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Yielding of Silt at High Temperature and Suction Magnitudes

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Abstract:	<p>This study presents an evaluation of yielding mechanisms for unsaturated, compacted silt using drained triaxial compression tests with control of elevated temperatures and high suction magnitudes. After anisotropic compression, some compacted silt specimens were heated by approximately 40 °C before a suction of approximately 300 MPa was applied, while others were heated after suction application. A frictional response was observed for the specimens sheared under high suction magnitudes, in the form of a consistent increase in peak shear strength with increasing net confining stress. An effective stress analysis was used to evaluate the trends in the peak shear stress and the role of stress history for the different specimens. A single peak failure envelope was observed when the shear strength data was interpreted in terms of the mean effective stress. Changes in preconsolidation stress were estimated by identifying the intersections between a thermo-elasto-plastic yield function and the experimental peak shear strength values. Soil specimens heated before application of high suction values had lower peak shear strengths than reference specimens at high suction and ambient temperature. This behaviour is consistent with thermal softening trends observed in soils heated under low suction values. However, soil specimens heated after suction application had greater peak shear strengths than the reference specimens. This indicates heating under high suction results in hardening. The impact of suction on the preconsolidation stress was found to be better represented by a power law model at high suction magnitudes than other available models. The estimated preconsolidation stress values were used to evaluate the impacts of stress history on the thermal volume change response, which matched well with data from tests on saturated specimens.</p>	
Response to Reviewers:	<p>Comment 1 :</p> <p>The paper presents interpretation of results from suction and temperature controlled triaxial testing of unsaturated silt. While the experimental results have been published previously by the authors, the interpretation here is original and valuable. The paper is very well written and presented. Background information is covered nicely. Recommendation is to publish. Authors should consider the following comments in a</p>	

revision.

Reponse 1: We thank the reviewer for the thorough evaluation of the paper. We have carefully addressed the reviewer's comments, and we believe that they have improved the presentation of the paper. We have also carefully reviewed the entire manuscript to check for any editorial issues.

Comment 2: Abstract, line 6: the phrase "even though" carries connotation that is not made clear in the abstract. Please clarify.

Response 2: We added a clarification sentence in the abstract as follows: "A frictional response was observed for the specimens sheared under high suction magnitudes, in the form of a consistent increase in peak shear strength with increasing net confining stress."

Comment 3: Why silt? It would be helpful to provide some rationale for selection of the particular soil tested.

Response 3: Here are some sentences that explain our choice of soil and the preparation details: "This soil was selected as it has low permeability and water retention over a wide range of suctions, but does not have diffuse-double layer effects expected in clay soils." and "The reason for selecting such a low compaction water content was to prepare specimens with a low enough initial degree of saturation such that their air permeability would be high enough to permit rapid gas flow through the specimen in the vapor flow technique, leading to faster suction equilibration."

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Yielding of Silt at High Temperature and Suction Magnitudes

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Abstract

This study presents an evaluation of yielding mechanisms for unsaturated, compacted silt using drained triaxial compression tests with control of elevated temperatures and high suction magnitudes. After anisotropic compression, some compacted silt specimens were heated by approximately 40 °C before a suction of approximately 300 MPa was applied, while others were heated after suction application. A frictional response was observed for the specimens sheared under high suction magnitudes, in the form of a consistent increase in peak shear strength with increasing net confining stress. An effective stress analysis was used to evaluate the trends in the peak shear stress and the role of stress history for the different specimens. A single peak failure envelope was observed when the shear strength data was interpreted in terms of the mean effective stress. Changes in preconsolidation stress were estimated by identifying the intersections between a thermo-elasto-plastic yield function and the experimental peak shear strength values. Soil specimens heated before application of high suction values had lower peak shear strengths than reference specimens at high suction and ambient temperature. This behaviour is consistent with thermal softening trends observed in soils heated under low suction values. However, soil specimens heated after suction application had greater peak shear strengths than the reference specimens. This indicates heating under high suction results in hardening. The impact of suction on the preconsolidation stress was found to be better represented by a power law model at high suction magnitudes than other available models. The estimated preconsolidation stress values were used to evaluate the impacts of stress history on the thermal volume change response, which matched well with data from tests on saturated specimens.

Keywords

Unsaturated soil; shear strength; temperature; high suction; yield stress

List of notation

a	is an empirical parameter of the power law model
b	is an empirical parameter of the power law model
M_w	is the molecular mass of water vapour
M_{CSL}	is the slope of the critical state line
M_{peak}	is the slope of the peak failure envelope
N	is a reference specific volume point on the virgin compression line
p	is the mean total stress
p'	is the mean effective stress
p'_c	is the mean effective preconsolidation stress
p'_{co}	is the initial mean effective preconsolidation stress
p_n	is the mean net stress
p'_{peak}	is the mean effective stress at peak failure
q_{peak}	is the principal stress difference at peak failure
R_h	is the relative humidity of the pore air in decimal form
R	is the universal (molar) gas constant
r	is the spacing ratio parameter of the Uchaipichat (2005) model
r_a	is a parameter of the Alonso et al. (1990) model
T	is the temperature
u_w	is the pore water pressure
u_a	is the pore air pressure

v_{κ}	is a reference specific volume point on the recompression line
α	is an empirical parameter of the Uchaipichat (2005) model
α_{GS}	is a parameter of the Grant and Salezadeh (1996) model
β	is a parameter of the Grant and Salezadeh (1996) model
β_a	is a parameter of the Alonso et al. (1990) model
ΔT	is the change in temperature
λ	is the slope of the virgin compression line
λ_{GS}	is a parameter of the Grant and Salezadeh (1996) model
κ	is the slope of the recompression line
ρ_w	is the density of water
σ_s	is the suction stress
σ'	is the effective stress
σ	is the total stress
σ_n	is the net stress
ψ	is the suction
ψ_{aev}	is the air entry suction
Ω	is an effective stress scaling parameter in the model of Khalili and Khabbaz (1998)

1. Introduction

Many types of thermally-active geotechnical systems involve application of elevated temperatures and high suction magnitudes to soils above the water table, including ground-source heat exchangers (Preene and Powrie 2009), energy piles (Brandl 2006; Laloui et al. 2006; Adam and Markiewicz 2009; Bourne-Webb et al. 2009; Murphy et al. 2015), heat dissipation embankments (McCartney 2012; Coccia and McCartney 2012; Stewart et al. 2014), containment systems for nuclear waste (Gens et al. 1998; Kanno et al. 1999), and backfills for buried electrical cables (Abdel-Hadi and Mitchell 1981; Brandon et al. 1989). Specifically, when heat is injected into these thermally-active geotechnical systems, thermally-induced water flow may cause unsaturated soil surrounding the heat exchangers to dry to the point that very high suction magnitudes may be encountered. During the coupled heat transfer and water flow processes, changes in the yield stress in the soil specimen may occur that will affect the deformation response of these systems to external or self-weight loading.

Data available on the thermo-hydro-mechanical response of compacted silt under low suction magnitudes (less than 300 kPa) indicates that application of suction will lead to a hardening effect, resulting in an increase in preconsolidation stress and a corresponding increase in stiffness and peak shear strength (Alonso et al. 1990; Lloret et al. 2003). However, application of elevated temperatures has been observed to lead to a softening effect on unsaturated soils, resulting in a decreasing the preconsolidation stress and a corresponding decrease in peak shear strength (Uchaipichat and Khalili 2009). However, temperature effects on the behaviour of

unsaturated soils under suctions on the order of hundreds of MPa may differ. Alsherif and McCartney (2014b) found that application of high suction magnitudes led to a hardening effect as expected, but Alsherif and McCartney (2015) found that the path of testing (i.e., the sequence of suction and temperature application) led to differing behavior from reference tests on unsaturated soils at room temperature. Soils heated under as-compacted (low suction) conditions before application of suction had lower peak shear strength while soils heated after application of suction showed higher shear strength. Despite the path dependency, Alsherif and McCartney (2014b, 2015) observed that the effective stress principle is still valid in unsaturated soils under high suction magnitudes and nonisothermal conditions similar to unsaturated soils under low suction magnitudes (Khalili and Khabbaz 1998; Lu et al. 2010). The primary objective of this paper is to better understand the impact of temperature on the hardening mechanisms of compacted, unsaturated silt specimens under high suction magnitudes by further interpretation of the shear strength and volume change results presented by Alsherif and McCartney (2014b, 2015). Understanding the impacts of changes in effective stress, yield stress, and volume change permits interpretation of the changes in soil behaviour for different testing paths.

2. Background

2.1 Thermal Volume Change Behaviour of Soils

Thermal effects on the volume change behaviour of saturated and unsaturated soils are complex due to coupling between temperature, suction and stress history. Accordingly, the approaches used by investigators include an evaluation of soil behaviour during changes in the temperature of a specimen under a constant effective stress state, or evaluation of the soil mechanical behaviour at various temperatures under isothermal conditions. A summary of the conclusions drawn from the different studies on the impact of temperature on the volume change behaviour of saturated soils is as follows:

- The compression index of soils is not sensitive to temperature for saturated soils (Campanella and Mitchell 1968; Eriksson 1989; Graham et al. 2001) and unsaturated soils (Saix et al. 2000; Uchaipichat and Khalili 2009).
- Heating of normally consolidated or lightly overconsolidated soils (overconsolidation ratio OCR less than 2) under constant effective stress and drained conditions leads to volumetric contraction (Paaswell 1967; Campanella and Mitchell 1968; Plum and Esrig 1969; Baldi et al. 1988; Hueckel and Baldi 1990; Towhata et al. 1993; Boudali et al. 1994; Delage et al. 2000, 2004), and thus a reduction in the void ratio. A greater amount of contraction is noted with increasing plasticity index (Demars and Charles 1982).
- Heating of heavily overconsolidated soils under constant effective stress and drained conditions leads to volumetric expansion (Demars and Charles 1982; Baldi et al. 1988; Hueckel and Baldi 1990; Belanteur et al. 1997; Delage et al. 2000, 2004; Sultan et al. 2002; Cekeravac and Laloui 2004), although heating of these soils above a threshold temperature

may lead to contraction (Hueckel and Baldi 1990; Delage et al. 2000, 2004; Cekeravac and Laloui 2004).

- Multiple cycles of heating and cooling may lead to a gradual accumulation of permanent volumetric contraction in soils regardless of the stress history due to thermal creep or kinetic thermal hardening (Campanella and Mitchell 1968; Burghignoli et al. 1992; Vega and McCartney 2014).

The volume change behaviour of unsaturated soils during heating has been investigated in several studies (Romero et al. 2003; Francois et al. 2007; Salager et al. 2008; Tang et al. 2008; Uchaipichat and Khalili 2009). Tang et al. (2008) performed thermo-mechanical tests on unsaturated, heavily-compacted MX80 bentonite following a wetting path from an initial suction of 110 MPa to a target suction of 9 MPa. The results from tests at constant confining stress showed that heating led to expansion under low values of confining stress and high suction, and led to contraction at high values of confining stress and low suction. Uchaipichat and Khalili (2009) performed drained heating tests on silt specimens at matric suctions up to 300 kPa and net confining stresses up to 200 kPa, and found that heavily overconsolidated specimens experienced reversible thermal expansion with introducing larger irreversible thermal contraction at lower overconsolidation ratio. Further, for a given net mean stress, the amount of thermal contraction increased with increasing suction. Although an effective stress analysis was not performed to calculate the OCR in either of these studies, their results indicate that stress history plays an important role in the thermal response of unsaturated soils in a similar manner to that of saturated soils.

2.2 Yielding of Unsaturated Soils under Nonisothermal Conditions

The effect of suction and/or temperature on the compression response of unsaturated soils in terms of the compressibility and the preconsolidation pressure has been studied by many researchers (Lloret et al. 2003; Blatz et al. 2005; Jotisankasa 2005; Francois et al. 2007; Ghembaza et al. 2007b; Tang et al. 2008; Uchaipichat and Khalili 2009; Mun and McCartney 2014). These studies indicate that an increase in suction leads to a hardening effect corresponding to an increase in the apparent preconsolidation (or yield) stress. The experimental results from the literature shown in Figure 1(a) indicate that the suction hardening continues to increase with increasing suction up to relatively high suction values. One of the early functional relationships between matric suction and preconsolidation stress in the suction mean net stress plane was introduced by Alonso et al. (1987) as the loading collapse (LC) curve. The area within the LC curve is considered an elastic zone, while any changes in suction or stress reaching the LC curve produce irreversible compressive volumetric strains.

Most studies on saturated soils and unsaturated soils under low suction magnitudes (less than 300 kPa) indicate that heating leads to a reduction in the apparent preconsolidation (or yield stress). The results in Figure 1(b) consistently show a log-linear decrease in yield stress with

1 increasing temperature. Cui et al. (2000) defined relationship between temperature and
2 preconsolidation stress (plotted in terms of mean net stress plane) as the thermal yield (TY)
3 curve, in a similar manner to the LC curve. In this approach, any changes in temperature
4 beyond the yield curve will lead to irreversible contraction. However, no experimental results
5 have confirmed the thermal softening behaviour of unsaturated soils under high suction
6 magnitudes.
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10 **2.3 Effect of Temperature on the Shear Strength of Soils**

11 The influence of temperature change on the shear strength and stress-strain characteristics of
12 saturated soils have been studied by several investigators. Houston et al. (1985) performed
13 consolidated undrained (CU) tests on saturated, normally consolidated specimens of Illite clay
14 at different temperatures, and observed that the pore water pressure increased and effective
15 stress decreased during heating. However, after drainage of these thermally-induced excess
16 pore water pressures, the peak shear strength of the soil was greater than at low temperatures.
17 Hueckel and Baldi (1990) performed drained triaxial compression tests on overconsolidated
18 specimens of Pontida silty clay, and observed a decrease in peak shear strength with
19 temperature. This was attributed to a reduction in the size of the yield locus (and consequently a
20 reduction in the overconsolidation ratio), but it may also be related to the increase in void ratio
21 during heating of the overconsolidated soil. The uniqueness of the critical state line (CSL) was
22 also investigated and confirmed by Hueckel and Baldi (1990) and Graham et al. (1995). These
23 studies also observed that temperature has a negligible effect on the critical state line.
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34 Uchaipichat and Khalili (2009) evaluated the shear strength and volume change of unsaturated
35 compacted silt under non-isothermal conditions for suction magnitudes less than 300 kPa. For a
36 given suction magnitude, they observed a decrease in peak shear strength with increasing
37 temperature. However, they observed that changes in matric suction led to greater changes in
38 the peak and critical state strength values than changes in temperature. Further, the shear
39 strength at critical state conditions was unaffected by temperature, similar to the conclusions
40 drawn for saturated clays. Similar reductions in peak shear strength and increases in shear
41 strength with suction were observed by Wiebe et al. (1998) and Ghembaza et al. (2007b).
42 Wiebe et al. (1998) prepared specimens of sand-bentonite at different degrees of saturation
43 using compaction at different water content, which could have led to different soil structures, but
44 Ghembaza et al. (2007b) incorporated the vapour equilibrium technique into a triaxial cell to
45 perform triaxial compression tests on normally consolidated, unsaturated sandy clay under a
46 temperature of 80°C and a suction value of 8.5 MPa.
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55 **3. Experimental Technique**

56 **3.1 Testing Apparatus**

57 This study uses the data from Alsherif and McCartney (2014a, 2014b, 2015) to evaluate
58 hardening mechanisms in unsaturated soils during temperature and suction changes. These
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studies used a specially-designed thermo-hydro-mechanical triaxial cell to measure the shear strength of unsaturated soils under elevated temperatures and high suction magnitudes. The vapour flow technique developed by Likos and Lu (2003) was employed in the triaxial cell to control the total suction in the specimen, along with a temperature control system to apply elevated temperatures. The vapour flow technique involves passing gas with known relative humidity in the specimen until the relative humidity of the air passing out of the specimen reaches the same value. The total suction in the specimen can then be calculated from the relative humidity using Kelvin's law, as follows:

$$\psi = \frac{\rho_w RT}{M_w} \ln(R_h)$$

1.

The molecular mass of water vapour, M_w , is assumed equal to 18.016 g/mol, and the universal (molar) gas constant, R , is equal to 8.31432 J/molK, ρ_w is the density of water, T is the temperature in kelvin, R_h is the relative humidity in percent. Electrical resistance heating elements inside the triaxial cell were used to reach a target elevated temperature and a flow pump was used to circulate water within the cell to ensure accurate and uniform temperature distributions. The mechanical loading system was adapted to operate in load-control conditions during suction application or temperature changes or in displacement-control condition during shearing of the soil specimen. A detailed description of the testing apparatus can be found in Alsherif and McCartney (2014b; 2015).

3.2 Specimen Preparation and Testing Procedures

The silt used in this study is Bonny silt, which has a USCS classification of ML and has a specific gravity of 2.65. This soil was selected as it has low permeability and water retention over a wide range of suctions, but does not have diffuse-double layer effects expected in clay soils. The model of Grant and Salehzadeh (1996) was used to evaluate the impact of temperature on the SWRC, as shown in Figure 2. The parameters of the model for room temperature were defined by fitting the model to a set of data defined using the axis translation and vapour flow techniques reported by Alsherif and McCartney (2014b). Heating causes the SWRC to shift to the left, corresponding to a reduction in water retention. Alsherif and McCartney (2015) attributed this shape of the SWRC to evaluate the behaviour of soils following the T-S path.

All of the specimens evaluated in the study were compacted to a dry unit weight of 15.7 kN/m³ and an initial degree of saturation of 0.41. The initial conditions for the specimens evaluated in the triaxial compression testing program are presented in Table 1. The low compaction water content was selected so that the initial degree of saturation was low enough to have continuous air voids through the specimen so that the vapour flow technique could be employed. The

compression response of unsaturated Bonny silt under as-compacted conditions was assessed using an oedometer test, with loading increments only applied until reaching the end of primary consolidation at each stress level. A preconsolidation stress of 400 kPa was estimated from the compression curve for an unsaturated specimen compacted to the target condition specified in this study.

After assembly of the triaxial cell, the tests all start with consolidation of the soil specimens under as-compacted conditions in anisotropic conditions ($K_0 = 0.5$) with confining stresses of 100, 200 and 300 kPa. Next, three different testing paths were performed as illustrated in Figure 3. The first set of tests was performed by applying a high suction magnitude to soil specimens under ambient laboratory temperature (approximately 23 °C). The second set of tests were performed by heating soil specimens to a target cell fluid temperature of 65 °C in a single stage under as-compacted conditions, then applying a high suction magnitude (T-S Path tests). The third set of tests were performed by applying a high suction magnitude to the soil specimens under ambient temperature, then heating them in three stages to target cell fluid temperatures of 35, 50, and 65 °C (S-T Path tests). Finally, a fourth type of test was performed following the S-T path but was cooled back to ambient temperature after heating. After following any of the testing paths in Figure 2, the soil specimens were sheared at a constant displacement rate of 1.27×10^{-4} m/min. During shearing, the pore air was permitted to drain freely. More details on the test procedures can be found in Alsherif and McCartney (2014b).

4. Results

Details on the suction equilibrium process and the shear stress-strain curves for the specimens tested at different temperatures and suctions following different paths are presented in Alsherif and McCartney (2014b; 2015), but a summary of the results is presented in Table 1. The drained failure envelopes for the specimens tested at ambient temperature and those tested following the T-S and S-T paths are shown in Figure 4(a). The peak shear strength increases linearly with net confining stress, and the total friction angle appears to not be affected by temperature or suction. However, the apparent cohesion in this total stress plot was affected by changes in both temperature and suction. The results in Figure 4(a) indicate that silt specimens tested following T-S path experienced thermal softening that led to lower peak shear strength values than those measured in tests on specimens at high suction magnitudes performed under ambient temperature. This behavior was consistent with that expected in the literature for overconsolidated soils. The same figure also indicates that the specimens tested following the S-T path consistently have a greater peak shear strength than the specimens tested at ambient temperature and suction of 291 MPa. Alsherif and McCartney (2015) proposed that this unexpected behaviour was attributed to the lower degree of saturation of the specimens tested following S-T path, and it is hypothesized that the degree of saturation has a different effect on the stress state at high suction magnitudes near residual saturation conditions than for nearly-saturated soils. For the test performed following the S-T path with a heating-cooling cycle, the

peak shear strength was found to be slightly higher than that from the test performed at the same confining stress and suction that had not experienced heating. This gain in peak shear strength can be attributed to the slight permanent contraction upon cooling. Because the peak shear strength increases with the confining pressure, the compacted silt specimens under high suction magnitudes show a frictional response, despite the high suction magnitude and brittle stress-strain curves reported in Alsherif and McCartney (2015).

The peak shear strength values plotted as a function of the mean effective stress calculated are shown in Figure 4(b). Details and discussion on how the effective stress was defined in this study using the suction stress characteristic curve (SSCC) concept can be found in Alsherif and McCartney (2014b, 2015). Regardless of the temperature and testing path, the peak shear strength values define a single peak failure envelope having a slope of $M = 2.1$. The slope of this peak failure envelope was found to be greater than the slope of the critical state line defined from tests on saturated specimens reported in the same Figure 4(b), which confirms that the brittle failure mechanism reported by Alsherif and McCartney (2014b) prevented the specimens from reaching critical state. Nonetheless, the fact that the peak shear strength of unsaturated specimens tested at different net normal stresses and high suction magnitudes and temperatures converge on a single failure envelope in mean effective stress space supports the validity of the effective stress principle for these conditions. The behaviour of the specimens at high suctions is consistent with the behaviour of heavily overconsolidated soils, which reach a peak shear strength value when the effective stress path intersects the steady-state boundary surface.

5. Analysis and Discussion

5.1 Characterization of the Preconsolidation Stress

The thermo-mechanical constitutive model developed by Uchaipichat (2005) was used to interpret the impact of temperature on the yield surface and the preconsolidation stress for the compacted unsaturated silt specimens evaluated in this study due to its incorporation of suction and effective stress state into the yield surface. As test results in this study can be interpreted in terms of mean effective stress, the constitutive relationships for saturated and unsaturated conditions are expected to be similar. The parameters used for modelling the yield locus and preconsolidation stresses for saturated and unsaturated compacted Bonny silt are listed in Table 2. The value of M was defined from isotropic triaxial compression tests on saturated specimens, and the values of λ and k were defined from the oedometer compression test. The preconsolidation stresses at failure for the different triaxial compression tests performed in this study were defined by assuming that the measured values of peak shear strength define points on the steady-state boundary surface ($q_{\text{peak}}, p'_{\text{peak}}$). Specifically, all three tests on specimens having the same suction, temperature and testing path are expected to fall on the same steady-state boundary surface. The preconsolidation stress p'_c can be estimated by fitting the model of

Uchaipichat (2005) to the experimental data points. The value of p'_c can be defined by satisfying the following equality:

$$\frac{(p' - (p'_c / r))^2}{((1 - 2/r)p' + (p'_c / r))^2} + \frac{q^2}{((1 - \alpha)Mp' + (\alpha/r)Mp'_c)^2} = 1$$

2.

In this equation, the spacing ratio parameter r is equal to p'_c/p'_{CSL} and the value of α was selected to be 0.7 to fit the experimental data. This value of α is similar to the value recommended by Uchaipichat (2005) for frictional soils.

The effects of suction on the yield locus for tests at ambient temperature are shown in Figure 5(a). The preconsolidation stress was defined so that the peak shear strength values intersect with the steady-state boundary surface. Although the shape of the yield surface is not particularly sensitive to the preconsolidation stress for low net stress values, the expansion of the yield surface reflects the trends in the peak shear strength values well. Similarly, the effects of temperature on the yield locus using Equation 2 for soil specimens tested following the T-S and S-T paths are shown in Figure 5(b). Assuming that temperature primarily controls the preconsolidation stress, the results of this analysis indicate that the yield locus shrinks with increasing temperature for specimens tested following the T-S path and expands with increasing temperature for specimens following the S-T path. Because all of the tests define a single peak failure envelope, the hypothesis of Hueckel et al. (2009) that temperature could affect the value of the slope of the CSL was not considered.

5.2 Constitutive Modelling of the Preconsolidation Stress

The most widely used elasto-plastic constitutive model for unsaturated soils by Alonso et al. (1990) was used to evaluate the role of suction hardening on the preconsolidation stress. Alonso et al. (1990) proposed that, under isotropic stress states, the shape of the LC yield curve in the suction: mean effective stress plane, and the development of this shape during yield curve expansion is expressed by the following equation:

$$\left(\frac{p'_c(\psi)}{p_0} \right) = \left(\frac{p'_c(0)}{p_0} \right)^{[\lambda(0)-k]/[\lambda(\psi)-k]}$$

3.

where $p'_c(\psi)$ is the isotropic yield value of mean stress at any general target suction ψ ; $p'_c(0)$ is the yield value at zero suction, p_0 is a reference stress, $\lambda(0)$ and k are the slopes of the normal

compression and recompression lines defined at zero suction and the parameter $\lambda(\psi)$ is assumed to vary exponentially with suction as follows:

$$\lambda(\psi) = \lambda(0)[(1 - r_a)\exp(-\beta_a\psi) + r_a]$$

4.

where r_a and β_a are soil constants. The experimental yield points measured from the compression tests performed at room temperature in this study for compacted Bonny silt along with the fitted LC curve by the model of Alonso et al. (1990) is shown in Figure 6(a). The value of the parameter r_a was suggested by Alonso et al. (1990) to be less than 1 where $\lambda(\psi)$ decreased with increasing suction. However, the parameters r_a and β_a in Figure 6(a) indicate values of 15 and 0.6 MPa⁻¹ respectively. This value of r_a is consistent with the observation by Wheeler et al. (2002), where they indicated that a value of $r_a > 1$ is possible for some soils and $\lambda(\psi)$ increases with increasing suction. The LC curve of Alonso et al. (1990) also implies that after a certain suction value is reached, there is no further hardening and that was indicated by flattening the fitting line at a high suction value of around 5000 kPa. This implication contradicts the behaviour shown by the experimental data from the literature reported in Figure 1(a) for soils under high suction values where a continuous hardening is observed with increasing suction. To overcome this limitation, a power law model is proposed to represent the LC curve over the entire range of suction as follows:

$$p'_c = a\psi^b$$

5.

where a and b are soil fitting parameters. The results of using the power law model with its fitting parameters shown in Figure 6(a) indicates that this model predicts a continuous hardening with suction increase up to 300 MPa suction value, which is consistent with the experimental data from tests performed at high suction values reported in Figure 1(a). To validate the effectiveness of using the power law model, the model was fitted to the Bonny silt data at ambient temperature, as shown in Figure 6(b). The data from Figure 1(a) is also shown for comparison, which confirm that the power law model captures the continuous increase in preconsolidation stress with suction.

In order to interpret the OCR of unsaturated soils, the impacts of suction on both the effective stress as well as the preconsolidation stress needs to be considered. The effective stress can be characterized by defining the suction stress term in the effective stress equation by Lu and Likos (2006) using the model of Khalili and Khabbaz (1998), as follows:

$$\sigma_s = \psi \left\{ \frac{\psi}{\psi_{aev}} \right\}^{-\Omega}$$

6.

where ψ is the suction, ψ_{aev} is the air entry suction, and Ω is an effective stress scaling parameter. Khalili and Khabbaz (1998) found that a value of $\Omega = 0.55$ fits well for most experimental data.

Using the predictive equations for effective stress (Equation 6) and preconsolidation stress (analysis in Figure 5), the changes in effective stress, preconsolidation stress as a function of suction are shown in Figure 7 for a specimen tested at a confining stress of 200 kPa. The results in Figure 7 indicate that during application of a suction of 291 MPa, the increase in preconsolidation stress is much greater than the increase in the mean effective stress. The OCR for the soil calculated as the ratio of preconsolidation stress to mean effective stress is also shown in Figure 7. The OCR is observed to increase nonlinearly during application of suction. The effects of suction on the preconsolidation stress for different temperatures and testing paths is shown in Figure 8 using the power law model to fit the data. As only a simple point was measured on the curves for the T-S and S-T paths, the shape of the curve was assumed to be the same as that at ambient temperature. The testing paths were observed to lead to a change in the exponent b (which affects the asymptote value), but not a change in the value of a (which affects the curvature). The OCR values are not shown in this figure for the different testing paths because it is not possible to assess the change in OCR of the specimen during heating following the T-S path because the preconsolidation stress before suction application is not known. However, it is expected to be lower than the value of 400 kPa measured in the oedometer test due to the effects of thermal softening.

5.3 Impacts of Temperature and Suction on the Void Ratio-Stress Paths

Using the understanding of the effective stress in unsaturated soil, along with the changes in the preconsolidation stress with suction and temperature, the effects of suction and temperature on the effective stress path in specific volume v versus the natural logarithm of the mean effective stress can be explored. In this analysis, the slopes of the CSL, recompression line (RCL), and virgin compression line (VCL) are assumed to be independent of temperature in effective stress space, but the VCL may shift to the left or right depending on path-dependent hardening or softening mechanisms. The parameters of the CSL, RCL, and VCL used in this evaluation are summarized in Table 2.

The effective stress paths for a test following the T-S path performed at a confining stress of 200 kPa is shown in Figure 9(a). In this figure, the change in the specific volume was calculated using the void ratios presented in Table 1 and reported in Alsherif and McCartney (2015), while

the effective stress was calculated using the suction stress calculated using Equation 6. For the specimen following the T-S path, a small volumetric contraction without a change in effective stress was observed during drained heating, which was followed by an increase in effective stress and a large volumetric contraction during suction application. A small dilation was observed during shearing, along with an increase in the mean effective stress during drained triaxial compression. Although the decrease in effective stress due to temperature effects on the SWRC is likely the reason for the lower shear strength in the T-S path test, it is also possible that a greater amount of softening on the preconsolidation stress occurred due to the larger degree of saturation for these tests.

The effective stress path for a specimen following the S-T path performed at a confining stress of 200 kPa is shown in Figure 9(b). The specific volume was calculated in the same way as in Figure 6(a), but the effective stress was calculated by assuming that the air entry value for the specimen following the S-T path increased. The effective stress path indicates that the specimen following the S-T path experienced an elastic contraction during initial confinement followed by a large plastic contraction during suction application. Application of a high suction also results in an increase in the mean effective preconsolidation stress, as shown in Figure 6. A small dilation was observed during shearing, along with an increase in the mean effective stress during drained triaxial compression.

The estimated values of effective stress and preconsolidation stress at failure were used to estimate the overconsolidation ratio (OCR) of the unsaturated specimens. This permitted evaluation of the role of stress history on the ratio of the thermal volume change at equilibrium to the corresponding change in temperature ($\Delta\varepsilon_v^T/\Delta T$) for unsaturated Bonny silt, as shown in Figure 10. The data for saturated Bonny silt reported by Vega and McCartney (2014) for multiple cycles of heating and cooling are shown in this figure for comparison. The trend in the saturated and unsaturated data for Bonny silt is consistent, and also follows the trends of the thermal volume change data of Uchaipichat and Khalili (2009) for compacted silt that was reinterpreted in terms of OCR by Stewart et al. (2014). During drained heating, the saturated and unsaturated specimens of Bonny silt expand under low mean effective stresses (high OCR), and contract under higher mean effective stresses (low OCR).

6. Conclusion

The results from drained triaxial compression tests on specimens of compacted silt were evaluated to investigate the impact of temperature on the hardening mechanisms associated with application of high suction magnitudes and elevated temperatures. Further, the results were evaluated to interpret the effective stress paths of the soils following different testing paths (sequence of suction and temperature application). The results emphasize the need to consider the path-dependent hardening and softening behaviour of unsaturated soils subjected to high suction magnitudes and elevated temperatures. A power law model was found to best represent

the increase in yield stress of unsaturated soils over a wide range of suction, both for soil data presented in this study as well as data from the literature. The preconsolidation stress values for the soil specimens were inferred from identifying the intersection of the peak shear strength values and a yield surface. The preconsolidation stress was found to increase with temperature for tests heated after suction application compared to reference tests, but decrease with temperature for specimens heated under as-compacted conditions before suction application. The changes in effective stress and preconsolidation stress with suction and temperature were found to be useful in interpreting the effective stress paths and volume change of the unsaturated soil specimens. Using the overconsolidation ratio defined using these variables, the thermal axial strains for the unsaturated silt specimens normalized by the change in temperature were observed to be similar in magnitude and follow a similar decreasing trend to those observed for saturated silt specimens.

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Table 1. Summary of the initial conditions and results from compression triaxial tests on unsaturated Bonny silt specimens at ambient and elevated temperatures

Test path	Suction ψ (MPa)	Temp. at bottom shearing T (°C)	Rel. hum. at bottom R_h (%)	Conf. stress at consol. σ_3 (kPa)	Void ratio at shear e_s	Princ. stress diff. $\sigma_1 - \sigma_3$ (kPa)	Exp. suction stress σ_s (kPa)	Initial mean net stress p_{net} (kPa)	Initial mean effective stress p'_0 (kPa)	Mean effective stress at peak p'_f (kPa)	Final void ratio e_f	Final degree of sat. S_r
Room	162	22.7	30.5	100	0.572	1965	202	113	128	937	0.576	0.10
	161	22.8	30.6	200	0.503	2679	202	247	261	1275	0.507	0.11
	165	23.8	29.8	300	0.508	3364	202	380	390	1603	0.513	0.12
Room	289	22.9	12.0	100	0.48	2392	263	113	128	1140	0.49	0.05
	292	23.2	11.8	200	0.49	3142	263	247	261	1490	0.51	0.08
	289	22.6	12.0	300	0.39	3563	263	380	390	1731	0.40	0.06
T-S	317	64.0	12.5	100	0.56	2235	235	113	128	1060	0.56	0.06
	317	64.0	12.5	200	0.52	2986	235	247	261	1410	0.53	0.06
	311	63.9	13.0	300	0.48	3408	235	380	390	1651	0.49	0.06
S-T	291	63.9	14.9	100	0.53	2707	313	113	128	1295	0.54	0.04
	290	63.9	15.0	200	0.468	3480	313	247	261	1653	0.469	0.03
	289	63.9	15.1	300	0.36	4135	313	380	390	1971	0.37	0.04

Table 2. Summary of the parameters for the yield locus of Uchaipichat (2005) and the Cam-clay model parameters for compacted Bonny silt

Parameter	Value
M_{CSL}	1.23
α	0.6
r	1.4
λ	0.11
κ	0.03
$N_{initial}$	2.33
$v_{\kappa, initial}$	1.86
$N_{suction-TS}$	2.54
$v_{\kappa-suction-TS}$	1.82
$N_{heat-ST}$	2.5
$v_{\kappa-heat-ST}$	1.76

Figure 3

Figure 8

Figure 10