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THE $3\pi^0$ DECAY OF THE K_L^0

Ivan R. Linscott
(Ph. D. Thesis)

July 1972

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ABSTRACT

The $3\pi^0$ decay of the K_L^0 offers the opportunity for observing a possible resonant $\pi^0 - \pi^0$ interaction. Six shower events from the decay $K_L^0 \rightarrow 3\pi^0$ have been reconstructed kinematically to obtain the $3\pi^0$ Dalitz plot, using a monoenergetic K_L^0 beam and a nearly 4π solid-angle, lead plate, spark chamber gamma ray detector. The Dalitz plot does not show any resonant structure. Representing the Dalitz position in polar coordinates (r, θ) , the Matrix element for the decay, expanded to second order, is of the form $M(3\pi^0) = 1 + \alpha r^2$. The radial distribution of these events has a slope $\alpha = .05 \pm .08$, consistent with a small $\pi^0 - \pi^0$ scattering length.

I. INTRODUCTION

The system of two interacting pions is one of the simplest and most fundamental in strong interaction physics, and yet presents one of the most elusive experimental problems. Since pions are spinless bosons, the interacting states are few and depend only on the quantum numbers of charge and overall angular momentum. The study of the two pion system has been extremely attractive, not only because the form of the interaction is particularly uncomplicated and thus the nature of the strong interaction readily observable, but also due to the resonant structure in the system. Since the pion is unstable, (lifetime of the order of 10^{-8} sec for the charged pions and 10^{-16} sec for the neutral pion) the production of pion targets has not been feasible to date, and so other means have been sought to observe the direct interaction of pions at low energy.

The presence of resonances in the $\pi - \pi$ interaction has been anticipated for some time. A resonant amplitude with a mass of about 600 MeV and a width of about 100 MeV for $I(J^P) = 1(1^-)$ was predicted to explain the total cross-section in π, p scattering in 1955.¹ Also about that time a similar resonant state was required to explain the electromagnetic form factor of the nucleon² and fit the electron nucleon scattering data.³ The subsequent study of the $\pi - \pi$ system in production reactions of the type $\pi N \rightarrow \pi\pi N$ revealed a large enhancement in the $\pi - \pi$ invariant mass at 760 MeV.⁴ This resonance, called the ρ , had three charged states and implied that it had an isospin quantum

number $I = 1$. Further resonances were discovered in the two pion system at higher energies, for example, the $f(1260)$ meson⁵ with $I(J^P) = 0(1^+)$.

A more detailed examination of the angular distribution in the two pion system uncovered asymmetries in the decay distribution of the $\pi^+\pi^-$. This asymmetry was attributed to the interference of the ρ with an $I = 0$, $J^P = 0^+$ resonance referred to as the epsilon or the sigma, of about the same mass as the ρ . Detailed examinations of the $\pi - \pi$ system since that time have always produced inconclusive results.

The optimum condition for the formation of such a spin zero resonance would be in the interaction of two neutral pions, since this resultant state must be even in isospin. Hence the desirability of observing a situation where neutral pions can interact at low energy makes the $3\pi^0$ decay mode of the K_L^0 meson attractive. In this experiment, a measurement of this interaction was performed by constructing the $3\pi^0$ Dalitz plot from $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ events detected in a lead plate spark chamber array.

Inherent in this $3\pi^0$ system are several difficulties. The weak transition of the K_L^0 into three neutral pions is not expected to significantly distort the phase space of the three pion system, but there is no direct way of verifying this. The energy range over which the pions can interact is very limited by the low Q value of the decay (about 100 MeV), and since each pion decays principally into two photons, all the resolution problems associated with the detection and observation of

high energy photons will be propagated into the $\pi - \pi$ system.

In a straightforward analysis, these errors are such as to obliterate any structure on the Dalitz plot. The principal effort of this experiment has been to formulate the kinematics and methods of analysis to obtain a Dalitz plot resolution capable of revealing a strong $\pi - \pi$ interaction. Using various techniques, the effects of a non-resonant, threshold $\pi - \pi$ interaction will be estimated. These show that a very high resolution would be necessary to measure such effects.

A. Phenomenology of $K_L^0 \rightarrow 3\pi^0$.

The K_L^0 meson decays into three neutral pions via the Weak interaction with a rate of $\Gamma = 4.81 \pm .41 \times 10^6 \text{ sec}^{-1}$. The matrix element for the decay process, defined in terms of the rate, is

$$d\Gamma = |M|^2 dR$$

with dR the phase space, and M the matrix element containing any dynamic energy dependence for the decay. The matrix element may be expanded in terms of the pion kinetic energies as,

$$M_{123} = a + \sum_i b_i T_i + \sum_i c_i T_i^2 + \sum_i d_i T_j T_k + \dots \quad (1)$$

$$i = 1, 3 \text{ and } k \neq j \neq i$$

where the expansion includes terms up to second order, and $T_i = E_i - m_\pi$, E_i is the pion energy in the K_L^0 rest frame. Other functions of the pion energies, such as relativistic invariants may be useful. Since these classes of functions are usually linearly related to each other, then any such class will suffice for a discussion of form. The symmetry due to Bose' statistics requires the matrix element to be invariant under the interchange of any two pions, so,

$$M_{123} = M_{132} = M_{213} = M_{231} = M_{321} = M_{312}$$

which reduces (1) to:

$$M(3\pi^0) = a + b(T_1 + T_2 + T_3) + c(T_1^2 + T_2^2 + T_3^2) + d(T_1 T_2 + T_1 T_3 + T_2 T_3).$$

Since $T_1 + T_2 + T_3 = Q$, $Q = M_K - 3m_\pi$, then

$$M(3\pi^0) = a_0' (1 + \alpha' \sum_i T_i^2).$$

Now the coefficients are, $a'_0 = a + bQ + \frac{dQ^2}{2}$ and $\alpha' = (c - d/2)/a_0$. With $\sum_i T_i^2 = \frac{Q^2}{3} (1 + \frac{1}{2} r^2)$, where r is the distance for the configuration (T_1, T_2, T_3) from the center of the Dalitz plot (see section on Dalitz plot analysis), the $3\pi^0$ matrix element, to second order, becomes,

$$M(3\pi^0) = a_0 (1 + \alpha r^2). \quad (2)$$

Hence, the determination of the Dalitz radial distribution is a measure of the effects of the weak transition and final state interaction on the three neutral pions. The sensitivity of α to the dynamics of the $\pi - \pi$ system may be understood by calculating α using several formalisms.

Since the K_L^0 is a superposition of the states of definite strangeness K^0 , $\bar{K}^0$ that are in turn members of an isotopic spin doublet, the amplitudes for other 3π modes of K decay are related to each other.

An estimate of α in (2) can be obtained, for example, from fits to a similar expansion of the matrix element for $K^+ \rightarrow \pi^+ \pi^+ \pi^-$. Also, the amplitude for $K_L^0 \rightarrow 3\pi^0$ may be related to the amplitude for $K_S^0 \rightarrow 2\pi^0$ in the limit of one of the pion four momenta vanishing, providing a direct calculation of α . Additionally, a simple phase shift model may be used to parameterize the $3\pi^0$ amplitude in terms of the $\pi\pi$ scattering lengths, allowing an estimate of α to be made.

Each of these methods will be detailed in the following sections.

B. Isotopic spin formalism.

Since K^0 and $\bar{K}^0$ are mesons of definite strangeness, and $\bar{K}^0 = CP K^0$, then states of definite lifetime K_1^0 and K_2^0 , may be formed from superpositions of the K^0 and $\bar{K}^0$ states.⁶

$$K_1^0 = \sqrt{1/2} [K^0 + \bar{K}^0],$$

$$K_2^0 = \sqrt{1/2} [K^0 - \bar{K}^0].$$

So $CP K_1^0 = K_1^0$, and $CP K_2^0 = -K_2^0$. CP is not conserved in K^0 decay, but the "amount" of CP non-conservation is small,⁷ and its effects will be negligible in the 3π amplitudes.

Since for the 3π state, $CP = (-1)^I$, then $I = 1$ or 3 if CP is conserved. The decay amplitudes may be expressed in factored form as,

$$M = \sum M_I M_F M_{JP}$$

M_I contains the isotopic spin transformation properties

M_F are form factors of the pion energy and momenta

M_{JP} contain the spin-parity transformation properties

where the sum is carried out over the various spin states. M_I may be constructed out of isospin vectors $\bar{a}_i = (a_{i1}, a_{i2}, a_{i3})$ which transform like rectangular coordinates under isospin rotations. Then, for example, the $I = 1$ amplitudes can be represented as,

$$\begin{aligned} M_{I=1} &= A_1 \bar{a}_1 (\bar{a}_2 \cdot \bar{a}_3) + A_2 \bar{a}_2 (\bar{a}_1 \cdot \bar{a}_3) + A_3 \bar{a}_3 (\bar{a}_1 \cdot \bar{a}_2) \\ &= -(A_1 + A_2 + A_3) \text{ for } 3\pi^0 \\ &= A_1 + A_2 \text{ for } \pi^+\pi^+\pi^- \end{aligned} \tag{3}$$

where the A_i may be regarded as symmetric functions of the pion momenta and are invariant under the interchange, $\pi_j \leftrightarrow \pi_k$, for $j \neq k \neq i$. For

decays with small Q , these functions may be expanded as,

$$A_i = a + b s_i + c s_i^2 + d s_j s_k$$

$$s_i = T_i - Q/3$$

$$s_1 + s_2 + s_3 = 0$$

where the expansion is up to order two.

Characterizing the decay amplitudes by their isospin transformation properties requires $M_I = \sum M_{I, \Delta I}$, since for $I=1$, $\Delta I = 1/2$, or $3/2$, and for $I=3$, $\Delta I = 5/2$ or $7/2$. Experimentally, the $I=1$, $\Delta I = 1/2$ amplitude is dominant,⁸ so neglecting contributions from states with $I \neq 1$, or $\Delta I \neq 1/2$, table 1 lists the amplitudes, to second order, for various $K \rightarrow 3\pi$ decays.

Table 1

Decay	Amplitude $I = 1, \Delta I = 1/2$
$K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$	$2a - b s_3 + c(s_1^2 + s_2^2) + d(s_2 s_3 + s_1 s_3)$
$K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$	$a + b s_3 + c s_3^2 + d s_1 s_2$
$K_L^0 \rightarrow 3\pi^0$	$-[3a + (s_1^2 + s_2^2 + s_3^2) \cdot (c - d/2)]$
$K_L^0 \rightarrow \pi^+ \pi^- \pi^0$	$-[a + b s_3 + c s_3^2 + d s_1 s_2]$

The matrix element $M(++-)$ for the decay $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ is then,

$$M(++-) = 4|a|^2 [1 + a_+ s_3 + b_+(s_1^2 + s_2^2) + c_+ s_3^2]$$

where

$$a_+ = \frac{-2(a^* b + ab^*)}{4|a|^2}, \quad b_+ = \frac{2(a^* c + ac^*)}{4|a|^2}, \quad c_+ = \frac{|b|^2 - 2(a^* d + ad^*)}{4|a|^2}.$$

Fits of this decay to matrix elements of the form $1 + g y + h y^2$, and $1 + r x^2$, give,⁹ $g = .24 \pm .01$, $h = -.023 \pm .019$, $r = -.04 \pm .02$, where x and y are the cartesian Dalitz variables. So from these projections, $a_+ = g$, $b_+ = (8/3)r$, $c_+ = h - \frac{1}{2}b_+$.

Similarly, the matrix element $M(ooo)$ for the decay $K_L^0 \rightarrow 3\pi^0$ is,

$$M(ooo) = 9|a|^2 \left[1 + a_n \sum s_i^2 \right]$$

with

$$\begin{aligned} a_n &= (1/3) \left[\frac{a^* c + a c^*}{|a|^2} - 1/2 \frac{a^* d + a d^*}{|a|^2} \right] \\ &= (1/3) [2b_+ + c_+ - (1/4)|b|^2/|a|^2]. \end{aligned}$$

If a and b are both mostly real then $|b|^2/|a|^2 \simeq a_+^2$. However, if a is mostly imaginary, and b is mostly real, or vice versa, then $|b|^2/|a|^2 \simeq 0$. So $|b|^2/|a|^2$ does not make a significant contribution to a_n , and,

$$a_n = -.02.$$

Thus $\alpha \simeq a_n/6 \simeq -.003$.

(4)

C. Single Pion reduction.

The algebra of currents as applied to the weak interaction and the notion of a partially conserved axial-vector current (PCAC), have given an impressive quantitative account of effects in β decay and K-meson decay. In particular, for the decay $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$, the matrix element depends only on the energy of the π^0 , to first order, and the calculation for the slope of that dependence is in good agreement with experiment. So by extending this technique of expressing the matrix element for the three pion process in terms of the matrix element for a two pion process (in other words reducing out one pion) the amplitude and matrix element for $K_L^0 \rightarrow 3\pi^0$ decay will be calculated.

Define the amplitude for a transition from a state of n non-interacting particles, $|(p_1, p_2, \dots, p_n)\rangle = |\alpha_{in}\rangle$, to a state of m non-interacting particles, $|(q_1, q_2, \dots, q_m)\rangle = |\beta_{out}\rangle$, as

$$S_{\beta\alpha} = \langle \beta_{out} | \alpha_{in} \rangle.$$

In particular, let $|\alpha'_{in}\rangle = |\alpha_{in}, p\rangle$ be the above assembly plus an additional scalar boson. Then,¹⁰

$$\langle \beta_{out} | \alpha'_{in} \rangle = \langle \beta_{out} | a_{in}^\dagger(p) | \alpha_{in} \rangle = \langle \beta_{out} | a_{out}^\dagger(p) | \alpha_{in} \rangle + \langle \beta_{out} | a_{in}^\dagger - a_{out}^\dagger | \alpha_{in} \rangle.$$

With $a(p)$, and $a^\dagger(p)$ the annihilation and creation operators for one particle states with momentum p expressed in terms of scalar fields $\phi(x)$ satisfying the Klein-Gordon equation as,¹¹

$$a(p) = i \int \frac{d^3x}{\sqrt{(2\pi)^3 2\omega_p}} e^{-ip \cdot x} \overleftrightarrow{\partial}_0 \phi(x).$$

Substitution and integration by parts gives for the matrix element,

$$S_{\beta, \alpha p} = \Delta_{\beta\alpha} + i \int \frac{d^4 x}{\sqrt{(2\pi)^3 2\omega_p}} e^{ip \cdot x} (\square^2 + m^2) \langle \beta_{out} | \phi(x) | \alpha_{in} \rangle$$

where $\Delta_{\beta\alpha} = \langle \beta_{out} | a_{out}^\dagger(p) | \alpha_{in} \rangle$ represents, for example, forward elastic scattering.

Now for the decay $K_L^0 \rightarrow 3\pi^0$, $M = S_{3\pi, K} = \langle (\pi\pi)\pi | H_w^\ell(0) | K_L^0 \rangle$, where $H_w^\ell(0)$ is the weak, strangeness changing non-leptonic Hamiltonian current density,¹² then

$$M = i \int d^4 x e^{-iq \cdot x} (\square^2 + m^2) \langle (\pi\pi) | T(\phi_{\pi_j}(x) H_w^\ell(0) | K_L^0 \rangle.$$

The factor $\sqrt{(2\pi)^3 2\omega_q}$ has been absorbed into the amplitude. Integrating by parts, with neglect of surface terms, and using the PCAC hypothesis,¹³

$$\partial_\lambda \mathcal{F}_j^{5\lambda}(x) = \frac{c}{\sqrt{2}} \phi_{\pi_j}(x), \quad c = F_\pi$$

(where $\mathcal{F}_j^{5\lambda}(x)$ is a member of the axial vector octet of currents having the quantum numbers of the π^j meson, $j = +, -, 0$; and

$c = 2g_a M_n m\pi^2 / g_r(0)$, with g_a the ratio of axial-vector to vector β -decay coupling constants, and g_r the strong pion-nucleon coupling constant), the decay amplitude becomes, in the limit of $q^2 \rightarrow 0, q_\lambda \rightarrow 0$,

$$M(q_{\pi_j} \rightarrow 0) = -i \frac{\sqrt{2}}{c} m^2 \int d^3 x e^{iq \cdot x} \langle (\pi\pi) | [\mathcal{F}_j^{50}(x), H_w^\ell(0)] | K_L^0 \rangle.$$

Using the commutation relation, $^{14} [\mathcal{F}_j^{5_0}(x), H_W^\ell(0)] = (1/2) \delta(x) H_W^\ell(0)$, for $j = 3$ only, then,

$$\begin{aligned} M(q_{\pi^0} \rightarrow 0) &= i \frac{\sqrt{2}}{c} m^2 \int d^3x e^{iq \cdot x} (1/2) \delta(x) \langle \pi\pi | H_W^+(0) | \sqrt{1/2} (K^0 - \bar{K}^0) \rangle \\ &= 1/2 \frac{\sqrt{2}}{c} m^2 \langle \pi^0 \pi^0 | H_W^+(0) | \sqrt{1/2} (K^0 + \bar{K}^0) \rangle. \end{aligned}$$

Note that the term in brackets on the right hand side is just the amplitude for $K_S^0 \rightarrow 2\pi^0$, so finally, denoting the amplitudes by A,

$$A_S(00) = [2g_a M_n / g_r(0)] A_L(000). \quad (5)$$

Experimentally, $^{15} A_S = .56 \times 10^{-6}$, and assuming A_L is a constant, $[2g_a M_n / g_r(0)] A_L = .4 \times 10^{-6}$, indicating some structure may be present in the decay amplitude. So let

$$A_L = A_0 \left[1 + \frac{a}{m_\pi^4} \left(\sum_i s_i^2 - B_0 - 4M_K^2 Q^2/3 \right) \right]$$

where s_i is the relativistic invariant for the i th pion, $s_i = (q_{4j} + q_{4k})^2 = (q_{4K} - q_{4i})^2$. When $q(\pi_1^0) \rightarrow 0$, then $s_1 = M_K^2$, $s_2 = m_\pi^2$, $s_3 = m_\pi^2$. The form of A_L is chosen because of the relation,

$$\sum_i s_i^2 = B_0 + 4M_K^2 Q^2/3 + (2M_K^2 Q^2/3)r^2$$

where $B_0 = 3(M_K - m_\pi)^4 - 4(M_K - m_\pi)^2 M_K Q$, and r is the radial distance on the Dalitz plot. So evaluating A_L when $q(\pi^0) \rightarrow 0$, and using Eq. (5), gives

$$a = \frac{m_\pi^4/2}{M_K^4 + 2m_\pi^4 - B_0 - 4M_K^2 Q^2/3}.$$

Thus, from Eq. (2),

$$\alpha = a \frac{2M_K^2 Q^2/3}{m_\pi^4} = .033. \quad (6)$$

D. Phase shift model.

If the pions are predominantly in an $I = 1$ state in the $K_L^0 \rightarrow 3\pi^0$ decay, then the amplitude for the process, Eq. (3), may be written,

$$A(3\pi) = -(A_1 + A_2 + A_3).$$

Let A_i be the amplitude for the scattering of two pions in the presence of a third pion as an observer. Then,

$$A_i = A_{\pi_j \pi_k} e^{iq_i r}$$

where $A_{\pi_j \pi_k}$ is the scattering of pions $j \neq k \neq i$, q_i is the momentum of pion i in the j, k frame, and r is the region in which the scattering occurs. Since two neutral pions are in an $I = 0$, or $I = 2$ state,

$$A_{\pi_j^0 \pi_k^0} = -\sqrt{\frac{1}{3}} A_i^{I=0}(\pi_j^0 \pi_k^0) + \sqrt{\frac{2}{3}} A_i^{I=2}(\pi_j^0 \pi_k^0).$$

Representing the $\pi\pi$ amplitude by a phase shift decomposition,

$$A(\pi\pi) = \frac{1}{k} \sum_{\ell} (2\ell + 1) \left(\frac{\eta_{\ell} e^{2i\delta_{\ell}} - 1}{2i} \right) P_{\ell}(\cos\theta)$$

and considering only S-wave, elastic scattering, $\ell = 0$, $\eta_0 = 1$, then,

$$A^I(\pi\pi) = \frac{1}{k} \frac{1}{\cot \delta - i} = \frac{1}{1/a_I - ik}$$

where a_I is the di-pion scattering length, and k is the momentum in the di-pion frame. Specifically, the di-pion mass is,

$$M_{\pi\pi}^2 = 4(m_{\pi}^2 + k^2)$$

so in the K_L^0 frame,

$$k^2 = (1/4)(M_K - m_\pi)^2 - (1/2)M_K T - m_\pi^2 \simeq m_\pi^2 \left[1 - 2 \frac{T}{m_\pi} \right].$$

Since $1/a$ is probably larger than m_π , and $0 \leq k \leq m_\pi$, then,

$$A_1^I(\pi\pi) \simeq a_I [1 + ia_I k_i].$$

For consistency, the approximation should be carried out to higher order, but since k^2 is linear in T , the sum of the three amplitudes A_i yields only a constant term of negligible effect. The third order term will produce non linear terms in T , but these are also negligible, being suppressed by a_0^3 . Now by approximating the amplitude for the spectator pion by unity, (since the momenta are small $e^{iqr} = 1$), the combined amplitude A_i becomes,

$$A_i(\pi\pi) = -\sqrt{\frac{1}{3}} (a_0 - \sqrt{2} a_2) [1 + ia_{02} k_i]$$

where $a_{02} = (a_0^2 - \sqrt{2} a_2^2) / (a_0 - \sqrt{2} a_2)$. Assuming $a_2 \ll a_0$, then $a_{02} \simeq a_0$. Approximating k_i by the expansion,

$$k_i \simeq m_\pi \left[1 - (T_i/m_\pi) - (1/2) (T_i/m_\pi)^2 \right].$$

By substituting the approximation for k_i into the amplitude A_i , summing the three amplitudes and using the relation,

$$\sum_i T_i^2 = \frac{Q^2}{3} \left(1 + \frac{r^2}{2} \right) \quad \text{where } r \text{ is the radius on the Dalitz plot,}$$

gives for the three pion amplitude,

$$A(3\pi) = a [3 + ig - i h r^2]$$

with

$$a = -\sqrt{\frac{1}{3}}(a_0 - \sqrt{2} a_2), \quad g = a_0 m_\pi [3 + Q/m_\pi - \frac{1}{6}(Q/m_\pi)^2],$$

$$h = a_0 m_\pi (Q/m_\pi)^2.$$

Evaluating the matrix element as $M = A^* A$, and keeping terms to order r^2 ,

$$M(3\pi) = |a'|^2 [1 - 2 \frac{gh}{9} r^2],$$

and using $a_0^{-1} \simeq 5m_\pi$,¹⁶ gives for the coefficient of r^2 in Eq. (2),

$$\alpha = -\frac{2}{9} \frac{1}{25m_\pi^2} \frac{m_\pi^2}{12} [3 - Q/m_\pi - \frac{1}{6}(Q/m_\pi)^2] (Q/m_\pi)^2$$

$$\simeq - .001. \quad (7)$$

The estimates, Eqs. (4), (6), (7), all show the slope of the Dalitz radial distribution to be near zero, indicating that the $3\pi^0$ Dalitz plot should be flat to within a few percent. However, a very large $\pi^0 - \pi^0$ scattering length, or the existence of a narrow, low mass $\pi^0 - \pi^0$ resonance would affect a much larger value for α and produce a significant enhancement at the periphery or at the center of the Dalitz plot. Assuming that the weak transition $K_L^0 \rightarrow 3\pi^0$ does not distort the $3\pi^0$ phase space significantly, any structure on the Dalitz plot is then a direct measure of the $\pi^0 - \pi^0$ interaction.

II. EXPERIMENTAL APPARATUS

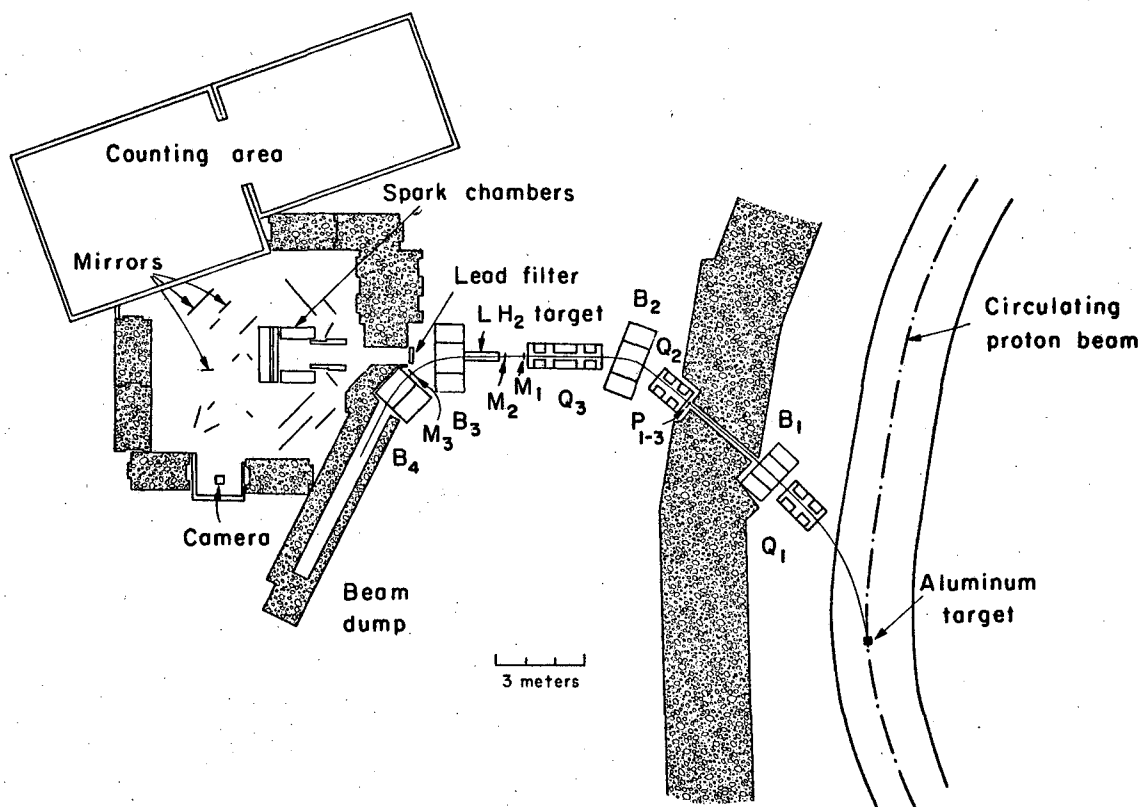
A. General procedure.

The $K_L^0 \rightarrow 3\pi^0$ decay was the principal background in the experiment designed to measure the rate of K_L^0 decay to $2\pi^0$ relative to $3\pi^0$.¹⁷ The experimental apparatus produced approximately monoenergetic K_L^0 mesons which decayed in flight within a nearly 4π solid-angle gamma ray detection system. The arrangement of the experimental apparatus is shown in Fig. 1.

To create K_L^0 mesons of known energy, a π^- beam was extracted from the Bevatron, momentum analyzed, and directed into a 1.2-m long, liquid hydrogen target. The momentum spectrum of the π^- beam was chosen to maximize K^0 production from the reaction $\pi^- p \rightarrow \Lambda^0 K^0$, with a contamination of K^0 's from the reaction $\pi^- p \rightarrow \Sigma^0 K^0$ of a few percent. The emerging beam pions and charged particle background produced in the hydrogen target were swept aside by two bending magnets. The neutral beam of forward-going K^0 's, neutrons, and gamma rays passed through 4-in. of lead where the gamma rays were absorbed.

The remaining K^0 's and neutrons drifted downstream for 5 m and entered a 1 m^3 air-filled decay volume in the shape of a five-sided cube enclosed by lead-plate spark chambers. A tunnel of lead-Lucite Cerenkov counters was placed at the entrance to the spark chamber array to enhance the gamma ray detection geometry.

Electron showers, produced in the spark chambers by the conversion of gamma rays originating from the decay of a K_L^0 meson into



XBL 695 - 2631

Fig. 1. Plan view of experiment.

neutral pions, were detected by two banks of scintillator-Cerenkov trigger counters placed in the downstream spark chamber. A K_L^0 decay into a neutral final state was sensed by requiring two gamma ray showers, separated by at least 11-in., be detected, a π^- meson entering but not leaving the liquid hydrogen target, detecting no charged particles in the lead filter, and finally, no charged particles entering the spark chambers. The separation requirement for the trigger counters discriminated against neutron interactions in the downstream spark chamber. In this mode of operation, 464 000 pictures were taken, with 25 000 K_L^0 neutral decays occurring within the fiducial volume. Nearly all of the events detected were $3\pi^0$ decays.

The response of the spark chambers to gamma rays of known energy was determined by making calibration runs to observe the $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ mode. In the 180 000 pictures taken with the charged signature, 1000 $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ decays, and 4000 leptonic decays were found. The calibration runs were distributed uniformly and frequently throughout the collection of the neutral data.

B. Beam.

Pions for the beam were produced by bombarding a $10 \times 1/4 \times 1/2$ -in. aluminum target with the 5.6 GeV internal circulating proton beam of the Bevatron. Pions were extracted in the forward direction to keep the apparent size of the pion source small. The forward direction was found by maximizing the electron contamination in the extracted negative beam as a function of the internal target position, since gamma

rays produced obliquely in the target more readily escape without producing electron-positron pairs. The electrons were monitored by a 1 atm, 3-ft long, Freon 12-filled, gas Cerenkov counter, placed, for this purpose, in the beam upstream from the liquid hydrogen target.

A flux of 5×10^{11} protons incident on the Al target produced a flux of 8×10^6 pions, 2.6×10^6 electrons, and 1.4×10^6 muons in the momentum range 980 ± 40 MeV/c (FWHM) incident on the liquid hydrogen target, as determined by the use of a high pressure (0 to 300 psi), Freon 13-filled, gas Cerenkov counter.

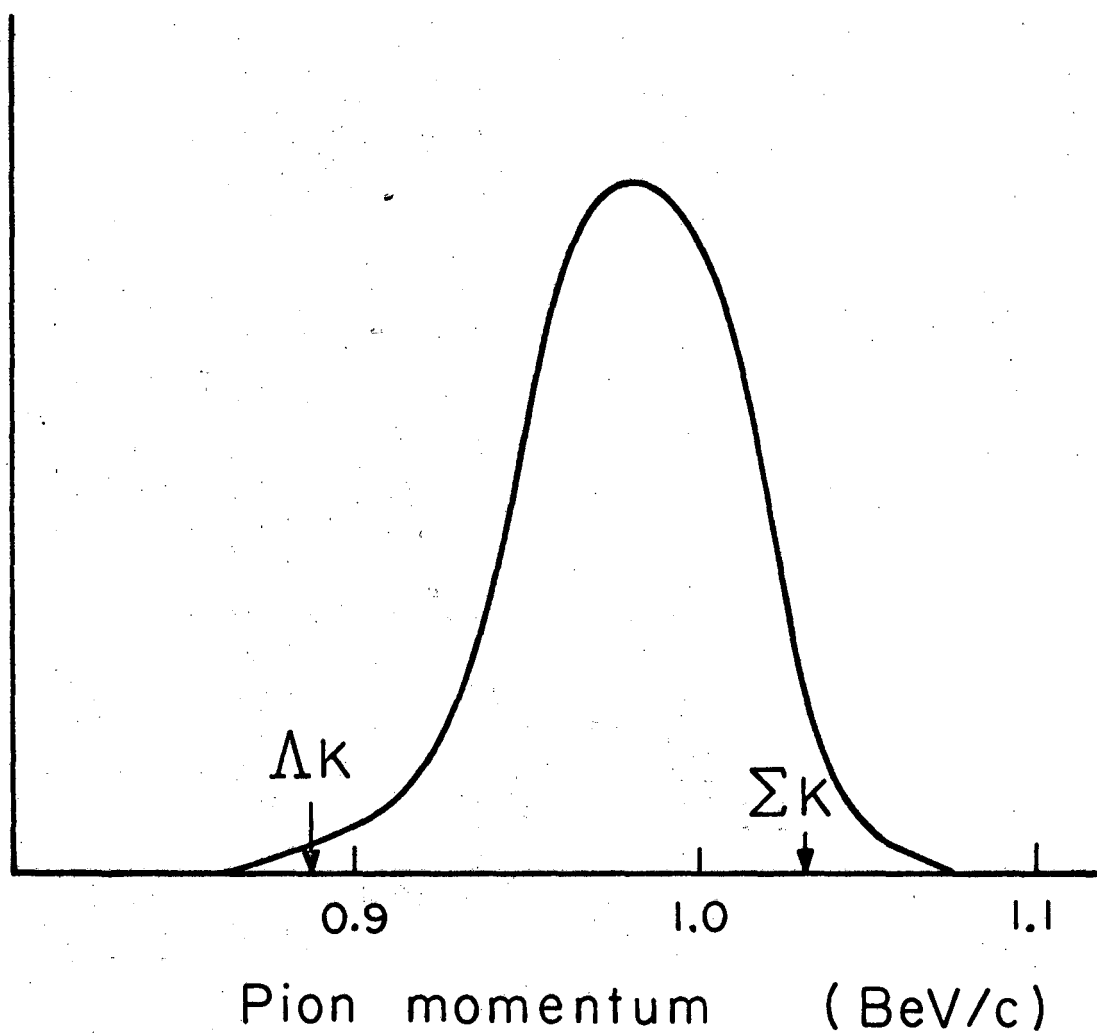
The elements of the beam transport system were:

- Q_1 - quadrupole doublet, 12-in. bore, 16, 16-in. long
- B_1 - H magnet, 12-in. gap, 18-in. wide, 36-in. long
- Q_2 - quadrupole doublet, 12-in. bore, 16, 16-in. long
- B_2 - H magnet, 8-in. gap, 18-in. wide, 36-in. long
- Q_3 - quadrupole triplet, 8-in. bore, 16, 32, 16-in. long
- B_3 - H magnet, 20-in. gap, 18-in. wide, 36-in. long
- B_4 - C magnet, 8-in. gap, 16-in. wide, 36-in. long

The foci of the target, for various momenta, produced by quadrupole Q_1 were dispersed horizontally across the entrance to Q_2 by the Bevatron field, and the field of magnet B_1 . Momentum selection was made here by inserting an 11-in. long $\times$ 1/2-in. diameter lead collimator into Q_2 . The selected momenta were recombined in a single focus at the hydrogen target using Q_2 as a field lens. Magnets B_3 and B_4 were used as sweeping magnets to remove charged particles from the neutral beam.

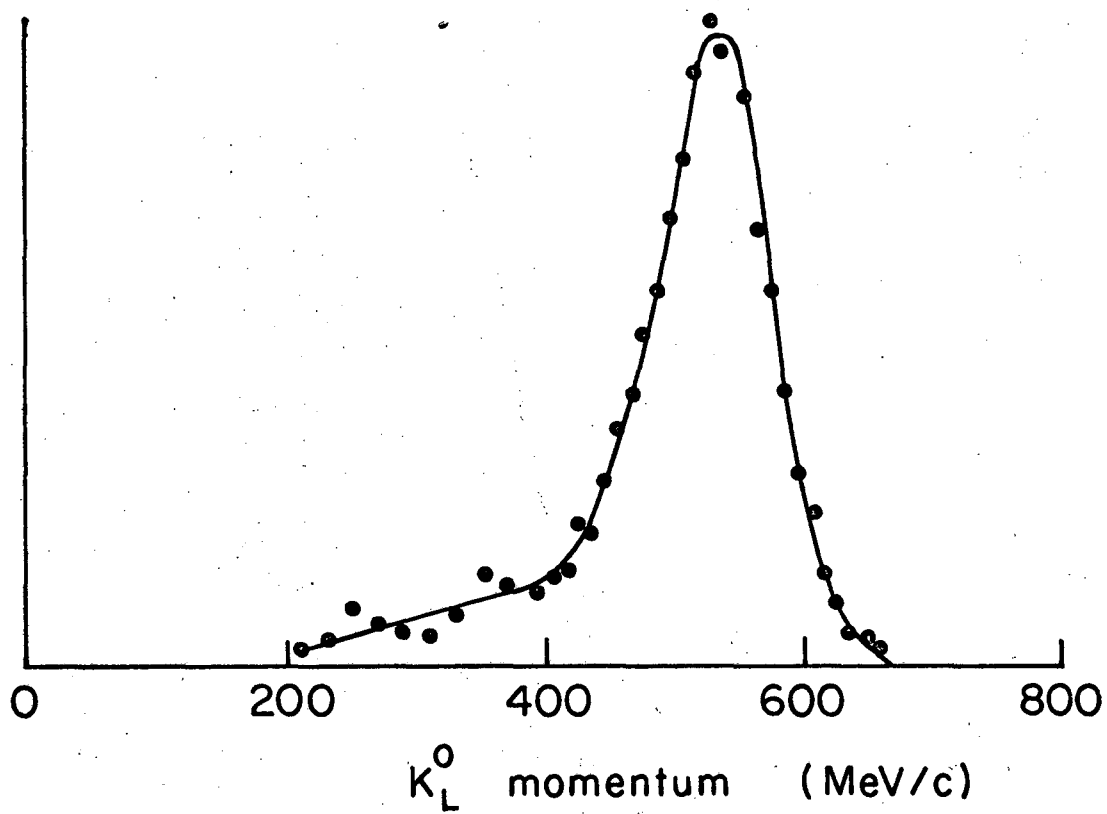
A momentum distribution of the pion beam was measured using magnet B_3 as an analyzer. Pion trajectories in the beam were sampled by two sets of finger counters, one, a set of ten 0.32-cm wide by 0.635-cm thick by 20.3-cm high, placed at the exit of quadrupole Q_3 , and the other, a similar set placed at the entrance to magnet B_3 . Together with an additional single finger counter, placed at the normal beam center 0.9-m downstream from the end of B_3 , trajectories through B_3 were determined. The momentum spectrum for the beam was constructed from particles traversing all possible pairs of the first two sets by measuring all triples rates as a function of B_3 current. Normalizing the triples rates to a monitor that covered the entire beam, and calculating the momentum value for the finger counter triplets by a wire orbit technique, gave a momentum resolution for a triplet of about 1%, and a momentum spectrum as shown in Fig. 2.

Using the known cross sections for K^0 production,¹⁸⁻¹⁹ the K_L^0 momentum distribution was calculated by Monte Carlo technique, taking into account the effect of ionization energy loss of the pions in the liquid hydrogen target. The variation in momentum due to the absorption of pions in the liquid hydrogen, and of the K_L^0 's in both the hydrogen and the lead filter, was assumed to be negligible. The momentum dependence of the K_L^0 lifetime was included in calculating the momentum distribution of the K_L^0 beam at the entrance to the spark chamber array as is shown in Fig. 3. An incident flux of 8×10^6 pions on the hydrogen target produced an average of 40 K_L^0 's traversing the decay volume.



XBL695-2632

Fig. 2. Measured momentum distribution of the π^- beam at the hydrogen target. Thresholds for Λ^0 and Σ^0 production are shown.



XBL695- 2662

Fig. 3. Calculated momentum distribution of the K_L^0 beam at the entrance to the spark chamber array. The points are the result of the Monte Carlo calculation.

C. Gamma ray detectors.

The 1 m^3 decay volume was surrounded on five sides by lead-plate spark chambers, with the upstream side left open for the entering K_L^0 mesons. The decay volume was further enclosed by a gamma shower counter, in the form of a four sided tunnel at the entrance to the spark chamber array. The total solid angle subtended by the gamma detection assembly was about 98% in the rest frame of the K_L^0 .

The spark chambers were assembled in modular form. The first module in each chamber contained five 0.048-in thick aluminum plates for the identification of entering charged particles. The remaining modules in each chamber contained only lead plates. The plates in the lead module were constructed from a 0.032-in lead sheet, stiffened by gluing 0.016-in thick Al plates to each side. Six of these plates followed by a single Al plate formed a six gap lead module with a total thickness of $2\frac{5}{16}$ -in, and 0.915 radiation lengths of lead. In the module, the plates were glued to Lucite frames to form gaps of $\frac{5}{16}$ -in, where the spacing was maintained by gluing $\frac{1}{4} \times \frac{1}{4} \times \frac{5}{16}$ -in Lucite blocks between the plates at intervals of 8 in. The faces of these blocks were optically polished and aligned parallel to the edges of the chamber.

The chambers were of two slightly different constructions. The four chambers axial to the beam were 4 by 5-ft and consisted of one Al module followed by seven lead modules. However, the downstream chamber was $6\frac{1}{2} \times 6\frac{1}{2}$ -ft, with one Al module followed by eight lead modules, and was electrically two different chambers. The chamber

was separated into halves by a 1-in wide vertical Lucite bar, staggered for the different modules, with one half $3\frac{1}{2}$ by $6\frac{1}{2}$ -ft, and the other 3 by $6\frac{1}{2}$ -ft.

The entrance tunnel to the spark chamber array contained an independent shower counter in each of its four sides. Each counter was 3-ft wide and 4-ft long in the beam direction, with the downstream 2-ft length containing 5.7 radiation lengths, and the upstream 2-ft section containing 8.5 radiation lengths. The tunnel, shown in Fig. 4, extended 6 in. into the region enclosed by the spark chambers. The counters were constructed by laminating sheets of $1/32$ -in lead and $1/8$ -in Lucite, with the laminate preceded by a $1/4$ -in sheet of scintillator. Cerenkov light generated in the Lucite was viewed by six 58 AVP phototubes located at the upstream edge of the counter, whose outputs were added to form a single output for each of the four counters.

Electron showers were detected by two banks of trigger counters placed in the downstream spark chamber. Each bank had 11 counters stacked vertically, with each counter $5\frac{1}{2}$ -in wide and 5-ft long, and with the first bank arrayed after the first lead module, and the second bank after the second lead module. Each trigger counter was a scintillator-Cerenkov pair, with the Cerenkov member a Lucite slab $1\frac{1}{2}$ -in thick. The efficiency for detecting a single highly relativistic particle at normal incidence was 96% with a variation of 2% over the counters' length. The Cerenkov pairs at the same height in the two banks were combined optically at a 58 AVP phototube, while the corresponding scintillators were viewed separately by two 56 AVP phototubes, but with the two scintillator signals added passively at their

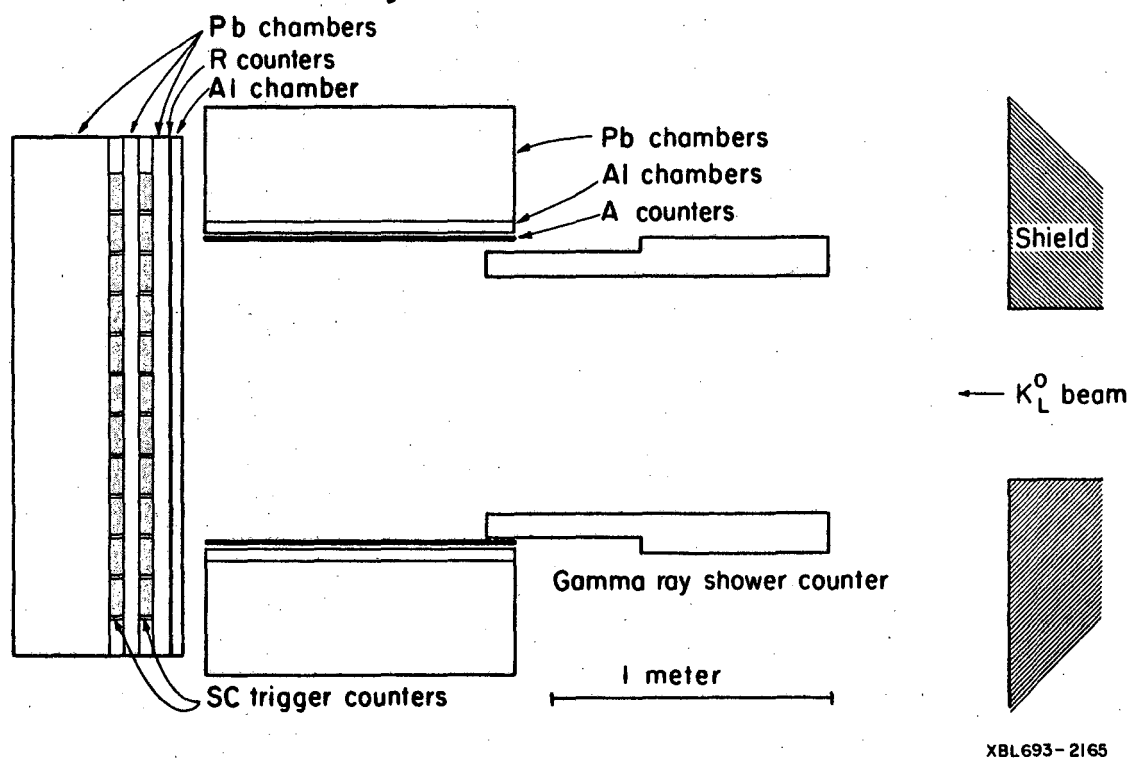


Fig. 4. Vertical section through gamma detector assembly.

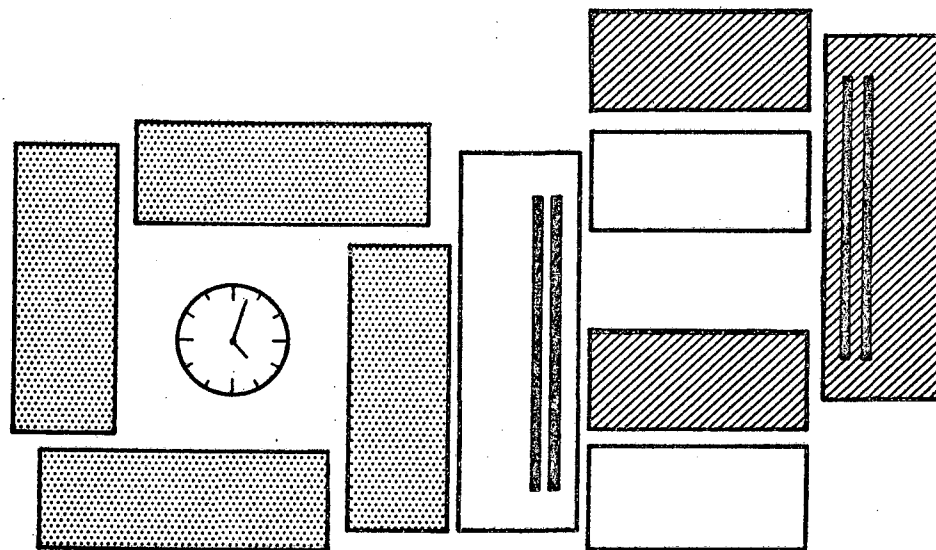
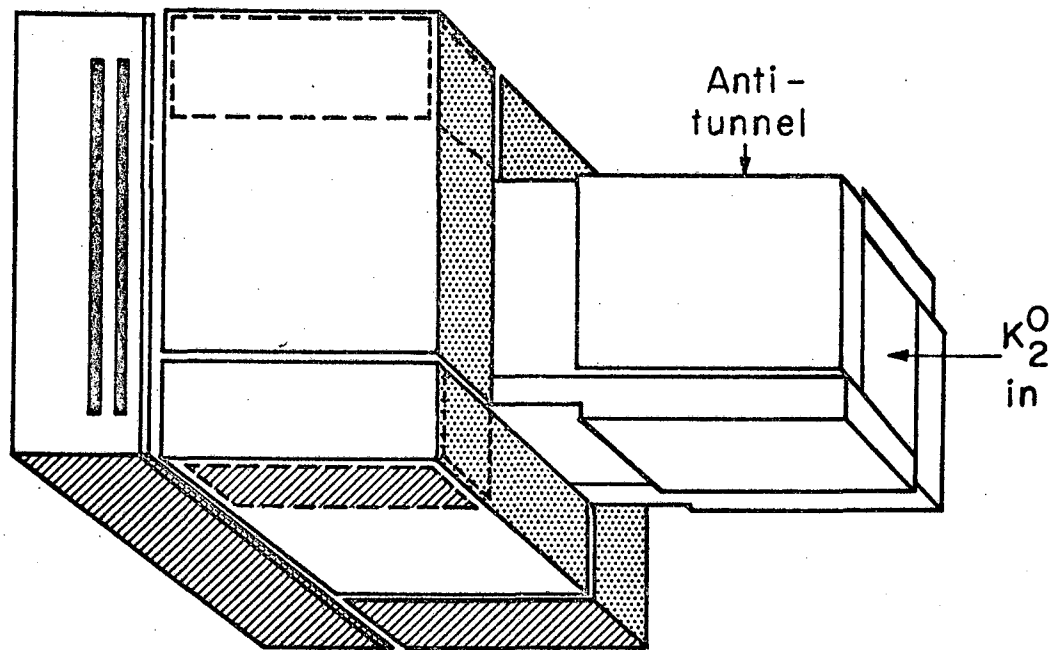
bases.

D. Optical system.

A system of 46 front-surfaced mirrors combined two orthogonal views of each of the spark chambers into one camera frame. The ten views were arranged on the film in three groups corresponding to a real-space view of the spark chamber array, as shown in Fig. 5. For each group the geometrical relationship among the views of the chambers was nearly the same on film as in real space, making it possible to count accurately the number of showers present in an event. In addition, the cases where a shower passed through more than one chamber could be correctly interpreted easily, and some accidental tracks could be eliminated because they clearly did not originate from the vertex formed by the other showers in the event.

To focus light leaving the chambers parallel to the optic axis of the camera, plano-convex Lucite field lenses of focal length 432-in were mounted on the optical faces of the chambers. The camera lens had a focal length of 75 mm and produced a demagnification of 138 from real space to film.

Reference marks were provided on the film for film to space transformations by fixing four fiducials at the corners of the spark chambers' optical faces. A non-negligible distortion of the spark images, arising from the passage of the light from a spark through a Lucite frame, a Lucite field lens, and reflection from four to six mirrors before reaching the camera, was correctable by the use of Mylar grids. These transparent grids, photographed at the front and back of



XBL681-1612

Fig. 5. (top) Perspective drawing of spark chamber array and gamma ray shower counter.
 (bottom) Layout on film of views of spark chamber array. Shading indicates the correspondence between the views in the two drawings.

each optical view of the chambers, were placed in known relationships to the fiducials so that the transformation from film back to real space could be precisely determined for many points distributed throughout the views. The distortions were typically less than 0.1 in. but in several locations were as large as 0.25 in.

E. Electronics.

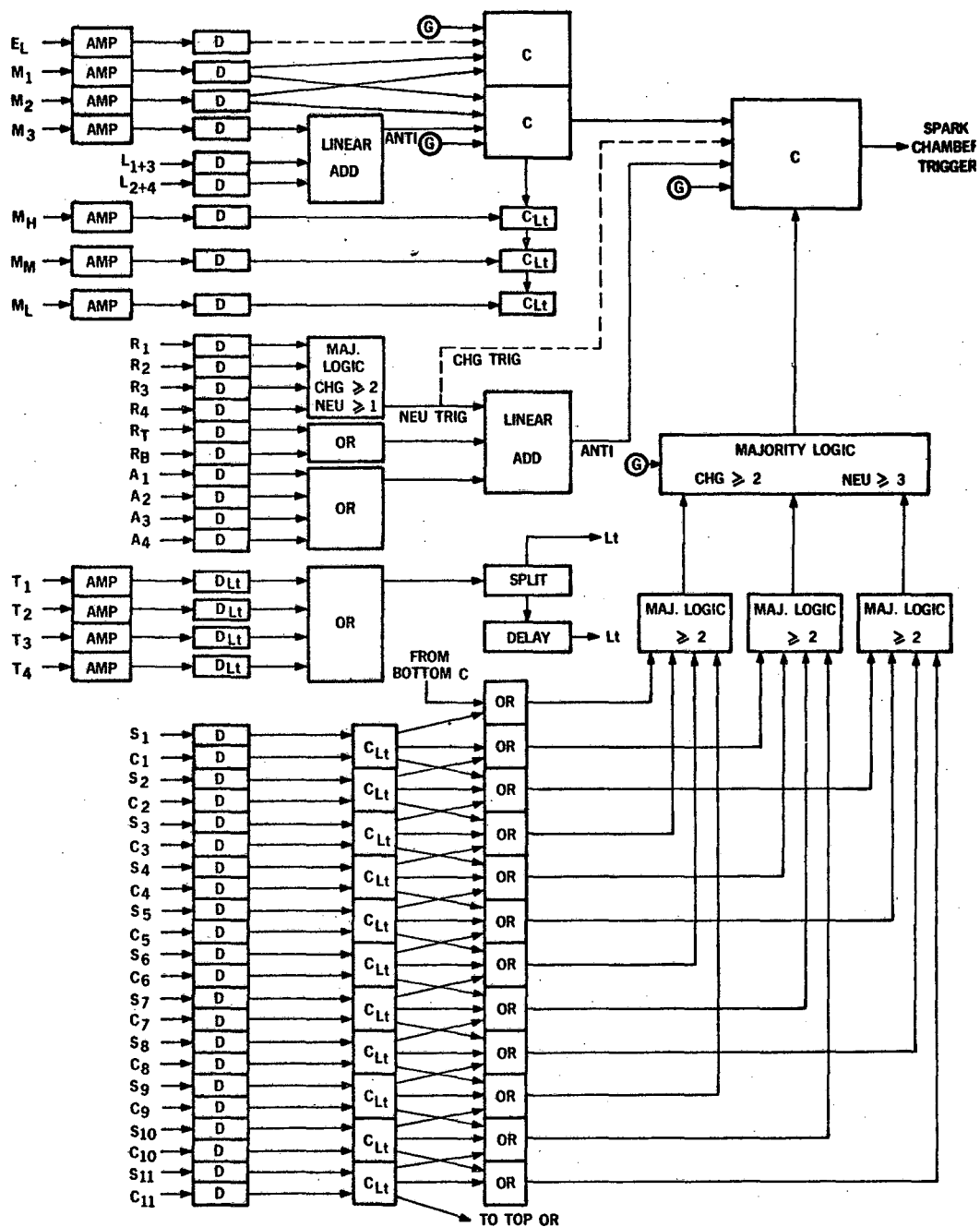
To monitor the flux of pions at the liquid hydrogen target, two beam defining, 8-in diameter, 1/4-in thick scintillation counters, M_1 and M_2 , were located at the exit of magnet Q_3 and at the entrance to the hydrogen target respectively. A similar, third scintillation counter, M_3 , detected pions on their way to the beam dump, having not interacted in the target, and was located at the entrance to magnet B_4 . Corrections to the pion momenta in the beam were obtained by monitoring the pion flux at the entrance to the momentum defining collimator in magnet Q_2 using three scintillators, P_{1-3} , 3-in high, 4-in wide, 1/4-in thick. They divided the beam into three momentum bands, each with a momentum spread of approximately half that of the whole beam.

A lead filter was placed in the neutral beam to absorb gamma rays produced in the target by background reactions such as $\pi^- p \rightarrow \pi^0 n$ and $\pi^- p \rightarrow \gamma n$. To veto events in which a shower in the filter was not completely contained and some gamma rays escaped out the downstream side to possibly trigger the chambers, four scintillation counters were placed at depths of 1, 2.25, 3.25, and 4 in. in the lead, L_{1-4} . Then the possible production of a K^0 was taken to have an electronic signature of $M_1 M_2 \bar{M}_3 \bar{L}_{1-4} = M$.

The desirability of taking data at beam fluxes of 10^7 sec^{-1} required circuitry capable of operating efficiently at high repetition rates. For this logic, the Cronetics 100 series, nominally capable of repetition rates of 100 MHz, was used extensively. The high rates also required that special precautions be taken with the beam counters. By introducing a wide-band dc amplifier with 1 ns risetime between the base and the input to the Cronetics 100 discriminator,²⁰ the current output required at the base of counters M_{1-3} , L_{1-4} could be reduced an order of magnitude. To further insure stable operation of the phototube bases, Zener diodes regulated the voltage for the last four stages of multiplication, and 10 ma of additional current was supplied to the last two stages.

The veto efficiency was kept high at these rates by combining the logic signals from M_3 and the L counters in a deadtime-free fashion. For this purpose, a linear dc amplifier with multiple inputs summed the veto signal, then this sum, through a dc coupling, was used in anti-coincidence with the $M_1 M_2$ coincidence.

The possibly decay of a K_L^0 into a neutral final state had a signature of: (1) no response from the scintillation counters lining the inside walls of the spark chambers, (2) no response from the scintillation counters embedded in the downstream chamber after the first aluminum gaps, and (3) response from at least two of the scintillator-Cerenkov trigger units, separated by a minimum distance of two trigger units. The signature for (3) was obtained by taking the output of each trigger counter, splitting it into three signals, and recombining these signals as shown in Fig. 6. The majority logic units require that a given number of input signals be present without specifying any in particular.



XBL 696-809

Fig. 6. Block diagram of fast electronics. Discriminators are denoted by the letter D; coincidence circuits by the letter C.

Low energy gamma ray showers which fired a trigger counter but produced only one or two sparks in the chamber were identified by installing display lights on the side of the downstream chamber directly over the ends of the corresponding counter. These lights were also useful in discriminating against accidental tracks in the downstream chamber. Similar lights recorded on film coincident counts in the entrance tunnel shower counters, T_{1-4} , and coincident counts in the momentum counters, P_{1-3} .

The counters mounted on or in the spark chambers had large variations in the light collection time since their lengths were of the order of two meters. The variations were largest in the Lucite Cerenkov slabs because Cerenkov light is emitted directionally and at low intensity, giving rise to statistical fluctuations in the low number of photons reaching the phototubes. To make the jitter times of the SC pairs comparable to that of the scintillator member alone, the Cerenkov discriminator output length was set equal to the maximum jitter time and shifted earlier. The scintillator member then determined the timing of the pair. The differences among the various particles and light collection paths gave the circuits combining the SC pairs unavoidably broad resolution times of 20 to 40 nsec. Nevertheless, the accidental trigger rate was fairly low, about 25% at maximum beam intensities.

The charged decay of a K_L^0 meson was given a signature of (1) no response from the A scintillation counters, (2) no response from the top or bottom R counters, (the R counters were scintillators lining the decay volume, see Fig. 4), (3) response from two or more of the

inner R counters, and (4) response from two or more of the trigger scintillators, separated by a minimum distance of one trigger unit. The signature could be easily changed between charged and neutral, and charged runs were regularly placed among the neutral runs, providing for several systematic checks and calibrations of the experimental apparatus.

When the overall signature for the production and decay of a K_L^0 meson was met, the logic was gated off and the spark chambers were fired. The repetition rate for taking pictures was limited by the time required for the high voltage supplies to recharge the chamber pulsers, about 70 msec. The data taking rate was below this capacity at one to two pictures per pulse for the neutral mode, and five to six pictures per pulse for the charged mode. The Bevatron pulse was usually 600 to 800 msec long.

A low inductance connection to the plates of the spark chambers was made possible by the geometry of the pulser²¹ design and broad connectors coupled directly to the spark chamber plates, and resulted in rise times of about 25 nsec. The capacitance of each pulser was 15 000 pf, while the capacitance of the gap it drove was 6000 pf. To prevent spurious breakdown in the chambers when no track was present in the gap, the voltage was dumped using a self triggering spark gap plugged into the chamber near critical areas like gas holes.²² By varying the spacing of these gaps the duration of the pulse on the plate could be adjusted. For full multiple track efficiency, the minimum pulse width was 40 nsec FWHM at a peak voltage of 8 kV, and with essentially no spurious breakdown occurring in the chambers.

The chamber pulsers were fired by a chain of triggered spark gaps. Initially, the logic-level output generated by the electronics when the signature for an event was satisfied fired a master pulser which in turn drove six slave gaps located at each of the chambers. Then the slave spark gaps triggered the 23 to 26 pulsers used to bring each of the spark chambers up to voltage. About 350 nsec elapsed from the time an event occurred until voltage appeared on the plates.

A dc clearing field of 20 volts on the chambers swept out electrons from old tracks. A further sweeping field of 150 volts was pulsed for 40 msec while the pulsers were being recharged to clear out the debris of heavy (or slow) ions.

III. DATA ANALYSIS

A. Scanning and Measuring

To locate all K_L^0 events each roll of film was scanned twice. For all events with three or more showers present in the chambers, the frame number, total number of showers, number of showers present in the back chamber, chamber number of any two spark showers, and numbers of any triggered tunnel or P counters were recorded. Due to the difficulty of correctly counting the number of showers in a $3\pi^0$ event, a list of six shower candidates was selected from any event where either scanner found four or more showers. A total of 25 120 of these events were found in the film.

The six shower candidates were reevaluated and classified as:

1. Unambiguous six shower event, 28% of all K_L^0 , or 7040 events total.
2. Questionable six shower event (confusing shower structure, or one or more showers possibly unassociated with event, or one or more showers may be non-shower tracks), 17% of all K_L^0 , or 4322 events total.
3. Non-six shower event:
 - a) clear five shower event, 27% of all K_L^0 , or 6724 events total.
 - b) seven or more shower event, 3% of all K_L^0 , or 755 events total.
 - c) poor five, or fewer shower event, 25% of all K_L^0 , or 6279 events total.

The efficiency for finding and placing a six shower event in the first two categories was better than 99%. Geometric reconstruction and kinematic fitting were attempted on the first two categories.

To assist the geometric reconstruction, a grid zone was encoded for each shower, indicating its chamber, beginning point (with an approximate three inch resolution), and quality (a five point scale measuring the confidence that the shower originated from the conversion of a K_L^0 decay photon). Geometric reconstruction consisted of determining the spatial positions of all the sparks in each of the showers in an event, and the subsequent decay point, or the intersection point of the showers.

Measurement of all the sparks in the events was made feasible by using SASS,²³ a high resolution CRT digitizer controlled by a DDP-24 computer. A control program for the SASS computer was developed, allowing the machine to scan along each gap of the ten views of the chambers.²⁴ The gap positions on the film for each chamber were calculated from the fiducial coordinates for that chamber, and the scan for each event referenced on four of the more reliable fiducials. The midpoint of each spark encountered in the gap scan was recorded along with the spark's width, intensity and gap number.

The local optical distortions on the film were severe enough to cause most views of the chambers to be scanned incorrectly for a small fraction of the area. Here the linear scan was either not in the intended gap or was between gaps. Since the average spark was almost two gaps in diameter, these aberrations did not cause many sparks to be missed,

however the larger sparks were digitized more than once. This double counting of the larger sparks did not occur often enough to seriously effect the calculations of the showers direction or energy. The minimum step in the SASS coordinate system is one part in four thousand, or six microns on the film, an uncertainty of 5% of an average spark diameter.

B. Geometric reconstruction.

The showers in each event were extracted from the SASS data using a pattern recognition technique assisted by the grid zone information. The grid zone locates the beginning of the showers. If an approximate shower direction can be established, then a search in a cone whose apex is centered at the shower origin will collect the sparks associated with that shower, provided the shower is clearly visible in both orthogonal views. The initial shower direction can be determined from the existence of a tight clump of sparks in the vicinity of the grid zone, or from the center of the grid zone together with the intersection point of two or more previously found showers. At the start of each event the decay point is assumed to be at the center of the fiducial volume, and the search cone's half angle is 15° . The decay point is reevaluated after the showers in each chamber have been found, provided more than two showers have been accumulated. At the end of the event, the cone is reset to 5° and the event is reprocessed with the decay point initialized to the best value of the previous iteration.

Having identified the sparks in a shower, the points in each view are correlated to give the spatial position of each spark. Consequently, for each shower the total number of sparks shower direction,

directional error, length and shape are known. Because of the relatively high probability (20% per shower) that two or more showers will be superimposed in one of the views, the total number of sparks, length and shape are taken from the clearest view when overlapping occurs.

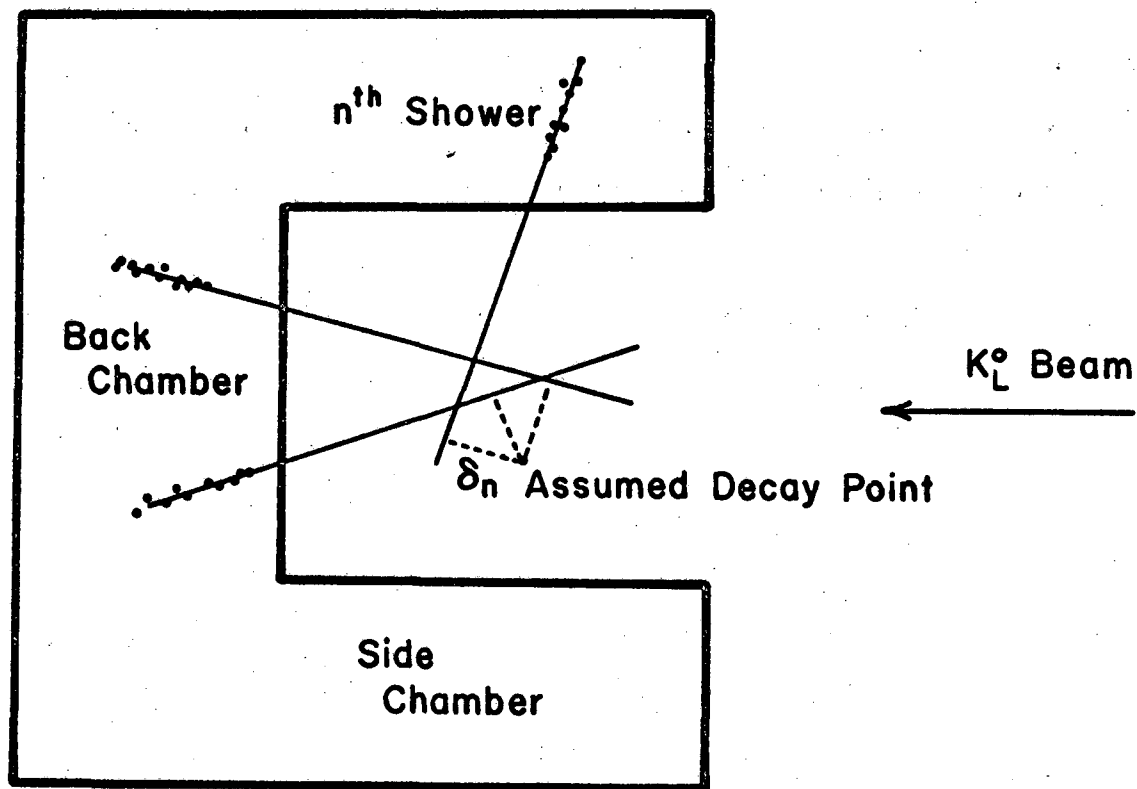
The shower intersection point used in the assembly of showers was analytically calculated by minimizing the quantity

$$\sum_n w_n \delta_n^2$$

where, with the aid of Fig. 7, w_n is the weight given to the n th shower, and δ_n is the perpendicular distance from the decay point to the extrapolated line in the direction of the n th shower. The weight of a shower was chosen to be the number of sparks in the shower.

C. Kinematics.

The kinematics of this experiment presented two essential problems. The complexity of the kinematics formed highly convoluted χ^2 topologies. In order to successfully fit the events, the kinematics had to be formulated to reduce the fitted solutions' sensitivity to detection and measurement errors, while keeping the χ^2 topology as simple as possible. Also, to choose the correct (or true), pairing of the photons from among the fifteen possible pairings for each event, required a complex and Monte Carlo dependent selection mechanism. To be free of bias, a relatively simple selection technique was needed.



XBL 725-150

Fig. 7. Cross section through spark chamber array showing some typical showers and their fitted directions.

The kinematics of the process $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ is completely determined by measuring the directions of all the initial and final state particles, assuming all particle masses are known. The seven equations of constraint, four of energy-momentum conservation, three of pairing gammas to the invariant mass of a pion, are sufficient to determine all energies and momenta. Since all directional vectors are tied to the decay point of the K_L^0 , the kinematics, using the spark positions and estimates of the photon energies, allow the decay point to be varied. In so doing, the geometric and kinematic factors may be optimized.

Events were fit by developing a basic three-dimensional χ^2 search and applying this to an increasingly complex set of kinematics. Since the six gammas may be paired into three pions in fifteen possible ways, the outstanding difficulty in the fit was determining the correct pairing. The fitting procedure has been structured to allow an optimum choice for the correct pairing. First a geometric fit to all the sparks in the showers was performed to locate a beginning point for the subsequent kinematic fits. Then using the photon energies as estimated from the spark counts, a fit was made to the best balance of energy and momentum. The photon energies so produced were used as reference in the remaining kinematic fits. The fifteen pairings were then appraised and the best five selected, for in this set the correct pairing, or an acceptable alternate, almost always resided. The existence of alternate pairings with nearly the same Dalitz radial position as the true pairing, and their preferential occurrence in this selected set, eliminated the need for developing a mechanism for choosing the true

pairing. The full kinematic fit was then applied to each of these best pairings.

The full fit was a costly procedure and practical considerations forbade its application to all fifteen pairings. Since the momentum of the K_L^0 was known to within ten percent, the kinematics was used to calculate the six photon energies, so only two of the three pairing equations were needed. This made the pions distinguishable, requiring an order for the pions to be chosen on the basis of the remaining pairing equation.

Each of the select pairings was then fit, measuring the quality of the fit by a combination of the third pairing equation, the reference photon energies, and the geometry. The best pairing was chosen from those pairings, if any, that survived the fitting with physical solutions and good quality.

A specific description of the fitting technique will be given considering each of these steps separately.

D. Geometric fitting.

Measuring the events on SASS allowed the positions of all sparks in each shower to be obtained. All the sparks were necessary to give the best estimate of the geometric decay point, (see section on pairing appraisal) which was found by locating the minimum on the surface

$$\chi_G^2 = \sum_n g_n \sum_{i=2}^{N_n} \delta_{ni}^2 (X, Y, Z) / \sigma_{n,i}$$

X, Y, Z is the spatial position of the decay point, and where, with the aid

of Fig. 8,

δ_i is the minimum distance from the i th point in shower n to the line through X, Y, Z to x_1, y_1, z_1

$$\delta_i^2 = r_i^2 - t_i^2$$

r_i is the distance from the decay point to the i th spark

$$r_i^2 = (X_i - X)^2 + (Y_i - Y)^2 + (Z_i - Z)^2$$

t_i is the distance from the decay point to the base of δ_i

$$t_i = \alpha(X_i - X) + \beta(Y_i - Y) + \gamma(Z_i - Z)$$

α, β, γ are the direction cosines for shower n , given by the first point

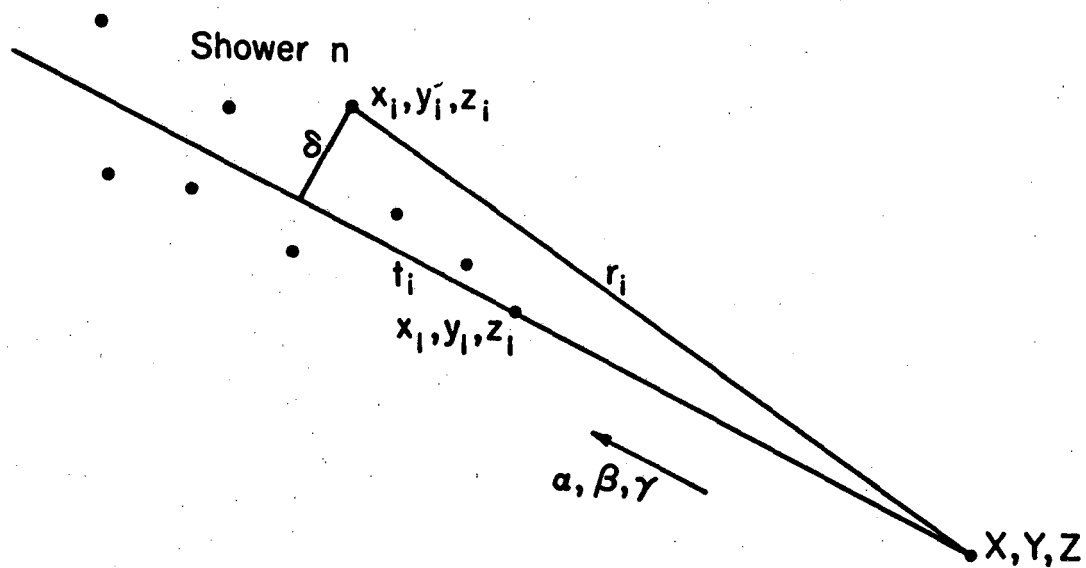
$$\alpha = (x_1 - x)/r, \quad \beta = (y_1 - y)/r, \quad \gamma = (z_1 - z)/r$$

$$r^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2.$$

The error in the spark position, σ_{ni} , reflects the assumption that the sparks lie in a cone whose opening angle is inversely proportional to the total number of sparks in the shower, or energy. Figure 9 shows the fitted opening half angle as a function of the estimated shower energy for real six shower events. The errors indicate the fluctuation of the showers' shape.

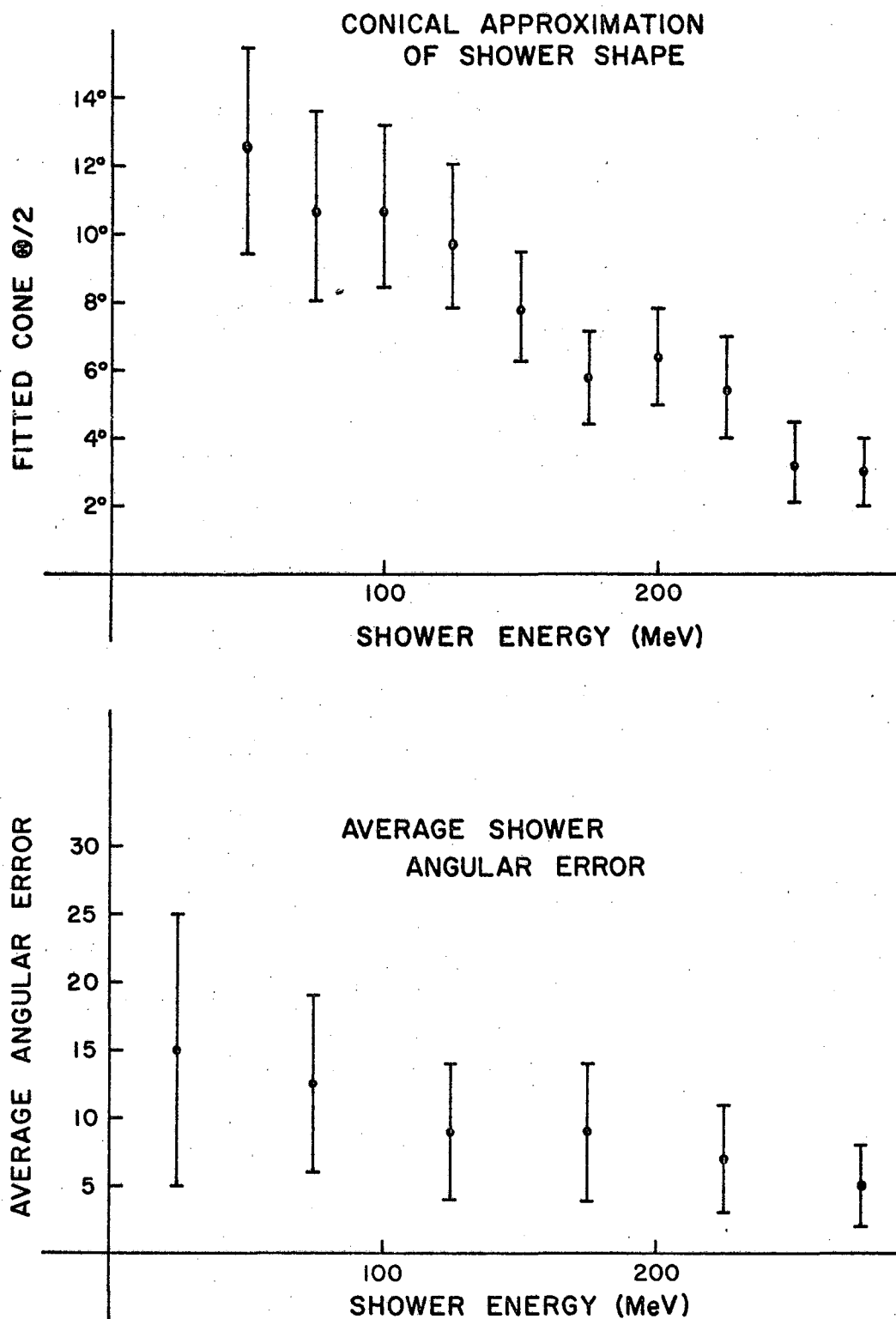
$$\sigma_{ni} = \sigma_0 + \Theta(E_n) S_i$$

with σ_0 a first spark error of .3 in. and S_i the distance along the shower direction of the i th spark from the first spark.



XBL 726-908

Fig. 8. Spark positions in a typical shower illustrating the spatial relationship among quantities used to determine the geometric decay point.



XBL 725-148

Fig. 9. (top) Energy dependence of cone angle fitted to showers. (bottom) Energy dependence of shower angular error. Bars enclose two-thirds of the showers in each energy interval.

The weighting factor g_n or inverse error for the n th shower reflects the fact that the average angular deviation for a shower is also a function of the photon energy. Figure 9 shows the angular deviation of a straight line fit to the sparks in a shower, from the estimated true shower direction as a function of shower energy, estimated by use of showers from the decay mode $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$. A choice of

$$1/g_n = g_0 / (1 + E/E_0)$$

with g_0 normalized to 20 degrees, and E_0 set to 100 MeV, gave a normal χ^2 distribution.

A sampling of the χ_G^2 surface along the X, Y plane, centered at the final decay point, and taken over an 8×8 -in area within the $20 \times 20 \times 25$ -in. decay volume, is shown in Fig. 10.

E. Momentum-Energy balance.

A fit to the photon energies, balancing momentum and energy, was made for each event, using the four conservation equations and shower energies estimated from the spark count. Figure 11 shows the distribution of total energy for the K_L^0 obtained by summing the shower energies E_n , determined by the following two methods.

$$1) E_n = A_1 S_n$$

where S_n is the total number of sparks in the n th shower. The spark count for a shower was obtained by averaging the number of sparks found in each view of the shower. If a shower was obscured in one view, the spark count was set to the number of sparks in the clearer view. A_1 has been adjusted to give the best fit of the distribution

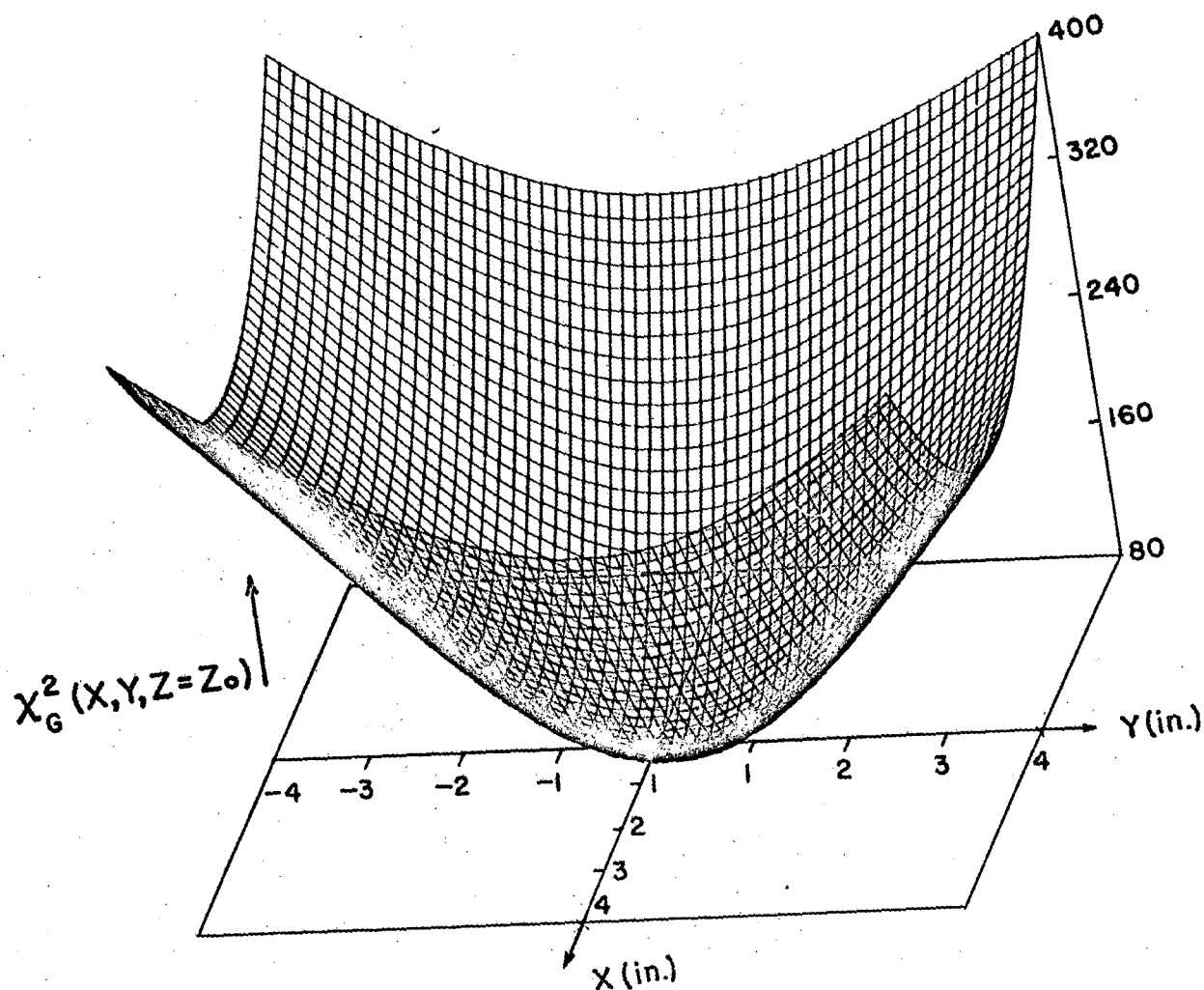
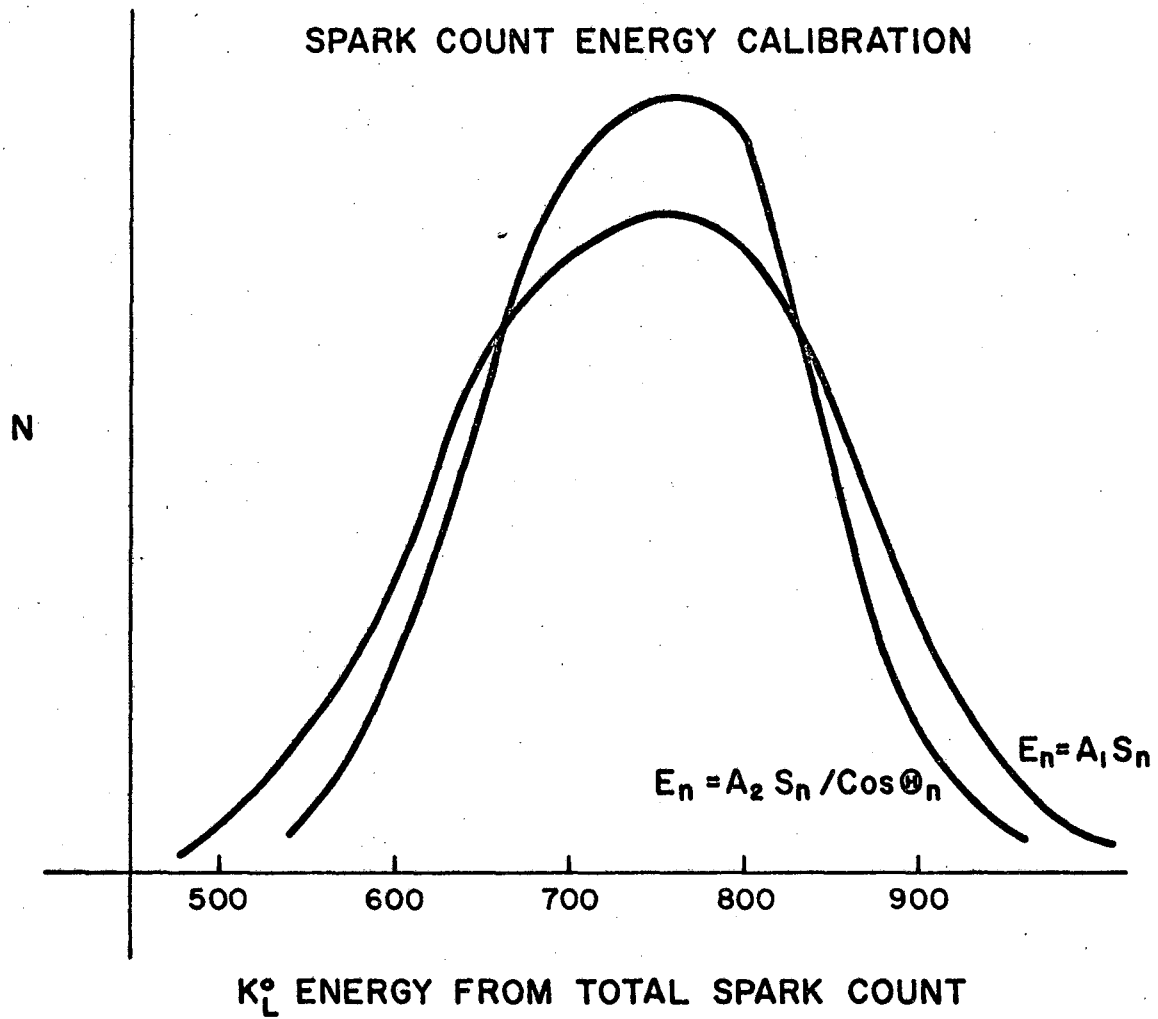


Fig. 10. Geometric chi-square surface sampled in the X, Y plane over an 8×8 -in. area centered at $(X, Y, Z) = (-12.0, 14.4, -12.1)$. The vertical scale is linear.



XBL 725-146

Fig. 11

$E_t = \sum_n E_n$ with the distribution of K_L^0 energy calculated from the beam momentum spectrum. This gives, $A_1 = 7.4$ MeV/spark.

$$2) E_n = A_2 S_n / \cos \theta$$

where θ is the angle between the fitted shower direction and the normal to the plates of the spark chamber, and A_2 has been adjusted as in (1) to give, $A_2 = 5.7$ MeV/equivalent spark. Choosing the angle θ as the direction of the shower, rather than the most probable direction of the photon, appears best (see section on pairing appraisal) although the analogous distribution E_t is not clearly distinguishable as best.

The data from the decay mode $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$ were measured on SASS and subject to the same shower analysis as the $3\pi^0$ data. The kinematics for this mode allows the photon energies to be calculated unambiguously. Figure 12 plots the energies calculated both from the spark count and kinematics for this mode, and indicates the photon energies may be estimated within a factor of two by using the spark count.

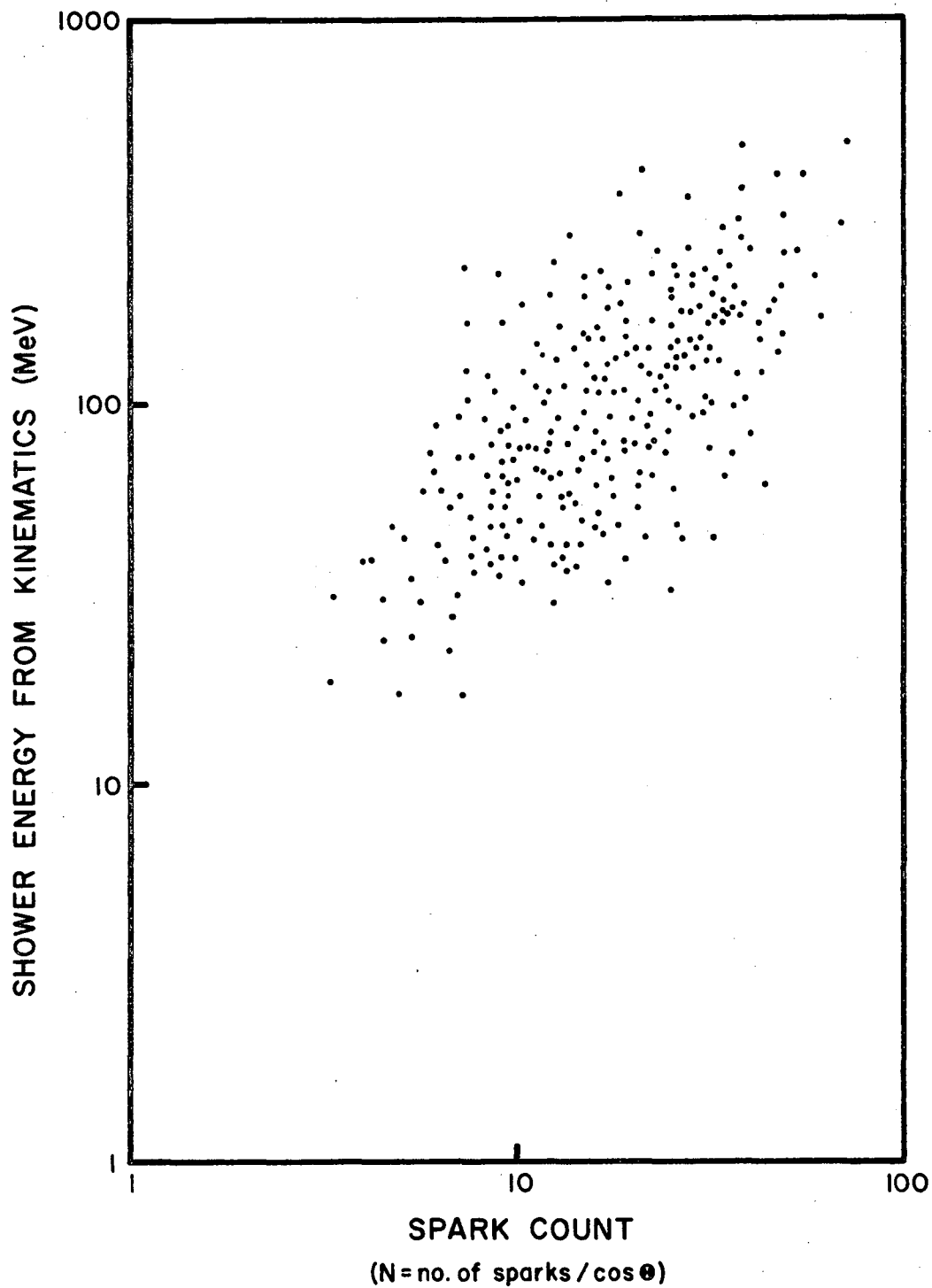
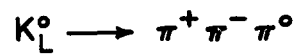
The shower energies estimated as in method (2) were used to find the photon energies k_n that best balance momentum and energy by formulating a correction e_n such that

$$k_n = E_n + e_n \quad n = 1, 6$$

Then stating the conservation equations as

$$\alpha k = P$$

where α is the four by six matrix of direction cosines, (using $\alpha_{4n} = 1$) and k is the six-dimensioned vector whose elements are the photon energies, and P is the four-dimensioned vector referring to the



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Fig. 12. Scatter plot of spark count vs energy for gamma rays from the π^0 in $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$ decays.

momentum-energy of the K_L^0 as (p_x, p_y, p_z, E_K) , the correction is

$$\alpha e = P - \alpha E = d.$$

Now the e_i may be considered as a function of any two of the corrections, say e_5 and e_6 . With $H_{ij} \equiv \alpha_{ij}$: $i = 1, 4$, $j = 1, 4$ then the remaining corrections are

$$H_{ij} e_j = d_i - \alpha_{i5} e_5 - \alpha_{i6} e_6 = f_i.$$

So

$$e = H^{-1} f.$$

The values of e_5 and e_6 may be fixed by minimizing

$$\begin{aligned} \chi_A^2 &= \sum_{i=1}^6 e_i^2 / E_i \\ &= \chi_A^2(e_5, e_6). \end{aligned}$$

For a fixed decay point (X, Y, Z) , $\nabla \chi_A^2 = 0$ gives an analytic solution for e_5 and e_6 from the linear equations:

$$F_{11} e_5 + F_{12} e_6 = G_1$$

$$F_{21} e_5 + F_{22} e_6 = G_2$$

where

$$F_{11} = \left(\sum_{n=1}^4 B_n / E_n \right) + 1/E_5 \quad F_{12} = F_{21} = \sum_n \frac{B_n C_n}{E_n}$$

$$F_{22} = \left(\sum C_n^2 / E_n \right) + 1/E_6$$

$$G_1 = \sum_n A_n B_n / E_n \quad G_2 = \sum_n A_n C_n / E_n$$

with

$$A_n = H_{nm}^{-1} d_m, \quad B_n = H_{nm}^{-1} \alpha_{m5} e_5, \quad C_n = H_{nm}^{-1} \alpha_{m6} e_6.$$

A fit may be performed allowing the decay point to vary with the interpretation

$$e_n = e(X, Y, Z).$$

So

$$\chi_A^2 = \chi_A^2(X, Y, Z)$$

and a decay point can be found which best balances energy and momentum by finding the minimum on the χ_A^2 surface. A two-dimensional sampling of such a surface is shown in Fig. 13.

The photon energies $k = E + e$, produced at the best balance point, are used to appraise the pairings, and reference the subsequent kinematic fits.

The large error in the photon directions, and the poor ability to estimate the photon energies are the principal factors limiting the Dalitz plot resolution. Knowing either the decay point or shower energies with more precision would greatly improve the quality of the kinematic fits, and consequently place the events on the Dalitz plot closer to their true positions. The emerging technology of multi-wire proportional chambers²⁵ holds the hope of solving both these problems. An experiment could be designed to observe $K_L^0 \rightarrow 3\pi^0$ decay by placing these chambers behind thin, high density photon converters and allowing long flight paths for the showering electrons and positrons. The reduced e^+ , e^- scattering would allow the shower directions, and hence the photon directions, to be determined to perhaps better than 1° , instead of the 5°

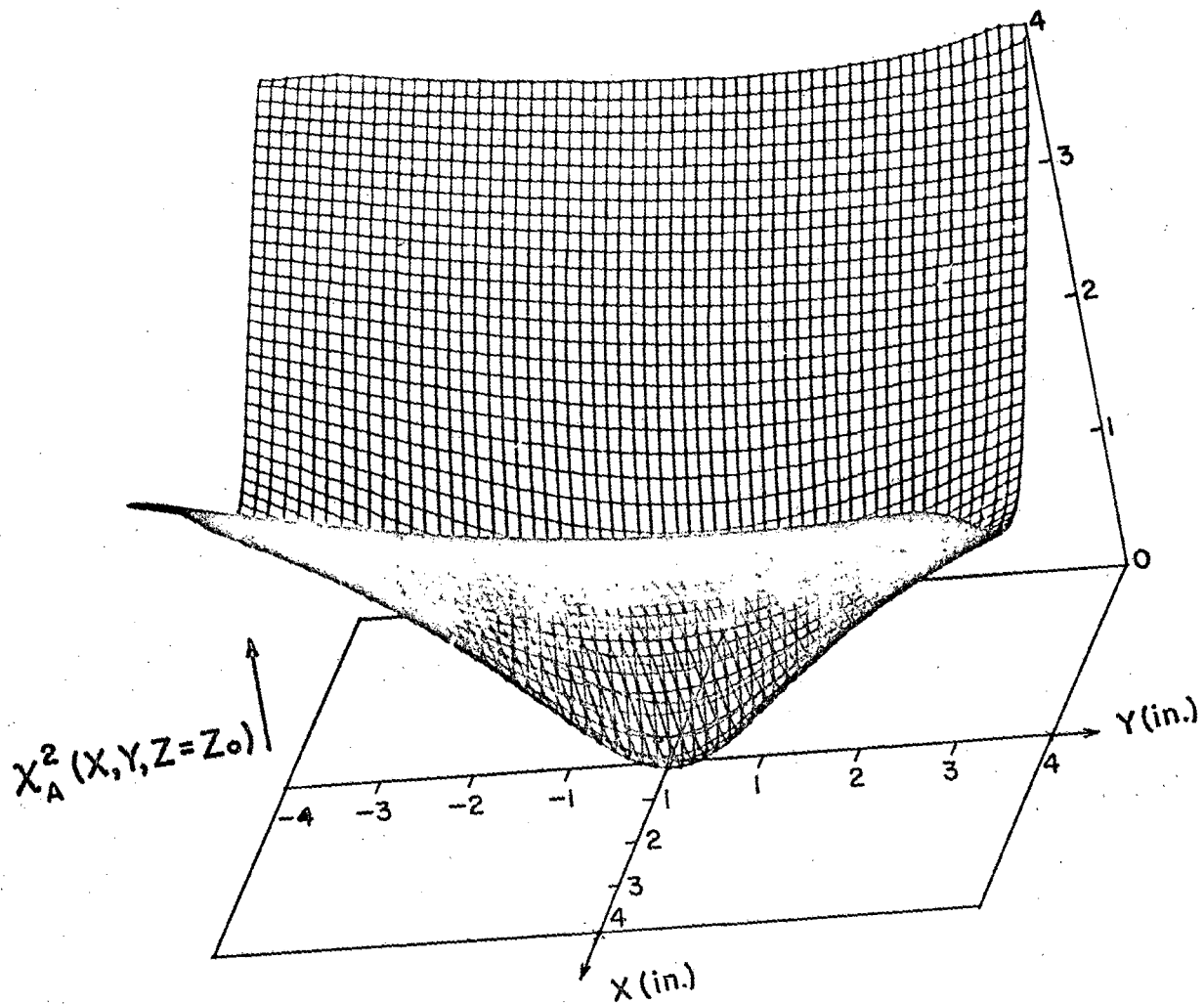


Fig. 13. Chi-square surface for momentum-energy balance sampled in the X, Y plane over an 8×8 -in. area centered at $(X, Y, Z) = (9.2, -2.6, -25.4)$ -in. The vertical scale is logarithmic to base 10.

to 10^0 error of this experiment. The signal pulse from the chambers should be able to measure the shower energy to within 30%, considerably better than the factor of two variation found here. Such precision would improve the Dalitz plot radial error by an order of magnitude, and with appropriate statistics, provide the means for a detailed measurement of the structure of the $3\pi^0$ system.

F. Pairing appraisal.

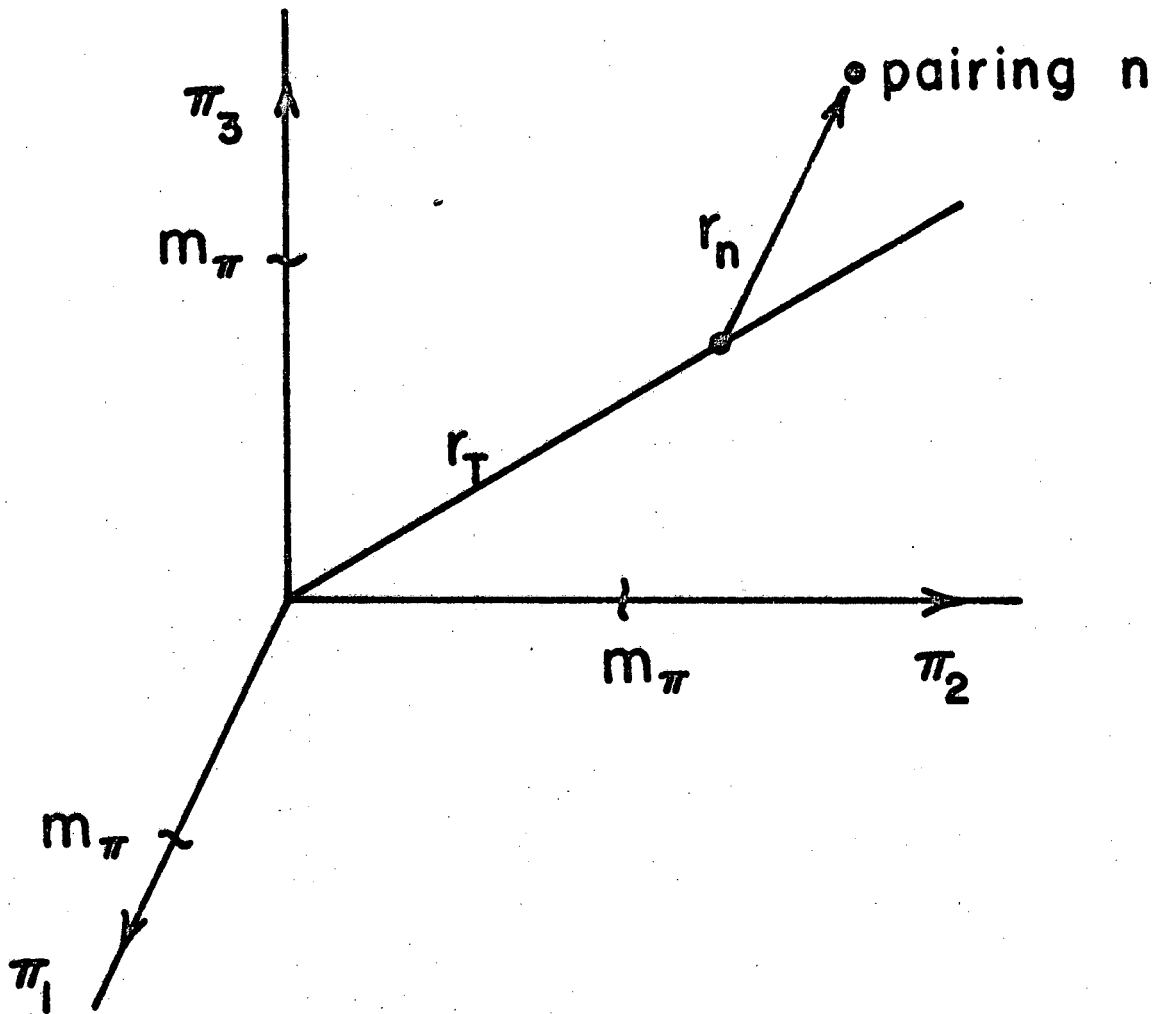
Six photons may be paired to form three pions in fifteen distinct combinations. Each pairing yields three invariant masses for the pions:

$$m_{\alpha}^2 = 2 k_i k_j (1 - \cos \theta_{ij}) \quad \alpha = 1, 3 \text{ and } i \neq j.$$

If each mass is interpreted as a distance along a coordinate axis, then an orthogonal, three dimensional space is defined. Each pairing is represented as a point in this space, as in Fig. 14. The correct or true pairing, in the absence of measurement error, would lie along the principal diagonal at a distance $r_T = \sqrt{3} m_{\pi}$ from the origin. A convenient description of points in this space is in terms of a polar coordinate system with origin at the 3π point.

Because of measurement error, the correct pairing will be displaced from the 3π point, with the magnitude of the displacement being a measurement of the combined error. For many events, the correct pairings should cluster about the 3π point, while the incorrect pairings should be dispersed somewhat uniformly throughout this space.

The geometry of this spherical system provides a natural weighting to the true or near-true pairings, and allows them to stand out against a background of many incorrect pairings. The "radial



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Fig. 14. Three pion mass space.

pairing density" or density of points from all pairings of many events as a function of r^3 , the radial distance from the 3π point in this space, has been an effective means of seeing "pions" in the data, and a good measure of the methods of analysis. For instance, Fig. 15 shows the advantage of using all the sparks in the geometric fit. The invariant masses have been evaluated at the decay points found when all, and then at most six, consecutive sparks were used in the fit. Figure 15 shows the difference between using the fitted shower direction, or the final photon direction in calculating the shower energy, with the invariant masses evaluated at the geometric decay point.

The density of points resulting from the evaluation of the invariant masses at the point of optimum momentum-energy balance is Fig. 16, and indicates that the estimates of the decay point, and shower energies have been significantly improved. If the peak contained only the correct pairings, the area of the peak should be $1/15$ th of the total area. However, the ratio of the area above background within the "pion peak" to the total area is greater than $1/15$, and suggests that in general, more than one pairing, in each event, is physically likely.

By letting the probability that the n th pairing is the true pairing be proportional to $1/r_n$, and evaluating r_n for all pairings in an event, a small subset of the best may be selected, within which the true pairing will generally reside. Table 2 lists the probability P , that the true pairing will be included in a subset of N of the best pairings, using Monte Carlo events.

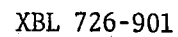
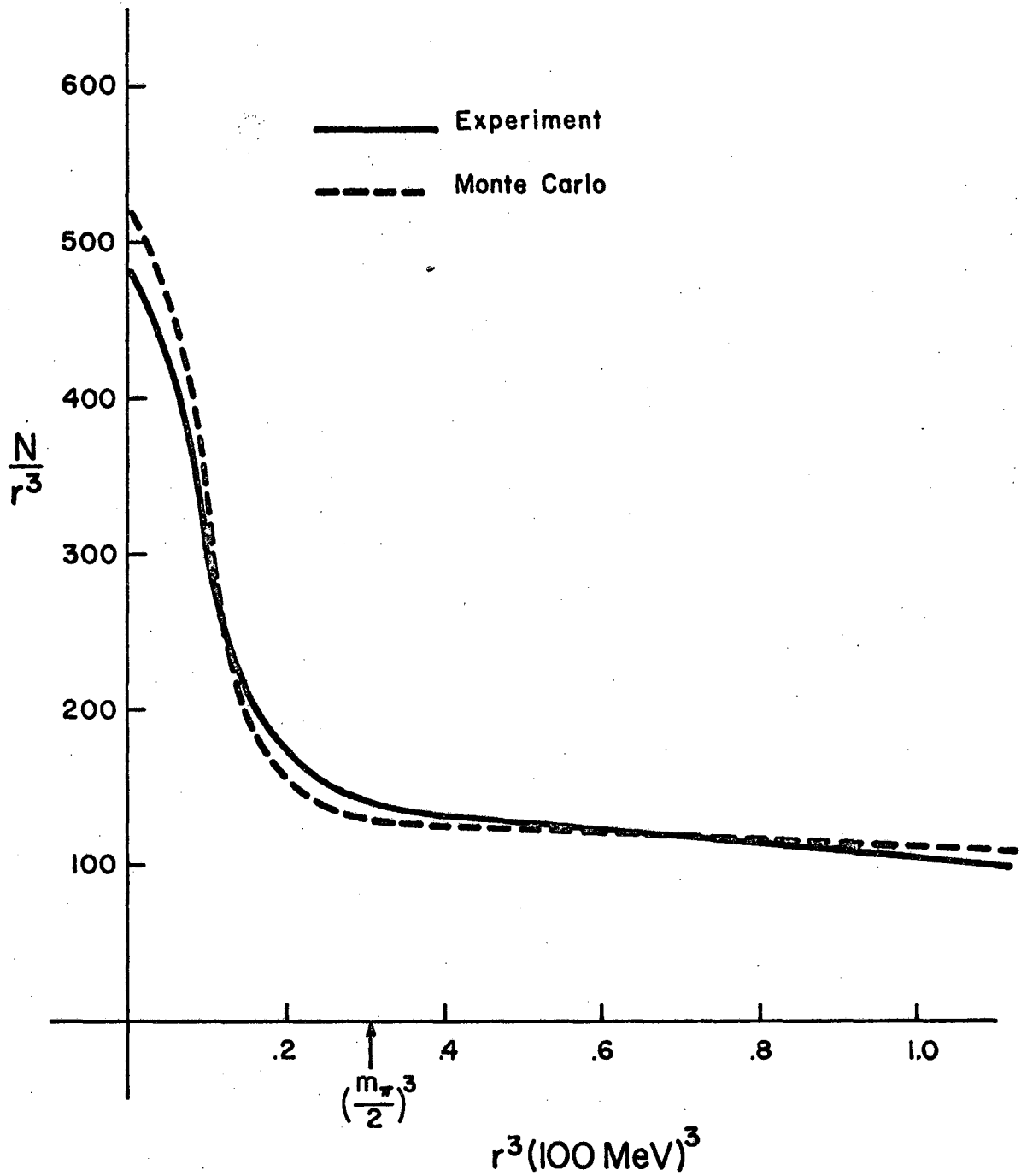


Fig. 15. Distribution in r^3 in the three pion mass space for all pairings.



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Fig. 16. Distribution in r^3 in three pion mass space for all pairings where each event is evaluated at the point in the decay volume of optimum energy-momentum balance.

Table 2

Best N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	51	65	77	89	94	96	96	99	99	99	99	99	99	99	100

These probabilities were established by analyzing data produced by a Monte Carlo simulation of the $K_L^0 \rightarrow 3\pi^0$ decay process.

1. Monte Carlo simulation.

To provide the pairing efficiencies, and in general, a detailed knowledge of the resolutions of the detection, measurement, analysis system, a Monte Carlo computer program was written to simulate the appearance of the $3\pi^0$ events in the experimental apparatus.

The K_L^0 mesons were produced with the experimental momentum spectrum, and passing through the fiducial volume, decayed in flight. The decay into three pions could be modified away from phase space distribution by a matrix element of arbitrary form. Severe alterations of the pion momenta away from three body phase space had no noticeable effect on the overall appearance of the detected events. The three pions were generated in a plane randomly oriented in the K_L^0 rest frame. The gamma rays were generated with equal and oppositely directed momenta in each of the pion rest frames, and their four vectors transformed to first, the K_L^0 rest frame and finally to the lab system.

The photons were converted to electron showers at points determined from Compton and pair-creation cross sections.²⁶ The showers were given a structure from a library of measured, real showers of known energy from the mode $K_L^0 \rightarrow \pi^+\pi^-\pi^0$. These shower

patterns were infused into the geometry of the spark chambers, allowing a modification, due to the structural features of the apparatus, of the showers to be made.

The use of the shower library freed the program of the necessity of using efficiency functions for the conversion and observation of gamma in the spark chambers. Predictions of the Monte Carlo of properties of the data, depending only on the chambers, were rigorous tests of the programs validity. Correct predictions were obtained for the observed distribution of gamma ray conversion points, the shower angular deviations from the various decay points, and the multiplicities of showers appearing in the chambers.¹⁷ Again, the pion mass space pairing distribution is a good test of the Monte Carlo reliability, and Fig. 16 shows agreement with the experimental data.

Thus with confidence that the true pairing was present in the set of the best five, full kinematic fits were attempted on each pairing in the select group.

G. Kinematic fitting.

Assuming a particular pairing for the gamma rays, the six photon momenta may be determined using a fixed value of the K_L^0 momentum at 530 MeV/c, the nominal value. The calculation requires the use of the four equations of energy-momentum conservation, together with two of the pairing equations, say

$$\begin{aligned} k_1 k_2 &= m_\pi^2 / [2 (1 - \cos \theta_{12})] = \eta_1 \\ k_3 k_4 &= m_\pi^2 / [2 (1 - \cos \theta_{34})] = \eta_2. \end{aligned} \tag{8}$$

This combination results in a fourth degree polynomial in one of the photon momenta.

The quartic equation may be easily obtained by reducing the four momentum-energy conservation equations to one equation in three unknown energies. Performing this reduction twice, keeping a different set of unknowns each time, produces a pair of comparable equations of the form,

$$\begin{aligned} Q_{11}k_1 + Q_{12}k_2 + Q_{13}k_3 &= P_1 \\ Q_{21}k_1 + Q_{22}k_2 + Q_{24}k_4 &= P_2. \end{aligned} \quad (9)$$

Multiplying Eqs. (9) together and using (8) gives the desired quartic.

$$k_1^4 + Q_a k_1^3 + Q_b k_1^2 + Q_c k_1 + Q_d = 0 \quad (10)$$

where the coefficients are

$$\begin{aligned} Q_a &= -(P_2 Q_{11} + P_1 Q_{21})/D \\ Q_b &= [(Q_{21} Q_{12} + Q_{22} Q_{11}) \eta_1 + P_1 P_2 - Q_{13} Q_{24} \eta_2]/D \\ Q_c &= -(Q_{12} P_2 + Q_{22} P_1) \eta_1 / D \\ Q_d &= Q_{22} Q_{12} \eta_1^2 / D, \quad D = Q_{21} Q_{11}. \end{aligned}$$

In general, four solutions of (10) are possible, but requiring k_1 to be physical, (k_1 real, and $10 \leq k_1 \leq 400$) usually selected only one or two of the quartic branches. Any choices between remaining branches could be made by picking that solution whose resultant m_3

$$m_3^2 = 2k_5 k_6 (1 - \cos \theta_{56})$$

was closest to the pion mass. The photon momenta $k_i, i = 1, 6$ determined by these kinematics can now be considered functions of the decay point, $k_i = k(X, Y, Z)$, and a fit performed.

Since only two of the three pairing equations are used in the fit, except for the choice of pion order, the pions are no longer indistinguishable, the pion formed by gammas five and six is distinct. A pion order is chosen which gives the best value for m_3 and physical solutions for all gamma energies. Approximately 40%, or two out of the five best pairings in each event, will not produce physical solutions for any choice of pion order or quartic branch.

The photon energies for the selected pairings in each event were fit by searching for a minimum on a chi-square surface formulated from the photon energies as,

$$\chi_S^2 = \left(\sum_n (k_n - E_n)^2 / E_n \right) \cdot \chi_G^2$$

Figure 17 shows this relatively simple surface sampled in the x, y plane centered at the minimum point for two pairings.

The χ_S^2 minimum point was used as the starting point for the final minimum search on the more complex surface formed by using m_3 , the last equation of constraint.

$$\chi_M^2 = (m_3 - m_\pi)^2.$$

A sampling of this surface for two pairings is in Fig. 18. With this method more than one pairing were usually found in each event with $dm = \text{Abs}(m_3 - m_\pi) \leq .1 \text{ MeV}$, essentially satisfying all kinematic constraints.

A criteria was devised for the Monte Carlo data which selected the true pairing from among the pairings surviving the kinematic fits,

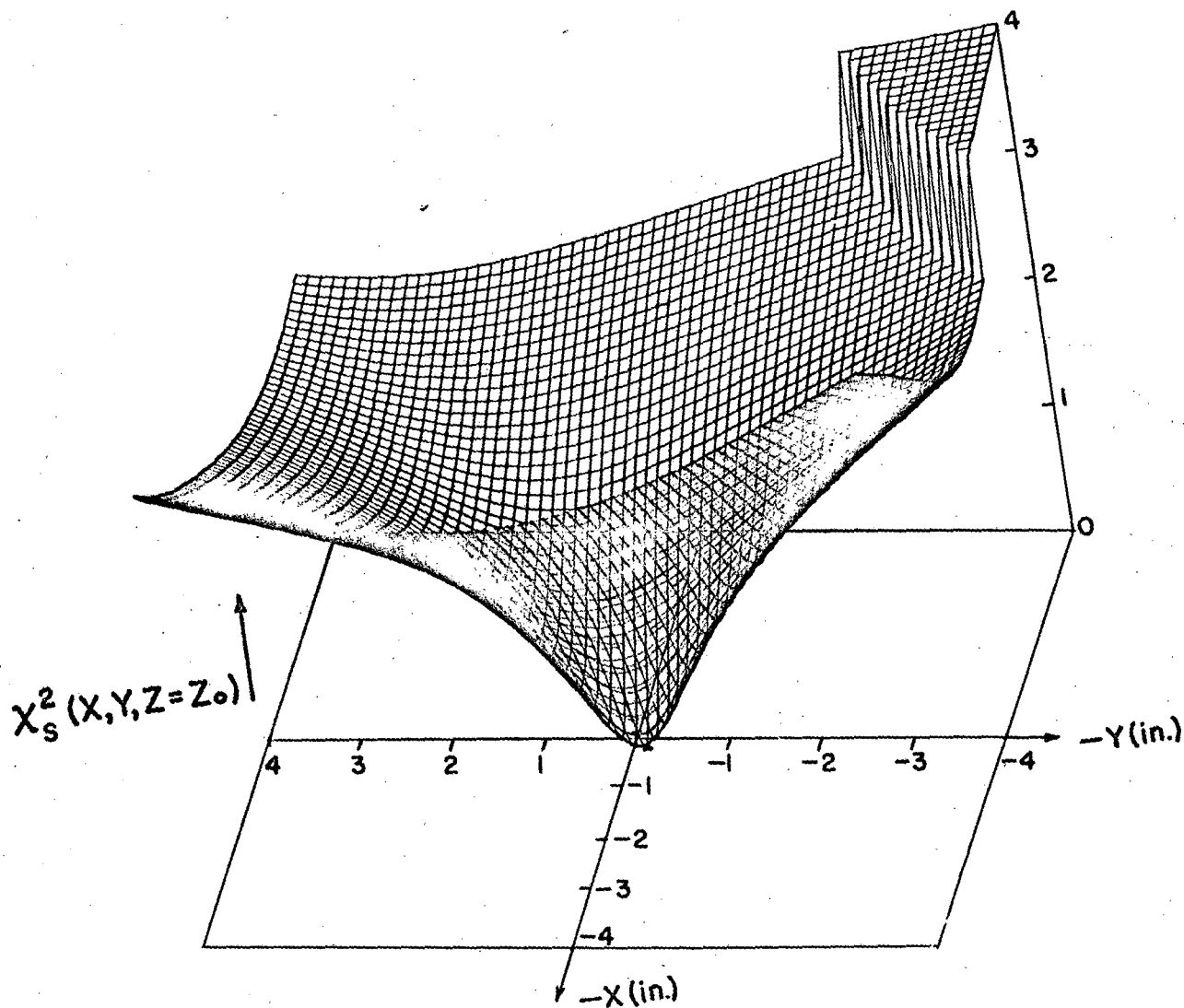


Fig. 17a. Chi-square surface for kinematically fit photon energies sampled in the X, Y plane over an 8×8-in. area centered at $(X, Y, Z) = (-.7, 18.1, -18.1)$ -in. The vertical scale is logarithmic to base 10.

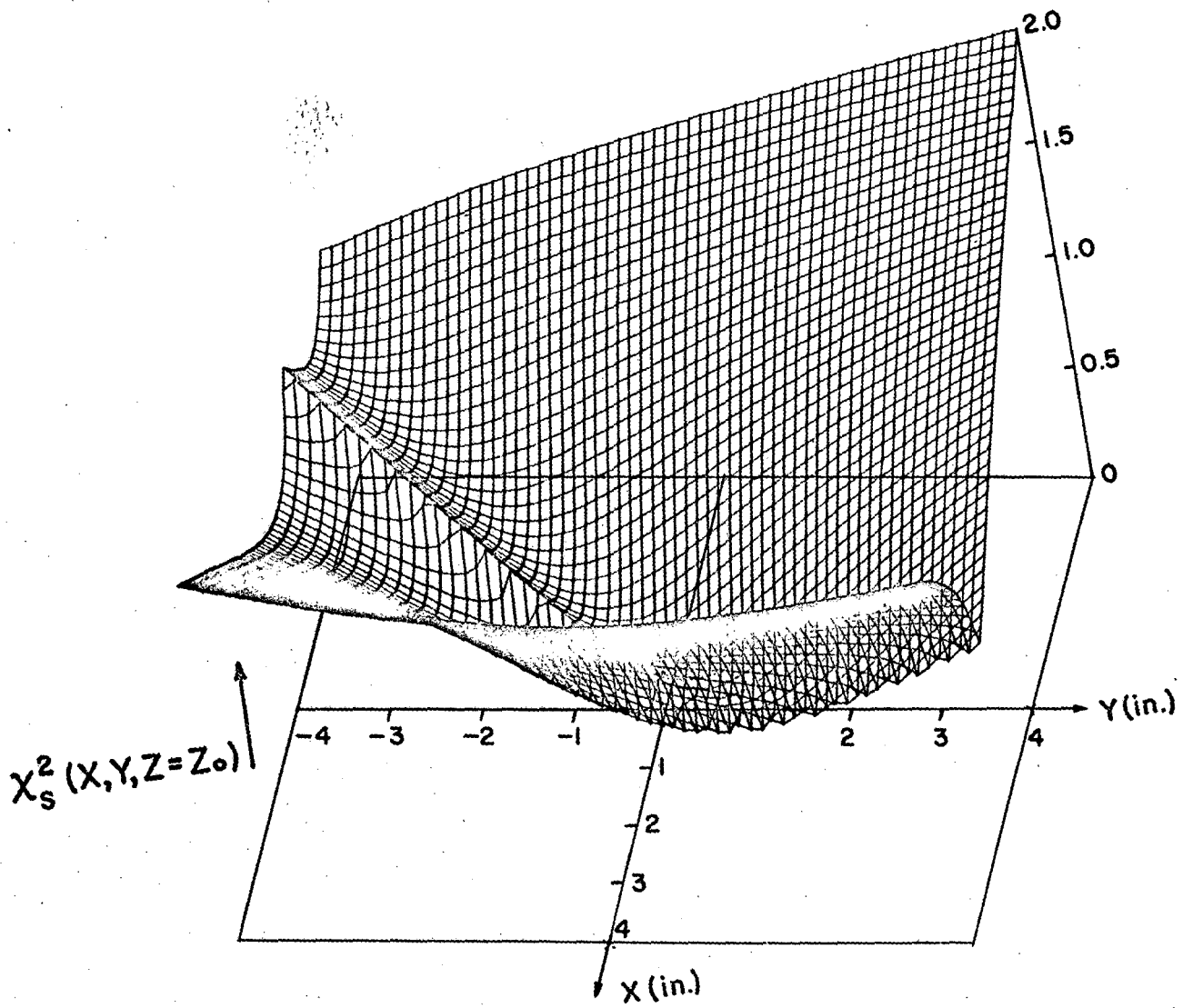


Fig. 17b. Chi-square surface for kinematically fit photon energies sampled in the X, Y plane over an 8×8 -in. area centered at $(X, Y, Z) = (-11.6, 13.6, -11.9)$ -in. The vertical scale is logarithmic to base 10.

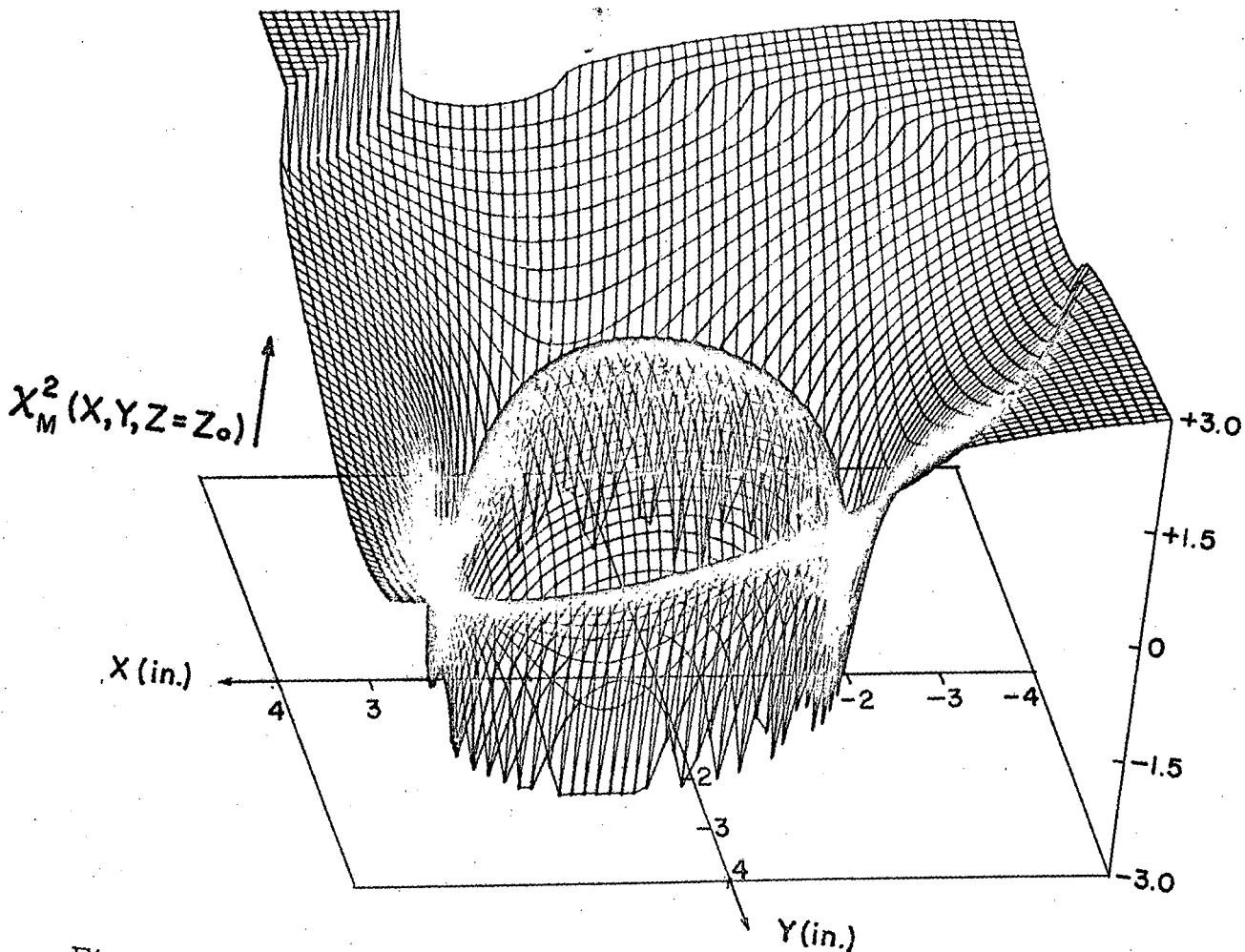


Fig. 18a. Chi-square surface for pion mass sampled in the X, Y plane over an 8X8-in. area centered at $(X, Y, Z) = (.2, 18.5, -18.7)$ -in. The vertical scale is logarithmic to base 10.

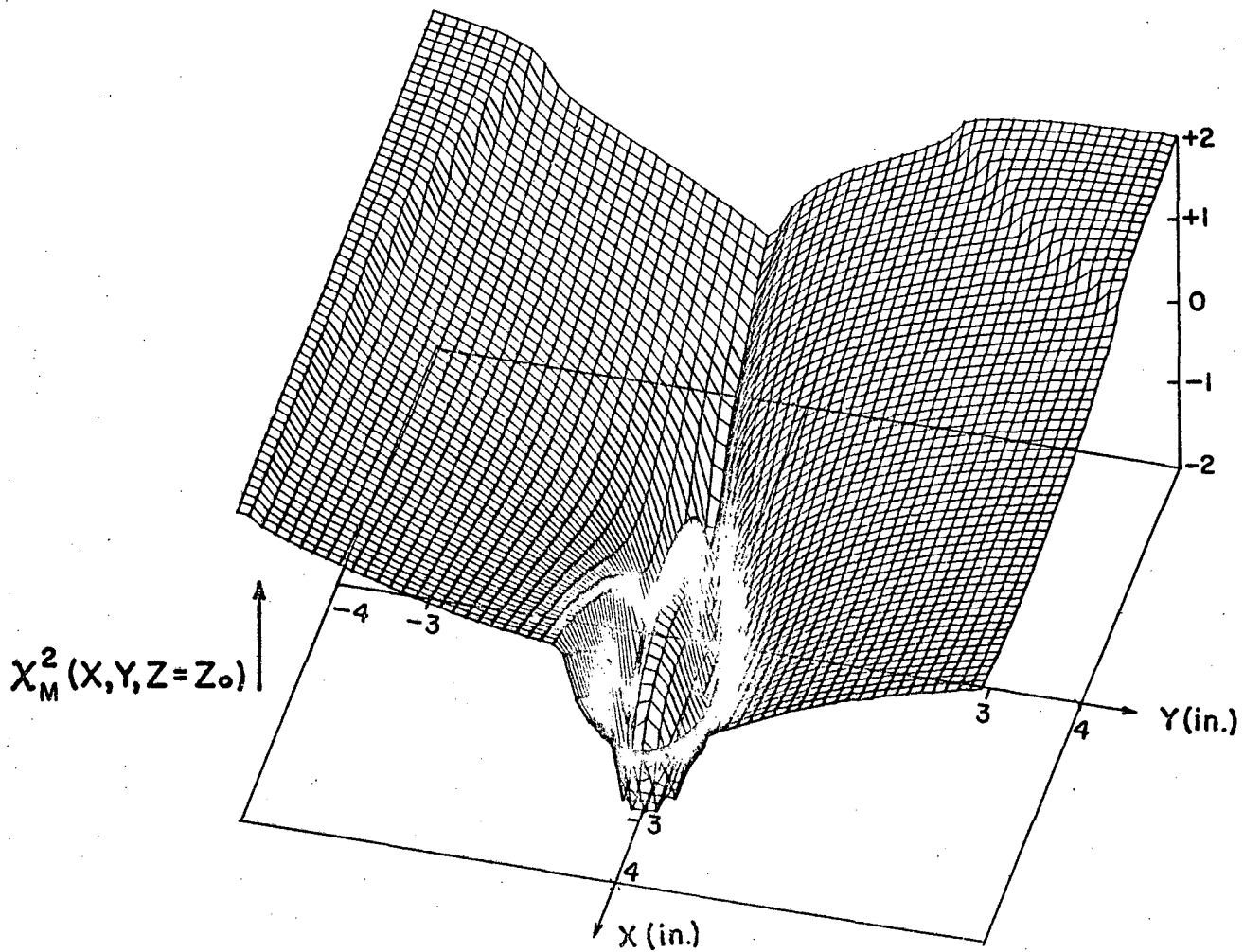


Fig. 18b. Chi-square surface for pion mass sampled in the X, Y plane over an 8×8 -in. area centered at $(X, Y, Z) = (13.2, 12.6, -17.7)$ -in. The vertical scale is logarithmic to base 10.

with a 70% efficiency. However, this efficiency was highly sensitive to the errors in the data and possible biases in the analysis. Fortunately, the radial position of an event on the Dalitz plot (see the following section on Dalitz plot analysis), is approximately the same for more than one of the surviving pairings. So simply choosing the best (lowest) value of χ_S , consistent with $dm \leq .1$, gave as good a resolution as the more detailed selection. The probability for selecting the true pairing was reduced to 50%, but the method is much less sensitive to error and bias and not dependent on the Monte Carlo.

The feature of the $3\pi^0 \rightarrow 6\gamma$ topology which allows than one pairing in an event to be at the same radial position on the Dalitz plot is studied in detail in the appendix.

H. Dalitz Plot Analysis.

The 3π final state of the K_L^0 decay has nine degrees of freedom. The four equations of energy-momentum conservation allow this state to be specified by five independent variables. Choosing among these the energies of any two of the pions, and integrating over all other directional variables gives the differential rate of decay $d^2\Gamma$, as

$$d^2\Gamma = \pi^2 [M]^2 dE_1 dE_2$$

where E_1 and E_2 are the pion energies, $[M]^2$ is the matrix element containing any energy dependence of the decay rate.

The phase space for the process, defined as

$$d^2R = \pi^2 dE_1 dE_2$$

is proportional to the area on the E_1, E_2 plane and is thus "flat." This suggests a planar representation for the decay, proposed by Dalitz, utilizing the fact that the sum of the pion energies must be constant. In the case of the pion kinetic energies $T_i = E_i - m_\pi$,

$$T_1 + T_2 + T_3 = Q, \quad Q = M_K - 3M_\pi = 92.8 \text{ MeV}$$

then the T_i 's may be interpreted as the distances from the sides of an equilateral triangle.

This Dalitz plot may be parameterized in terms of other properties of the 3π system, with these various parameters linearly related to each other. In addition to the total energy and kinetic energy representations, invariant mass description may be formulated as,

$$s_i = [P_{4i} + P_{4k}]^2$$

where P_{4i} and P_{4k} are the pion and K_L^0 four-momentum vectors.

Alternatively, these parameters are related to the kinetic energy by,

$$s_i = (M_K + m_\pi)^2 + 2M_K T_i.$$

Taking the invariant mass of pion pairs gives another description,

$$S_i = [P_{4j} + P_{4k}]^2 \quad (i, j, k) = 1, 2, 3 \text{ and } i \neq j \neq k. \quad (11)$$

Again these are related to the kinetic energies,

$$S_i = (M_K - m_\pi)^2 - 2M_K T_i.$$

The kinetic energy representation may be conveniently transformed into a polar coordinate system r, θ using,

$$T_i = \frac{Q}{3} (1 + r \cdot \cos \theta_i),$$

with θ_i referenced from the T_i axis. (See Fig. 23).

Since the matrix element for the $3\pi^0$ process is expressible, to lowest order, as

$$M(K \rightarrow 3\pi^0) = 1 + \frac{a}{m_\pi} \sum T_i^2$$

then in terms of the polar system,²⁷

$$\sum T_i^2 = \frac{Q^2}{3} (1 + r^2/2) \quad (12)$$

and

$$M(K \rightarrow 3\pi^0) = A(1 + a' r^2),$$

a function of the radius squared only. The plot boundary, in the polar system $r_b(\theta)$, is

$$r_b^2 = 1/[1 + g(1 + r_b \cos 3\theta)]$$

where $g = 2(Q/M_K)/(2 - Q/M_K)^2$, and due to the small value of Q , r_b deviates from a circle of unit radius by only 10%.

IV. CONCLUSIONS

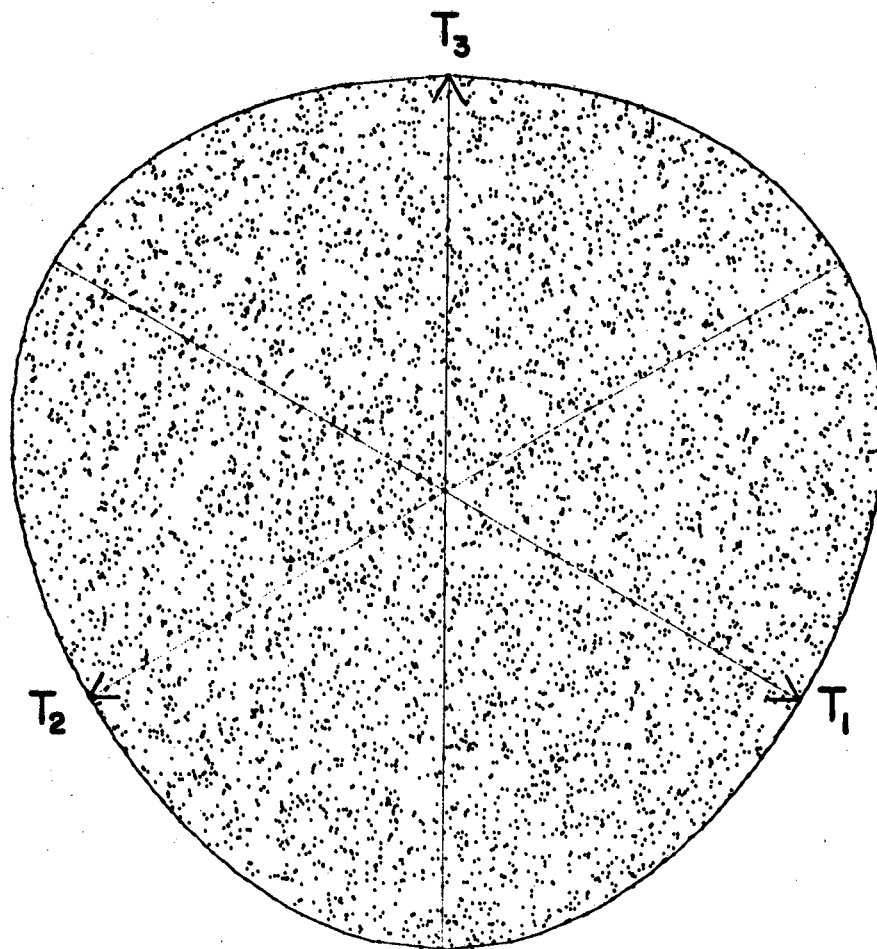
Of the unambiguous six shower events, 5629 survived the fit to the $K_L^0 \rightarrow 3\pi^0$ kinematics with a confidence level of 1% or better, and their Dalitz plot is shown in Fig. 19. The Dalitz plot for $3\pi^0$ has six-fold symmetry, due to the indistinguishability of the pions, so in principle, all the events could be combined into one segment. The full Dalitz plot is presented here to show that no bias exists in the treatment of the pions by the analysis. Since the Dalitz plot deviates from a perfect circle by about 10%, the radial distribution was constructed as a function of R^2 , where,

$$R^2 = (r/r_0)^2,$$

r is the Dalitz position, and r_0 is the radial distance to the boundary at the azimuthal position θ . The radial distribution is shown in Fig. 20 and the azimuthal distribution, combined into one segment of the Dalitz plot due to its six-fold symmetry, is shown in Fig. 20. Fitting the radial distribution to the form $\rho(R) = A_0 [1 + a R^2]$, gives for the coefficient,

$$a = .03 \pm .04.$$

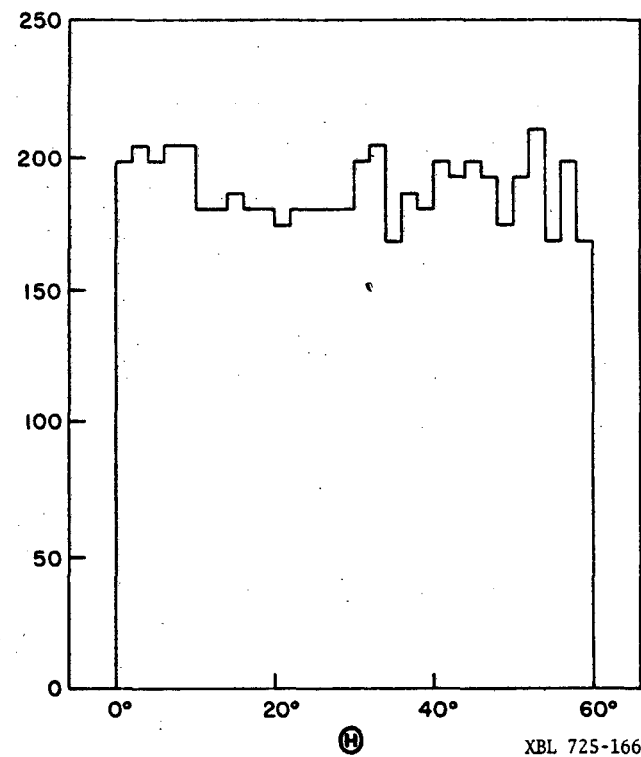
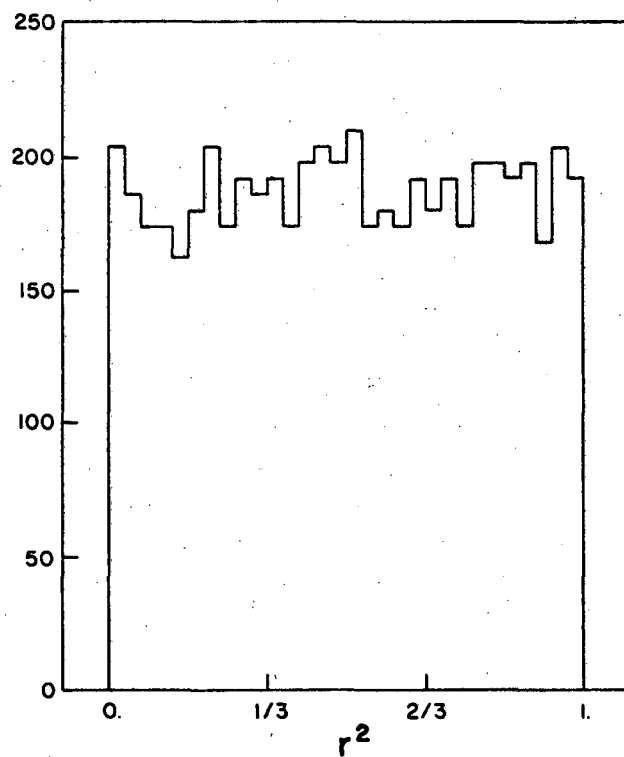
Polynomials of higher order in R were fit. Terms in $\cos \theta$ were not included due to the poor θ resolution. For $\rho(R) = A_0 [1 + a R^2 + b R^3]$, the fit gave $a = .04 \pm .2$, $b = .001 \pm .2$, and for $\rho(R) = A_0 [1 + a R^2 + b R^3 + c R^4]$, find $a = .0002 \pm .5$, $b = .1 \pm 1.1$, $c = -.07 \pm .7$.



DALITZ PLOT

 $K_L^0 \rightarrow 3\pi^0$

Fig. 19



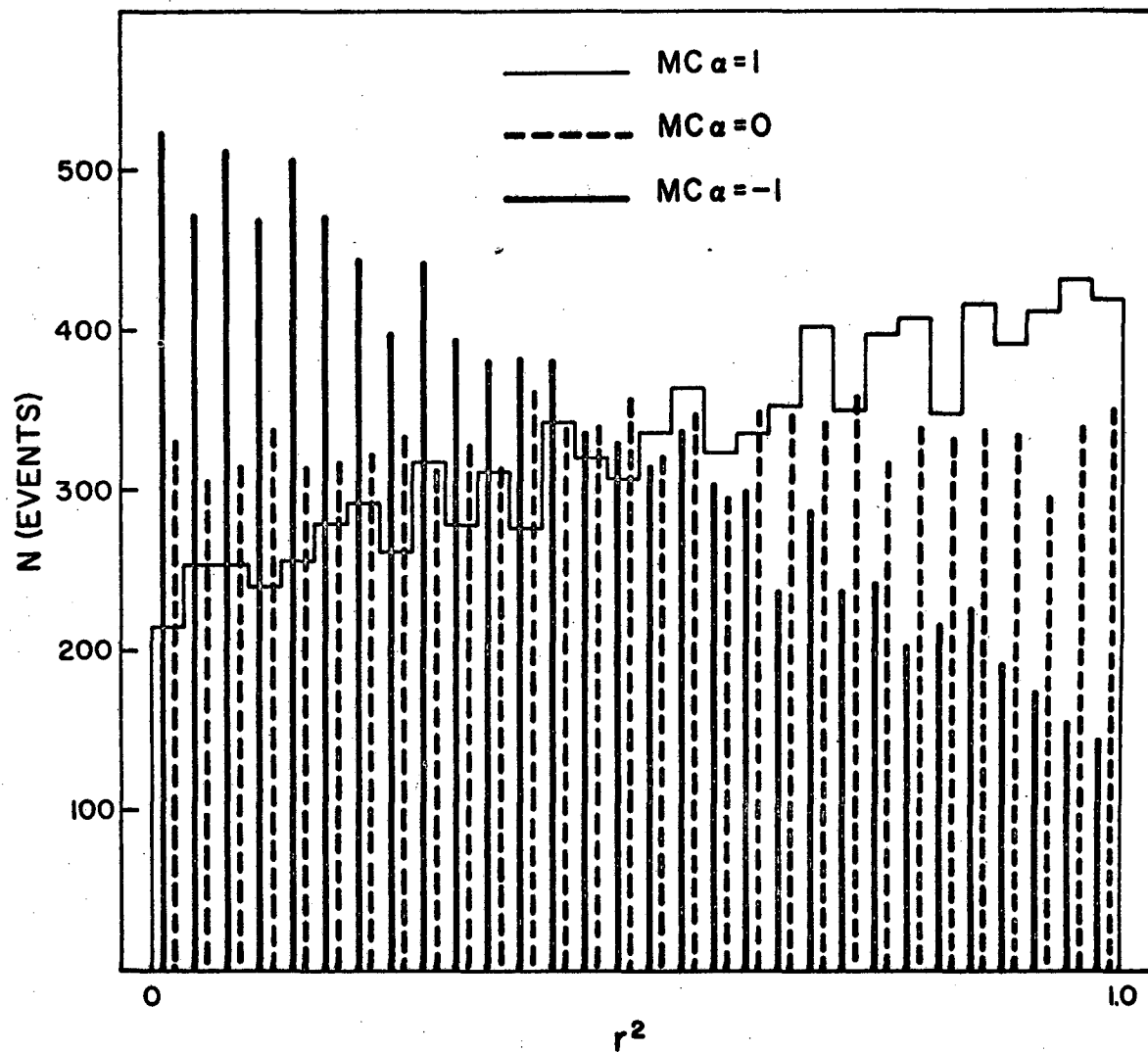
XBL 725-166

Fig. 20. (left) Radial distribution of fitted events on Dalitz plot
(right) Azimuthal distribution of fitted events folded into one segment of the Dalitz plot.

Monte Carlo data generated using a matrix element of the form, $M = 1 + \alpha r^2$, for $\alpha = -1, 0, +1$, were kinematically fit and analyzed by a procedure identical to that for the real data. The R^2 distributions for these data are presented in Fig. 21, and the azimuthal distributions in Fig. 22. The Dalitz plot resolution for these data is drawn in Fig. 23 as an error ellipse. Fitting these data to the above polynomial forms gives values for the coefficients as shown in the following table.

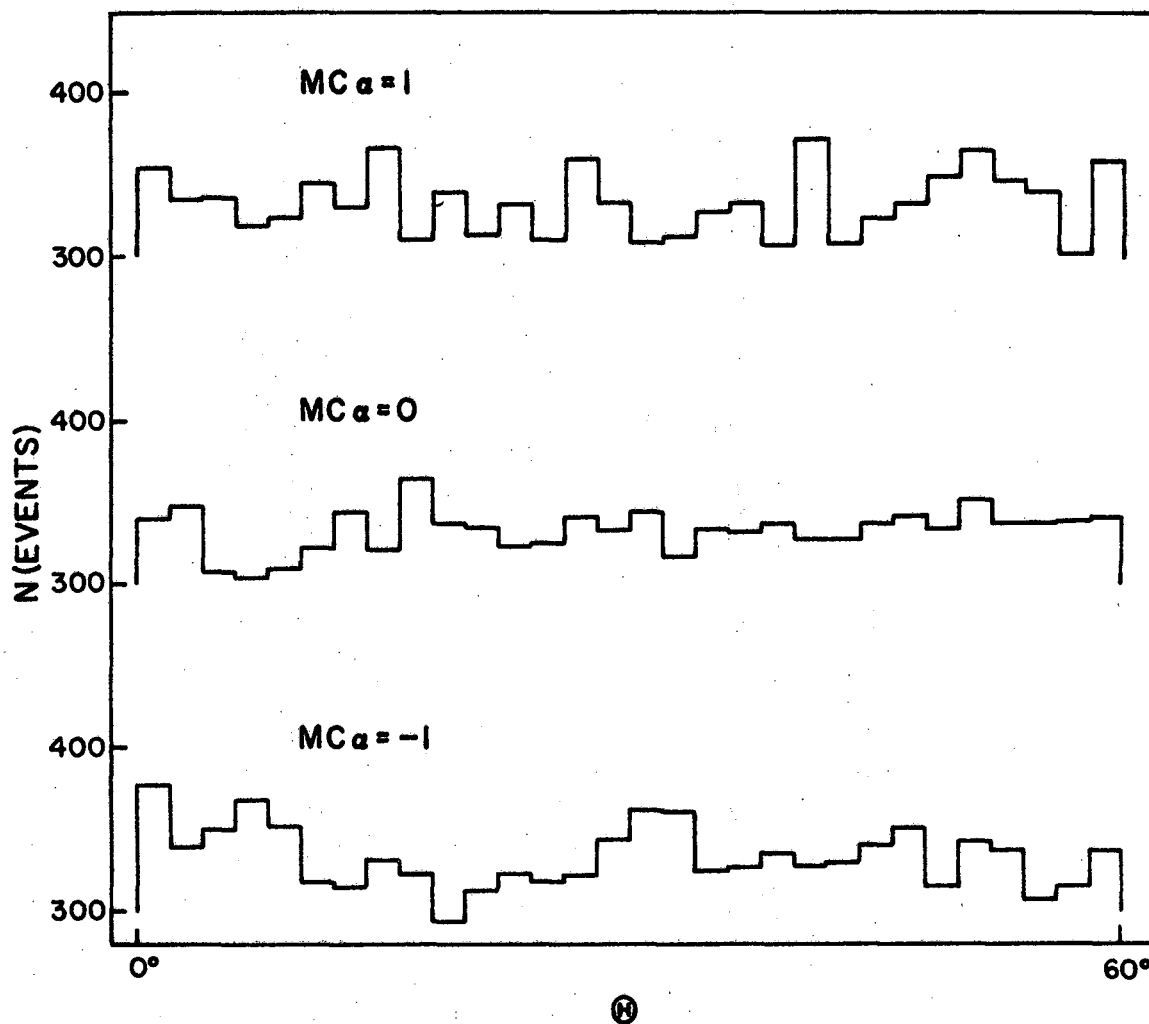
Monte Carlo data (10 000 fitted events per set)

	<u>a</u>	<u>b</u>	<u>c</u>
$\alpha = -1$	$-.66 \pm .031$		
	$-.77 \pm .110$	$.085 \pm .085$	
	$-.94 \pm .266$	$.509 \pm .537$	$.271 \pm .34$
$\alpha = 0$	$.01 \pm .033$		
	$.07 \pm .15$	$.01 \pm .14$	
	$.04 \pm .36$	$.07 \pm .83$	$-.05 \pm .55$
$\alpha = +1$	$.84 \pm .037$		
	$.81 \pm .220$	$.06 \pm .193$	
	$.62 \pm .49$	$.51 \pm 1.13$	$-.311 \pm .760$



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Fig. 21. Radial distribution of Monte Carlo events on Dalitz plot, 10 000 events per set.



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Fig. 22. Azimuthal distribution of Monte Carlo events folded into one segment of the Dalitz plot, 10 000 events per set.

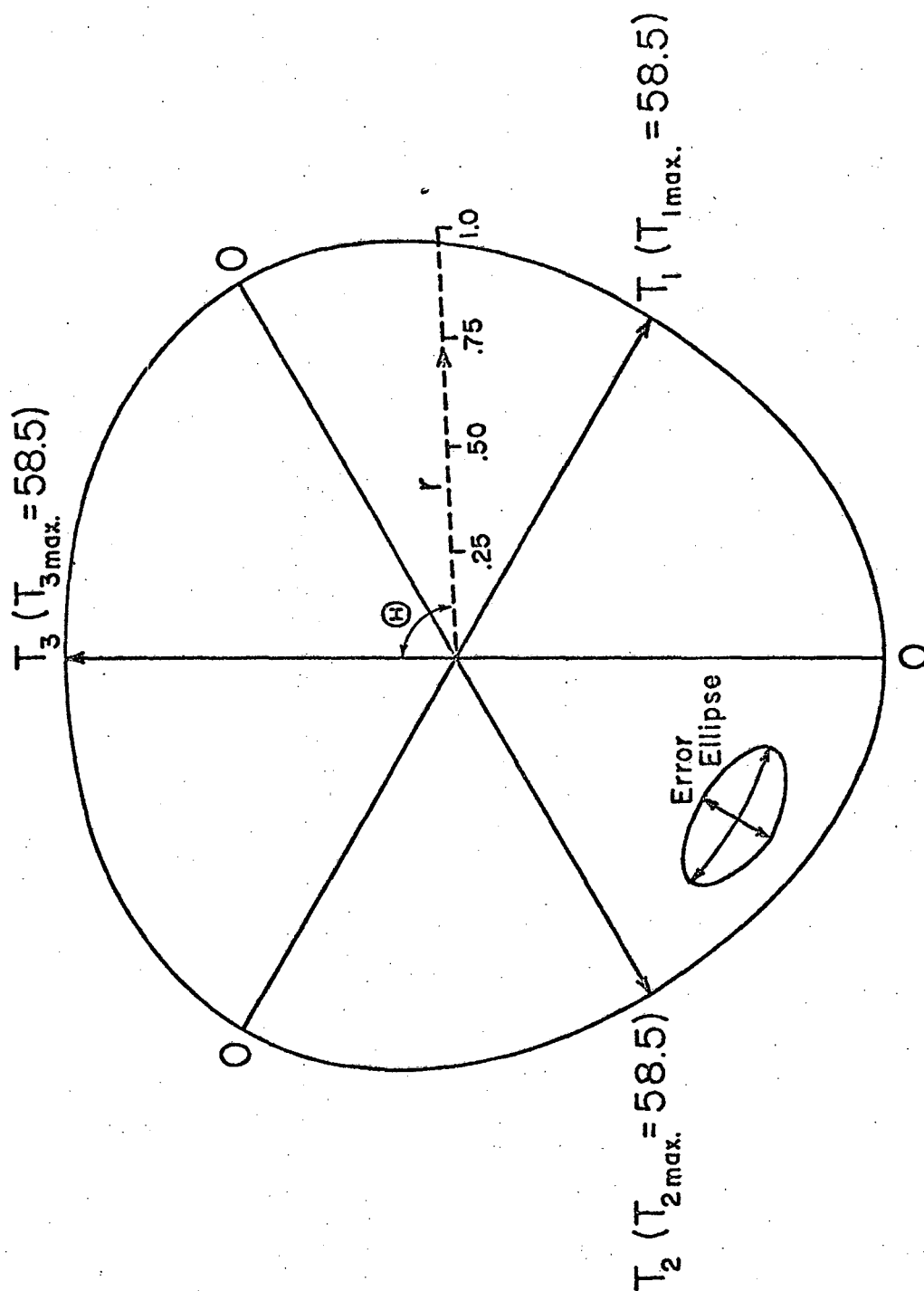


Fig. 23. Dalitz plot resolution.

Using this table, a correction to the slope of the R^2 distribution may be deduced of the form,

$$\alpha = a/a_0 + \delta$$

where $a_0 = .7$, and $\delta = .04$. The coefficient in Eq. (2), the matrix element for the $K_L^0 \rightarrow 3\pi^0$ decay, is thus,

$$\alpha = .05 \pm .08$$

where the error has been enlarged to cover any systematic biases.

This measurement is consistent with the estimates of α derived earlier, Eqs. (4), (6), and (7).

The lack of any measurable structure on the Dalitz plot implies the absence of any low mass, narrow $\pi^0 - \pi^0$ resonances, or a strong low energy $\pi^0 - \pi^0$ interaction. The statistics and resolution of this experiment make the effects of a broad low mass $\pi^0 - \pi^0$ resonance unobservable. An experiment designed to improve statistics by two orders of magnitude, and resolution by an order of magnitude would be capable of seeing the structure of such a resonance, as well as provide a good measurement of the model dependent $\pi^0 - \pi^0$ scattering length. Since the estimates of α are not in substantial agreement as to magnitude or sign, such a measurement could appraise the various models for the $\pi^0 - \pi^0$ interaction, as well as the weak transition of the K_L^0 . A comparison with an identical analysis for the process $\eta \rightarrow 3\pi^0$ would help in separating the effects of the $\pi^0 - \pi^0$ interaction from the weak transition.

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The guidance and encouragement of Professor A. Carl Helmholz as my adviser and chairman of my dissertation committee, and the support of Professor Burton J. Moyer is deeply appreciated.

The ingenuity of design and the high quality of experimental technique came largely from the contribution of Dr. Sherwood Parker, Dr. Charles Rey, and Dr. Robert Kenney. I have learned much by my association with them.

For the many contributions and efforts of Dr. William P. Oliver, my fellow graduate student, I am very grateful.

The identification and encoding of the six shower events were largely performed by Garth Burns and Tim Daly, who also drew several of the figures appearing in this thesis. I am indebted to them for this vital service.

The SASS programs were written by Max Leavitt to whose considerable contributions of time and interest I am extremely grateful.

Ellen Epstein's efforts in organizing the scanning data are very much appreciated, and the stamina and vitality of Emily Friedman in performing many checks of the state of the experimental apparatus during the run is greatly admired.

To Alice, my wife, whose faith and encouragement have sustained me on a long and often difficult road, I offer my deepest appreciation.

I am grateful to Mrs. Loretta Lizama for helping me put this thesis into good technical form and to Carmen Hubbard for typing the manuscript.

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APPENDIX

Pairing Redundancy.

The existence of alternate pairings with approximately the same radial Dalitz position can be understood by separating the expression for the radius into pairing independent and pairing dependent terms. The sensitivity of the pairing dependent term under the permutations of the pairing operation determines then the number of pairings sufficiently similar to the true pairing. Using Eq. 12, the radius, in terms of the pion kinetic energies, is

$$r^2 = 2\left(\frac{3}{Q^2} \sum_i T_i^2 - 1\right)$$

Since

$$\begin{aligned} T_i^2 &= (E_i - m)^2 \\ &= (k_{i_1} + k_{i_2} - m)^2 \end{aligned}$$

where k_{i_1} and k_{i_2} are the momenta of the two photons forming the ith pion. Then, using the above relation,

$$\sum T_i^2 = 3m^2 - 2mM_K + \sum_{n=1}^6 k_n^2 + 2 \sum_{i=1}^3 k_{i_1} k_{i_2}.$$

Only the last term is pairing dependent, so writing

$$r^2 = R_{\text{ind}} + R_{\text{dep}}$$

gives

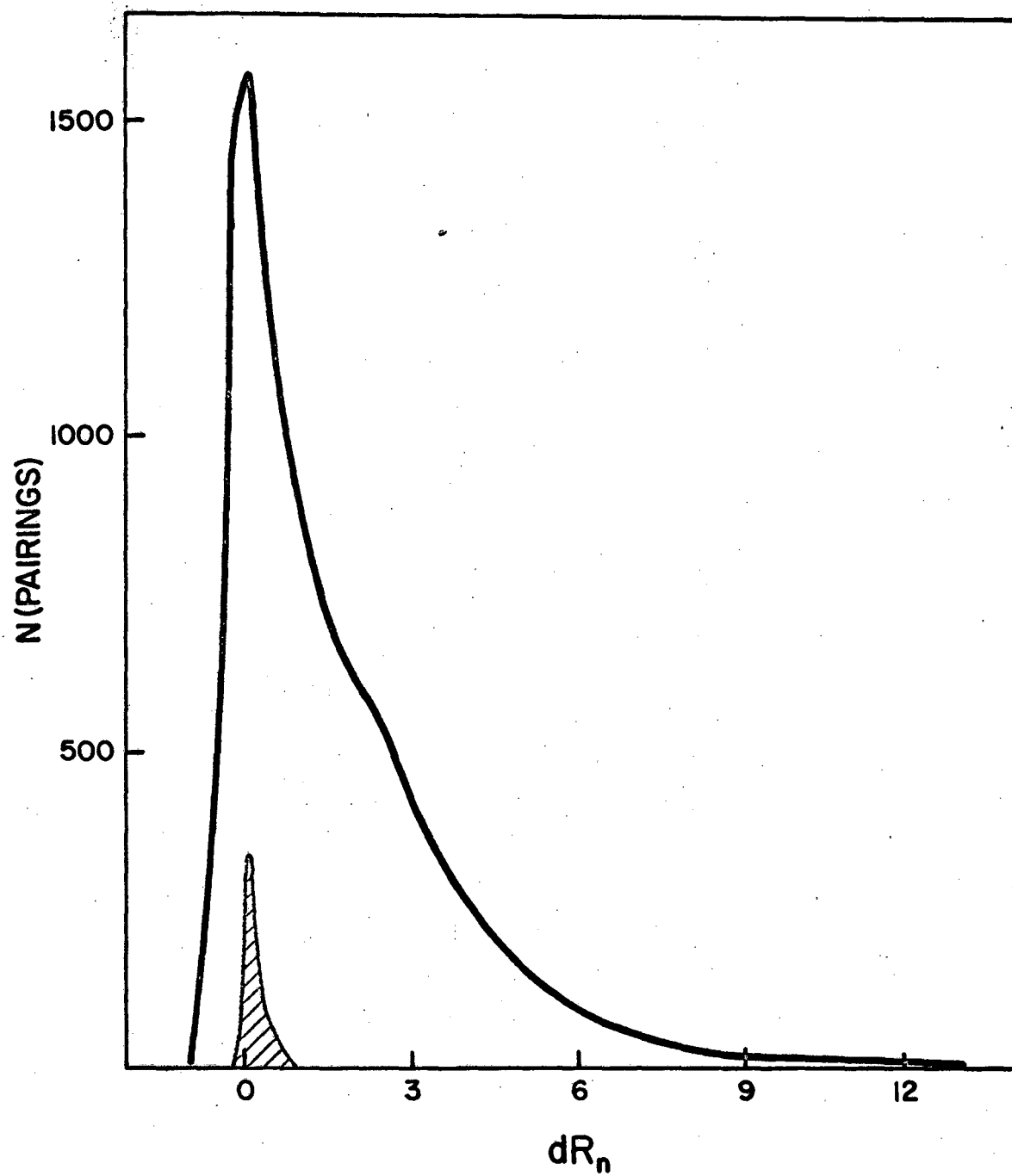
$$R_{\text{dep}} = \frac{12}{Q^2} \sum k_{i_1} k_{i_2}.$$

For each event, fifteen values of R_{dep} may be calculated by choosing the various pairing combinations. Let n label the particular pairings, $1 \leq n \leq 15$, and denote these values by $R_{\text{dep}}(n)$. The error in $R_{\text{dep}}(n)$ for the fourteen incorrect pairings, expressed as,

$$dR_n = R_{\text{dep}}(n) - R_{\text{dep}}(\text{true}) \quad n \neq \text{true}$$

appears quite insensitive to most of the pairings, as indicated by placing all fourteen dR_n in a single distribution, as in Fig. 24, and observing that the distribution peaks strongly at $dR_n = 0$. Since few of the incorrect pairings combine the photons so as to form "pions" with invariant masses near m_π , the Dalitz plot for these is not necessarily bounded by $r = 1$. So radial error will probably exceed unity and, additionally, since $R_{\text{dep}}(n)$ is not the full expression for the Dalitz radius, dR_n has no constraint to be less than unity. If the requirement that these incorrect pairings survive the kinematic fitting procedure described in the analysis is made, the Dalitz radii for these pairings is bounded by $r=1$. The error dR_n need not in principle be less than unity, but as the shaded distribution in Fig. 24 (obtained by making a separate distribution of dR_n for those pairings surviving the kinematic fit) shows, the bound of unity is not exceed.

To show that each event may have more than one pairing with near zero dR_n , the dR_n are ordered by magnitude and placed in fourteen



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Fig. 24. Error in pairing dependent term for all incorrect pairings. Pairings surviving the kinematic fit are within the shaded inset.

distributions. The distribution in which a particular dR_n is placed is determined by its position in the ordered sequence. These distributions are displayed in Fig. 25 as a line of points, where the density of points on the line corresponds to the height of the distribution, and the distance along the line measures the magnitude of dR_n . In Fig. 25, ordered sequence 1 is the distribution of the best (or smallest) dR_n for each event, ordered sequence 2 is the distribution of the next best (or second largest) dR_n , and so on. Here then can be seen that each event has on the average two or three incorrect pairings with dR_n very near zero, and quite probably, with the radial error with respect to the true pairing, acceptably small. The first distribution in the sequence, the best of dR_n as shown separately in Fig. 26, is again strongly peaked at zero, indicating a preferential agreement by this pairing for the true Dalitz radius.

To show the dependence between the radial error on the Dalitz plot and dR_n , for each of these best dR_n in the distribution in Fig. 26, the radius on the Dalitz plot for that incorrect pairing was determined, and the radial difference with respect to the true pairing taken. A scatter plot of this radial difference versus the corresponding dR_n is shown in Fig. 27, and indicates that the relationship between them is fairly linear. In fact, the Dalitz radial error is essentially dR_n , and 80% of the events have a radial error of less than .2 for this best alternate pairing. By numbering the pairings from one to fifteen, with the true pairing number one, then the distribution of the pairing number (or n) for the best alternate pairing, shown in Fig. 28, reveals that certain pairings will very likely resemble the topology of the true pairing.

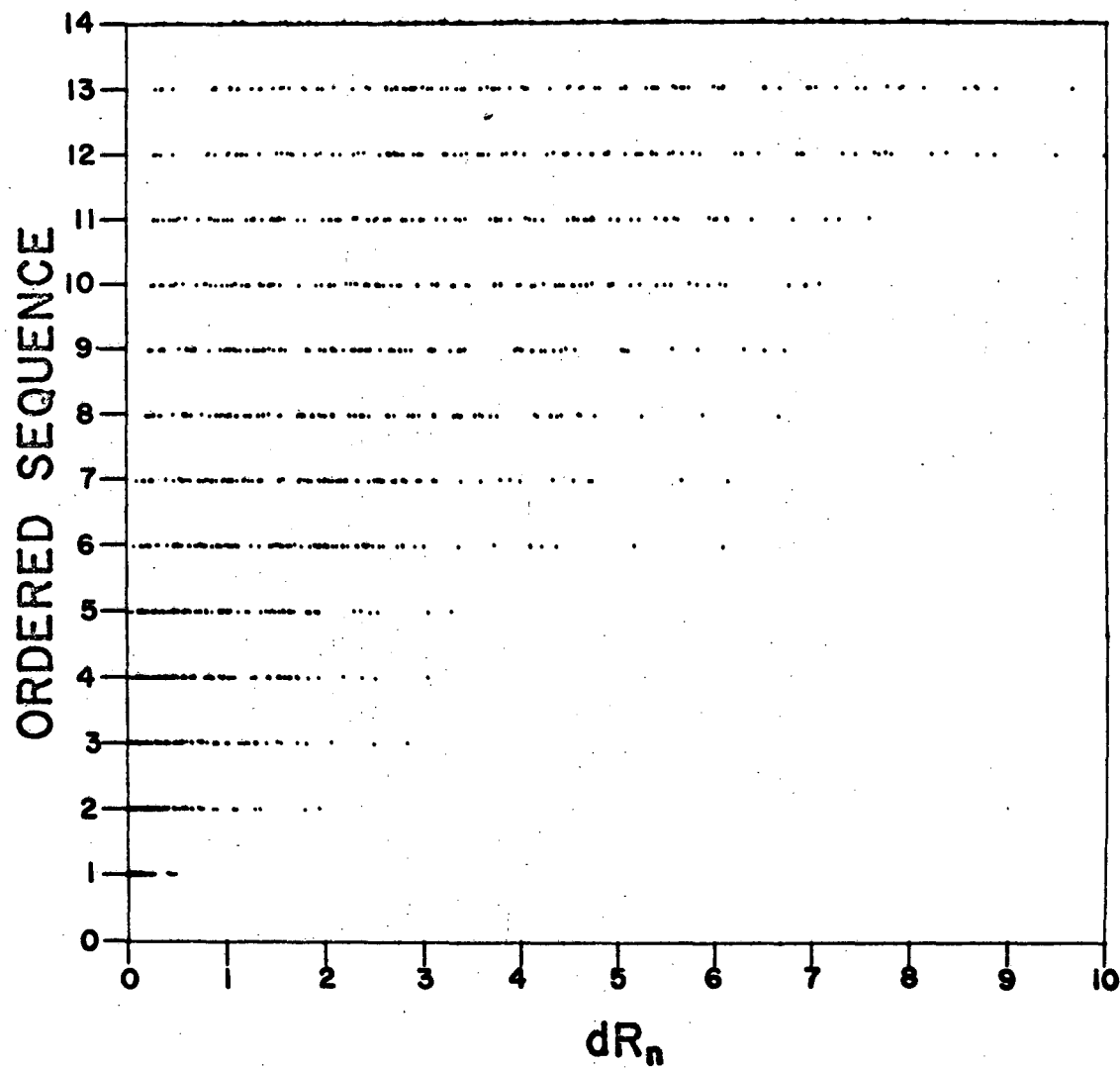
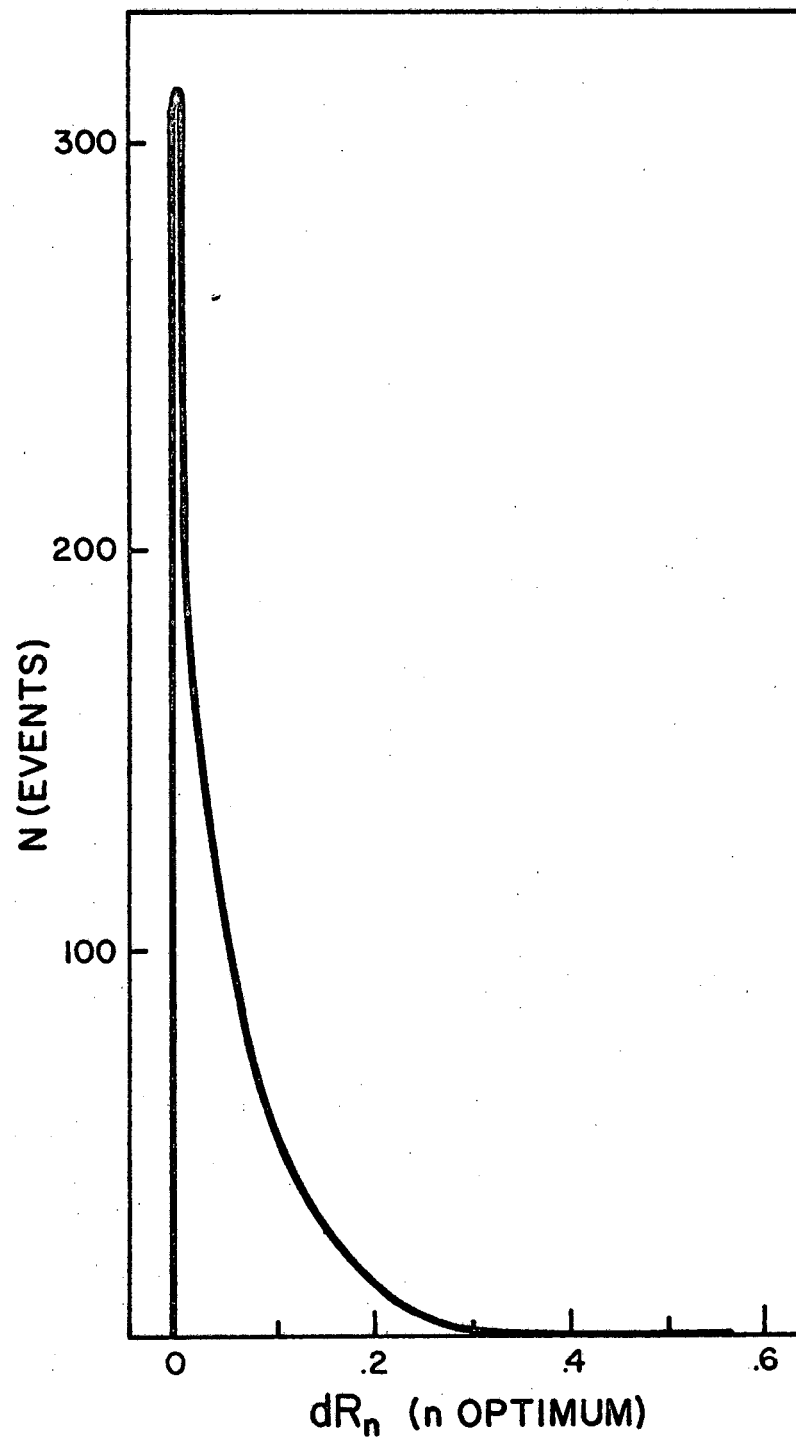


Fig. 25. Error in pairing dependent term, where for each event the fourteen errors are ordered and placed in separate distributions. These distributions are displayed here as line densities.



XBL 726-906

Fig. 26. Error in pairing dependent term for pairing with smallest error. All pairings here are incorrect pairings.

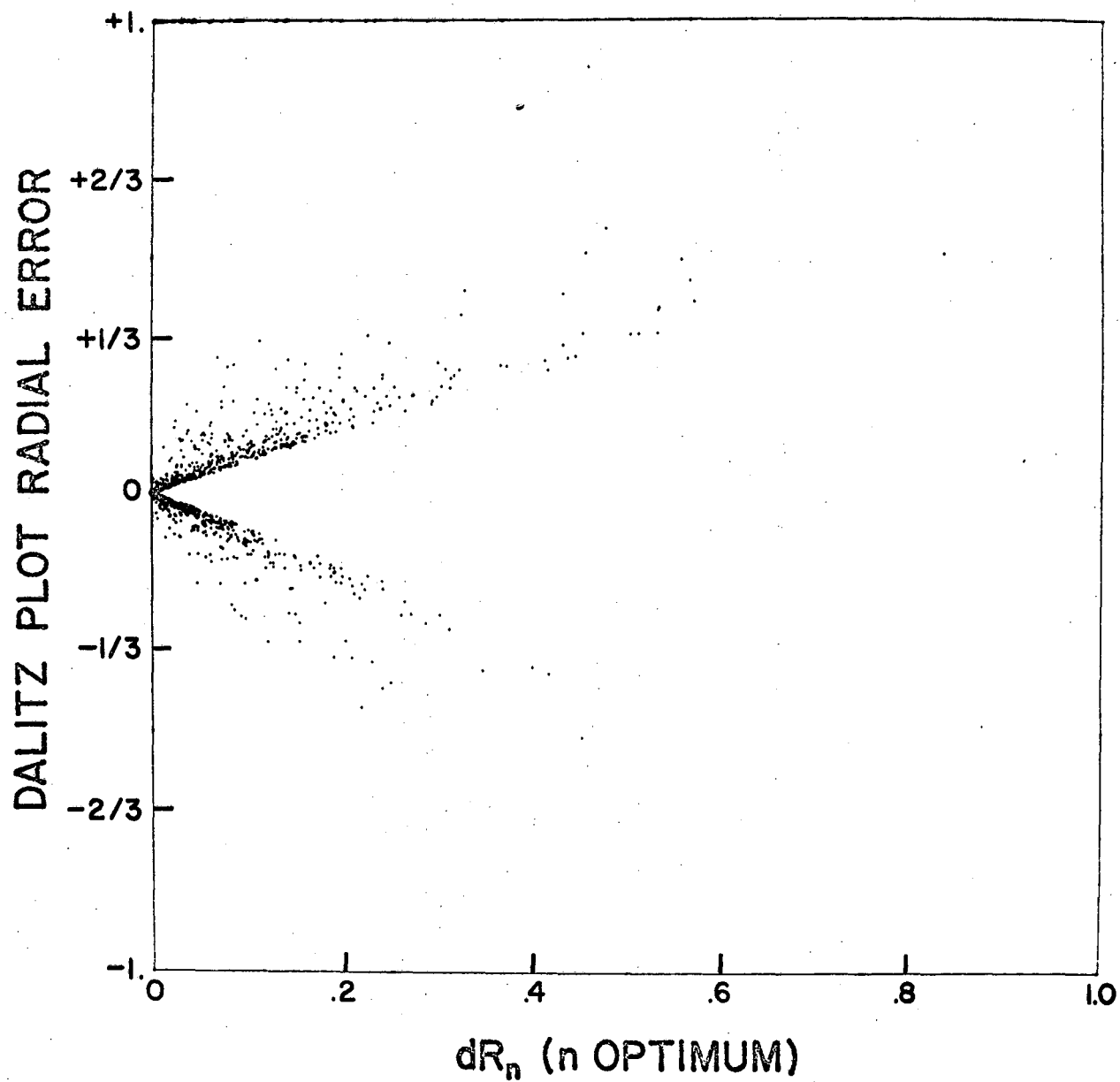
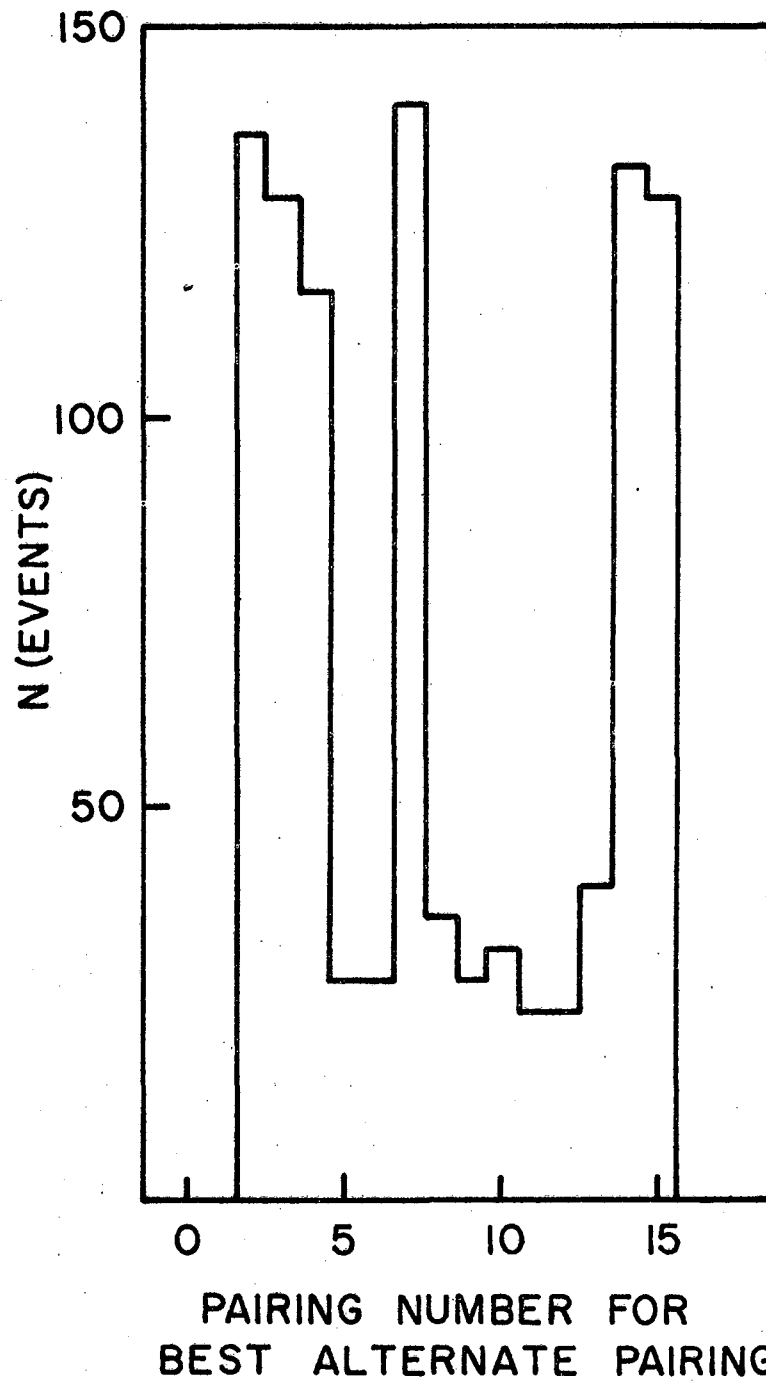


Fig. 27. Scatter plot of Dalitz plot radial error vs the error in the pairing dependent term for the pairing with smallest error. All pairings here are incorrect pairings.

-85-



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Fig. 28. Distribution of the number for the incorrect pairing with smallest error.

To further illustrate the topological resemblance among the pairings, use may be made of an additional Dalitz formulation. From Eq. 11 the radius can be expressed in terms of the invariant masses:

$$\sum S_i^2 = A_0 + 4M_k^2 \sum T_i^2$$

$$\text{with } A_0 = 2(M_k - m)^4 - 4(M_k - m)^2 M_k Q$$

then

$$r^2 = \frac{1}{2M_k^2 Q^{2/3}} \left[\sum S_i^2 - A_0 - 4M_k^2 Q^{2/3} \right].$$

In terms of the photon momenta,

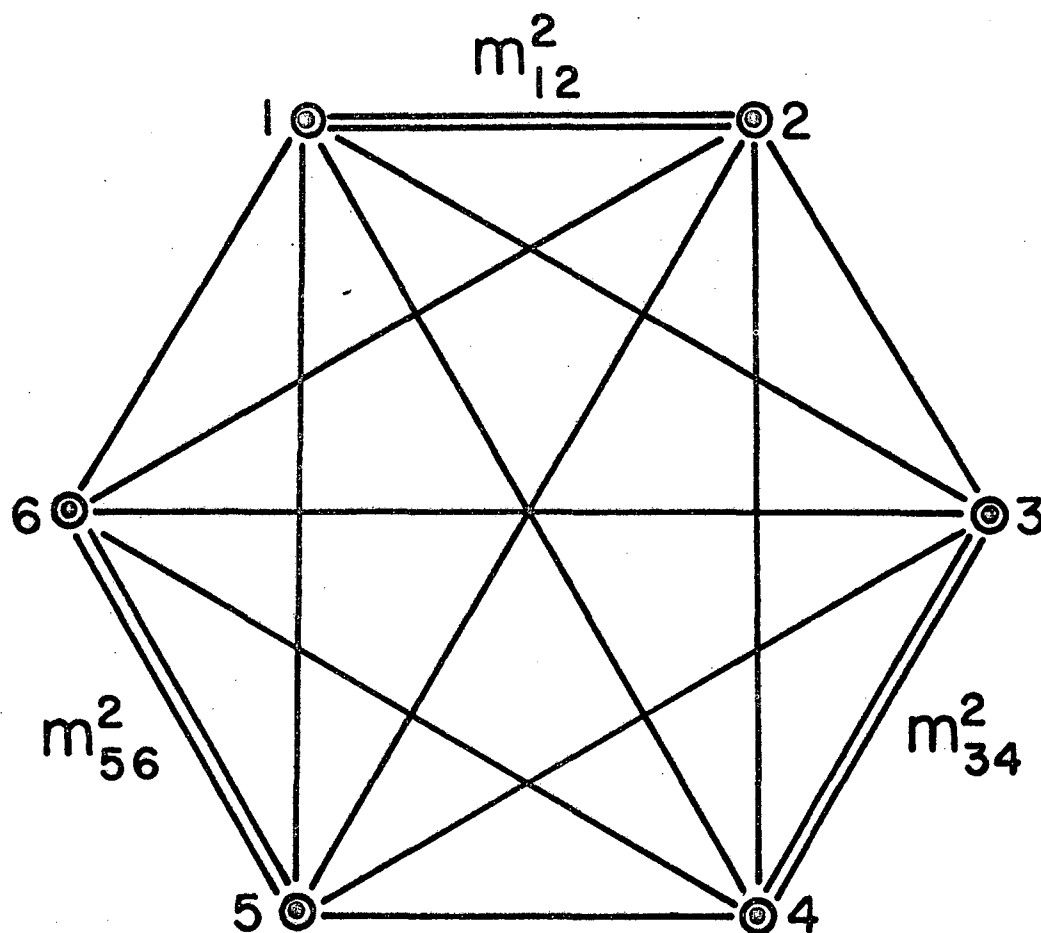
$$\begin{aligned} S_i^2 &= [(k_{j_1} + k_{j_2} + k_{k_1} + k_{k_2})^2 - (\bar{k}_{j_1} + \bar{k}_{j_2} + \bar{k}_{k_1} + \bar{k}_{k_2})^2]^2 \\ &= [m_{j_1 j_2}^2 + m_{k_1 k_2}^2 + m_{j_1 k_1}^2 + m_{j_2 k_1}^2 + m_{j_2 k_1}^2 + m_{j_2 k_2}^2]^2. \end{aligned}$$

Or,

$$\sum S_i^2 = \sum_{i=1}^3 \left(\sum_{n=1}^6 m_{ab}^2 \right)^2.$$

Hence r^2 is related to the fifteen invariant masses formed by taking all possible combinations of the six photons in pairs.

The six m_{ab}^2 's per pion may be grouped by their sensitivity in determining the radius. These group assignments are shown by representing each m_{ab}^2 as a line segment connecting any two points (representing the photons), placed at the corners of a hexagon, as in Fig. 29.



DI-GAMMA INVARIANT MASS GROUPS

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Fig. 29. Illustrating the two-photon invariant mass combination arising from a particular choice of pairing.

Assuming a pairing of (12) (34) (56), where (ij) means gammas i and j form a pion, Fig. 25 shows that the invariant masses of the hypothesized pions have greater weight in the r^2 determination. So the two groupings are,

$$M_1(i) = m_{j_1 j_2}^2 + m_{k_1 k_2}^2$$

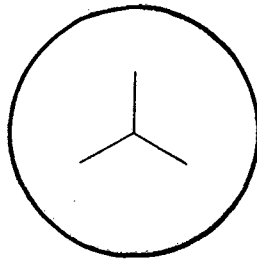
$$M_2(i) = m_{j_1 k_1}^2 + m_{j_1 k_2}^2 + m_{j_2 k_1}^2 + m_{j_2 k_2}^2.$$

These parameters may be used to produce a two dimensional representation of the multi-dimensional six photon-three pion topology. Each of the pions is now described by two variables, M_1 and M_2 . To produce a distinctive pattern for the three pion system, break a unit circle into three equal arcs. Using each arc as an axis, plot a two parameter function with the boundary condition that at each end of the arc segment the functions all have a common value. So for R_{π} , the function for the i th pion, take for example,

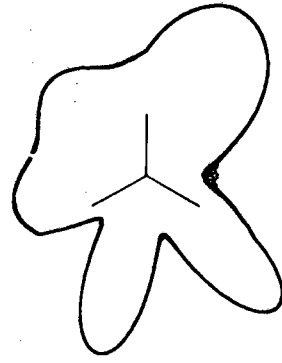
$$R_{\pi_i} = 1 + A_i \sin[(2\pi/3)b_i\Phi] \quad i = 1, 3.$$

Set $A_i = C_1 \text{ Abs}[M_1(i) - 2m^2]/(2m^2)$, and $b_i = 1 + C_2 \text{ Abs}[M_2(i) - 4m^2]/(4m^2)$,

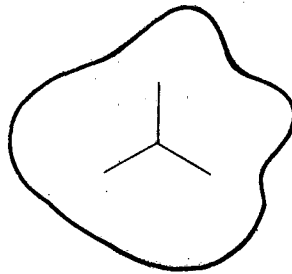
an integer. The values of C_1 and C_2 were adjusted to give a good display of M_1 and M_2 and set to $C_1 = C_2 = 4$. If R_{π_i} and Φ are interpreted as polar coordinates, and the three pion functions are joined in sequence, then a flower-like pattern emerges. Figure 30 shows all pairings for a typical Monte Carlo event. This two dimensional representation of a six dimensional system allows similar topologies to be identified. The true pairing is a circle, thus pairing 3, which is very nearly circular is near congruent in topology,



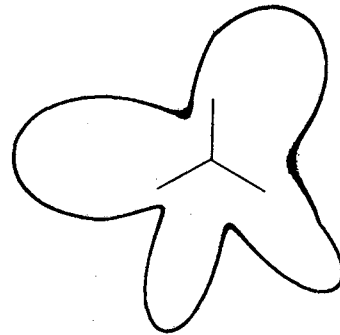
PAIRING 1



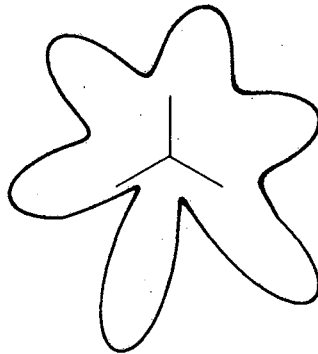
PAIRING 2



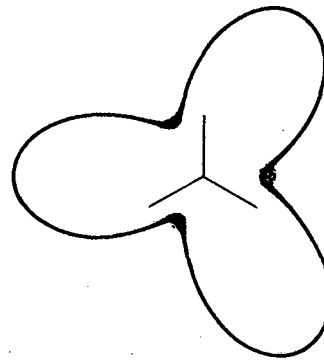
PAIRING 3



PAIRING 4



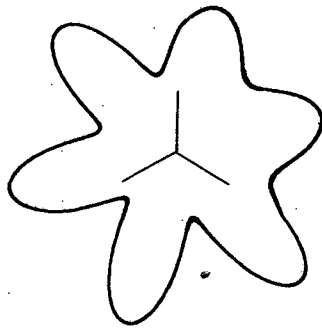
PAIRING 5



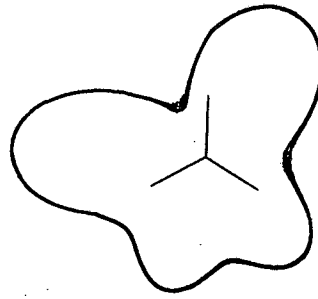
PAIRING 6

XBL 725-142

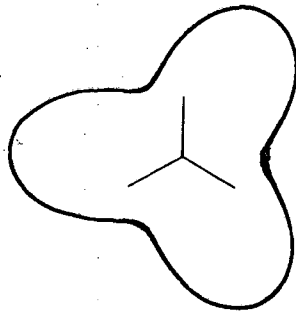
Fig. 30a. Pion flower topological representation for each of the fifteen pairings in a typical Monte Carlo event.



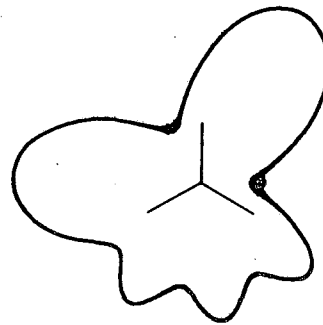
PAIRING 7



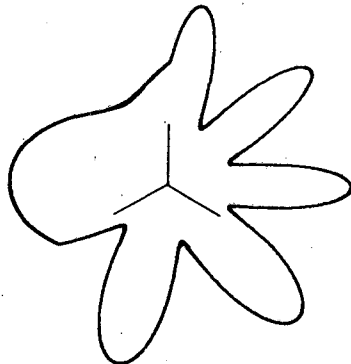
PAIRING 8



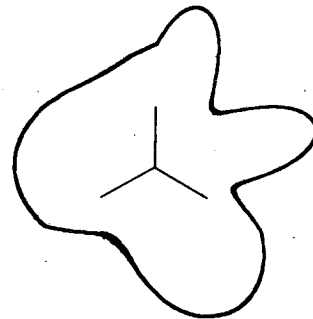
PAIRING 9



PAIRING 10



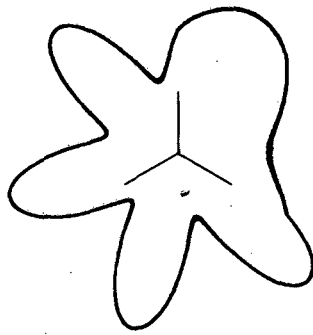
PAIRING 11



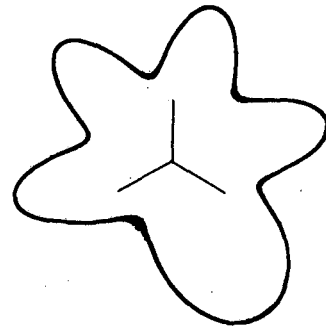
PAIRING 12

XBL 725-140

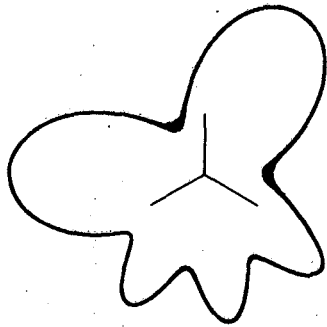
Fig. 30b



PAIRING 13



PAIRING 14



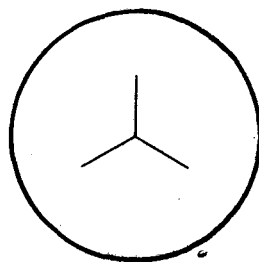
PAIRING 15

XBL 725-143

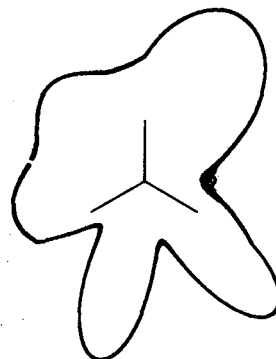
Fig. 30c

and at approximately the same Dalitz radial position as the true pairing. Not coincidentally, pairing 3 is among the more frequent in the distribution in Fig. 28.

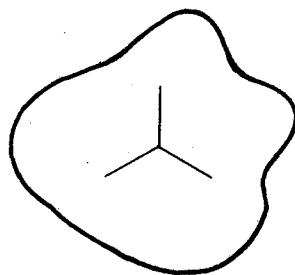
Since the pairings near the same Dalitz radial position have, as patterns in Fig. 30 show, similar topologies and kinematics, they will be preferentially selected by any pairing selection mechanism biased toward selecting the true pairing. Thus a relatively simple selection criteria can produce good radial resolution on the Dalitz plot. However, the azimuthal position for these alternate pairings is not approximately that of the true pairing. The azimuthal resolution is poor, then, for these selection procedures. Figure 31 are these patterns for a typical event of $K_L^0 \rightarrow 3\pi^0$, and shows the familiar similarities among the pairings.



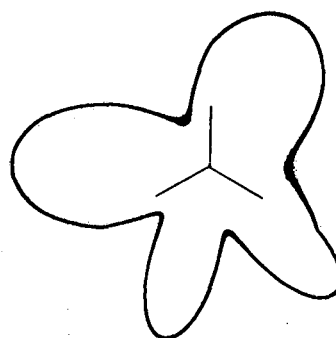
PAIRING 1



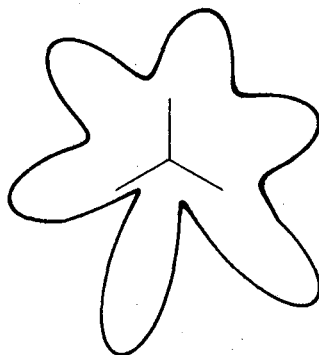
PAIRING 2



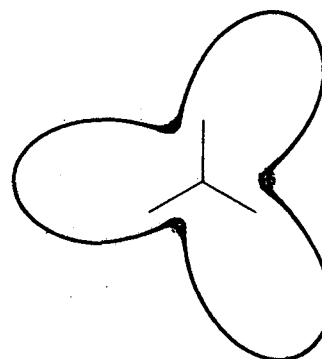
PAIRING 3



PAIRING 4



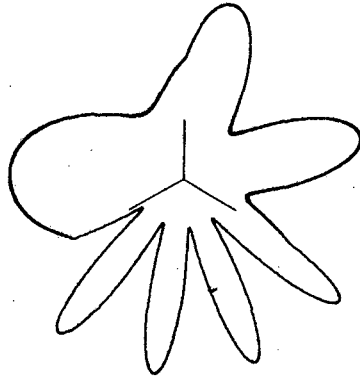
PAIRING 5



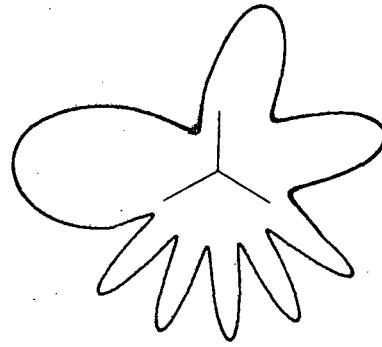
PAIRING 6

XBL 725-142

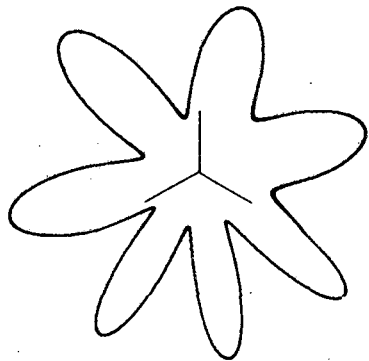
Fig. 31a. Pion flower topological representation for each of the fifteen pairings in a typical detected $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ decay.



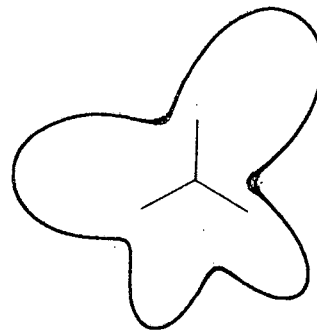
PAIRING 7



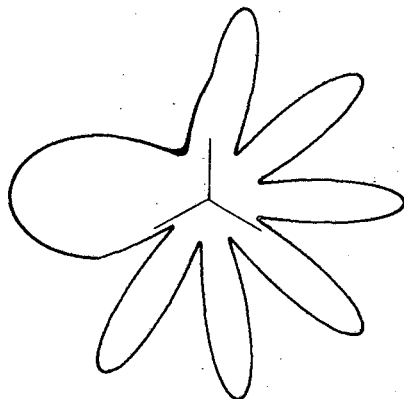
PAIRING 8



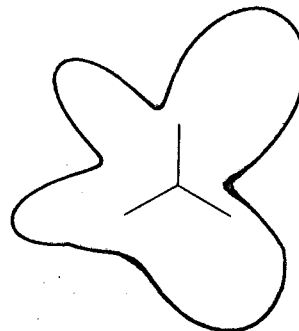
PAIRING 9



PAIRING 10



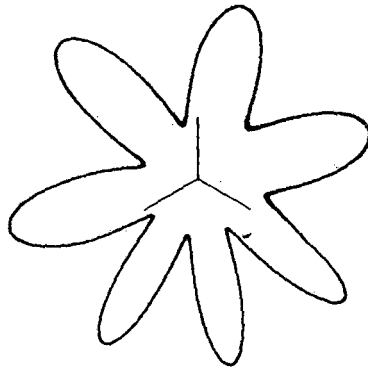
PAIRING 11



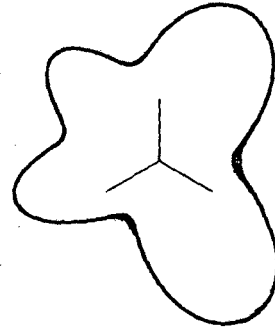
PAIRING 12

XBL 725-141

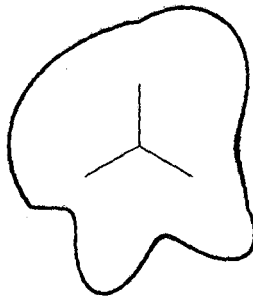
Fig. 31b



PAIR 15



PAIR 14



PAIR 16

XBL 725-144

Fig. 31c

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