabbaz 1998; Wheeler et al. 2003; Sivakumar 1993; Gallipoli et al. 2003; Lu et al. 2010).

Another form of equations used the concept of independent stress state variables to define the value of G_{max} along the drying and wetting paths of the SWRC in by considering the effects of the mean net stress and suction independently (Mancuso et al. 2002; Mendoza et al. 2005; Oh and Vanapalli 2009; Ng et al. 2009; Sawangsuriya et al. 2009), as follows:

$$G_{\max} = Af(e)p_n{}^{n} \quad + B\psi \tag{3}$$

where A and B are the model parameters describing the rate of change of G_{max} with respect to the mean net stress and matric suction, respectively, n is a material dependent fitting parameter and *f*(e) is a void ratio function. When fitting Eqs. (1) and (3) to experimental G_{max} data, the relationship for *f*(e) is typically incorporated empirically by considering that e and p′ are uncoupled or defined by Hardin and Black (1969) and Hardin (1978) for saturated soils, as shown in Fig. (1).

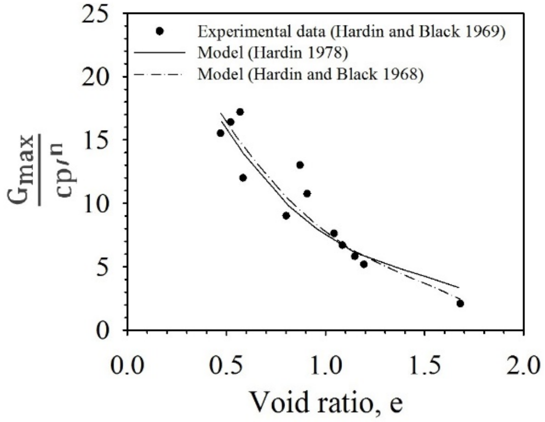

Fig. 1. Evaluation of empirical relationships between void ratio and normalized G_{max} for saturated clays (Khosravi 2011)

Khosravi and McCartney (2012) argued the void ratio is closely linked to hydro-mechanical loading through elasto-plastic constitutive models and indicated that more research into f(e) is needed for unsaturated soils, where the behavior of soils may change upon wetting and drying due to hydraulic hysteresis.

2. BACKGROUND: A SEMI-EMPIRICAL RELATIONSHIP FOR G_{MAX} OF UNSATURATED SOILS

Khosravi and McCartney (2012) incorporated the concept of double hardening for unsaturated soils introduced by Wheeler et al. (2003) and Tamagnini (2004) into the G_{max} expression presented by Hardin (1978) for saturated soils to describe changes in G_{max} of unsaturated soils during two coupled physical processes: plastic compression, arising from slippage between the particles, and the hydraulic process of water flow during wetting and drying and developed a semi-empirical relationship for G_{max} of unsaturated soils as follows:

$$G_{\max} = AP_a\left[\frac{p_c{'}_0}{p_n}\exp\left(\frac{\Delta e^p}{\lambda-\kappa}\right)\right]^K \left[\frac{p_n}{p'}\exp\left(b[S_{e0}-S_e]\right)\right]^{K'}\left(\frac{p'}{P_a}\right)^n \tag{4}$$

where P_a is the atmospheric pressure, A and n are stress dependency parameters, $p_c{'}$ is the mean apparent preconsolidation stress (i.e., the mean yield stress), p′ is the mean effective stress, p_n is the net stress, Δe^p is a plastic change in void ratio, λ and κ are the slopes of the virgin compression and the elastic rebound curves, respectively, K′ and K are hardening constant, $p_c{'}_0$ is the initial mean apparent preconsolidation stress, b is referred to as the double-hardening parameter which governs the rate of change in $p_c{'}$ caused by changes in soil saturation, S_e is the effective saturation which is defined as:

$$S_e = \frac{S_r - S_{r,res}}{1 - S_{r,res}} \tag{5}$$

and S_{e0} is the initial effective saturation. In Eq. (5), S_r and $S_{r,res}$ are the values of S_r at current and residual saturation conditions.

Khosravi and McCartney (2012) validated their model against experimental data under different values of mean net stress and matric suction, and the model was found to fit well with the experimental data. However, the specimens mostly stayed on the elastic unloading-reloading curve of e-p' throughout the tests so the effect of void ratio on the measured SWRC and G_{max} relationship was not fully investigated. In the study presented herein, the contribution of void ratio to the small strain shear modulus at unsaturated state is further investigated by re-interpreting experimental results of two soils reported in the literature. The literature data is also used to examine the validity of Eq. (4) for a wider spectrum of soil types and effective stress.

3. EXPERIMENTAL RESULTS

In this study, two soils including quartz silt

(Cabarkapa et al. 1999), and clayey silt (Ng. et al. 2008; Ng et al. 2009) were identified from the literature for which data on the SWRC, void ratio and G_{max} were available. The SWRC measurements of corresponding soils are presented in Fig. (2) and some of their characteristics are summarized in Table 1.

Table 1. Soil Properties

Experimental data	Soil type	G_s	w_l (%)	w_p (%)	D_{50} (mm)	C_u	C_c
Cabarkapa et al. (1999)	quartz silt	2.67	31	0	0.02	-	-
Ng et al. (2008, 2009)	clayey silt	2.73	43	14	-	4.55	0.61

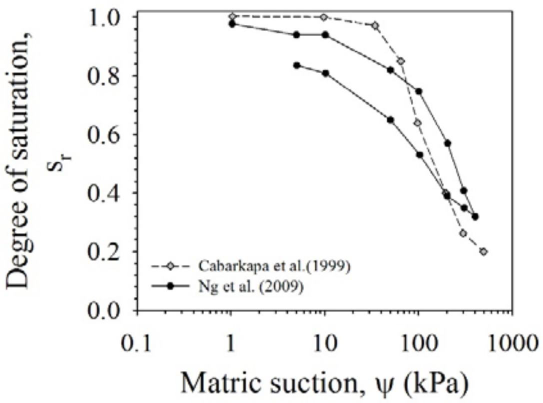

Fig. 2. The SWRC measurements of soils analyzed in this study

Figs. (3) and (4) describe the effect of void ratio and various state parameters on the small strain shear modulus of different unsaturated soils. The state parameters are the matric suction and mean effective stress, p'. The mean effective stress employed in this study was defined using an approach similar to that used by Lu et al. (2010), Khosravi and McCartney (2012), and Haeri et al. (2014) as follows:

$$p' = p_n + S_e \times \psi$$
(6)

This equation is similar to Bishop's (1959) single-value effective stress variable where the effective stress parameter χ is equal to S_e. The value of S_e is obtained from Eq. (5) and using the SWRCs for each soil presented in Fig. 2. The normalized void ratio in Figs. (3) and (4) is defined as e/e_0, where e_0 is the void ratio corresponding to a mean net stress of 25 kPa for quartz silt and 110 kPa for clayey silt and the mean effective stress was defined using.

Evaluation of the isotropic compression curves in Figs. (2a) and (3a) indicate that all of the soil specimens exhibit a nonlinear decrease in volume with increasing mean effective stress, as expected. However, the void ratio measurements of the soil specimens subjected to higher levels of suction were consistently higher for the same values of effective stress than those in low suction testing. This observation indicates a hardening response in the specimens as a result of suction increase. This hardening response leads to an increase in G_{max} increase during drying.

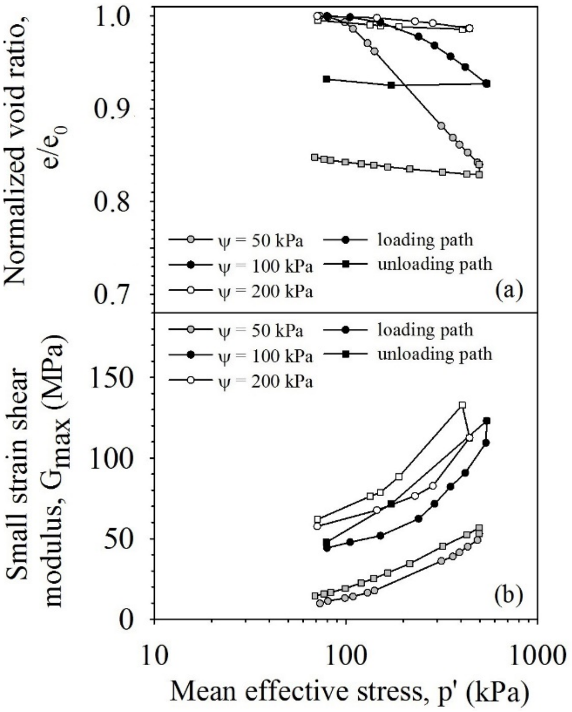

Fig. 3. Variation in (a) void ratio and (b) G_{max} with p' for quartz silt (Cabarkapa et al. 1999)

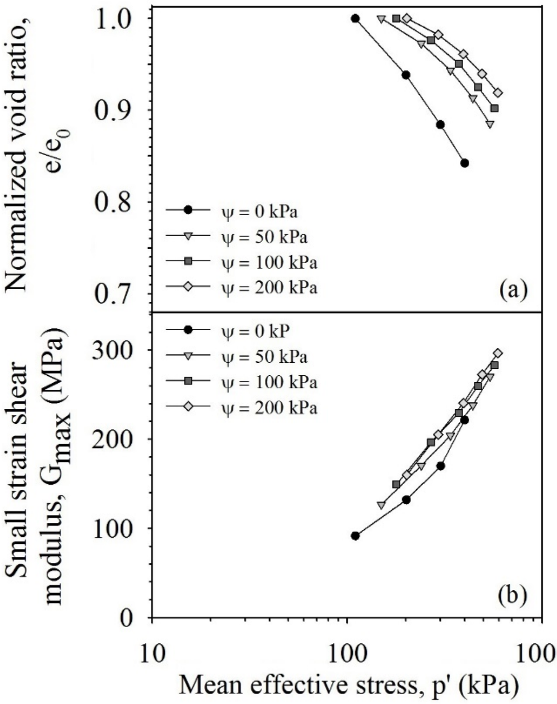

Fig. 4. Variation in (a) void ratio and (b) G_{max} with p' for clayey silt (Ng et al. 2008)

During the loading process, the G_{max} measurements follow an increasing path with p' increase. However, the rate of changes in G_{max} varied depending on the magnitude of the applied mean effective stress and suction. The rate of changes in G_{max} was lower at p' less than the mean apparent preconsolidation stress where the specimens experienced smaller volume change during loading. However, after the mean effective stress exceeded the mean apparent preconsolidation stress during loading, G_{max} increased at a greater rate with increasing p'. Therefore, it may be concluded that both e-log(p') and G_{max}-log(p') curves are almost composed of two linear sections with the intersection near the mean apparent preconsolidation stress. During unloading, the small strain shear modulus followed a decreasing path with p' decrease. However, the rate of changes in G_{max} during unloading was different from that during loading and a greater shear modulus was measured along the unloading path. It was also noted that the value of G_{max} was not fully recovered once the initial applied effective stress was reached.

In this study, the effect of hydraulic hysteresis on G_{max} of unsaturated soils was also investigated using the results of bender element tests which were conducted by Ng et al. (2008) at different stress state conditions (Fig. 5). The SWRCs of the tested specimens are shown in Figure 5(a) and the variations of G_{max} with suction are presented in Figure 5(b).

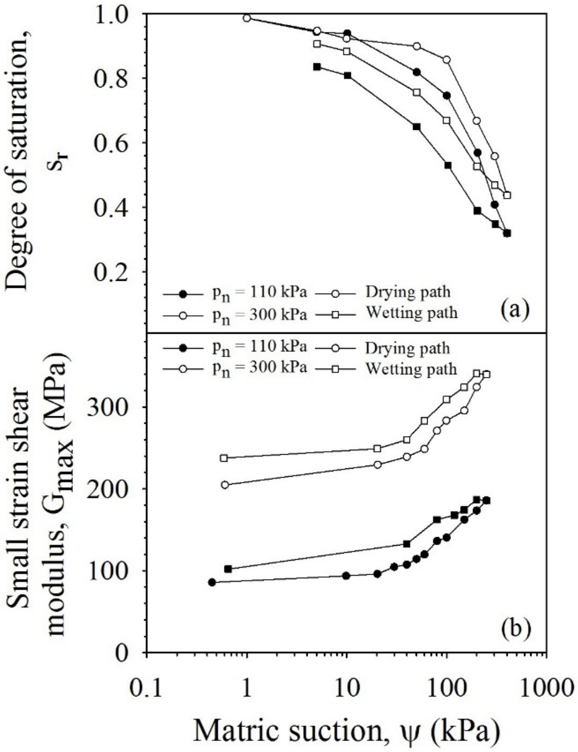

Fig. 5. Variation in (a) degree of saturation and (b) G_{max} with matric suction for clayey silt under different total stresses (Ng et al. 2009)

As observed in Figure 5, and also as noted by other researchers (Ng. et al. 2009; Khosravi and McCartney 2012), G_{max} follows an increasing path with increasing matric suction. However, the rate of changes in G_{max} for the different soils was lower at suctions below the air entry value. During wetting, G_{max} decreased as matric suction decreased, with the greatest reduction between the water-entry value and the air-expulsion value, where the soil started to absorb greater amount of water.

4. ANALYSIS

The measurements of unsaturated small strain shear modulus reported in the literature were also used to assess the validity of the proposed relationship by Khosravi and McCartney (2012). Experimental G_{max} data is shown in Fig. (6) in the effective stress space along with associated predictive relationships for G_{max} following different paths of hydro-mechanical loadings. The model parameters required to predict the variation of G_{max} were obtained following the methodology proposed by Khosravi and McCartney (2012).

The value of K for each soil was defined using guidance from Hardin (1978), and the fitting parameters, A and n, were determined from fitting a curve to G_{max} data at zero matric suction (saturation condition) under different mean net stresses. The hardening parameters K' and b were determined from the results of G_{max} tests along the drying path of the SWRC at a constant p_n using least squares minimization. Table 2 summarizes the model parameters for different soils which were used in this study. The data in this figure indicate that the model shows a good fit with the data for the particular fitting values presented in Table 2. Evaluation of the results in Fig. 6 indicated that there are still some discrepancies between the data and the model and additional tests under different stress state conditions are recommended to further evaluate the reliability of the proposed approach.

Table 2. Model parameters required to solve the evolution of G_{max} for different soils

Experimental data	K	K′	A	n	b
Cabarkapa et al. (1999)	0.252	0.13	0.314	0.96	2.3
Ng et al. (2008, 2009)	0.138	0.743	0.0879	0.72	1.37

5. CONCLUSION

G_{max} is an important parameter to properly model the behavior of geotechnical systems under dynamic loading. This study aimed to improve our understanding of the trend between G_{max} of unsaturated soils with void ratio and state parameters using the data from literature. The results presented in this study reflected the relative impacts of e, S_r and p' on G_{max}. Similar to e vs. log(p') curves, the G_{max} trends showed a bilinear behavior in the log(p') space with the intersection near the mean apparent preconsolidation stress. From these trends, it may be concluded that changes in G_{max} of unsaturated soils correspond highly to changes in the void ratio and accordingly, trends established based on an uncoupled behavior between e and G_{max} may not be fully representative of unsaturated soil behavior.

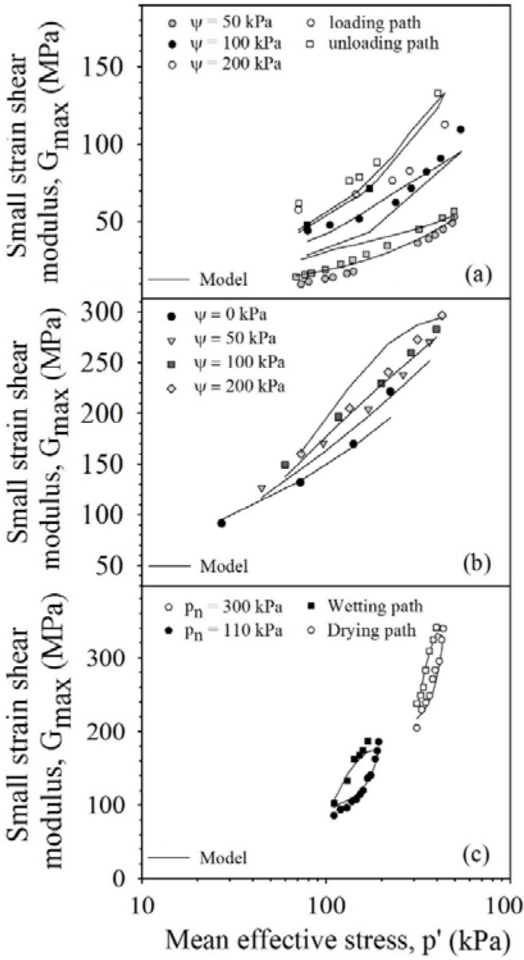

Fig. 6. Relationships between G_{max} and mean effective stress of different soils during (a) isotropic loading (Cabarkapa et al. 1999); (b) isotropic loading (Ng et al.2008); and (c) hydraulic hysteresis (Ng et al. 2009) predicted using parameters presented in Table 2

The small strain shear modulus data of unsaturated soils obtained from the literature was also used to assess the reliability of a semi empirical approach in predicting G_{max} of unsaturated soils during hydro-mechanical loading. The model was observed to provide adequate prediction of the G_{max} data upon different stress paths.

REFERENCES

1) *Bishop, A.W. (1959). "The principle of effective stress." Teknisk Ukeblad I Samarbeide Med Teknikk 106(39), 859–863.*

2) *Cabarkapa, Z., Cuccovillo, T., and Gunn, M. (1999). "Some aspects of the pre-failure behaviour of unsaturated soil." Proc. 2nd Int. Symp. Pre-failure Deform. Character. of Geomat., Torino, Italy, Vol. 1, 159–165.*

3) *Gallipoli, D., Gens, A., Sharma, R., and Vaunat, J., 2003, "An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behavior." Géotechnique, 53(1), 123–135.*

4) *Haeri, S. M., Garakani, A., Khosravi, A., Meehan, C.L., 2014. "Assessing the hydro-mechanical behavior of collapsible soils using a modified triaxial test device." ASTM Geotechnical Testing Journal. 37(2), 190-204.*

5) *Hardin, B.O. (1978). "The nature of stress strain behavior of soils." Earthquake Engineering and Soil Dynamics. 1, 3–90.*

6) *Hardin, B.O., and Black, W.L. (1968). "Vibration modulus of normally consolidated clay." J. Soil Mech. Found. Div. 94(2), 353–369.*

7) *Hardin, B.O., and Black, W.L. (1969). "Vibration modulus of normally consolidated clay: Closure." J. Soil Mech. Found. Div. 95(6), 1531–1537.*

8) *Hardin B. O. and Drnevich, V. P. (1972). "Shear modulus and damping in soils: measurement and parameter effects." J. Soil Mech. Found. Eng. Div. 98(6), 603–624.*

9) *Inci, G., Yesiller, N., and Kagawa, T. (2003). "Experimental investigation of dynamic response of compacted clayey soils." ASTM Geotech. Test. J. 26(2), 125–141.*

10) *Iwasaki, T., Tatsuoka, F. & Yoshikazu, T. (1978). "Shear moduli of sands under cyclic torsional shear loading." Soils and Foundations. 18(1), 39-56.*

11) *Khalili, N., and Khabbaz, M. H., (1998). "A unique relationship for the determination of the shear strength of unsaturated soils." Géotechnique. 48(5), 681–687.*

12) *Khosravi, A., and McCartney, J. S. (2009). "Impact of stress state on the dynamic shear modulus of unsaturated, compacted soils." 4th Asia- Pacific Conf. Unsat. Soils. Newcastle, Australia. 1-6. (CD-ROM)*

13) *Khosravi, A., Ghayoomi, M., McCartney, J.S., and Ko, H.-Y. (2010). "Impact of effective stress on the dynamic shear modulus of unsaturated sands." GeoFlorida 2010 (GSP 199). Feb. 20-24. ASCE. 410-419.*

14) *Khosravi, A. and McCartney, J.S. (2011). "Resonant column test for unsaturated soils with suction–saturation control." ASTM Geotechnical Testing Journal. 34(6), 730-739.*

15) *Khosravi, A. and McCartney, J.S. (2012). "Impact of hydraulic hysteresis on the small-strain shear modulus of unsaturated soils." ASCE Journal of Geotechnical and Geoenvironmental Engineering. 138(11), 1326–1333.*

16) *Kramer S.L., (1996). Geotechnical Earthquake Engineering. Prentice Hall, New Jersey.*

17) *Lu, N., Godt, J., and Wu, D., 2010, "A closed-form equation for effective stress in unsaturated soil." Water Res., Vol. 46, W05515.*

18) *Mancuso, C., Vassallo, R., and d'Onofrio, A. (2002). "Small strain behavior of a silty sand in controlled-suction resonant column-torsional shear tests." Can. Geotech. J. 39(1), 22–31.*

19) *Marinho, F.A.M., Chandler, R.J., and Crilly, M.S. (1995). "Stiffness measurements on an unsaturated high plasticity clay using bender elements." Proc. 1st Int. Conf. Unsat. Soils, A.A. Balkema, Rotterdam, Netherlands, Vol. 2, 535–539.*

20) *Mendoza, C.E., Colmenares, J.E., and Merchan, V.E. (2005). "Stiffness of an unsaturated compacted clayey soil at very small strains." Conf. Adv. Exp. Unsat. Soil Mech., Trento, Italy, 199–204.*

21) *Ng, C.W.W. and Yung. S,Y. (2008). Determination of the anisotropic shear stiffness of an unsaturated decomposed soil. Géotechnique. 58(1):23–35.*

22) *Ng, C. W. W., Xu, J., and Yung, S. Y. (2009). "Effects of*

imbibition-drainage and stress ratio on anisotropic stiffness of an unsaturated soil at very small strains." Can. Geotech. J., 46(9), 1062–1076.

23) Oh, W. T., and Vanapalli, S. K. (2009). "A simple model for predicting the shear modulus of unsaturated sandy soils." 4th Asia-Pacific Conf. Unsat. Soils. Newcastle, Australia. 1-6. (CD-ROM).

24) Sivakumar, V., (1993). "A Critical State Framework for Unsaturated Soil," Ph.D. thesis, University of Oxford, Oxford, UK.

25) Stokoe, K. H., II, Darendeli, M. B., Andrus, R. D., and Brown, L. T. (1999). "Dynamic soil properties: Laboratory, field and correlation studies." 2nd Int. Conf. Earthquake Geotech. Eng., Lisbon, Portugal. Vol. 3.

26) Sawangsuriya, A., Edil, T. B., and Bosscher, P. J. (2009). "Modulus-suction- moisture relationship for compacted soils in post compaction state." J. Geotech. Geo environ. Eng., 132(2), 131–142.

27) Tamagnini, R. (2004). "An extended cam-clay model for unsaturated soils with hydraulic hysteresis." Géotechnique, 54(3), 223–228.

28) Vassallo, R., Mancuso, C., and Vinale, F. (2007). "Effects of net stress and suction history on the small strain stiffness of a compacted clayey silt." Can. Geotech. J., 44(4), 447–462.

29) Wheeler, S.J., Sharma, R.S., and Buisson, M.S.R. (2003). "Coupling of hysteresis and stress–strain behaviour in unsaturated soil." Géotechnique, 53(1), 41–54.