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Paul Hoyer

April 28, 1976

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SISTER TRAJECTORIES IN THE DUAL RESONANCE MODEL*

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April 28, 1976

ABSTRACT

The existence of a Regge trajectory $\gamma = \frac{1}{3}\alpha - 1$ in the eight-point amplitude of the Dual Resonance Model is demonstrated. It is suggested that the three presently known trajectories α , β and γ are part of an infinite set of "sister" trajectories, the expected properties of which are discussed.

The recent discovery [1] of a new Regge trajectory β in the Dual Resonance Model (DRM) six-point amplitude raises several interesting questions. One of these is whether the two trajectories α and $\beta = \frac{1}{2}\alpha - \frac{1}{2}$, together with their daughters, represent the entire Regge pole content of the DRM. The study of the B_6 amplitude shows that α and β are indeed the only reggeons with nonvanishing couplings to two or three particles (at the tree-diagram level). The question is therefore whether there are further trajectories that couple only to four (or more) particles, and so appear only in eight-point (or higher) amplitudes.

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In this letter we study a high-energy limit of the DRM B_8 amplitude, in which reggeons with a nonzero coupling to four particles can be exchanged. We find that a new trajectory $\gamma = \frac{1}{3}\alpha - 1$ contributes in this limit. This result makes it appear very plausible that α , β and γ are the first three members of an infinite set of "sister" trajectories in the DRM,

$$\alpha_k(t) = \frac{1}{k} \alpha(t) - \frac{1}{2} (k - 1), \quad (k = 1, 2, \dots) \quad (1)$$

where α_k couples only to $k + 1$ (and more) particles. Each trajectory α_k is furthermore accompanied by an infinite set of daughters, that are parallel to it and spaced by $1/k$. In fig. 1 we show the Chew-Frautschi plot with the α , β and γ trajectories together with their daughters. In general, two consecutive sister trajectories α_k , α_{k+1} intersect at $\alpha_k = \alpha_{k+1} = -k$. A given trajectory α_k thus represents the leading exchange in the region $-k \leq \alpha_k \leq -(k - 1)$, provided it contributes to the amplitude.

In isolating the γ exchange contribution to B_8 we shall draw on our experience with β exchange in B_6 . It is clear that both the β and γ contributions must vanish in limits where all asymptotic variables approach $-\infty$, since such limits have been extensively studied previously [2]. In order to simplify the problem we shall begin by taking the four-fold helicity-pole (HF) limit of B_8 shown in fig. 2a. Because only α -type exchanges α_a , α_b , α_c and α_d can couple to each pair of external particles in fig. 2a, we expect that this limit can be calculated by first taking the relevant variables to $-\infty$, and then analytically continuing them to $+\infty$, if required. The analogous method for B_6 was explicitly shown [1] to give correct results.

The HP limit in fig. 2a implies that the following ratios of large variables vanish:

$$\frac{s_{26}}{s_{24}} = \frac{s_{46}}{s_{13}} = \frac{s_{35}}{s_{68}} = \frac{s_{37}}{s_{57}} = \frac{s_{27}}{s_{24}s_{57}} = \frac{s_{45}}{s_{13}s_{68}} = 0. \quad (2)$$

By explicitly constructing the four-momenta of all particles one may easily verify that the conditions (2) are compatible with the four-dimensionality constraints. Taking the limit $\alpha_{24}, \alpha_{13}, \alpha_{68}, \alpha_{57} \rightarrow -\infty$ under the constraint (2) one obtains in the standard way*

$$B_8 \simeq (-\alpha_{24})^{\alpha_a} (-\alpha_{13})^{\alpha_b} (-\alpha_{68})^{\alpha_c} (-\alpha_{57})^{\alpha_d} \Gamma(-\alpha_a) \Gamma(-\alpha_d) F, \quad (3a)$$

$$F = \int_0^1 dy y^{-\alpha_s-1} (1-y)^{-\alpha_t+\alpha_b+\alpha_c-1} \int_0^\infty dz_1 dz_2 z_1^{-\alpha_b-1} z_2^{-\alpha_c-1} e^{-z_1-z_2} \times \left[1 - y - z_1/\kappa_{23} - z_2/\kappa_{25} \right]^{\alpha_a} \left[1 - y - z_2/\kappa_{67} - z_1/\kappa_{47} \right]^{\alpha_d}. \quad (3b)$$

Here the ratios of asymptotic variables which remain finite in the helicity pole limit are given by

$$\begin{aligned} \kappa_{23} &= \alpha_{24}\alpha_{13}/\alpha_{23} & \kappa_{67} &= \alpha_{68}\alpha_{57}/\alpha_{67} \\ \kappa_{25} &= \alpha_{24}\alpha_{68}/\alpha_{25} & \kappa_{47} &= \alpha_{13}\alpha_{57}/\alpha_{47}. \end{aligned} \quad (4)$$

The four-dimensionality constraints imply that all the κ variables are numerically equal:**

* It is helpful to start from the expression for the two-reggeon, four-particle amplitude given in ref. 3.

** This is most easily seen by explicitly constructing the four-momenta of all particles using eq. (2), cf. ref. 1.

$$\kappa = \kappa_{23} = \kappa_{67} = \kappa_{25} = \kappa_{47}. \quad (5)$$

However, for the discussion below concerning the analytic structure of B_8 in the asymptotic variables it is important to treat them as being independent.

After the (analogous) HP limit had been imposed on B_6 , the amplitude could be expressed [1] as a sum of terms with distinct singularity structures in the large variables. This was a direct consequence of the generalized Steinmann relations, which forbid simultaneous discontinuities in overlapping variables. The β trajectory was present in each of the terms, even in limits where all asymptotic variables approached $-\infty$. Thus the vanishing of the β contribution to the full amplitude in such limits was due to a cancellation between the various terms. This suggests that the simplest way of seeing γ exchange is by first isolating a single term in the corresponding decomposition of B_8 , and then going to the di-triple Regge limit of fig. 2b by letting $\alpha_s \rightarrow -\infty$. This is the procedure that we follow here.

When considering the analytic structure of B_8 in the eight large variables of the HP limit (fig. 2a), it is helpful to observe that the function F of eq. (3b) depends on these variables only through the four ratios defined by eq. (4). Thus, the term in the decomposition of B_8 that has a simultaneous discontinuity in α_{24} and α_{57} must correspond to a term in F of the form

$$(-\kappa_{23})^{-\alpha_b} (-\kappa_{67})^{-\alpha_c} E(\kappa_{23}, \kappa_{67}, \kappa_{25}, \kappa_{47})$$

where E is an entire function of all four κ variables. This follows because $\alpha_{25}(\alpha_{47})$ is overlapping with $\alpha_{57}(\alpha_{24})$, so that

there can be no cuts in κ_{25} or κ_{47} . The specific dependence on κ_{23} and κ_{67} is required in order that the singularities of eq. (3a) in α_{13} and α_{68} cancel out. Hence, if we isolate this term in B_8 by taking a double discontinuity in α_{24} and α_{57} , it will have the form

$$D_{24} D_{57} B_8 = (-\alpha_{23})^{\alpha_b} (-\alpha_{67})^{\alpha_c} \alpha_{24}^{\alpha_a - \alpha_b} \alpha_{57}^{\alpha_d - \alpha_c} \frac{(2\pi i)^2}{\Gamma(1 + \alpha_a) \Gamma(1 + \alpha_d)} I \quad (6)$$

where the integral I is entire in all the κ variables. It is straightforward to take this double discontinuity using the expression (3) for B_8 , by writing the two brackets in eq. (3b) as linear functions of α_{24} and α_{57} , respectively. After a change of variables $x_1 = z_1/\kappa_{23}$, $x_2 = z_2/\kappa_{67}$ one obtains

$$I = \int_0^1 dy y^{-\alpha_s - 1} (1 - y)^{-\alpha_t + \alpha_b + \alpha_c - 1} \int_0^1 dx_1 dx_2 x_1^{-\alpha_b - 1} x_2^{-\alpha_c - 1} e^{-x_1 \kappa_{23} - x_2 \kappa_{67}} \times \left[1 - y - x_1 - x_2 y \kappa_{67}/\kappa_{25} \right]^{\alpha_a} \left[1 - y - x_2 - x_1 y \kappa_{23}/\kappa_{47} \right]^{\alpha_d} \quad (7)$$

The upper limits of the x_1, x_2 integrations are fixed by the requirement that the brackets should be positive inside the integration region.

At this stage we can simplify the expression (7) for I by imposing the kinematic constraints (5). In four dimensions κ is moreover a linear function [1] of α_s , so that $\kappa/\alpha_s \rightarrow 1$ as $|\alpha_s| \rightarrow \infty$.^{*} Thus I may be regarded as a function of a single complex variable α_s (for fixed momentum transfers), whose only singularities

^{*} In higher dimensions this constraint should be viewed as following from the definition of the Regge limit.

are poles on the real axis. The Phragmén-Lindelöf theorem then guarantees that the limits $\alpha_s \rightarrow +\infty$ and $\alpha_s \rightarrow -\infty$ are simply related. Since the integral representation (7) is convergent for $\alpha_s < 0$, it is technically easier to derive the $\alpha_s \rightarrow -\infty$ limit of I . In this limit also $\kappa \rightarrow -\infty$, so that some other large variables except α_{24} and α_{57} will have to approach $+\infty$. For definiteness, we shall take $\alpha_{23}, \alpha_{67}, \alpha_{25}, \alpha_{47} \rightarrow \infty + i\epsilon$ and $\alpha_{13}, \alpha_{68} \rightarrow -\infty$. This gives rise to a phase factor $\exp[-i\pi(\alpha_b + \alpha_c)]$ in eq. (6).

The $\alpha_s \rightarrow -\infty$ limit of I can be derived using methods that are analogous to the calculation [1] of β exchange in B_6 . We shall therefore only indicate the main steps. The integrand in eq. (7) contains two factors with an exponential dependence on α_s : $\exp(-\alpha_s \log y)$ and $\exp[-\kappa(x_1 + x_2)]$. One can easily verify that the maximum value of $x_1 + x_2$ is reached when both brackets vanish simultaneously, at which point $x_1 = x_2 = (1 - y)/(1 + y)$. For any y in the range $0 \leq y < 1$ we have $-\log y > 2(1 - y)/(1 + y)$. Thus the integrand will be exponentially damped as $\alpha_s \rightarrow -\infty$ with $\kappa/\alpha_s \rightarrow 1$ unless $y \rightarrow 1$. We can therefore expand in $(1 - y)$:

$$-\log[1 - (1 - y)] = (1 - y) + \frac{1}{2} (1 - y)^2 + \frac{1}{3} (1 - y)^3 + \dots \quad (8a)$$

$$2 \frac{1 - y}{1 + y} = (1 - y) + \frac{1}{2} (1 - y)^2 + \frac{1}{4} (1 - y)^3 + \dots \quad (8b)$$

At the maximum value of $x_1 + x_2$ the exponential behavior of the integrand is thus given by $\exp[\alpha_s(1 - y)^3/12]$, leading to the exchange of a γ trajectory with slope $1/3$. For smaller values of $x_1 + x_2$ one has to distinguish between the region where only the term $(1 - y)$ in (8a) is canceled (β exchange) and that in which no term is canceled (α exchange).

The above arguments indicate that it is advantageous to use an integration variable proportional to $x_1 + x_2$ in eq. (7). In terms of the new variables $t = (x_1 + x_2)/(1 - y)$ and $u = x_2/(x_1 + x_2)$ the integration region can in fact be conveniently divided into four regions, each of which contributes to the exchange of a single Regge pole in the limit $\alpha_s \rightarrow -\infty$:

$$\left. \begin{aligned} (\alpha): \quad 0 \leq t \leq 1 - (-\alpha_s)^{-\epsilon} \\ (\beta 1): \quad 1 - (-\alpha_s)^{-\epsilon} \leq t \leq 1 \end{aligned} \right\} \quad 0 \leq u \leq 1$$

$$\left. \begin{aligned} (\beta 2): \quad 0 \leq t - 1 \leq \frac{1}{2} (1 - y) [1 - (-\alpha_s)^{-\eta}] \\ (\gamma): \quad \frac{1}{2} (1 - y) [1 - (-\alpha_s)^{-\eta}] \leq t - 1 \leq \frac{1 - y}{1 + y} \end{aligned} \right\} \quad t - 1 \leq ut(1 - y) \leq 1 - yt.$$

Here ϵ and η are arbitrary numbers in the range $0 < \epsilon < \frac{1}{2}$ and $0 < \eta < \frac{1}{3}$. The region giving rise to β exchange has to be divided into two parts ($\beta 1$) and ($\beta 2$) due to the varying range of integration in u . After the high energy limit has been taken, the β residue coming from the two regions is given by nonfactorizable integrals. However, the integrals can be combined to give a sum of two factorizable terms. All together we find in the limit of fig. 2b:

$$D_{24} D_{57} B_8 \simeq (2\pi i)^2 e^{-i\pi(\alpha_b + \alpha_c)} \alpha_{23}^{\alpha_b} \alpha_{67}^{\alpha_c} \alpha_{24}^{\alpha_a - \alpha_b} \alpha_{57}^{\alpha_d - \alpha_c} (-\alpha_s)^{-\alpha_a - \alpha_d} \times [R(\alpha) + R(\beta) + R(\gamma)] \quad (9a)$$

$$R(\alpha) = (-\alpha_s)^{\alpha_t} \frac{\Gamma(-\alpha_t + \alpha_a + \alpha_d)}{\Gamma(1 + \alpha_a) \Gamma(1 + \alpha_d)} \Gamma(1 + \alpha_t) \frac{\Gamma(-\alpha_b) \Gamma(-\alpha_c)}{\Gamma(1 + \alpha_t - \alpha_b - \alpha_c)} \quad (9b)$$

$$R(\beta) = (-\alpha_s)^{\beta_t} \Gamma(-1 - \beta_t) \Gamma(2 + \beta_t) \Gamma(-\beta_t + \alpha_a + \alpha_d) 2^{-\beta_t - 1 + \alpha_b + \alpha_c} \times \left[\frac{\Gamma(-\alpha_c)}{\Gamma(1 + \alpha_d) \Gamma(\alpha_a - \beta_t) \Gamma(2 + \beta_t - \alpha_c)} \times {}_2F_1(-1 - \beta_t, -\alpha_d; \alpha_a - \beta_t; -1) {}_2F_1(2 + \beta_t, 1 + \alpha_b; 2 + \beta_t - \alpha_c; -1) + (\alpha_a \leftrightarrow \alpha_d; \alpha_b \leftrightarrow \alpha_c) \right] \quad (9c)$$

$$R(\gamma) = (-\alpha_s)^{\gamma_t} \Gamma(-2 - \gamma_t) 2^{\alpha_a + \alpha_b + \alpha_c + \alpha_d} (12)^{-\gamma_t/54} \quad (9d)$$

where $\beta_t = \frac{1}{2} \alpha_t - \frac{1}{2}$ and $\gamma_t = \frac{1}{3} \alpha_t - 1$. The function ${}_2F_1(a, b; c; x)$ is the ordinary hypergeometric function [4].

The above result is in several respects analogous to what was previously found for B_6 , and provides a rather clear hint as to what the general pattern will be as one considers higher point functions. First, we note that the constraint $\kappa/\alpha_s = 1$ was crucial in obtaining eqs. (9). If we had taken $\kappa/\alpha_s < 1$, the cancellations between the terms in (8a) and (8b) could not have occurred, and only α -exchange would have resulted. If, on the other hand, we had assumed $\kappa/\alpha_s > 1$, then the exponentially increasing term in the integrand of eq. (7) would have dominated, resulting in an exponential behavior of $D_{24} D_{57} B_8$ as $\alpha_s \rightarrow -\infty$.

It is natural to conjecture that α , β and γ are the first three members of an infinite set of sister trajectories α_k with progressively diminishing slopes $1/k$ ($k = 1, 2, \dots$). Assuming that two

The signature of the γ trajectory is not clear, since we here considered only a discontinuity of B_8 . However, in order that γ should cancel the spurious poles of β also in the signaturized amplitude, γ must have (at least a component of) positive signature. Similarly, the degeneracy level of the trajectories is not determined. From its factorization properties in eq. (9c), β must be at least doubly degenerate (this was seen already at the B_6 level [1].). Presumably γ is likewise degenerate.

$2 \rightarrow 2$ reactions should curve upward from the linear α trajectory as $-t$ increases. This effect would in fact be qualitatively similar to that expected from multiple Regge cuts.

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FIGURE CAPTIONS

Fig. 1. Trajectories of the α , β and γ families as they appear on a Chew-Frautschi plot. The horizontal scale (and hence the intercepts) are arbitrary. Heavy lines denote parent trajectories.

Fig. 2. (a) A four-fold helicity pole limit of the eight-point amplitude. We use the "inclusive" notation common in the DRM, e.g., $\alpha_{13} = \alpha(s_{13}) = \alpha[(p_1 + p_2 + p_3)^2]$. We also use the alternative notation as indicated:

$$\alpha_s = \alpha_{14}, \quad \alpha_t = \alpha_{36}, \quad \alpha_a = \alpha_{12}, \quad \alpha_b = \alpha_{34}, \quad \alpha_c = \alpha_{56}, \\ \alpha_d = \alpha_{78}.$$

(b) The di-triple Regge limit obtained from the limit in (a) when $|\alpha_s| \rightarrow \infty$.

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Fig. 1

(a)

(b)

Fig. 2

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