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Publication Date
1984-07-01

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ABSTRACT

This paper investigates the possibility that the observed deviations of major bilateral exchange rates from values implied by market fundamentals is a consequence of rational asset market bubbles. Using a new econometric methodology for detecting asset market bubbles, the no bubble hypothesis is soundly rejected for the dollar-Deutschemark and dollar-pound rates using monthly data over the period 1973-1982.

I. INTRODUCTION

In sharp contrast to the 1970's, the 1980's has witnessed a dramatic dollar apprecation. As calculated by Morgan Guaranty Trust, the dollar's real effective exchange rate appreciated by roughly 30% against the Deutsche-mark and sterling and 15% against the yen over the period of early 1980 to late 1982. The strong dollar has frustrated forecasters (Levich (1983)) and lent credence to the view that exchange rate changes are governed by more than simple market fundamentals (relative money supply growth rates, interest differentials, deviations from purchasing power parity (PPP), etc.) Some economists have argued (McKinnon (1976)) that frequent and large exchange rate fluctuations can be explained by speculative runs that may reflect a self-sustaining speculative mentality on the part of market participants. There exists, however, very little academic empirical evidence on which we might assess the validity of the "bandwagon" hypothesis. In important early work Dooley and Shafer (1975) document the transactions cost adjusted profitability of simple filter rules applied to daily exchange rate data. This 15-20% annualized profitability remains in more recent studies: (Dooley and Shafer (1983) and Sweeney (1984a,b)). The latter author finds that risk-adjusted filter rules profits are significant.[1]

It is well known that time differences of spot and forward exchange rates are well approximated by random walks (Poole (1967), Mussa (1979), Meese-Rogoff (1983a,b), among others). While the paucity of serial correlation in exchange rate changes has proved unstable and difficult to predict, the filter rule studies highlight the fact that the first order autocorrelation coefficient of daily exchange rate changes has remained positive over the floating rate period. Given the apparent profitability of filter rules on daily data, it would appear that a major omission of empirical

academic exchange rate research has been to ignore analysis of intra-day exchange rate movements, since banks frequently maintain zero net open positions in each currency overnight.[2] What evidence that exists for daily, weekly, monthly and quarterly exchange rates suggest that the filter rule profitabilty disappears as the time horizon increases from daily to less finely sampled data.

The profitability of filter rules on daily exchange rate data does not necessarily imply asset market bubbles in the sense to be defined below. Filter rule profits are suggestive of bubbles, and, in the context of a well specified model of risk, can indicate market inefficienty (Sweeney (1984a)). Arbitrage does not by itself prevent bubbles (Blanchard and Watson (1982)), and bubbles need not be associated with irrationality of market participants. Rational asset market bubbles can exist in a world of risk averse agents with heterogeneous information sets, again see the discussion in Blanchard and Watson (1982). There are, however, theoretical models in which bubble paths can be ruled out on economic grounds, as demonstrated by Gray (1982) and Obstfeld and Rogoff (1983).

Flood and Garber (1980) were the first to attempt empirical tests of bubbles in the context of a rational expectations model of the German hyper-inflation. Their methodology is appropriate for a deterministic bubble. In the study of exchange markets a deterministic bubble is unrealistic, for to be rational, the bubble (and hence the value of a currency in terms of another) must increase indefinitely. A second problem with empirical studies of deterministic bubbles is that conventional asymptotic distribution theory precludes exponentially growing regressors. Flood and Garber (1980) circumvent the latter problem by assuming normality of disturbances.

In this paper we provide econometric evidence for the hypothesis that point sample monthly exchange rate data are consistent with the existence of bubbles in currency markets. The econometric methodology is suggested by West (1984), who applies it to equity markets. The methodology admits stochastic bubbles, and gives rise to a condition that validates standard asymptotic distribution theory for parameter estimates under the alternative hypothesis of exchange rate bubbles. Our test for bubbles in exchange markets is conditioned on a hybrid monetary exchange rate model; a model consistent with the observed long term deviations of exchange rates from PPP values. The model and the testing methodology are described in the next section, results comprise section three, and section four concludes.

II. TESTING FOR BUBBLES IN A HYBRID MONETARY EXCHANGE RATE MODEL

Following Bilson (1978, 1979), Dornbusch (1976), Frankel (1979, 1981), Frankel (1976) or Mussa (1982), assume transactions-type money demand equations of the following form:[3]

$$(1) \qquad m_t - p_t = a_1 y_t - a_2(i_t - i_t^*) ,$$

where m_t, p_t, y_t are the logs of relative (U.S. to foreign) money supplies, price levels, real incomes, and $i_t - i_t^*$ is the short term interest differential (U.S. minus foreign). Woo (1984) provides evidence for the equality of income elasticities (a_1) and interest rate semi-elasticities ($-a_2$) for the U.S.

and Germany when a transactions money demand equation is appended with a stock adjustment mechanism. We shall assume uncovered interest parity (UCIP)

$$(2) \qquad i_t - i_t^* = E(s_{t+1}|\Phi_t) - s_t \ ,$$

where $E(s_{t+1}|\Phi_t)$ denotes the linear least squares projection of the time (t+1) spot exchange rate s_{t+1} (natural logarithm of dollars per foreign currency units) based on information dated t. The information set Φ_t contains at least the current and past values of all variables introduced thus far. While econometric techniques have proved sufficiently powerful to reject (2) (Geweke and Feige (1979), Hansen and Hodrick (1980)) no one has demonstrated that deviations from UCIP are large or explainable in the context of a stable financial model of risk (Frankel (1982), Hodrick and Srivastava (1983)). We take (2) to be a reasonable first order approximation. Finally, we shall impose the condition that deviations from PPP follow a random walk:

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$$(4) \qquad s_t = m_t - a_1 y_t + a_2(E(s_{t+1}|\Phi_t) - s_t) + u_t \ .$$

Define $b = \frac{a_2}{1+a_2}$; $0 < b < 1$ as a_2 is minus the interest semi-elasticity of money demand. Equation (4) can be written more usefully as

$$(5) \qquad s_t = (1-b)(m_t - a_1 y_t) + bE(s_{t+1}|\Phi_t) + u_t \ .$$

There exists overwhelming evidence that the level of s_t follows a borderline nonstationary process (Meese and Singleton (1982), Meese and Rogoff (1983a,b) among others). As a consequence, we shall rely on the first difference of (5) for empirical applicatons:

$$(6) \qquad \Delta s_t = (1-b)(\Delta m_t - a_1 \Delta y_t) + b(E(s_{t+1}|\Phi_t) - E(s_t|\Phi_{t-1})) + \varepsilon_t \ ,$$

where Δ denotes the first difference operator. To promote notational simplicity, define the "market fundamentals" process Δx_t as

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where δ_t is a white noise with variance σ^2_δ. We are implicitly assuming that a_1 can be treated as known for expositional purposes. Sample determination of the order of the Δx_t process is discussed below. For given value(s) of a_1, (6) can be estimated by McCallum's (1976) technique: the unobservable expectation $E(s_{t+1}|\Phi_t)$ is replaced by its actual value s_{t+1} minus a forecast error, η_{t+1}, uncorrelated with Φ_t. This substitution creates a first order moving average (MA(1)) composite disturbance process for (6). Nevertheless, an instrumental variables estimator of b is consistent when

instruments are chosen appropriately, as shown below.

Equation (6) may also be solved recursively forward to obtain

$$(8) \qquad \Delta s_t = (1-b) \sum_{i=0}^{\tau-1} b^i (E(x_{t+i}|\Phi_t) - E(x_{t-1+i}|\Phi_{t-1})) + b^\tau (E(s_{t+\tau}|\Phi_t) - E(s_{t+\tau-1}|\Phi_{t-1})) + \sum_{i=0}^{\tau-1} b^i \varepsilon_t$$

If the transversality condition,

$$(9) \qquad \lim_{\tau \to \infty} b^\tau (E(s_{t+\tau}|\Phi_t) - E(s_{t+\tau-1}|\Phi_{t-1})) = 0 ,$$

holds, then the unique, no bubbles solution to (6) is

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From (7), the optimal prediction formula for x_{t+i} conditional on the information set Φ_t is

$$(11) \qquad E(x_{t+i}|\Phi_t) = x_t + \sum_{j=1}^{i} c^j \Delta x_t .$$

The rational expectations, no bubbles solution to (6) can be shown to be

$$(12a) \qquad \Delta s_t^* \equiv \Delta s_t = \Delta x_t + \frac{bc}{1-bc} (\Delta x_t - \Delta x_{t-1}) + \varepsilon_t/(1-b),$$

(12b) $$\Delta x_t = c\Delta x_{t-1} + \delta_t ,$$

where Δs_t^* will be used to distinguish the so-called market fundamentals solution. Under the null hypothesis of no bubbles the estimate of b that can be extracted from the system (12) is asymptotically efficient. The estimate of b obtained using McCallum's technique on (6) is inefficient, as demonstrated below. If the transversality condition (9) is violated, then any solution of the form

(13) $$s_t = s_t^* + d_t, \text{ where } E(d_{t+1} \mid \Phi_t) = \frac{1}{b} d_t,$$

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To see why solutions of the form (13) invalidate maximum likelihood estimation of (12), note that ordinary least squares (OLS) estimation of (12a) is asymptotically efficient if Δx_t is exogenous with respect to Δs_t. Under this assumption b and c are just identified, the system (12) is recursive, and equation by equation OLS is the efficient estimation strategy. Define

$\gamma = bc/(1-bc)$. Using (13) it can be shown that

$$(14) \qquad \operatorname*{plim}_{T\to\infty} \hat{\gamma} = \gamma + \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \Delta d_t(\Delta x_t - \Delta x_{t-1})/(2\sigma_\delta^2/(1+c)).$$

The probability limit of $\hat{\gamma}$ is not necessarily γ since there is no presumption that

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i.e., that the bubble is uncorrelated with the fundamentals process $\{\Delta x_t\}$.

It is useful to rewrite equation (6) in a more convenient form for applying McCallum's technique:

$$(15) \qquad (\Delta s_t - \Delta x_t) = b(\Delta s_{t+1} - \Delta x_t) + (\varepsilon_t - b(\eta_{t+1} - \eta_t)),$$

where the composite disturbance, call it θ_t, can be expressed as

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$$\varepsilon_t - b\left(\frac{1}{1-bc}(\delta_{t+1} - \delta_t) + \frac{1}{1-b}(\varepsilon_{t+1} - \varepsilon_t)\right)$$

under the null hypothesis of no bubbles. It is clear from (15) and (16) that only Δx_s, for $s \leq (t-1)$, are legitimate instruments for $(\Delta s_{t+1} - \Delta x_t)$, as the composite disturbance contains δ_t.[4] Under our assumptions, and using just Δx_{t-1} as an instrument of $(\Delta s_{t+1} - \Delta x_t)$, the limiting distribution of the instrumental variables estimator can be expressed as

(17) $$\sqrt{T}\,(\hat{b}_{IV} - b) \xrightarrow{L} N(0, \sigma_\theta^2 Q_{IV}),$$

where $\hat{b}_{IV}$ is the instrumental variables estimator of b, $\sigma_\theta^2 = \text{var}(\theta_t)$,

$$Q_{IV} = \frac{(1+c)^2(1-bc)^2}{\sigma_\delta^4 c^2}\left[\frac{\sigma_\delta^2}{1-c^2} + \frac{2c_\delta^2\phi}{1-c^2}\right],$$

$\phi = \text{cov}(\theta_t,\theta_{t-1})/\text{var}(\theta_t)$.[5] The limiting distribution of $\hat{b}$ extracted from the estimators of δ and c in (12) can be shown to be:

(18) $$\sqrt{T}(\hat{b} - b) \xrightarrow{L} N\left(0, \left[\frac{(1-c^2)b^2}{c^2} + \frac{\sigma_\varepsilon^2(1-bc)^4(1+c)}{\sigma_\delta^2 2(1-b)^2c^2}\right]\right)$$ [6]

The Hausman specification error test statistic, distributed as a chi-square with one degree of freedom has the following form:

(19) $$= T\,(\hat{b}_{IV}-\hat{b})^2 \Big/ \left[\frac{b^2(1+c)^2}{c^2} + \sigma_\varepsilon^2\,\frac{(1+c)^3(1-bc)^2((1-bc)^2 + 2b(1-c))}{\sigma_\delta^2\,2(1-c^2)(1-b)^2c^2}\right].$$

While it is not explicitly necessary to derive the population expression for $\text{var}(\hat{b}_{IV}) - \text{var}(\hat{b})$, direct application of the Hausman test using the estimated variance of $\hat{b}_{IV}$, $S^2(\hat{b}_{IV})$, of the McCallum procedure, and the estimated variance of $\hat{b}$, $S^2(\hat{b})$, from (12) does not necessarily produce an estimate of the variance of the difference of $(\hat{b}_{IV}-\hat{b})$ that is positive.[7] To avoid this common difficulty, we have explicitly derived the test statistic

(19) whose denominator must be positive.

It is possible to test for the existence of bubbles in a model where a_1, the income elasticity of money demand, is explicitly estimated; see below. There does not, however, appear to be any general form for the test statistic (19), as the derivation of (19) relied heavily on the assumed form of the relationship between current spot rates and expected future spot rates, the assumptions on the driving process x_t, and the behavior of the structural disturbances.

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$$(18) \qquad \sqrt{T}(\hat{b} - b) \xrightarrow{L} N\left(0,\ \left[\frac{(1-c^2)b^2}{c^2} + \frac{\sigma_\varepsilon^2(1-bc)^4(1+c)}{\sigma_\delta^2 2(1-b)^2c^2}\right]\right)^{6}$$

The Hausman specification error test statistic, distributed as a chi-square with one degree of freedom has the following form:

$$(19) \qquad = T\,(\hat{b}_{IV}-\hat{b})^2 \Big/ \left[\frac{b^2(1+c)^2}{c^2} + \sigma_\varepsilon^2\,\frac{(1+c)^3(1-bc)^2((1-bc)^2+2b(1-c))}{\sigma_\delta^2\,2(1-c^2)(1-b)^2c^2}\right].$$

While it is not explicitly necessary to derive the population expression for $\mathrm{var}(\hat{b}_{IV})$ - var $(\hat{b})$, direct application of the Hausman test using the estimated variance of $\hat{b}_{IV}$, $S^2(\hat{b}_{IV})$, of the McCallum procedure, and the estimated variance of $\hat{b}$, $S^2(\hat{b})$, from (12) does not necessarily produce an estimate of the variance of the difference of $(\hat{b}_{IV}-\hat{b})$ that is positive.[7] To avoid this common difficulty, we have explicitly derived the test statistic

(19) whose denominator must be positive.

It is possible to test for the existence of bubbles in a model where a_1, the income elasticity of money demand, is explicitly estimated; see below. There does not, however, appear to be any general form for the test statistic (19), as the derivation of (19) relied heavily on the assumed form of the relationship between current spot rates and expected future spot rates, the assumptions on the driving process x_t, and the behavior of the structural disturbances.

III. UNCORKING EVIDENCE OF BUBBLES IN BILATERAL DEUTSCHEMARK AND STERLING RATES WITH THE U.S. DOLLAR

Tests of the no bubbles hypothesis were conducted for a grid of reasonable parameter values on a_1, the income elasticity of money demand. The interval [.2, 1.0] was explored in steps of (0.1). This encompasses values from theoretical transactions demand for money models; roughly .3 to 1.0 depending on integer constraints and the assumptions regarding the size versus the frequency of transactions as income rises.[8] The range also includes Goldfeld's (1973) short run (roughly .2) and long run (approximately .7) income elasticity estimates. While this coefficient is estimated directly in a more elatorate version of the model than the one described in section 2, point estimates of the other focus parameters, c and b or a_2, are qualitatively uneffected by the use of this more complicated procedure. Therefore, only detailed results are given for the model of section 2 over a range of plausible a_1 values. The implicit constraint imposed on the driving processes Δm_t and Δy_t in the theoretical development of the previous section is that both variables are adequately represented by univariate autoregressions with the same lag (L) polynomial. In other words, in the bivariate autoregression

$$\Delta m_t(1 - \sum_{i=1}^{N} a_i L^i) + \Delta y_t \sum_{i=1}^{N} b_i L^i = \Delta m_t^* \tag{20}$$

$$\Delta y_t(1 - \sum_{i=1}^{N} e_i L^i) + \Delta m_t \sum_{i=1}^{N} f_i L^i = \Delta y_t^*,$$

where $(\Delta m_t^*, \Delta y_t^*)$ is a bivariate white noise orthoganal to Φ_t, it is the case that $N = 1$, $b_1 = 0 = f_1$ and $a_1 = e_1$. These constraints cannot be rejected for bilateral U.S. - German and U.S. - U.K. data sets. In addition, the condition that Δs_t not Granger cause $\Delta x_t = (\Delta m_t - a_1 \Delta y_t)$ is accepted for the \$/yen, \$/DM and \$/L rates for all values of a_1 considered.[9] These Granger causality tests contained a constant, 11 seasonal dummes, and eight lags of Δs_t and Δx_t. All regressions discussed thus far are based on 110 observations of the dependent variables: October 1973 to November 1982 inclusive. Variables at the beginning and end of the sample period (January 1973 - December 1982) are required for leads and lags in the various regressions. The data are described in the appendix.

The results are presented by country, but first, the form of the test statistic (19) merits further comment. The denominator of the test statistic can be computed using values of b, c, σ_ε^2 and σ_δ^2 extracted entirely from the estmation of the system (12), entirely from the estimation of (15) and (12b), or a mixture of the two. Only the estimates based on (15) and (12b) are consistent for their population values under the alternative hypothesis of bubbles. This distinction turns out to be critical. In terms of economic significance the estimates of b derived from (15) are reasonable and imply values of interest rate semi-elasticities consistent with those reported in the literature. The estimates of b implied by the system (12) are, however, always too large ($\hat{b} > 1$), suggesting a positive interest rate semi-elasticity: $\hat{a}_2 = \hat{b}/(1-\hat{b})$, where a_2 is minus the interest semi-elasticity of money demand from (1). Since the derivation of the test statistic (19) requires the assumption $b<1$, it makes little sense to evaluate the statistic

using the implausible (no bubbles) estimate of b. For comparatives purposes two values of the test statistic are reported in Table 1, the first is based on parameter estimates of (15) and (12b) while the second is based on a mixture of (15) and (12b) with (12a).[10]

The nice feature of McCallum's technique in this context is that under the alternative hypothesis of bubbles, (1) parameter estimates are consistent for their population counterparts, (2) conventional asymptotic distribution theory still applies to (15) when |c| < |b|, and (3) the model (15) should pass conventional diagnostic checks for goodness of fit if the hybrid monetary exchange rate model of section 2 provides a reasonable in-sample description of the U.S. - German and U.S. - U.K. data sets under the alternative hypothesis of bubbles. We have already discussed point (1) above. To confirm the validity of point (2), note that (13) implies

$$(21) \qquad d_t = \frac{1}{b}\, d_{t-1} + \zeta_t \,, \qquad 0 < b < 1 \,,$$

where ζ_t is orthogonal to Φ_t. Assuming the process (ζ_t, δ_t) is a serially independent vector white noise with a (2x2) covariance matrix Σ, Q_{IV} (defined in footnote 4) is finite and nonzero provided |c| < |b|. This condition stems from the interaction of the driving process Δx_t with $\Delta s_t = (\Delta s_t^* + \Delta d_t)$, and is a condition for a convergent sum of a geometric series.[11] This condition is satisfied for all models reported below, and highlights and additional advantage of estimating both the structural exchange rate equation and the driving process in first differences. If the driving process autoregression were run in levels, then the first order autoregressive parameter would be roughly (1+c); we find c small, but significantly negative.

The exact form of σ_θ^2 , the distrubance variance of (15), in terms of

b, c, σ_δ^2 , and σ_ε^2 is not known under the alternative hypothesis of bubbles. Nevertheless, it is still the case that we a priori expect the disturbances of (15), based on an instrumental variables estimator of b, to exhibit MA(1) behavior, as there is no reason to rule out contemporaneous correlation of η_t and ε_t. Before correction for the MA(1) error process (see the estimator described in footnote 4), the autocorrelation function of the residuals of (15) exhibit precisely this behavior. As such, examination of the autocorrelation function of (15) provides a diagnostic check of the adaquecy of the model under the alternative, point (3) above.

Tables 1-5 are organized as follows. The value of the income elasticity of money demand and the bilateral data set are indicated by the table heading. For each value of a_1 a test statistic (an F(8,82)) for the test that Δs_t doesn't Granger cause Δx_t is reported, two versions of the bubble test (19) (both $X^2(1)$) are given, and the individual regression results for (12a), (12b) and (15) are displayed. Results of equation (15) include the residual sample autocorrelation function, lags 1-13. For the U.S.-German data set, regressions based on a_1 = .3, .4, and .5 are reported: these bracket the estimate of a_1 (.39) obtained from an unconstrained version of the model of section 2.[12] The income elasticity range of the U.S.-U.K. data set, a_1 = .3 and .4, also brackets the unrestricted estimate of a_1 = .35. For convenience, in the row labeled average interest elasticity, we have converted the implied estimates of a_2, the interest semi-elasticity, into elasticities by dividing by the sample mean of i_t.

All versions of the test (19) indicate rejection of the no bubbles hypothesis at very small significance levels. Estimates from the model (15) indicate an adequate fit under the alternative for both data sets except the

U.S.-German model with $a_1 = 3$. For this model the $\bar{R}^2$ is low and the interest elasticity is quite large by comparison to extant empirical work. Strong evidence of bubbles also emerges from an analysis of the DM/£ cross rate, although the autocorrelation function of the residuals of (15) for this data set contains a significant positive spike at lag seven. This suggests that the hybrid monetary model may be misspecified on the German-U.K. data set.[13] Tests for bubbles were also performed using the U.S.-Japanese exchange rate. While test statistics here also provide strong evidence of bubbles, the adequacy of (15) as a description of the U.S.-Japan data set is seriously in doubt. The $\bar{R}^2$ of (15) is low, the focus parameters are imprecisely estimated, and the disturbance term appears to possess a complicated serial correlation pattern. For these reasons the U.S.-Japan bubble tests (not reported) are inconclusive.

The most menancing empirical regularity that confronts exchange rate modelers is the failure of the current generation of empirical exchange rate models to provide stable results across subperiods of the modern floating rate period. The results of this paper provide no exception to the rule. Tests for bubbles were conducted for two subperiods of our sample: the pre (October 1973 to September 1979) and post (October 1979 to November 1982) change in Federal Reserve operating procedures. For the U.S.-German data set, the first subperiod estimation results of (12a, b) and (15) are qualitatively the same as those reported in Table 1, when the income elasticity of money demand is greater than or equal to .40. In the second subperiod, an AR(1) process is no longer appropriate for Δx_t. Nevertheless, these results using an AR(4) to approximate Δx_t, again provide evidence of bubbles for $a_1 \geq .40$. Smaller values of the income elasticity result in the failure of equation (15) to provide an adequate description of the data, as

the estimated b exceeds one for small values of a_1.

Split sample results from the U.S.-U.K. data bear little resemblance to the full sample results reported in Table 1. While the driving process Δx_t appears to be relatively stable, equation (15) fails to provide sensible estimates of the interest semi-elasticity of money demand for all values of a_1 in either subperiod.

IV. CONCLUSION

While many economists believe that asset prices reflect the values of the underlying market fundamentals, asset market participants often express the view that fundamentals are just part of the story. Characterizations of asset price movements from the latter group otfen include discussions of "extraneous events"; see for example the frequent explanations of exchange rate or other asset price movements given in the "What's News" column of The Wall Street Journal. As Blanchard and Watson (1982, p.1) point out, "...economists have overstated their case. Rationality of both behavior and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value, rational bubbles."

This paper provides the first strong evidence of asset market bubbles in exchange markets, using a monthly monetary model of the dollar-Deutsche-mark and dollar-pound exchange rates. The model appears to be better suited to the dollar-Deutschemark rate, as split sample tests highlight the inability of the monetary model to explain the dollar-pound rate across two

different U.S.monetary policy regimes. Since the statistical test for bubbles exployed in this paper is model specific, it would be useful to find a model or intertemporal equilibrium condition that offers a better approximation to the mechanism generating the dollar-pound exchange rate for different policy regime subperiods of the current floating rate period. When this is done, a more convincing test for the existence of bubbles may be conducted for these currencies.

TABLE 1

U.S. - German data: income elasticity $a_1 = .3$

Granger test: $F(8,82) = .640$
Bubble test 1: $\chi^2(1) = 56.6^*$ Bubble test 2: $\chi^2(1) = 6.52^*$

Parameter	Equation (12a)	Equation (12b)	Equation (15)	Residual auto-correlations of (15)		
				lag	rho	(std. error)
$-a_2$	1.49		-26.2			
	(.828)		(31.6)	1	-.423*	(.0953-1)
c		-.251*		2	-.171-1	(.111)
		(.982-1)				
average interest elasticity	0.18	----	-3.09	3	.203-2	(.111)
σ^2_ε	.103-2			4	.125	(.111)
σ^2_δ		.343-3		5	.121	(.112)
σ^2_θ			.139-2	6	-.182	(.114)
Jan	-.337-1	-.120	-.441-1	7	.185	(.116)
	(.184-1)	.110-1)	(.669-1)			
Feb	.587-2	.223-1*	.396-2	8	-.483-1	(.119)
	(.170-1)	(.953-2)	(.430-1)			
March	_296-1	.213-3	.369-1	9	.335-1	(.119)
	(.159-1)	(.942-2)	(.364-1)			
April	-.200-1	.135-2	-.424-2	10	.698-1	(.119)
	(.171-1)	(.105-1)	(.253-1)			
May	-.959-2	-.427-1*	-.259-1	11	.806-1	(.120)
	(.193-1)	(.102-1)	(.800-1)			
June	.699-2	-.368-1*	-.176-1	12	-.187	(.120)
	(.161-1)	(.881-2)	(.594-1)			
July	.117-1	-.560-1*	-.180-1	13	-.212-1	(.123)
	(.181-1)	(.916-2)	(.859-1)			
Aug	.123-1	-.455-1*	-.343-1			
	(.161-1)	(.874-2)	(.856-1)			
Sept	-.198-1	.309-1*	-.194-1			
	(.152-1)	(.898-2)	(.167-1)			
Oct	-.350-1	.145-1	-.127-1			
	(.184-1)	(.124-1)	.306-1)			
Nov	.489-2	-.520-1*	-.281-1			
	(.201-1)	(.103-1)	(.108)			
Constant	.934-2	.174-1*	.193-1			
	(.130-1)	(.734-2)	(.494-1)			
$\bar{R}^2$	.37	.68	.156			
DW	1.55	2.02	NA			
Q(13)	22.7	14.5	39.0*			

NOTE: Standard errors in parantheses
(*): Indicates significance at a 5% level.
NA: Statistic was not calculated

TABLE 2
U.S. - German data: Income elasticity $a_1 = .4$

Granger test: $F(8,82) = .656$
Bubble test 1: $x^2(1) = 71.5^*$ Bubble test 2: $x^2(1) = 9.13^*$

Parameter	Equation (12a)	Equation (12b)	Equation (15)	Residual correlations of auto-		(15)
				lag	rho	(std. error)
$-a_2$	1.87		-8.26			
	(.786)		(7.16)	1	-.429*	(.953-1)
c		-.270*		2	-.111-1	(.111)
		(.976-1)				
avg. interest elasticity	0.22	---	-.97	3	.595-3	(.111)
σ^2_ε	.103-2			4	-.121	(.111)
σ^2_δ		.393-3		5	.121	(.112)
σ^2_θ			.129-2	6	-.188	(.113)
Jan	-.257-1	-.205-1	-.395-1	7	.192	(.116)
	(.178-1)	(.109-1)	(.511-1)			
Feb	.101-1	-.208-1*	.578-2	8	-.520-1	(.119)
	(.159-1)	(.958-2)	(272-1)			
March	-.294-1	.272-2	-.352-1	9	-.331-1	(.119)
	(.156-1)	(.982-2)	(.248-1)			
April	-.198-1	.534-2	-.361-2	10	.676-1	(.119)
	(.163-1)	(.106-1)	(183-1)			
May	-.833-2	-.420-1*	-.213-1	11	.890-1	(.120)
	(.183-1)	(.105-1)	(.531-1)			
June	.108-1	-.392-1*	-.142-1	12	-.184	(.120)
	(.157-1)	(.935-2)	(.409-1)			
July	.217-1	-.687-1*	-.118-1	13	-.208-1	(.123)
	(.177-1)	(.950-2)	(.655-1)			
Aug	.239-1	-.560-1*	-.287-1			
	(.156-1)	(.948-2)	(.613-1)			
Sept	-.225-1	.439-1*	-.213-1			
	(.156-1)	(.936-2)	(.216-1)			
Oct	-.404-1*	.218-1	-.115-1			
	(.180-1)	(.132-1)	(.210-1)			
Nov	.134-2	-.423-1*	-.227-1			
	(.183-1)	(.105-1)	(.650-1)			
Constant	.709-2	.170-1*	.166-1			
	(.121-1)	(.730-2)	(.322-1)			
$\bar{R}^2$	.44	.71	.30			
DW	1.57	2.03	NA			
Q(13)	22.5	15.7	39.0*			

Note: Standard errors in parantheses
(*): Indicates significance at a 5% level
NA: Statistic was not calculated

TABLE 3

U.S. - German data: Income elasticity $a_1 = .4$

Granger test: $F(8,82) = .693$
Bubble test 1: $X^2(1) = 90.40^*$ Bubble test 2: $x^2(1) = 14.17^*$

Parameter	Equation (12a)	Equation (12b)	Equation (15)	Residual auto-correlations of (15).		
				lag	rho	(std. error)
$-a_2$	1.56* (.740)		-6.37 (4.29)	1	-.415*	(.953-1)
c		-.287* (.971-1)		2	-.990-2	(.111)
avg interest elasticity	0.18	--	-.75	3	-.177-2	(.111)
σ^2_ε	.104-2			4	-.119	(.111)
σ^2_δ		.456-3		5	.120	(.112)
σ^2_θ			.126-2	6	-.189	(.113)
Jan	-.177-1 (.173-1)	-.294-1* (.111-1)	-.369-1 (.440-1)	7	.194	(.116)
Feb	.145-1 (.154-1)	-.200-1 (.101-1)	.582-2 (.207-1)	8	-.529-1	(.119)
March	-.291-1 (154-1)	.495-2 (.104-1)	-.350-1 (.201-1)	9	-.322-1	(.119)
April	-.196-1 (.158-1)	.894-2 (.110-1)	-.395-2 (.162-1)	10	.664-1	(.119)
May	-706-2 (.176-1)	-.416-1* (.109-1)	-.196-1 (407-1)	11	.937-1	(.119)
June	.148-1 (.155-1)	-.419-1* (.101-1)	-.128-1 (.319-1)	12	-.183	(.120)
July	.316-1 (.173-1)	-820-1* (.101-)	-809-2 (.326-1)	13	-.189-1	(.122)
Aug	.357-1* (.153-1)	-.671-1* (.107-1)	-.259-1 (.563-1)			
Sept	-.248-1 (.165-1)	.564-1* (.101-1)	-.241-1 (.501-1)			
Oct	-.459-1* (.177-1)	.291-1* (.141-1)	-.116-1 (.260-1)			
Nov	-.209-2 (.170-1)	-.330-1* (.108-1)	-.219-1 (.448-1)			
Constant	.475-2 (.116-1)	.169-1* (.747-2)	.158-1 (.243-1)			
$\bar{R}^2$	.52	.74	.41			
DW	1.59	2.03	NA			
Q(13)	22.4	16.9	38.7*			

Note: Standard errors in parantheses
(*): Indicates significance at a 5% level
NA: Statistic was not calculated

TABLE 4

U.S. - U.K. data: income elasticity $a_1 = .3$

Granger test: $F(8,82) = 1.615$
Bubble test 1: $x^2(1) = 148.05^*$ Bubble test 2: $x^2(1) = 23.52^*$

Parameter	Equation (12a)	Eqauation (12b)	Equation (15)	Residual auto-correlations of (15) lag	rho	(std. error
$-a_2$	1.22		-5.99	1	-.401*	(.953-1)
	(.802)		(.310)			
c		-.174		2	-.497-1	(.110)
		(.101)				
avg interest elasticity	0.14	----	-.71	3	.358-1	(.110)
σ^2_ε	.347-3			4	-.121	(.110)
σ^2_δ		.286-3		5	.414-1	(.111)
σ^2_θ			.873-3	6	.335-2	(.111)
Jan	.783-2	.545-1*	.142-1	7	.512-1	(.111)
	(.172-1)	(.963-2)	(.371-1)			
Feb	-.135-1	.211-1*	.219-1	8	-.995-1	(.111)
	(.124-1)	(.803-2)	(.246-1)			
March	-.139-1	.276-1*	.363-2	9	.808-1	(.112)
	(.142-1)	(.869-2)	(.297-1)			
April	-.795-2	.613-2	-.132-2	10	.206-1	(.113)
	(.127-1)	(.822-2)	(.172-1)			
May	.206-1	.958-2	.170-1	11	.765-1	(.113)
	(.131-1)	(.913-2)	(.159-1)			
June	-.476-2	.370-1*	-.695-2	12	-.156	(.113)
	(.162-1)	(.974-2)	(.285-1)			
July	.302-2	.562-2	.196-1	13	.772-1	(.115)
	(.125-1)	(.800-2)	(.180-1)			
Aug	.176-1	-.117-1	.124-1			
	(.131-1)	(.926-2)	(.156-1)			
Sept	-.219-1	.715-1*	-.636-2			
	(.188-1)	(.984-2)	(458-1)			
Oct	-.347-1*	.392-1*	.889-2			
	(.121-1)	(.830-2)	(272-1)			
Nov	-.251-1	.529-1*	-.229-2			
	(.143-1)	(.801-2)	(.348-1)			
Constant	.659-2	-.294-1*	-.650-2			
	(.103-1)	(.601-2)	(.227-1)			
$\bar{R}^2$	.74	.72	.44			
DW	1.93	1.93	NA			
Q(13)	9.92	13.0	28.1*			

Note: Standard error in parantheses
(*) Indicates significance at a 5% level.
NA: Statistic was not calculated

TABLE 5

U.S.-U.K. data: income elasticity a_1 = .4

Granger test: F(8,82) = 1.664

Bubble test 1: $x^2(1)$ = 151.09* Bubble test 2: $x^2(1)$ = 26.47*

Parameter	Equation (12a)	Equation (12b)	Equation (15)	residual auto-correlations of (15)		
				lag	rho	(std. error)
$-a_2$	1.25 (.723)		-6.74* (3.06)	1	-.403*	(.953-1)
c		-.200* (.100)		2	-509-1	(.110)
avg interest elasticity	0.15	-----	-.80	3	.375-1	(.110)
σ^2_ε	.700-3			4	-.122	(.110)
σ^2_δ		.347-3		5	.413-1	(.111)
σ^2_θ			.885-3	6	.136-2	(.111)
Jan	.108-1 (.176-1)	.574-1* .110-1	.144-1	7	.526-1	(.111)
Feb	-.166-1 (.127-1)	.319-1* (.879-2)	.211-1 (.362-1)	8	-.101	(.112)
March	-.172-1 (.143-1)	.340-1* (.944-2)	.351-2 .297-1)	9	.810-1	(.112)
April	-.504-2 (.128-1)	.343-2 (.912-2)	-.821-3 (.162-1)	10	.207-1	(.113)
May	.239-1 (.137-1)	-.652-2 (.105-1)	.164-1 (.170-1)	11	.749-1	(.113)
June	-.399-2 (.161-1)	.387-1* (.107-1)	-.670-2 (.275-1)	12	-.155	(.113)
July	.563-2 (.127-1)	.643-2 (890-2)	.197-1 (.178-1)	13	.772-1	(.115)
Aug	.240-1 (.133-1)	-.179-1 (.104-1)	.131-1 (.146-1)			
Sept	-.239-1 (.202-1)	.871-1* (.114-1)	-.759-2 (.488-1)			
Oct	-.439-1* (.122-1)	.519-1* (.933-2)	.830-2 (.285-1)			
Nov	-.300-1* (.146-1)	.639-1* (.881-2)	-.284-2 (.360-1)			
Constant (.107-1)	.698-2 (.673-2)	-.343-1* (.230-1)	-.631-2			
$\bar{R}^2$	.63	.74	.53			
DW	1.63	1.93	-----			
Q(13)	20.8	9.92	28.4*			

Note: Standard error in parantheses
(*): Indicates significance at a 5% level
NA: Statistic was not calculated

FOOTNOTES

1. Sweeney (1984a) finds that the filter rule profitability, in excess of a buy and hold strategy, is significant; his results constitute a violation of a simple capital asset pricing model.

2. See Shapiro (1982) or Fieleke (1980). In the later part of our sample, banks became more aggressive in attempting to profit from exchange rate movements. Over the early part of our sample, banks' foreign exchange departments were more service oriented.

3. For simplicity, we will suppress deterministic terms - a constant and seasonal dummy variables - throughout the theoretical analysis. Such terms are included in the empirical work reported in the next section. A stochastic disturbance term can be appended to the money demand equation as well, without altering the discussion below. While it is customary to omit money demand disturbances in derivations of monetary exchange rate models, this practice runs counter to the rational expectations literature where the source of disturbances is deemed to be crucial. In the present context, an appropriate disturbance process can be assumed for (1) so that after suitable use of "Granger's lemma" (see Harvey(1981, p. 43)), the disturbance processes assumed for the estimating equations below remain intact.

4. Note that Δs_{t-i}, $i \geq 1$ are also legitimate instruments. Our concern here is <u>not</u> with the choice of an optimal instrument set.

5. Define the (1XT) vectors $A' = (\Delta x_0, \ldots, \Delta x_{T-1})$ and $B' = (\Delta s_2 - \Delta x_1, \ldots (\Delta s_{T+1} - \Delta x_T)$. Let V be a symmetric banded matrix for a MA(1) error process with ϕ as the off diagonal entries and ones down the diagonal.

Then

$$Q_{IV} = \underset{T\to\infty}{\text{plim}} \left(\frac{1}{T} B'A\right)^{-1} \left(\frac{1}{T} A' VA\right) \left(\frac{1}{T} A'B\right)^{-1} .$$

Under the null hypothesis of no bubbles this expression has the limiting form given in the text where

$$\sigma_\theta^2 = \sigma_\varepsilon^2 \left[\frac{1+b^2}{(1-b)^2}\right] + \sigma_\delta^2 \left[\frac{2b^2}{(1-bc)^2}\right]$$

and $\text{cov}(\theta_t, \theta_{t-1}) = \sigma_\varepsilon^2 \frac{-b}{(1-b)^2} + \sigma_\delta^2 \frac{-b^2}{(1-bc)^2}$.

5. Define the function $b = g(\gamma,c) = \gamma/c(1+\gamma)$, $c \neq 0$, $\gamma \neq -1$. Under our assumptions,

$$\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{L} N(0, \frac{\sigma_\varepsilon^2 (1+c)}{2(1-b)^2\sigma_\delta^2})$$

and $\sqrt{T}(\hat{c} - c) \xrightarrow{L} N(0, (1-c^2))$. By application of the Mann-Wald theorem (Rao(1973 p. 124))

$$\sqrt{T}(g(\hat{\gamma},\hat{c}) - b) \xrightarrow{L} N(0, \Omega): \quad \Omega = \left[\frac{dg}{d\gamma}\right]^2 \frac{\sigma_\varepsilon^2 (1+c)}{2(1-b)^2\sigma_\delta^2} +$$

$$\left[\frac{dg}{dc}\right]^2 (1-c^2) = \frac{(1-c^2)b^2}{c^2} + \frac{\sigma_\varepsilon^2 (1-bc)^4(1+c)}{\sigma_\delta^2\, 2(1-b)^2\, c^2} .$$

In deriving this expression, we have used the fact that $\text{cov}(\hat{\gamma}, \hat{c}) = 0$.

7. For the experiments reported below, the difference of $S^2(\hat{b}_{IV})$ and $S^2(\hat{b})$ was always negative, hence the need for deriving $\text{var}(\hat{b}_{IV}) - \text{var}(\hat{b})$ explicitly. A reason for the sample violation of the population variance ordering is suggested is section 3.

8. See the discussions in Barro (1976), Miller-Orr (1966) and Whalen (1966).

9. The condition that Δs_t not Granger cause Δx_t is necessary but not sufficient for Δx_t to be exogenous with respect to Δs_t. We choose to report Granger tests of Granger causality as Geweke, Meese and Dent (1983) report strong evidence for preferring these tests to other variants proposed in the literature.

10. The first test statistic for bubbles reported in Table I relies on parameter estimates of b, c, σ_ε^2 and σ_δ^2 from (15) and (12b). An estimate of σ_ε^2 can be extracted from σ_θ^2; see footnote 4. The second test statistic is calculated from estimates of b from (15), c and σ_δ^2 from (12b) and σ_ε^2 from (12a).

11. Since conventional asymptotic distribution theory precludes exponentially growing regressors (Theil (1971)) it is important to demonstrate that Q_{IV} is finite and nonzero under the alternative of bubbles. In this case

$$Q_{IV} = (\sigma_\delta^2 (1 + 2\phi c)/(1-c^2))/ \left(\underset{T\to\infty}{\text{plim}} \left(\frac{1}{T} \sum_{t=2}^{T-1} (\Delta s_{t+1} - \Delta x_t)\, \Delta x_{t-1}\right)\right)^2 .$$

Substituting $\Delta s_t = (\Delta s_t^* + \Delta d_t)$ into the denominator introduces the term

$$\underset{T\to\infty}{\text{plim}} \frac{1}{T} \sum_{t=2}^{T-1} (\Delta d_{t+1} - \Delta x_t)\, \Delta x_{t-1} ,$$

which must be finite if Q_{IV} is to be nonzero. The expression

$$\operatorname*{plim}_{T\to\infty} \frac{1}{T} \sum_{t=2}^{T-1} \Delta d_{t+1}\, \Delta x_{t-1} = \operatorname*{plim}_{T\to\infty} \frac{1}{T} \sum_{t=2}^{T-1} \Big(\sum_{i=0}^{\infty} (1/b)^i \zeta_{t+1-i} \Big) * \Big(\sum_{j=0}^{\infty} c^j\, \delta_{t+1-j} \Big).$$

Since the (ζ_t, δ_t) process is contemporaneously but not serially correlated, the probability limit of the above expression is finite when $\left|c/b\right|<1$. In this case, the limit is $\sigma_{12}/b^2(1-c/b)$, where σ_{12} is the off diagonal element of Σ.

12. The more general model of section 2 would have the form:

$$(15') \quad \Delta s_t - \Delta x_t = -a_1 (1-b)\Delta y_t + b(\Delta s_{t+1} - \Delta m_t) + \theta_t$$

$$(12a') \quad \Delta s_t - \Delta x_t = -a_1 \Delta y_t + \frac{bc}{1-bc} (\Delta x_t - \Delta x_{t-1}) - \frac{a_1 be}{1-be} (\Delta y_t - \Delta y_{t-1}) + \frac{\varepsilon_t}{(1-b)}$$

$$(12b') \quad \Delta y_t = e\Delta y_{t-1} + \delta_{1,t}$$
$$\Delta m_t = c\Delta m_{t-1} + \delta_{2,t},$$

where Δy_{t-1} and Δm_{t-1} are used as instruments for the two regressors of (15'). The form of the bubbles test statistic for this model (a statistic comparable to (19)) is a computational nightmare. Since we accept e = c, there is no loss of generality in exploiting the simple form of the model presented in section 2

13. No explanation for the significant seventh order autocorrelation coefficient comes readily to mind. The bilateral German-U.K. data set was constructed from the U.S.-German and U.S.-U.K. data sets. These bilateral data sets with the U.S. are aligned for different days of the month. As such, the improper alignment of the German-U.K. data series might explain the peculiar autocorrelation.

DATA APPENDIX

The raw data consist entirely of seasonally unadjusted monthly observations over the period from January 1973 to December 1982. All data are taken from the publicly available sources listed below; a more detailed description of the data set can be fpund in Meese and Rogoff (1983b).

United States data series

Three-month Treasury bill rates, CPI, industrial production, and M_1-B; Federal Reserve Board data base.

Foreign data series

Spot rates: Federal Reserve Board data base.

Monetary aggregates and industrial productions: O.E.C.D. Main Economic Indicators.

Consumer prices: Monthly Report of the Deutsche Bundesbank (Germany), Financial Times and Employment Gazette (U.K.), Economics Statistics Monthly (Japan).

Interest rates (three-month): Frankfurter Allgemeine Zeitung (three month German interbank rate), Financial Times (three-month British local authorities deposits), FRB data base ("Over two-month ends" bill discount rate, Tokyo stock exchange).

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