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Sensor Deployment with Limited Communication Range in Homogeneous and Heterogeneous Wireless Sensor Networks

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Abstract—We study the heterogeneous wireless sensor networks (WSNs) and propose the necessary condition of the optimal sensor deployment. Similar to that in homogeneous WSNs, the necessary condition implies that every sensor node location should coincide with the centroid of its own optimal sensing region. Moreover, we discuss the dynamic sensor deployment in both homogeneous and heterogeneous WSNs with limited communication range for the sensor nodes. The purpose of sensor deployment is to improve sensing performance, reflected by distortion and coverage. We model the sensor deployment problem as a source coding problem with distortion reflecting sensing accuracy. However, when the communication range is limited, a WSN is divided into several disconnected sub-graphs under certain conditions as we will discuss in this paper. In such a scenario, neither the conventional distortion nor the coverage represents the sensing performance as the collected data in disconnected sub-graphs cannot be communicated with the access point. By defining an appropriate sensing performance measure, we propose a Restrained Lloyd (RL) algorithm and a Deterministic Annealing (DA) algorithm to optimize sensor deployment in both homogeneous and heterogeneous WSNs. Our simulation results show that both DA and RL algorithms outperform the existing algorithms when communication range is limited.

Index Terms—Sensor deployment, homogeneous, heterogeneous, source coding, coverage.

I. INTRODUCTION

As a bridge between the physical world and the virtual information world, wireless sensor networks (WSNs) collect data from the physical world and communicate it with the virtual information world, such as computers. Proper sensor deployment improves monitoring and controlling the physical environment. To accomplish their tasks, WSNs should address two needs: (i) Sensing in the target area and (ii) Communication between the sensor nodes. WSNs are utilized to collect physical information, such as temperature, humidity, voice and so on. But, the collected data is useless if it cannot be transmitted to the access point (AP) and to the outside information world through the AP node. When sensors are connected by wire lines, the connectivity is provided automatically. On the other hand, the connectivity of WSNs is not guaranteed. In this paper, we consider the sensing and connectivity together and redefine the goal of WSN design

accordingly. In particular, we develop a general model for both homogeneous and heterogeneous WSNs.

A huge body of literature exists on the topic of sensor deployment. Coverage as an important performance measurement in WSNs has been deeply studied in the literatures [1]–[18]. One survey [4] classifies coverage into three types: area coverage, barrier coverage, and target coverage in terms of the subject to be detected. Furthermore, according to mobility, sensors can be classified as static sensors and mobile sensors [4]. In this paper, we focus on area detection and mobile sensors. To evaluate sensing quality, the binary coverage model [6]–[12] and the probabilistic model [13]–[18] are widely used in WSNs. In the binary coverage model, sensors can only detect the points within a range of R_s . The range R_s is called the sensing range. In the probabilistic coverage model, the probability that a sensor detects an event depends on the distance between them. When the number of sensors is large enough to cover the whole sensing region, the coverage degree in [19], [20] is used to evaluate sensing performance. On the contrary, when the sensing region is too large to be covered by the given sensors, the coverage area is used as the performance measurement [6]. In [6], the authors treat every point in the sensing field equally. However, in real world, some regions are more important or dangerous than others. To consider such differences, a density function [21] reflecting the point importance should be taken into account. Even if the entire region is covered, sensors have different sensing accuracies at different points in the covered region. In fact, sensing capability diminishes as the distance increases [3], [21]–[29]. To be more realistic, the authors in [23] study the relationship between sensing capability and sensing distance by analyzing the signal attenuation characteristics of radar sensors. Distortion, as an important parameter in source coding, models the sensing accuracy via a function of distance [21], [27]. One can minimize distortion in WSNs through vector quantization techniques in [30], [31]. Lloyd algorithm is one of the tools to minimize distortion in homogeneous WSNs [21]. The convergence of the Lloyd algorithm has been studied in [32]–[34].

Connectivity is another important requirement in WSNs. The binary communication model has been widely used in [19]–[21], [35]–[38] to deal with connectivity. In this model, each sensor node has a limited communication range that affects the connectivity of the network [37], [38]. The Critical Sensor Density (CSD) in [2] is the number of nodes per unit area required to provide full coverage when the com-

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munication range is limited. Moreover, [19], [20] take both coverage and connectivity into consideration. A geometric analysis of the relationship between coverage degree and connectivity is proposed in [20]. In [35], the authors have come up with some deployment patterns to achieve both desired coverage degree and full connectivity. The distributed sensor deployment algorithms, such as Lloyd Algorithm, can converge to a deployment without connectivity. However, to the authors' best knowledge, neither minimizing distortion nor maximizing coverage area with connectivity has been studied. When sensor nodes are divided into several disconnected sub-graphs, the Lloyd algorithm in [21] cannot converge to a proper deployment. In this paper, we propose a method, named Restrained Lloyd (RL) Algorithm, to distribute sensor nodes and to minimize distortion with full connectivity. Then, a more complex approach, named Deterministic Annealing (DA) Algorithm, is designed to avoid sub-optimal solutions.

In many practical situations, the sensors in the WSN may have different characteristics such as computational power, sensing range, and sensing accuracy. The deployment and topology control of such heterogeneous WSNs that include the sensor nodes with different communication or sensing ranges have been studied in [39], [40]. Similar to [12], three movement-assisted protocols in [39] are designed to avoid coverage holes in heterogeneous WSNs. However, [40] deploys sensor nodes one-by-one and deploys a new sensor by using the location information of all previously deployed nodes. Furthermore, when distortion is used to evaluate the sensing accuracy, Weighted Voronoi Diagrams (WVD) [29] rather than conventional Voronoi diagrams [41] will provide the best sensing regions as we will discuss in this paper. An algorithm to construct WVDs for a different application has been proposed in [42]. However, because of the non-convex and complex geometry of the WVD, the optimal sensor deployment in heterogeneous WSNs has not been studied, even if the communication range is infinite.

In this paper, we study the sensor deployment in heterogeneous WSNs and make the following contributions: (i) We present a new general accuracy model for heterogeneous WSNs that includes coverage area in homogeneous WSN as a special case; (ii) Based on the geometry of the optimal cell partitioning WVDs, we calculate the gradient of the global sensing performance in heterogeneous WSNs; (iii) We propose the necessary condition for the optimal sensor deployment in heterogeneous WSNs with and without communication constraint. The necessary condition helps us to design the sensor deployment algorithms; (iv) We propose a method, which depends only on local information, to keep connectivity in WSNs. Although we concentrate on Lloyd Algorithm in this paper, this effective method can be extended to other distributed sensor deployment algorithms; (v) We design two algorithms RL and DA to distribute sensors with connectivity guarantee in both homogeneous and heterogeneous WSNs. RL and DA obtain better performance (lower distortion or larger coverage area) compare to other numerical algorithms, such as Lloyd Algorithm [21], Genetic Algorithm (GA) [6], and Virtual Force Algorithm (VFA) [6]. Furthermore, RL Algorithm and DA Algorithm have been extended to non-

convex environments [26]–[28].

In the rest of this paper, we first introduce the system model for both homogeneous and heterogeneous WSNs and formulate the problems of sensing and connectivity in Section II. Section III analyzes the optimal deployment in heterogeneous sensor networks with and without communication constraint. Section IV proposes RL and DA algorithms to improve sensing performance and maintain connectivity. Section V presents simulation results and Section VI provides the conclusions.

II. SYSTEM MODEL AND GENERAL PROBLEMS

Let Q be a simple convex polygon in $\mathbb{R}^2$ including its interior. Given n sensors in the target area Q , sensor deployment is defined by $P = (p_1, \dots, p_n) \subset Q^n$, where p_i is Sensor i 's location. For any point $q \in Q$, $\lambda(q)$ is the probability density function of an event at point q . A cell partition R of Q is a collection of disjoint subsets of $\{R_i(P)\}_{i \in 1, \dots, n}$ whose union is Q . Let $B(c, r) = \{q \mid \|q - c\| \leq r\}$ be a disk centered at c with radius r in two-dimensional space. For two points a and b , let equation $Eq + F = 0$, where $E \in \mathbb{R}^{1 \times 2}$ is a 1×2 matrix and $F \in \mathbb{R}$ is a constant, define the perpendicular bisector hyperplane between the two points. Then, the equations $Eq + F \geq 0$ and $Eq + F \leq 0$ define two half spaces. we denote the half space that contains point a by $HS(a, b)$.

As mentioned before, we define the AP as the sensor node that can communicate with the outside information world. Let $S(P)$ be the set of sensor nodes that can communicate with the AP when the sensor deployment is P . Note that in general not all nodes can communicate with the AP and $\text{card}(S(P)) \leq n$, where $\text{card}(A)$ is the number of elements in set A . We define a new sensor deployment, which is a subset of the all sensor locations, $H(P)$ as the vector of sensor locations for the $\text{card}(S(P))$ sensor nodes connected to the AP. When $S(P)$ includes all sensor nodes, we have $P = H(P)$ and $\text{card}(S(P)) = n$. Let T be the set of sensor deployments that provide full connectivity, i.e., $T = \{P \mid \text{card}(S(P)) = n\}$. In our model, two sensor nodes can communicate with each other within one hop if and only if the distance between the two is smaller than R_c , where R_c is referred to as the communication range. A sensor node can transfer data outside if and only if there exists a path from the sensor to the AP. The path consists of a sequence of sensor nodes where each hop distance is smaller than the communication range R_c . Sensor nodes that are connected to the AP construct the backbone network. Specifically, we can choose the AP as the root and run Breadth First Search (BFS) or Depth First Search (DFS) to obtain the spanning tree. Obviously, sensors in the spanning tree construct the backbone network. If all sensors are included in the backbone network, we call the network fully connected. Otherwise, the network is divided into several disconnected sub-graphs.

Another important factor in analyzing the performance of a WSN is its sensing distance. Sensing performance directly depends on distance [3], [21]–[29]. Therefore, to represent

the average sensing accuracy in the target area, we define the following general performance measure:

$$\Phi(P) = \sum_{i=1}^n \int_{R_i(P)} f^{(i)}(\|q - p_i\|) \lambda(q) dq, \quad (1)$$

where performance function $f^{(i)} : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a non-increasing function of the distance between p_i and q . When sensors have an identical performance function, i.e., $f^{(i)}(\cdot) = f(\cdot)$, Eq. (1) becomes the performance measure in homogeneous WSNs and has been widely used in different applications, such as the precipitation estimation problem in [29]. The partition $\{R_i(P)\}_{i=1,\dots,n}$ in the above definition include all sensor nodes. However, as explained previously, when the communication range is limited, some sensor nodes cannot transfer their data back to the AP. As a result, only the sensor nodes in the backbone network can contribute to the sensing and therefore the performance should be revised as

$$\Phi(P) = \sum_{i \in S(P)} \int_{R_i(H(P))} f^{(i)}(\|q - p_i\|) \lambda(q) dq. \quad (2)$$

Note that to derive Eq. (2) from Eq. (1), one has to replace P with $H(P)$, i.e., one has to consider only the sensor nodes that are in the backbone network. We reiterate that in the case of a fully connected network, $H(P) = P$, Eqs. (1) and (2) are identical. Given a fully connected network, the distortion $\Phi(P) = \Phi(P, R(P))$ is determined by the sensor deployment P and the cell partition $R(P)$. In homogeneous WSNs, every point should be detected by the nearest sensor in order to make the largest contribution to the total performance. This is because (i) all sensors in homogeneous WSNs have the same performance function $f(x)$ which is only determined by the Euclidean distance $x = \|q - p_i\|$, and (ii) the sensing ability diminishes as the distance increases. Therefore, given the sensor deployment, Voronoi partitions [29], [41] provide the optimal performance. The Voronoi partition of Q generated by P with respect to the Euclidean norm is the collection of sets $\{V_i(P)\}_{i=1,\dots,n}$ defined by

$$V_i(P) = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \forall j \in 1, \dots, n\}, \quad (3)$$

where $\|\cdot\|$ is the Euclidean norm.

However, different sensors with different complexity, power, and sensing ability are used in heterogeneous WSNs. As we will show later, the optimal partitioning in this case is WVD [44]. The WVD of Q generated by P is the collection of sets $V_i^H(P)_{i=1,\dots,n}$ defined by

$$V_i^H(P) = \{q \in Q \mid \eta_i \|q - p_i\|^2 \leq \eta_j \|q - p_j\|^2, \forall j \in 1, \dots, n\}, \quad (4)$$

where the cost parameters $\{\eta_i \in \mathbb{R}^+\}_{i=1,\dots,n}$ are constants that depend on the sensor characteristics, indicating the quality of sensor nodes. For example, the transmission power of Radar sensors, like the distance between the sensor and the events, has a direct influence on the sensing performance [23]. Therefore, radar sensors with different transmitting power will have different sensing abilities. In this case, η_i depends on the transmitting power. The smaller the cost parameter, the stronger the sensing ability. Both Voronoi regions $V_i(P)_{i=1,\dots,n}$ and WVDs $V_i^H(P)_{i=1,\dots,n}$ are functions of P . Since the Voronoi partitioning can be considered as a special case of the weighted

Voronoi partitioning, in which $\eta_i = 1, i = 1, \dots, n$, we simply use $V_i^H(P)_{i=1,\dots,n}$ to represent both. Placing Eq. (4) back to Eq. (2), we will have

$$\Phi(P) = \sum_{i \in S(P)} \int_{V_i^H(H(P))} f^{(i)}(\|q - p_i\|) \lambda(q) dq. \quad (5)$$

Obviously, choosing different functions $f^{(i)}(\cdot)$ in Eq. (5) results in different problem formulations. One natural choice for the function $f^{(i)}(\cdot)$ is a continuous function defined by

$$f^{(i)}(x) = -\eta_i x^2. \quad (6)$$

Eq. (6) represents the negative of the mean squared error (MSE) in source coding and we refer to it as the **negative weighted MSE model**. By this definition, each sensor can detect all points in its sensing region $R_i(P)$. Using WVDs, every event is sensed by the node with the smallest cost. Therefore, given the sensor deployment, WVDs provide the minimum distortion. In this model, maximizing $\Phi(P)$ is equivalent to minimizing the distortion.

Another choice for the function $f^{(i)}(\cdot)$ is the one that results in the **binary coverage model**. In this model, Sensor i can detect events within a circle with a fixed radius $\frac{R_s}{\sqrt{\eta_i}}$. The covered area with respect to density function is $\int \bigcup_{i \in S(P)} B(p_i, R_s) \lambda(q) dq$. This model is equivalent to Eq. (5) if the performance function is defined by

$$f^{(i)}(x) = \begin{cases} 1, & \text{for } \eta_i x^2 < R_s^2 \\ 0, & \text{for } \eta_i x^2 \geq R_s^2 \end{cases}. \quad (7)$$

This is easily shown by replacing $f^{(i)}(\cdot)$ from Eq. (5) by Eq. (7) and using the fact that

$$\bigcup_{i \in S(P)} B(p_i, R_s) = \bigcup_{i \in S(P)} \left[V_i^H(H(P)) \cap B(p_i, \frac{R_s}{\sqrt{\eta_i}}) \right]. \quad (8)$$

The proof of Eq. (8) is trivial and is omitted here. The resulting coverage area for the homogeneous case, $\eta_i = 1$ for all i , is the same as the model used in [6]–[11].

A third choice for the function $f^{(i)}(\cdot)$ is the one that results in the **exponential coverage model**. Our exponential coverage model is a modified version of the probabilistic coverage model for homogeneous WSNs in the literature [13]–[18], where we assume each event is only sensed by the sensor with highest probability of sensing. In the probabilistic coverage model, events within the confident range of $\frac{R_s}{\sqrt{\eta_i}}$ can be sensed without loss. For the events outside of the confident range, the probability of correctly sensing event q by Sensor i is a non-increasing function of the distance $x = \|q - p_i\|$. We model this sensing probability via

$$P_i^s(x) = \begin{cases} 1, & \text{for } \eta_i x^2 < R_s^2 \\ e^{-K(\eta_i x^2 + R_s^2)}, & \text{for } \eta_i x^2 \geq R_s^2, \end{cases} \quad (9)$$

where $K \in \mathbb{R}^+$ is a positive constant. The exponential coverage model approximates the binary coverage model when $K \rightarrow +\infty$. The covered area with respect to density function is $\int_Q \max_i P_i^s(\|q - p_i\|) \lambda(q) dq$. Our exponential coverage model, that allows sensing each event by only one sensor, is equivalent to Eq. (5) with performance function $f^{(i)}(x) = P_i^s(x)$.

Without loss of generality, we represent every performance

function $f^{(i)}$ as a piece-wise continuous and differentiable function with l discontinuities, i.e., jumps, at $r_1^{(i)}, \dots, r_l^{(i)} \in \mathbb{R}^+$, with $r_1^{(i)} < \dots < r_l^{(i)}$. For convenience, we set $r_0^{(i)} = 0$ and $r_{l+1}^{(i)} = +\infty$ and rewrite $f^{(i)}(x)$ as

$$f^{(i)}(x) = \sum_{\alpha=1}^{l+1} f_{\alpha}^{(i)}(x) 1_{[r_{\alpha-1}, r_{\alpha})}(x), \quad (10)$$

where $f_{\alpha}^{(i)}(\cdot)$ is a continuous and differentiable function and $1_s(x)$ is the indicator function defined by $1_s(x) = 1$ if $x \in s$, and $1_s(x) = 0$ if $x \notin s$. Applying this representation to the specific models presented before, we have no discontinuity, i.e., $l = 0$ for negative weight MSE and $l = 1$ for binary coverage and exponential coverage. Our main goal is to find a sensor deployment that maximizes $\Phi(P)$ defined in Eq. (2). It is easy to show that a necessary condition for such an optimal sensor deployment is to have a fully connected network. Moreover, WVDs are the optimal partitions for the three models in this section. Using WVDs, the performance is determined by the sensor deployment only. This is the topic of the discussion in the next section.

III. OPTIMAL DEPLOYMENT IN HETEROGENEOUS WSNs

In this section, we study the optimal deployment in heterogeneous WSNs with infinite or limited communication range. Finding the global optimal deployment is difficult because one needs to compare all the local maxima. Let $P^*(R_c)$ be a deployment set including all the critical deployments that obtain the performance maxima when communication range is R_c . Instead of finding the global optimal deployment, one should at least find a deployment $P \in P^*(R_c)$. The connectivity is guaranteed when the communication range R_c is infinite. Under such circumstance, the objective function is continuous and differentiable. Therefore, the critical deployment set $P^*(\infty)$ is just the set of deployment with zero-gradient. However, the wireless communication range is limited due to the finite transmitting power, and the network will be divided into several subgraphs when some sensors are out of the AP's spanning tree. It is self-evident that the objective function is a discontinuous and indifferentiable function when communication range is limited. The jumps result from the changes of nodes in the backbone network. In this case, the critical deployment set becomes $P^*(R_c) = \{P = (p_1, \dots, p_n) \mid \frac{\partial \Phi(P)}{\partial p_i} \text{ is 0 or it does not exist, } \forall i \in \{1, \dots, n\}\}$. Next, we provide the necessary condition for the optimal deployment in heterogeneous WSNs.

Lemma 1. *The necessary condition for the optimal deployment in heterogeneous WSNs with communication range R_c is $P \in P^*(R_c)$.*

Proof: Let $\Delta = \sum_{i=1}^{2n} \Delta_i e_i$ be a $2n$ -dimensional vector, where $e_i, i \in \{1, \dots, 2n\}$, are standard basic vectors. For any local maximum point P^* , there exists some $\epsilon > 0$ such that $\Phi(P) \geq \Phi(P^* + \Delta)$ for all Δ with $\|\Delta\| \leq \epsilon$. Specially, $\Phi(P^*) \geq \Phi(P^* + \epsilon e_i)$ for all $i \in \{1, \dots, 2n\}$. As a result, the partial derivative $\frac{\partial \Phi(P)}{\partial p_i} \big|_{P=P^*}$ is 0 or does not exist for $i \in \{1, \dots, n\}$. Specially, when $R_c = +\infty$, the partial derivative $\frac{\partial \Phi(P)}{\partial p_i} \big|_{P=P^*} = 0$ for all $i \in \{1, \dots, n\}$. ■

Now, we can easily obtain the the critical deployment set by calculating the gradient of the objective function $\Phi(P)$. In homogeneous WSNs, every sensor uses the same performance function and Eq. (5) can be rewritten as

$$\Phi(P) = \sum_{i \in S(P)} \int_{V_i(H(P))} f(\|q - p_i\|) \lambda(q) dq \quad (11)$$

The partial derivatives of Eq. (11) are calculated in [21] as

$$\frac{\partial \Phi(P)}{\partial p_i} = \int_{V_i(P)} \frac{\partial}{\partial p_i} f(\|q - p_i\|) \lambda(q) dq + \sum_{\alpha=1}^l \left[(f_{\alpha}(r_{\alpha}) - f_{\alpha+1}(r_{\alpha})) \int_{\text{arc}_i(P, r_{\alpha})} n^t(q) \lambda(q) dq \right], \quad (12)$$

where $\text{arc}_i(P, r_{\alpha})$ consists of arcs in the boundary of $V_i(P) \cap B(p_i, r_{\alpha})$, $n^t(q)$ is the unit outward normal to $\bigcup_{\alpha=1}^l \text{arc}_i(P, r_{\alpha})$ at point q . Before we discuss the partial derivatives in heterogeneous WSNs, we need to present the following definitions and lemmas.

Definition 1. *A set $S \subseteq \mathbb{R}^n$ is called star-shaped if and only if there exists a point $p \in \text{int}(S)$ such that for all $s \in \partial S$ and all $\lambda \in (0, 1]$, one has $\lambda p + (1 - \lambda)s \in \text{int}(S)$, where $\text{int}(S)$ is the interior of S and ∂S is the boundary of S . The point p is the reference point.*

Definition 2. *A set $S \subseteq \mathbb{R}^n$ is called a convex region if and only if for every pair of points $x, y \in S$ and all $\lambda \in (0, 1)$, one has $\lambda x + (1 - \lambda)y \in \text{int}(S)$.*

Lemma 2. *If a set $S \subseteq \mathbb{R}^n$ is convex, then S is star-shaped.*

Proof: For any convex region $S \subseteq \mathbb{R}^n$, pick a point $p \in \text{int}(S) \subset S$. For any point $s \in \partial S \subset S$ and all $\lambda \in (0, 1)$, one has $\lambda p + (1 - \lambda)s \in S$. When $\lambda = 1$, $\lambda p + (1 - \lambda)s = p \in \text{int}(S)$. Therefore, the set S is star-shaped. ■

We will use the fact that the intersection of any collection of convex sets is convex [43], [44] and as a result star-shaped according to Lemma 2.

Lemma 3. *The union of star-shaped sets that are associated with the same reference point p is star-shaped.*

Proof: Given m star-shaped sets $S_i, i = 1, \dots, m$ with the same reference point p , the corresponding union is $S = \bigcup_{i=1}^m S_i$. Since point p is the reference point of star-shaped sets S_i , where $i \in \{1, \dots, m\}$, we have $p \in \bigcap_{i=1}^m S_i$ and therefore $\bigcap_{i=1}^m S_i \neq \emptyset$. The boundary of S comes from the boundaries of m star-shaped sets S_i , where $i \in \{1, \dots, m\}$. Thus, for any point $s \in \partial S$, we have $s \in \bigcup_{i=1}^m \partial S_i$. Because of m star-shaped sets, for all $s \in \partial S_i$ and all $\lambda \in (0, 1]$, we will have $\lambda p + (1 - \lambda)s \in \text{int}(S_i)$. Thus, for all $s \in \partial S$ and for all $\lambda \in (0, 1]$, one can find a subset S_i such that $s \in \partial S_i$ and so $\lambda p + (1 - \lambda)s \in \text{int}(S_i) \subset \text{int}(S)$. ■

Lemma 4. *Let $S(x) = \bigcup_{i=1}^m S_i(x)$ be a star-shaped set that consists of m disjoint sub-sets S_i , where $i \in \{1, \dots, m\}$. We then have*

$$\int_{\partial S} \varphi(\gamma) n^t(\gamma) d\gamma = \sum_{i=1}^m \int_{\partial S_i} \varphi(\gamma) n^t(\gamma) d\gamma, \quad (13)$$

where $\varphi(\cdot)$ is a continuous function of γ , and $n^t(q)$ is the unit outward normal to $\bigcup_{i=1}^m \partial S_i$ at q .

Proof:

$$\begin{aligned} & \sum_{i=1}^m \int_{\partial S_i} \varphi(\gamma) n^t(\gamma) d\gamma \\ &= \sum_{i=1}^m \left[\sum_{j \neq i} \int_{\partial(S_i \cap S_j)} \varphi(\gamma) n^t(\gamma) d\gamma + \int_{\partial(S_i \cap S)} \varphi(\gamma) n^t(\gamma) d\gamma \right] \end{aligned} \quad (14)$$

For any i and j such that $S_i \cap S_j = \emptyset$, the corresponding curve integral $\int_{\partial(S_i \cap S_j)} \varphi(\gamma) n^t(\gamma) d\gamma$ is 0. On the other hand, for any i and j such that $S_i \cap S_j \neq \emptyset$, the corresponding curve integral $\int_{\partial(S_i \cap S_j)} \varphi(\gamma) n^t(\gamma) d\gamma = -\int_{\partial(S_j \cap S_i)} \varphi(\gamma) n^t(\gamma) d\gamma$ because of opposite unit outward normal. Therefore, we have $\sum_{i=1}^m \int_{\partial S_i} \varphi(\gamma) n^t(\gamma) d\gamma = \sum_{i=1}^m \left[\int_{\partial(S_i \cap S)} \varphi(\gamma) n^t(\gamma) d\gamma \right] = \int_{\partial S} \varphi(\gamma) n^t(\gamma) d\gamma$ ■

Now, we have enough tools to derive the main results in this section. The calculation of the gradient in homogeneous WSNs, Eq. (12), relies on Proposition 1.6 in [21], which requires star-shaped integral regions. In homogenous WSNs, the integral regions are star-shaped Voronoi diagrams. However, the integral regions in WVDs can be non-star-shaped [42], and therefore we need to show how to calculate the partial derivatives in heterogeneous WSNs. The following proposition is needed to calculate these partial derivatives in $\mathbb{R}^2$.

Proposition 1. *In a heterogeneous sensor network including different types of sensors, let $P = [p_1, p_2, \dots, p_n]$ be the sensor deployment, and $W \in \mathbb{R}^2$ be an arbitrary convex set. Let a series of functions $\varphi_i : \mathbb{R}^2 \times (a, b) \rightarrow \mathbb{R}$, where $i = 1, \dots, n$, be continuous on $\mathbb{R}^2 \times (a, b)$, continuously differentiable with respect to its second argument for all $p_i \in (a, b)^2$, where $i \in 1, \dots, n$, and almost all $q \in \mathbb{R}^2$, and such that for each $p_i \in (a, b)^2$, the maps $q \mapsto \varphi_i(q, p_i)$ and $q \mapsto \frac{\partial \varphi_i}{\partial x}(q, x)$ are measurable, and integrable on $\mathbb{R}^2$. Then the function*

$$\int_{V_i^H(P) \cap W} \varphi_i(q, p_i) dq \quad (15)$$

is continuously differentiable and

$$\begin{aligned} & \frac{\partial \int_{V_i^H(P) \cap W} \varphi_i(q, p_i) dq}{\partial p_j} \\ &= \int_{V_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq + \int_{\partial[V_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} dq. \end{aligned} \quad (16)$$

Proof: The proof is provided in Appendix A. ■

Theorem 1. *Given a density function λ and n piece-wise continuous and differentiable functions $f^{(i)}, i = 1, \dots, n$, the multiple-center function $\Phi(P)$ is continuously differentiable on Q^n , and for each $i \in 1, \dots, n$,*

$$\begin{aligned} \frac{\partial \Phi(P)}{\partial p_i} &= \int_{V_i^H(P)} \frac{\partial}{\partial p_i} f^{(i)}(\|q - p_i\|) \lambda(q) dq \\ &+ \sum_{\alpha=1}^l \left[\left(f_{\alpha}^{(i)}(r_{\alpha}) - f_{\alpha+1}^{(i)}(r_{\alpha}) \right) \int_{\text{arc}_i(P, r_{\alpha})} n^t(q) \lambda(q) dq \right], \end{aligned} \quad (17)$$

where $\text{arc}_i(P, r_{\alpha})$ consists of arcs in the boundary of $V_i^H(P) \cap B(p_i, r_{\alpha})$, $n^t(q)$ is the unit outward normal to $\bigcup_{\alpha=1}^l \text{arc}_i(P, r_{\alpha})$ at point q .

The proof is similar to that of Theorem 2 in [21] and then is omitted here. Particularly, when we fix the partitions as $V_i^H(P'), i \in \{1, \dots, n\}$, for a given deployment P' , the constrained performance becomes

$$\Phi(P, V^H(P')) = \sum_{i=1}^n \int_{V_i^H(P')} f^{(i)}(\|q - p_i\|) \lambda(q) dq \quad (18)$$

By Proposition 1, for each $i \in \{1, \dots, n\}$, the partial derivative of $\Phi(P, V^H(P'))$ becomes Eq. (19), on the top of the next page, where $\text{ARC}_i(P, P', r_{\alpha})$ consists of arcs in the boundary of $V_i^H(P') \cap B(p_i, r_{\alpha})$. Obviously, $\frac{\partial \Phi(P)}{\partial p_i}$ and $\frac{\Phi(P, V^H(P'))}{\partial p_i}$ will have the identical value at the deployment P' . Next, we provide the corresponding gradients for specific performance functions defined in Section II.

Negative weighted MSE model:

The performance function in this model is continuous without jump discontinuities. As a result, all the terms in the second summand of Eq. (17) vanish and we have

$$\frac{\partial \Phi(P)}{\partial p_j} = 2\eta_j(p_j - c_j(P))M_j(P), \quad (20)$$

where $M_j(P) = \int_{V_j^H(P)} \lambda(q) dq$ and $c_j(P) = \frac{\int_{V_j^H(P)} q \lambda(q) dq}{M_j(P)}$ are, respectively, the mass and the center of WVD $V_j^H(P)$ with respect to the density function λ . The critical deployment set $P^*(\infty) = \{P = (p_1, \dots, p_n) | p_i = c_i(P), \forall i \in 1, \dots, n\}$ is similar to the centroid Voronoi configurations [21] although the critical deployments in heterogeneous WSNs are based on WVDs instead of Voronoi diagrams.

Binary coverage model:

The performance function in this model is a step function with one jump discontinuity. In this case, the first term in Eq. (17) vanishes and we have

$$\frac{\partial \Phi(P)}{\partial p_j} = \int_{\text{arc}_i(P, r_{\alpha})} n^t(q) \lambda(q) dq. \quad (21)$$

Exponential coverage model:

The performance function in this model is continuous with one indifferentiable point. All the terms in the second summand of Eq. (17) vanish and we have

$$\frac{\partial \Phi(P)}{\partial p_j} = \int_{V_j^H(P) \setminus B(p_j, \frac{R_s}{\sqrt{\eta_j}})} 2(p_j - q) e^{-\eta_j \|q - p_j\|^2 + R_s^2} \lambda(q) dq. \quad (22)$$

Using the above gradients, we can design algorithms, such as gradient descent and Lloyd Algorithm, to optimize the performance. Moreover, zero-gradient can help us to check if a given deployment provides a local optimal performance (minimum distortion or maximum coverage area). Again, given the gradient of the objective function, we can get the critical deployment set as we discussed the beginning of this section. Furthermore, in order to get a locally optimal solution, the designed algorithm should converge to a deployment $P \in P^*(R_c)$. In the next section, we will discuss how to

$$\frac{\partial \Phi(P, V^H(P'))}{\partial p_j} = \int_{V_i^H(P')} \frac{\partial}{\partial p_i} f^{(i)}(\|q - p_i\|) \lambda(q) dq + \sum_{\alpha=1}^l \left[\left(f_{\alpha}^{(i)}(r_{\alpha}) - f_{\alpha+1}^{(i)}(r_{\alpha}) \right) \int_{ARC_i(P, P', r_{\alpha})} n^t(q) \lambda(q) dq \right] \quad (19)$$

design algorithms to achieve this convergence requirement.

IV. RESTRAINT LLOYD ALGORITHM AND DETERMINISTIC ANNEALING ALGORITHM

In this section, we focus on the negative weighted MSE model because of its trackable critical deployment set. Two algorithms are designed to optimize the performance of this model in heterogeneous WSNs. Although the algorithms are designed for the negative weight MSE model, their main ideas can be applied to the other two models too. First, we quickly review the conventional Lloyd algorithm. Lloyd Algorithm has two basic steps in each iteration: (i) Sensor nodes move to their centroid; (ii) Partitioning is done by assigning the optimal partitions, WVDs, to each sensor node. Lloyd Algorithm provides good performance and is simple enough to be implemented distributively. It converges to a critical deployment when the communication range is infinite [32], [33]. Unfortunately, Lloyd Algorithm also has three shortcomings. First, since maximizing multi-center performance is a non-convex optimization problem, Lloyd Algorithm ends at a local maximum rather than the global maximum. Second, Lloyd Algorithm results in a disconnected network when the spanning tree rooted at the AP does not include all sensors due to the limited communication range. Third, when WSNs are divided into several disconnected sub-graphs, some sensors cannot collect the complete local information and as a result Lloyd Algorithm is not feasible. In other words, since there is no global information available about the sensor locations, each sub-graph will run the algorithm separately. To deploy a network with full connectivity and smaller distortion (or larger coverage area), we add some restraints on sensors' movements. We design a class of algorithms based on the Lloyd algorithm, referred to as RL Algorithm.

A. Restrained Lloyd Algorithm

Before we introduce the details of our RL Algorithm, we introduce two concepts: (1) local performance; (2) desired region. First, the global performance is the sum of local performances defined by

$$\Phi_i(P) = \int_{V_i^H(H(P))} f^{(i)}(\|q - p_i\|) \lambda(q) dq, i \in S(P). \quad (23)$$

Second, to define the desired region, without loss of generality, let us assume we are trying to move Sensor i at a given step. Our goal is to keep the connectivity of the backbone network after moving Sensor i . Therefore, all Sensor i 's locations that result in connecting Sensor i to the backbone network is defined as Sensor i 's desired region, denoted by $L_i(P)$. In our RL Algorithm, if Sensor i is in the backbone network, we will restrain its movement within its desired region. To achieve this goal, we need to find the desired region $L_i(P)$. Given a deployment P , if Sensor i from the backbone network is removed, the rest of the sensor nodes in the backbone network will be divided into K_i components:

$U_{i1}(P), U_{i2}(P), \dots, U_{iK_i}(P)$, where $U_{ij}(P)$ is a set of sensors included in the j th component. Then, we can calculate the desired region as

$$L_i(P) = \bigcap_{k=1}^{K_i} \left[\bigcup_{j \in U_{ik}} B(p_j, R_c) \right]. \quad (24)$$

We provide an example of the desired region that is not star shaped in Fig. 1. In this simple example, $i = 1$,

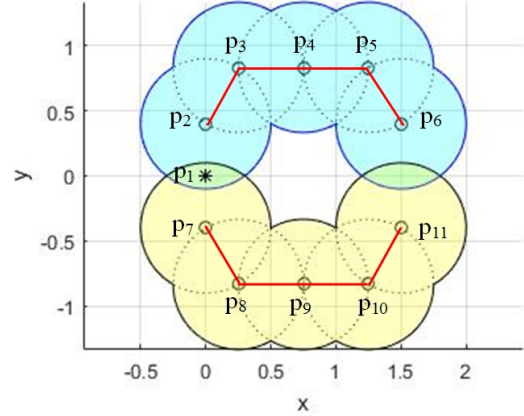


Fig. 1: A non-starshaped desired region.

i.e., we are trying to move Sensor 1, $R_c = 0.5$, $K_1 = 2$, $U_{11}(P) = \{2, 3, 4, 5, 6\}$, and $U_{12}(P) = \{7, 8, 9, 10, 11\}$. According to the definition of the desired region, the green overlaps between blue region ($\bigcup_{j=2}^6 B(p_j, R_c)$) and yellow region ($\bigcup_{j=7}^{11} B(p_j, R_c)$) construct Sensor 1's desired region. As shown in Fig. 1, the desired region $L_1(P)$ has two disconnected parts, indicating a non-star-shaped region.

Since the desired region is primarily influenced by the neighboring sensor nodes, we can approximate it by

$$\tilde{L}_i(P) = \bigcap_{k=1}^{K_i} \left[\bigcup_{j \in U_{ik} \cap N_i(P)} B(p_j, R_c) \right], \quad (25)$$

where $N_i(P)$ consists of Sensor i 's neighbors when the deployment is P . Note that the approximation in (25) can be calculated locally, but to calculate the exact desired region, one needs global information. Also, according to Lemma 3, the approximate desired region $\tilde{L}_i(P)$ is a star-shaped set.

Now, we provide the details of our RL Algorithm. The algorithm iterates between two steps:

(i) Sensors in the backbone network move one by one. Every sensor in the backbone network calculates its own approximate desired region $\tilde{L}_i(P)$ and moves to a critical location with maximum local performance. Sensors outside the backbone network move randomly and check if there is a path to the AP. Unlike the conventional Lloyd algorithm, these new locations may not be the centroid of the partition regions; (ii) The target

area, Q , is partitioned to WVDs for sensors in the backbone network, $S(P)$.

The main difference between RL Algorithm and the Lloyd algorithm is in the first step. Given the derivatives of each local performance, the existing gradient descent can be used to find the critical locations in Step (i). For negative weighted MSE model, we can simplify the algorithm by moving Sensor i to the closest point to its centroid $c_i(P)$ within $\tilde{L}_i(P)$. In what follows, we show that Step (i) in RL Algorithm provides the smallest local distortion.

According to the parallel axis theorem, the local performance for negative weighted MSE model can be rewritten as Eq. (26), on top of the next page, where $c_i(P) = \frac{1}{\int_{V_i^H(H(P))} \lambda(q) dq} \int_{V_i^H(H(P))} q \lambda(q) dq$ is the centroid of the partition region $V_i^H(H(P))$ with respect to the probability density function. Both $\int_{V_i^H(H(P))} \|q - c_i(P)\|^2 \lambda(q) dq$ and $\int_{V_i^H(H(P))} \lambda(q) dq$ are constants when the integral area $V_i^H(H(P))$ is fixed. In other words, the local distortion is a monotonously increasing function of $\|p_i - c_i(P)\|$, i.e., the sensor's distance to its centroid. Therefore, the movements in Step (i) minimize the local distortion. As the sum of local distortions, the global distortion will not increase. Since the sequence of the global distortion values is a non-increasing sequence with a lower bound of zero, it will converge. It is self-evident that the sensor deployment in RL Algorithm converges to an element in $P^*(R_c)$.

We also show that our RL Algorithm guarantees the connectivity of the network with high probability after enough number of iterations. Note that once a sensor node finds a path to the AP, our RL Algorithm will keep it in the backbone network. Intuitively, as we have more iterations, the sensors outside the backbone network will move randomly and eventually connect to the AP as well. Quantitatively, for the deployment after k iterations, the area in which a sensor can communicate with the backbone network can be calculated by $A_k = \text{AREA}(Q \cap [\bigcup_{i \in \text{backbone}} B(p_i, R_c)])$. Then, the probability that a sensor outside the backbone network is not connected to the AP in its next move is $\frac{\text{AREA}(Q) - A_k}{\text{AREA}(Q)}$. After N iterations, the probability that a sensor is still out of the backbone network can be calculated by $P_{\text{out}}(N) = \prod_{k=1}^N \left[\frac{\text{AREA}(Q) - A_k}{\text{AREA}(Q)} \right] \leq \left[\frac{\text{AREA}(Q) - \min A_k}{\text{AREA}(Q)} \right]^N$ and then $\lim_{N \rightarrow \infty} P_{\text{out}}(N) = 0$ because of $\min A_k > 0$. In other words, as long as the number of iterations is large enough, almost all sensor nodes will be included in the backbone network, indicating full connectivity, with high probability.

B. Deterministic Annealing Algorithm

Like any other steepest-descent algorithm, RL Algorithm converges to a local optimum. One approach to improve the sub-optimal solution or find the global optimal solution, is to use annealing methods. Simulated Annealing (SA) [45], [46] is a method in which a candidate sensor movement is generated randomly. However, SA ignores the characteristics of the objective function and requires burdensome computations. In this paper, we design a DA algorithm which combines RA with annealing to minimize the distortion. Unlike SA, the proposed

DA generates two new sensor positions deterministically at each iteration; however, choose one of the two options randomly.

Like RL Algorithm, our DA Algorithm iterates between two steps. The second step is identical to that of RL Algorithm. In the first step, the algorithm creates two candidate locations for each node in the backbone network. One candidate is the RL Algorithm's candidate that minimizes the local distortion. On the other hand, the second candidate increases the local distortion. It is easy to show that to maximize the local distortion of Sensor i in the backbone network for negative weighted MSE model, one should move it to the point o on the boundary of the desired region $\tilde{L}_i(P)$ that has the largest distance to the centroid $c_i(P)$. But the goal of the second candidate is to increase the distortion and not necessarily maximize it. Moreover, the performance is more sensitive to the sensors with smaller cost parameters. In order to avoid increasing distortion too fast, Sensor i is moved to the point $p_i + \frac{\eta_i}{\min_j(\eta_j)}(o - p_i)$. The algorithm will choose the first candidate with probability p and increases p from 0 to 1. Otherwise, the algorithm will choose the second candidate. In our algorithm, the probability p is increased in proportion to $\log k$, where k is the iteration number and for the last M iterations we force the probability $p = 1$. Like RL Algorithm, DA Algorithm guarantees connectivity and convergence. The proof is similar to that of RL Algorithm and is omitted. Furthermore, both RL and DA can be easily extended to non-convex environments [26]–[28]. We only need to revise the desired region to

$$L'_i(P) = \bigcap_{k=1}^{K_i} \left[\bigcup_{j \in U_{ik} \cap N_i(P)} B(p_j, R_c) \right] \cap \Omega_o^c, \quad (27)$$

where Ω_o is the obstacle region. To avoid the geometric calculation of $L'_i(P)$, one can calculate $\tilde{L}_i(P)$ and then move Sensor i towards the critical point by TangentBug [27].

V. PERFORMANCE EVALUATION

We compare the performance of DA Algorithm, RL Algorithm, Lloyd Algorithm [21], OPTGA [6], and VFA [6] using the negative weighted MSE model. We provide simulation in three sensor networks: (1) WSN1: A homogeneous WSN in which all sensors have the same cost parameter $\eta_i = 1, i = 1, \dots, 16$; (2) WSN2: A heterogeneous WSN including 2 kinds of sensors: four strong sensors with $\eta_i = 1$ and twelve weak sensors with $\eta_j = 16$; (3) WSN3: A heterogeneous WSN including three kinds of sensors: two strong sensors with $\eta_i = 1$, four medium sensors with $\eta_j = 4$ and ten weak sensors with $\eta_k = 16$. Sixteen sensors are provided in each sensor network. The parameter values are summarized in Table I. The AP is chosen from the sixteen sensors randomly. However, when we report the distortion or coverage area for the Lloyd algorithm, we calculate the distortion and the coverage area of each connected subgraph and report the best. Obviously, this will be advantageous for the Lloyd algorithm, but our proposed algorithms still outperform the Lloyd algorithm. We use ten random initial deployments for each algorithm. To have a fair comparison, we consider the same target domain Q as in

$$\Phi_i(P) = - \int_{V_i^H(H(P))} \|q - c_i(P)\|^2 \lambda(q) dq - \int_{V_i^H(H(P))} \lambda(q) dq \cdot \|p_i - c_i(P)\|^2, i \in S(P) \quad (26)$$

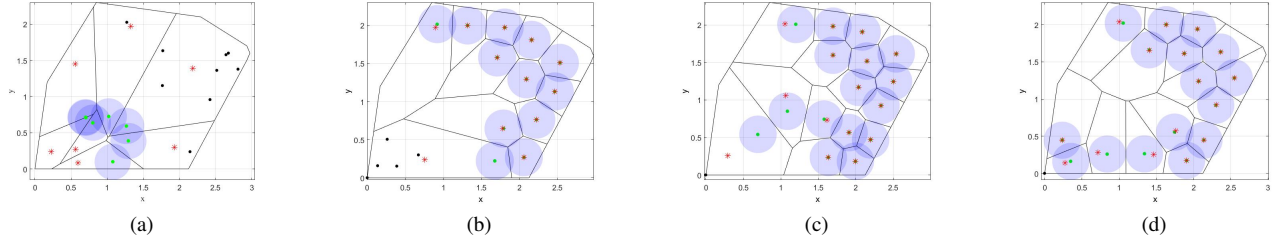


Fig. 2: Sensor deployments in WSN1. (a) The initial sensor deployment and the corresponding Voronoi regions. (b) The final deployment of Lloyd Algorithm. (c) The final deployment of RL Algorithm. (d) The final deployment of DA Algorithm.

[21]. Q is determined by the polygon vertices (0,0), (2.125,0), (2.9325,1.5), (2.975,1.6), (2.9325,1.7), (2.295,2.1), (0.85,2.3), (0.17,1.2).

TABLE I: Simulation Parameters

Parameters	WSN1	WSN2	WSN3
n	16	16	16
$\eta_1 - \eta_2$	1	1	1
$\eta_3 - \eta_4$	1	1	4
$\eta_5 - \eta_6$	1	16	4
$\eta_7 - \eta_{16}$	1	16	16
R_c	0.5	0.5	0.5
$R_s = R_c/2$	0.25	0.25	0.25

The distribution of the events is also the same as [21]. The probability density function is the sum of five Gaussian functions of the form $5\exp(6(-(x - x_{center})^2 - (y - y_{center})^2))$. The centers (x_{center}, y_{center}) are (2,0.25), (1,2.25), (1.9,1.9), (2.35,1.25) and (0.1,0.1). We use 0.5 as the communication range R_c . Also, when reporting the coverage area using (7), we use 0.25 as the sensing range R_s . In DA Algorithm, the first candidate is accepted at the i th iteration by a probability of $p(i) = \log(i+1)/\log(N+1)$, where N is the number of regular iterations. Additional $M = 25$ iterations are used in DA Algorithm to avoid ending with a process that increases the local distortions. Given the final deployment generated by different iterative algorithms, we can calculate the coverage area in the binary coverage model. Specially, sensor i has a sensing range $\frac{R_s}{\sqrt{\eta_i}}$ in the heterogeneous WSNs. In following figures, strong sensors, medium and weak sensors in the backbone network are, respectively, denoted by green circles, yellow circles and magenta circles. Sensors out of the backbone network are denoted by black dots. The corresponding centroid for sensors in the backbone network are denoted by red stars. The radius of each blue circle is $R_c/2 = 0.25$. Each algorithm runs 500 iterations.

Figs. 2a and 2b show one example of the initial and the final deployments of Lloyd Algorithm in WSN1. Lloyd Algorithm assumes an infinite communication range and requires the global knowledge of the sensor locations. Otherwise, disconnected sub-graphs run Lloyd Algorithm independently and there is no guarantee for convergence. Nonetheless, the calculation of the final distortion only considers sensors in the backbone network. In the final deployment of the example in Fig. 2b, there are four sensors disconnected from the backbone

network, resulting in a large distortion $D(P) = 2.21$. Fig. 2c shows the outcome of RL Algorithm in WSN1. After 500 iterations, the distortion is decreased from 11.30 to 0.60. Simultaneously, the coverage area is increased from 0.15 to 6.26 and the final deployment is connected. Fig. 2d shows the final deployment of DA Algorithm in WSN1. After 500 iterations, the distortion is decreased from 11.30 to 0.32, which is better than that of RL Algorithm. Simultaneously, the coverage area is increased from 0.15 to 6.99 and full connectivity is provided. Unlike Lloyd Algorithm, both RL Algorithm and DA Algorithm guarantee connectivity.

Fig. 3a illustrates the performance of VFA, Lloyd Algorithm, RL Algorithm, and DA Algorithm for 10 random initial deployments. We also add a line for the performance of OPTGA [6] which is independent of the initial deployment. While [6] reports the average of the 100 surviving deployments, the line in Fig. 3 reports the best performance. As can be seen from the figure, the performance of DA Algorithm is not sensitive to the initial deployment. In other words, DA Algorithm avoids most poor local minimum solutions. Fig. 3a shows that DA Algorithm has the best performance among the five algorithms.

Fig. 3b compares the final coverage area of RL Algorithm and DA Algorithm with that of Lloyd Algorithm, OPTGA and VFA. In most cases, decreasing the distortion results in increasing the coverage area as well. Intuitively, this behavior can be explained by considering coverage area as a hard-decision version of distortion. Next, the relationship between performance (distortion and coverage area) and communication range R_c in homogeneous WSN1 using DA Algorithm is depicted in Fig. 4.

Figs. 5a and 5b show one example of the initial and the final deployments of Lloyd Algorithm in WSN2. As usual, the final distortion only considers sensors in the backbone network. Initially, two strong sensors and four weak sensors are consisted in the backbone network shown in Fig. 5a. In Fig. 5b, only one strong sensor and four weak sensors are included in the backbone network, resulting in a large distortion $D(P) = 12.67$ which is only 0.48 smaller than the initial distortion. Fig. 5c shows the outcome of RL Algorithm in WSN2. After 500 iterations, the distortion is decreased from 13.15 to 5.52. Simultaneously, the coverage area is increased from 0.08 to 1.46 and the final deployment is connected. Fig. 5d shows the final deployment of DA Algorithm in WSN2. After 500 iterations, the distortion is decreased from 13.15 to 1.09, which is better than that of RL Algorithm.

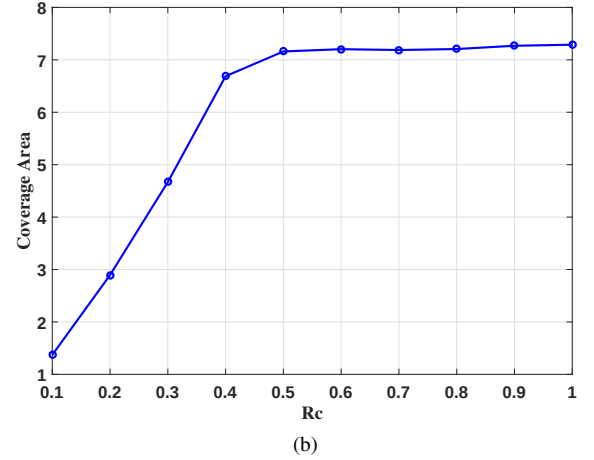
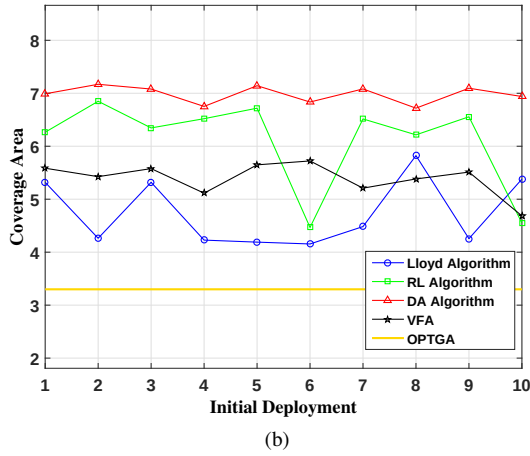
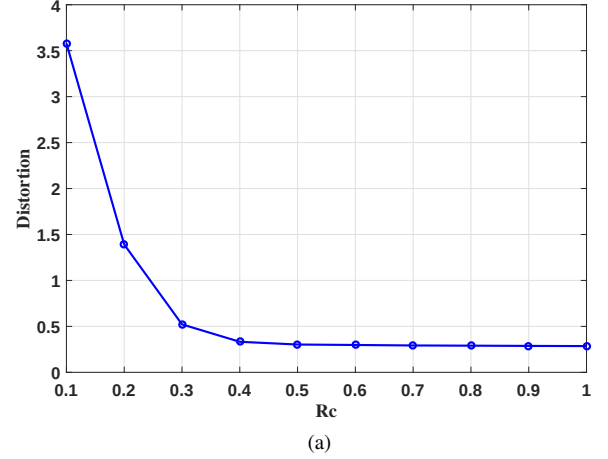
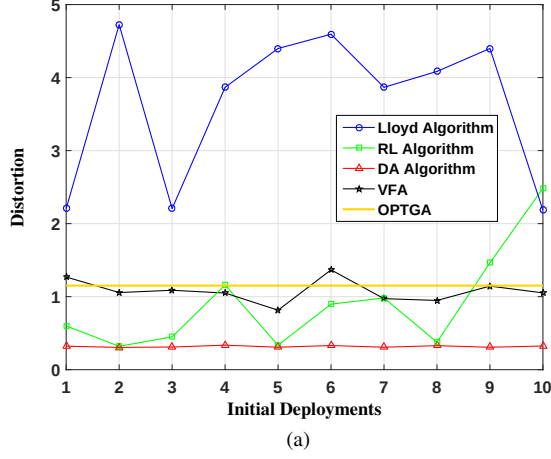


Fig. 3: Comparison of performance in WSN1. (a) Comparison of distortion for different algorithms in WSN1. (b) Comparison of coverage area for different algorithms in WSN1.

Fig. 4: Relationship between performance and R_c in WSN1. (a) Relationship between distortion and R_c in WSN1. (b) Relationship between coverage and R_c in WSN1.

Simultaneously, the coverage area is increased from 0.08 to 2.48 and full connectivity is provided.

Figs. 6a and 6b show one example of the initial and the final deployments of Lloyd Algorithm in WSN3. Initially, one strong sensor, two medium sensors and four weak sensors are consisted in the backbone network shown in Fig. 6a. In Fig. 6b, two strong sensors, one medium sensors and one weak sensors are disconnected from the backbone network, resulting in a large distortion $D(P) = 19.83$. Fig. 6c shows the outcome of RL Algorithm in WSN3. After 500 iterations,

the distortion is decreased from 10.96 to 3.22. Simultaneously, the coverage area is increased from 0.08 to 1.51 and the final deployment is connected. Fig. 6d shows the final deployment of DA Algorithm in WSN3. After 500 iterations, the distortion is decreased from 10.96 to 1.35, which is better than that of RL Algorithm. Simultaneously, the coverage area is increased from 0.03 to 2.07 and full connectivity is provided.

Figs. 7a and 7b illustrate the distortion and coverage area of five different algorithms (OPTGA, VFA, Lloyd Algorithm, RL Algorithm, and DA Algorithm) in WSN2. The DA Al-

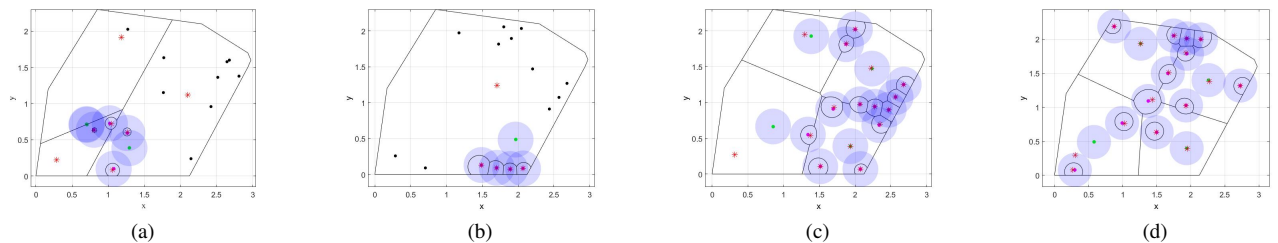


Fig. 5: Sensor deployments in WSN2. (a) The initial sensor deployment and the corresponding weighted Voronoi regions. (b) The final deployment of Lloyd Algorithm. (c) The final deployment of RL Algorithm. (d) The final deployment of DA Algorithm.

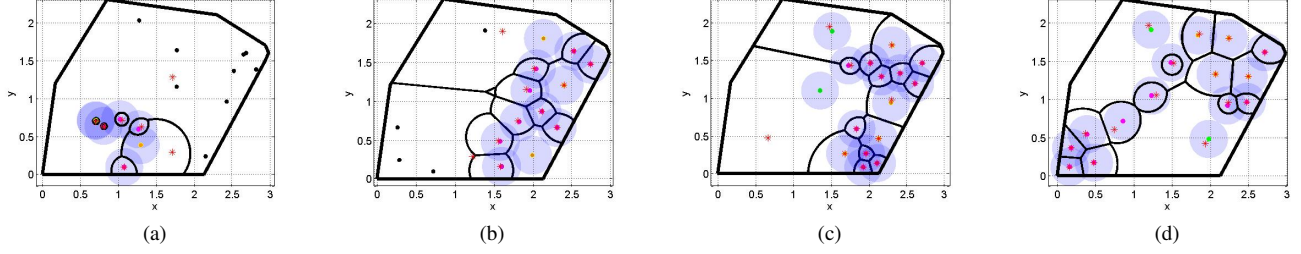
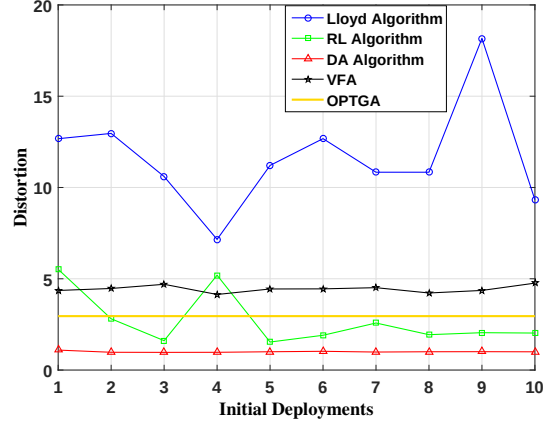
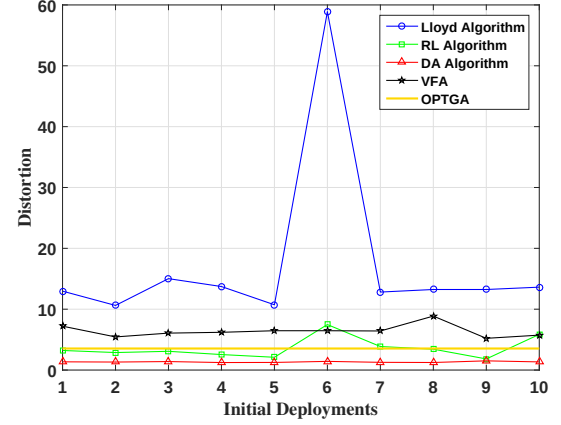


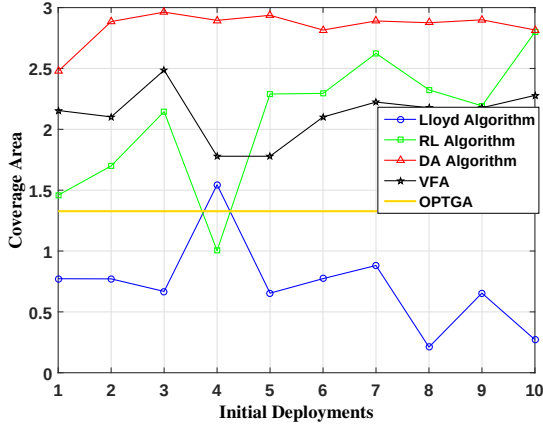
Fig. 6: Sensor deployments in WSN3. Figure (a) The initial deployment and corresponding weighted Voronoi regions. (b) The final deployment of Lloyd Algorithm. (c) The final deployment of RL Algorithm. (d) The final deployment of DA Algorithm.



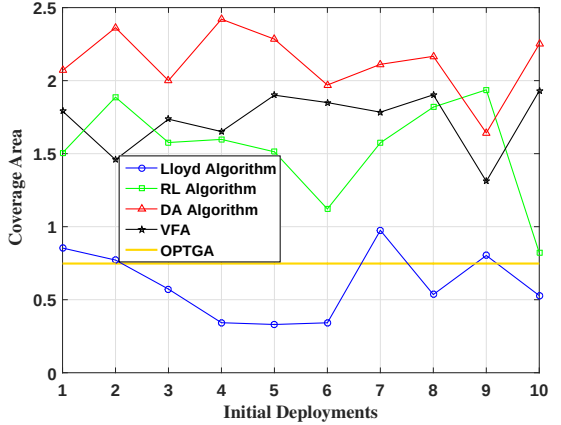
(a)



(a)



(b)



(b)

Fig. 7: Comparison of performance in WSN2. (a) Comparison of distortion for different algorithms in WSN2. (b) Comparison of coverage for different algorithms in WSN2.

Fig. 8: Comparison of performance in WSN3. (a) Comparison of distortion for different algorithms in WSN3. (b) Comparison of coverage area for different algorithms in WSN3.

gorithm provides both lower distortion and higher coverage area compare to the other four algorithms. Moreover, our RL Algorithm also provides lower distortion compare VFA and Lloyd Algorithm due to its connectivity guarantee. Figs. 8a and 8b show similar performance comparison in WSN3. The trends in heterogeneous WSNs 2 and 3 are similar to those in homogeneous WSN1. In other words, our RL Algorithm and DA Algorithm provide the performance improvement in both homogeneous and heterogeneous WSNs.

Furthermore, Table II provides the statistics of the per-

formance for different algorithms after 500 iterations. The statistics is reported for the 100 surviving deployments of the OPTGA and over the 10 different initial deployments for the other algorithms. For VFA, Lloyd, RL, and DA, a smaller standard deviation indicates less sensitivity to the initial condition. Again, DA Algorithm provides the best performance in both Homogeneous and Heterogeneous WSNs.

Fig. 9 shows the sensor deployments in a non-convex environment that includes two obstacles. Sensors can only move out of obstacles. Figs. 9a and 9b show one example

TABLE II: Distortion Comparison
(μ : the average distortion, σ : the standard deviation)

Scenario	VFA		OPTGA		Lloyd		RL		DA	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
WSN1	1.08	0.15	1.18	0.01	3.65	1.04	0.91	0.68	0.32	0.01
WSN2	4.44	0.19	3.04	0.51	11.76	2.80	2.72	1.45	1.00	0.04
WSN3	6.41	1.04	3.54	0.11	17.49	14.61	3.63	1.75	1.33	0.09

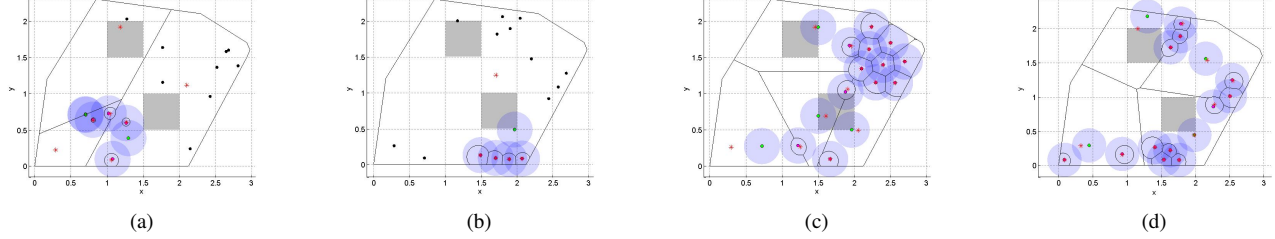


Fig. 9: Sensor deployments in WSN2. (a) The initial sensor deployment and the corresponding weighted Voronoi regions. (b) The final deployment of Lloyd Algorithm. (c) The final deployment of RL Algorithm. (d) The final deployment of DA Algorithm. Two obstacles are shown by the gray region.

of the initial and the final deployments of Lloyd Algorithm in WSN2 with two obstacles. Initially, two strong sensors and four weak sensors are consisted in the backbone network shown in Fig. 9a. In Fig. 9b, only one strong sensor and four weak sensors are included in the backbone network. Fig. 9c shows the outcome of RL Algorithm in WSN2 with two obstacles. After 500 iterations, the distortion is decreased from 13.15 to 1.67. Simultaneously, the coverage area is increased from 0.08 to 1.73 and the final deployment is connected. Fig. 9d shows the final deployment of DA Algorithm in WSN2 with two obstacles. After 500 iterations, the distortion is decreased from 13.15 to 1.10, which is better than that of RL Algorithm. Simultaneously, the coverage area is increased from 0.08 to 2.60 and full connectivity is provided.

Figs. 10a and 10b illustrate the distortion and coverage area of four different algorithms (VFA, Lloyd Algorithm, RL Algorithm, and DA Algorithm) for 10 random initial deployments in WSN2 with two obstacles. The DA Algorithm provides both lower distortion and higher coverage area compare to the other three algorithms.

VI. CONCLUSIONS

We studied the deployment of sensors for three kinds of model in heterogeneous wireless sensor networks. The optimal partitioning and gradient are supplied to search optimal sensor locations. In the negative weighted MSE model, similar to homogeneous WSNs, the necessary condition for optimal deployment implies that every sensor node location should coincide with the centroid of its own optimal sensing region. Moreover, we considered a limited communication range for the sensor nodes and modeled the sensor deployment problem as a multi-center optimization problem with different performance functions. By defining an appropriate performance measure, we proposed a Restrained Lloyd algorithm and a Deterministic Annealing algorithm to optimize sensor deployment in both homogeneous and heterogeneous WSNs. Our simulation

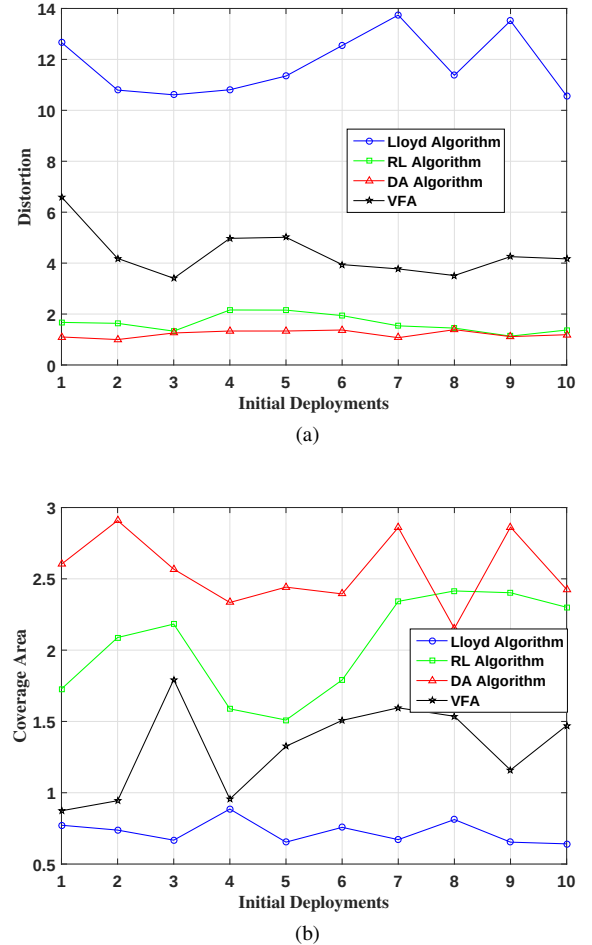


Fig. 10: Comparison of performance in WSN2. (a) Comparison of distortion for different algorithms in WSN2. (b) Comparison of coverage for different algorithms in WSN2.

results show that both DA and RL algorithms provide a fully connected network and outperform Lloyd algorithm, Genetic

Algorithm, and Virtual Force Algorithm when communication range is limited. The DA is not sensitive to initial conditions. Moreover, both RL and DA have been applied to solve sensor deployment in non-convex environments. Our future work includes the optimal sensor deployment with the random connection model in heterogeneous WSNs.

APPENDIX A PROOF OF PROPOSITION 1

Proof: At the beginning, it is necessary to study the shape of WVDs. Let $\text{Part}_{ij} = \{q \mid \eta_i \|q - p_i\|^2 \leq \eta_j \|q - p_j\|^2\}$ be the pairwise weighted Voronoi region of Sensor i when we only consider Sensors i and j . Then, the exact WVD of Sensor i is the intersection of these pairwise WVDs, i.e., $V_i^H(P) = \bigcap_{j \neq i} \text{Part}_{ij}$. We define the coordinates of $q = (x, y)$, $p_i = (p_{ix}, p_{iy})$ and $p_j = (p_{jx}, p_{jy})$ and define $\eta = \sqrt{\eta_i/\eta_j} > 0$. Then, expanding the hyperplane equation $\eta_i \|q - p_i\|^2 = \eta_j \|q - p_j\|^2$ results in Eq. (28), at the bottom of the page. When $\eta = 1$, the hyperplane equation is $2(p_{jx} - p_{ix})x + 2(p_{jy} - p_{iy})y + p_{ix}^2 + p_{iy}^2 - p_{jx}^2 - p_{jy}^2 = 0$. This hyperplane is the boundary of the half space $HS(p_i, p_j)$. When $\eta < 1$, Part_{ij} is defined by

$$(q - c)^2 \geq r^2. \quad (29)$$

When $\eta > 1$, Part_{ij} is defined by

$$(q - c)^2 \leq r^2, \quad (30)$$

where $c = (\frac{p_{jx} - \eta^2 p_{ix}}{1 - \eta^2}, \frac{p_{jy} - \eta^2 p_{iy}}{1 - \eta^2})$, $r = \frac{\eta}{|1 - \eta^2|} \|p_i - p_j\|$. Therefore, we have

$$\begin{aligned} V_k^H(P) &= \{q \in Q \mid \eta_k \|q - p_k\| \leq \eta_t \|q - p_t\|, \forall t \in 1, \dots, n\} \\ &= \left[\bigcap_{i: \eta_i < \eta_k} B(c_{ik}, r_{ik}) \right] \cap \left[\bigcap_{l: \eta_l = \eta_k} HS(p_k, p_l) \right] \cap \left[\bigcap_{j: \eta_j > \eta_k} [B(c_{kj}, r_{kj})]^c \right] \\ &= \left[\bigcap_{i: \eta_i < \eta_k} B(c_{ik}, r_{ik}) \right] \cap \left[\bigcap_{l: \eta_l = \eta_k} HS(p_k, p_l) \right] \cap \left[\bigcup_{j: \eta_j > \eta_k} B(c_{kj}, r_{kj}) \right]^c \end{aligned} \quad (31)$$

where A^c denotes the complementary set of A .

Let $\bar{V}_k^H(P)$ be the WVD of Sensor k when we ignore sensors with larger cost parameters. Accordingly, $\bar{V}_k^H(P)$ is defined by $\bar{V}_k^H(P) = \{q \in Q \mid \eta_k \|q - p_k\|^2 \leq \eta_i \|q - p_i\|^2, \forall i \text{ such that } \eta_k \geq \eta_i\}$. Review the definition of $V_k^H(P)$ in Eq. (4), we have Eq. (32), on top of the next page. According to Eq. (31), $\bar{V}_k^H(P)$ is the intersection of convex regions and therefore star-shaped. The relationship between $\bar{V}_k^H(P)$ and $V_k^H(P)$ is

$$\bar{V}_k^H(P) = V_k^H(P) \bigcup W_k, \quad (33)$$

where $W_k = \{q \in Q \mid \eta_k \|q - p_k\|^2 \geq \eta_j \|q - p_j\|^2, \eta_k \|q - p_k\|^2 \leq \eta_i \|q - p_i\|^2 \text{ for any } i, j \text{ such that } \eta_j > \eta_k, \eta_k \geq \eta_i\}$. For any point q such that $\eta_j \|q - p_j\|^2 \leq \eta_k \|q - p_k\|^2$ and $\eta_k \|q - p_k\|^2 \leq \eta_i \|q - p_i\|^2$, we have $\eta_j \|q - p_j\|^2 \leq \eta_i \|q - p_i\|^2$. So we can rewrite W_k as Eq. (34), on the top of the next page. Due to the definitions of $\bar{V}_k^H(P)$ and $V_k^H(P)$, we have

$$W_k = \left[\bigcup_{j: \eta_j > \eta_k} V_j^H(P) \right] \cap \bar{V}_k^H(P) = \bigcup_{j: \eta_j > \eta_k} [V_j^H(P) \cap \bar{V}_k^H(P)] \quad (35)$$

Replacing W_k from Eq. (35) in Eq. (33), we get the final result

$$\bar{V}_k^H(P) = V_k^H(P) \bigcup_{j: \eta_j > \eta_k} [V_j^H(P) \cap \bar{V}_k^H(P)] \quad (36)$$

The elements in the right side are disjoint subsets. Without loss of generality, let us assume that the M disjoint cost parameters are ordered such that the k -level cost parameter is larger than the $k + 1$ -level cost parameter. We also call the set including the indices of all sensors with a k -level cost parameter Z_k . Then:

(1) For any level-1 sensor i , $V_i^H(P)$ is a convex set. Accordingly, the intersection $V_i^H(P) \cap W$ is a convex set and therefore Eq. (16) holds due to Proposition 1.6 in [21].

(2) Assume the equation holds for any sensor whose level is smaller than or equal to k . Consider a sensor i whose level is $k + 1$, by using the relationship between $\bar{V}_i^H(P)$ and $V_i^H(P)$ in Eq. (36), we rewrite the objective function as Eq. (37), on the top of the next page. Since $V_i^H(P) \cap W$ is a convex set and therefore star-shaped, the partial derivative of the first term can be solved by proposition 1.6 in [21]. Sensors' levels in the second term are smaller than k and thus the partial derivative of the second term can be solved by our assumption in Step (2). Therefore, the partial derivative becomes

$$\begin{aligned} & \frac{\partial \int_{V_i^H(P) \cap W} \varphi_i(q, p_i) dq}{\partial p_j} \\ &= \int_{\bar{V}_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq + \int_{\partial[\bar{V}_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} d\gamma \\ & \quad - \sum_{l=1}^k \sum_{t \in Z_l} \left[\int_{V_t^H(P) \cap \bar{V}_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq \right] \\ & \quad - \sum_{l=1}^k \sum_{t \in Z_l} \left[\int_{\partial[V_t^H(P) \cap \bar{V}_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} d\gamma \right]. \end{aligned} \quad (38)$$

Note that $\bar{V}_i^H(P) \cap W = [V_i^H(P) \cap W] \cup G(P)$, where $G(P) = \left[\bigcup_{l=1}^k \bigcup_{t \in Z_l} (V_t(P) \cap V_i^H(P) \cap W) \right]$, is a star-shaped set consisting of several disjoint subsets. By using Lemma 4, we have Eq. (39), on the top of the next page. Also, we have Eq. (40), on the top of the next page. Eq. (16) is derived by replacing Eqs. (39) and (40) in Eq. (38). In other words, Eq. (16) is correct for sensors whose level is smaller than or equal to $k + 1$. In summary, Eq. (16) is correct for sensors in all levels of heterogeneous WSNs. ■

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$$(\eta^2 - 1)(x^2 + y^2) + 2(p_{jx} - \eta^2 p_{ix})x + 2(p_{jy} - \eta^2 p_{iy})y = p_{jx}^2 - \eta^2 p_{ix}^2 + p_{jy}^2 - \eta^2 p_{iy}^2 \quad (28)$$

$$V_k^H(P) = \{q \in Q \mid \eta_k \|q - p_k\|^2 \leq \eta_i \|q - p_i\|^2, \eta_k \|q - p_k\|^2 \leq \eta_j \|q - p_j\|^2, \forall i, j \text{ such that } \eta_j > \eta_k, \eta_k \geq \eta_i\} \quad (32)$$

$$W_k = \{q \in Q \mid \eta_j \|q - p_j\|^2 \leq \eta_i \|q - p_i\|^2, \eta_j \|q - p_j\|^2 \leq \eta_k \|q - p_k\|^2, \eta_k \|q - p_k\|^2 \leq \eta_i \|q - p_i\|^2 \text{ for any } i, j \text{ such that } \eta_j > \eta_k, \eta_k \geq \eta_i\}. \quad (34)$$

$$\int_{V_i^H(P) \cap W} \varphi(q, p_i) dq = \int_{\bar{V}_i^H(P) \cap W} \varphi(q, p_i) dq - \sum_{l=1}^k \sum_{t \in Z_l} \left[\int_{V_t^H(P) \cap \bar{V}_i^H(P) \cap W} \varphi(q, p_i) dq \right]. \quad (37)$$

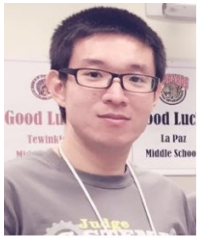
$$\int_{\partial[\bar{V}_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} d\gamma - \sum_{l=1}^k \sum_{t \in Z_l} \left[\int_{\partial[V_t^H(P) \cap \bar{V}_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} d\gamma \right] = \int_{\partial[V_i^H(P) \cap W]} \varphi_i(\gamma, p_i) n^t(\gamma) \frac{\partial \gamma}{\partial p_j} d\gamma. \quad (39)$$

$$\int_{\bar{V}_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq - \sum_{l=1}^k \sum_{t \in Z_l} \left[\int_{V_t^H(P) \cap \bar{V}_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq \right] = \int_{V_i^H(P) \cap W} \frac{\partial \varphi_i(q, p_i)}{\partial p_j} dq. \quad (40)$$

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