

# UC Berkeley

## Working Papers

### Title

An Enhancement to Speed Estimation with Single Loops

### Permalink

<https://escholarship.org/uc/item/3q21h5tg>

### Authors

Lin, Wei-Hua  
Dahlgren, Joy  
Huo, Hong

### Publication Date

2003-10-01

CALIFORNIA PATH PROGRAM  
INSTITUTE OF TRANSPORTATION STUDIES  
UNIVERSITY OF CALIFORNIA, BERKELEY

# **An Enhancement to Speed Estimation with Single Loops**

**Wei-Hua Lin, Joy Dahlgren, Hong Huo**

**California PATH Working Paper  
UCB-ITS-PWP-2003-14**

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department Transportation, Federal Highway Administration.

The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California. This report does not constitute a standard, specification, or regulation.

Report for Task Order 4141

October 2003

ISSN 1055-1417

# **An Enhancement to Speed Estimation With Single Loops**

*Wei-Hua Lin*

*Department of Systems and Industrial Engineering  
The University of Arizona  
Tucson, AZ 85721*

*Joy Dahlgren*

*The PATH program  
University of California at Berkeley,  
Berkeley, CA, 94804-4698*

*Hong Huo*

*Department of Systems and Industrial Engineering  
The University of Arizona  
Tucson, AZ 85721*

*July 18, 2003*

## **Executive Summary**

Traffic data from single loop detectors are one of the dominant data sources widely used in many traffic operation centers and traveler information systems. Speed estimation from single loop detectors is mainly based on occupancy data, a conversion factor from occupancy to density (which is related to vehicle length), and the assumed relationship between flow, speed, and density. This paper investigates the discrepancy between the speed estimated with single loops and the speed measured directly from double loops. It was found that the inaccuracy of speed estimation with single loops is mainly caused by the irregular behavior of vehicle pace. Under congested or unstable traffic conditions, the distribution of vehicle pace within a given time interval often exhibits a large variance accompanied by a strong skewness. Accuracy in speed estimation can be improved by computing occupancy in a different way, using the median vehicle passage time over the detector as opposed to the mean vehicle passage time often used in the conventional method. The performance of the enhanced speed estimation method is very encouraging. The use of the median vehicle passage time reduces the skewness of pace data.

**KEY WORDS:** Traffic sensors, traffic flow theory, traffic management systems, traveler information systems, and traffic control.

## **Acknowledgements**

The research on which this report was based was sponsored by the Caltrans Division of Research and Technology, MOU 4141. Data for the research was provided by the Berkeley Highway Lab.

# TABLE OF CONTENTS

- 1. INTRODUCTION.....6
- 2. CONVENTIONAL METHODS FOR SPEED ESTIMATION WITH SINGLE LOOPS .....8
- 3. PROPOSED METHOD FOR SPEED ESTIMATION BASED ON THE MEDIAN OF THE VEHICLE PASSAGE TIME.....12
- 4. PERFORMANCE EVALUATION .....14
- 5. CONCLUDING REMARKS .....18
- REFERENCES.....20

## LIST OF FIGURES

<i>Figure 1: Plot of speed vs. time (each time unit = 1 minute).</i>	12
<i>Figure 2: Comparison of <math>(l/v)_M</math> and <math>l_M/v_M</math></i>	14
<i>Figure 3: Schematic representation of data collected from double loop vehicle detectors.</i>	15
<i>Figure 4: Comparison of the proposed method (labeled as the median method) and the conventional method (labeled as the mean method).</i>	16

## LIST OF TABLES

<i>Table 1: Comparison of the results from the proposed model and the conventional model: Mean relative errors (MRE).</i>	17
<i>Table 2: Comparison of the results from the proposed model and the conventional model: mean squared error (MSE).</i>	18

## 1. INTRODUCTION

Estimation of prevailing vehicle speeds for traffic streams plays a crucial role in traffic management centers and traveler information systems since speed is an important indicator of traffic conditions and a key variable in determining travel time. This is especially true for Intelligent Transportation Systems (ITS). In recent years, many algorithms and procedures developed for ITS applications, such as the variable message sign, ramp metering, and dynamic traveler information systems, have been using vehicle speed information in one way or another on a real time basis. An improvement to the accuracy in speed estimation will greatly enhance the performance of these algorithms and procedures.

For the past decade or so, traffic surveillance technologies have been improved at a rapid pace. Emerging sophisticated technologies have made it possible to capture the characteristics of vehicles in ways that were not possible in the past. Probes and other devices have been adopted for collecting traffic data relevant to traffic control. However, traffic data from loop detectors remain one of the primary data source used in practice. There are two types of loop detectors, double loops and single loops. Though double loops are better in capturing vehicle speed of a traffic stream, detectors for traffic counts used in many places are predominantly single loops. The focus of this paper is on speed estimation with single loops. It is expected that the use of loop detectors, especially single loops, will still be dominant for traffic data collection in the foreseeable future. Therefore, the study of how to obtain accurate speed measurement from single loops is not only of interest to the research world, but also of interest for field application. The enhancement to the quality of loop detector data in estimating traffic states has become a challenging task that many traffic operation centers have to deal with on a daily basis.

It is often contended that the estimation of speed is very challenging for congested traffic. When traffic is congested, occupancy data become very noisy, making it difficult to capture the relationship between flow, density, and speed. To address this issue, various models were proposed in the past, including the one suggesting a discontinuous flow-density relationship [1]. For a detailed review of both recent and past empirical research on congested flow, see [2]. Unlike speed measurement using double loop detectors, speed



measurement using single loop detectors is indirect. In practice, speed calculated from single loops is based on flow, occupancy, and a conversion factor,  $g$ , i.e., the estimated speed = flow / (occupancy \*  $g$ ), based on the assumption that occupancy is linearly proportional to density. This assumption has been challenged. Hall and Persaud argued that the  $g$  value is not a constant [3]. Rather, it is a function of occupancy. A better fit can be obtained when  $g$  changes in accordance with the change in the occupancy level. The data used in their research are conventional data aggregated for a pre-specified time interval. Cassidy and Coifman [4], however, contended that a constant  $g$  is reasonable if traffic parameters are measured in the manner consistent with the one defined by Edie [5]. They suggested the use of non-conventional data aggregated on a pre-specified number of vehicles, which effectively allows variable lengths for sampling intervals.

In this paper, we first examine the various assumptions associated with the existing methods for speed estimation with single loops. We will show that the high variance and the skewness of the pace data under congested traffic have a profound effect on the accuracy in speed estimation. The variance of pace is very high when traffic is congested. This is supported both by analytical and empirical evidence. Based on this finding, we proceed to propose a method that uses the median of the vehicle passage time over the detectors instead of the mean of the vehicle passage time to represent the occupancy information for speed estimation. This simple modification reduces the skewness in data and yields more robust estimation compared with the method using occupancy calculated in the conventional way. The method appears to be promising when tested with the limited data sets available to this study. It consistently outperformed the method based on the occupancy data processed in a conventional way.

The report is organized as follows. In the next section, we describe briefly how speeds are estimated using single loop detectors and the key factors that would affect the accuracy in estimation. Some empirical evidence showing the behavior of the variance of pace and how it would affect the accuracy in estimating the prevailing vehicle speeds of a traffic stream are provided. Sec. 3 describes the proposed method for speed estimation and its properties. Field data were then used to compare the performance of the proposed method and the performance of the conventional method based on the commonly used

performance measures, the mean squared error and the mean relative error. The results are summarized in Sec. 4. Finally, Sec. 5 summarizes the proposed method and identifies the direction for future research.

## 2. CONVENTIONAL METHODS FOR SPEED ESTIMATION WITH SINGLE LOOPS

The following is a set of notation used in the paper:

$l_i$  : the length of the  $i$ th vehicle;

$v_i$  : the speed of the  $i$ th vehicle;

$v_s$  : the actual space-mean speed;

$\hat{v}_s$  : the estimated space-mean speed;

$t_i$  : the passage time over the detector for the  $i$ th vehicle;

$k$  : vehicle density (veh/distance unit);

$q$  : vehicle flow (veh/time unit);

$o$  : vehicle occupancy, percentage time detector is covered by vehicles;

$p_i$  : the pace of  $i$ th vehicle, which is the reciprocal of the vehicle speed;

$T$  : time interval during which traffic parameters are aggregated.

With single loops, speed estimation is based on occupancy since occupancy to speed is one-to-one relationship. Occupancy is defined as the percentage of time that the loop is

covered by vehicles. Mathematically, occupancy for a given time interval  $T$  is  $o = \frac{\sum_{i=1}^n \frac{l_i}{v_i}}{T}$ .

If we assume that vehicle length ( $L$ ) and vehicle speed ( $V$ ) are two independent random variables, i.e.  $E\left(\frac{L}{V}\right) = E(L)E\left(\frac{1}{V}\right)$ , we can then rewrite the occupancy as

$o = \frac{\sum_{i=1}^n \frac{l_i}{v_i}}{T} \approx \frac{\sum_{i=1}^n l_i}{n} \frac{\sum_{i=1}^n \frac{1}{v_i}}{T}$ . The estimation of the space-mean speed for a given time interval,

$T$ , with  $n$  vehicles becomes

$$\hat{v}_s = \frac{n}{\left(\sum_{i=1}^n \frac{1}{v_i}\right)} = \left(\frac{n}{T}\right) \frac{\frac{\sum_{i=1}^n l_i}{n}}{\frac{\sum_{i=1}^n l_i}{n} \frac{\sum_{i=1}^n \frac{1}{v_i}}{n}} \approx \left(\frac{n}{T}\right) \frac{g(\cdot)}{o}. \quad (1)$$

The g-factor is the average vehicle length within a time interval. The approximation is good if we deal with a sufficient number of vehicles in a time period during which traffic is stationary and vehicle speed is independent of vehicle length. Errors of this approximation may arise from two sources. First, while the independency assumption is quite reasonable and is distribution free, bias due to statistical fluctuations is likely to occur for a small sample. Secondly, in practice, we use a time-invariant g-factor to represent the average

vehicle length for all time intervals. An approximation to  $\frac{\sum_{i=1}^n l_i}{n}$  is usually obtained off-line

if we assume that the average length of vehicles on the road does not vary substantially at different sampling intervals. For a specific time interval, there is no guarantee that a time-invariant  $g()$  or a time specific  $g$  is the most appropriate. Deviation could occur, especially for the time intervals with a small portion of long trucks or unstable vehicle speeds. It will affect the accuracy of speed estimation under both light and congested traffic conditions. For example, suppose four vehicles were recorded during a time interval. Their speeds are 60, 55, 65, 62 mph, respectively. Their lengths are 70, 20, 21, 19 feet, respectively. The space-mean speed is 60 mph. However, the speed estimated from the single loop is 37 mph, if we choose  $g=20$  feet which is the effective length of passenger cars. This is a light traffic situation in which the bias in estimation is caused by a long vehicle. How  $g()$  should change in accordance with traffic conditions is still an ongoing research topic. The studies done by Hall and Persaud proposed to use a time dependent  $g()$  for different traffic conditions as discussed in the previous section [3]. For short time intervals (e.g. less than 5 minutes), the estimation based on a uniform g-factor could indeed lead to very inaccurate results. Unfortunately, for practical purposes, speed

estimation for short time intervals are used much more frequently than for long time intervals.

The accuracy of speed estimation based on the conventional method can be assessed without making the independency assumption. We will show this in the following since it will shed more light on the root of the problem. Note that occupancy is qualitatively

related to vehicle density, which is calculated by  $k = \frac{\sum_{i=1}^n \frac{1}{v_i}}{T}$ . The exact relationship

between occupancy and density can be written as

$$o = \frac{\sum_{i=1}^n \frac{l_i}{v_i}}{T} = \frac{\left( \sum_{i=1}^n \frac{1}{v_i} \right)}{T} \sum_{i=1}^n \left( \frac{\frac{1}{v_i}}{\sum_{i=1}^n \frac{1}{v_i}} \right) l_i = k \sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i. \quad \text{The speed estimation based on the}$$

occupancy now becomes:

$$\hat{v}_s = \frac{q}{k} = \frac{\frac{T}{o}}{\frac{\sum_{i=1}^n \frac{1}{v_i}}{T}} = \frac{n}{\left( \sum_{i=1}^n \frac{1}{v_i} \right)} \frac{g(\cdot)}{\sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i} = v_s \frac{g(\cdot)}{\sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i}. \quad (2)$$

Comparing Eqs. (1) and (2), we can see that the g-factor in (2) has a different physical meaning. Instead of approximating the average vehicle length during a time

interval as shown in (1), the g-factor in (2) approximates  $\sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i$ . The estimation is

accurate if  $g(\cdot) = \sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i$  for every time interval. However, neither  $p_i$  nor  $l_i$  can be

measured directly with single loops. Recognizing that similar to Eq. 1,  $g(\cdot)$  in (2) is in fact to approximate a linear combination of vehicle length weighed by pace, it is natural to use

the average vehicle length instead in hopes that  $\frac{\sum_{i=1}^n l_i}{n} \approx \sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i$ . This assumption is

again a quite strong one that would directly affect the accuracy of our estimation. Unlike the derivation based on the assumption of independency, Eq. (2) further reveals that an

accurate measurement of  $\frac{\sum_{i=1}^n l_i}{n}$  would not necessarily lead to a good approximation to

$\sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i$ , even if  $\frac{\sum_{i=1}^n l_i}{n}$  can be updated at every sampling interval on a real-time basis.

It is intuitive that the accuracy in speed estimation counts greatly on the variance structure

of the pace. Generally speaking, the difference between  $\frac{\sum_{i=1}^n l_i}{n}$  and  $\sum_{i=1}^n \left( \frac{p_i}{\sum_{i=1}^n p_i} \right) l_i$  should be

small if the variance of pace remains small since  $\frac{p_i}{\sum_{i=1}^n p_i} \approx \frac{1}{n}$ . This is certainly not the case

when traffic becomes congested. One can show that the variance of pace grows much quickly than the variance of speed when traffic becomes congested. Using Taylor expansion and reducing terms with higher orders, the variance of pace can be expressed as a function of speed:  $\text{var}(p) \approx \frac{\text{var}(v)}{E(v)^4}$ . The ratio of the variance of pace to variance of speed

is  $\frac{\text{var}(p)}{\text{var}(v)} \approx \frac{1}{E(v)^4}$ . Let indices  $h$  and  $l$  represent heavy and light traffic respectively. The

ratio of the change in variance is  $\frac{\text{var}(p_h)/\text{var}(p_l)}{\text{var}(v_h)/\text{var}(v_l)} \approx \left( \frac{E(v_h)}{E(v_l)} \right)^4$ . As an example, a reduction

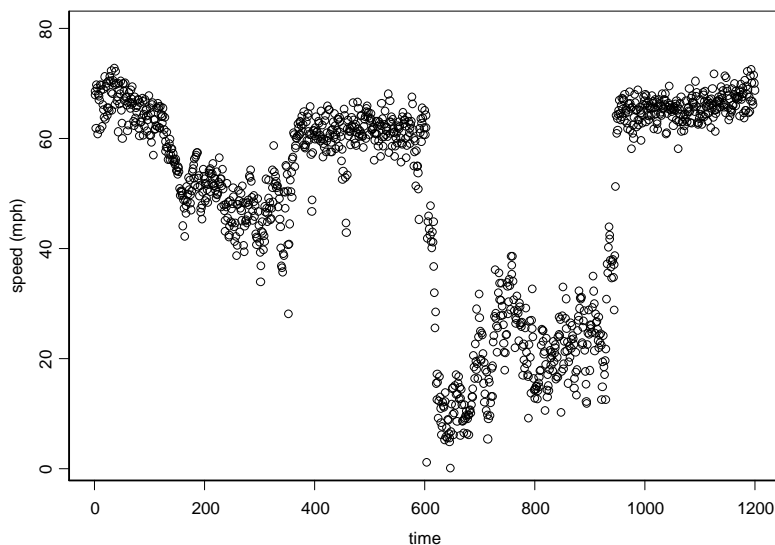
of speed from 75 mph to 25 mph would result in the ratio to be as large as 81. This is consistent with our observations of the field data. Fig. 1 is a plot of vehicle speeds for a

time period of 1200 minutes, covering both congested and uncongested traffic. One can see that traffic has a high speed between  $t=410$  to  $510$ , and low speed for  $t = 750$  to  $850$ . The ratio of the variance of the speed for the time period  $(410,510)$  to the variance of the speed for the time period  $(750, 850)$  is 1.26, whereas the ratio of the variance of the pace for these two time periods is as large as 316.8. Even if we consider the speed difference during these two time periods and adjust the variance with respect to the mean speed, we still get 3.37 for the ratio of the variance of the speed and 103 for the ratio of the variance of the pace. This is a rather extreme case. We consistently found that the variance of the vehicle pace increases very quickly with the increase in congestion level. We have also tested some other data sets. It appeared that in all of the cases we have tested pace tends to have a much higher variance than speed for congested traffic.

**Figure 1: Plot of speed vs. time (each time unit = 1 minute).**

### **3. PROPOSED METHOD FOR SPEED ESTIMATION BASED ON THE MEDIAN OF THE VEHICLE PASSAGE TIME**

In the previous section, we have shown the potential complexity of choosing an appropriate  $g$ -factor that works for all the cases. The complication intensifies when we deal with congested traffic and short time intervals. This motivates us to find alternative



methods to avoid the bias resulting from short-term irregularity in traffic. We describe our method as follows. Let  $t_i = \frac{l_i}{v_i}$  be the passage time over the detector for vehicle  $i$  and  $\bar{t}$  be the average vehicle passage time. We rewrite occupancy as follows:

$$o = \frac{\sum_{i=1}^n l_i}{T} = \frac{\sum_{i=1}^n t_i}{n} = \frac{n\bar{t}}{T}.$$

The speed estimation is thus:

$$\hat{v}_s = \frac{\frac{n}{T}}{\frac{o}{g(\cdot)}} = \frac{g(\cdot)}{\bar{t}}.$$

As shown in Fig. 4,  $t_i$  can be very noisy under congested traffic conditions. If we replace  $\bar{t}$  by the median of the vehicle passage times over detectors,  $t_M$ , we then have a new estimator for the space mean speed:

$$\hat{v}_s = \frac{g(\cdot)}{t_M},$$

where  $t_M$  is the median of  $t_i$ , i.e.,  $P\{t_i < t_M\} \leq 1/2$  and  $P\{t_i \geq t_M\} \geq 1/2$ . If  $t_M = (l/v)_M$  is a good approximation of  $l_M/v_M$ , then the estimation of the space mean speed becomes:

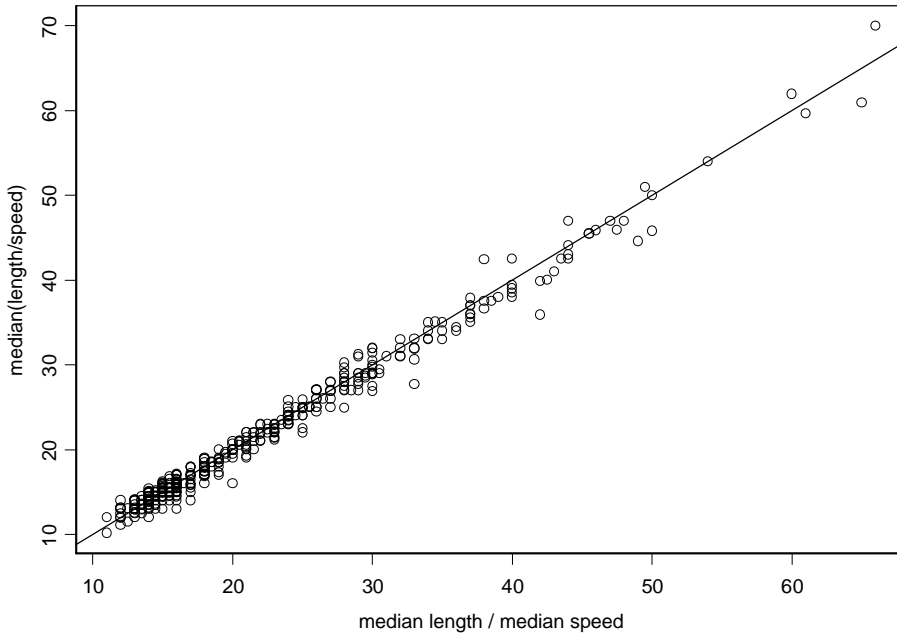
$$\hat{v}_s = v_M \frac{g(\cdot)}{l_M}.$$

The above estimation would perform well if the following two conditions are satisfied:

- (a)  $v_M$  varies with traffic conditions and reflects the prevailing traffic speed; and
- (b)  $l_M$  remains relatively stable and is invariant of traffic conditions.

Based on condition (b), we can simply use a constant  $g$  as a representation for  $l_M$ . The use of the median vehicle passage time over detectors, however, would effectively reduce the skewness of pace that would arise under stop-and-go traffic and the presence of long vehicles. A variation of this scheme is to use the average vehicle passage times for a range

of  $t_i$  chosen from the middle 30% percentile. We have tested this modified scheme with the field data. For the limited data sets available to us, the gain from this alternative method over the median method seems to be very limited. Furthermore, our limited empirical evidence shows that  $t_M = (l/v)_M$  is indeed a good approximation of  $l_M/v_M$ . For the example shown in Sec. 2, the speed estimated with the median method is 58 mph, with only 3% of relative error. A plot comparing  $(l/v)_M$  and  $l_M/v_M$  is displayed in Fig. 2.



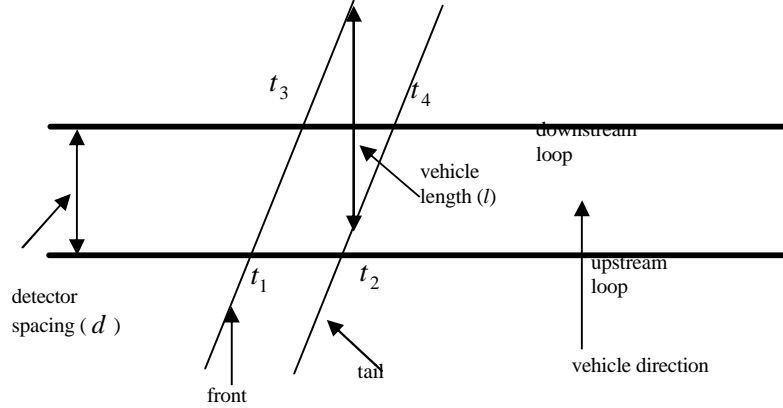
**Figure 2: Comparison of  $(l/v)_M$  and  $l_M/v_M$**

#### 4. PERFORMANCE EVALUATION

The data available to this study is 1/60<sup>th</sup> second data from a 4 mile-long stretch of freeway along Interstate 80 of California, provided by the Berkeley Highway Laboratory. They were collected from double loop detectors. A schematic representation of the double loop for a single site is shown in Fig. 3. The data kept at the tick level provides very detailed



information about each individual vehicle as it crosses a given detector station. Each record in the data file contains the information of a single vehicle crossing a specific double loop, including the information about the station number, the lane number, and the four time stamps that track the times for the vehicle to cross the upstream ( $t_1$  and  $t_2$ ) and downstream ( $t_3$  and  $t_4$ ) loops with its front and rear bumpers, respectively.



**Figure 3: Schematic representation of data collected from double loop vehicle detectors.**

With data of this resolution, we can get very detailed information about each individual vehicle. Exact speed can be calculated either by:

$$v_i = \frac{d}{t_3 - t_1} \text{ (measured with the front bumper)}$$

or  $v_i = \frac{d}{t_4 - t_2}$  (measured with the rear bumper)

Vehicle length can be calculated by:

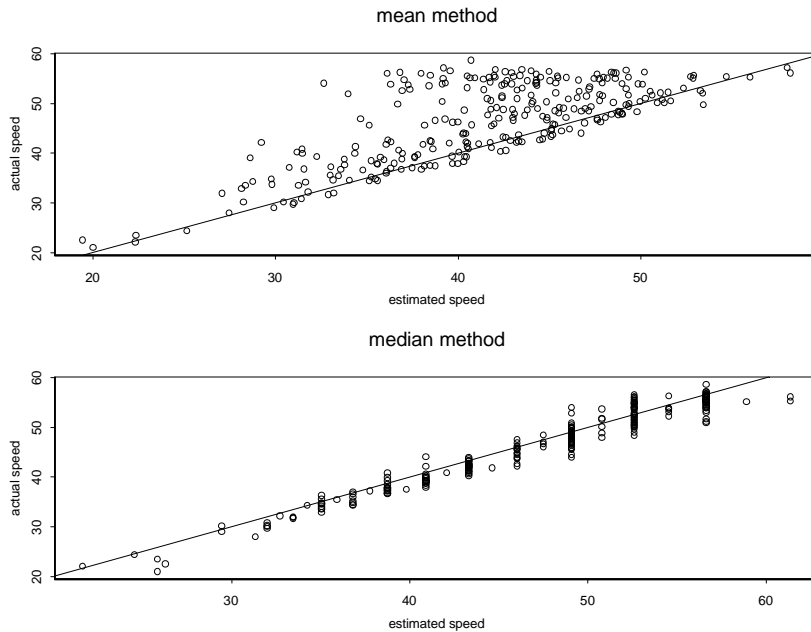
$$l_i = v_i(t_2 - t_1)$$

or  $l_i = v_i(t_4 - t_3)$ .

With data of this resolution, we can measure the density, time-mean speed or the space mean speed directly with the double loops. In this study, the variables that are measured directly are used as the “ground truth” data. We then mimic the data collection process of single loops by considering only one of the loops. Aggregated data such as the

commonly used flow and occupancy within a chosen sampling interval were produced from the single loop. The speed calculated from the single loops is then compared with the “ground truth” speed.

In Fig. 4, we compare the result from the proposed method with the result from the conventional method based on the one-minute data. The plots were generated with data sets from lane 3 at station 3. As shown in the Figure, our method (labeled as the median method) performs significantly better than the conventional method (labeled as the mean method).



**Figure 4: Comparison of the proposed method (labeled as the median method) and the conventional method (labeled as the mean method).**

We compare the proposed model with the conventional method based on the two commonly used measures of performance, the mean relative error (MRE) and the mean squared error (MSE). The data used for comparison were collected at one-minute sampling interval. Seven data sets were used. The results for MRE and MSE are displayed in Tables 1 and 2, respectively. For the mean relative error measurement, the

proposed method seems to be consistently better than the conventional method. The differences between the two in some cases are more significant than others. For the mean squared error measurement, the effectiveness of the proposed method seems to be more pronounced. This is intuitive since the proposed method should be more robust than the conventional method. By choosing the median of the vehicle passage time, the proposed method eliminates the extreme cases in which detector errors, acceleration or deceleration of vehicles, or excessively long vehicles may introduce excessive long passage time (long pace as well). Yet the use of median preserves the prevailing passage time.

**Table 1: Comparison of the results from the proposed model and the conventional model: Mean relative errors (MRE).**

Data Source	The proposed method	The conventional method
Station 5, lane 3	0.133	0.199
Station 5, lane 7	0.110	0.228
Station 3, lane 7	0.068	0.202
Station 3, lane 3	0.036	0.107
Station 6, lane 2	0.158	0.169
Station 6, lane 4	0.106	0.141
Station 1, lane 3	0.098	0.151

**Table 2: Comparison of the results from the proposed model and the conventional model: mean squared error (MSE).**

Data Source	The proposed method	The conventional method
Station 5, lane 3	31.233	83.713
Station 5, lane 7	16.226	94.458
Station 3, lane 7	5.683	86.116
Station 3, lane 3	3.844	52.576
Station 6, lane 2	27.019	28.534
Station 6, lane 4	22.842	49.111
Station 1, lane 3	18.687	65.799

## 5. CONCLUDING REMARKS

This report proposes an enhancement to the existing methods for estimating speeds using single loop detectors. Like most existing methods, it computes the speed by converting occupancy to density first and then making use of the fundamental relationship between flow, density, and speed. The proposed method departs from the existing methods in that: instead of using the average vehicle passage time over detectors for computing occupancy, it uses the median of the vehicle passage time over detectors. This simple modification yields speed estimation with smaller mean relative errors and mean squared errors compared with the estimation generated from conventional methods. For the test cases shown in the paper, the proposed method consistently outperforms the conventional method. The proposed method requires the raw data to be processed in a non-conventional way. The requirement, however, can easily be incorporated into the microprocessors in the controller box for processing loop detector data.

In the report, we use a constant  $g(=18)$  as a conversion factor to represent the median vehicle length. The results from our preliminary studies (not shown in this paper) indicate that the choice of the  $g$ -factor might be related to the detector spacing, which could lead to a bias in the “ground truth” speed data from double loops, which subsequently, would affect the results displayed in Tables 1 and 2. Studies are ongoing to

explore this issue. For future research, we will explore the potential use of  $g$  as a function of occupancy as suggested in [3]. We will also explore the use of variable sampling length as suggested in [4]. Though the proposed method outperforms the conventional method, its performance seems to vary quite substantially with different data sets obtained at different locations. Studies are ongoing to investigate this large variation. We also need to explore further the situation in which the use of median information to represent the prevailing traffic is not satisfactory, i.e., the situation in which  $(l/v)_M$  is not representative for  $l_M/v_M$ .

## REFERENCES

- [1] Koshi, M., M. Iwaski, and I. Ohkura. (1983) Some findings and an overview on vehicular flow characteristics. In Proceeding of 8<sup>th</sup> International Symposium on Transportation and Traffic Theory. Pp. 8 – 20.
- [2] Banks, J. H. (1999) Investigation of some characteristics of congested traffic flow, In *Transportation Research Record*. no.1678, pp.128-134. National Research Council.
- [3] Hall, F. L. and B. N. Persaud (1989) Evaluation of speed estimates made with single-detector data from freeway traffic management systems, IN: *Transportation Research Record*. no. 1232, pp.9 – 16. National Research Council.
- [4] Cassidy, M. and B. Coifman (1997). Relation among average speed, flow, and density and analogous relation between density and occupancy, IN: *Transportation Research Record*, no. 1591, pp. 1-6, National Research Council.
- [5] Edie, L. C. (1963) Discussion of traffic stream measurements and definitions. IN: Proceedings of the Second International Symposium on the Theory of Traffic Flow, pp. 139 – 154. Paris.