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Bayesian Nonlinear Finite Element Model Updating of a Full-Scale Bridge-Column using Sequential Monte Carlo

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ABSTRACT
Digital twin-based approaches for structural health monitoring (SHM) and damage prognosis (DP) are emerging as a powerful framework for intelligent maintenance of civil structures and infrastructure systems. Model updating of nonlinear mechanics-based Finite Element (FE) models using input and output measurement data with advanced Bayesian inference methods is an effective way of constructing a digital twin. In this regard, the nonlinear FE model updating of a full-scale reinforced-concrete bridge column subjected to seismic excitations applied by a large shake table is considered in this paper. This bridge column, designed according to US seismic design provisions, was tested on the NEES@UCSD Large High-Performance Outdoor Shake Table (LHPOST). The column was subjected to a sequence of ten recorded earthquake ground motions and was densely instrumented with an array of 278 sensors consisting of strain gauges, linear and string potentiometers, accelerometers and Global Positioning System (GPS) based displacement sensors to measure local and global responses during testing. This heterogeneous dataset is used to estimate/update the material and damping parameters of the developed mechanics-based distributed plasticity FE model of the bridge column. The sequential Monte Carlo (SMC) method (set of advanced simulation-based Bayesian inference methods) is used herein for the model updating process. The inherent architecture of SMC methods allows for parallel model evaluations, which is ideal for updating computationally expensive models.

Keywords: Bayesian Inference, Digital Twin, Finite Element, Model Updating, Sequential Monte Carlo, Structural Health Monitoring, Full-Scale Structural Systems, Earthquake.

1 INTRODUCTION

Structural health monitoring (SHM) is the general process of making an assessment, utilizing measurement data, about the current ability of the system to perform its intended design functions. Damage prognosis or prognostics (DP) extends this process by combining it with a probabilistic description of future loading to estimate metrics such as remaining useful life (RUL) of the system [1]. With the tremendous increase in computational capabilities and with the advent of new algorithms to solve complex machine learning tasks, the statistical pattern recognition paradigm for SHM/DP of civil structures is gaining popularity among researchers. This paradigm is especially attractive because it offers the possibility of automating the SHM process, i.e., removing the need for interventions of human experts as far as possible. One main objective of the SHM system is to detect, localize, classify and quantify the damage on the structure of interest [2]. To achieve this, in a pattern recognition perspective, the data corresponding to all the conceivable damage states/mechanisms of the structure of interest are required [3]. One of the potential ways of obtaining this data for civil infrastructure systems is using digital twins/cyber models (hybrid data-physics models). A potential way of constructing digital twins for full-scale structural systems is by using the finite element (FE) model updating framework.

The last few decades have witnessed tremendous progress in nonlinear modeling and analysis methods for civil engineering structural, geo-structural, and soil-foundation-structural systems subjected to static, quasi-static, and dynamic loading, particularly from natural hazards such as earthquakes. Mechanics-based nonlinear FE models (of various complexities) of civil engineering systems (e.g., buildings, bridges, dams, miter gates) are now able to capture the damage and failure mechanisms developing in such systems in critical loading environment. The current state-of-the-art nonlinear FE modeling techniques allow reasonably accurate predictions of the actual response of civil structural systems if realistic and "well-calibrated" values are used for the unknown parameters of the FE model. These parameters for civil systems generally include inertial, damping, hysteretic material law, loading, boundary conditions, and geometric parameters. When input-output measurement data are available, the FE model updating framework allows to estimate/update the unknown parameters of the FE model [4]–[7]. The Bayesian approach to model updating is attractive because it accounts for various sources of uncertainties observed in the real world (i.e., noisy output measurements, unknown/partially known/noisy input measurements, uncertainty in FE model parameters, FE model form uncertainty) during estimating/updating the unknown parameters and characterizes the remaining estimation uncertainty. This confidence level associated with the parameter estimates is extremely useful for SHM/DP as it supports rigorous decision-making. In Bayesian model updating, the modeler needs to specify the probabilistic description of unknown parameters (referred to as prior knowledge). The prior knowledge is then updated (referred to as posterior) accounting for the measurement data using Bayesian inference. The FE model characterized by the posterior probabilistic description of the unknown parameters is referred to as the updated FE model. This updated model, which is essentially a fusion of heterogeneous measurement data and a physics-based FE model, is the digital twin/cyber model and can be utilized for SHM/DP purposes.

Bayesian Nonlinear FE Model updating of a full-scale reinforced-concrete bridge column tested on the large high-performance outdoor shake table (LHPOST) at UC San Diego (UCSD) [8] is considered in this paper. The bridge column was subjected to a sequence of ten recorded earthquake ground motions (uniaxial excitation) and was densely instrumented with an array of 278 sensors consisting of strain gauges, linear and string potentiometers, accelerometers and Global Positioning System (GPS) based displacement sensors to measure local and global responses during testing. First, a frame-type FE model with distributed plasticity of the bridge column is developed. Then, the measurement data from the first earthquake excitation (EQ1) are used to update five key structural parameters (Young's modulus of concrete E_c and steel E_s; Rayleigh damping model parameters a_1 and a_2; and tensile strength of concrete f_{ct}) of the nonlinear FE model using the sequential Monte Carlo (SMC) method [9], [10] – a class of Bayesian inference methods. The novelty of this study lies in the use of real-world input-output measurement data of a full-scale structural system to update a detailed mechanics-based nonlinear finite element model using a "fully" Bayesian inference technique (SMC).

2 FINITE ELEMENT MODEL UPDATING USING BAYESIAN INFERENCE

Let $\mathbf{u}_k \in \mathbb{R}^{n_\mathbf{u}}$ and $\mathbf{y}_k \in \mathbb{R}^{n_\mathbf{y}}$ denote the vector of measured input and output responses, respectively, of the structure of interest at the time t_k (or time step k). These measurements are obtained from the heterogeneous sensor array mounted on the real structure. Assuming the measurement responses are obtained for N time steps, the measured/observed input-output dataset is $\mathcal{D} = (\mathbf{u}, \mathbf{y})$, where $\mathbf{u} = \left[\mathbf{u}_1^T, \mathbf{u}_2^T, ..., \mathbf{u}_N^T\right]^T \in \mathbb{R}^{(n_\mathbf{u} \times N) \times 1}$ and $\mathbf{y} = \left[\mathbf{y}_1^T, \mathbf{y}_2^T, ..., \mathbf{y}_N^T\right]^T \in \mathbb{R}^{(n_\mathbf{y} \times N) \times 1}$.

At time step k, let $\mathbf{y}_k^{FE} = \mathbf{h}_k(\mathbf{u}_{1:k}; \boldsymbol{\theta}) \in \mathbb{R}^{n_\mathbf{y}}$ denote the response predicted by the FE model $\mathbf{h}$ parameterized by the unknown parameter vector $\boldsymbol{\theta} \in \mathbb{R}^{n_\boldsymbol{\theta}}$ when subjected to the measured input time-history $\mathbf{u}_{1:k} = \left[\mathbf{u}_1^T, \mathbf{u}_2^T, ..., \mathbf{u}_k^T\right]^T \in \mathbb{R}^{(n_\mathbf{u} \times k) \times 1}$. For the methodology described in this paper, the parameter vector $\boldsymbol{\theta}$ can include any unknown time-invariant parameters such as inertial, damping, hysteretic material law, loading, boundary conditions and geometric parameters of the FE model. In practice, the measured response $\mathbf{y}$ and the FE predicted response $\mathbf{y}^{FE}$ do not match due to numerous sources of uncertainty [11]. These include

a) Uncertainty in the measured output due to sensor noise
b) Unmeasured/partially measured input and sensor noise associated with measured inputs

c) Uncertainty about the structure/form of the model, i.e., the selected model class cannot represent the real system. This model form error, if not accounted for, can introduce bias in estimation and handicap the predictive utility of the model.
d) Uncertainty about the parameters of the model, assuming the structure/form of the model is known.

Put in simple terms, the goal of Bayesian model updating is to estimate/update the unknown parameter vector $\boldsymbol{\theta}$ accounting for all the pertinent sources of uncertainties and characterize the remaining estimation uncertainty. To achieve this, first, the likelihood function should be constructed using a measurement equation, i.e., a model of the measurement process. This is also referred to as a joint statistical-physical model that relates model parameters to measurements/observations. The following measurement equation is used in this paper

$$\text{Measument equation at time step } k \rightarrow \quad \underset{\textit{measured output}}{\mathbf{y}_k} = \underbrace{\mathbf{h}_k\left(\mathbf{u}_{1:k};\boldsymbol{\theta}\right)}_{\textit{FE predicted response}} + \underset{\textit{error/noise}}{\mathbf{w}_k} \tag{1}$$

where $\mathbf{w}_k = \left[w_{1,k}, w_{2,k}, ..., w_{n_\mathbf{y},k}\right]^T$ is the measurement error/noise at time step k and $w_{i,k}$ denotes the discrepancy between measured and FE predicted responses corresponding to the i^{th} output measurement channel at time step k. In the measurement equation, the noise term, lumping all the sources of uncertainties, is assumed additive to the FE predicted response. Therefore, the accurate statistical description model of the noise process $\mathbf{w} = \left[\mathbf{w}_1^T, \mathbf{w}_2^T, ..., \mathbf{w}_N^T\right]^T$ is crucial in Bayesian inference. In this paper, the noise process $\mathbf{w}$ is assumed temporally white ($\mathbf{w}_1, \mathbf{w}_2, ...$ are statistically independent) and random vector $\mathbf{w}_k$ is assumed to follow a zero-mean Gaussian probability density function (PDF) with independent components (i.e., noise/error terms across all measurement channels are assumed statistically independent).

$$\mathbf{w}_k \sim \mathcal{N}\left(\underset{n_\mathbf{y}\times 1}{\mathbf{0}}, \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{n_\mathbf{y}}^2 \end{pmatrix}_{n_\mathbf{y}\times n_\mathbf{y}}\right) \quad \text{and} \quad \mathbf{w} \sim \mathcal{N}\left(\underset{(n_\mathbf{y}\times N)\times 1}{\mathbf{0}}, \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma} \end{pmatrix}_{(n_\mathbf{y}\times N)\times(n_\mathbf{y}\times N)}\right) \tag{2}$$

where σ_i^2 denote the variance of the discrepancy between measured and predicted responses of the i^{th} output measurement channel. These noise variances are also typically unknown in the real world. Therefore, in this paper, the unknown vector $\boldsymbol{\sigma} = \left[\sigma_1, \sigma_2, ..., \sigma_{n_\mathbf{y}}\right]^T$ is estimated/updated jointly with the unknown FE model parameter vector $\boldsymbol{\theta}$. With the measurement model described by Equations (1) and (2), the likelihood function for the unknown parameters $\boldsymbol{\theta}$ and $\boldsymbol{\sigma}$ is given by

$$p\left(\mathbf{y}\,|\,\mathbf{u},\boldsymbol{\theta},\boldsymbol{\sigma}\right) = \prod_{k=1}^{N} p\left(\mathbf{y}_k\,|\,\mathbf{u}_{1:k},\boldsymbol{\theta},\boldsymbol{\sigma}\right) = \prod_{k=1}^{N} \frac{1}{\left(2\pi\right)^{n_\mathbf{y}/2}} |\boldsymbol{\Sigma}|^{-1/2} \exp\left\{-\frac{1}{2}\left[\mathbf{y}_k - \mathbf{h}_k\left(\mathbf{u}_{1:k};\boldsymbol{\theta}\right)\right]\boldsymbol{\Sigma}^{-1}\left[\mathbf{y}_k - \mathbf{h}_k\left(\mathbf{u}_{1:k};\boldsymbol{\theta}\right)\right]^T\right\} \tag{3}$$

The modeler also needs to specify the prior PDF of the unknown parameter vector , $p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\right)$. The prior PDF is then updated to obtain the posterior PDF, which accounts for the prior knowledge and the observed data, using Bayes rule as

$$p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\,|\,\mathcal{D}\right) \equiv \underbrace{p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\,|\,\mathbf{u},\mathbf{y}\right)}_{\textit{posterior}} = \frac{\overbrace{p\left(\mathbf{y}\,|\,\mathbf{u},\boldsymbol{\theta},\boldsymbol{\sigma}\right)}^{\textit{likelihood}} \times \overbrace{p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\right)}^{\textit{prior}}}{\underbrace{p\left(\mathbf{u},\mathbf{y}\right)}_{\textit{evidence}}} \tag{4}$$

However, determining analytically the complete joint posterior is an intractable problem. Many numerical approximations methods such as Bayesian Kalman filters, particle filters, Markov chain Monte Carlo methods (MCMC), sequential Monte Carlo (SMC) methods (SMC), etc., have been developed to perform this computation and most of these methods rely on sampling the joint posterior. In this paper, SMC is used to sample the joint posterior PDF defined in Equation (4).

2.1 SEQUENTIAL MONTE CARLO

SMC methods are a class of Bayesian inference techniques that sample the joint posterior PDF of the unknown parameters. In the literature, there are several closely related algorithms that are referred to as transitional Markov chain Monte Carlo, particle filters, bootstrap filters, condensation algorithm, survival of the fittest and population Monte Carlo [12]. SMC methods do not require the Gaussian assumption about the prior and posterior PDFs of the unknown parameters, an inherent assumption in Bayesian Kalman filters (e.g., unscented Kalman filter, extended Kalman filter). Unlike standard MCMC methods, SMC methods are parallelizable, they can be used to perform model updating of high-fidelity large-scale nonlinear FE models using high-performance computing (HPC) resources.

The idea of SMC is to sample from a series of simpler intermediate PDFs that converge to the target posterior PDF, thus circumventing the need to directly sample the target posterior. To achieve this, SMC samplers proceed through a series of stages, starting from the prior distribution until the posterior distribution. All these distributions (called tempered posterior distributions) are controlled by the tempering parameter β as

$$p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\,|\,\mathbf{u},\mathbf{y}\right)_{\beta} \propto p\left(\mathbf{y}\,|\,\mathbf{u},\boldsymbol{\theta},\boldsymbol{\sigma}\right)^{\beta} \times p\left(\boldsymbol{\theta},\boldsymbol{\sigma}\right) \tag{5}$$

When $\beta = 0$, the tempered posterior is just the prior PDF and when $\beta = 1$ the tempered posterior is the true posterior PDF. The SMC sampler starts with $\beta = 0$ and progresses by monotonically increasing the value of β, at each stage, until it reaches the value of 1. The tempered posterior distribution at every stage in SMC is represented by a set of weighted samples (also called particles). Also, at each stage, SMC uses independent Markov chains (which start at the samples of the current tempered posterior distribution) to reach the next tempered posterior distribution. Therefore, the SMC sampling algorithm can also be thought of as a parallel MCMC algorithm that can effectively sample high-dimensional parameter spaces [9]. Due to its inherent parallel nature, SMC can be used to efficiently perform model updating of high-fidelity large-scale nonlinear FE models using high-performance computing resources. In contrast to MCMC, SMC can effectively sample from posterior distributions with flat peaks and multiple peaks which arise in non-identifiable and locally-identifiable problems, respectively. SMC also computes the model evidence (denominator of Equation (4)) as a by-product, which can then be used for Bayesian model class selection and model averaging [10]. The SMC algorithm used in this paper to sample the joint posterior in Equation (4) is presented in Table 1.

Table 1: Sequential Monte Carlo Algorithm

Notation:	N_p : number of particles , ESS : effective sample size , β : tempering parameter , j : stage number $\boldsymbol{\alpha} = \left[\boldsymbol{\theta}^T, \boldsymbol{\sigma}^T\right]^T$: unknown parameter vector to be updated , $\boldsymbol{\alpha}_j^i$: i^{th} particle of $\boldsymbol{\alpha}$ at stage j
Initialize:	N_p , $j = 0$, $ESS_0 = N_p$, $\beta_0 = 0$, Generate N_p samples $\left\{\boldsymbol{\alpha}_{j=0}^i;\ i = 1,...,N_p\right\}$ from the prior PDF $p\left(\boldsymbol{\alpha}\right)$
while $\beta_j < 1$: stage number $j = j+1$ choose $\tilde{\beta}_j$ such that $ESS_j = 0.95 \times ESS_{j-1}$, $\beta_j = \min\left(\tilde{\beta}_j, 1\right)$ weighting: $w_j^i = p\left(\mathbf{y}\,|\,\mathbf{u},\boldsymbol{\alpha}_{j-1}^i\right)^{\beta_j - \beta_{j-1}}$ for $i = 1,...,N_p$ resampling: $\tilde{\boldsymbol{\alpha}}_j^i = \boldsymbol{\alpha}_{j-1}^l$ with probability w_j^l for $i = 1,...,N_p$	

perturbation:	start an MCMC chain at $\tilde{\boldsymbol{\alpha}}_j^i$ and take N_{MCMC} steps with target distribution $p(\boldsymbol{\alpha} \mid \mathbf{u}, \mathbf{y})_{\beta_j}$ for each $i = 1, \ldots, N_p$. Gather last sample of each MCMC chain to obtain $\{\boldsymbol{\alpha}_j^i; i = 1, \ldots, N_p\}$
end	
save last stage $m = j$	
$\{\boldsymbol{\alpha}_m^i; i = 1, \ldots, N_p\}$ are the samples of the target posterior $p(\boldsymbol{\alpha} \mid \mathbf{u}, \mathbf{y})$	

3 FULL-SCALE REINFORCED-CONCRETE BRIDGE COLUMN

A full-scale reinforced-concrete bridge column was tested on the large high-performance outdoor shake table (LHPOST) at the University of California, San Diego (UCSD) from July through September 2010 [8] (see Figure 1). The 24ft high and 4ft diameter column was designed and detailed according to the California Department of Transportation (Caltrans) seismic design guidelines. The objective of the test was to validate the current seismic design guidelines in terms of the structural seismic response of bridge columns. For this purpose, the column was tested under dynamic loading conditions by subjecting it to a series of ten earthquake ground motion records (uniaxial excitation along the table's longitudinal axis or east-west direction). A concrete superstructure block weighing 522 kips was cast on top of the column for mobilizing the inertial forces during the dynamic tests. This block was designed such that its center-of-mass coincided with the top of the column. This test specimen was densely instrumented with an array of 278 sensors consisting of strain gauges, linear and string potentiometers, accelerometers and Global Positioning System (GPS) based displacement sensors to measure local and global responses during testing [8].

Figure 1: Full-scale reinforced-concrete bridge column tested on the LHPOST@UCSD

3.1 FINITE ELEMENT MODEL OF THE COLUMN

With a height-to-diameter ratio of 6, the test specimen was intended to respond in the nonlinear range with predominant flexural behavior. FE models using beam-column elements with distributed plasticity have been proven to capture the observed nonlinear behavior of such flexural dominated systems extremely well. Due to their accuracy in matching experimental results, formulation simplicity, and computation feasibility and efficiency, such FE model types are widely used in research and engineering practice [13]. In this paper, the 24ft long bridge column is modeled using two nonlinear fiber-section Euler-Bernoulli force-based beam-column elements with three Gauss-Lobatto integration points (monitored cross-sections) along the length of each element (see Figure 2). Each element cross-section is discretized into longitudinal fibers as shown in Figure 2. The section nonlinear response behavior is simulated from the uniaxial material constitutive laws used for the fibers. Uniaxial Popovics material model [14] is used for modeling confined and unconfined concrete behavior at the fiber level and the reinforcing steel material model [15] is used for modeling longitudinal steel fibers as shown in Figure 2. The effects of nonlinear geometry are accounted for using the corotational formulation. The inherent damping properties representing sources of energy dissipation beyond the hysteretic energy dissipated through inelastic material behavior are modeled using Rayleigh damping (proportional to the mass matrix $\mathbf{M}$ and tangent stiffness matrix at the last converged step of analysis $\mathbf{K}_T$). The footing is approximated as a fixed restraint and the inertial effect of the superstructure is lumped on the top node of the column.

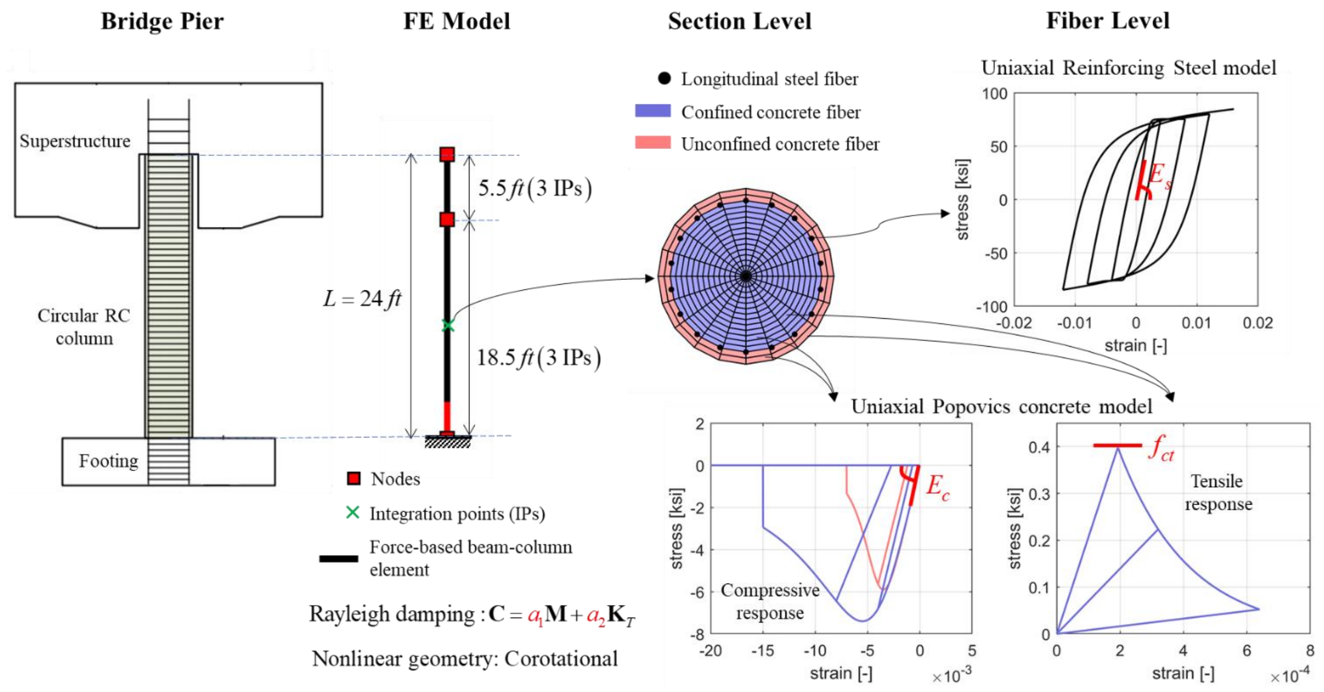

Figure 2: Finite Element Model Hierarchy of Bridge Column

3.2 FE MODEL UPDATING SETUP

In this paper, the measured input-output data corresponding to the first earthquake excitation (EQ1) are utilized to update the developed FE model of the column. During EQ1 excitation, it was observed that the column response was essentially linear elastic with no observable damage. Only hairline cracks (less than 0.1mm wide) were observed at the base of the column (above the footing) [8]. Therefore, only the parameters that govern the linear elastic behavior of the column (Young's modulus of concrete E_c and steel E_s; Rayleigh damping parameters a_1 and a_2) along with the tensile strength of concrete, parameter f_{ct} , are assumed unknown and estimated using the measured input-output data; thus, $\boldsymbol{\theta} = \left[E_c, E_s, a_1, a_2, f_{ct}\right]^T \in \mathbb{R}^{5\times1}$ (see Figure 2).

The parameters of concrete (E_c and f_{ct}), steel (E_s) and Rayleigh damping model (a_1 and a_2) were determined experimentally by testing 6 *in* x 12 *in* concrete cylinders, reinforcing steel bars and subjecting the column to white noise excitation, respectively. These experimentally determined parameter values (reported in [8]) are shown in Table 2 and will be referred to as parameter values reported in the PEER report or PEER parameters in brief. These reported values are utilized to construct the prior PDF.

The accelerometers mounted at the footing of the column are used as the measured EQ1 input excitation to the column (see Figure 3). This measured column base acceleration is then used as input data $\mathbf{u}$ in model updating. The accelerometers and displacement string potentiometers mounted on the superstructure mass are used to reconstruct the absolute acceleration response and relative (to the base of the column) displacement response at the top of the column along the direction of shaking. This reconstructed absolute acceleration and relative displacement responses are used as output data $\mathbf{y}$ in model updating. Thus,

$$\mathbf{y}_k = \begin{bmatrix} \text{Absolute Acceleration of the top of the column at time step } k \\ \text{Relative Displacement at the top of the column at time step } k \end{bmatrix} \in \mathbb{R}^{2\times 1} \quad (6)$$

The measurement equation described in Equation (1) is utilized for model updating. The noise/error term $\mathbf{w}_k$ is assumed to have Gaussian independent components white in time, i.e., $\mathbf{w}$ is modeled as a vector Gaussian white noise, i.e.,

$$\mathbf{w}_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma} = \begin{pmatrix} \sigma_a^2 & \mathbf{0} \\ \mathbf{0} & \sigma_d^2 \end{pmatrix}\right) \quad \text{and} \quad \mathbf{w} \sim \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \mathbf{\Sigma} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Sigma} \end{pmatrix}\right) \quad (7)$$

where σ_a^2 and σ_d^2 denote the variance of the discrepancy between measured and FE predicted responses for the acceleration and displacement response quantities, respectively.

The five parameters of the FE model (E_c, E_s, a_1, a_2 and f_{ct}) together with the noise standard deviations σ_a and σ_d are estimated using the SMC algorithm described in Table 1. The prior PDF is constructed by assuming that the seven parameters are mutually statistically independent. Normal distributions with mean values taken as the parameter values reported in the PEER report are used to construct the prior PDF of the FE model parameters. The standard deviations of the priors are selected to obtain coefficients of variation of 0.30, 0.20, 0.60, 0.60 and 0.40 for E_c, E_s, a_1, a_2 and f_{ct}, respectively. Since all the FE parameters are positive, one-sided truncation at the lower tail is performed at zero. Half normal distributions of standard deviations $0.1\,g$ and $1.2\,in$ are used as priors for noise parameters σ_a and σ_d, respectively. At each stage of SMC, the tempered posterior is represented using $N_p = 1000$ particles and during the perturbation phase, the number of MCMC steps is set as $N_{MCMC} = 10$. Parallel computing across 10 cores (20 threads) is used for evaluating the likelihood function at every step of the perturbation phase of each SMC stage.

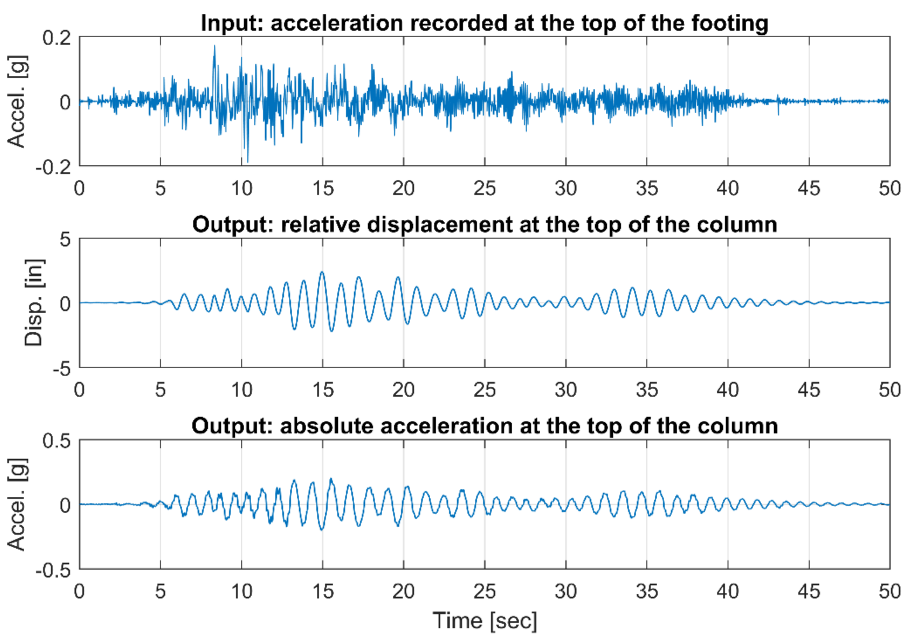

Figure 3: Input (acceleration at top of footing) and output (absolute acceleration and relative displacement at the top of the column) measurement data for model updating

3.3 RESULTS

The pair plot of all seven unknown parameters constructed using the posterior samples obtained from SMC is shown in Figure 4. The plots along the diagonal show the histogram and kernel density estimates of the marginal posterior PDF of each parameter. The marginal posterior PDFs are very sharp (conveyed by very small coefficients of variation, CV), implying that the remaining estimation uncertainty after model updating is very low. The plots above the diagonal show the posterior samples in the space of every parameter pair and the plots below the diagonal show the contour plots of the corresponding kernel density estimates where r is the Pearson correlation coefficient. The sample mean values of the posterior SMC samples are compared in Table 2 with the corresponding parameter values reported in the PEER report.

Table 2: Parameter values reported in PEER report vs the mean values of the posterior SMC samples

Parameter	Values from PEER report	Sample mean of SMC posterior samples
E_c	3320.0 *ksi*	0.59×3320.0 *ksi*
E_s	28400.0 *ksi*	0.96×28400.0 *ksi*
a_1	0.737 sec^{-1}	0.27×0.737 sec^{-1}
a_2	0.0002016 *sec*	4.78×0.0002016 *sec*
f_{ct}	0.306 *ksi*	2.43×0.306 *ksi*
σ_a		$7.7\ in/s^2\ (2.01\%\ g)$
σ_d		$0.09\ in\ (0.23\ cm)$

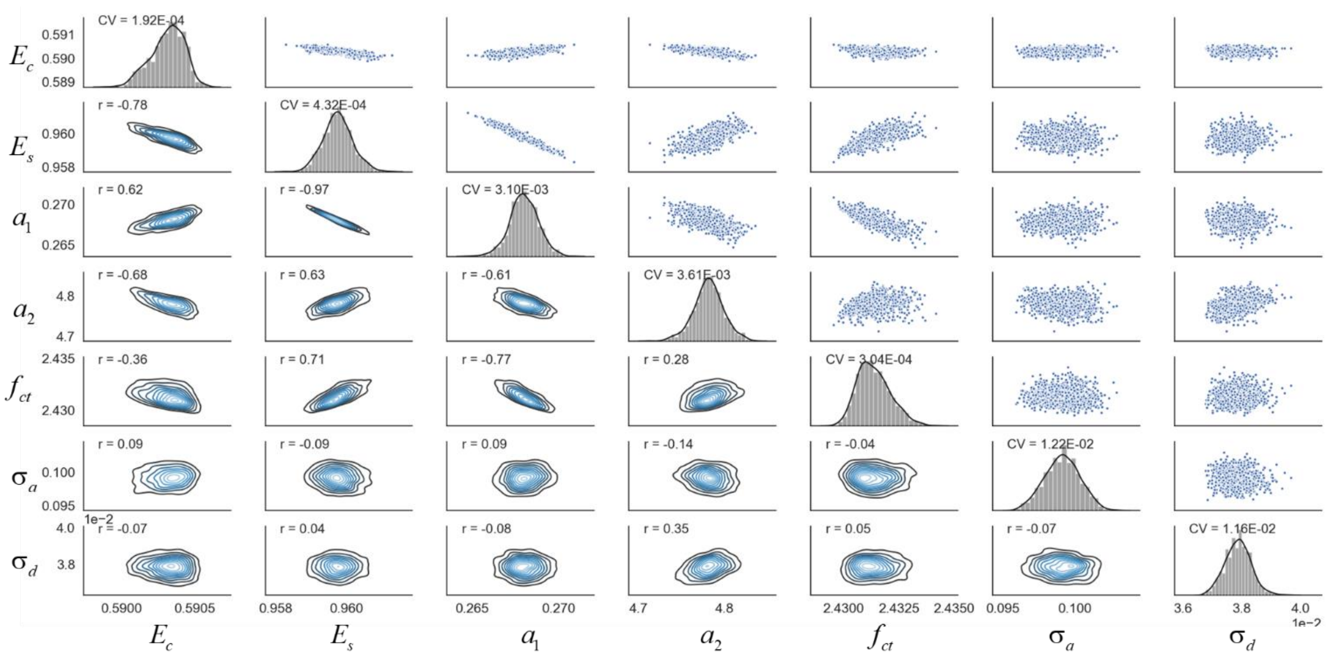

Figure 4: Pair plot using posterior samples of all seven unknown parameters obtained using sequential Monte Carlo

The FE predicted acceleration and relative displacement responses of the bridge column obtained using the parameter values reported in the PEER report (i.e., before FE model updating) and using the sample mean values of the posterior SMC samples (i.e., after FE model updating) are compared with the experimentally measured responses in Figure 5. The relative-root-mean-square error (RRMSE) is used as a metric to measure the discrepancy between two time series. The FE responses predicted using the PEER parameters (i.e., before FE model updating) match poorly the corresponding experimental responses (high RRMSE), while the responses predicted using the sample mean of the posterior samples (i.e., after FE model updating) match the experimental responses extremely well (low RRMSE). This demonstrates that by fusing detailed mechanics-based FE modeling with input-output measurement data through Bayesian inference, one can capture the observed response of a full-scale structural system extremely well. The predictive capabilities of the updated FE model when subjected to subsequent input excitations are yet to be studied, which is beyond the scope of this paper.

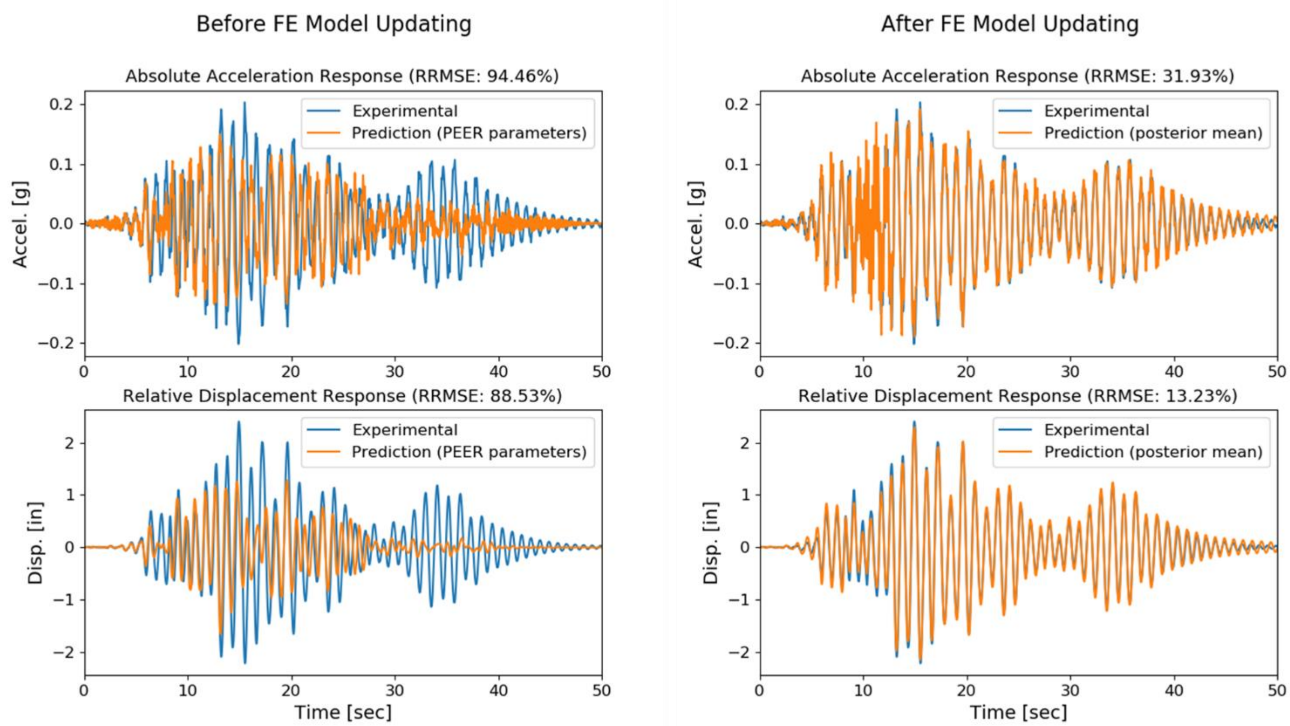

Figure 5: Response prediction of the FE model vs experimentally measured response, before and after model updating

4 CONCLUSIONS

This paper focuses on model updating of civil structures using Bayesian inference. A full-scale reinforced-concrete bridge column tested on the LHPOST@UCSD is selected as the testbed structure. The parameters of the FE model are updated using the sequential Monte Carlo method, a fully Bayesian inference method, using measured input-output data corresponding to the earthquake excitation applied by the shake table. The framework shown here can be used to tune the unknown parameters of the FE model to match the measured response (model calibration). The updated nonlinear FE model acts as a digital twin of the structure of interest and can subsequently be interrogated for the presence, location, type, and extent of damage (i.e., losses in stiffness, strength, ductility capacity, …) in the structure. The updated FE model can also be used to better predict the future performance/functionality of the structure by using it in conjunction with probabilistic descriptions of future loading (damage prognosis).

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