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Asset Pricing, Slow-Moving Capital, Monetary Policy, and Inflation

A dissertation submitted in partial satisfaction of the

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in Management

by

Matthias Fleckenstein

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ABSTRACT OF THE DISSERTATION

Asset Pricing, Slow-Moving Capital, Monetary Policy, and Inflation

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Professor Francis A. Longstaff, Chair

This dissertation focuses on a major challenge to neoclassical asset pricing theory – the existence of persistent arbitrage mispricing in financial markets. Many scholars, e.g. Liu and Longstaff (2004) and Shleifer and Vishny (2007), have challenged the neoclassical no-arbitrage paradigm. However, the nature of arbitrage mispricing is not yet fully understood and requires further study.

The first chapter 'The TIPS–Treasury Bond Puzzle', jointly written with Francis A. Longstaff and Hanno Lustig, analyzes the relative pricing between U.S. Treasury Bonds and Treasury Inflation-Protected Securities (TIPS). We document that Treasury bonds are consistently overpriced relative to TIPS. The price of a Treasury bond can exceed that of an inflation swapped TIPS issue exactly matching the cash flows of the Treasury bond by more than $20

per \$100 notional amount. The relative mispricing of TIPS and Treasury bonds represents one of the largest examples of arbitrage ever documented and poses a major puzzle to classical asset pricing theory. We find direct evidence that the mispricing narrows as additional capital flows into the markets. This provides strong support for the slow-moving-capital explanation of arbitrage persistence.

In the second chapter, I extend the analysis in the first chapter to the G7 government bond markets and document new stylized facts about the dynamics and determinants of arbitrage mispricing in and across financial markets. The new insight for the slow-moving capital theory is that capital available to specific types of arbitrageurs is significantly related to the inflation-linked–nominal bond mispricing (ILB mispricing). Specifically, returns of hedge funds following fixed income strategies strongly predict subsequent changes in ILB mispricing, whereas other hedge fund categories lack statistically significant forecasting power. Furthermore, I analyze the effects of monetary policy on arbitrage mispricing and find that central banks have exacerbated mispricing through large-scale asset purchase programs.

The third chapter extends the analysis of inflation-linked securities markets. The magnitude of deflation risk and the economic and financial factors that contribute to deflation risk are not well studied. This chapter, jointly written with Francis A. Longstaff and Hanno Lustig, presents a new market-based approach for measuring deflation risk. This approach allows us to solve directly for the market's assessment of the probability of deflation for horizons of up to 30 years using the prices of inflation swaps and options. We find that the market prices the economic tail risk of deflation very similarly to other types of tail risks such as catastrophic insurance losses. In contrast, inflation tail risk has only a relatively small premium.

The dissertation of Matthias Fleckenstein is approved.

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To my parents Dr. Reinhard Fleckenstein and Andrea Fleckenstein.

To my sister Christine.

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Chapter 1: The TIPS–Treasury Bond Puzzle

1.1 Introduction

The Treasury bond and the Treasury Inflation-Protected Securities (TIPS) markets are two of the largest and most actively traded fixed-income markets in the world. Despite this, we find that there is persistent mispricing on a massive scale across these two markets. Furthermore, this mispricing is almost invariably in one directionTreasury bonds are consistently overpriced relative to TIPS. For example, we show that the price of a Treasury bond can exceed that of an inflation swapped TIPS issue exactly matching the cash flows of the Treasury bond by more than $20 per $100 notional amount. The relative mispricing of TIPS and Treasury bonds represents one of the largest examples of arbitrage ever documented and poses a major puzzle to classical asset pricing theory.[1]

We proceed by first describing the TIPS–Treasury arbitrage strategy. The logic behind this strategy is simple. The inflation-linked cash flows from a TIPS issue can be converted into fixed cash flows using inflation swaps. The resulting cash flows can be structured to match exactly the cash flows from a Treasury bond with the same maturity date as the TIPS issue. Hence, we have created a synthetic nominal Treasury bond from the TIPS issue. Price differences between the synthetic Treasury bond and the nominal Treasury bond therefore represent straightforward arbitrage opportunities. The data set includes daily prices for 29 maturity-matched pairs of TIPS issues and Treasury bonds from 2004 to 2009.

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[1] For examples of significant mispricing in other financial markets, see Dammon, Dunn, and Spatt (1993), and Lamont and Thaler (2003).

We find mispricing across all pairs of TIPS and Treasury bonds. For individual pairs, the mispricing often exceeds $10 to $20.[2] Translated into yields, the average size of the mispricing is 54.5 basis points, but can exceed 200 basis points for some pairs. The average size of this mispricing is orders of magnitude larger than the transaction costs of executing the arbitrage strategy. While other instances of Treasury mispricing have been documented, these have all been much smaller in size. One prominent example is the yield spread between old and new Treasury bonds, commonly referred to as the on-the-run/off-the-run spread.[3] The TIPS-Treasury mispricing we find is much larger and more persistent than the on-the-run/off-the run spread for Treasuries.[4]

We also provide clear evidence that our results are not simply due to mispricing in the inflation swaps market since we find no mispricing on average when the same arbitrage strategy is applied to corporate fixed-rate and inflation-linked bonds. Thus, the mispricing is directly attributable to the relative prices of TIPS and Treasury bonds – Treasuries are expensive relative to TIPS. We also consider the potential impact of transaction costs, differential taxation, credit risk, institutional and foreign ownership of Treasury bonds and TIPS, collateralization, the ability to short Treasury bonds, market liquidity, and other factors. None of these factors is able to provide a fully satisfactory explanation for the existence of this mispricing.

Is the TIPS–Treasury arbitrage strategy truly an arbitrage in the textbook sense? Or is

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[2]For simplicity, all bond prices and dollar mispricing values will be expressed in terms of dollars per $100 notional or par amount throughout the paper.

[3]For a description of the properties of on-the-run bonds, see Krishnamurthy (2002) and Vayanos and Weill (2008).

[4]For example, Krishnamurthy (2002) finds that this spread averages 6.05 basis points for 30-year Treasuries between 1995 and 1999. Furthermore, the on-the run/off-the-run spread typically decreases to close to zero right before auctions.

it a risky leveraged strategy that could result in losses for an arbitrageur in some states of the world? The answer to both of these questions is yes. As shown by Shleifer and Vishny (1997), Liu and Longstaff (2005), and others, even a textbook arbitrage can generate mark-to-market losses that might force an arbitrageur facing constraints to unwind a position at a loss prior to convergence. In this paper, we distinguish between the general question of whether arbitrage mispricing exists and the specific question of whether a particular hedge fund could profitably implement the arbitrage strategy. We focus on the first since it depends only on market prices, and abstract from the second since it depends entirely on the idiosyncratic set of constraints faced by the arbitrageur. We observe, however, that many hedge funds and institutional asset managers have in fact implemented trading strategies that exploit the divergence between the prices of TIPS, Treasuries, and inflation swaps.

The primary objective of this paper, however, is not just to document a major violation of the law of one price in the financial markets. Rather, our goal is to also shed light on two fundamental issues in asset pricing. First, why is the mispricing there in the first place, and what accounts for its size and sign? Second, why does mispricing persist?

Turning to the first issue, previous papers argue that investors value the liquidity and safety of U.S. Treasury bonds and are willing to forgo returns as a result, likening these bonds to money (see, for example, Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2010a), and Bansal, Coleman, and Lundblad (2010)). These special attributes drive down the yield on Treasury bonds relative to other similar securities not issued by the Treasury, especially when the Treasury securities are in short supply. Krishnamurthy and Vissing-Jorgensen (2010a) refer to this yield spread between Treasuries and similar non-Treasury securities as

a Treasury convenience yield. Our findings suggest that only nominal securities issued by the Treasury are perceived to have these attributes, not the inflation-indexed ones. This could help explain why nominal Treasury bonds are consistently expensive relative to inflation-indexed securities issued by the Treasury, and why this differential increases during times of financial distress when demand for these attributes increases.

Turning next to the second issue of the persistent nature of mispricing, important recent theoretical work by Gromb and Vayanos (2002), Duffie (2010), Ashcraft, Gârleanu, and Pedersen (2010), Brunnermeier and Pedersen (2009), and others stresses that slow-moving capital may play a key role in propagating mispricing in financial markets. Motivated by this work, we explore the implications of the slow-moving-capital hypothesis by studying the relation between changes in TIPS-Treasury mispricing and changes in capital available to arbitrageurs. The results provide direct evidence that the mispricing narrows as additional hedge fund capital flows into the market. This novel result provides strong support for the slow-moving capital explanation of the persistence of arbitrage mispricing in the market.

Another implication of the slow-moving-capital literature is that these types of frictions may induce correlations across different types of arbitrages. To see the intuition behind this implication, imagine that there was a large downward shock in the aggregate amount of capital available to arbitrageurs in the market. As a result, we might observe the amount of mispricing between securities widening in multiple markets simultaneously. To investigate this correlated arbitrage implication, we regress changes in TIPS–Treasury mispricing on changes in the corporate bond/CDS arbitrage described by Duffie (2010), the CDX index/component arbitrage, the on-the-run/off-the-run spread (Krishnamurthy (2002)), and

the Refcorp–Treasury spread (Longstaff (2004)). Although these mispricings occur in very different markets, we find that there is strong commonality across these mispricings, consistent with the theory.

An additional implication of the slow-moving-capital literature is that changes in capital may have forecasting power for subsequent changes in mispricing. Specifically, if capital flows slowly to arbitrageurs, then an increase in capital today will tend to reduce mispricing in the market, but only with a lag. Thus, we could predict future changes in mispricing conditional on current changes in aggregate investor wealth. To explore this implication, we regress changes in TIPS–Treasury mispricing on ex-ante measures of changes in aggregate investor wealth such as stock, bond, and hedge fund returns. Consistent with theory, we find that changes in mispricing are strongly forecastable and are negatively related to these ex-ante returns. Finally, we also find that TIPS–Treasury mispricing is affected by funding liquidity factors such as the availability of Treasury collateral in the primary dealer repo market.

The results in this paper also have public finance implications. While there may be legitimate reasons for why the Treasury chooses to issue TIPS, our results imply that the Treasury faces some costly tradeoffs in doing so. In particular, our results suggest that the Treasury could have saved billions of dollars by issuing nominal bonds instead of TIPS over the past decade. On average, the U.S. government has to levy $2.92 more in taxes, in present discounted value, to repay $100 of debt issued if the debt is indexed rather than nominal. Furthermore, nominal debt allows for state contingency in real returns by creating inflation. In response to an adverse fiscal shock, the government can exploit this state contingency to smooth taxes

either through surprise inflation or the announcement of inflation at some point in the future before the current nominal debt matures. In contrast, indexed debt does not allow for this type of state contingency. Thus, by issuing TIPS, the government clearly gives up a valuable fiscal hedging option.

Finally, our findings of persistent arbitrage mispricing in these markets also imply that the Treasury–TIPS price differentials cannot be used to back out the market's inflation expectations, a common practice. In fact, the implied measure is biased downwards, and the bias worsens in times of increased volatility in financial markets.

This paper contributes to the literature on the pricing of inflation-linked bonds. Other important papers on real bonds include Roll (1996, 2004), Barr and Campbell (1997), Evans (2003), Seppälä (2004), Bardong and Lehnert (2004), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007, 2008), Campbell, Shiller, and Viceira (2009), Dudley, Roush, and Steinberg Ezer (2009), Fleming and Krishnan (2009), Adrian and Wu (2009), Barnes et al. (2009), Gürkaynak, Sack, and Wright (2010), Christensen, Lopez, and Rudebusch (2010a, 2010b), Andonov, Bardong, and Lehnert (2010), Pflueger and Viceira (2011a, 2011b), and many others. This paper differs from the previous literature by being the first to formally study the no-arbitrage relation between TIPS and Treasury bonds and explore the determinants of the mispricing.[5]

The remainder of this paper is organized as follows. Section 1.1 provides a brief introduction to the TIPS and inflation swap markets and describes the TIPS–Treasury arbitrage strategy. Section 1.2 describes the data. Section 1.3 examines the size of the TIPS–Treasury

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[5]Our key findings have also been confirmed in subsequent studies. For example, see Fleckenstein (2012) and Haubrich, Pennacchi, and Ritchken (2012).

mispricing. Section 1.4 discusses the risks that an arbitrageur might face in implementing the strategy. Section 1.5 examines whether these results are simply an artifact of mispricing in the inflation swap market. Section 1.6 considers additional factors that might drive a wedge between the pricing of TIPS and Treasury bonds. Section 1.7 explores the determinants of TIPS–Treasury mispricing. Section 1.8 examines the relation between TIPS–Treasury mispricing and other types of arbitrage mispricing. Section 1.9 investigates the forecastability of TIPS–Treasury mispricing. Section 1.10 summarizes the results and presents concluding remarks.

1.2 TIPS–Treasury Arbitrage

In this section, we provide brief introductions to the TIPS and inflation swap markets. We then describe the arbitrage strategy that links the theoretical prices of Treasury bonds, TIPS, and inflation swaps.

1.2.1 The TIPS Market

TIPS are direct obligations of the U.S. Treasury and are similar in most respects to Treasury bonds.[6] The key difference is that the principal amount of a TIPS issue is adjusted over time to reflect changes in the CPI. Since the fixed coupon rate for the TIPS issue is applied to its principal amount, the actual semiannual coupon received varies over time as the principal amount changes in response to the realized inflation or deflation rate. Similarly, the

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[6]For expositional convenience, we generally refer to all nominal debt obligations of the Treasury (including Treasury bills and Treasury notes) simply as Treasury bonds throughout the paper.

final principal amount paid to the bondholder equals the maximum of the original principal amount or the inflation-adjusted principal amount. Thus, TIPS investors' principal is protected against deflation (although the same is not true for coupon payments).

The principal amount of a TIPS issue is adjusted daily based on the CPI for All Urban Consumers, known as CPI-U. Let I_t denote the inflation adjustment for a TIPS issue as of date t. The inflation adjustment is computed as the ratio of the reference CPI at the valuation date t divided by the reference CPI at the issuance date, which we designate as time 0. The reference CPI for a particular date during a month is linearly interpolated from the CPI reference index for the beginning of that month and the CPI reference index for the beginning of the subsequent month. The CPI reference index for the first day of any calendar month is the CPI-U index for the third preceding calendar month. Thus, the reference CPI for April 1 would be the CPI-U index for the month of January, which is reported by the Bureau of Labor Statistics during February. Details on how TIPS are adjusted for inflation are described on the U.S. Treasury's website.[7]

The total principal amount of all TIPS outstanding at the end of the sample period is in excess of $550 billion. The Treasury first began auctioning TIPS in January 1997. As of the end of our sample period, 34 separate TIPS issues have been auctioned. Currently, the Treasury issues five-year, 10-year, and 30-year TIPS on a regular cycle.

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[7]See http://www.treasurydirect.gov/instit/statreg/auctreg/auctreggsr31cfr356.pdf.

1.2.2 The Inflation Swap Market

Beginning with the first TIPS auction in 1997, market participants began making markets in inflation swaps as a way of hedging inflation risk. As the TIPS market has grown, the inflation swap market has become liquid and actively traded, particularly in the U.S. and the U.K.[8] Inflation swaps have also become widely used among institutional investment managers because of their high correlation with realized CPI.[9] The notional size of the inflation swap market is estimated by Pond and Mirani (2011) to be on the order of hundreds of billions. Conversations with inflation swap traders confirm that these instruments are fairly liquid with typical bid-ask spreads on the order of five basis points.[10]

In this paper, we focus on the most basic and widely used type of inflation swap, namely, a zero-coupon swap. This swap is executed between two counterparties at time 0 and has only one cash flow that occurs at the maturity date of the swap. For example, imagine that at time 0, the five-year zero-coupon inflation swap rate is 200 basis points. As is standard with swaps, there are no cash flows at time 0 when the swap is executed. At the maturity date of the swap in five years, the counterparties to the inflation swap exchange a cash flow of $(1+0.0200)^5 I_t$, where I_t is again the inflation adjustment factor. Thus, if the realized inflation rate was 1.50% per year over the five-year horizon of the swap, $I_t = 1.015^5 = 1.077284$. In this case, the net cash flow from the swap would be $(1 + 0.0200)^5 - 1.077284 = 0.026797$ per

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[8]Kerkhof (2005) provides an excellent introduction to the inflation swap market. Also see Jarrow and Yildirim (2003) and Hinnerich (2008). Fleckenstein (2012) extends our analysis to other inflation-linked bond markets including the U.K.

[9]As one example, inflation swaps are a key element of J.P. Morgan's Columbus Fixed Income Inflation Managed Bond Strategy.

[10]This estimate of the bid-ask spread is consistent with Schulz and Stapf (2011), who find that the median bid-ask spreads for seven-year inflation swaps near the height of the 2008 crisis period were on the order of four to seven basis points, with a few values exceeding 10 basis points. Typical values during noncrisis periods would presumably be lower.

dollar notional of the swap. The timing and index lag construction of the index I_t used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

1.2.3 The Arbitrage Strategy

The idea behind the TIPS–Treasury arbitrage strategy is very simple. Imagine that an investor buys a TIPS issue at par that has a coupon rate of s per semiannual period. Because of the inflation adjustment, the coupon paid at time t will be sI_t. Now imagine that the investor executes a zero-coupon inflation swap with a maturity date and notional amount matching that of the coupon payment for the TIPS issue. At date t, the inflation swap pays a cash flow of $s(1 + f)^t - sI_t$, where f is the fixed inflation swap rate. The sum of the two cash flows is now just $sI_t + s(1 + f)^t - sI_t = s(1 + f)^t$, which is a constant. Similarly, by executing zero coupon inflation swaps with maturities and notional amounts matching the indexed cash flows from the TIPS issue, the investor can convert all of these indexed cash flows into fixed cash flows.

To make the mechanics of this arbitrage strategy more clear, Table 1.1 shows the various components of the strategy and their associated cash flows. The first part of the table shows the cash flows associated with a Treasury bond purchased at price P and with a coupon rate of c. The Treasury bond pays a semiannual coupon of c per period, and then makes a principal payment of 100 at maturity date T.

Table 1.1 – Cash Flows from the Treasury Bond and the Synthetic Treasury Bond Replicating Strategy. This table shows the cash flow generated each period from the indicated positions. P denotes the price of the Treasury bond with coupon c, V denotes the price of the TIPS bond with the same maturity date as the Treasury bond and a coupon rate of s, and $D(t)$ denotes the price of a Treasury STRIP with a maturity of t. F_t denotes the fixed payment on a zero-coupon inflation swap of maturity t (calculated as $(1+f)^t$, where f is the corresponding inflation swap rate). The inflation index I_t denotes the ratio of the CPI-U index at time t divided by the CPI-U index at time zero.

Strategy	0	1	2	3	...	T
Buy Treasury	$-P$	c	c	c	...	$c+100$
Buy TIPS	$-V$	sI_1	sI_2	sI_3	...	$(s+100)I_T$
Inflation Swap$_1$	0	$s(F_1-I_1)$	0	0	...	0
Inflation Swap$_2$	0	0	$s(F_2-I_2)$	0	...	0
Inflation Swap$_3$	0	0	0	$s(F_3-I_3)$	...	0
...	...	...	...	...	...	...
Inflation Swap$_T$	0	0	0	0	...	$(s+100)(F_T-I_T)$
STRIPS$_1$	$-(c-sF_1)D(1)$	$c-sF_1$	0	0	...	0
STRIPS$_2$	$-(c-sF_2)D(2)$	0	$c-sF_2$	0	...	0
STRIPS$_3$	$-(c-sF_3)D(3)$	0	0	$c-sF_3$	...	0
...	...	...	...	...	...	...
STRIPS$_T$	$-((c+100)D(T)-$ $(s+100)F_TD(T))$	0	0	0	...	$(c+100)-$ $(s+100)F_T$
Total Cash Flow	$-(\sum_{i=1}^{T}(c-sF_i)\,D(i)+$ $100(1-F_T)D(T)+V)$	c	c	c	...	$c+100$

The second part of the table shows how the cash flows from the Treasury bond can be replicated exactly from a TIPS position. First, the arbitrageur purchases a TIPS issue with a coupon rate of s and the same maturity date as the Treasury bond for a price of V. The TIPS bond pays coupons of sI_t each period, and then makes a principal payment of $100I_T$ at maturity. The arbitrageur then enters into an inflation swap for each coupon payment date with a notional amount of s (or $s + 100$ for the final principal payment date). This converts all of the indexed cash flows from the TIPS into fixed cash flows. To match exactly the cash flows from the Treasury bond, however, the arbitrageur also needs to go long or short a small amount of Treasury STRIPS for each coupon payment date. As shown at the bottom of the second part of the table, the net result is a portfolio that exactly replicates the cash flows from the Treasury bond in the first part of the table.[11]

To provide a specific example, Table 1.2 shows the actual cash flows that would result from applying the arbitrage strategy on December 30, 2008 to replicate the 7.625% coupon Treasury bond maturing on February 15, 2025. As shown, the price of the Treasury bond is $169.479. To replicate the Treasury bond's cash flows, the arbitrageur buys a 2.375% coupon TIPS issue with the same maturity date for a price of $101.225. Since there are 33 semiannual coupon payment dates, 33 inflation swaps are executed with the indicated notional amounts. Finally, positions in Treasury STRIPS of varying small notional amounts are also taken by the arbitrageur. The net cash flows from the replicating strategy exactly

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[11]There are alternative ways in which some parts of the arbitrage strategy could be implemented. For example, an investor could enter into an asset swap as an alternative to taking a position in a Treasury bond or TIPS issue directly. Asset swaps can be viewed as equivalent to taking a long position in the asset and financing the transaction at Libor plus a spread. Market participants often refer to the difference between Treasury and TIPS asset swap spreads as the breakeven inflation spread and contrast it with the inflation swap spread.

match those from the Treasury bond, but at a cost of only $146.379. Thus, the cash flows from the Treasury bond can be replicated at a cost that is $23.10 less than that of the Treasury bond.

To evaluate whether the arbitrage would be profitable after considering transaction costs, we obtain estimates of the bid-ask spreads for the various elements of the strategy. Fleming (2003) shows that the average cost of trading a 10-year Treasury bond is on the order of 0.78 ticks (32nds) and is rarely more than 1.5 ticks. He also shows that the cost is lower for shorter maturity Treasury notes and bonds. Fleming and Krishnan (2009) estimate that the bid-ask spreads for a five-year, 10- year, and 20-year TIPS issues are 2.6, 2.7, and 7.3 ticks, respectively. Daves and Ehrhardt (1993) estimate the average bid-ask spread for Treasury STRIPS at about three ticks. This is consistent with Grinblatt and Longstaff (2000), who provide estimates ranging from one to four ticks. Similar estimates are given in Jordan, Jorgensen, and Kuipers (2000). Finally, as described above, the average bid-ask spread for inflation swaps is estimated to be five basis points in terms of yields. We have also confirmed these estimates of transaction costs through discussions with a number of Treasury bond traders.

To provide specific estimates of the cost of implementing an arbitrage strategy similar to that shown in Table 1.2, we do the following. First, we assume that the bid-ask spread for Treasury bonds, TIPS, and STRIPS, are two, six, and four ticks, respectively. In addition, we assume that the bid-ask spread for inflation swaps is six basis points. These values are clearly very conservative estimates (overestimates) of the actual transaction costs. Second, we apply these estimates to the strategy shown in Table 1.2. To provide additional perspective, we also

Table 1.2 – **A Specific Example of the Synthetic Treasury Bond Replicating Strategy**. This table shows the cash flows associated with the 7.625% Treasury bond with maturity date February 15, 2025 and the cash flows from the replicating strategy using the 2.375% TIPS issue with the same maturity date that replicates the cash flows of the Treasury bond. The example is based on market prices for December 30, 2008. Cash flows are in dollars per \$100 notional. I_t denotes the realized percentage change in the CPI Index from the inception of the strategy to the cash flow date. Date refers to the number of the semiannual period in which the corresponding cash flows are paid.

Date	Treasury	TIPS	Inflation Swaps	STRIPS	Total
0	-169.4793	-101.2249	0	-45.6367	-146.3786
1	3.8125	$1.1875\,I_1$	$1.1856 - 1.1875\,I_1$	2.6269	3.8125
2	3.8125	$1.1875\,I_2$	$1.1638 - 1.1875\,I_2$	2.6487	3.8125
3	3.8125	$1.1875\,I_3$	$1.1480 - 1.1875\,I_3$	2.6645	3.8125
4	3.8125	$1.1875\,I_4$	$1.1467 - 1.1875\,I_4$	2.6658	3.8125
5	3.8125	$1.1875\,I_5$	$1.1307 - 1.1875\,I_5$	2.6818	3.8125
6	3.8125	$1.1875\,I_6$	$1.1376 - 1.1875\,I_6$	2.6749	3.8125
7	3.8125	$1.1875\,I_7$	$1.1566 - 1.1875\,I_7$	2.6559	3.8125
8	3.8125	$1.1875\,I_8$	$1.1616 - 1.1875\,I_8$	2.6509	3.8125
9	3.8125	$1.1875\,I_9$	$1.1630 - 1.1875\,I_9$	2.6495	3.8125
10	3.8125	$1.1875\,I_{10}$	$1.1773 - 1.1875\,I_{10}$	2.6352	3.8125
11	3.8125	$1.1875\,I_{11}$	$1.1967 - 1.1875\,I_{11}$	2.6158	3.8125
12	3.8125	$1.1875\,I_{12}$	$1.2095 - 1.1875\,I_{12}$	2.6030	3.8125
13	3.8125	$1.1875\,I_{13}$	$1.2248 - 1.1875\,I_{13}$	2.5877	3.8125
14	3.8125	$1.1875\,I_{14}$	$1.2466 - 1.1875\,I_{14}$	2.5659	3.8125
15	3.8125	$1.1875\,I_{15}$	$1.2683 - 1.1875\,I_{15}$	2.5442	3.8125
16	3.8125	$1.1875\,I_{16}$	$1.2866 - 1.1875\,I_{16}$	2.5259	3.8125
17	3.8125	$1.1875\,I_{17}$	$1.3058 - 1.1875\,I_{17}$	2.5067	3.8125
18	3.8125	$1.1875\,I_{18}$	$1.3304 - 1.1875\,I_{18}$	2.4821	3.8125
19	3.8125	$1.1875\,I_{19}$	$1.3556 - 1.1875\,I_{19}$	2.4569	3.8125
20	3.8125	$1.1875\,I_{20}$	$1.3792 - 1.1875\,I_{20}$	2.4333	3.8125
21	3.8125	$1.1875\,I_{21}$	$1.4009 - 1.1875\,I_{21}$	2.4116	3.8125
22	3.8125	$1.1875\,I_{22}$	$1.4225 - 1.1875\,I_{22}$	2.3900	3.8125
23	3.8125	$1.1875\,I_{23}$	$1.4427 - 1.1875\,I_{23}$	2.3698	3.8125
24	3.8125	$1.1875\,I_{24}$	$1.4635 - 1.1875\,I_{24}$	2.3490	3.8125
25	3.8125	$1.1875\,I_{25}$	$1.4806 - 1.1875\,I_{25}$	2.3319	3.8125
26	3.8125	$1.1875\,I_{26}$	$1.4979 - 1.1875\,I_{26}$	2.3146	3.8125
27	3.8125	$1.1875\,I_{27}$	$1.5126 - 1.1875\,I_{27}$	2.2999	3.8125
28	3.8125	$1.1875\,I_{28}$	$1.5277 - 1.1875\,I_{28}$	2.2848	3.8125
29	3.8125	$1.1875\,I_{29}$	$1.5407 - 1.1875\,I_{29}$	2.2718	3.8125
30	3.8125	$1.1875\,I_{30}$	$1.5548 - 1.1875\,I_{30}$	2.2577	3.8125
31	3.8125	$1.1875\,I_{31}$	$1.5676 - 1.1875\,I_{31}$	2.2449	3.8125
32	3.8125	$1.1875\,I_{32}$	$1.5823 - 1.1875\,I_{32}$	2.2302	3.8125
33	103.8125	$101.1875\,I_{33}$	$135.9861 - 101.1875\,I_{33}$	-32.1736	103.8125

compute the transaction costs for two-year, five-year, and 10-year versions of the strategy in which we hold the cash flows fixed, but vary the assumed maturity date of the strategy. The estimated transaction costs for the two-year, five-year, 10-year, and actual strategies are approximately 20.2, 29.5, 46.3, and 69.1 cents per $100 notional amount, respectively. These transactions costs are clearly orders of magnitude smaller than the arbitrage. Thus, transaction costs cannot begin to account for mispricing of this magnitude.

1.3   The Data

The data for the study consist of daily closing prices for U.S. Treasury bonds, TIPS, STRIPS, and inflation swaps for the period from July 23, 2004 to November 19, 2009. All data are obtained from the Bloomberg system. The TIPS and Treasury pairs in the data set have maturities ranging from 2007 to 2032. Daily closing prices for TIPS and Treasury bonds are adjusted for accrued interest following standard market conventions.

Inflation swaps are quoted in terms of the constant rate on the contract's fixed leg. The traded maturities are one, two, three, four, five, six, seven, eight, nine, ten, 12, 15, 20, 25, and 30 years. To obtain swap rates for intermediate maturities, we use cubic spline interpolation. For maturities that include fractional years (e.g., 2.3 years), seasonal patterns in inflation must be taken into account. To do this, we first estimate seasonal weightings for the CPI-U for each month of the year by regressing the CPI-U index values for the January 1980 to October 2009 period on monthly indicator variables. The estimated weights are normalized to ensure than there is no seasonal effect for full-year swaps and then used to adjust the interpolated inflation swap curve (seasonal adjustments are not used for maturities

less than one year). Details about the algorithm used to compute synthetic Treasury bond prices are provided in the Appendix.

For our analysis, we match TIPS and Treasury bonds based on their respective maturities. We define maturity mismatch as the number of days between the maturity of a TIPS issue and the maturity of a Treasury bond with the closest maturity to that of the TIPS issue. We only include pairs of TIPS and Treasury bonds in the sample if the maturity mismatch is less than or equal to 31 days. This leads to a total of 29 TIPS–Treasury bond pairs.[12] In particular, there are seven exact matches, nine mismatches of 15 days, and 13 mismatches of 31 days. The 31-day mismatches occur only for maturities of February 2015 or later. Thus, these mismatches represent a very small percentage mismatch in the maturities of the TIPS and Treasury bonds. To adjust for the maturity mismatches, we calculate the yield to maturity on the synthetic fixed-rate bond formed from the TIPS issue and the inflation swaps, and then apply this yield to calculate the price of a synthetic bond that would exactly match the maturity of the Treasury bond in the pair.

1.4   How Large is the Mispricing?

Table 1.3 provides summary statistics for the mispricing for each of the 29 pairs of TIPS and Treasury bonds in the sample. The first two columns show the maturity date and coupon rate for the TIPS issue in each pair. The next two columns show the maturity date and coupon rate for the Treasury bond in each pair. The column labeled Mismatch in Days denotes the

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[12]Specifically, the Treasury issued 34 TIPS bonds prior to the end of the sample period. One of these issues had matured by the beginning of the sample period. Four issues had maturity mismatches in excess of 31 days.

Table 1.3 – Summary Statistics for TIPS-Treasury Mispricing. This table reports summary statistics for TIPS-Treasury mispricing for the 29 pairs of TIPS and Treasury bonds shown. Mismatch in Days denotes the maturity mismatch of the pair. The left central panel reports summary statistics for the mispricing measured in dollars per $100 notional. The right central panel reports summary statistics for the mispricing measured in basis points. The sample period is from July 23, 2004 to November 19, 2009.

TIPS		Treasury		Mismatch in Days	Dollar Mispricing					Basis-Point Mispricing					N
					Mean	SDev	Min	Max	ρ	Mean	SDev	Min	Max	ρ	
Jan-15-07	3.375	Dec-31-06	3.000	15	0.18	0.39	−0.76	1.10	0.97	34.57	92.03	−255.56	357.23	0.98	506
Jan-15-08	3.625	Dec-31-07	4.375	15	0.34	0.34	−0.25	1.26	0.96	53.82	66.57	−80.99	270.41	0.96	502
Jan-15-09	3.875	Jan-15-09	3.250	0	0.67	0.46	−0.34	2.56	0.95	72.54	135.34	−25.55	723.29	0.98	1109
Jan-15-10	4.250	Jan-15-10	3.625	0	0.85	0.59	−1.05	4.69	0.91	55.14	71.91	−64.47	420.39	0.97	1215
Apr-15-10	0.875	Apr-15-10	4.000	0	1.09	0.65	−1.18	4.51	0.93	58.25	57.84	−69.20	316.69	0.96	1161
Jan-15-11	3.500	Jan-15-11	4.250	0	1.32	0.71	−0.03	4.94	0.92	50.24	33.67	−1.07	231.07	0.94	971
Apr-15-11	2.375	Mar-31-11	4.750	15	1.67	0.70	−0.37	5.03	0.91	56.13	33.04	−15.24	213.25	0.94	736
Jan-15-12	3.375	Jan-15-12	1.125	0	1.84	0.75	0.79	4.64	0.96	72.32	24.20	31.10	163.04	0.95	215
Apr-15-12	2.000	Apr-15-12	1.375	0	1.42	0.41	0.62	2.32	0.91	54.11	14.90	21.83	90.97	0.90	154
Jul-15-12	3.000	Jul-15-12	1.500	0	1.66	0.37	0.94	2.89	0.86	60.25	12.44	35.72	104.19	0.83	91
Apr-15-13	0.625	Mar-31-13	2.500	15	2.19	1.18	−1.07	6.37	0.95	55.44	28.02	−24.54	156.69	0.95	395
Jul-15-13	1.875	Jun-30-13	3.375	15	4.02	1.83	1.77	9.36	0.98	96.27	39.99	49.04	212.92	0.97	353
Jan-15-14	2.000	Dec-31-13	1.500	15	4.38	1.50	2.30	7.86	0.98	103.66	30.32	59.34	173.67	0.97	225
Apr-15-14	1.250	Mar-31-14	1.750	15	1.76	0.30	1.07	2.58	0.85	41.24	6.97	23.77	56.82	0.85	143
Jul-15-14	2.000	Jun-30-14	2.625	15	3.01	0.48	2.04	4.04	0.95	67.20	9.76	46.45	88.47	0.93	101
Jan-15-15	1.625	Feb-15-15	4.000	31	3.36	2.04	1.22	12.52	0.99	55.48	37.53	15.62	214.11	0.99	1204
Jul-15-15	1.875	Aug-15-15	4.250	31	3.61	2.18	1.54	13.24	0.99	56.39	36.45	22.68	207.57	0.99	1079
Jan-15-16	2.000	Feb-15-16	4.500	31	4.01	2.29	1.63	13.14	0.99	59.66	35.41	22.46	206.56	0.99	950
Jul-15-16	2.500	Jun-30-16	3.250	15	3.76	0.59	2.46	4.99	0.98	62.34	9.63	40.75	82.58	0.98	101
Jan-15-17	2.375	Feb-15-17	4.625	31	4.27	2.35	1.51	12.56	0.98	58.22	31.97	18.92	166.06	0.98	698
Jul-15-17	2.625	Aug-15-17	4.750	31	4.43	2.34	1.70	11.20	0.97	57.29	29.83	20.51	143.82	0.97	573
Jan-15-18	1.625	Feb-25-18	3.500	31	5.00	2.51	2.13	12.05	0.98	65.33	31.57	26.99	147.04	0.97	446
Jul-15-18	1.375	Aug-15-18	4.000	31	5.38	2.62	1.78	12.31	0.98	65.78	29.84	21.72	137.22	0.97	320
Jan-15-19	2.125	Feb-15-19	2.750	31	5.32	2.08	2.56	10.14	0.99	68.36	24.60	33.66	123.37	0.99	194
Jul-15-19	1.875	Aug-15-19	3.625	31	3.94	0.78	2.40	5.09	0.99	47.98	9.44	29.05	62.51	0.99	68
Jan-15-25	2.375	Feb-15-25	7.625	31	4.27	3.57	−0.89	23.06	0.98	29.40	23.45	−5.51	138.97	0.98	1342
Jan-15-26	2.000	Feb-15-26	6.000	31	4.90	3.16	−0.06	18.49	0.97	36.85	21.96	−0.50	118.59	0.96	961
Jan-15-27	2.375	Feb-15-27	6.625	31	5.30	3.46	0.54	18.53	0.97	36.42	22.03	3.70	108.12	0.96	709
Jan-15-29	2.500	Feb-15-29	5.250	31	6.84	3.49	1.68	15.22	0.98	48.43	23.69	12.22	103.74	0.98	205

maturity mismatch between the two bonds. The central panel of the table reports summary statistics for the mispricing. The rightmost panel of the table reports summary statistics for the mispricing measured as the basis point difference between the yield of the synthetic Treasury bond and the actual Treasury bond for each pair.

The mispricing reported in Table 1.3 is stunning in magnitude and is likely the largest ever documented in any fixed-income market.[13] For example, the mispricing for many of the

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[13]Examples of fixed-income arbitrage mispricing reported in the literature include Cornell and Shapiro (1990), Amihud and Mendelson (1991), Boudoukh and Whitelaw (1991), Longstaff (1992, 2004), Daves and Ehrhard (1993), Kamara (1994), Jordan, Jorgensen, and Kuipers (2000), Grinblatt and Longstaff (2000), Longstaff, Santa Clara, and Schwartz (2001), Yu (2006), Duarte, Longstaff, and Yu (2007), and many others.

TIPS–Treasury pairs with maturities of 2015 or later reach values in excess of $10. In fact, the mispricing for the TIPS–Treasury pair maturing in 2025 reaches a level in excess of $23. What makes these findings even more dramatic is that the TIPS and Treasury markets are two of the largest and most liquid financial markets in the world. In almost every case, the value of the Treasury bond is larger than its synthetic equivalent constructed from the matching TIPS issue and the inflation swap. Thus, Treasury bonds appear to be almost uniformly "rich" relative to the portfolios of Treasury securities that replicate their cash flows.

The average sizes of the mispricing shown in Table 1.3 are equally astonishing. For example, the average size of the mispricing between the TIPS and Treasury bondsmaturing in January 2029 and February 2029, respectively, is $6.84. Similarly, the average basis-point size of the mispricing between the TIPS and Treasury bonds maturing in January 2014 and December 2013, respectively, is 103.66 basis points. We note that the average basis-point size of the mispricing is fairly uniform across all maturities. Thus, there does not appear to be any relation between the maturity of the TIPS–Treasury bond pair and the average size of the mispricing.

To illustrate the average size of the TIPS–Treasury mispricing, we compute the TIPS notional-weighted mispricing for each date during the sample period, where the average is taken over all TIPS–Treasury pairs in the sample on that date. Figure 1.1 plots the weighted-average dollar mispricing for the TIPS–Treasury pairs. Figure 1.2 plots the corresponding weighted-average basis-point mispricing for these pairs. As can be seen, the mispricing is evident throughout the entire sample period, not just during the crisis period
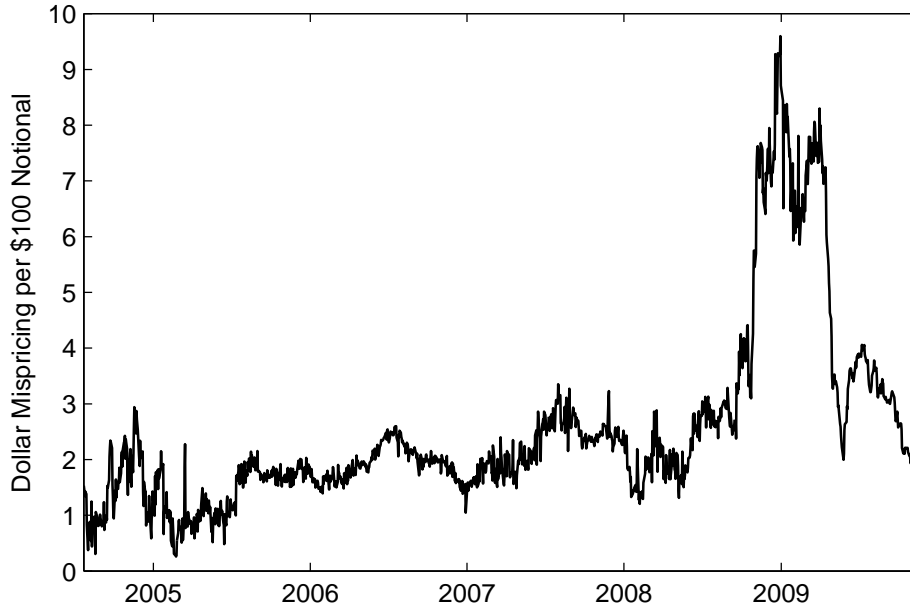
Figure 1.1 – **TIPS-Treasury mispricing**. This figure plots the time series of the weighted-average TIPS-Treasury mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample, where the average is weighted by the notional amount of the TIPS issue.

from 2008 to 2009. In particular, while the amount of mispricing peaked at $9.60 or 175 basis points around the time of the Lehman bankruptcy in the Fall of 2008, there were clearly earlier periods when the average mispricing was in excess of $3 or about 60 basis points. In addition, Figures 1.1 and 1.2 show that there is significant time-series variation in TIPS–Treasury mispricing throughout the sample period. The overall average size of the mispricing is $2.92. The overall average basis-point size of the mispricing is 54.5 basis points.

We note that there are a few cases of negative mispricing. However, these represent only 2.56% of the total observations. We investigated these cases and found that the vast majority were associated with the first four pairs of bonds in Table 1.3. The negative mispricings were fairly evenly distributed throughout the sample period rather than clustered in time. Furthermore, there appeared to be relatively little correlation in the incidence of negative

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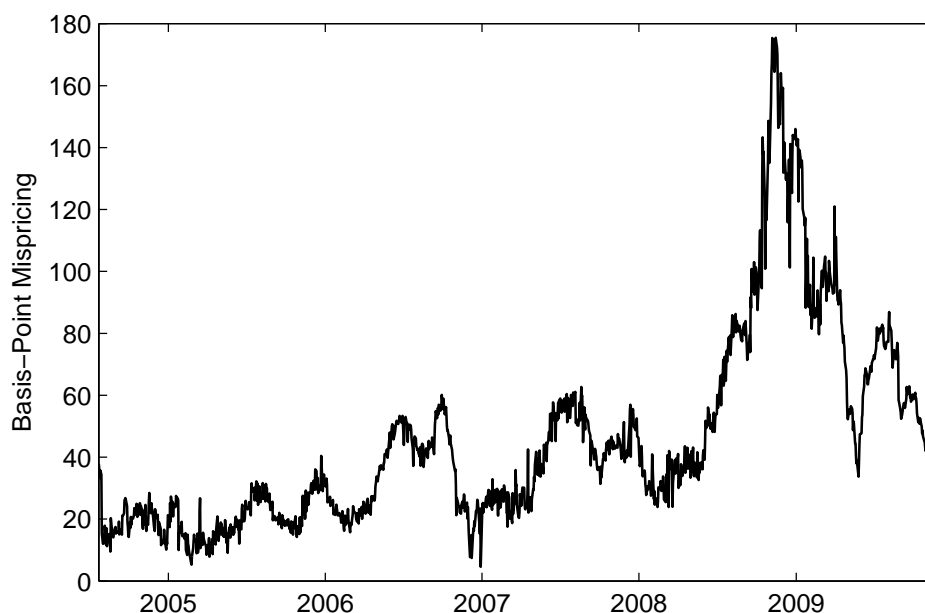
Figure 1.2 – **Weighted average TIPS-Treasury mispricing in basis points**. This figure plots the time series of the average TIPS-Treasury mispricing, measured in basis points, across the pairs included in the sample, where the average is weighted by the notional amount of the TIPS issue.

mispricing across bonds; it was rare to have more than one case of negative mispricing at a time. We checked the data carefully to make sure that the negative mispricing was not due to errors or outliers. Since the first four pairs of bonds involve TIPS with the highest coupons, the negative mispricings could potentially reflect an investor preference for short-term high-coupon TIPS issues.[14]

It is important to acknowledge that practitioners have long recognized that breakeven inflation spreads appear mispriced relative to inflation swaps.[15] These discussions, however, have generally attributed the discrepancy to some form of risk premium. An important implication of our findings is that the discrepancy cannot be due to a risk premium (defined in the rigorous theoretical sense as a pricing effect arising from the interaction of a security's cash

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[14]We are grateful to the referee for suggesting this analysis.

[15]For example, see the discussion in United States Governmental Accountability Office (2009) and Pond and Mirani (2011).

flows with a pricing kernel) since we show that TIPS–Treasury mispricing is a violation of the law of one price and therefore cannot be reconciled with an equilibrium model of asset pricing.[16]

On the other hand, it is not uncommon to see deviations from the law of one price, which we define formally as mispricing, described using alternative terminology such as liquidity effects, liquidity risk premia, arbitrage risk premia, etc. For example, recent papers by D'Amico, Kim, and Wei (2010), Christensen and Gillan (2011a, 2011b, 2011c), and Haubrich, Pennachi, and Ritchken (2012) use the term liquidity risk premia to characterize the component of TIPS prices that cannot be explained within the context of a formal asset pricing model. Thus, the difference between what we term mispricing and what these papers call a "liquidity risk premium" is simply a semantic one, and there is no fundamental conflict between their results and ours.[17]

As discussed earlier, the total notional amount of TIPS outstanding has increased significantly over time. In particular, the total amount of TIPS outstanding at the beginning of the sample period in July 2004 was $222.60 billion, but increased to $567.51 billion by the end of the sample period in November 2009. At the end of the sample period, TIPS accounted for 7.91% of the total notional value of marketable U.S. Treasury debt.

From the Treasury's perspective, TIPS–Treasury mispricing represents a potential opportunity for reducing Treasury debt. For example, if Treasury bonds have a higher market

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[16]Haubrich, Pennachi, and Ritchken (2012) provide an excellent example of an equilibrium model of Treasury and TIPS pricing in which term premia as well as inflation risk premia are explicitly defined.

[17]We are grateful to the referee for pointing out this distinction. We note that there are formal asset pricing models in which liquidity risk premia arise through the interaction between the timing of cash flows and a pricing kernel. As one example, see Longstaff (2009). These types of liquidity risk premia, however, are fundamentally different from those in the papers cited above.

valuation than the equivalent inflation-swapped TIPS issues, then the Treasury could potentially generate significant savings by buying back all the outstanding TIPS issues, issuing Treasury bonds with the same maturity, and hedging out the inflation risk in the inflation swap market. The evidence in Han, Longstaff, and Merrill (2007) suggests that the Treasury is able to buy back large quantities of its debt with only minor market impact costs. To evaluate the potential savings from this type of a debt exchange, we multiply the TIPS–Treasury mispricing by the notional amount of TIPS outstanding and total this value over all pairs of bonds available during the sample period (including the four with maturity mismatches in excess of 31 days).

The total savings from the debt exchange follows a pattern similar to that in Figures 1.1 and 1.2. The total increases secularly over the sample because of the increase in the issuance of TIPS. Moreover, it spikes towards the end of 2008 in the wake of the global financial crisis and reaches a peak of $56.4 billion on December 30, 2008. By the end of the sample period, the total savings is $11.2 billion.

Another perspective on this issue is given by computing the cost to the Treasury of issuing TIPS rather than Treasury bonds. This is perhaps a more realistic measure of the costs incurred, because the Treasury could clearly have simply issued Treasury bonds rather than TIPS. Figure 1.3 plots the cumulative total cost to the Treasury of the 27 TIPS issuances during the sample period. The total cost of new issuances during the sample period is $9.6 billion.[18] On January 30, 2009, the Treasury issued $14.01 billion of 20-year TIPS at a cost of $12.00 per $100 notional. This issuance alone cost the Treasury $1.68 billion. Clearly,

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[18]This number does not include the 0.875% TIPS issue with maturity April 15, 2010 issued on October 29, 2004 because there is not a good match with a Treasury bond for the first part of the sample period.
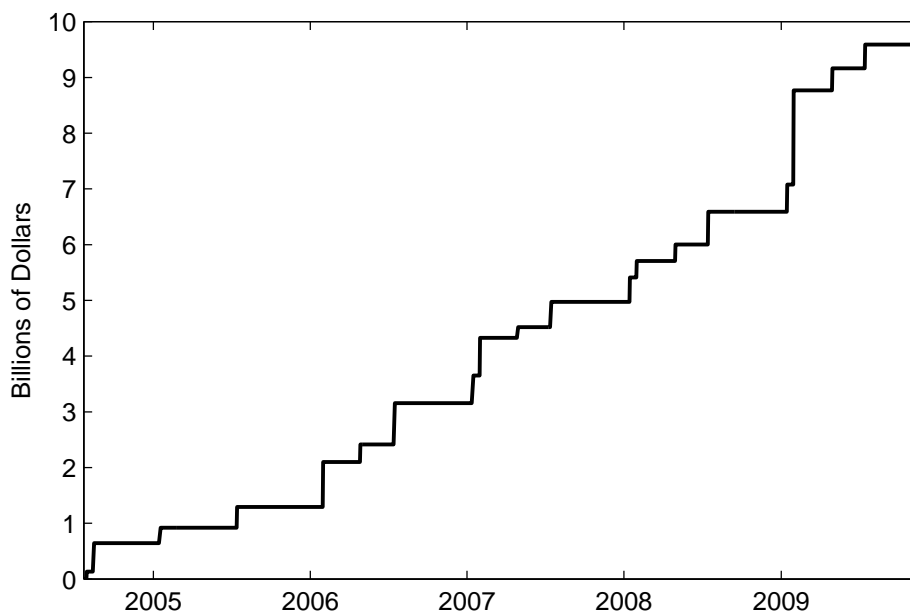
Figure 1.3 – **Cumulative total cost to the Treasury from issuing TIPS rather than Treasury bonds**. This figure plots the cumulative total cost to the Treasury (measured in billions of dollars) of issuing TIPS rather than Treasury bonds measured across all TIPS auctions during the sample period.

issuing TIPS during periods of increased volatility in the financial markets and flights to nominal Treasury bonds implies that large new TIPS issuance can be very costly from the taxpayers' vantage point.

1.5   Is the Arbitrage Truly Riskless?

The answer to this question is that it depends on the investor. As shown earlier, the arbitrage strategy is in fact an arbitrage in the textbook sense. As is well known, however, even a textbook arbitrage can be a risky venture for an arbitrageur facing constraints. For example, Shleifer and Vishny (1997), Liu and Longstaff (2005), and others show that an arbitrageur subject to margin constraints could suffer mark-tomarket losses and be forced to liquidate a position in a textbook arbitrage at a loss prior to the date of convergence. Thus, an arbitrage

could essentially be riskless from the perspective of a relatively unconstrained arbitrageur such as a sovereign wealth fund, yet risky from the perspective of a highly leveraged and constrained hedge fund.

Could a hedge fund successfully implement the TIPS–Treasury arbitrage strategy described in this paper? Many hedge funds have already done so. As one example, we quote from recent Financial Times blogs by Kaminska (2010) and Jones and Kaminska (2010) about Barnegat Fund Management:[19]

> But as Barnegat explain: ''We will buy the TIPS, short the nominal
>
> bond, and lock in the inflation rate with the inflation swap. The
>
> result is that the net initial payment is zero, but until 2014 this
>
> trade yields up to 2.5 percent per year of the notional.''
>
> For a small group of savvy traders the pricing discrepancies at their
>
> widest led to one of the most successful hedge fund trades in recent
>
> memory. One of the biggest beneficiaries was the low-profile New
>
> Jersey-based $450 million Barnegat fund founded in 1999. Barnegat
>
> acquired TIPS bonds shortly after the collapse of Lehman Brothers
>
> and then shortedbet on a fall in ratesregular Treasury bonds of an
>
> equivalent maturity. As the pricing discrepancy narrowed, the fund
>
> realised huge gains. The fund returned 132.6% to investors in 2009.

We have also had numerous discussions with traders, researchers, and portfolio managers at a variety of hedge funds and investment management firms confirming that similar strategies

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[19]See Izabella Kaminska, 2010, Who played the largest ever arbitrage? http://ftalphaville.ft.com/blog/tag/barnegat-fund-management.html and Sam Jones and Izabella Kaminska, 2010, Bond strategy led to big win after Lehman, http://www.ft.com/intl/cms/s/0/a9832c1e-c109-11df-99c4-00144feab49a,s01=1.html.

are widely used in practice.

On the other hand, could every hedge fund make money following the TIPS–Treasury arbitrage strategy described in this paper? Probably not. The reason is that some arbitrageurs face constraints that limit their ability to fully realize arbitrage profits from violations of the law of one price.[20] Examples of these types of limits to arbitrage include the costs and funding risks of financing securities positions in the repo markets, as well as the regulatory, mark-to-market, and capital costs of keeping Treasury security positions on the balance sheet. As discussed earlier, our approach in this paper is to focus primarily on the broad implications of TIPS–Treasury mispricing, while abstracting from the narrower issue of the risks that a specific arbitrageur might face in implementing the arbitrage strategy.

As a final note, we observe that there has been a recent increase in market interest in TIPS–Treasury strategies, which are often referred to as breakeven inflation trades. For example, in late 2011, both ProShares Advisors and State Street announced plans to offer ETFs based on long-short positions in TIPS and Treasuries.

1.6   Inflation Swap Mispricing?

We have shown that a simple no-arbitrage argument imposes a strong restriction on the relative prices of Treasury bonds, TIPS, and inflation swaps, and that this restriction is

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[20]One possible example of this might be Morgan Stanley. From a June 29, 2011 Bloomberg article, "The bank's interest-rates trading group lost at least tens of millions of dollars on the trade, which the firm has been unwinding ... Traders at the bank bet that inflation expectations for the next five years would rise in Treasury markets ... Such wagers on so-called breakeven rates involve paired purchases and short sales of Treasuries and Treasury Inflation Protected Securities, or TIPS, in both maturities." See http://www.bloomberg.com/news/2011-06-29/morgan-stanley-said-to-suffer-trading-loss-after-wager-on-u-s-inflation.html.

frequently violated in the market. It is important to observe, however, that since there are three legs to the arbitrage strategy, mispricing in any one of these three could be responsible for the TIPS–Treasury arbitrage. Because inflation swaps are less familiar to many market participants, it is perhaps natural to suspect that distortions in the pricing of inflation swaps may be the underlying explanation for the results.

In this section, we provide conclusive evidence that the mispricing of inflation swaps cannot explain more than a small portion of TIPS–Treasury mispricing. Specifically, we repeat our analysis by applying the same arbitrage strategy to matching corporate fixed-rate and inflation-linked bonds and using the same set of inflation swap prices as before. If inflation swap mispricing were the underlying reason for the TIPS–Treasury results, then we would expect to see the same type of mispricing between corporate fixed-rate and inflation-linked debt since identical inflation swap prices are used in both cases. In actuality, however, we find little or no evidence of systematic mispricing between corporate fixed-rate and inflation-linked debt. Thus, we can definitively rule out that mispricing in the inflation swap market is the source of the TIPS–Treasury mispricing.

1.6.1 Corporate Inflation-Linked Debt Arbitrage

During the past decade, a number of corporations have issued inflation-linked debt (linkers). For the most part, these firms have tended to be in the financial sector. Since many of these firms have fixed-rate debt as well, we can directly apply the arbitrage strategy to compare the price of a fixed-rate corporate bond to that of an inflation-swapped corporate inflation-linked bond with cash flows that exactly replicate those of the fixed-rate bond. Note that in

doing so, we use the same inflation swap prices as we used in calculating the TIPS–Treasury mispricing.

Specifically, we search through the Bloomberg system for all corporate inflation-linked debt issues for which we can find a fixed-rate bond for the same firm with a matching maturity date. When there is more than one matching fixed-rate and inflation-linked pair for a firm, we choose the most-liquid pair (defined in terms of the number of days on which prices are available). This process results in a sample of fixed-rate and inflation-linked pairs for the following firms: Bank of America, Citigroup, JP Morgan, Morgan Stanley, Prudential, and Sallie Mae. The original maturities of the inflation-linked debt issues are all 10 years. The mismatch in the maturities of the fixed-rate and inflation-linked debt issues ranges from zero days to a maximum of 31 days.

In general, corporate fixed-rate and inflation-linked debt is much less liquid than Treasury debt. This is particularly true during periods in which the underlying firm experiences serious credit issues as is the case for a number of the financial firms in our sample during the Lehman crisis. To address this issue, our approach is to focus on the periods during which the risk of a default for the underlying firm is viewed as small by market participants as reflected by the firm's credit default swap (CDS) spread being below some threshold.

Table 1.4 reports summary statistics for the yield differences between the corporate fixed-rate bonds and the corresponding inflation-swapped portfolio that exactly replicates the cash flows of the fixed-rate bond. For comparison, we also provide summary statistics for the contemporaneous TIPS–Treasury mispricing on the dates when we have an observation for a corporate fixed-rate and inflation-linked pair. The table reports the results using CDS

Table 1.4 – Comparison of Corporate Fixed-Rate and Inflation-Linked Debt Mispricing with TIPS-Treasury Mispricing. This table reports summary statistics for the mispricing of maturity-matched pairs of corporate fixed-rate and inflation-linked debt using the same arbitrage strategy as described in Table 1.1 and the same set of inflation swap data used to compute TIPS-Treasury mispricing. The sample consists of pairs of fixed-rate and inflation-linked debt for Bank of America, Citigroup, JP Morgan, Morgan Stanley, Prudential, and Sallie Mae. For perspective, the table also reports summary statistics for TIPS-Treasury mispricing for the same dates as the corporate fixed-rate and inflation-linked mispricing observations. Corporate fixed-rate and inflation-linked mispricing observations are computed when simultaneous pricing data for both types of debt are available and when the CDS spread for the underlying firm is less than or equal to the indicated CDS threshold (measured in basis points). Corr. denotes the correlation between the corporate fixed-rate and inflation-linked mispricing observations and the corresponding TIPS-Treasury mispricing observations. The sample period is from July 23, 2004 to November 19, 2009.

CDS Threshold	Corporate Mispricing			TIPS–Treasury Mispricing			Corr.	N
	Mean	Median	Std. Dev.	Mean	Median	Std. Dev.		
25	−6.11	−7.15	56.26	31.78	26.97	13.23	−0.281	465
50	−0.28	−4.34	58.96	31.76	26.88	13.41	−0.164	542
75	7.55	0.68	65.92	32.45	28.06	13.56	−0.052	581
100	9.17	1.09	68.11	32.95	28.35	14.26	0.019	598

spread thresholds for the underlying firm of 25, 50, 75, and 100 basis points.

As shown, the mispricing between corporate fixed-rate and inflation-linked debt is much smaller than the contemporaneous TIPS–Treasury mispricing for all of the CDS thresholds considered. For example, the average corporate mispricing is only 0.28 basis points when the credit threshold is 50 basis points. In contrast, the average value of the TIPS–Treasury mispricing on the same dates is 31.76 basis points. Note that the median values of the corporate mispricing are all either negative or nearly zero.[21] This provides direct evidence that mispricing in the inflation swap market cannot be the explanation for the TIPS–Treasury mispricing. This follows simply since the same inflation swap prices are used in computing

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[21]The standard deviation for the corporate mispricing is several times larger than for the TIPS–Treasury mispricing. The primary reason for this is that the daily TIPS–Treasury mispricing estimates are weighted averages of the mispricing across many TIPS–Treasury pairs. In contrast, the corporate mispricing estimates are based on individual pairs (it is rare to have more than one corporate mispricing estimate per day). Thus, since there is no averaging across different pairs, the daily corporate mispricing estimates appear more volatile.

both the corporate and TIPS–Treasury mispricing. Finding that corporate mispricing is nearly zero on average shows that the mispricing is unique to the TIPS–Treasury pairs.

As an alternative way of exploring this issue, observe that if inflation swaps were mispriced, then corporate and TIPS–Treasury mispricing would be highly correlated over time because of their common dependence on the prices of inflation swaps. Table 1.4, however, shows that there is very little correlation between the corporate and TIPS–Treasury mispricing series. In fact, the correlation between the two time series is negative in sign for three of the four credit thresholds, and nearly zero for the fourth. This provides additional evidence against the notion that mispricing in the inflation swap market is the source of the TIPS–Treasury mispricing.

As a final diagnostic check, we also use the following approach suggested by Ashton (2006). For each corporate fixed-rate and inflation-linked pair, we identify a TIPS–Treasury pair with closely matching maturities. Given these two pairs, we can then estimate the credit spread for the fixed-rate corporate bond by subtracting from its yield the yield on the matching Treasury bond. We refer to this as the fixed-rate credit spread. Similarly, we can then estimate the credit spread for the inflation-linked corporate bond by subtracting from its yield the yield on the matching TIPS issue. We refer to this as the inflation-linked credit spread. Intuitively, the two credit spreads should have values that are very similar to each other if the bonds are all fairly priced.

In actuality, however, we find that the fixed-rate credit spread is substantially higher than the inflation-linked spread. The mean difference between the two spreads ranges between 59 basis points to 86 basis points, depending on the CDS threshold. The hypothesis that the

difference in spreads is zero is strongly rejected by a simple test of the mean. The implication of these results is that spreads measured relative to Treasury bonds are higher than spreads measured relative to TIPS, consistent with the view that Treasury bonds are rich relative to TIPS.

It is important to recognize that this last comparison does not use any inflation swaps data; this analysis simply compares fixed-rate and inflation-linked yields. Thus, the inference that Treasury bonds are rich relative to TIPS is clearly not an artifact of the pricing of inflation swapsthe same inference holds even when we do not use inflation swaps data in the analysis.[22]

1.6.2 Other Potential Factors Affecting Inflation Swaps

The above analysis shows that inflation swap mispricing is not the explanation for the TIPS–Treasury mispricing. It is important to stress, however, that this conclusion does not necessarily imply that inflation swaps are always correctly priced. It simply means that whatever mispricing there may be in the inflation swap market is too small to explain the magnitude of TIPS–Treasury mispricing. For the sake of completeness, however, it is worthwhile to consider the potential impact of other market factors and frictions that have been discussed in the financial press.

Corporate Inflation-Linked Issuance A number of market participants have argued that the issuance of inflation linked debt by corporations creates an artificial inflation-hedging demand among Wall Street dealers. It is also argued that this demand could temporarily

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[22]Our results are also consistent with Ashton (2006), who finds that inflation swaps cannot explain the mispricing between Treasuries and TIPS.

distort prices in the inflation swap market.

To explore the implications of this hypothesis, we collected weekly data on all U.S. inflation-linked corporate debt issuance during the sample period from the Bloomberg system. We then regress weekly changes in inflation swap rates on weekly inflation-linked debt issuance. We find no evidence that inflation-linked debt issuance affects inflation swap prices.

In addition, we also regress weekly changes in the TIPS–Treasury mispricing on weekly inflation-linked debt issuance. Again, we find no relation between the two time series. Note that TIPS–Treasury mispricing is largest during the financial crisis, and that corporate inflation-linked issuance is almost nonexistent during this period. In summary, these results provide little or no support for the hypothesis that inflation swap pricing or TIPS–Treasury mispricing is driven by corporate inflation-linked debt issuance.[23]

Counterparty Credit Risk   The financial crisis has focused significant attention on the role of counterparty credit risk in the pricing of derivative contracts. This raises the question of how inflation swap contracts might be affected by the credit risk of inflation swap dealers in the market.[24]

In a recent paper, Arora, Gandhi, and Longstaff (2012) study the effect of counterparty credit risk on the pricing of CDS contracts. They document that differences in the credit risk of dealers selling credit protection have only a very small effect on the pricing of CDS contracts. They argue that the market practice of requiring full collateralization of swap liabilities

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[23]We acknowledge, however, that we have not included every possible factor driving inflation swap rates in these univariate regressions. Thus, we cannot fully rule out the possibility of omitted variables bias.

[24]Note that with bilateral counterparty credit risk, it is not obvious which direction the effect on CDS spreads would be. We are grateful to the referee for this insight.

results in counterparty credit risk having only a tiny effect on the pricing of swap contracts. Their evidence is also consistent with theoretical models of the effect of counterparty credit risk on swap contracts such as Duffie and Huang (1996) and others. Given that there is much less volatility in inflation rates than in credit spreads, the effect of counterparty credit risk on inflation swaps would be even smaller than is the case for CDS contracts. In light of this, it is unlikely that counterparty credit risk has much of an effect on the pricing on inflation swaps.

Hedging Costs and Illiquidity Another argument is that inflation swap dealers may face additional costs related to the hedging of their positions that may be impounded into inflation swap prices. Examples of these types of costs might include the cost of financing long and short TIPS and Treasury positions in the Treasury repo markets, the costs of using asset swaps to replicate TIPS and Treasury hedging positions, or the cost of holding collateral. Similarly, it is also argued that since inflation swaps may be less liquid than Treasuries, inflation swap pricing may reflect this illiquidity.

While it is undoubtably true that swap dealers may face hedging costs, the extent to which this could affect market inflation swap rates is unclear for a number of reasons. For example, dealers in other swap markets such as interest rate swaps and foreign exchange swaps are also exposed to these types of hedging costs. Studies of swap pricing, however, have discovered that these costs have at most a minor effect on equilibrium swap rates.[25] The liquidity of inflation swaps, while less than that of Treasury securities, is still relatively high. As

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[25]For example, see Duffie and Singleton (1997), Liu, Longstaff, and Mandell (2006), and Johannes and Sundaresan (2007).

described earlier, industry estimates of the notional size of the inflation swap market are on the order of several hundred billion dollars. Thus, the notional size of the inflation swap market approximates the size of the TIPS market.

1.7   TIPS and Treasury Bonds

The results above provide strong evidence that TIPS–Treasury mispricing is not due to the pricing of inflation swaps. Thus, TIPS–Treasury mispricing must be driven by the relative valuations of Treasury bonds and TIPS issues. Before exploring the determinants of TIPS–Treasury mispricing, however, is it important to consider whether there are institutional or economic factors that might drive a wedge between the market prices of Treasury bonds and TIPS. In this section, we consider a list of possibilities and briefly evaluate their potential impact. A number of these factors are addressed in the analysis.

1.7.1   Tax Differences

The federal and state income taxation of Treasury bonds is identical to that of TIPS in all but one small respect. Specifically, since the notional amount of TIPS accretes over time with realized inflation, taxable investors must treat this "phantom income" as if it were interest income for federal tax purposes. In contrast, taxable investors holding Treasury bonds only include coupons as interest income (abstracting from original issue discount (OID) and premium amortization issues). Interest income from both Treasury bonds and TIPS (including any accreted notional amounts) is exempt from state income taxation.

Although we do not have specific information about the ownership of TIPS, discussions with market participants suggest that a large portion of outstanding TIPS issues are held either directly or indirectly by tax-sheltered entities such as pension plans and retirement funds. Thus, the phantom income provision is irrelevant for many of these investors. This view is consistent with a survey by the Bond Market Association in which 79% of respondents indicated that the current tax status of TIPS is not a deterrent to buying TIPS, some indicating that this was because of the tax-free status of their funds.[26] Finally, it is important to observe that if the taxation of phantom income were to affect the valuation of TIPS, it should do so uniformly across all issues since the accretion rate is the same for all TIPS. Furthermore, the effects should also be present in the pricing of Treasury STRIPS since they are also subject to the phantom income provisions. In actuality, however, studies of the pricing of Treasury STRIPS have not found evidence of phantom income-related tax effects.[27]

1.7.2 Credit Risk

In recent years, it has become clear that the market attaches some positive probability to the event that the U.S. Treasury defaults on its debt. For example, Euro-denominated CDS contracts on the U.S. Treasury traded at spreads as high as 100 basis points during early 2009 (see Ang and Longstaff (2011)). There is an extensive literature on sovereign default risk including Duffie, Pederson, and Singleton (2003), Pan and Singleton (2008), Buraschi, Sener, and Mengütürk (2010), Longstaff et al. (2011), and many others. A key point often

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[26]See http://archives1.sifma.org/research/tipssurvey.pdf.
[27]For example, see Grinblatt and Longstaff (2000) and Jordan, Jorgensen, and Kuipers (2000).

made in this literature is that default risk for foreign currency-denominated sovereign debt may differ from that for local currency-denominated debt.

This foreign versus local distinction is relevant for Treasury bonds and TIPS since one can imagine scenarios in which the U.S. might be able to honor its nominal debt by simply "printing more money," but then not be able to pay off its inflation linked debt. In essence, inflation-linked TIPS can be viewed as equivalent to foreign currency-denominated debt from a sovereign default-risk perspective. If the market views the default risk of Treasury bonds as lower than that of TIPS, then TIPS might trade at prices lower than those implied by the no-arbitrage model.[28]

1.7.3 Bid-Ask Spreads

Another possible difference between Treasury bonds and TIPS might be in their trading costs. In reality, however, the costs of trading Treasury bonds and TIPS are both very small. As discussed in Section 1.2.3, the difference in the bid-ask spreads between Treasury bonds and TIPS is probably on the order of three to four ticks, or roughly 15 cents. Together with the earlier results, this implies that TIPS–Treasury mispricing greater than, say, five basis points cannot be explained in terms of transaction costs; the transaction costs are very small relative to the typical size of the pricing differences between Treasury bonds and TIPS.

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[28]CDS contracts on the U.S. Treasury currently do not distinguish between defaults of nominal bonds and TIPS. Industry sources such as ISDA suggest that a default of either type of bond would trigger payment on a U.S. Treasury CDS contract.

1.7.4 Deflation Floor

As discussed earlier, the principal amount of a TIPS issue is protected against deflation since the principal amount received by a TIPS holder at maturity cannot be less than par. Thus, there is an embedded option or deflation floor incorporated into the TIPS issues. Because of this, the value of a TIPS issue may be somewhat higher than it would be if there were no protection against deflation.

The analysis in the previous sections abstracts from the value of the deflation option. It is clear, however, that if we were to adjust observed TIPS prices by subtracting out the value of the deflation option, then the estimated TIPS–Treasury mispricing would be potentially much larger than reported. Thus, the deflation floor in TIPS prices goes in the wrong direction to explain TIPS–Treasury mispricing.

1.7.5 Repo Financing

A difference in an investor's ability to obtain repo financing for TIPS relative to Treasury bonds might induce pricing differences between the two types of Treasury debt. Discussions with bond traders, however, indicate that both types of debt are treated similarly by repo dealers. In particular, both Treasury bonds and TIPS can be financed at government general collateral repo rates with similar levels of haircuts. One trader estimated that the typical haircut applied to Treasury bonds or TIPS issues by large institutional participants in the repo market is on the order of 2% to 3%.

This evidence is consistent with a number of other sources. For example, the Fixed Income

Clearing Corporation of the Depository Trust and Clearing Corporation (DTCC) allows dealers to trade general collateral repos through their system and explicitly allows TIPS as a generic security type along with Treasury bonds and STRIPS.[29] The Security Industry and Financial Markets Association (SIFMA) provides repo trading practices guidelines for TIPS.[30] The only difference between their guidelines for Treasury bond repo and TIPS repo is their recommendation that "prices for repurchase agreement transactions involving Treasury Inflation-Indexed Securities be quoted on an 'all-in' price – including the inflation adjustment to the principal amount and the accrued interest on such inflation-adjusted principal" This technical accounting distinction, however, should have no effect on the availability of repo financing for TIPS. Finally, the Federal Reserve Bank of New York explicitly includes TIPS as eligible general collateral for dealer repo transactions with the System Open Market Account.[31] In summary, there is no material difference between Treasury bonds and TIPS in terms of an investor's ability to obtain repo financing.

1.7.6   Special Repo Rates

As discussed by Duffie (1996), Fisher (2002), Krishnamurthy (2002), Moulton (2004), Banerjee and Graveline (2011), and many others, holders of on-the-run Treasury bonds may be able to finance their positions at special repo rates that are below general repo rates. This feature confers a potential benefit on the owner of an on-the-run Treasury bond that might be incorporated into the price of the bond and help explain some of the richness of Treasury

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[29]See http://www.dtcc.com/products/documentation/cs/ficc/gov/GCFCollateralTypes.pdf.

[30]See SIFMA Restated Repo Trading Practices Guidelines, Update No. 1997-1, TIPS Repurchase Agreement Transactions at http://www.sifma.org/services/standard-forms-and-documentation/government-securities/.

[31]See http://www.newyorkfed.org/aboutthefed/fedpoint/fed04.html.

bonds relative to TIPS.

Special repo financing, however, cannot fully account for TIPS–Treasury mispricing. First, special repo financing is limited primarily to on-the-run Treasury bonds, while TIPS–Treasury mispricing occurs for virtually all Treasury bond and TIPS pairs. Second, discussions with TIPS traders indicate that on-the-run TIPS issues can also be financed at special repo rates. Finally, the present value of the special repo financing benefit for on-the-run Treasuries is much smaller than the average TIPS–Treasury mispricing for these securities. Specifically, Duffie (1996), Moulton (2004), and Banerjee and Graveline (2011) provide estimates of the differences between overnight/term general and special repo rates ranging from about 30 to 125 basis points. A back-of-the-envelope upper bound calculation shows that even if a Treasury bond could be financed at a special repo rate 125 basis points below general collateral rates for as long as six months (the maximum time between auctions), the present value of this would only be 62.5 cents per \$100 notional. This upper bound is substantially lower than the average size of the TIPS–Treasury mispricing.

1.7.7 Collateral Value

Since the principal and interest from both Treasury bonds and TIPS is fully guaranteed by the U.S. Treasury, both types of debt are acceptable collateral for almost all forms of public, private, and banking obligations. To provide some examples, TIPS are equally acceptable as collateral for the Treasury Tax and Loan Program and the Treasury Term Investment Option (see 31 CFR Parts 202 and 203), as collateral for bonds secured by government obligations in lieu of bonds with sureties (see 31 CFR Part 225), and as collateral for uninsured deposits (see

12 CFR 550.320). Similarly, Treasury bonds and TIPS are equally acceptable as collateral for virtually all state and local government purposes. One hedge fund, however, told us that some banks were reluctant to accept TIPS as collateral during the crisis.

1.7.8 Eligibility for the Treasury STRIPS Program

Both Treasury bonds and TIPS are eligible for stripping under the Treasury's STRIPS program. The key difference is that stripped coupon from different TIPS issues is not fungible since each issue has its own CPI reference level. The U.S. Treasury's Statement of the Public Debt reports that on December 31, 2009, 21.22% of the notional amount of all Treasury bonds, 0.49% of the notional amount of all Treasury notes, and 0.03% of the notional amount of all TIPS were held in stripped form. These percentages are fairly stable throughout the sample period.

1.7.9 Futures Contracts

Futures contracts on Treasury notes and bonds are traded at the Chicago Board of Trade. Each contract specifies a list of Treasury notes and bonds that are deliverable in settlement of futures positions. In contrast, futures contracts on TIPS are not currently traded on any futures exchange. This distinction likely has little impact on the relative pricing of most Treasury bonds and TIPS. This is because forward purchases or sales of both Treasury bonds and TIPS can be readily executed by institutional participants in the over-the-counter (OTC) market. The key exception might be the case of a cheapest-to-deliver bond at or near the expiration of a futures contract. Market participants, however, indicate that any cheapest-

to-deliver effect on Treasury bond prices would typically be very small in magnitude since the Treasury bond/futures basis is actively traded and arbitraged by many financial institutions.

1.7.10 Foreign Ownership

We attempt to obtain data on whether Treasury bonds and TIPS differ in terms of the foreign ownership of these securities. Unfortunately, only aggregate foreign ownership data for Treasury bonds and TIPS are available. As of November 2009, the largest foreign holders of U.S. Treasury bonds and TIPS are China and Japan, with holdings of $789.6 billion and $757.3 billion, respectively. We note, however, that an August 2008 report by the Office of Debt Management of the U.S. Treasury Department provides a graph indicating that during the 2000 to 2008 period, roughly 60% of TIPS were auctioned to dealers and brokers, 30% to investment firms, and 10% to foreign entities. Similarly, Gongloff (2010) reports that foreign demand at TIPS auctions averages about 39%.[32]

1.7.11 Institutional Ownership

To explore whether there are differences in the pattern of institutional ownership between Treasury bonds and TIPS, we note that some data on institutional ownership are available via SEC Form 13F filings. In particular, Section 13(f) of the Securities Exchange Act of 1934 requires that institutional investment managers using the U.S. mail (or any other means or instrumentality of interstate commerce) in the course of their business and exercising

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[32]See Gongloff, Mark, 2010, TIPS given the cold shoulder, *Wall Street Journal*, April 27, p. C8. Fleming (2007) finds that indirect bidders represent a larger percentage of buyers at TIPS auctions than is the case for Treasury bond auctions.

investment discretion over \$100 million or more in Section 13(f) securities must file Form 13F. In making these filings, many of these institutional investors provide information about their holdings of Treasury and TIPS bonds.

Information about institutional holdings of Treasury bonds and TIPS included in these Form 13F filings is compiled by Bloomberg and is summarized for each bond or TIPS issue. We collect data on the TIPS issues in the sample from the Bloomberg system and then collect data for a sample of Treasury bonds with maturities closely matching those of the TIPS issues. We then compare the percentages of the notional amounts held by the institutions filing Form 13F. In doing this, it is important to note that the coverage of Treasury bonds and TIPS issues provided by these Form 13F filings and tabulated by the Bloomberg system may not necessarily be comprehensive.

On average, 31.58% of the notional amount of the TIPS bonds in the sample are reported on Form 13F. The corresponding value for a set of maturity-matched Treasury bonds is 25.02%. Thus, the total percentage amounts reported are similar. A more detailed analysis, however, indicates that there are some intriguing differences in the institutional ownership patterns. In particular, investment firms (mutual funds, investment advisors, etc.) hold 20.69% of the TIPS, but only 4.71% of the matching Treasury bonds. In contrast, the Federal Reserve Bank of New York holds 8.41% of the TIPS, but 17.35% of the matching Treasury bonds. Thus, while the total reported institutional ownership of TIPS and Treasury bonds is similar, the data indicate that investment funds hold a much larger fraction of the TIPS than the Federal Reserve Bank of New York, while the reverse is true for Treasury bonds. Insurance companies hold 2.48% of the TIPS and 2.96% of Treasury bonds. This evidence of partial

segmentation in the ownership of Treasury bonds and TIPS is consistent with results in Section 1.7 supporting the slow-moving-capital hypothesis.

1.7.12 Bond Dealers and Market Microstructure

We also investigate whether there are differences between Treasury bonds and TIPS in the number and types of institutions functioning as bond dealers. The Federal Reserve Bank of New York maintains a list of primary government securities dealers. This list currently includes BNP Paribas, Bank of America, Barclay's Capital, Cantor Fitzgerald, Citigroup, Credit Suisse, Daiwa, Deutsche Bank, Goldman Sachs, HSBC, Jefferies, J.P. Morgan, Mizuho, Morgan Stanley, Nomura, RBC, RBS, and UBS.

The Federal Reserve Bank of New York also lists the standards expected of primary dealers. For example, primary dealers are expected to meet a $150 million minimum net capital requirement. Furthermore, primary dealers are expected to participate consistently as a counterparty to the New York Fed in its execution of open market operations. Primary dealers are also required to participate in all auctions of U.S. government debt and to make reasonablemarkets in these securities. These rules make clear that there is no difference between Treasury bonds and TIPS in how these primary dealers are expected to conduct their operations. This is also confirmed by discussions with Treasury bond and TIPS traders who indicate that there is little difference in how bond dealers make markets in the two types of securities. The OTC market microstructure is very similar across the Treasury bond and TIPS markets.

1.7.13 Supply Considerations

One clear distinction between Treasury bonds and TIPS issues is in terms of the supply of these securities to the financial markets. To provide some background on the relative size of the TIPS market to the total Treasury bond market, we refer to Table FD-2 of the March 2010 Federal Reserve Bulletin. The ratio of TIPS notional debt outstanding to the total amount of Treasury debt held by the public was 6.67% at the end of 2005, 8.17% at the end of 2006, 9.05% at the end of 2007, 9.02% at the end of 2008, and 7.30% at the end of 2009. Thus, the notional amount of TIPS was less than 10% of the total amount of Treasury debt held by the public during recent years. The ratio increased significantly during the 2005 to 2007 period, but declined during the recent financial crisis as total Treasury debt issuance accelerated.

1.7.14 TIPS Liquidity

As one measure of the relative liquidity of TIPS and Treasury bonds, we can examine the average trading volume of the two types of securities by primary dealers. This information is tabulated and reported online by the Federal Reserve Bank of New York as well as SIFMA. Focusing on 2011, the total average daily trading volume of nominal Treasury notes and bonds was $212.6 billion, which is 2.78% of the total notional amount of these securities outstanding at the end of 2011. The total average daily trading volume of TIPS during 2011 was $9.5 billion, which is 1.29% of the total amount of TIPS outstanding at the end of 2011. Thus, the trading activity of TIPS is about 46% that of Treasury notes and bonds. These results suggest that while TIPS are not as actively traded as Treasury notes and bonds,

TIPS have a high degree of liquidity. In contrast, using the same metric, the trading activity of all municipal bonds is only 11% that of Treasury notes and bonds during 2011. Similarly, the trading activity of all corporate bonds is only 9.5% that of Treasury notes and bonds during the same period. Similar results hold for the other years in the study period.

We also interview Treasury bond and TIPS traders who confirm this assessment of the relative liquidity of the two markets. In particular, one trader told us that there are roughly 15 dealers who were competitive in providing quotes and would be able to quickly execute purchases and sales of Treasury bonds. In contrast, the same trader indicated that there were only about five dealers who would be able to provide the same level of liquidity for TIPS. Despite this, however, the trader felt that TIPS were liquid and that trades could be executed rapidly.

1.7.15 Costs of Shorting Treasury Bonds

To short a Treasury bond, an investor must first borrow the bond through a reverse repo arrangement. In return, the investor allows the owner of the bond to borrow funds at some market determined rate. Typically, this rate is slightly below the market rate and the difference represents the borrowing cost of the bond. Discussions with traders indicate that it was always possible to short Treasury bonds throughout the sample period.

In extreme situations, however, this spread could widen. For example, during the depths of the financial crisis in the Fall of 2008, an arbitrageur wishing to short a bond might have needed to allow the owner of the bond to borrow at a cost of zero. Since short-term repo rates were on the order of only 25 basis points during this period, however, the effective cost

to the arbitrageur of allowing the owner of the bond to borrow at zero was relatively minor.

In mid-2009, SIFMA mandated that repo failures result in the security lender being able to borrow at an annual rate of 300 basis points. This change increased the maximum potential cost to an arbitrageur of short selling Treasury issues in the extreme situation in which the arbitrageur was not able to find a repo dealer willing to lend him the security. Given the timing of this provision, however, it is unlikely to have had much impact on the results reported in this paper.

1.7.16 Quantitative Easing

On March 18, 2009, the Federal Open Market Committee (FOMC) of the Federal Reserve announced an unprecedented program to purchase up to $300 billion of longer-dated Treasury bonds through a series of competitive auctions.[33] Over the course of the program the FOMC purchased $11 billion in nominal Treasury securities maturing in one to two years, $242 billion maturing in two to 10 years, $42 billion maturing in 10 to 30 years, and $5 billion in TIPS. This quantitative easing program (now known as QE 1) affected the tradable supply of Treasury securities in the market, which in turn could potentially affect the relative pricing of Treasury bonds and TIPS issues. For a discussion of the QE1 program, see Krishnamurthy and Vissing-Jorgensen (2010b).

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[33]Permanent open market operations include purchases or sales of securities on an outright basis that add to or diminish reserves. These are different from temporary open market operations that consist of short-term repurchases or reverse repurchase agreements.

1.8 What Drives the Mispricing?

The evidence of significant and persistent mispricing between TIPS and Treasury bonds presents a major puzzle to our understanding of how these markets function. In this section, we explore whether variation in the mispricing is linked to a number of economic and financial variables suggested by the literature or motivated by the discussion in the previous section. By doing this, we hope to shed light on the underlying reasons for the mispricing via the identification of factors that may drive the mispricing.

1.8.1 The Variables

A number of possible factors might influence the size of TIPS–Treasury mispricing over time. We discuss each of these in turn and describe the specific variables used in the regression analysis.

Supply The supply of Treasury securities available in the financial markets may be a key factor affecting the ability of arbitrageurs to exploit pricing differences between the TIPS and Treasury bond markets. In particular, it may be easier to execute arbitrage strategies in a market when there is an increase in the supply of on-the-run or recently auctioned bonds. This follows from Kamara (1988, 1994), Cammack (1991), Boudoukh and Whitelaw (1991), Amihud and Mendelson (1991), Krishnamurthy (2002), Han, Longstaff, and Merrill (2007), and others who document that on-the-run bonds differ in terms of their trading and pricing characteristics. To explore the effects of supply on TIPS–Treasury mispricing, we include the total notional amount of all TIPS and all Treasury bonds auctioned each month during

the sample period. These data are obtained from the Treasury website.

Liquidity   An extensive literature documents that liquidity patterns can have significant effects on the valuation of securities. For example, see Boudoukh and Whitelaw (1993), Vayanos and Vila (1999), Acharya and Pedersen (2005), Amihud, Mendelson, and Pedersen (2005), Brunnermeier and Pedersen (2009), Longstaff (2009), Huang and Wang (2010), and many others.

To study the effects of changes in liquidity on TIPS–Treasury mispricing, we include two variables in the analysis. The first is the total notional amount of repo fails experienced by primary bond dealers. Repo fails represent a measure of market disruption caused by investors' inability to find specific Treasury securities in the markets, and directly reflects a breakdown in market liquidity.

Specifically, a repo fail occurs when a primary dealer is not able to deliver a Treasury security that the dealer had previously committed to deliver as part of a securities repurchase agreement. Alternatively, a repo fail occurs when the primary dealer does not receive back a Treasury security pledged as collateral on a repurchase agreement. In either case, the failure indicates that market participants are not able to locate specific Treasury securities. Thus, repo fails should increase during stressed periods in which liquidity and available supply of Treasury securities in the markets dries up. Information on repo fails is reported by the Federal Reserve Bank of New York.

The second liquidity measure is the ratio of total TIPS trading volume by U.S. primary dealers to total coupon-bearing Treasury note and bond trading volume by U.S. primary

dealers. Intuitively, changes in this ratio may capture variation in the liquidity of TIPS relative to that of Treasury bonds. Information on trading activity by primary bond dealers is also reported by the Federal Reserve Bank of New York.

We note that we also considered a number of alternative liquidity measures including several suggested by Lesmond, Ogden, and Trzcinka (1999) and Chen, Lesmond, and Wei (2007). In particular, we considered using bid-ask spreads, the percentage of zero returns, and the Lesmond, Ogden, and Trzcinka measure constructed from the frequency of zero returns. Since we do not have reliable time series of bid-ask spreads, we are not able to use bid-ask spreads as a measure of liquidity. Also, given the high level of trading activity in the TIPS market on a daily basis, the observed frequency of zero returns over a month in this market is essentially zero. Thus, measures of liquidity based on the incidence of zero returns do not appear to be applicable in our analysis.

Credit Risk   As discussed earlier, another possibility might be that the market perceives the credit risk of TIPS as being slightly higher than that of Treasury bonds. In this case, TIPS might appear to be underpriced relative to Treasury bonds. On the other hand, even if the market viewed the credit risk of TIPS and Treasury bonds as equivalent, changes in aggregate credit risk in other markets might influence the relative pricing of TIPS and Treasury bonds. This is because TIPS and Treasury bonds might not be viewed as equally attractive safe havens in the event of a credit induced flight to quality in the financial markets.

To explore the effects of credit risk on TIPS–Treasury mispricing, we use the 10-year swap spread. Swap spreads are one of the most important indicators of the credit risk of the

banking system, and have been widely used as measures of aggregate credit risk.[34] We

obtain data on the 10-year swap spread from the Bloomberg system.

Slow Moving Capital   A number of recent papers have put forward potential explana-

tions for the existence of persistent mispricing in financial markets. Mitchell, Pedersen, and

Pulvino (2007) and Duffie (2010) discuss the role that slow-moving capital may play in allow-

ing arbitrage opportunities to exist for extended periods of time. Shleifer and Vishny (1997),

Gromb and Vayanos (2002), Liu and Longstaff (2005), Fostel and Geanakoplos (2008), Gor-

ton and Metrick (2009), and Ashcraft, Gârleanu, and Pederson (2010) argue that margins,

haircuts, and other collateral-related frictions may allow arbitrage or deviations from the

law of one price to occur. Brunnermeier and Pedersen (2009) emphasize the role that the

availability of funding may play in liquidity effects on security prices.

To explore the implications of the slow-moving-capital literature, we examine whether TIPS–

Treasury mispricing is affected by a measure of the amount of capital available in the market

that could potentially be directed towards arbitraging mispricing. Specifically, we include

changes in total global hedge fund net asset values as estimated by Hedge Fund Research

Inc. and reported in the Bloomberg system.

1.8.2   Regression Analysis

To explore the relation between the above variables and TIPS–Treasury mispricing, we

regress monthly changes in the mispricing on the corresponding changes or values of the

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[34]For example, see Duffie and Singleton (1997), and Liu, Longstaff, and Mandell (2006).

Table 1.5 – Results from Regression of Monthly Changes in Average Basis-Point Mispricing on Supply, Liquidity, Credit, and Capital Flow Factors. This table reports summary statistics for the regression of monthly changes in TIPSTreasury mispricing on the indicated variables. TIPS Issuance denotes the total notional amount of TIPS (in $billions) issued during the month. Treasury Issuance denotes the total notional amount of Treasury notes and bonds (in $billions) issued during the month. Repo Fails denotes the total notional amount (in $billions) of repo failures reported by primary dealers during the month. Trading Ratio denotes the percentage of total monthly TIPS trading volume by primary dealers to total monthly Treasury note and bond trading volume by primary dealers. Swap Spread denotes the monthly basis point change in the 10-year USD swap spread. Hedge Fund Flows denotes the percentage change in total global assets held by hedge funds. The superscript ** denotes significance at the 5% level; the superscript ˆ* denotes significance at the 10% level. The sample period is June 2004 to November 2009.

Explanatory Variable	Regression Coefficient	Newey-West t-Statistic
Intercept	3.6174	0.29
TIPS Issuance	−0.5304	−1.85*
Treasury Issuance	−0.0579	−1.71*
Repo Fails	0.0011	2.23**
Trading Ratio	1.9019	0.27
Swap Spread	0.3732	0.99
Hedge Fund Flows	−2.2727	−2.32**
Adj. R^2		0.175
N		66

explanatory variables. Rather than doing this at the level of individual TIPS and Treasury pairs, however, we focus on the average yield mispricing across all pairs, where the average is weighted by the outstanding notional amount of the TIPS issue (taking into account the accretion in the notional amount). Table 1.5 reports the regression results.

The results in Table 1.5 provide a number of interesting insights into the determinants of TIPS–Treasury mispricing. First, the results indicate that the mispricing is affected by the supply of new Treasury securities. In particular, the notional amounts of both TIPS and Treasury issuance are statistically significant (at the 10% level). Surprisingly, both regression coefficients are negative in sign. Thus, TIPS–Treasury mispricing decreases in magnitude

whenever the Treasury issues TIPS or Treasury bonds. An important implication of this result is that it is not the relative amounts of TIPS and Treasury bonds in the market that affects mispricing. Rather, it is the presence of liquid on-the-run Treasury securities of either type in the market that allows arbitrageurs to drive prices closer together.

This implication is reinforced by the results for the liquidity measures. Specifically, the amount of repo fails is significantly related to TIPS–Treasury mispricing. Thus, when the market experiences liquidity disruptions as primary dealers are unable to receive or deliver Treasury securities that they have purchased or sold TIPS–Treasury mispricing increases. These results provide clear evidence that the supply or liquidity of the securities involved in the arbitrage is directly linked to the size of the arbitrage. As far as we are aware, this is the first time that such a result has been documented in the literature.

In contrast, the results indicate that systemic credit risk, as measured by the ten-year swap spread, is not significantly related to the TIPS–Treasury mispricing. We repeat this analysis using other measures of credit risk such as the CDX index of CDS spreads for U.S. investment grade firms as well as the sovereign CDS spread on the U.S. Treasury. Neither of these credit risk measures is significantly related to TIPS–Treasury mispricing.[35] We also test whether monthly changes in mispricing are related to the total amount of Treasury debt purchased as part of the QE1 program by the Federal Reserve, but find that there is no relation.[36]

Finally, the results provide strong direct support for the implications of the slow-moving-capital hypothesis. In particular, changes in the amount of capital managed by hedge funds

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[35]For a discussion of U.S. sovereign CDS, see Ang and Longstaff (2011).

[36]We note, however, that we did not test whether there were pair-specific effects on mispricing based on which bonds were actually purchased by the Federal Reserve.

are significantly related to TIPS–Treasury mispricing. The coefficient estimate indicates that as the amount of hedge fund capital increases by 1%, TIPS–Treasury mispricing decreases by 2.27 basis points. This result is particularly striking since not all hedge funds would be willing to take significant positions in long-maturity Treasury bonds or TIPS.

1.9 Correlated Arbitrage

One important implication of the slow-moving-capital explanation of the persistence of mispricing is that arbitrage in different markets could be driven by a common factor. For example, if capital returns slowly to the fixed-income arbitrage hedge fund sector after periods of flat performance, then arbitrages arising in various types of fixed-income markets could display significant commonality. Motivated by this, we explore the extent to which TIPS–Treasury mispricing is correlated with other types of fixed-income arbitrage.

In this section, we focus on four well-known types of fixed-income arbitrage strategies or forms of mispricing. The first is the CDS-Corporate bond basis strategy discussed by Duffie (2010). In this strategy, the spread for a CDS contract on a firm is compared with the spread on corporate bonds issued by that firm. In theory, the two spreads should be very similar. In reality, there is often a significant difference between the two spreads, which is termed the CDS/corporate-bond basis (Longstaff, Mithal, and Neis (2005)). Duffie argues that this basis may be a result of slow moving capital. In particular, entering into a CDS contract requires little capital, while purchasing a corporate bond requires the use of significant capital. Thus, if arbitrage capital in the market is relatively scarce, then the CDS/corporate-bond basis may be able to persist. We were given access to a proprietary time series by a major investment

management firm of the CDS/corporate-bond basis for the firms included in the CDX index for the period from January 2005 to November 2009. We average the basis across these firms and compute the monthly changes in the average basis. For simplicity, we refer to this arbitrage as the CDS arbitrage.

The second arbitrage strategy is based on the difference between the CDX index and the average CDX spreads for the 125 firms included in the CDX index. Since contracts on the CDX index trade separately, the market price for a contract on the CDX need not always equal the average value of the CDS spreads in the index. This failure of the index to equal the sum of its parts is similar in concept to the difference between the price of an ETF and the value of the ETF's holdings (Tucker and Laipply (2010)). We were also given access to these data for the same time period as for the CDS/corporate-bond basis strategy described above. We refer to this arbitrage as the CDX arbitrage.

It is important to observe that these two arbitrage strategies are different in terms of their use of capital. To implement the CDS/corporate-bond basis strategy requires an investment in the underlying corporate bond and use of the arbitrageur's capital. In contrast, the CDX index arbitrage strategy only requires taking long and short positions in CDS contracts, involving little or no capital. Note also that neither of these two arbitrage strategies involves Treasury bonds or TIPS. Thus, finding that these strategies and TIPS–Treasury mispricing are correlated would provide evidence in support of the implications of the literature described above.

The third type of mispricing is the yield difference between on-the-run and off-the-run Treasury bonds of similar maturities (see Krishnamurthy (2002)). Specifically, we use the on-the-

run/off-the-run Treasury yield curve computed in the Bloomberg system (Curve USD Z111), and take the difference between the on-the- run 10-year yield and the off-the-run nine-year yield.

The fourth type of mispricing is the yield difference between 10-year Refcorp and Treasury STRIPS (see Longstaff (2004)). Since Refcorp bonds are guaranteed by the Treasury, these bonds should have identical yields to those of comparable maturity Treasury bonds. Thus, their difference represents a simple violation of the law of one price. Longstaff shows that the Refcorp-Treasury spread is influenced by a number of market liquidity measures.

Table 1.6 reports the results from the regression of the monthly changes in the TIPS–Treasury mispricing index on the current and lagged monthly changes in the CDX and CDX arbitrage measures as well as the on-the-run/off-the-run spread and the Refcorp-Treasury spread. As shown, there is a strong relation between changes in TIPS–Treasury mispricing and changes in the other arbitrage or mispricing measures. In particular, nine of the 12 current or lagged values of these mispricing measures are statistically significant. Overall, changes in these mispricing measures explain a major fraction of the variation in the TIPS–Treasury mispricing; the R^2 for the regression is 0.691 and the adjusted R^2 is 0.616.[37]

These results provide strong support for the hypothesis of correlated arbitrage. The fact that measures of arbitrage in the corporate bond and CDS markets as well as in the Refcorp market are able to explain such a large proportion of TIPS–Treasury mispricing argues that there is some strong commonality in observed arbitrage mispricing that transcends markets.[38]

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[37]Note that the signs of the regression coefficients are arbitrary since, for example, an arbitrage could be defined as cash versus synthetic, or just as easily, synthetic versus cash. For this reason, we focus primarily on the R^2 from the regressions.

[38]There are other examples of fixed-income mispricing measures that might be interesting to explore in

Table 1.6 – Results from the Regression of Monthly Changes in Average Basis-Point Mispricing on Monthly Changes in CDS Mispricing, CDX Mispricing, On/Off-the-Run Spreads, and Refcorp/ Treasury Spreads. This table reports summary statistics for the regression of monthly changes in TIPS–Treasury mispricing on the indicated contemporaneous and lagged monthly changes in the CDS and CDX arbitrage measures, and on the On/Off-the-Run and Refcorp-Treasury spreads. All mispricings and spreads are measured in basis points. The superscript ** denotes significance at the 5% level; the superscript * denotes significance at the 10% level. The sample period is June 2004 to November 2009.

Explanatory Variable	Regression Coefficient	Newey-West t-Statistic
Intercept	0.1698	0.12
CDS_t	0.0986	1.77*
CDS_{t-1}	0.0599	1.25
CDS_{t-2}	−0.1937	−3.59**
CDX_t	−0.1940	−1.85*
CDX_{t-1}	−0.4390	−3.60**
CDX_{t-2}	0.3335	3.97**
On/Off_t	0.5093	0.84
On/Off_{t-1}	−1.2045	−2.02**
On/Off_{t-1}	1.1413	2.39**
$Refcorp_t$	0.3445	2.11**
$Refcorp_{t-1}$	−0.4088	−3.85**
$Refcorp_{t-2}$	0.2014	1.23
Adj. R^2		0.616
N		66

Clearly, the notion of slow-moving capital in financial markets is one possible explanation for why there could be correlated arbitrage in multiple markets at the same time.

1.10 Predictable Arbitrage

Another important implication of the slow-moving-capital explanation is that mispricing in the market should be predictable on the basis of changes in the amount of capital invested in the financial markets. For example, imagine that investors collectively experience a major increase in their wealth as the stock market rallies. With the increased availability of financial capital, some of this capital would move slowly into mispriced markets and lead to a subsequent decline in the size of the mispricing. Thus, there should be an inverse relation between increases in current wealth and future declines in arbitrage mispricing.

To explore this implication of the slow-moving-capital explanation, we regress ex-post changes in TIPS–Treasury mispricing on three ex-ante measures of changes in the amount of capital available in the markets. The first is simply the excess return on the value-weighted CRSP index (data provided by Ken French). The second is the return on a portfolio of all Treasury bonds (index computed and reported by the Bloomberg system). The third is the return on the value-weighted Hedge Fund Research (HFR) index of all hedge funds. Table 1.7 reports the results from the forecasting regression.

As shown, the lagged stock, bond, and hedge fund returns have very strong forecasting ability for changes in TIPS–Treasury mispricing. The R^2 for the regression is 0.523; the adjusted

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future work including the Hu, Pan, and Wang (2011) measure of dispersion of Treasury bond yields around a smooth yield curve.

Table 1.7 – Results from the Forecasting Regression of Monthly Changes in Average Basis-Point Mispricing on Lagged Stock, Bond, and Hedge Fund Returns. This table reports summary statistics for the regression of monthly changes in TIPS-Treasury mispricing on the indicated lagged stock, bond, and hedge fund returns. Stock denotes the excess return on the CRSP value-weighted index. Bond denotes the return on the Bloomberg index of all Treasury debt with maturities in excess of one year. HedgeFund denotes the return on the HFRI value-weighted index of all hedge funds. The superscript ** denotes significance at the 5% level; the superscript * denotes significance at the 10% level. The sample period is June 2004 to November 2009.

Explanatory Variable	Regression Coefficient	Newey-West t-Statistic
Intercept	5.6081	2.61**
Stock_{t-1}	−1.8203	−2.95**
Stock_{t-2}	2.1812	3.33**
Bond_{t-1}	−4.3274	−3.24**
Bond_{t-2}	−3.5386	−2.82**
HedgeFund_{t-1}	0.3471	0.23
HedgeFund_{t-2}	−3.8906	−2.62**
Adj. R^2		0.471
N		66

R^2 is 0.471. Thus, roughly half of the variation in TIPS–Treasury mispricing during a month is predictable on the basis of returns in other markets during the previous two months.

Table 1.7 shows that five of the six lagged returns are statistically significant. The first lagged stock return has a negative sign. Similarly, both the first and second lagged bond market returns are significant and negative in sign. Finally, the second lagged hedge fund index return is significant and negative in sign. These negative relations between changes in TIPS–Treasury mispricing and prior returns are consistent with the slow-moving-capital hypothesis. In particular, the results provide evidence consistent with the intuition that as investor wealth in the stock, bond, and hedge fund markets increases, more capital is available to arbitrage away mispricing in the TIPS and Treasury markets. Note that these

results are consistent with the view that the movement of capital may take place via a gradual reallocation of capital from a variety of asset classes to a mispriced asset class, rather than simply the slow deployment of fixed-income arbitrage hedge fund capital to fixed-income arbitrage opportunities.

Another dimension that might be interesting to explore in future work is the notion that mispricing might not only be a function of the supply of arbitrage capital, but also the demand for arbitrage capital. In particular, the demand for arbitrage capital might depend on the number and types of arbitrages available in the financial markets. It is certainly true that there were many other types of fixed-income mispricings in the market around the timeframe of our study period besides those we discuss. See, for example, Froot (2001), Krishnamurthy (2010), Buraschi, Sener, and Mengütürk (2010), Stanton and Wallace (2011), and Bai and Collin-Dufresne (2011).

1.11 Conclusion

In this paper, we study the relative pricing of TIPS and Treasury bonds. A simple no-arbitrage argument places a strong restriction on the relation between the prices of these securities. We show that this no-arbitrage relation is frequently violated in the markets. The mispricing, which can exceed $20 per $100 notional amount, is among the largest ever documented in the literature. Furthermore, the sheer magnitude of this mispricing in markets as deep and actively traded as the Treasury bond and TIPS markets presents a serious challenge to conventional asset pricing theory.

We use this mispricing as a vehicle to explore the implications of recent theoretical work on the role of slow-moving capital. We find strong support for this explanation of the persistence of mispricing in financial markets. In particular, we show that TIPS–Treasury mispricing narrows as additional capital flows into the hedge fund sector, that TIPS–Treasury mispricing is correlated with arbitrage mispricing in other markets, and that TIPS–Treasury mispricing can be forecast using measures of changes in aggregate investor wealth.

Finally, we show that TIPS are almost always too "cheap" relative to Treasury bonds. An immediate implication of this is that the Treasury could have reduced the cost of the public debt by issuing only nominal bonds, or alternatively, by actually buying back TIPS and replacing them with nominal bonds. Thus, while there may be solid reasons for issuing TIPS, our results suggest that the policy may be far more costly than previously recognized. This is because the Treasury not only gives up a fiscal hedging option by issuing TIPS, but also leaves billions of dollars on the table by issuing securities that are not as highly valued by the market as nominal Treasury Bonds.

1.12   Appendix

A.1 Measuring TIPS–Treasury Mispricing

This section of the Appendix describes how we compute the size of the TIPS–Treasury mispricing. In addition to the pricing data for TIPS, Treasury bonds, and STRIPS issues, we also download daily closing prices of inflation swaps with maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25 and 30 years for the period from July 23, 2004 to November 19, 2009 from the Bloomberg terminal. Inflation swaps are identified on the Bloomberg system by the ticker USSWITn, where n denotes the maturity of the swap. For a few of these swaps, inflation swap data are missing for several days. In these cases, we replace missing data points by the last available observation.

To implement the arbitrage strategy, we set the notional amount of each inflation swap to match the corresponding semiannual coupon payment (before inflation adjustment) on the TIPS issue which we designate s. At date t, the inflation swap pays a cash flow of $s\left(1+f_t\right)^t - sI_t$, where I_t is the indexed leg and f_t is the fixed inflation swap rate for maturity t.

Implementing the arbitrage strategy requires interpolating the quoted inflation swap rates for all maturities ranging from 0 to 30 years. Furthermore, seasonal patterns in inflation must be taken into account for swap maturities that include fractional years (e.g. 2.3 years). To interpolate the inflation swap rate curve, we first fit a standard cubic spline through the quoted maturities using a grid size of one month. Let the interpolated swap rates be denoted by $f_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$, where the first index refers to the year and

the second to the month.

We then estimate seasonal components in inflation from the monthly non-seasonally adjusted U.S. CPI index (CPI-U NSA) series between January 1980 and October 2009 by estimating an OLS regression of monthly log changes in the CPI index on month dummies. More specifically,

$$\Delta CPI_t \equiv \log \left(\frac{CPI_t}{CPI_{t-1}} \right) = \sum_{i=1}^{12} \beta_i \, d_i \; + \; \varepsilon_i, \tag{A1}$$

where t is measured in months. The month dummies d_i, $i = 1, 2, \ldots, 12$ are defined as

$$d_i = \begin{cases} 1, & \text{for month i,} \\ \\ 0, & \text{otherwise.} \end{cases} \tag{A2}$$

and $d_1 =$January, $d_2 =$February, $\ldots$, $d_{12} =$December. We obtain an estimate of the seasonal effect in month i by subtracting the average of the coefficients $\bar{\beta} = \frac{1}{12} \sum \hat{\beta}_i$ from the estimated coefficients $\hat{\beta}_i$, $i = 1, 2, \ldots, 12$. Let this estimate be denoted by $\hat{b}_i = \hat{\beta}_i - \bar{\beta}$, $i = 1, 2, \ldots, 12$.

Next, we construct monthly forward rates $H_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$ from the interpolated swap rates $f_{i,j}$. Then, we normalize the seasonal factors $\hat{b}_i$ so that their product is unity. Let the normalized monthly adjustment factors be denoted by $\hat{m}_i$, $i = 1, 2, \ldots, 12$, where $\prod_{i=1}^{12} \hat{m}_i = 1$. We then multiply the forward rates $H_{i,j}$ by the corresponding adjustment factor $\hat{m}_j$, $j = 1, 2, \ldots, 12$ to obtain seasonally adjusted forward rates $\tilde{H}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$. By construction, there will be no seasonal effects for full-year

swaps. In the last step, we obtain the seasonally adjusted inflation swap curve by converting the forward rates $\tilde{H}_{i,j}$ into inflation swap rates $\tilde{f}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$. We do not interpolate or adjust maturities smaller than one year, but use the one-year swap rate instead, because the interpolated rates are sensitive to short-term inflation assumptions in that case. We set $\tilde{f}_{0j} = f_1$, $j = 1, 2, \ldots, 12$.

With the inflation swap curve, we can now implement the TIPS–Treasury arbitrage strategy and compute the size of the mispricing in the following way. First, we take a position in a TIPS issue with a semi-annual coupon rate of s and maturity T for a price of V. Each period, the TIPS issue pays coupons of sI_t and makes a principal payment of $100I_T$ at maturity.

Next, we enter into an inflation swap for each coupon payment date $t = 1, 2, \ldots, T$ with notional amount of s for $t < T$ and $s + 100$ for the final principal payment at time T. Let f_t denote the fixed rate on the inflation swap for date $t = 1, 2, \ldots, T$ obtained from the interpolated inflation swap curve. At each coupon payment date t, the inflation swap pays a cash flow of $s(1 + f_t)^t - sI_t$ and $(s + 100)(1 + f_T)^T - (s + 100)I_T$ at maturity T. The sum of the cash flows at date t from the TIPS issue and the inflation swap is constant, since $sI_t + s(1 + f_t)^t - sI_t = s(1 + f_t)^t$. Similarly, at maturity $(s + 100)I_T + (s + 100)(1 + f_T)^T - (s + 100)I_T = (s + 100)(1 + f_T)^T$. This converts all of the indexed cash flows from the TIPS bond into fixed cash flows.

Let P and c denote the price and the semiannual coupon payment for the Treasury bond, respectively. To match the cash flows c from the Treasury bond exactly, the replicating portfolio must include a small long or short position in Treasury STRIPS for each coupon payment date t and the maturity date T, such that $s(1 + f_t)^t + x_t = c$ and $(s + 100)(1 + f_T)^T + x_T = c + 100$,

where x_t denotes the notional amount of STRIPS for date $t = 1, 2, \ldots, T$. This step converts the indexed bond into a synthetic security with fixed cash flows that exactly replicate the magnitude of the cash flows from the Treasury bond. Given the fixed cash flows and the value of the replicating portfolio, we then calculate the yield to maturity for the replicating portfolio.

In the last step, we use the yield to maturity for the replicating portfolio to determine the price of a synthetic Treasury bond with the same maturity, coupon rate, and cash flows as the matched Treasury bond. The difference between the prices of the synthetic Treasury bond and the matched Treasury bond represents the TIPS–Treasury mispricing.

A.2 Measuring Corporate Fixed-Rate and Inflation-Linked Mispricing

Next, we describe how we implement the arbitrage strategy in the corporate bond market to compute the mispricing between corporate fixed-rate bonds and corporate inflation-indexed bonds (linkers). The strategy is analogous to the TIPS–Treasury arbitrage strategy. We create a synthetic fixed-rate bond from a corporate inflation-linker issue by converting the inflation-indexed coupons to fixed payments using inflation swaps. We obtained daily closing prices for corporate inflation-linked and fixed-rate bonds issued by the same companies from the Bloomberg terminal for the period from July, 2004 to November 19, 2009.

Analogous to the TIPS–Treasury analysis, we define maturity mismatch as the number of days between the maturity of a corporate fixed-rate bond issue and that of an inflation-linked bond by the same company. We only include pairs of corporate inflation-linked and fixed-rate bonds in the sample if the maturity mismatch is less than or equal to 31 days. For many inflation-linked issues there is either no pricing data available or the pricing history

consists only of a few observations. Due to this limitation and the 31-day restriction on the maturity mismatch between corporate inflation-linked and fixed-rate bonds, the final dataset consists of six pairs from six distinct companies: Bank of America, Citigroup, J.P. Morgan, Morgan Stanley, Prudential, and Sallie Mae. The maturities range from July, 15 2011 for the Prudential issue to December, 2017 for the Morgan Stanley pair. The maturity mismatches are between zero days for the Bank of America issue and 31 days for the Sallie Mae pair.

In contrast to TIPS, all inflation-linked corporate bonds in the sample pay monthly coupons linked to the realized year-on-year inflation rate. The reference index is the non-seasonally adjusted U.S. CPI index (CPI-U NSA). Furthermore, the corporate inflation-linked issues are not capital indexed, and only the monthly coupon rate varies with realized year-on-year inflation. For all inflation-indexed bonds in the sample the coupon rate is bounded below by zero. For example, the coupon rate for the J.P. Morgan index-linked bond with maturity March, 15 2014 is the maximum of the realized year-on-year inflation rate + 145 bps and zero.

To replicate exactly the semi-annual cash flows of the fixed-rate bond, we use inflation swaps to swap out the inflation-indexed payments of the indexed bond. Implementing the arbitrage strategy requires interpolating the quoted inflation swap rates. Analogous to the TIPS–Treasury analysis, we interpolate the inflation swap rate curve by fitting a standard cubic spline through the quoted maturities using a grid size of one month. Let the interpolated swap rates be denoted by $f_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$, where the first index refers to the year and the second to the month. Analogous to the TIPS–Treasury analysis, we estimate seasonal components in inflation from the monthly nonseasonally adjusted U.S.

CPI index (CPI-U NSA) series between January 1980 and October 2009 by estimating an OLS regression of monthly log changes in the CPI index on month dummies. The seasonally-adjusted forward rates $\tilde{H}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$ are obtained in the same way as in the TIPS–Treasury analysis.

Next, we construct an implied CPI index from the seasonally adjusted forward rates obtained in the last step. Let the implied index level be denoted by $\overline{CPI}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$. Lastly, we compute one-year forward rates from the implied index values $\overline{CPI}_{i,j}$. For $\overline{CPI}_{0,j}$, we use the known reference CPI index level at that date as the base index in the calculation of the forward rates. This reflects that we do not interpolate or adjust maturities smaller than one year and use the one-year swap rate instead as in the TIPS–Treasury analysis. Let the one-year forward rates be denoted by $\tilde{f}_{i,j}$. From now on, we write $f_{i,j}$ instead of $\tilde{f}_{i,j}$.

We then implement the arbitrage strategy and compute the size of the mispricing for corporate bonds in the following way. First, we take a position in a corporate inflation-linked issue with a semi-annual coupon rate of s and maturity T for a price of V. Let I_t denote the year-on-year inflation rate realized at time t. Each period the inflation-linked issue pays a coupon of sI_t and makes a terminal payment of $100 + sI_T$ at maturity. Next, we enter into an inflation swap for each coupon payment date $t = 1, 2, \ldots, T$ with notional of s. At each coupon payment date t, the inflation swap pays a cash flow of $sf_t - sI_t$ and $sf_T - sI_T$ at maturity T. The sum of the cash flows at date t from the index-linked bond and the inflation swap is constant, since $sf_t - sI_t + sI_t = sf_t$. Similarly, at maturity $sf_T - sI_T + 100 + sI_T = 100 + sf_T$. This converts all of the indexed cash flows from the indexed bond into fixed cash flows.

Let P and c denote the price and the semiannual coupon payment for the fixed-rate bond of the same company, respectively. To match the cash flows c from the fixed-rate bond exactly, the replicating portfolio must include short positions in Treasury STRIPS for each monthly coupon of the inflation-linked issue when the coupon payment dates of the inflation-linked and the fixed-rate bonds do not coincide. Furthermore, to ensure that the cash flows of the synthetic security reflect the same credit risk as those from the fixed-rate bond, we adjust the STRIPS prices using credit default swaps spreads. We collect CDS spreads for all six companies during the period from July, 2004 to November 19, 2009 for maturities of 0.5, 1, 2, 3, 4, 5, 6, 7, and 10 years. Intermediate maturities are obtained by linear interpolation. To adjust the STRIPS prices, we first calculate the yields on the STRIPS and then add the CDS spread for the matching maturity. In the last step, we convert the adjusted yields back to obtain CDS-adjusted STRIPS prices. Henceforth, we refer to the adjusted STRIPS simply as STRIPS.

Let the set of dates t at which only the inflation-linked issue pays a coupon be denoted by $\tilde{T}_l$. Similarly, the set of dates t at which both the inflation-linked and fixed-rate bonds have scheduled coupon payments be denoted by $\tilde{T}_f$. More specifically, for all $t \in \tilde{T}_l$, $(sf_t - sI_t) + sI_t + x_t = sf_t + x_t = 0$, where x_t denotes the notional amount of STRIPS for date $t \in \tilde{T}_l$. Similarly, for all $t \in \tilde{T}_f$, $(sf_t - sI_t) + sI_t + x_t = sf_t + x_t = c$. At maturity $T \in \tilde{T}_f$, $(sf_t - sI_t) + 100 + sI_t + x_t = sf_t + 100 + x_t = 100 + c$. To obtain STRIPS prices for all dates t, we linearly interpolate between the quoted prices obtained from the Bloomberg system.

The last step converts the indexed bond into a synthetic security with fixed cash flows that exactly replicate the magnitude of the cash flows from the fixed-rate bond. Given the fixed

cash flows and the value of the replicating portfolio, we then calculate the yield to maturity for the replicating portfolio. The difference in yields between the synthetic and the fixed-rate bond defines the corporate mispricing in yield space. Analogous to the TIPS–Treasury analysis, we use the yield to maturity for the replicating portfolio to determine the price of a synthetic fixed-rate bond with the same maturity, coupon rate, and cash flows as the matched fixed-rate bond. The difference between the prices of the synthetic bond and the matched fixed-rate bond defines the corporate inflation-linked debt mispricing in price space.

Chapter 2: The Inflation-Linked Bond Puzzle

2.1   Introduction

The government bond markets in the United States, the United Kingdom, Japan, Canada, France, Italy, and Germany are among the largest and most actively traded fixed-income markets in the world. Despite this, there are persistent violations of the law of one price within these markets. The inflation-linked bond (ILB) arbitrage strategy is executed by converting the index-linked cash flows from an inflation-indexed bond issue into fixed cash flows using inflation swaps such that the resulting cash flows match exactly the cash flows from a nominal bond with the same maturity date as the index-linked bond issue. Price differences between the inflation-swapped index-linked bond and the actual nominal bond represent violations of the law of one price which will be referred to as ILB mispricing.

Arbitrage mispricing presents a major challenge to classical asset pricing theory. Still, several such puzzles have been documented in the literature.[39] However, ILB arbitrage mispricing in the G7 government bond markets is unique in that it represents one of the largest examples of arbitrage ever documented. In all countries, prices of nominal bonds almost always exceed those of inflation-linked bonds (ILB). The magnitudes are stunning: aggregate mispricing between nominal and inflation-linked bonds (ILB mispricing) in the G7 government bond markets is in excess of $22 billion on average during the period from July 2004 to September 2011. In the aftermath of the 2008 financial crisis, it peaks at $101 billion which represents more than eight percent of the total size of the G7 inflation-linked bond markets. In the

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[39]For examples of significant mispricing in financial markets, see Dammon, Dunn, and Spatt (1993), and Lamont and Thaler (2003), Duffie (2010), Krishnamurthy (2002)), and Longstaff (2004).

United Kingdom, the price of a nominal gilt and an inflation-swapped index-linked gilt issue exactly replicating the cash flows of the nominal gilt can differ by more than $20 per $100 notional.[40]

This paper presents new insights into the properties, dynamics and determinants of arbitrage mispricing in and across seven of the world's largest and most liquid financial markets. Specifically, ILB no-arbitrage violations are positively correlated across markets contemporaneously and in the time series. Furthermore, VAR results reveal that an increase in the mispricing in the United States, for instance, is associated with subsequent increases in ILB mispricing in the other G7 countries, but with a lag of about one to two months. After about twelve months the increase due to the initial shock disappears across all countries. ILB mispricing in any of the G7 countries is strongly forecastable from lagged mispricings in the other countries. Therefore, this paper presents direct evidence that that there is a channel through which arbitrage mispricing propagates across global financial markets. Although these pricing violations occur in very different markets, there is strong commonality between them which is consistent with the existence of a common factor driving these arbitrages. Furthermore, ILB arbitrage produces positively-skewed risk-adjusted excess returns across all countries ranging from 0.51 percent per month in France to 0.69 percent per month in the United States, contrary to the notion that ILB arbitrage is merely a strategy that earns small positive returns most of the time, but occasionally experiences dramatic losses, similar to "picking up nickels in front of a steamroller"[41] or writing deep out-of-the-money puts.

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[40]For simplicity, all bond prices and dollar mispricing values will be expressed in terms of dollars per $100 notional or par amount throughout the paper.

[41]Duarte, Longstaff, and Yu (2007).

Recent theoretical work has put forward potential explanations for the existence of persistent mispricing in financial markets. In particular, Mitchell, Pedersen, and Pulvino (2007) and Duffie (2010) discuss the role that slow-moving capital may play in allowing arbitrage opportunities to exist for extended periods of time. The slow moving capital hypothesis attributes the persistence of arbitrage to various market frictions. It captures the notion that capital constraints, liquidity problems and other frictions limit the speed at which investors can take advantage of mispricings in the market.

This paper not only analyzes ILB mispricing in the context of the slow-moving capital theory, but its findings broaden our understanding of the nature of slow moving capital and the dynamics of arbitrage mispricing. Specifically, changes in the supply of capital available to specific types of arbitrageurs is inextricably linked to subsequent changes in mispricing in specific markets. This captures the notion that not all arbitrageurs are equally able to exploit arbitrage opportunities due to idiosyncratic constraints which may include special knowledge and abilities required in implementing such strategies, available funds, and the ability to take on leverage.[42] The key contribution is that capital available to specific types of arbitrageurs is significantly related to ILB mispricing across all G7 government bond markets. Returns of hedge funds following fixed-income strategies strongly predict subsequent changes in ILB mispricing, whereas returns of hedge funds following non fixed-income related investment styles lack statistically significant forecasting power. Furthermore, ex-ante measures of changes in aggregate investor wealth such as stock, bond, and hedge fund returns predict subsequent changes in ILB mispricing in all countries.

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[42]See, for example, Longstaff, Duarte, and Yu (2007).

This paper also presents new insights into the effects of monetary policy on arbitrage mispricing. In the aftermath of the 2008 financial crisis, central banks implemented large scale asset purchase programs to stabilize financial markets. Coincidentally, ILB mispricing spikes in the G7 countries during the same time period. This paper provides evidence that central bank liquidity programs may have exacerbated ILB mispricing. In the United States, the announcement effect of monetary policy measures is associated with an increase in the mispricing by 94.7 cents per $100 notional, which is consistent with the notion that large scale asset purchases by the major central banks during and in the aftermath of the financial crisis have affected market prices of the government bonds involved in ILB arbitrage, allowing mispricing to persist, and even to increase.

The remainder of this paper is organized as follows. Section 2.2 reviews the extant literature. The ILB arbitrage strategy is described in Section 2.3. Section 2.4 examines the size of the mispricing between cash-flow matched inflation-linked and nominal bonds for all G7 fixed-income markets and in aggregate. Section 2.5 analyzes ILB mispricing in the context of the slow-moving capital theory. Section 2.6 analyzes the impact of monetary policy interventions on the mispricing. The risk-and-return characteristics of ILB arbitrage are analyzed in Section 2.7. Section 2.8 discusses whether there are other factors that could account for the mispricing. Section 2.9 summarizes the results and presents concluding remarks. The Appendix provides introductions to the inflation-linked bond markets in the United States, the United Kingdom, Japan, Canada, France, Italy, and Germany, and describes the dataset for each country. The Appendix also discusses the inflation swaps markets in these countries and describes details on the implementation of the ILB arbitrage strategy.

2.2 Literature Review

This paper contributes to the literature on the pricing of inflation-linked bonds and limits to arbitrage. Other important papers on real bonds include Roll (1996, 2004), Barr and Campbell (1997), Evans (2003), Seppälä (2004), Bardong and Lehnert (2004), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007, 2008), Campbell, Shiller, and Viceira (2009), Dudley, Roush, and Steinberg Ezer (2009), Fleming and Krishnan (2009), Adrian and Wu (2009), Barnes, Bodie, Triest, and Wang (2009), Gürkaynak, Sack, and Wright (2010), Christensen, Lopez, and Rudebusch (2010a, 2010b), Andonov, Bardong, and Lehnert (2010), Pflueger and Viceira (2011a, 2011b), and many others. Fleckenstein, Longstaff and Lustig (2012) were the first to formally study the no-arbitrage relation between TIPS and Treasury bonds and to explore determinants of the mispricing. Their key findings have also been confirmed in subsequent studies, for example, in Haubrich, Pennacchi, and Ritchken (2011).

Examples of fixed-income arbitrage mispricing reported in the literature include Cornell and Shapiro (1990), Amihud and Mendelson (1991), Boudoukh and Whitelaw (1991), Daves and Ehrhard (1993), Kamara (1994), Longstaff (1992, 2004), Grinblatt and Longstaff (2000), Longstaff, Santa Clara, and Schwartz (2001), Yu (2006), Duarte, Longstaff, and Yu (2007), Jordan, Jorgensen, and Kuipers (2000) and many others.

The strand of literature on limits to arbitrage is large. Shleifer and Vishny (1997), Gromb and Vayanos (2002), Liu and Longstaff (2005), Fostel and Geanakoplos (2008), Gorton and Metrick (2009), and Ashcraft, Gârleanu, and Pederson (2010) argue that margins, haircuts,

and other collateral-related frictions may permit arbitrage or deviations from the law of one price to occur. Brunnermeier and Pedersen (2009) emphasize the role that the availability of funding may play in allowing liquidity effects on security prices. Mitchell, Pedersen, and Pulvino (2007) and Duffie (2010) discuss the role that slow-moving capital may play in allowing arbitrage opportunities to exist for extended periods of time. Specifically, Shleifer and Vishny (1997), Liu and Longstaff (2005), and others show that an arbitrageur subject to margin constraints could suffer mark-to-market losses and be forced to liquidate a position in a textbook arbitrage at a loss prior to the date of convergence

The risk-and-return characteristics of arbitrage strategies has received significant attention by academics and practitioners alike. Closest to section 2.7 are the important studies of fixed income arbitrage returns by Duarte, Longstaff, and Yu (2007) and on equity arbitrage strategies by Mitchell and Pulvino (2001) and Mitchell, Pulvino, and Stafford (2002). Duarte, Longstaff, and Yu (2007) study returns on fixed income arbitrage strategies. Important work on equity arbitrage strategies include Mitchell and Pulvino (2001) and Mitchell, Pulvino, and Stafford (2002). There is a large literature focusing on the actual returns reported by hedge funds. These papers include Fung and Hsieh (1997, 2001, 2002), Ackermann, McEnally, and Ravenscraft (1999), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann, and Park (2000), Dor and Jagannathan (2002), Brown and Goetzmann (2003), Getmansky, Lo, and Makarov (2004), Agarwal and Naik (2004), Malkiel and Saha (2004), Chan, Getmansky, Haas, and Lo (2007) and Chan, et al. (2005). There is strand of literature that investigates the profitability of breakeven inflation (BEI) trades in the U.S. market. Bardong and Lehnert (2004a, 2004b, and 2008) study breakeven trades and find that investors were able to earn

positive average excess returns over the 1997–2003 period. Breakeven trades are based on differences between the BEI and inflation forecasts. For instance, if the estimated future inflation rate is higher than BEI, it can be interpreted as a signal of future increases in the BEI, which would lead to an increased demand for TIPS. The trading strategy would then go long in a TIPS position. These results are not unique to the U.S., as Bardong and Lehnert (2004b) find similar results for breakeven trades in French OATi bonds. Andonov, Bardong and Lehnert (2010) find that the break-even strategy is consistently profitable across different forecasting horizons and over three, six and twelve month holding periods even after accounting for trading costs.

This paper also contributes to the literature on the effects of monetary policy actions in the aftermath of the financial crisis. Examples include Baba and Packer (2009), Bauer and Rudebusch (2011), Christensen and Rudebusch (2012), Christensen, Lopez and Rudebusch (2009), D'Amico and King (2011), Gagnon et al. (2011), Goldberg, Kennedy and Miu (2010), McAndrews (2009), McAndrews, Sarkar and Wang (2008), Sarkar and Shrader (2010), Taylor and Williams (2009), Wu (2008), and many others.

A number of important recent papers have studied the impact of liquidity programs that were implemented by the Federal Reserve and other central banks. Taylor and Williams (2009) find that the actual lending from the Federal Reserve's Term Auction Facility (TAF) had no significant impact on easing credit markets. McAndrews, Sarkar and Wang (2008), on the other hand, present evidence that announcements about the TAF did significantly lower credit spreads.

The strand of literature focusing on inflation expectations and breakeven rates includes Sack

and Elsasser (2004), Gürkaynak, Sack, and Wright (2008), Christensen, Lopez and Rude-busch (2010), Gürkaynak, Sack and Wright (2010), Hördahl and Tristani (2007), Hördahl and Tristani (2010).

2.3 The ILB Arbitrage Strategy

The arbitrage strategy is executed in the same way for all seven countries in this study. Without loss of generality, it is described for the U.S. TIPS market. The exact algorithm is described in the Appendix.

An investor buys a TIPS issue at par which has a coupon rate of s per semiannual period. Due to the inflation adjustment, the coupon paid at time t will be sI_t. Concurrently, the investor executes a zero-coupon inflation swap with a maturity date and notional amount matching that of the coupon payment for the TIPS issue. At date t, the inflation swap pays a cash flow of $s(1 + f)^t - sI_t$, where f is the fixed inflation swap rate. The sum of the two cash flows is $sI_t + s(1 + f)^t - sI_t = s(1 + f)^t$ which is constant. Similarly, by executing zero-coupon inflation swaps with maturities and notional amounts matching the indexed cash flows from the TIPS issue, the investor converts all indexed cash flows into fixed cash flows.

Table 2.1 shows the various components of the strategy and their associated cash flows. The first part of the table shows the cash flows associated with a Treasury bond purchased at price P and with a coupon rate of c. The Treasury bond pays a semiannual coupon of c per period with a principal payment of 100 at maturity date T.

Table 2.1 – Cash Flows from the Treasury Bond and the Synthetic Treasury Bond Replicating Strategy. This table shows the cash flow generated each period from the indicated positions. P denotes the price of the Treasury bond with coupon c, V denotes the price of the TIPS bond with the same maturity date as the Treasury bond and a coupon rate of s, and $D(t)$ denotes the price of a Treasury STRIP with a maturity of t. F_t denotes the fixed payment on a zero-coupon inflation swap of maturity t (calculated as $(1+f)^t$, where f is the corresponding inflation swap rate). The inflation index I_t denotes the ratio of the CPI-U index at time t divided by the CPI-U index at time zero.

Strategy	0	1	2	3	...	T
Buy Treasury	$-P$	c	c	c	...	$c+100$
Buy TIPS	$-V$	sI_1	sI_2	sI_3	...	$(s+100)I_T$
Inflation Swap$_1$	0	$s(F_1-I_1)$	0	0	...	0
Inflation Swap$_2$	0	0	$s(F_2-I_2)$	0	...	0
Inflation Swap$_3$	0	0	0	$s(F_3-I_3)$	...	0
...	...	...	...	...	...	...
Inflation Swap$_T$	0	0	0	0	...	$(s+100)(F_T-I_T)$
STRIPS$_1$	$-(c-sF_1)D(1)$	$c-sF_1$	0	0	...	0
STRIPS$_2$	$-(c-sF_2)D(2)$	0	$c-sF_2$	0	...	0
STRIPS$_3$	$-(c-sF_3)D(3)$	0	0	$c-sF_3$	...	0
...	...	...	...	...	...	...
STRIPS$_T$	$-((c+100)D(T)-$ $(s+100)F_TD(T))$	0	0	0	...	$(c+100)-$ $(s+100)F_T$
Total Cash Flow	$-(\sum_{i=1}^{T}(c-sF_i)\,D(i)+$ $100(1-F_T)D(T)+V)$	c	c	c	...	$c+100$

The second part of the table shows how the cash flows from the Treasury bond are replicated exactly from a TIPS position. First, the arbitrageur purchases a TIPS issue with a coupon rate of s and the same maturity date as the Treasury bond for a price of V. The TIPS bond pays coupons of sI_t each period, and then makes a principal payment of $100I_T$ at maturity. The arbitrageur then enters into an inflation swap for each coupon payment date with a notional amount of s (or $s + 100$ for the final principal payment date). This converts all indexed cash flows from the TIPS into fixed cash flows. To match exactly the cash flows from the Treasury bond, the arbitrageur also goes long or short a small amount of Treasury STRIPS for each coupon payment date. As shown at the bottom of the second part of the table, the net result is a portfolio that exactly replicates the cash flows from the Treasury bond in the first part of the table.

Inflation-indexed bonds and nominal bonds are matched based on their respective maturities. The number of days between the maturity of an inflation-indexed bond issue and that of a nominal bond with the closest maturity to that of the index-linked issue is defined as maturity mismatch. To adjust for differences in maturity, the yield to maturity on the synthetic fixed rate bond formed from the inflation-linked bond issue and the inflation swaps is applied to obtain the price of a synthetic nominal bond that would exactly match the maturity of the Treasury bond in the pair. It is important to note that for any maturity mismatch, the cash flows of the synthetic nominal bond always exactly match those of the underlying nominal bond by construction.

Dollar mispricing is defined as the price difference per \$100 notional between the nominal bond and the synthetic nominal bond formed from the inflation-linked bond and the inflation

swaps. Similarly, basis-point mispricing is defined as the difference in yields to maturity between the nominal bond and the synthetic nominal bond formed from the inflation-linked bond and the inflation swaps.

In most of the study, ILB mispricing is aggregated at the country level. The ILB dollar mispricing index for each country is constructed for each day during the sample period as the linker–notional-weighted average mispricing per $100 notional for all individual nominal–inflation-linked pairs in that country on that day. Similarly, the ILB basis-point mispricing index for each country is constructed for each day during the sample period as the linker–notional-weighted average basis-point mispricing per $100 notional for all individual nominal–inflation-linked pairs in that country on that day. The G7 dollar mispricing index is constructed by taking the notional-weighted average of the individual country's index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country. The G7 basis-point mispricing index is constructed by taking the notional-weighted average of the individual country's basis-point mispricing expressed in basis points across the pairs included in the sample for that country.

2.4 Magnitude of ILB Mispricing

This section describes the magnitude of ILB mispricing in the United Kingdom, Japan, Canada, France, Germany, Italy, and the United States. Table 2.2 reports summary statistics for the indexed-bond-nominal bond mispricing for all G7 countries separately, and in aggregate. The left panel shows summary statistics for the mispricing measured in dollars per $100 notional, and the right panel for the mispricing measured in basis points. The

mispricing in each country is the weighted-average index-linked-nominal bond mispricing, expressed in units of dollars per \$100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked bond issue. The basis-point mispricing for each country is the weighted-average index-linked-nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked bond issue. The middle panel in Table 2.2 shows summary statistics for the period from June 14, 2007 to September 20, 2011 in which all countries are included. The bottom panel in Table 2.2 shows summary statistics for the period from May 22, 2006 to September 20, 2011 which excludes Canada and Japan, since both countries enter the sample period at later dates.

Table 2.2 shows that in all countries, prices of nominal bonds always exceed those of their inflation-linked counterparts on average. In the aftermath of the financial crisis of 2008, ILB mispricing peaks at \$101 billion which represents more than eight percent of the total size of the inflation-linked bond markets in the study. On average, aggregate G7 ILB mispricing is \$22.09 billion which represents 1.40% of the total G7 Index-Linked notional amount outstanding. Over the sample period from July 2004 until September 2012, nominal bonds are always relatively more expensive than their cashflow-matched inflation-linked counterparts: aggregate G7 mispricing is always in excess of \$1.8 billion. The right panels in the fourth row of Figures 2.1 and 2.2 plot the time-series of the G7 dollar and basis-point mispricing indices, respectively. In aggregate, the average G7 mispricing is \$1.93 for the dollar and 33.54 basis points for the basis-point mispricing, respectively. In early 2009, the mispricing peaked

Table 2.2 – **Summary Statistics for Indexed-Bond–Nominal Bond Mispricing.**
This table reports summary statistics for the indexed-bond–nominal bond mispricing for all G7 countries separately, and in aggregate. The left panel reports summary statistics for the mispricing measured in dollars per $100 notional, and the right panel for the mispricing measured in basis points. The mispricing in each country is the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked bond issue. The basis-point mispricing for each country is the weighted-average index-linked–nominal Bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked bond issue.

Country	Dollar Mispricing				Basis-Point Mispricing				N
	Mean	SDev	Min	Max	Mean	SDev	Min	Max	
United States	2.23	1.79	0.18	11.77	39.81	42.36	2.31	292.72	1868
United Kingdom	1.14	2.36	−2.17	14.09	16.36	40.51	−29.24	203.66	1868
Japan	2.16	1.73	−3.24	9.63	31.46	24.67	−44.33	139.64	1187
Canada	1.02	2.95	−5.27	9.60	7.23	20.42	−39.35	67.99	1114
France	0.40	0.73	−1.04	4.01	6.87	17.23	−32.06	103.46	1868
Italy	0.56	0.96	−0.77	6.43	8.77	18.48	−37.35	114.24	1868
Germany	1.62	0.97	−0.37	5.41	28.13	22.03	−8.39	99.80	1392
G7	1.93	1.56	0.02	9.46	33.54	34.48	2.72	101.24	1868

June 2007 – September 2011

Country	Dollar Mispricing				Basis-Point Mispricing				N
United States	2.68	2.19	0.24	11.77	50.17	51.87	2.31	292.72	1114
United Kingdom	1.77	2.83	−2.17	16.22	28.18	48.53	−29.24	203.66	1114
Japan	2.28	1.71	−3.24	9.63	33.24	24.36	−44.33	139.64	1114
Canada	1.02	2.95	−5.27	9.60	7.23	20.42	−39.35	67.99	1114
France	0.58	0.87	−1.04	4.01	10.34	21.38	−32.06	103.46	1114
Italy	0.86	1.12	−0.77	6.43	13.05	10.32	−37.35	114.24	1114
Germany	1.71	1.12	−0.37	5.41	31.28	19.90	−8.39	99.80	1114
G7	2.08	1.86	0.02	9.46	31.72	39.19	2.91	101.24	1114

May 2006 – September 2011

Country	Dollar Mispricing				Basis-Point Mispricing				N
United States	2.51	1.99	0.24	11.77	46.42	47.16	2.31	292.72	1392
United Kingdom	1.45	2.62	−2.17	16.22	22.04	45.18	−29.24	203.66	1392
France	0.52	0.80	−1.04	4.01	9.10	19.38	−32.06	103.46	1392
Italy	0.75	1.03	−0.77	6.43	11.50	20.56	−37.35	114.24	1392
Germany	1.62	0.97	−0.37	5.41	28.13	22.03	−8.39	99.80	1392
G7	1.98	1.76	0.02	9.46	36.78	37.09	2.91	101.24	1392

at \$9.46, or 226.87 basis points which amounts to \$101 billion dollars in terms of the total notional amount of G7 Index-Linked debt outstanding. These findings are significant since the G7 government bond markets are among the largest and most-liquid financial markets in the world.

In the United States, the overall average sizes of the dollar and basis-point mispricing are \$2.23 and 39.81 basis points, respectively. The amount of mispricing peaked at \$11.77 or 292.72 basis points around the time of the Lehman bankruptcy in the Fall of 2008, but there were clearly earlier periods when the average mispricing was in excess of \$3 or 60 basis points. The top left panel in Figure 2.1 plots the weighted-average dollar mispricing for the TIPS-Treasury pairs. There is significant time series variation in TIPS-Treasury mispricing throughout the sample period. ILB mispricing for individual TIPS–Treasury pairs reaches values in excess of \$10 for many of the TIPS-Treasury pairs with maturities exceeding 2015. In fact, the mispricing for the TIPS-Treasury pair maturing in 2025 reaches a level in excess of \$23. In almost every case, the value of the Treasury bond is larger than its synthetic equivalent constructed from the matching TIPS issue and the inflation swap.

In the United Kingdom, ILB mispricing is most significant during the crisis period of 2008–2009. In particular, the amount of mispricing peaked at \$14.07 or 203.66 basis points around the time of the Lehman bankruptcy in the fall of 2008. Figures 2.1 and 2.2 show significant time series variation in index-linked–nominal gilt mispricing throughout the sample period. These results do not depend on whether the index-linked bond has an eight-month indexation or a three-month lag. The average basis-point size of the mispricing is fairly uniform across all maturities and lags of the individual pairs. During the period leading up to the financial
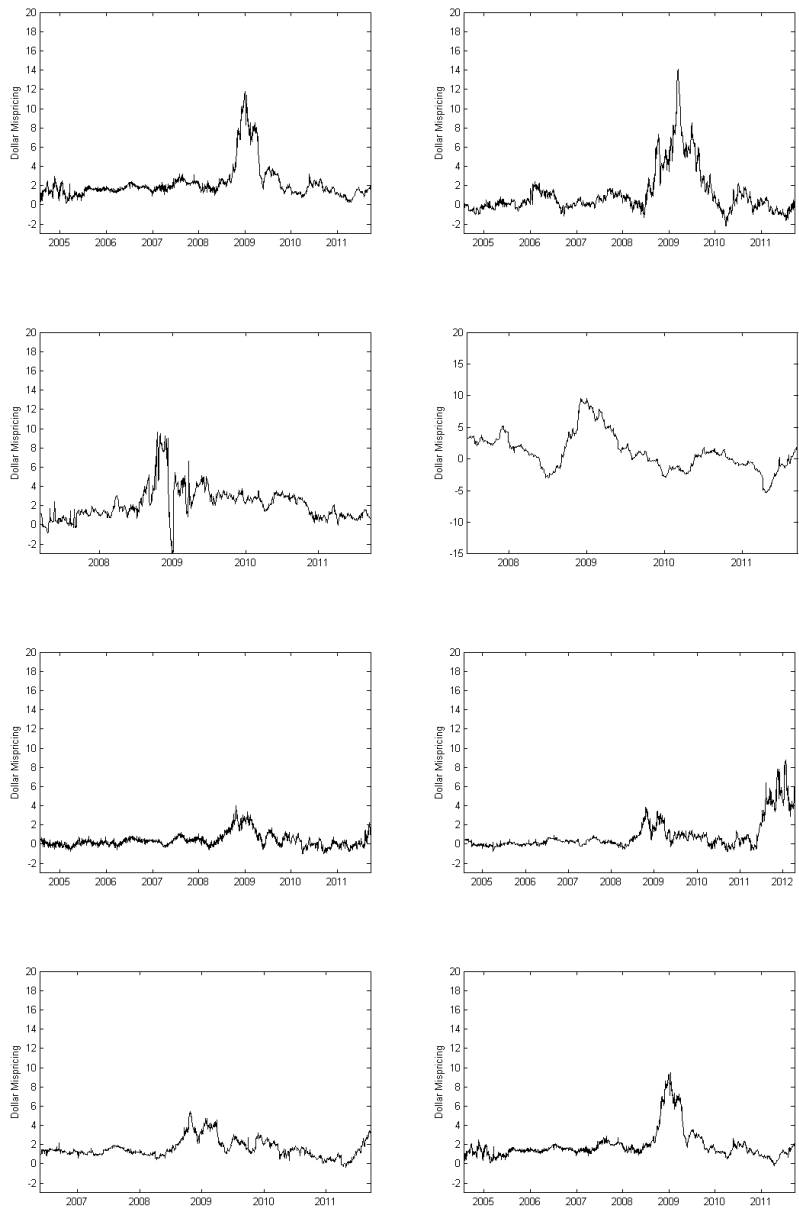
Figure 2.1 – ILB Mispricing. This figure plots the time series of the weighted-average nominal–index-linked bond mispricing for all countries in the study, as well as the aggregate ILB mispricing in the G7 countries. From the top-left to the bottom-right: United States, United Kingdom, Japan, Canada, France, Italy, Germany, Aggregate G7. ILB mispricing is expressed in units of dollars per $100 notional, across the pairs included in the sample, where the average is weighted by the notional amount of the index-linked issue.
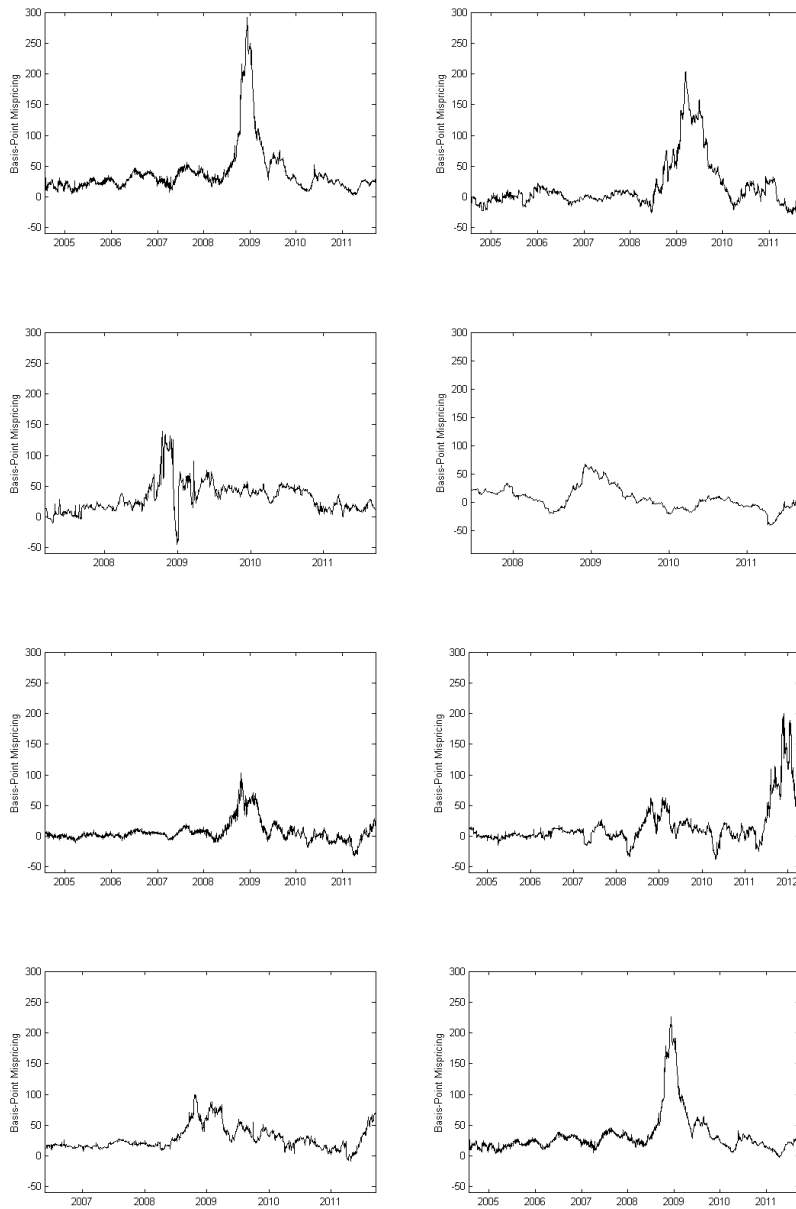
Figure 2.2 – ILB Basis-Point Mispricing. This figure plots the time series of the weighted-average nominal–index-linked bond mispricing for all countries in the study, as well as the aggregate ILB mispricing in the G7 countries. From the top-left to the bottom-right: United States, United Kingdom, Japan, Canada, France, Italy, Germany, Aggregate G7. ILB mispricing is expressed in basis points, across the pairs included in the sample, where the average is weighted by the notional amount of the index-linked issue.

crisis, the mispricing in the U.K. is only about half that observed in the United States. The overall average size of the mispricing is $1.14 compared to $2.23 in the United States. Similarly, the overall average basis-point size of the mispricing is 16.36 basis points in the U.K. versus 39.81 basis points in the United States. Hence, the mispricing is about twice as large in the United States compared to the United Kingdom. In the aftermath of the financial crisis, there are times when the mispricing switches sign. This never occurs in the United States during the whole sample period. Similar observations hold for the basis-point mispricing. However, the magnitudes of these sign reversals are small, and compared to the mispricing observed during the financial crisis, hardly different from zero. Specifically, the overall average size of the negative mispricing is only -0.49 basis points and the maximum in absolute terms is -2.17 basis points. The standard deviation is 0.3877 basis points. A simple test of means applied to the sample period excluding the financial crisis, defined as the time span from September 1, 2008 until January 1, 2010, does not reject the null hypothesis at the five percent significance level that the mean of the basis-point mispricing is zero during that period.

The government bond market in Japan exhibits mispricing throughout the entire sample period, not just during the crisis period of 2008-2009. In particular, while the amount of mispricing peaked at $9.63 or 139.64 basis points around the time of the Lehman bankruptcy in the Fall of 2008, there were clearly earlier periods when the average mispricing was in excess of $2 or about 40 basis points. In addition, Figure 2.1 and 2.2 show particularly strong time series variation in JGB-JGBi mispricing throughout the sample period. The large decline during the financial crisis stands out. The overall average sizes of the dollar and basis-point

mispricing are \$2.16 and 31.46 basis points, respectively. While the mispricing in the U.S. is uniformly positive, ILB mispricing in Japan reverses sign when the Japan Ministry of Finance introduced index-linked bonds and in the aftermath of the financial crisis. The dollar and basis-point mispricing reached levels of $-\$3.24$ and -44.33 basis points around December 2008, respectively. When this period is excluded from the sample, the mispricing is almost always positive. A few sign reversals occur during the first half of 2007, but a simple test of the mean cannot reject that the mispricing is zero during that period at the five percent significance level. The stark decline in the dollar and basis-point mispricing around December 2008 is a result of policy intervention by the Ministry of Finance, when it started to aggressively buy back Japanese inflation-indexed bonds. However, Figure 2.1 and 2.2 show that the mispricing quickly changes sign again after the policy intervention in the first half of 2010 which suggests that the buy-back programs only provided temporary relief.

The middle right panels of Figure 2.1 and 2.2 show ILB mispricing in Canada. In contrast to the other G7 countries, ILB mispricing in the Canadian government bond market exhibits strong time variation with frequent sign reversals. The mispricing is most evident during the crisis period of 2008-2009 where the amount of mispricing peaked at \$9.60 or 67.99 basis points around the time of the Lehman bankruptcy in the fall of 2008. However, ILB mispricing reaches levels in excess of \$5 (35 basis points) even before the financial crisis. Furthermore, the mispricing switches sign during the first half of 2008 before rising dramatically at the onset of the financial crisis. Towards the end of 2009 the mispricing switches sign again, but reverses to a level of around \$2 around July 2011. A simple test of means applied to the sample period excluding the financial crisis, defined as the time span

from September 1, 2008 until January 1, 2010, does not reject the null hypothesis at the five percent significance level that the mean of the basis-point mispricing is zero during that period. Therefore, while there is overwhelming evidence of ILB mispricing in Canada during the financial crisis, there is no statistically significant evidence of arbitrage mispricing apart from this period.

Figures 2.1 and 2.2 show that ILB mispricing in France is significantly smaller in magnitude than in the United States, the United Kingdom, and Japan. ILB mispricing is most significant during the 2008–2009 crisis where the amount of mispricing peaked at $4.01 or 3.91 basis points around the time of the Lehman bankruptcy in the fall of 2008. These results do not depend on whether the index-linked bond has the French CPI or the Europen HICPX as reference index. The average basis-point size of the mispricing is fairly uniform across all maturities and the two reference indices. In contrast to the United States and Japan, there is significantly less time series variation in ILB mispricing before and after the financial crisis. In the period leading up to the financial crisis, the average mispricing in France is only about 18 percent of the U.S. mispricing. The overall average sizes of the dollar and basis-point mispricing are only $0.40 and 6.87 basis points, respectively, compared to $2.23 and 39.81 basis points in the United States. In the United States, the average size of the mispricing between the TIPS and Treasury bonds maturing in January 2027 and February 2027, respectively, is $4.49. In the French government bond market, by contrast, the average mispricing never exceeds $1.80 for any pairs. While the mispricing in the U.S. is uniformly positive, the dollar and basis-point mispricing in the French market switch sign several times during the sample period. Even before the financial crisis, there are periods

where the basis-point mispricing becomes negative, and in the aftermath of the financial crisis, the mispricing is negative for prolonged periods. Interestingly, the mispricing surges again around June 2011 at the onset of the European debt crisis. The overall mean of the negative ILB mispricing is -3.91 basis points and the standard deviation is 6.30 basis points. The French inflation-linked bonds have a par floor. Section 2.8.5 discusses in detail that the negative mispricing is overestimated in absolute terms due to this feature. Specifically, if the deflation floor is taken into account, a simple back-of-the envelope calculation using the estimates in Heider, Li, and Verma (2012) for the value of the par floor (mean estimate of 3.22 basis points with the maximum estimated at 5.5 basis points) shows that the average ILB mispricing is $-3.91 + 3.22 = -0.69$ basis points during the periods when ILB mispricing is negative. A simple test of means cannot reject the null hypothesis that the mean of the basis-point mispricing is zero before and after the financial crisis at the five percent significance level. Therefore, there is overwhelming evidence of ILB mispricing in France during the financial crisis and at the onset of the European debt crisis. However, there is no statistically significant evidence of arbitrage mispricing apart from this period when the value of the deflation floor is accounted for.

ILB mispricing in the Italian government bond market is most significant during the crisis period of 2008–2009, and at the onset of the European debt crisis in Summer 2011 where it surges even higher, peaking at $6.43 (114.24 basis points). The bottom left panels in Figure 2.1 and 2.2 show that there is significant time series variation in index-linked–nominal BTP mispricing throughout the sample period. In the Italian government bond market, the average sizes of the mispricing across all BTP pairs are significantly smaller than in the

U.S. government bond market. First, during the financial crisis the mispricing in the United States peaks at \$11.77 or 292.72 basis points around the time of the Lehman bankruptcy in the fall of 2008. The maximal dollar and basis points mispricing for the index-linked–nominal BTP pairs never exceed \$4 or and 70 basis points during that period. Second, in the period leading up to the financial crisis, the average mispricing in Italy is only about one quarter that observed in the United States. The overall average size of the mispricing is only \$0.56 compared to \$2.23 in the United States. The overall average basis-point size of the mispricing is 8.77 basis points in Italy and 39.81 basis points in the U.S. While the mispricing in the U.S. is uniformly positive, the dollar and basis-point mispricing in the Italian government bond market switch sign several times during the sample period. Even before the financial crisis, there are periods where the basis-point mispricing becomes negative. and in the aftermath of the financial crisis, the mispricing is negative for prolonged periods. The overall mean of the negative ILB mispricing is -3.55 basis points and the standard deviation is 8.74 basis points. Inflation-linked BTP have a par floor. Section 2.8.5 discusses that the negative mispricing is overestimated in absolute terms as a result of this feature. If the deflation floor is taken into account, a simple calculation using the estimates in Heider, Li, and Verma (2012) for the value of the par floor (mean estimate of 2 basis points with the maximum estimated at 3 basis points) shows that the average ILB mispricing is $-3.55 + 2 = -1.55$ basis points during the periods when ILB mispricing is negative. A simple test of means cannot reject the null hypothesis that the mean of the basis-point mispricing is zero before and after the financial crisis at the five percent significance level. Therefore, there is overwhelming evidence of ILB mispricing in Italy during the financial crisis and at the onset of the European debt crisis.

In Germany, ILB mispricing is highest during the crisis period of 2008–2009 where it peaks at $5.41 and 99.80 basis points. Furthermore, at the onset of the European debt crisis in 2011, the mispricing surges again reaching values in excess of $3 (70 basis points). The average sizes of the mispricings across all Bund pairs are significantly smaller than in the United States. The average mispricing never exceeds $6.06 for any pair, and the overall average basis-point size of the mispricing is 28.13 basis points in Germany compared to 39.81 basis points in the United States. Furthermore, there is little variation in the average dollar mispricing across all maturities which never exceeds $2. The dollar and the basis-point mispricing in the German government bond market switch sign several times during the sample. Even before the financial crisis, there are periods where the basis-point mispricing becomes negative, and in the aftermath of the financial crisis, the mispricing is negative for prolonged periods. The overall mean of the negative ILB mispricing is -3.01 basis points and the standard deviation is 2.20 basis points. Inflation-linked Bunds feature a par floor. A simple back-of-the envelope calculation using the estimates in Heider, Li, and Verma (2012) for the value of the par floor (mean estimate of 2.80 basis points with the maximum estimated at 3.80 basis points) shows that the average ILB mispricing is $-3.01 + 2.80 = -1.55$ basis points during the periods when ILB mispricing is negative. A simple test of means cannot reject the null hypothesis that that the mean of the basis-point mispricing is zero before and after the financial crisis at the five percent significance level. Therefore, there is overwhelming evidence of ILB mispricing in Germany during the financial crisis and at the onset of the European debt crisis. However, there is no statistically significant evidence of arbitrage mispricing apart from this period. This stands in stark contrast to the observations in the United States where there is clear evidence of mispricing even before and after the financial crisis.
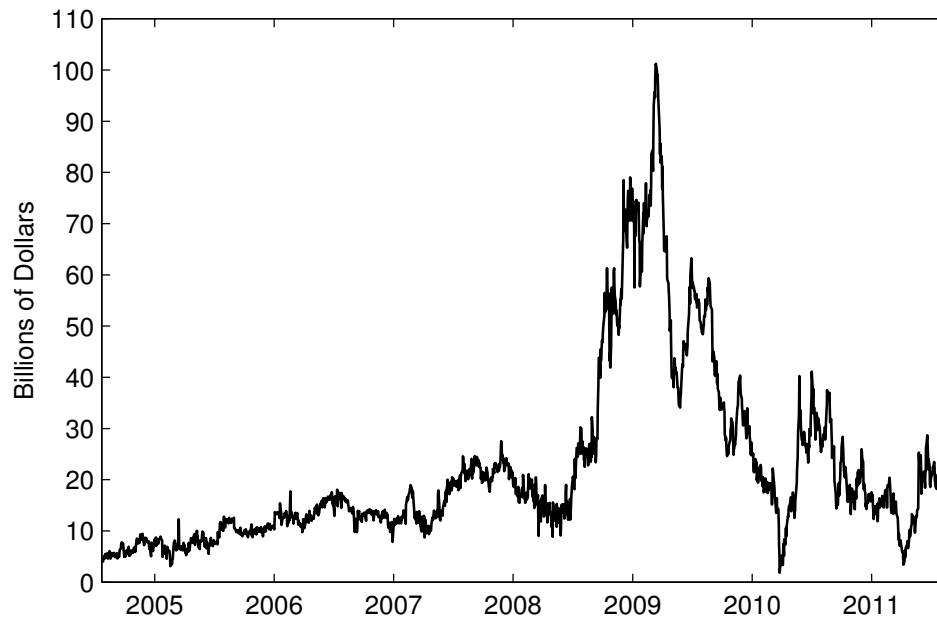
Figure 2.3 – **Aggregate G7 ILB Mispricing.** This figure plots the time series of the aggregate nominal–index-linked bond mispricing in all G7 countries, measured in billions of dollars.

2.5 Tests of the Slow-Moving Capital Theory

This section analyzes ILB mispricing in the context of the implications from the slow moving capital literature. At the core of the theory is the notion that arbitrage opportunities may persist because capital constraints, liquidity problems, and other frictions limit the speed at which investors can take advantage of mispricings in the market.

The key finding is that capital available to specific types of arbitrageurs is significantly related to ILB mispricing across all G7 government bond markets. Returns of hedge funds following fixed-income strategies strongly predict subsequent changes in ILB mispricing,

whereas returns of hedge funds following non fixed-income related investment styles lack statistically significant forecasting power. This broadens our understanding of the nature of slow moving capital and the dynamics of arbitrage mispricing. Specifically, changes in the supply of capital available to specific types of arbitrageurs is inextricably linked to subsequent changes in mispricing in specific markets. This is consistent with the notion that not all arbitrageurs are equally able to exploit arbitrage opportunities due to idiosyncratic constraints which may include special knowledge and abilities required in implementing such strategies, available funds, and the ability to take on leverage.[43]

The slow-moving capital theory predicts that in response to a downward shock in the aggregate amount of capital available to arbitrageurs, the amount of mispricing between securities is expected to widen in multiple markets simultaneously, even if the violations of the law of one price occur in vastly different markets. Therefore, the slow moving capital literature predicts that different types of arbitrages may be correlated. All correlations between the dollar and basis-point mispricings for all G7 countries during the period from June 14, 2007 to September 20, 2011 using daily data are positive and large in magnitude. Canada, France, Germany, and the U.K. all have correlation coefficients in excess of 0.7 to the United States during the period June 14, 2007 to September 20, 2011. Similar results hold for the subperiods May 22, 2006 to September 20, 2011 which excludes Canada and Japan, and the period July 23, 2004 to September 20, 2011, which excludes Germany, Canada and Japan. Furthermore, ILB mispricings in all other countries are strongly positively correlated with correlation coefficients in excess of 0.30. These observations are consistent with the notion that violations of the law of one price are correlated, even across global financial markets.

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[43]See, for example, Longstaff, Duarte, and Yu (2007).

Table 2.3 reports summary statistics for the regression of monthly changes in average basis-point ILB mispricing in each country on changes in average basis-point mispricing in the previous month in the other G7 countries. All regression coefficients are positive and the R^2 test statistics are all in excess of 0.5, with exception of the United States where the R^2 is 0.48. Furthermore, there is evidence of regional clustering. Within the European Union, for example, each country's mispricing is statistically significantly related to the other member countries' ILB mispricings.

Table 2.3 – Results from the Forecasting Regression of Monthly Changes in Average BPS Mispricing on Lagged ILB Mispricing. This table reports summary statistics for the regression of monthly changes in average basis-point ILB mispricing in each country on one-month lagged changes in average basis-point ILB mispricing in the other G7 countries. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The explanatory variables correspond to the rows in the table. For each country, the left column shows the regression coefficient associated with the explanatory variable in that row, and the right column shows the corresponding Newey-West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from June 14, 2007 to September 20, 2011.

	Canada		France		Germany		Italy		Japan		UK		USA	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
Intercept	0.11	0.12	0.21	0.22	0.15	0.36	0.18	0.14	0.12	1.02	0.13	0.81	0.17	0.19
Canada	–	–	0.11	0.87	0.01	1.11	0.04	0.66	0.14	0.73	0.38	1.44	0.22	0.98
France	0.21	0.22	–	–	0.97	2.36**	1.02	2.23**	0.38	1.23	0.69	1.61*	0.63	1.43
Germany	0.29	1.54	0.98	2.36**	–	–	0.92	2.56**	0.46	1.59*	0.81	1.78*	0.79	1.69**
Italy	0.14	1.23	0.85	1.99**	0.84	2.13**	–	–	0.12	1.11	0.52	1.39	0.61	1.23
Japan	0.81	1.58*	0.29	1.51	0.09	1.42	0.11	1.34	–	–	0.61	1.67*	0.85	1.82*
UK	0.97	1.78*	0.77	1.65*	0.78	1.81*	0.72	1.89*	0.84	1.76*	–	–	0.91	2.01**
USA	1.23	2.89**	1.01	2.25**	0.94	2.46**	0.97	2.17**	1.09	1.99**	1.11	2.19**	–	–
Adj. R^2	0.69		0.62		0.57		0.60		0.52		0.55		0.48	
N	51		51		51		51		51		51		51	

The question naturally arises whether there is a channel through which mispricing propagates across global financial markets. Specifically, following a sudden increase in ILB mispricing in the United States, will there be a subsequent spike in ILB mispricing in the other G7 countries? Furthermore, how long does it take for ILB mispricing to manifest itself in the other countries? To investigate these questions, I estimated a vector auto regression on the dollar and basis point mispricing indices for the United States, Europe, the United Kingdom, and Japan using four lags on monthly data. The number of autoregressive lags is determined based on the Akaike information criterion. The top and bottom panels in Figure 2.4 show the impulse response of the dollar and basis-point mispricing to a one standard-deviation shock to the mispricing in the United States. The horizontal axis denotes the number of months after the shock and the vertical axis shows the relative change in the mispricing in the United States, Europe, Japan, and the United Kingdom to the case when there is no shock in the United States. In response to a shock in the United States, the mispricing increases in all other G7 countries, and peaks approximately after two months. In Europe, the basis point mispricing is about five percent larger two months after the initial shock compared to when the United States experiences no shock. The other G7 countries exhibit similar dynamics. At the twelve month mark, the impact of the initial shock to the mispricing in the U.S. has almost died out. In summary, there is strong evidence that the occurrence of ILB arbitrage is strongly linked across markets. ILB arbitrage mispricings are positively correlated, and there is a channel through which arbitrage mispricing is transmitted across markets. Although ILB arbitrages occur in very different markets, there is strong commonality among them which suggests that these types of arbitrages are driven by a common factor.
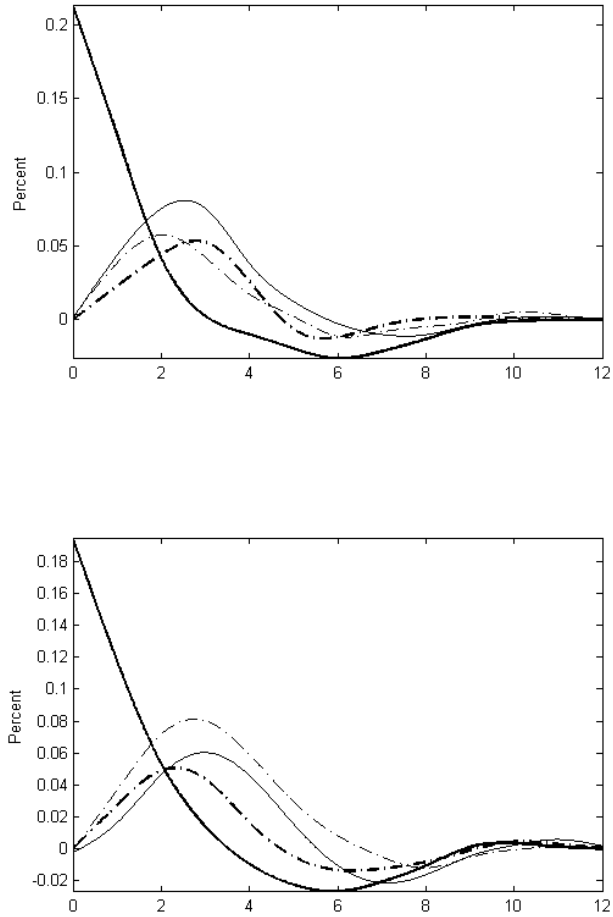
Figure 2.4 – Impulse Response of Nominal–Index-Linked Bond Mispricing. This figure plots the impulse response of the average nominal–index-linked bond mispricing in United States, Europe, Japan, and the United Kingdom to a one standard deviation shock to the mispricing in the United States, expressed as percent change to the base case when no shock occurs. The solid thick line represents the United States, the thick dashed line Europe, the solid thin line represents the United Kingdom, and the thin dashed line represents Japan. The top panel shows the dollar and the bottom panel the basis point mispricing.

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The slow-moving capital theory implies that changes in the supply of capital to arbitrageurs may have forecasting power for subsequent changes in ILB mispricing. Specifically, if capital flows slowly to global arbitrageurs after a negative shock to aggregate arbitrage capital, then a sudden increase in capital will tend to reduce mispricing, but only with a lag as arbitrage capital is being deployed. To empirically test this prediction, Table 2.4 presents results from the regression of monthly changes in average basis-point ILB mispricing of each country on lagged stock, bond, and volatility index returns. The equity index for each country is the MSCI Index for that specific country in the previous month. The bond index is the Bloomberg EFFA government bond index for that country with maturity exceeding two years in the previous month. Hedge Fund denotes the return on Bloomberg BAIF Government and Corporate Bonds Hedge Fund Index in that specific country in the previous month. All member funds for each of the seven country indices are incorporated in that specific country. The JP Morgan G7 Volatility Index is a measure of market distress and captures the notion that arbitrageurs may be more constrained in times of financial turbulence. I regress changes in the average basis point mispricing for each country on one month lagged returns on the three indices. Although not shown, similar results hold for the dollar mispricing. Table 2.4 shows summary statistics of the regression results. In the United States, the U.S. basis point mispricing narrows by 2.33 basis points when the MSCI index returns one percent in the previous month, and in France the basis point mispricing widens by 1.89 basis points when the one month prior G7 volatility index increased by one percent. The regression coefficients in all countries are consistent with the notion that when capital flows slowly to arbitrageurs, then an increase in capital today will tend to reduce mispricing in the markets with a lag.

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Table 2.4 – Results from the Forecasting Regression of Monthly Changes in Average BPS Mispricing on Lagged Stock, Bond, and Hedge Fund Returns. This table reports summary statistics for the regression of monthly changes in average basis-point ILB mispricing of each country on the indicated lagged stock, bond, and volatility index returns. Stock denotes the return on the MSCI Index for that specific country in the previous month. Bond denotes the return on the Bloomberg EFFA government bond index with maturity exceeding two years in the previous month. Hedge Fund denotes the return on Bloomberg BAIF Government and Corporate Bonds Hedge Fund Index in that specific country in the previous month. All member funds for each of the seven country indexes are incorporated in that specific country. Vola denotes the return on the JP Morgan G7 Volatility Index in the previous month. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country, the left column shows the regression coefficient, and the right column shows the Newey-West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from June 14, 2007 to September 20, 2011.

	Canada		France		Germany		Italy		Japan		UK		USA	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
Intercept	0.63	1.43	0.42	1.07	0.56	1.28	0.88	1.51	0.49	1.16	0.82	1.59	0.91	1.61
Stock	−1.51	−1.72*	−2.07	−1.81*	−2.46	−2.52**	−2.61	−2.86**	−2.11	−1.94*	−2.81	−2.91**	−2.33	−2.71**
Bond	−3.67	−1.64*	−3.88	−1.99**	−4.22	−2.55**	−3.71	−1.97**	−3.11	−1.91*	−4.49	−3.01**	−4.39	−2.89**
Hedge Fund	0.34	1.04	−3.29	−2.02**	−3.58	−3.19**	−2.19	−1.52	−2.88	−3.09**	−3.68	−3.52**	−3.41	−3.19**
Vola	1.26	1.51	1.89	1.88*	1.99	1.89*	1.41	1.54	2.36	1.61*	2.16	1.53	2.68	2.21**
Adj. R^2	0.376		0.451		0.413		0.397		0.448		0.468		0.492	
N	51		86		64		86		54		86		86	

The ILB arbitrage could essentially be riskless from the perspective of a relatively uncon-strained arbitrageur, such as a sovereign wealth fund, yet risky from the perspective of a highly leveraged and constrained hedge fund. Therefore, it is natural to expect that changes in the supply of capital available to specific types of arbitrageurs should be inextricably linked to subsequent changes in ILB mispricing in specific markets. Specifically, changes in the supply of capital to fixed-income arbitrageurs should forecast subsequent changes in fixed-income mispricing. This captures the notion that not all arbitrageurs are equally able to exploit arbitrage opportunities due to idiosyncratic constraints which may include spe-cial knowledge and abilities required in implementing such strategies, available funds, and the ability to take on leverage.[44] To empirically test this predication, I regress monthly changes in the average basis-point mispricing on one-month lagged monthly returns on the HFRX Hedge Fund indices. These hedge fund indices are classified by investment style.[45] The HFRX Macro Strategy Index consists of strategies in which the investment process is predicated on movements in underlying macroeconomic variables and the impact these have on equity, fixed income, currency and commodity markets. Funds in the Equity Hedge category maintain positions both long and short in primarily equity and equity derivative securities. The HFRX Event Driven Strategy Index consists of funds that maintain positions in companies currently or prospectively involved in corporate transactions of a wide variety including but not limited to mergers, restructurings, financial distress, tender offers, share-holder buybacks, debt exchanges, security issuance or other capital structure adjustments. The HFRX Relative Value index consists of funds that maintain positions in which the in-

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[44]See, for example, Duarte, Longstaff, and Yu (2007)

[45]For detailed description of these style categories see http://www.hedgefundresearch.com/index.php?fuse=indices-str#2889.html

vestment thesis is predicated on realization of a valuation discrepancy in the relationship between multiple securities ranging across equity, fixed income, derivatives or other security types. Intuition suggests that relative value funds would be more likely to engage in ILB arbitrage. Table 2.5 presents summary statistics of the regression results. For each country in the study, the relative value funds significantly predict changes in ILB mispricing at the five percent level. Consistent with theory, all regression coefficient are negative. The Equity Hedge and Event Driven indices are not statistically predictors of changes in ILB mispricing. These results confirm the notion, that a negative wealth shock at time $t-1$ will constrain specific arbitrageurs and mispricing widens. Conversely, after a positive shock to the the wealth of specific types of arbitrageurs capital may be more abundant to that specific type and mispricing decreases as these arbitrageurs align market prices. More succinctly, it is capital available to specific types of arbitrageurs that matters for the ILB mispricing, not just arbitrage capital in general.

Table 2.5 – Results from the Forecasting Regression of Monthly Changes in Average BPS Mispricing on Lagged HFRX Index Returns. This table reports summary statistics for the regression of monthly changes in the basis-point mispricing on one-month lagged monthly returns of the HFRX Hedge Fund Indices. MCR denotes the HFRX Macro Strategy Index, EH the HFRX Equity Hedge Index, EDr the HFRX Event Driven, and RV the HFRX Relative Value index. Int denotes the regression intercept. G7 denotes the notional–weighted mispricing index constructed from all countries in the study. The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country, the left column shows the regression coefficient, and the right column shows the Newey-West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

| | Canada | | France | | Germany | | Italy | | Japan | | UK | | USA | | G7 | |
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
Int	−1.18	−0.53	0.28	0.36	0.59	0.70	1.04	0.83	−0.36	−0.19	0.35	0.24	1.06	0.57	0.56	0.46
MCR	−1.24	−1.45	−1.06	−1.63*	−0.94	−1.71*	−1.61	−1.81*	−1.18	−1.49	−0.98	−1.93*	−1.15	−1.63*	−1.09	−2.02**
EH	−0.15	−0.84	−0.53	−0.54	−0.24	−1.60	−0.88	−1.04	−0.95	−0.51	−0.75	−0.49	−0.94	−0.83	−0.88	−0.79
EDr	0.08	0.70	−0.15	−0.39	0.03	0.11	−0.54	−0.87	0.44	0.66	0.05	0.99	−0.78	−0.85	−0.46	−1.09
RV	−1.92	−1.99**	−1.79	−2.09**	−1.64	−2.17**	−2.13	−2.71**	−2.13	−1.97**	−2.85	−2.51**	−3.73	−2.72**	−3.02	−3.26**
Adj. R^2	0.356		0.432		0.394		0.381		0.424		0.457		0.471		0.428	
N	52		86		65		86		55		86		86		86	

The HFRX Relative Value index is subdivided into the style categories Convertible Arbitrage, Volatility Strategies, Multi-Strategy, Corporate Fixed Income, Asset Backed Securities, Sovereign Fixed Income, Real Estate, Fixed Income Alternative Yield, and Energy. Of particular interest are the Sovereign Fixed Income and Convertible Arbitrage subcategories. Convertible Arbitrage includes strategies in which the investment thesis is predicated on realization of a spread between related fixed income instruments. Fixed Income Sovereign strategies are predicated on the realization of a spread between related instruments in which one or multiple components of the spread is a sovereign fixed income instrument. Strategies employ an investment process designed to isolate attractive opportunities between a variety of fixed income instruments, typically realizing an attractive spread between multiple sovereign bonds or between a corporate and risk free government bond. Funds in these two subcategories present likely candidates that would engage in ILB arbitrage. To assess this predication empirically, I regress monthly changes in the average basis-point mispricing on one-month lagged monthly returns on these subcategory indices. Table 2.6 presents summary statistics of the regression results.

Table 2.6 – Results from the Forecasting Regression of Monthly Changes in Average BPS Mispricing on Lagged Substrategy Index Returns of the HFRX Relative Value Index. This table reports summary statistics for the regression of monthly changes in the average basis-point mispricing on one-month lagged monthly returns on the substrategy indices of the HFRX Relative Value Hedge Fund Index. Conv. denotes the HFRX Fixed-Income Convertible Arbitrage Index, Vola denotes the HFRX Volatility Strategies Index, M.Str. denotes the HFRX Multi-Strategy Index, Corp. denotes the HFRX Fixed Income Corporate Index, AB denotes the HFRX Fixed Income Asset Backed Securities Index, Sov. denotes the HFRX Fixed Income Sovereign Index, RE denotes the HFRX Real Estate Index, A.Yld. denotes the HFRX Fixed Income Alternative Yield Index, and Enrgy denotes the HFRX Energy Index. G7 denotes the notional–weighted mispricing index constructed from all countries in the study. The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per \$100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country, the left column shows the regression coefficient, and the right column shows the Newey–West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

	Canada		France		Germany		Italy		Japan		UK		USA		G7	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
Int.	0.40	0.85	−0.61	−0.23	−0.59	−0.35	0.51	1.36	−0.57	−0.24	−0.77	−0.16	0.39	−0.26	0.03	0.17
Conv.	−2.07	−2.37**	−1.01	−1.75*	−0.83	−1.84*	−2.13	−3.06**	−1.71	−1.98**	−1.95	−2.63**	−2.83	−3.05**	−2.11	−2.65**
Vola	−1.60	−2.04**	−1.91	−1.97**	−1.79	−1.91*	−1.82	−2.02**	−1.85	−1.81*	−1.96	−2.04**	−1.77	−2.19**	−1.83	−1.98**
M.Str.	−0.79	−0.46	−0.82	−0.33	−0.68	−0.40	−0.72	−0.81	−1.31	−0.86	−1.02	−0.64	−0.98	−0.74	−0.94	−1.60
Corp.	0.30	0.67	0.26	0.56	0.12	0.67	0.27	0.58	0.17	0.71	−0.90	0.64	0.39	0.74	0.26	0.88
AB	0.09	0.57	0.13	0.97	0.18	0.88	0.11	0.97	0.22	1.11	0.08	0.44	0.14	0.61	0.13	0.58
Sov.	−2.74	−3.03**	−1.91	−2.52**	−1.80	−2.77**	−2.86	−3.20**	−2.73	−2.02**	−2.79	−3.09**	−2.99	−3.22**	−2.89	−3.11**
RE	−0.31	−0.38	−0.90	−0.41	−0.56	−0.59	−0.75	−0.69	−0.31	−0.66	−0.57	−0.77	−0.47	−0.51	−0.50	−0.76
A.Yld.	−1.79	−1.75*	−1.36	−1.79*	−1.57	−1.82*	−1.49	−1.94*	−1.52	−1.72*	−1.59	−1.70*	−1.63	−1.80*	−1.58	−1.75*
Enrgy	0.18	0.97	0.29	1.01	0.11	0.88	−0.46	−0.86	0.43	0.65	0.42	0.53	−0.51	−0.71	0.47	0.52
Adj. R^2	0.345		0.423		0.381		0.372		0.414		0.445		0.461		0.409	
N	52		86		65		86		55		86		86		86	

The sovereign and convertible substrategy indices are statistically significant predicators for ILB mispricing at the five percent. The regression coefficients are negative which is consistent with the notion, that a negative wealth shock at time $t-1$ will constrain arbitrageurs in these two subcategories and mispricing widens for specific arbitrages that these types engage in. Conversely, after a positive shock to the the wealth of specific types of arbitrageurs, capital becomes more abundant to that specific type and mispricing decreases as these arbitrageurs are then able to align market prices. Almost none of the other subcategories have statistically significant explanatory power for ILB mispricing. These results provide additional evidence, that it is capital available to specific types of arbitrageurs that matters for the specific types of arbitrages, such as the ILB mispricing.

Instead of using hedge fund returns, an alternative proxy for the wealth of different types of arbitrageurs is capital invested in specific types of hedge funds. To explore this, I regress monthly changes in ILB Mispricing on one-month lagged percentage changes in total Hedge Fund assets for the HFRX reference hedge fund indices. Table 2.7 presents summary statistics of the regression results for the basis-point and dollar ILB mispricing. The results confirm that the relative value funds category is a statistically significant predicator for ILB mispricing at the five percent level. In the United States, for example, a decline of total assets of relative value hedge funds by one percent is associated with an increase in ILB mispricing by $1.08, or 21.26 basis points. In summary, this section provides strong empirical evidence for the slow moving capital theory of arbitrage mispricing in global markets. As specific types of arbitrageurs become wealth constrained, the mispricing widens. It is capital available to these specific types of arbitrageurs that is crucial for explaining arbitrage mispricing, not just arbitrage capital in general.

Table 2.7 – Results from the Forecasting Regression of Monthly Changes in Average ILB Mispricing on Lagged Percentage Changes in HFRX Total Hedge Fund Assets. This table reports summary statistics for the regression of monthly changes in ILB Mispricing on one-month lagged percentage changes in total Hedge Fund assets of the HFRX reference hedge fund indices. MCR denotes the HFRX Macro Strategy Index, EH the HFRX Equity Hedge Index, EDr the HFRX Event Driven, and RV the HFRX Relative Value index. Int denotes the regression intercept. The first row for each regressor presents results for the basis-point mispricing, and the second row for the mispricing per $100 notional. G7 denotes the notional–weighted mispricing index constructed from all countries in the study. The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country, the left column shows the regression coefficient, and the right column shows the Newey–West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

	Canada Coeff.	Canada t-Stat	France Coeff.	France t-Stat	Germany Coeff.	Germany t-Stat	Italy Coeff.	Italy t-Stat	Japan Coeff.	Japan t-Stat	UK Coeff.	UK t-Stat	USA Coeff.	USA t-Stat
Int	-0.640	-0.223	0.676	0.236	0.694	0.243	-0.605	-0.211	0.626	0.218	0.648	0.226	0.726	0.253
	-0.073	-0.325	0.078	0.343	0.080	0.352	-0.069	-0.307	0.072	0.317	0.074	0.328	0.083	0.368
EQ	0.016	1.333	0.014	1.195	0.011	0.907	0.017	1.387	0.018	1.493	0.012	0.800	0.016	1.355
	0.006	1.470	0.005	1.317	0.004	0.998	0.006	1.529	0.007	1.646	0.003	0.882	0.006	1.493
EDr	-0.103	-1.527	-0.093	-1.416	-0.057	-0.878	-0.092	-1.049	-0.084	-1.332	-0.074	-1.055	-0.108	-1.404
	-0.013	-1.474	-0.017	-1.327	-0.072	-0.893	-0.006	-0.982	-0.011	-1.256	-0.008	-0.994	-0.030	-1.447
MCR	-7.732	-1.549*	-8.339	-1.604*	-9.805	-1.804*	-9.013	-1.708*	-10.736	-1.915*	-8.765	-1.755*	-7.904	-1.554
	-0.333	-1.704*	-0.351	-1.842*	-0.426	-1.871*	-0.387	-1.940*	-0.465	-1.993**	-0.366	-1.935*	-0.337	-1.715*
RV	-8.179	-1.678*	-19.846	-2.156**	-21.629	-1.996**	-16.687	-2.790**	-20.758	-2.911**	-22.580	-2.412**	-21.264	-3.083**
	-0.234	-1.797	-1.007	-3.158**	-1.101	-2.961**	-0.850	-2.533**	-1.056	-3.089**	-1.144	-3.419**	-1.081	-3.276**
Adj. R^2	0.321		0.441		0.481		0.381		0.415		0.519		0.478	
	0.313		0.430		0.469		0.371		0.411		0.489		0.469	
N	52		86		65		86		55		86		86	

2.6   Effects of Monetary Policy on ILB Mispricing

In August 2007, the world was hit by what Alan Greenspan, former Chairman of the Fed, described in Congressional testimony as a "once-in-a-century credit tsunami". The tsunami from the 2007–2009 financial crisis, not only flattened economic activity, producing the most severe world-wide economic contraction since the Great Depression, but it also seemed to sweep away confidence in the ability of central bankers to successfully manage the economy. Therefore, monetary policy played a key role in restoring confidence in the world's capital markets.

During and in the aftermath of the financial crisis of 2008, policymakers took a number of extraordinary steps to improve the functioning of financial markets and to stimulate the economy which resulted in huge expansions of their balance sheets. With interest rates already at the zero bound, the Federal Reserve and other central banks initiated large scale asset purchases to provide support to strained capital markets. The U.S. Fed, in particular, started an unprecedented expansion of its balance sheet by purchasing large amounts of Treasury debt and federal agency securities of medium and long maturities. The Federal Reserve's purchases of large quantities of government backed securities in the secondary market, conventionally known as the Large Scale Asset Purchase –or"LSAP"– programs were among the most important measures, in terms of both scale and prominence. The LSAPs included debt obligations of the government-sponsored housing agencies, mortgage-backed securities (MBS) issued by those agencies, and coupon securities issued by the U.S. Treasury, and they collectively amounted to $1.7 trillion over a period of about 15 months–the single largest government intervention in financial-market history. The Bank of England

also purchased longer-term debt securities during the financial crisis. On March 5 2009, the Bank of England announced plans to purchase £75 billion in assets, mainly gilts with residual maturities between five and twentyfive years. The program was extended multiple times: to £125 billion in May 2009, to £175 in August 2009, and to £200 in Noveber 2009. In February 2010, the BOE stated that it would make additional purchases if necessary. The Bank of England's gilt purchases, at 14 percent of U.K. GDP, were similar in scale to the Federal Reserve's LSAPs which amounted to 12 percent of U.S. GDP. Asset purchases by the Bank of Canada were $75b in 2008. In Japan, purchases by the Bank of Japan were not large as a share of GDP and they were skewed toward bonds with short residual maturities. McCauley and Ueda (2009) show that the BOJ purchases were mainly seasoned JGBs with short residual maturities; the average maturity of the BOJ's holdings of JGBs fell from more than five years to less than four years. However, as discussed further below, the Bank of Japan retired a significant amount of inflation-linked debt during the financial crisis. The ECB used outright bond purchases less frequently as a monetary policy tool to regulate the money supply. Instead, the ECB relied on refinancing facilities, in which, instead of purchasing bonds, the ECB uses bonds as collateral in repo transactions and collateralized loans. However, with the onset of the European sovereign-debt crisis, direct bond purchases became more frequent, primarily for Spanish and Italian government debt. In May 2009, the European Central Bank (ECB) announced plans to purchase €60 billion of covered bonds, and in May 2010, the ECB announced plans to purchase sovereign bonds of its member countries in order to improve market depth and liquidity.

There was particular cause for skepticism regarding the program to purchase Treasury se-

curities. The market for U.S. government debt is among the largest and most liquid in the world, and it was not obvious that even such a sizeable intervention-the $300 billion purchased by the Fed constituted about eight percent of the market at the time-would have significant effects, given the array of other securities that serve as potential substitutes for Treasuries. Given the unprecedented size and nature of these programs and the speed with which they were proposed and implemented, policymakers could have had, at best, only a very rough ex ante sense of their potential impact. The minutes of the December 2008 Federal Open Market Committee meeting summarized the prospects thus: "The available evidence indicated that [LSAP] purchases would reduce yields on those instruments, and lower yields on those securities would tend to reduce borrowing costs for a range of private borrowers, although participants were uncertain as to the likely size of such effects." In the second part of 2010 the Fed implemented a second round of monetary stimulus (LSAP 2) by both reinvesting principal payments from its securities holdings and carrying out new purchases in longer-term Treasury securities in order to jump-start the sluggish economic recovery and to avoid undershooting the inflation target. The objective of the Fed's large-scale asset purchases (LSAPs) was to reduce long-term yields in order to ease financial markets and spur economic growth. Several studies provide evidence that the LSAP program was effective in lowering interest rates below levels that otherwise would have prevailed in the market.

Coincidentally, ILB mispricing spikes consistently across all countries during the same time period which raises the question whether central banks have affected mispricing in the markets through active monetary policy. This Financial Times blog post from April 4, 2012 by

Sam Jones (Jones(2012)) provides anecdotal evidence in support of this notion:[46]

> Wide pricing anomalies in European bond markets caused by the ECB's longer-term refinancing operations have led to bumper profits for a small group of arbitrage hedge funds in recent months. "It's a good environment for them to generate alpha [a measure of hedge fund managers skill-based returns] because of the actions of central banks," said Ermanno Dal Pont, head of Barclays' European capital solutions business, which deals with hedge funds.

> Italian bond markets, for example, exhibited unprecedented price discrepancies between different classes of bond issued by the government as a result of the ECB's LTRO liquidity injection. In January, investors dumped inflation-protected Italian bonds, fearful that they would automatically drop out of key European bond indices if the country's credit rating was downgraded, while at the same time Italian banks snapped up regular Italian bonds with LTRO cash. Hedge funds bought the cheap inflation-protected bonds, wrote swaps to offset inflation and then shorted expensive regular Italian bonds, thereby completely hedging out credit risk and inflation and locking in the supply and demand-driven difference between the two bonds. The spread between them was more than 200 basis points, according to Bob Treue, the founder of Barnegat, a US-based fixed income arbitrage hedge fund that has made 18 per cent on its investments so far this year.

There is a growing literature on the impact of monetary policy on financial markets, in particular in the aftermath of the 2008 financial crisis. Gagnon et al. (2011) examine changes in the ten-year Treasury yield and Treasury yield term premium. They document that after eight key LSAP announcements the ten-year yield fell by a total of 91 basis points, while their

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[46]See http://www.ft.com/intl/cms/s/0/cb74d63a-7e75-11e1-b009-00144feab49a.html

measure of the ten-year term premium fell by 71 basis points. The authors argue that the Fed's asset purchases primarily lowered long-term rates through a portfolio balance channel that reduced term premia. Furthermore, they examine the effects of similar asset purchase programs in Japan and the United Kingdom and find effects that are generally consistent with those found in the United States. Krishnamurthy and Vissing-Jorgensen (2011) evaluate the effect of the Federal Reserve's purchase of long-term Treasuries and other long-term bonds (QE1 in 2008-2009, and QE2 in 2010-2011) on interest rates using an event-study methodology. Treasuries-only purchases in QE2 had a significantly larger effect on Treasuries and Agency Securities relative to corporate bonds and mortgage-backed securities, with yields on the latter falling primarily through the market's anticipation of lower future federal funds rates. Rosa (2011a and 2011b) examine the effects of decisions and statements by the FOMC on the level and volatility of U.S. stock and volatility indices, and the U.S. dollar exchange rates using an intraday event-study analysis. Rosa (2012) examines the impact of large-scale asset purchases (LSAP) on U.S. nominal and inflation-indexed bonds, stocks, and U.S. dollar spot exchange rates, and finds that LSAP announcements had economically large and highly significant effects on the prices of these assets. Particularly, these asset pricing effects are similar to an unanticipated cut in the fed funds target rate. Furthermore, the response of U.K. asset prices the Bank of England's gilt purchases is quantitatively similar to the reaction of U.S. asset prices to the Fed's asset purchases. Kuttner (2001) estimates the impact of monetary policy actions on Treasury bill, note, and bond yields, and finds that there is only a small effect of anticipated target rate changes on interest rates. However, there is a large and significant effect from unanticipated changes. Beechey and Wright (2009) study the response of nominal and index-linked bond yields to macroeconomic and

monetary news announcements. They document an increase in yields in response to stronger-than-expected data and a decrease on the weaker-than-expected data. Bernanke and Kuttner (2005) analyze the impact of changes in monetary policy on equity prices, and document that, on average, a hypothetical unanticipated 25-basis-point cut in the Federal funds rate target is associated with about a 1 percent increase in broad stock indexes. Andersen, Bollerslev, Diebold and Vega (2003) and Faust, Rogers, Wang and Wright (2007) examine the intraday response of the U.S. spot exchange rate to real-time U.S. monetary and macroeconomic news. Christensen and Rudebusch (2012) analyze the declines in government bond yields that followed the announcements of plans by the Federal Reserve and the Bank of England to buy longer-term government debt by decomposing these declines into changes in expectations about future monetary policy and changes in term premia. Joyce et al. (2011) investigate the impact of the Bank of England's quantitative easing policy on U.K. asset prices and find that asset purchases by the central bank have depressed medium to longterm government bond yields by about 100 basis points.

This section is different from prior studies on quantitative easing programs in that it studies the effects of monetary policy announcements on arbitrage mispricing. These announcements represent the initiation of government bond purchase programs and liquidity facilities, changes in interest rates, and other quantitative easing measures. In the United States, these events dates include QE1 and QE2 announcements on November 25, 2008, December 1, 2008, December 16, 2008, January 28, 2009, March 18, 2009, August 8, 2010, and September 21, 2010. The reason for studying announcement effects it that with forward-looking financial markets, a policy of asset purchases, for instance, is expected to impact

asset prices not at the time that the purchases are actually made, but rather at the time that investors learn that they will take place. Large Scale Asset Purchases (LSAPs) are announced ahead of time, in the statements that follow FOMC meetings. These statements are in turn anticipated to some extent by investors, whose expectations have been guided by speeches and other comments by FOMC members. Furthermore, whereas the federal funds futures market gives a fairly clear measure of investors' real-time expectations for changes in the target federal funds rate, there is no such measure for other policy measures such as the expectations of the size of LSAPs. FOMC statements and days with other announcements can change investors' views about the likely extent of monetary policy actions and about the underlying state of the economy. The methodology in this paper is similar to the event-study in Krishnamurthy and Vissing-Jorgensen (2011). To analyze the impact of monetary policy on arbitrage mispricing in the markets, I regress changes in the ILB mispricing on indicator variables representing monetary policy announcements by the central banks of Canada, Europe, Japan, the United Kingdom, and the United States. Periods of financial turmoil, such as the period from Fall of 2008 to Spring 2009, make inference from an event-study more difficult. To the extent that inflation-linked bonds are less liquid than their nominal counterparts, the prices of the less liquid assets may react slowly in response to an announcement. I address this issue by using both one and two day changes (from the day prior to the day after the announcement) in arbitrage mispricing and find that the results are qualitatively similar in both cases. Therefore, Table 2.8 presents regression results for one-day changes only. A second issue is the identification of monetary policy announcements as causative events for changes in ILB mispricing.

Table 2.8 – Results from the Regression of Changes in BPS Mispricing on Monetary Policy Announcements. The table reports summary statistics for the regression of changes in the ILB mispricing on indicator variables representing monetary policy announcements by central banks using daily data. These announcements reflect quantitative easing programs by central banks such as purchases of government bonds, changes in interest rates, and the establishment of liquidity facilities. Europe comprises France, Germany, and Italy. For each regressor, the first row presents results for the dollar mispricing (labeled USD), the second row for the basis-point mispricing (labeled BPS). The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country/region, the left column shows the regression coefficient, and the right column shows the Newey-West t-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The two rows for each R^2 statistic correspond to the dollar and basis-point mispricing, respectively. The sample period is from July 23, 2004 to September 20, 2011.

	Canada		Europe		Japan		UK		USA	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
USD	0.493	1.252	0.761	2.361**	−0.895	1.591*	1.098	2.803**	0.947	3.077**
BPS	9.892	1.343	17.032	2.216**	−12.588	1.627*	18.716	2.951**	21.016	3.091**
Adj. R^2	0.1193		0.2864		0.3087		0.2912		0.3205	
	0.1250		0.2981		0.3206		0.3010		0.3379	
N	1114		1868		1187		1868		1868	

Arrival of other economic news around announcement dates could potentially create measurement errors. Krishnamurthy and Vissing-Jorgensen (2011) address this issue by presenting intraday movements in Treasury yields and trading volume for each of the QE event dates in the United States. They show that the events identify significant movements in Treasury yields and Treasury trading volume and that the announcements do appear to be the main piece of news released on the event days.

Table 2.8 reports summary statistics from the regression of changes in the ILB mispricing on indicator variables representing monetary policy announcements by central banks using daily data. Monetary policy announcements increased ILB mispricing in Canada, France, Germany, Italy, the United Kingdom, and the United States. In the United States, the announcement effect of monetary policy measures is associated with an increase in the dollar mispricing by 94.7 cents per $100 notional, and a 21.061 basis points increase in the basis-point mispricing. These results are significant at the five percent level. Although not reported, this represents an increase the mispricing on average by $38 million. These results are consistent with Wright (2011) who finds evidence of a rotation in breakeven rates from Treasury Inflation Protected Securities (TIPS), with short-term breakevens rising and long-term forward breakevens falling. If the policy announcement dates were predictable, this would represent a significant wealth transfer to arbitrageurs. As for the United States, one may be sceptical of these results because the Federal Reserve's asset purchase programs included Treasury Bonds as well as TIPS – neither the Bank of England nor the ECB have included inflation-linked bonds in their purchase programs. The System Open Market Account (SOMA) managed by the Federal Reserve Bank of New York shows an increase in

TIPS holdings from \$44.5 billion on August 26, 2008 to \$65.9 billion on August 24, 2011. Holdings in Treasury bonds and notes increased from \$676 billion to \$ 1.554 trillion over the same period. However, the share of TIPS in the SOMA decreased since QE1 from 3.01% on August 26, 2009, to 2.02% on August 25, 2010. By contrast, the share of U.S. Treasury Notes and Bonds has increased from 45.70% on August 26, 2009 to 58.86% on August 24, 2011. FLL (2012) provide clear evidence that the supply or liquidity of both TIPS and Treasuries is directly linked to the size of the arbitrage. The Federal Reserve's Asset Purchase programs decreased the total supply of both TIPS and Treasuries in the market and shifted the relative supply of both securities. Therefore, the increase in ILB mispricing in response to the Federal Reserve's LSAPs provides further support for the notion that the size of the arbitrage is expected to widen as supply or liquidity of both TIPS and Treasuries is reduced. Furthermore, Krishnamurthy and Vissing-Jorgensen (2011) show that decreases in TIPS yields is partially offset by changes in inflation swap rates, and the direction is consistent with an increase in ILB mispricing.

In contrast to the other G7 countries, monetary policy in Japan is associated with a decrease in ILB mispricing. In January 2007, the Ministry of Finance declared inflation-indexed bonds eligible for their buyback operations. In each of five buyback operations the Ministry of Finance retired about ¥45 billion of outstanding linkers until April 2008. In the wake of the financial crisis, prices of inflation-linked bonds declined significantly (in March 2008 the on-the-run 10 year JGBi breakeven rate approached -2 basis points) which lead to increased buybacks by the Ministry of Finance. During the 2008 financial crisis the liquidity of the 10-year inflation indexed bonds dropped precipitously. As breakeven rates continued

to decline during the peak of the financial crisis, reaching a low of -323 basis points in mid-December 2008, the Ministry of Finance stepped up their buyback operations and cancelled two linker auctions scheduled for October 2008 and February 2009. From October 2008, average monthly inflation-linked bond buybacks were in the order of ¥215 billion. Over 39 linker buyback operations took place between 2007 and 2009, bringing the total amount retired from the secondary inflation-indexed market to about ¥3.74 trillion. This represents nearly 40% of the total linker issuance and the secondary market declined in size from around $87 billion at the end of 2008 to around $61 billion at the end of 2009. After additional linker buybacks the size of the secondary inflation-linked bond market was around $32 billion at the end of 2010. However, breakeven rates at the long-end of the maturity curve, recovered to near -50 basis points by the end of 2009, and liquidity improved significantly as well with bid/ask spreads in the 10-year maturities narrowing by over 50%.

Measures of inflation expectations are important for conducting monetary policy and for assessing its credibility. In particular, the differential between yields on nominal Treasury securities and on TIPS of comparable maturities, often called the breakeven inflation (BEI) rate, has often been used in policy circles and the financial press as a proxy for the market's inflation expectations (see, for example, D'Amico, Kim and Wei (2010)). However, using breakeven inflation rates as measures of inflation expectations can be problematic as many researchers have pointed out. Most studies base their criticism on the notion that the yield differential between nominal and index-linked bonds reflects, besides expected inflation, other components such as risk and liquidity premia. Among them are, for example, Christensen and Gillan (2011c), Hördahl (2008), Grishchenko and Huang (2008), Carlstrom and Fuerst

(2004), Trehan (2010), and Shen (2006, 1998). It is also important to acknowledge that practitioners have long recognized that breakeven inflation spreads appear mispriced relative to inflation swaps. These discussions, however, have generally attributed the discrepancy to some form of risk premium. The findings in this paper imply that this explanation can be ruled out since the ILB mispricing is a violation of the law of one price and, therefore, cannot be reconciled with an equilibrium model of risk premia. This section sheds new light on this issue by relating ILB mispricing and breakeven inflation rates. To the extent that breakeven rates are correlated with the ILB mispricing, the common market practice of using breakeven rates to gauge the market's inflation expectations is flawed and provides a noisy measure of inflation expectations at best. Specifically, the implied measure is biased downwards and, moreover, the bias worsens in times of increased volatility in financial markets.

Sack and Elsasser (2004) report that the spread between ten-year yields on nominal securities and TIPS was, on average, about 50 basis points below the long-run inflation expectations reported in the Survey of Professional Forecasters which implies that the breakeven inflation rate has been 50 basis points below the expected long-term rate of inflation. Gürkaynak, Sack, and Wright (2008) provide evidence that breakeven inflation rates are not a pure measure of inflation expectations due to the presence of inflation risk premium and liquidity premium components. Christensen, Lopez and Rudebusch (2010) decompose breakeven rates into inflation expectation and inflation risk premia using an affine arbitrage-free term structure model and propose to adjust breakeven inflation rates by subtracting inflation risk premia from the breakeven rates. Gürkaynak, Sack and Wright (2010) also argue that breakeven-inflation rates do not solely reflect inflation expectations and provide evidence that

the interpretation of breakeven rates as expected inflation is complicated by inflation risk premia and the differential liquidity premia between TIPS and nominal securities. Hördahl and Tristani (2007) decompose breakeven inflation rates in the U.S. using a macrofinance model with monthly data on nominal and real yields, inflation, and the output gap. Their results suggest that fluctuations in breakeven rates have mostly reflected variations in the inflation risk premium, while long-term inflation expectations have remained anchored from 1999 to 2007. Hördahl and Tristani (2010) extend their prior work and estimate inflation risk premia in the United States and the Euro area.

The correlations between the basis-point mispricing for each country and the ten-year breakeven rates at daily frequency are -0.93, -0.28, -0.65, -0.51, -0.85, -0.67, and -0.87 for Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States, respectively. All p-Values are zero. For all countries in the sample, the basis point mispricing is significantly negatively correlated with the ten-year breakeven rate. Since changes in breakeven rates are negatively correlated with arbitrage mispricing, the findings in Wright (2011) and Krishnamurthy and Vissing-Jorgensen (2011) are also cosistent with an increase in the mispricing in response to monetary policy actions. To further analyze the question of whether breakeven rates are appropriate proxies for inflation expectations, I regress the time-series of ten-year breakeven inflation rates on the ILB mispricing for all G7 countries. Table 2.9 shows the regression results. All regression coefficients are negative in sign, and all adjusted R-Squared test statistics are larger than 0.70. In the United States, the adjusted R-Square test statistic is 0.8506 and in the United Kingdom it is 0.7650. This shows that ILB mispricing accounts for a large fraction of the variation in breakeven inflation rates.

Table 2.9 – Results from the Regression of Daily Changes in 10-year Breakeven Rates on Changes in Basis-Point Mispricing. This table reports summary statistics for the regression of daily changes in ten-year breakeven inflation rates on changes in ILB mispricing measured in basis-points for all G7 countries using daily data. The basis-point mispricing index for each country is constructed as the weighted-average index-linked–nominal Bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country, the left column shows the regression coefficient, and the right column shows the Newey-West t-Statistic. The superscript $**$ denotes significance at the five-percent level; the superscript $*$ denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

	Canada		France		Germany		Italy		Japan		UK		USA	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
	-0.761	-22.94^{**}	-0.465	-21.59^{**}	-0.283	-20.08^{**}	-0.246	-18.63^{**}	-0.271	-21.28^{**}	-0.189	-25.12^{**}	-0.885	-26.76^{**}
Adj. R^2	0.6285		0.6519		0.686		0.6047		0.6582		0.7013		0.7107	
N	1114		1868		1392		1392		1187		1868		1868	

In summary, this section provides evidence that monetary policy interventions by central banks, in particular the large scale asset purchase programs that were implemented in the aftermath of the financial crisis, may have exacerbated the mispricing. Furthermore, this section adds to the literature that criticizes the market practice of using breakeven inflation rates as a proxy for inflation expectations by relating the variation in breakeven inflation rates to variation in ILB mispricing. The results provide additional evidence that breakeven rates are a noisy measure of inflation expectations at best and hence that using breakeven inflation rates as gauge of the market's inflation expectations is flawed.

2.7 Risk and Return Characteristics of ILB Mispricing

Even an arbitrage in the text-book sense can generate mark-to-market losses that might force an arbitrageur facing constraints to unwind a position at a loss prior to convergence.[47] An arbitrage could essentially be riskless from the perspective of a relatively unconstrained arbitrageur such as a sovereign wealth fund, yet risky from the perspective of a highly leveraged and constrained hedge fund. This raises the question, whether ILB arbitrage is truly riskfree or whether it is risky leveraged strategy that could result in losses for an arbitrageur trying to take advantage of the mispricing between nominal and index-linked bonds. Several hedge funds and institutional asset managers have in fact implemented trading strategies that exploit the divergence between the prices of nominal bonds, inflation-indexed bonds, and inflation swaps in the United States and Europe. To quote from a recent Financial Times blog by Sam Jones (Jones (2012)):

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[47]See, for example, Liu and Longstaff (2005).

> Hedge Funds bought the cheap inflation-protected bonds, wrote swaps to offset inflation
> and then shorted expensive regular Italian bonds, thereby completely hedging out credit
> risk and inflation and locking in the supply and demand-driven difference between the
> two bonds. The spread between them was more than 200 basis points, according to Bob
> Treue, the founder of Barnegat, a US-based fixed income arbitrage hedge fund that has
> made 18 per cent on its investments so far this year.

Furthermore, as reported by in Financial Times blogs by Kaminska (2010) and Jones and Kaminska (2010) about Barnegat Fund Management:

> But as Barnegat explains: "We will buy the TIPS, short the nominal bond, and lock in
> the inflation rate with the inflation swap. The result is that the net initial payment is
> zero, but until 2014 this trade yields up to 2.5 percent per year of the notional."
> For a small group of savvy traders the pricing discrepancies at their widest led to one
> of the most successful hedge fund trades in recent memory. One of the biggest ben-
> eficiaries was the low-profile New Jersey-based $450 million Barnegat fund founded in
> 1999. Barnegat acquired TIPS bonds shortly after the collapse of Lehman Brothers and
> then shorted-bet on a fall in rates-regular Treasury bonds of an equivalent maturity. As
> the pricing discrepancy narrowed, the fund realised huge gains. The fund returned 132.6
> percent to investors in 2009.

Furthermore, there has been a recent increase in market interest in nominal–inflation-linked bond trading strategies, which are often referred to as breakeven inflation trades. In late 2011, both ProShares Advisors and State Street announced plans to offer ETFs based on long-short positions in TIPS and Treasuries. Nonetheless, there is empirical evidence suggesting that ILB arbitrage strategy may expose constrained arbitrageurs to substantial risks. Such constraints include costs and funding risks of financing securities positions in the repo

markets, as well as the regulatory, mark-to-market, and capital costs of keeping security positions on the balance sheet. In the context of breakeven trades one example may be Morgan Stanley. From a June 29, 2011 Bloomberg article[48]

> The banks interest-rates trading group lost at least tens of millions of dollars on the trade, which the firm has been unwinding ... Traders at the bank bet that inflation expectations for the next five years would rise in Treasury markets ... Such wagers on so-called breakeven rates involve paired purchases and short sales of Treasuries and Treasury Inflation Protected Securities, or TIPS, in both maturities.

In light of this anecdotal evidence, this section specifically analyzes the risk and return characteristics of the ILB trade. It is of particular interest, whether ILB arbitrage earns small positive returns most of the time, but occasionally experiences dramatic losses and whether the strategy earns positive excess returns, or "alpha", on a risk-adjusted basis.

For each day of the sample period when the mispricing is positive, the ILB arbitrage trade is initiated by going long the synthetic nominal bond and shorting the actual bond. After a one-month holding period, the trade is unwound. This is to reflect that a hedge fund actually implementing the trade may not be able to hold the position until convergence. Monthly returns are calculated for each index-linked–nominal bond pair and then notional-weighted to construct the return index for each country.

Summary statistics of daily ILB arbitrage returns are reported in Table 2.10. The average monthly excess returns from the individual countries are all statistically significant and range from about 0.43 to 0.56%.

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[48]See http://www.bloomberg.com/news/2011-06-29/morgan-stanley-said-to-suffer-trading-loss-after-wageron-u-s-inflation.html.

Table 2.10 – ILB Arbitrage Strategy Returns. This table reports summary statistics for the monthly percentage excess returns on the ILB arbitrage strategy for all G7 countries. Each day during the sample, the arbitrage trade is implemented if ILB mispricing is positive and the trade is unwound after a one-month holding period. The returns on the ILB arbitrage strategy are calculated from the average monthly mispricing in dollar terms. The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. SDev denotes the standard deviation of the monthly returns, ρ denotes the serial correlation of the monthly returns, and SR represents the annualized Sharpe Ratio. RN is the proportion of negative excess returns. Gain/loss is the Bernardo and Ledoit (2000) gain/loss ratio for the strategy. The sample period is from July 23, 2004 to September 20, 2011.

	Canada	France	Germany	Italy	Japan	UK	USA
Mean	0.501	0.486	0.524	0.437	0.519	0.523	0.558
SDev	3.875	2.987	3.121	3.013	3.645	2.998	2.571
Min	−8.878	−6.365	−8.307	−9.223	−5.667	−6.994	−7.264
Max	9.011	10.557	11.464	14.571	11.329	11.965	12.678
Skewness	0.569	1.112	0.984	2.156	1.112	1.548	1.995
Kurtosis	4.552	3.172	4.036	4.953	6.269	8.451	7.778
RN	0.301	0.337	0.281	0.401	0.347	0.346	0.327
ρ	−0.059	−0.088	−0.121	−0.158	−0.084	−0.108	−0.112
Gain/Loss	1.887	1.645	2.102	2.525	1.798	2.257	2.336
SR	0.448	0.564	0.582	0.502	0.493	0.604	0.752
N	1084	1838	1362	1838	1157	1838	1838

Furthermore, as shown in Table 2.10, ILB arbitrage trade generates positively-skewed excess returns on average. Thus, despite producing large negative returns from time to time, the ILB arbitrage strategy generates even larger offsetting positive returns.

To analyze the extent to which these positive excess returns represent compensation for bearing market risk, I regress excess returns on the ILB arbitrage strategy on the Fama-French factors. The analysis does not include controls for credit risk because subsection 2.8.3 shows evidence that ILB mispricing is not significantly related to systemic credit risk. Table 2.11 presents the regression results for all G7 countries, and by region for Europe, Japan, and North America. For all regions, the market risk factor is statistically significant at the five percent level and the sign of the regression coefficient is negative. The fact that the ILB trade exhibits equity market risk may seem counterintuitive given that it is a pure fixed-income strategy. Previous research by Campbell (1987), Fama and French (1993), Campbell and Taksler (2002), and others, however, documents that there are common factors driving returns in both bond and stock markets. Duarte, Longstaff, and Yu (2007) also show that the fixed income arbitrage strategies in their study exhibit high equity market risk. The negative loading on the market risk factor in the United States is consistent with Roll (2004) who finds finds that TIPS were negatively correlated with equities during the 1997–2004 period. The factor loadings on SMB, HML, and WML are not statistically significant. The regression intercept is positive and statistically significant at the the five percent level in all regions, except in Canada.

Table 2.11 – ILB Arbitrage Returns Regression on Fama–French Factors. This table reports summary statistics for the regression of monthly percentage excess ILB arbitrage returns on the Fama-French Factors denoted by SMB, HML, and WML, and on the excess return on the market, denoted by R_m. Each day during the sample, the arbitrage trade is implemented if ILB mispricing is positive and the trade is unwound after a one-month holding period. The dollar mispricing index for each country is constructed as the weighted-average Index–Linked–Nominal Bond mispricing, expressed in units of dollars per $100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

	Canada	France	Germany	Italy	Japan	UK	USA
Alpha	0.3761	0.5105	0.5348	0.5944	0.6038	0.6438	0.6910
t-Stat	1.4364	2.0972**	2.5608**	2.7126**	3.1712**	2.7878**	3.5805**
				t-**Statistics**			
R_m	-1.8019	-2.4914	-2.882	-2.4752	-2.9201	-3.0876	-3.2521
SMB	1.2375	0.9114	1.0766	0.5544	1.1009	1.0868	-0.6068
HML	-0.2475	0.2156	-0.3201	0.0693	-0.0505	-0.34848	-0.0761
WML	0.1584	0.1372	0.8342	0.2079	0.2525	0.90816	0.2284
Adj. R^2	0.0564	0.0735	0.0911	0.1158	0.0979	0.0712	0.0892
N	1084	1838	1362	1838	1157	1838	1838

There is clear evidence that after risk adjusting for equity market factors, the ILB arbitrage strategy produces significant "alpha" in all countries ranging from 0.51 percent per month in France to 0.69 percent per month in the United States. Although not shown, the results qualitatively still hold if a conservative estimate of transactions costs is applied. Duarte, Longstaff, and Yu (2007) document that arbitrage strategies requiring "more intellectual" capital to implement show significant positive alpha. The results in this section are consistent with this notion. Furthermore, the fact that a number of these factors share sensitivity to financial market "event risk" argues that the positive alpha are not merely compensation for bearing the risk of an as-yet-unrealized "peso" event. Furthermore, these results are consistent with the slow moving capital theory according to which arbitrage opportunities are expected to arise in multiple markets at the same time. In summary, the ILB arbitrage strategies generate significant risk-adjusted excess returns. The returns are positively skewed, contrary to the notion that arbitrage strategies generate small positive returns most of the time, but experience infrequent heavy losses as a result of bearing risk of a "peso" event.

2.8   Is ILB Mispricing driven by other Factors?

The following subsections address whether ILB mispricing is a violation of the law of one price or whether there are other factors that may drive a wedge between the prices of nominal and inflation-indexed government bonds. Subsection 2.8.1 discusses transaction costs. Subsection 2.8.2 analyzes the potential impact of mispricing in the inflation swaps market. Subsection 2.8.3 studies whether market-wide liquidity and liquidity differences between nominal and inflation-indexed bonds can account for ILB mispricing. Subsection 2.8.4 discusses differences

in taxation between nominal and index-linked bonds. Finally, subsection 2.8.5 discusses the effects of the embedded deflation floor in many index-linked government bonds on ILB mispricing. This section presents clear evidence that neither a single one of these factors nor all factors in ensemble are able to account for ILB mispricing in the magnitude observed in the G7 financial markets.

2.8.1 Transaction Costs

The question naturally arises whether the ILB arbitrage strategy would be profitable after accounting for transaction costs. Fleckenstein, Longstaff and Lustig (2012) provide an analysis for the U.S. market. A conservative estimate in the United States for two-year, five-year, and ten-year horizons are 20.2, 29.5, 46.3 cents per $100 notional amount, respectively. These transaction costs are clearly orders of magnitude smaller than the arbitrage. Subsection 2.8.3 shows that market liquidity in the other G7 markets is not significantly different compared to the United States. Even if transaction costs were twice or three times as large as in the U.S. market, they cannot begin to account for mispricing of the magnitude observed in the G7 countries.

2.8.2 Mispricing of Inflation Swaps

As described in section 2.3, the relative prices of inflation-linked bonds, nominal bonds, and inflation swaps are tied together by no-arbitrage restrictions. Section 2.4 shows that this restriction is frequently violated in all G7 government bond markets. The arbitrage strategy consists of three legs, however, and mispricing in any one of these three could cause ILB

arbitrage to occur. Since inflation swaps are relatively recent financial innovations, it is natural to explore whether potential mispricing of inflation swaps may be the underlying explanation for the ILB arbitrage results.

In the United States, beginning with the first TIPS auction in 1997, market participants began making markets in inflation swaps as a way of hedging inflation risk. As the TIPS market has grown, the inflation swaps market has become liquid and actively traded.[49] Inflation swaps have also become widely used among institutional investment managers because of their high correlation with realized CPI.[50] The notional size of the inflation swap market is estimated by Pond and Mirani (2011) to be on the order of hundreds of billions.

In the United Kingdom, there is about £100 billion of inflation swaps outstanding, according to estimates from the Royal Bank of Scotland. As in the U.S. market, inflation swaps have also become popular among institutional investment managers and are widely used by U.K. pension funds and insurance companies to hedge inflation-linked liabilities because. Inflation swaps pay out according to the retail price index, which is the standard measure of inflation in the U.K. for pension schemes.

In Japan, inflation swaps are among the most frequently traded inflation derivatives as well. Inflation swaps are linked to the same reference index as the JGBis and feature the same three-month indexation lag with interpolation to the tenth of the month. Liquidity in the zero-coupon inflation swaps market was initially tilted towards the six to ten year maturities, but trading in the shorter and longer maturities has significantly picked up since

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[49]See Kerkhoff (2005).

[50]As one example, inflation swaps are a key element of J.P. Morgan's Columbus Fixed Income Inflation Managed Bond Strategy.

2006. Bid/Ask spreads are of the same order of magnitude as those of index-linked Japanese government bonds. At the short end of the maturity spectrum, all tenors including the one year inflation swaps are traded, and at the long end, both the fifteen and twenty year maturities are traded, whereas trading activity for the thirty-year inflation swaps has been limited. However, the maturity range of the JGBi in this study is such that only the actively traded inflation swaps are needed to implement the arbitrage strategy. Therefore, limited trading activity in the 30-year maturity swaps cannot have any impact on the mispricing results in Japan.

In France, French CPIx inflation swaps first began trading in 1998 even before the first OATi was issued. Euro HICP inflation swaps started trading with the introduction of the Euro currency in 1999. Anecdotal reports by market practitioners indicate that liquidity in both the French CPIx and the Euro HICPx inflation swaps markets is comparable to that of their inflation-linked bonds counterparts. Zero-coupon inflation swaps referenced to the Euro HICPx are among the most common inflation derivatives. The contracts have a lag of three months, meaning that the base inflation index for the swap is the value of the HICPx three months before settlement. As swaps on the next base month also start to trade towards to end of the month, there is a discontinuity in the quoted rates at the time of the roll from one month to the next which reflects typical monthly seasonality in the CPI index. The most commonly-traded maturities are the five year and ten year swaps, but the maturity range above two years is traded frequently out to the thirty year maturity contracts. As Italy and Greece started issuing long-dated fifty year bonds linked to Euro HICPx in 2007, inflation swaps with the same maturities started to trade. The depth and liquidity in that maturity

range is limited. However, since these longer maturities are not needed to implement the arbitrage strategy for the inflation-linked bonds in the sample, the results cannot be due to stale prices and limited trading activity in inflation swaps. In 2005, monthly volumes were around €5 billion, up from €500 million in mid-2002. By 2007 monthly broker volumes were around €15 billion.

To strengthen the argument against mispricing in the inflation swap market even further, one may ask what the inflation swap rates would have to be in order to make the mispricing disappear. One way to explore this is to solve for the size of the parallel shift in the inflation swap curve that would be required to eliminate the mispricing. This analysis is carried for the United Kingdom and the United States. Although not presented here, similar results apply for all other countries in this study. The results argue strongly against the mispricing being due entirely to inflation swap mispricing. In particular, inflation swap rates which are already viewed by many market participants as anomalously low would actually need to be significantly lower to explain ILB mispricing. The top panel of Figure 2.5 plots the term structure of U.K. RPI inflation swap rates that would be needed to reconcile the March 17, 2009 mispricing with magnitude of $19.45 between the 4.25 percent gilt with maturity date December 27, 2027 and the 1.25 percent index–linked gilt issue with the same maturity. The market would need to be anticipating significant deflation for sixteen years to reconcile the mispricing. The bottom panel of Figure 2.5 plots the term structure of inflation swap prices that would be needed to reconcile the December 30, 2008 mispricing between the 7.625 percent Treasury bond with maturity date February 15, 2025 and the 2.375 percent TIPS issue with the same maturity. Similarly to the case of the United Kingfom, the U.S.

market would need to be anticipating significant deflation for ten years to reconcile TIPS–Treasury mispricing. Furthermore, the maximum inflation swap rate over the entire horizon of the strategy would only be 0.28 percent. It is very implausible that ILB mispricing could be explained by mispricing in inflation swaps of this magnitude and in this direction. Furthermore, in the case of the U.S., inflation swap rates would need to be 51.5 basis points lower on average to explain TIPS-Treasury mispricing between July 23, 2004 and November 19, 2009. This is roughly ten times as large as the bid-ask spread for inflation swaps. Furthermore, 51.5 basis points represents an average error of more than 21 percent of the average level of the five-year inflation swap during the sample period. Inflation swap pricing errors of this magnitude seem very implausible.

Fleckenstein, Longstaff and Lustig (2012) applied the same arbitrage strategy to matching corporate fixed-rate and inflation-linked bonds with the same set of inflation swap prices. The mispricing between corporate fixed-rate and inflation-linked debt is much smaller than the contemporaneous TIPS-Treasury mispricing. Furthermore, there is very little correlation between the corporate and TIPS-Treasury mispricing series. In fact, the correlation between the two time series is negative in sign. There is little or no evidence of systematic mispricing between corporate fixed-rate and inflation-linked debt, so that the notion that mispricing in the U.S. inflation swap market is the source of the TIPS mispricing can be ruled out. In this study, a similar comparison is not feasible because only a few countries have developed corporate index-linked debt markets. In conclusion, there is strong evidence suggesting that there is a liquid market for zero-coupon inflation swaps in all G7 countries. While it cannot be ruled out that inflation swaps may be occasionally mispriced, it is highly unlikely that
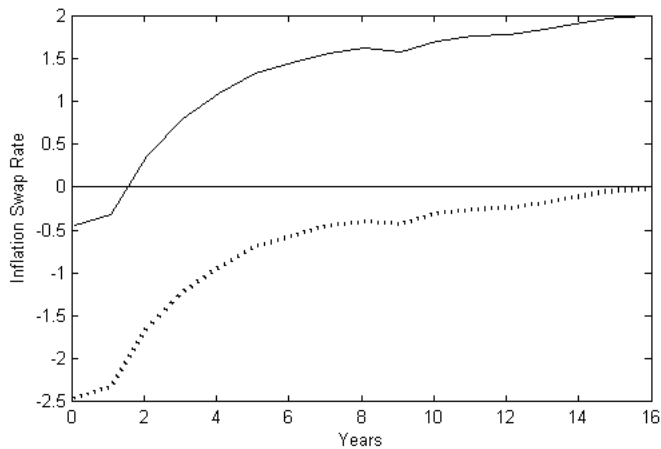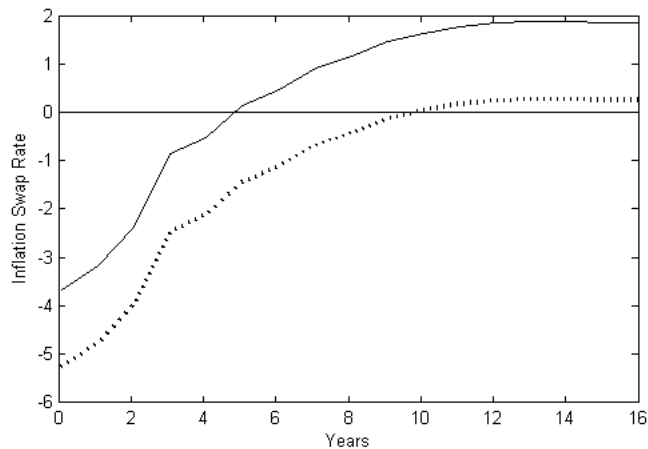
Figure 2.5 – Implied Inflation Swap Curve that Reconciles ILB Mispricing. The figure on the top panel plots the actual U.K. RPI inflation swap curve for March 17, 2009 (solid curve) and the implied inflation swap curve (dotted curve) for the same date that would reconcile the pricing of the U.K. gilt with maturity date December 27, 2027 and the corresponding index–linked gilt issue. The figure on the bottom panel plots the actual U.S. CPI inflation swap curve for December 30, 2008 (solid curve) and the implied inflation swap curve (dotted curve) for the same date that would reconcile the pricing of the Treasury bond with maturity date February 15, 2025 and the corresponding TIPS issue.

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mispricing in the inflation swaps markets is large enough to account for ILB mispricing: whatever mispricing there may be in the inflation swaps market is too small to explain the magnitude of ILB arbitrage mispricing documented in Section 2.4.

2.8.3 Illiquidity

The notion that liquidity patterns can have significant effects on the valuation of securities is well established in the literature. For example, see Boudoukh and Whitelaw (1993), Vayanos and Vila (1999), Acharya and Pedersen (2005), Amihud, Mendelson, and Pedersen (2005), Huang and Wang (2008), Brunnermeier and Pedersen (2009), Longstaff (2009), Huang and Wang (2010), and many others.

To study the effects of changes in liquidity on ILB mispricing, five variables proxy for liquidity conditions in the market. The first is the swap spread which is the difference between the current ten-year interest rate swap yield and the yield of the current reference ten-year bond future. The second is the swap rate which represents the current ten-year interest rate swap rate for a period ending at the maturity of the bond underlying the next-expiring ten-year bond future. The third is an index of implied volatilities on index options (VIX in the United States). The fourth is the price of a swaption on a one-year straddle on ten-year interest rate swaps with the strike price reset to the current at-the-money swap rate at the beginning of every roll period. The fifth is the current five-year CDX index by Markit. The choice of these liquidty proxies is motivated by the Citigroup CLX index which uses the same set of variables to construct the CLX liquidity index. Table 2.12 presents summary statistics of the regression results. Across all liquidity variables, a worsening of market liquidity as measured

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by the liquidity proxy variable is associated with an increase in ILB mispricing. The swap rate and the swaption variables are statistically significant at the ten percent level across all regions, and the volatility index is significant at the five percent level across all regions. These results provide strong evidence, that market-wide liquidity conditions have significant effects on ILB mispricing. The swap spread and the CDX index of CDS spreads are not significantly related to the mispricing. For the United States, these results are consistent with those first reported in Fleckenstein, Longstaff, and Lustig (2012). The latter variables, however, are also measures of credit risk.[51] Thus, to the extent that the swap spread and CDX index of CDS spreads capture credit risk, the ILB mispricing is not significantly related to systemic credit risk.

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[51]For a discussion of U.S. sovereign CDS, see Ang and Longstaff (2011).

Table 2.12 – Liquidity Factors Regression. This table reports summary statistics for the regression of changes in the average monthly basis-point ILB mispricing on liquidity factors. NORTH AMERICA comprises the United States and Canada, EUROPE consists of the United Kingdom, France, Germany and Italy, EUROPE EX. UK represents France, Germany, and Italy. Swap Spread denotes the difference between the current ten-year Interest Rate Swap yield and the yield of the current reference ten-year Bond Future. Swap Rate denotes the current ten-year interest rate swap rate for a period ending at the maturity of the bond underlying the next-expiring ten-year Bond future. Vola denotes the index of implied volatilities on index options (VIX in the United States). Swaption denotes a one-year straddle on ten-year interest rate swaps with the strike price reset to the current at-the-money swap rate at the beginning of every roll period. CDX index denotes the current five-year CDX index by Markit. The dollar mispricing index for each country is constructed as the weighted-average index-linked–nominal bond mispricing, expressed in units of dollars per \$100 notional, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. The basis-point mispricing index for each country is constructed as the weighted-average Index-Linked–Nominal Bond mispricing, expressed in basis points, across the pairs included in the sample for that country, where the average is weighted by the notional amount of the index-linked issue. For each country/region, the left column shows the regression coefficient, and the right column shows the Newey-West *t*-Statistic. The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is from July 23, 2004 to September 20, 2011.

	USA		Europe		Europe w/o UK		Japan		UK	
	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat	Coeff.	t-Stat
Intercept	3.592	1.099	1.077	1.350	−0.248	−0.338	−0.097	−0.068	−0.317	−0.187
Swap Spread	0.356	1.104	0.221	1.024	0.293	1.143	0.372	1.015	0.362	1.234
Swap Rate	1.385	1.691*	1.433	1.736*	1.747	1.709*	1.936	1.563	1.481	1.802*
Vola	0.004	1.983**	0.005	2.086**	0.007	1.986**	0.008	2.315**	0.006	2.189**
Swaption	0.085	1.927*	0.076	1.993**	0.077	1.947*	0.129	1.918*	0.065	2.039**
CDX	0.249	1.001	0.162	0.232	0.168	1.276	0.268	0.822	0.195	0.678
Adj. R^2	0.140		0.171		0.281		0.326		0.150	
N	87		87		87		55		87	

Another possible difference between nominal bonds and their index-linked counterparts could be in their trading costs. However, all seven countries in this study have liquid bond markets and the costs of trading nominal bonds and inflation-linked bonds are both small. Indexed-linked bonds in Japan were restricted to specific investors and therefore may have been illiquid. However, the restrictive requirements on the investor clientele were significantly relaxed by the time Japanese inflation-linked bonds enter the study. For the U.S. market, Fleckenstein, Longstaff and Lustig (2012) provide evidence that on average, a large financial institution would typically face a bid-ask spread for Treasury bonds on the order of a quarter of a basis point in yield. In terms of price, this would translate into a two or three cent bid-ask spread for a ten-year Treasury note. The same financial institution would generally face a bid-ask spread for a TIPS issue of about one basis point in yield during much of the study period. Bid-ask spreads for TIPS, however, roughly doubled with the onset of the financial crisis in 2008 and 2009 and are now about two basis points in yield. This would translate into roughly a 15 cent bid-ask spread for a 10-year TIPS issue. The bid-ask spreads for Treasury STRIPS would be on the same order of magnitude as that of TIPS. Finally, the bid-ask spread on inflation swaps is on the order of five basis points. Taken together, these values imply that TIPS-Treasury mispricing greater than about eight basis points cannot be explained in terms of transaction costs; the transaction costs are very small relative to the typical size of the pricing differences between Treasury bonds and TIPS. Qualitative similar results hold true for the United Kingdom and the Euro-Zone countries. Fleckenstein, Longstaff and Lustig (2012) also address the issue of index-linked bond illiquidity in the U.S. market. As one measure of the relative liquidity of TIPS and Treasury bonds, the average trading volume of the two types of securities by primary dealers was examined. This

information is tabulated and reported online by the Federal Reserve Bank of New York.

Focusing on 2009, the total average daily trading volume in Treasury bonds with maturities

of three years or more by primary dealers was about $207 billion. In contrast, the same

measure for TIPS bonds was roughly $5 billion. TIPS bonds, however, represent less than

ten percent of the total amount of Treasury debt held by the public. Thus, while TIPS may

not be as intensively traded as Treasury bonds, these results suggest that the average daily

trading volume for TIPS is still very substantial. Furthermore, Treasury bond and TIPS

traders confirmed this assessment of the relative liquidity of the two markets. In particular,

anecdotal reports by TIPS traders confirm that TIPS were liquid and that trades could be

executed rapidly. The European and UK markets are very similar in terms of liquidity to the

U.S. markets and it is safe to conclude that transaction costs and illiquidity cannot explain

the magnitude of the mispricing between index-linked and nominal bonds observed in these

developed bond markets.

2.8.4   Differences in Taxation

In the United States, the Federal and State income taxation of Treasury bonds is identical

to that of TIPS in all but one small aspect. Specifically, since the notional amount of TIPS

accretes over time with realized inflation, taxable investors must treat this "phantom income"

as if it were interest income for Federal tax purposes. In contrast, taxable investors holding

Treasury bonds only include coupons as interest income (abstracting from original issue

discount (OID) and premium amortization issues). Interest income from both Treasury

bonds and TIPS (including any accreted notional amounts) is exempt from State income

taxation. A large portion of outstanding TIPS issues are held either directly or indirectly by tax-sheltered entities such as pension plans and retirement funds. Thus, the phantom income provision is irrelevant for many of these investors. This view is consistent with a survey by the Bond Market Association in which 79 percent of respondents indicated that the current tax status of TIPS is not a deterrent to buying TIPS, some indicating that this was because of the tax-free status of their funds.[52] Finally, it is important to observe that if the taxation of phantom income were to affect the valuation of TIPS, it should do so uniformly across all issues since the accretion rate is the same for all TIPS. Furthermore, the effects should also be present in the pricing of Treasury STRIPS since they are also subject to the phantom income provisions. In actuality, however, studies of the pricing of Treasury STRIPS have not found evidence of phantom income related tax effects.[53]

In the United Kingdom, inflation-linked gilts are granted a more favorable tax treatment compared to their nominal counterparts because the inflation accumulated between tax year-ends is tax-exempt. Investors are only taxed on the real return, not on inflation compensation. In effect, it is as if the inflation increase in the principal is not taxable. The U.K. pension and insurance sectors are key investors in and holders of index-linked Gilts. In 2003 insurance companies and pension funds held over 90% of all outstanding index-linked debt, but their share has decreased significantly since then. However, according to the NAPF, pension fund allocations to inflation linked gilts increased to 12.3%, from 7.9% per cent in 2010 (NAPF Annual Survey (2010)). U.K. pension funds hold more than £100 billion of U.K. Gilts which represents around 11% of total issuance.[54] In contrast to the government bond market, the

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[52]See http://archives1.sifma.org/research/tipssurvey.pdf.

[53]For example, see Grinblatt and Longstaff (2000) and Jordan, Jorgensen, and Kuipers (2000).

[54]See http://www.ons.gov.uk/ons/rel/fi/mq5--investment-by-insurance-companies--pension-

inflation uplift for corporate inflation linked bonds is taxed. Consequently, similar to the United States, most index-linked debt is held by pension funds or within the pension business lines of life assurance companies, which are tax-exempt. Tax differences between nominal and index-linked gilts are irrelevant for many of these investors. Therefore, the tax treatment of index-linked bonds cannot drive a wedge between the market prices of index-linked and nominal gilts.

In Japan, prior to 2006, the principal portion of inflation-indexed bonds was classified as a derivative security by the Accounting Standards Board of Japan (ASBJ), since investors may be repaid less than par at maturity in case of persistent deflation. This implied that mark-to-market gains and losses had to be recognized on the income statement instead of on the balance sheet, as is the case for most nominal bonds. Therefore, inflation-indexed bonds were less attractive to domestic investors compared to nominal bonds. In 2006, however, the accounting classification was changed so that mark-to-market gains and losses did not have to be recognized on the income statement. Furthermore, since October 2008, holdings of off-the-run inflation indexed bonds could be recorded at theoretical value rather than at market value on the balance sheet. The coupon payments on index-linked bonds are subject to income tax. As is the case in the United States, the inflation-uplift in the principal is treated as an interest payment and is taxed as such. However, pension funds and life insurance companies are not major holders of indexed bonds in Japan. There are at least two explanations. First, Japan has not experienced inflationary pressures in the past decade. Second, Japan's public pension is based on a pay-as-you-go system, so that hedging demand from public pension funds is small. Importantly, however, more than 50% of index-linked

funds-and-trusts/q3-2012/index.html.

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bonds are held by foreign investors such as life insurance, pension funds and hedge funds according to the Ministry of Finance. For these investors interest and the gains or losses on principal would be exempt from withholding taxes. Therefore, it is highly unlikely that differences in taxation is causing ILB mispricing in Japan.

The tax treatment for Canadian Real Return Bonds is comparable to that of U.S. TIPS in that the full nominal interest payments, including the inflation uplift, are taxable. For any given tax year, bondholders must declare as income the amount by which the compensation for inflation on the principal has increased, even though this accrual is not paid out until the bond matures. Capital gains are not taxed until realized. For non-residents, the Canadian Treasury is not ordinarily required to withhold tax from interest or principal paid on RRBs.[55] As in the United States, a large portion of outstanding Real Return Bond issues are held either directly or indirectly by tax-sheltered entities such as pension plans and retirement funds. The phantom income provision is irrelevant for most of this investor class. Therefore, the tax treatment of index-linked bonds is highly unlikely to be the source of ILB mispricing in Canada.

In the Euro Zone, the tax treatment of inflation-indexed OATs, Bunds, and BTPs is similar to that of these countries' respective nominal bonds. The annual uplifted coupons and any capital gains realized at redemption or when the bond is sold are taxed. Taxation rules slightly differ between institutional and retail investors in that institutional investors pay tax on capital gains before redemption or sale. The investor class is similar to that in the United States, the United Kingdom and Canada. Therefore, tax differences between nominal bonds

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[55]See http://www.fin.gc.ca/invest/taxtreat-eng.asp.

and index-linked bonds are unlikely to be able to account for ILB mispricing in Europe.

In conclusion, while it cannot be completely ruled out that tax differences may have a slight effect on the relative prices of nominal and index-linked bonds, it is highly unlikely that tax differences can account for ILB mispricing of the magnitude documented in Section 2.4.

2.8.5 Deflation Floor

The implementation of the arbitrage strategy abstracts from the fact that many inflation-indexed bonds in the study, such as TIPS bonds in the U.S., feature an embedded deflation floor. As discussed earlier, the principal amount of a TIPS issue is protected against deflation since the principal amount received by a TIPS holder at maturity cannot be less than par. Thus, there is an embedded option or deflation floor incorporated into the TIPS issues. Because of this, the value of a TIPS issue may be somewhat higher than it would be if there was no protection against deflation. The analysis in the previous sections abstracts from the value of the deflation option. It is clear, however, that prices of inflation-linked bonds were adjusted by subtracting out the value of the deflation option, then the estimated ILB mispricing would be potentially much larger than reported. Thus, the deflation floor goes in the wrong direction to explain inflation-indexed/nominal mispricing when the indexed bonds feature a deflation floor. Furthermore, the countries in the sample, except Japan, have not experienced prolonged periods of deflation. Therefore, abstracting from the par floor is a non-issue for the robustness of the quantitative results in this paper. Even for Japan, any impact would be negligible since it enters the sample period in 2007 when deflationary pressures were not a concern.

The results in section 2.4 showed that there are times when the mispricing in the European countries switches sign. Inflation-linked bonds in France, Germany, and Italy feature a par floor, similar to TIPS in the United States. Clearly, in the case of negative mispricing, the fact that the analysis abstracts from the embedded inflation floor, works in the other direction. If the results were adjusted by the value of the deflation floor, the negative mispricing would be smaller in absolute terms, or, "less negative".

To provide a back-of-the-envelope calculation for the impact of the inflation floor on the ILB mispricing in the United States, I collected data on inflation options on the U.S. CPI from the Bloomberg terminal. These are zero-coupon floors whose payoff is tied to the realized inflation rate. The market for these securities developed first in the Euro area and the U.K., but has expanded in the U.S. during the last few years. A zero-coupon inflation floor is a contract entered into at time t. The seller of the floor is contractually obligated to pay a fraction $max\left((1+k)^n - (1+\pi(n))^n\right)$ of the contract's notional amount as a single payment in n years from the inception of the contract, where $\pi(n)$ denotes the average annual CPI inflation rate from t to $t+n$, and k denotes the strike of the floor. Without loss of generality, the notional amount is normalized to \$100, so that it matches the face value of the underlying index-linked bond. In exchange, the buyer makes an up-front payment of $P_t(k,n)$. If realized inflation is less then k at maturity of the contract, then the option expires out-of-the money. A zero-coupon inflation cap is identical, except that the payoff is $max\left((1+\pi(n))^n - (1+k)^n\right)$. The dataset consists of daily quotes on zero-coupon inflation floors at strike prices of 0 percent. The sample period is from October 5, 2009 until September 23, 2011. Since the average notional-weighted maturity of the TIPS bonds was 8.951 years

during that period, the analysis use zero-coupon inflation floors with a matching ten year maturity. During the period from October 5, 2009 until September 23, 2011 the average price of a zero-strike inflation floor was 8.55 basis points per annum. Therefore, the ILB mispricing could have potentially been around nine basis points higher during that period. This rough estimate illustrates that the deflation floor goes in the wrong direction to explain inflation-indexed–nominal bond mispricing. To provide a back-of-the-envelope calculation for the impact of the inflation floor on the ILB mispricing in the Euro Zone, I collected data on inflation options on the Euro HICPX from the Bloomberg terminal. These are zero-coupon floors whose payoff is tied to the realized inflation rate as measure by the Euro HICPX. The sample period is from October 5, 2009 until September 23, 2011. Since the average notional-weighted maturity of the Euro-Zone bonds was 8.561 years during that period, the analysis uses zero-coupon inflation floors with a closely matching ten year maturity. During the period from October 5, 2009 until September 23, 2011 the average price of a zero-strike inflation floor was 6.26 basis points per annum. Therefore, the ILB mispricing could have potentially been around six basis points higher during periods of positive mispricing. These back-of-the envelope calculations are consistent with results reported in Heider, Li, and Verma (2012) who estimate the value of the embedded deflation floor for the U.S., France, Germany, and Italy. Their reported maximum values of the par floor during the sample period are 7, 5.5, 3, and 3.8 basis points for the United States, France, Italy, and Germany, respectively.

2.8.6 Asset Swaps

Inflation asset swaps are widely used by inflation dealers to reduce balance sheets costs. Inflation dealers are typically short inflation because they sell structured inflation products, such as inflation swaps, to their customers. In order to hedge against the short exposure, dealers buy inflation-linked bonds which need to be recorded on their balance sheets. By selling asset swaps, inflation dealers get the inflation-linked bonds off their balance sheets while still keeping the exposure to the inflation-linked payments. Several variations of inflation asset swaps exist. In the simplest form, the par inflation asset swap, the asset swap buyer receives an inflation-linked bond from the asset swap seller. The buyer then enters into a series of inflation swaps to pay the asset swap seller inflation linked coupons equal to that of the inflation linked bond. In return, the asset swap seller pays floating rate payments of Libor plus (or minus) an agreed fixed spread, referred to as the asset swap spread. At maturity, there is an exchange of principal, the seller receiving the inflation uplifted notional and asset swap buyer receiving par. In this structure, counterparty credit risk plays a role. The notional on all the floating payments remains the same because the asset swap seller receives the inflation-accreted notional and pays par. However, the swap seller is exposed to counterparty credit risk because the inflation-uplifted redemption can be substantially higher than par. For example, with an annual inflation rate of two percent, the notional accretion over thirty years equals 81.1 percent. By contrast, Fleckenstein, Longstaff, and Lustig (2012) argue that it is unlikely that counterparty credit risk has much of an effect on the pricing of inflation swaps. Pflueger and Viceira (2011b) suggest that ILB arbitrage is related to the relative cost of financing a TIPS position versus a Treasury Bond position.

They measure these costs as the asset swap spread differential between inflation-linked and nominal bond asset swap spreads. The asset swap spread rose sharply during the financial crisis, reaching 130 basis point in December 2008. ILB basis-point mispricing peaks at 175 basis points around the time of the Lehman bankruptcy in the Fall of 2008 in the United States. First, one potential pitfall in comparing the ILB arbitrage trade against the asset swap trade is that the asset swap trade reflects counterparty risk, whereas counterparty risk has a negligible effect on the ILB arbitrage trade (FLL (2012)). Second, TIPS asset swaps are also driven by inflation swap demand. Furthermore, there were clearly earlier periods when the average mispricing was in excess of 60 basis points, whereas the asset swap spread varies within a relatively narrow range of 21 to 41 basis points from January 2004 through December 2006. It is even more critical to point out that the asset swap argument applies to levered investors. A non-levered investor who perceives TIPS to be undervalued relative to Treasury Bonds can enter a net zero portfolio, which is long one dollar of TIPS and short one dollar of nominal Treasuries. The levered investor would enter one TIPS asset-swap and go short one nominal Treasury asset swap. Even if levered investors could not take advantage of ILB mispricing in the U.S. market, it still remains a puzzle as to why unlevered investors would not engage in the TIPS–Treasury arbitrage trade and align prices. While it cannot be ruled out that the relative financing costs of a TIPS position compared to a position in Treasury bonds may have played a role for ILB mispricing during the financial crisis, it is highly unlikely that financing costs could explain arbitrage mispricing in the magnitude observed in the United States during and in the aftermath of the financial crisis.

2.9 Conclusion

The government bond markets in the United States, the United Kingdom, Japan, Canada, France, Italy, and Germany are among the largest and most actively traded fixed-income markets in the world. Despite this, there are persistent violations of the law of one price within these markets. In all countries, prices of nominal bonds exceed those of their inflation-linked counterparts. In the United Kingdom, the price of a nominal gilt and an inflation-swapped index-linked gilt issue exactly replicating the cash flows of the nominal gilt can differ by more than $20 per $100 notional. In the aftermath of the 2008 financial crisis, ILB mispricing peaks at $101 billion which represents more than eight percent of the aggregate size of the inflation-linked bond markets in the study. On average, aggregate G7 ILB mispricing is in excess of $22 billion.

This paper is the first to document mispricing between nominal and inflation-linked bonds and to analyze the properties and dynamics of arbitrage mispricing in and across seven of the largest fixed-income markets. Although these arbitrages occur in different global markets, ILB mispricing is significantly correlated contemporaneously and in the time series. Furthermore, ILB mispricing in the G7 countries is forecastable based on lagged ILB mispricing in the other countries. There is evidence that many hedge funds have indeed profited from the ILB trade. This paper shows that the ILB arbitrage strategy consistently earns positively-skewed excess returns in all G7 countries, and therefore it is not merely similar to "picking up nickels in front of a steam-roller," or writing deep out-of-the money puts.

This paper provides key new insights into the role slow-moving capital plays for the persis-

tence and the dynamics of arbitrage mispricing. Specifically, it presents evidence consistent with the notion capital that available to specific types of arbitrageurs is significantly related to the mispricing: returns of hedge funds following fixed-income strategies strongly predict subsequent changes in ILB mispricing, whereas returns of other types of hedge funds lack statistically significant forecasting power.

In the aftermath of the financial crisis, central banks around the world have taken measures to stabilize financial markets. This paper also presents new insights into the effects of monetary policy on arbitrage mispricing. Specifically, during the 2008 financial crisis, central banks around the world may have exacerbated ILB mispricing through large-scale asset purchase programs.

2.10 Appendix

2.10.1 The G7 Inflation–Linked Bond Markets

This section provides an overview of the inflation-linked and nominal bond markets in the United States, the United Kingdom, Japan, Canada, France, Italy, and Germany, and describes the dataset for each country. For expositional convenience, all nominal debt obligations of any country will be referred to as bonds and inflation-indexed debt will be referred to as inflation-index bonds (IIB), inflation-linked bonds (ILB), or linkers. Table 2.13 gives an overview of the key features of the G7 inflation-linked government bond markets.

Table 2.13 – G7 Inflation-Indexed Bond Markets. This table presents an overview of the inflation–linked bond markets in the G7 countries: Canada (CAN), France (FRA), Germany (GER), Italy (ITA), Japan (JPN), the United Kingdom (GBR), and the United States (USA). "Prc.Tot.Mrkt.Debt" denotes the ratio of ILB notional outstanding to total marketable debt for each country. All data is as of the September 2011.

	USA	GBR	FRA	GER	ITA	CAN	JPN
Inception	Jan 97	Mar 81	Sep 98	Mar 06	Sep 03	Dec 91	Mar 04
Index	CPI-U NSA	UK RPI	EUR HICPX	EUR HICPX	EUR HICPX	CPI NSA	JPN CPI
No. Bonds Outstanding	28	16	10	3	7	5	16
Notional Outstanding	$563.235b	$341.074b	$218.692b	$43.239b	$131.650	$43.474b	$69.012
Prc.Tot.Mrkt.Debt	7.3%	22.3%	22%	3.52%	6.8%	5.3%	1.62%
Indexation Lag (months)	2–3	8 or 2–3	2–3	2–3	2–3	2–3	2–3*
Floor	Par Floor	No Floor	Par Floor	Par Floor	Par Floor	No Floor	no Floor
Coupon Frequency	Semi-Annual	Semi-Annual	Annual	Annual	Semi-Annual	Semi-Annual	Semi-Annual

The United States Treasury Inflation-Protected Securities (TIPS) are direct obligations of the U.S. Treasury and are similar in most respects to Treasury bonds. The key difference is that the principal amount of a TIPS issue is adjusted over time to reflect changes in the price level as measured by the non-seasonally adjusted consumer price index (CPI). Since the fixed coupon rate for the TIPS issue is applied to its principal amount, the actual semiannual coupon received varies over time as the principal amount changes in response to the realized inflation or deflation rate. Similarly, the final principal amount paid to the bondholder equals the maximum of the original principal amount or the inflation-adjusted principal amount. Thus, TIPS investors' principal is protected against deflation (although the same is not the case for coupon payments). Practitioners refer to this feature as the "par floor".

The principal amount of a TIPS issue is adjusted daily based on the Consumer Price Index for All Urban Consumers, known as CPI-U. Let I_t denote the inflation adjustment for a TIPS issue as of date t. The inflation adjustment is computed as the ratio of the reference CPI at the valuation date t divided by the reference CPI at the issuance date which is normalized to be time zero. The reference CPI for a particular date during a month is linearly interpolated from the CPI reference index for the beginning of that month and the CPI reference index for the beginning of the subsequent month. The CPI reference index for the first day of any calendar month is the CPI-U index for the third preceding calendar month. Thus, the reference CPI for April 1 would be the CPI-U index for the month of January, which is reported by the Bureau of Labor Statistics during February. Details on how TIPS are adjusted for inflation are described on the U.S. Treasury's website[56].

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[56]See http://www.treasurydirect.gov/instit/statreg/auctreg/auctreg/gsr31cfr356.pdf

The Treasury first began auctioning TIPS in January 1997. As of the end of the sample period, 40 separate TIPS issues have been auctioned. Currently, the Treasury issues 5-year, 10-year, and 30-year TIPS on a regular cycle. The increase in the frequency of TIPS auctions has made the inflation–linked market more dense thus improving continuous market making.

Since inception of the program, large structural changes have affected the cash market for inflation protection. At the beginning of the sample period in July 2004, the notional amount of TIPS outstanding was $222.61 billion, representing 5.8 percent of total marketable debt. Towards the end of the sample period in September 2011, the total notional outstanding was at $681 billion, representing a 7.3 percent share of total marketable U.S. government debt. Since 2006, the outstanding amount of TIPS have almost doubled, from $393 billion to $782.0 billion in July 2012. With total marketable debt at $10,607.35b, TIPS represented a share of 7.4 percent in July 2012. In 2002, by contrast, TIPS made up only 4.5 percent of total marketable U.S. debt. In terms of issuance, TIPS made up 3.4% of total bond issuance at year end 2010, up from 3% at the end of 2009. Furthermore, the maturity spectrum has widened significantly, particularly through the re-introduction of 30-year TIPS since 2010. With the 2040, 2041, and 2042 TIPS issues, the 30-year sector represents 7.7 percent of the overall TIPS market.

Monthly turnover in TIPS has also increased significantly since inception of the program in 1997. In January 2000, monthly turnover was at $25 billion. Since then, monthly turnover has increased tenfold to $250 billion on January 2012. During the financial crisis, monthly turnover decreased significantly. In January 2008, it was at $225 billion. During the crisis, monthly turnover dipped below $125 billion.

Even with the explosive growth in the U.S. inflation-linked bond market, TIPS only represent around 7.5 percent of total marketable U.S. debt which compares to 23.3 percent for the United Kingdom, 13 percent for France, and 7.9 percent for Italy. Germany falls behind the United States, with index-linked debt only making up around 5 percent of total marketable debt.

The investor base in TIPS is tilted towards domestic investors. Investment fund accounted for 30.2% of TIPS sold at auction, but only 11.5% of notes and bonds. Foreign investors accounted for 8.2% of TIPS sold at auction but 21.1% of nominal notes and bonds. However, recent auction results show signs that the investor clientele in TIPS is broadening with foreign investors becoming more involved. Still, the mutual fund industry still dominates the U.S. TIPS inflation market. The total amount held by these institutions in 2012 was around \$150 billion, almost 20% of the total notional of TIPS outstanding. Strikingly, this is not the case in Europe where the mutual funds industry only holds around €3.5 billion which is less than one percent of the total notional outstanding.

The data for the United States consist of daily closing prices for U.S. Treasury bonds, TIPS, STRIPS, and inflation swaps for the period from July 23, 2004 to September 20, 2011. All data are obtained from the Bloomberg system. The TIPS and Treasury pairs in the dataset have maturities ranging from 2007 to 2041. Daily closing prices for TIPS and Treasury bonds are adjusted for accrued interest following standard market conventions.

For the analysis, TIPS and Treasury bonds are matched based on their respective maturities. Maturity mismatch is defined as the number of days between the maturity of a TIPS issue and that of a Treasury bond with the closest maturity to that of the TIPS issue. Only pairs

of TIPS and Treasury bonds with a maturity mismatch of less than or equal to 31 days are included in the sample. This leads to a total of 36 TIPS-Treasury bond pairs. Specifically, the Treasury issued 41 TIPS bonds prior to the end of the sample period. One of these issues had matured by the beginning of the sample period. Four issues had maturity mismatches in excess of 31 days. In particular, there are 13 exact matches, 12 mismatches of 15/16 days, and 11 mismatches of 31 days. The 31-day mismatches occur only for maturities of February 2015 or later. Thus, these mismatches represent a very small percentage mismatch in the maturities of the TIPS and Treasury bonds.

The United Kingdom In the United Kingdom, inflation-linked bonds have been in existence since 1981. The first index-linked gilt, the 2% September 1996, was auctioned in a single price auction by the U.K. Treasury on 27 March 1981, for £1 billion. In September 1998, a specialist index-linked market maker list was introduced and in November 1998, the issuance method was changed to a uniform price auction. The U.K. DMO's stated rationale for index-linked gilt issuance was to reinforce the U.K. Government's anti-inflationary credibility, to reduce debt servicing costs by offering a lower real return in exchange for inflation protection, to increase the DMO's flexibility to borrow even in times of high inflation, and to provide flexibility to the pension industry. At the end of 2010, the size of the inflation linked bond market was $420 billion, up from $320 billion in 2009. Total marketable debt outstanding at the end of 2010 was $1.9 trillion with linkers representing 22%, up from 21.5% in 2009, but down from 24% at the end of 2008. Inflation-linked bonds represented 23% of the gilt market at the end of 2010. Gross inflation-linked bonds issuance was $53 million in 2010 which made up 8.92% of gross total marketable debt issuance of $592 million, and

20% of total gilt issuance of $259 million. In uplifted nominal terms, the index-linked gilt market was at £284 billion in August 2012, and accounted for 23% of the total gilt market. The three-month lag U.K. gilts accounted for 64% of the DMO's index-linked gilt portfolio as of August 2012. As in the United States, the index-linked gilt market has seen significant growth. Between 2006 and 2012, the total notional amount of index-linked gilts outstanding has tripled from £108.7 to £282 billion. The number of index-linked Gilt-edged market makers also more than doubled from 9 in 1999–2000 to 21 in 2011–2012.

In the United Kingdom, inflation-linked bonds reference the U.K. Retail Price Index (RPI). Transportation (14.1%), food, beverages and tobacco (25.8%), and housing (23.6%) represent close to two thirds of the index. The RPI is constructed with arithmetic rather than geometric aggregation which results in an upward bias compared to a geometric aggregation because it uses the average of relative prices rather than a ratio of averages. Since 2003, the RPI is no longer used as the inflation measure targeted by monetary policy. The inflation target is measured against the U.K. CPI which is a harmonized index of consumer prices constructed similarly to the Eurostat's HICP index. Despite this, however, the U.K. real bond market remains linked to the RPI because most inflation-linked liabilities reference the RPI. The majority of pension fund liabilities, for example, accrue on a limited price indexation basis to the RPI.[57] Around 74% of pension indexation is explicitly linked to the RPI, and only about 3% is linked to a different index.

Prior to September 2005, all issued index-linked gilts were using an eight-month indexation lag with no interpolation. Initially, the justification for an eight-month lag was to allow two

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[57]This means that the liabilities increase each year by the rate of RPI inflation capped at a certain level, e.g. 5% with an implicit floor of 0%.

months for the compilation and publication of the RPI and a further six months to ensure that the nominal size of the next coupon payment is known at the start of each coupon period in order to compute accrued interest. However, to conform to other major inflation-linked bond markets, index-linked gilts issued from September 2005 onwards employ the three-month indexation lag structure first used in the Canadian real return bond market. In fact, in September 2005 the U.K. DMO issued the world's first 50-year sovereign index-linked bond which also was the first index-linked gilt to use a three-month indexation lag. In uplifted nominal terms, the three-month lag design accounted for 53% of the index-linked gilt market at the end of March 2011. The breakeven-point was reached in July 2010. U.K. inflation-linked gilts have no deflation floor and hence can be redeemed below par if the RPI falls over the lifetime of the bond. The eight-month lag linkers also have no deflation floor, but they have accreted a considerable amount of inflation since issuance. With the introduction of the three-month lag design, index-linked gilts also trade on a real clean price basis as in the U.S. market.

In contrast to the 3-month lagged gilts which trade in real space with a real price and with settlement amounts uplifted to account for the inflation accreted over the life of the bond, the linkers using an eight month lag trade in clean price cash terms, with the traded price rising and falling to reflect inflation that has occurred. In a positive inflation environment, the clean price of the old-style linker increases over time with inflation. Consequently, linkers first issued in the 1980s trade at prices above £200. Since the price of an eight-month linker already incorporates accrued inflation, no index ratio is used to determine the settlement price.

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Monthly turnover in index-linked gilts has also increased significantly since inception of the program in 1981. In April 2008, monthly turnover was at £6 billion. Since then, monthly turnover has almost tripled to £18 billion in August 2011. In August 2012, monthly turnover was around £14 billion, up from recent low of just above £8 billion in February 2012.

The U.K. pension and insurance sectors are key investors in and holders of index-linked gilts.[58] In 2003 insurance companies and pension funds held over 90% of all outstanding index-linked debt, but their share decreased significantly since then.[59] However, according to the NAPF, pension fund allocations to inflation linked gilts increased to 12.3%, from 7.9% in 2010. U.K. pension funds hold more than £100 million of U.K. gilts which represents around 11% of total issuance.

The data for the United Kingdom consist of daily closing prices for U.K. gilts, index-linked gilts, U.K. STRIPS, and inflation swaps for the period from July 23, 2004 to September 20, 2011. All data are obtained from the Bloomberg system. The gilt and inflation-linked gilt pairs in the dataset have maturities ranging from October 2004 to November 2055. Daily closing prices for gilts and inflation-indexed gilts are adjusted for accrued interest following standard market conventions. Note that accrued interest is calculated differently for three-month and the eight-month lag gilts.[60]

For the analysis, inflation-linked gilts and nominal gilts are matched based on their respective maturities. The maturity mismatch is defined as the number of days between the maturity

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[58]See http://www.ons.gov.uk/ons/rel/fi/investment-by-insurance-companies--pension-funds-and-trusts/mq5/quarter-4--2010.pdf

[59]See http://research.dwp.gov.uk/asd/asd5/WP102.pdf

[60]See http://www.dmo.gov.uk/documentview.aspx?docname=/giltsmarket/formulae/igcalc.pdf&page=Formulae/Calc/index.html

of an inflation-linked gilt issue and that of a nominal gilt with the closest maturity to that of the real gilt. For the U.K., only pairs of gilts and inflation-linked gilts with maturity mismatch less than or equal to 55 days are included in the sample. This leads to a total of nine nominal gilt–real gilt bond pairs, five of which have an eight-month indexation lag, and four with a three-month indexation lag. Throughout most of the sample, there are four bonds with an eight-month indexation lag and three bonds with a three-month indexation lag.

Japan   Japan issued the first 10-year maturity index-linked bonds (JGBi) in March 2004 in an amount of ¥100 billion. Initially, the issue traded at a 15 basis points breakeven rate while year-on-year inflation was negative. Despite prolonged periods of deflation in Japan, inflation-linked bonds did not have a deflation floor. However, the Ministry of Finance is taking a deflation floor for future issues into consideration. By the end of 2008, there were a total of 16 bonds outstanding with a total capitalization of close to ¥10 trillion ($90 billion).

Net issuance increased from around $18 billion in 2005 to around $26 billion in 2007. The total amount outstanding peaked at $87 billion in 2008. At that time the inflation-linked bond market represented around 1.62% of total marketable debt and around 3.5% of total long-term bonds outstanding. Inflation-linked bonds represented about 3% of total bond issuance in 2007 and made up around 1.37% of total bonds outstanding at the end of 2008.

Japan JGBi pay semiannual coupons and reference the Japan nationwide CPI index ex-fresh food (Japan Core CPI). The indexation lag is three months and the indexation style follows the Canadian model with linear interpolation to the tenth of the month. The refer-

ence index for inflation-linked Japanese government bonds and inflation swaps is the Japan non-seasonally adjusted consumer price index excluding perishable food items. The CPI is calculated using the Laspeyres method which is based on year-on-year changes in prices of goods and services with respect to the base year 2005. The base year fixes the weights of goods and services included in the index. The Laspeyres method is biased upwards as time passes from the base year. Due to fixed weightings, goods that increase in price are weighted more heavily than goods which decline in price. Japan JGBi do not feature a deflation floor and are not strippable. Similar to U.S. TIPS, Japan inflation-indexed bonds are quoted in real price terms (without inflation adjustment). Identical settlement and day-count conventions as for nominal bonds (3 day settlement, actual/365 day-count) apply to the linker market. The Ministry of Finance typically scheduled auctions on a bimonthly basis using a Dutch style auction process identical to that of nominal bonds (since 2007 re-openings were held as price-competitive auctions).

Initially, the investor base of inflation-indexed bonds was restricted to financial institutions and foreign governments: non-financial corporations, foreign investors, and individuals were not eligible to hold inflation-linked debt. Since 2005, foreign juridical persons including foreign governments, foreign local governments, foreign central banks, international organizations and foreign government agencies are eligible for proprietorship except when subject to taxation on interest from Japanese Government Bonds (JGBs).[61]

In January 2007, the Ministry of Finance declared inflation-indexed bonds eligible for their buyback operations. In each of five buyback operations the Ministry of Finance retired about

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[61]See   http://www.mof.go.jp/english/jgbs/publication/debt_management_report/2010/index.htm

¥45 billion of outstanding linkers until April 2008. In the wake of the financial crisis, prices of inflation-linked bonds declined significantly (in March 2008, the on-the-run 10 year JGBi breakeven rate approached -2 basis points) which lead to increased buybacks by the Ministry of Finance. During the 2008 financial crisis the liquidity of the 10-year inflation indexed bonds dropped precipitously. As breakeven rates continued to decline during the peak of the financial crisis, reaching a low of -323 basis points in mid-December 2008, the Ministry of Finance stepped up their buyback operations and cancelled two linker auctions scheduled for October 2008 and February 2009. From October 2008, average monthly inflation-linked bond buybacks were on the order of ¥215 billion. Over 39 linker buyback operations took place between 2007 and 2009, bringing the total amount retired from the secondary inflation-indexed market to about ¥3.74 trillion. This represents nearly 40% of the total linker issuance and the secondary market declined in size from around $87 billion at the end of 2008 to around $61 billion at the end of 2009. After additional linker buybacks, the size of the secondary inflation-linked bond market was around $32 billion at the end of 2010. However, breakeven rates at the long-end of the maturity curve, recovered to near -50 basis points by the end of 2009, and liquidity improved significantly as well with bid/ask spreads in the 10-year maturities narrowing by over 50%.

Nonetheless, the liquidity in the secondary inflation-linked bond market did not recover to pre-crisis levels with bid-ask spreads occasionally widening to levels seen during the financial crisis. In response to market conditions, new issuances of 10-year inflation-indexed bonds were suspended after August 2008. The planned issuance of ¥0.3 trillion in ten-year inflation-linked bonds in 2010 was put on hold in light of market conditions and in 2011 their issuance

was dismissed again as opinions calling for suspension of issuance remain in the majority at the Meeting of JGB Market Special Participants.[62]

The Bank of Japan targets the Japanese core CPI, along with the corporate goods price index (CGPI), in conducting monetary policy. Prior to 2011, however, the Bank of Japan did not narrowly define long-run price stability based on the core CPI describing medium- to long-term price stability as an "approximate range [of the year-on-year increase in the CPI] between zero and two percent".[63] However, in their 2011 annual review, the Bank of Japan revised their stance on monetary policy and set a goal of one percent for year-on-year increases in the consumer price index.[64]

The data for Japan consist of sixteen maturity-matched inflation-linked–nominal Japanese government bonds, JGB STRIPS and inflation swaps. All data are obtained from the Bloomberg system. Daily closing prices for JGBs and JGBis are adjusted for accrued interest following standard market conventions. Inflation-linked JGBi and JGB are matched based on their respective maturities. Maturity mismatch is defined as the number of days between the maturity of an inflation-linked bond and that of the nominal bond closest in maturity to the indexed bond. In contrast to other countries, the maturity mismatch for all sixteen pairs is ten days. The maturity range for the sixteen pairs is from March 2014 until June 2018, reflecting the fact the Japan Ministry of Finance has only been issuing ten-year maturity bonds to date. The sample period is from March 7, 2007 until September 20, 2011.

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[62]See Ministry of Finance Japan, 2011 Annual Debt Management Report, p. 12
[63]See http://www.boj.or.jp/en/announcements/release_2006/data/mpo0603a1.pdf
[64]See http://www.boj.or.jp/en/announcements/release_2012/k120214b.pdf

Canada In December 1991, the Government of Canada began issuing inflation-indexed debt securities called Real Return Bonds(RRB), and as of September 2011, there are six issues outstanding. Issuance has been concentrated in the thirty year maturity. With an adjusted principal amount outstanding of $30.4 billion as of December 2010, RRBs made up 5.3% of total marketable Canadian government debt, 8.25% of Canadian government bonds, and 21.11% of long-term bonds with maturities exceeding 20 years. The notional amount outstanding, as of March, 31, 2011, is Can$30.8 billion in real terms and Can$37.7 billion in inflation adjusted terms. The initial real return bond issue, the 4.25% 2021, was a 30-year maturity and as of September 2011 is the shortest RRB bond on maturity spectrum. The Treasury issues new bonds at four-year intervals; the 4.25% 2026 being issued in 1995, the 4% 2031 in 1999, the 3.0% 2036 in 2003, the 2.0% 2041 in 2007, and the 1.5% 2044 in 2010. Hence, each new issue extends the initial maturity by one year. In 2011, RRBs made up 5.33% of total marketable debt issuance and 21.72% of all long-term bonds issuance by the Canadian government.

Canadian RRBs are indexed against the non-seasonally adjusted all-items consumer price index. The index constituents represent goods and services from transportation, clothing, housing, food and recreation, and are weighted according to consumer spending patterns. The fixed basket price index is an arithmetic average of price relatives for all index commodities contained in the basket.

The Bank of Canada currently operates under a quarterly funding schedule with one 30-year RRB auction every three months. RRBs tend to trade at general collateral levels in the repo

market.[65]

The indexation methodology for Canada real return bonds is referred to as the "Canadian Model". At the time, the established indexation methodology was the U.K. model with an eight month lag. The Canadian indexation process uses a more contemporaneous measure of inflation by shortening the indexation lag to three months. The innovation in the indexation structure was the use of an index ratio to inflate principal and coupon payments for a given settlement date. With few exceptions, this methodology has been followed by all subsequent major issuers, including the U.K., which has been issuing all inflation-linked gilts with a three-month lag since 2005.

The index ratio for a given settlement date is defined to be the ratio of the reference CPI at that date divided by reference CPI at issue date of the bond. A reference CPI value is calculated for every day based upon the CPI values for three months and two months prior to the month containing the settlement date. The reference CPI for the first day of any calendar month is defined to be the published CPI index level for the month three months prior. The reference CPI for any other date is calculated by linear interpolation. Coupons accrue on an actual/actual basis and are paid semiannually. Canadian RRBs do not have a par floor on the inflation adjusted principal.

The data for Canada consist of daily closing prices Canadian nominal bonds, real-return bonds, STRIPS, and inflation swaps. Daily closing prices for real-return and nominal bonds are adjusted for accrued interest following standard market conventions. The real return bonds are the 4.25% December 2021 and the 4.25% December 2026 real return bonds. The

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[65]See http://www.bankofcanada.ca/stats/cars/f/bd_auction_schedule.html

corresponding matches are the 9.75% June 2021 and the 8% June 2027 nominal bonds. The maturity mismatches are 183 and 182 days, respectively. The 4% December 2031 bond is excluded because the mismatch is 548 days. The 3% December 2036, the 2 December 2041, and the 1.5% December 2044 real returns bonds are excluded due to lack of a good nominal bond match. The sample period is from June 14, 2007 until September 20, 2011. Real Return Bond data prior to June 14, 2007 is not included because inflation swap data is not available in the Bloomberg system.

France   The Agence France Treśor (AFT) issued the first index-linked government bond, the 3% OATi 2009, in September 1998. The issue was linked to the French CPI index excluding tobacco (CPIx) using the Canadian model. As with TIPS, the AFT decided to include a deflation floor which guarantees that the redemption payment on the linker bonds would not be less than the original par value in case of persistent deflation over the lifetime of the bond. The same protection, however, does not apply to the coupon payments. The inaugural issue was brought to market through a syndication process led by Banque Nationale de Paris, Barclays Capital and Société Générale. With the introduction of the Euro, the OATi 2009 was re-denominated in January 1999, as were all fixed-rate French government bonds, thus creating the first Euro denominated inflation-indexed bond issue. In the second half of 1999, the AFT issued a second inflation-indexed bond, the OATi 3.4% July 2029, again linked to the French CPI index excluding tobacco. With the creation of the monetary union, there was demand for securities indexed to the European inflation index (Euro HICPx). In October 2001, the AFT issue €6.5 billion of the OAT €i 3% July 2012, linked to the HICPx. The indexation methodology was identical to that of the OATi securities using a three-month

lag, a deflation floor and annual coupons.

The Euro HICPx is a weighted average of harmonized price indices of the individual Euro area countries with weights determined according to each country's share of consumption expenditure within the Euro area as measured by the household final monetary consumption expenditure. By construction, country weights change over time and also when new countries enter the European Monetary Union. The HICP is published by Eurostat on a non-seasonally adjusted basis. The European Central Banks (ECB) main reference for monetary policy, however, is the headline all-items HICP Index, referred to as the Monetary Union Index of Consumer Prices (MUICP). By mandate, the ECB has to maintain price stability defined as a level of MUICP inflation close to but below two percent.

With the successful launch of the first Euro HICP linker bond, the AFT started issuing new indexed bonds each year, while auctioning existing issues nearly every month. The OAT€i July 2032 was syndicated in 2002 and the OATi July 2013 was the first issue to be launched via auction in 2003. In 2004, the OAT€i 2020 was brought to market via syndication, but the OATi11, OAT€i15, OATi16, and the OATi17 were launched via auction in the second half of 2004 and 2005, respectively. In April 2006, the first BTAN linked to Euro HICPx, the BTAN€i10, was auctioned off. The OAT€i40 and the OATi23 were again brought to market via syndication in the first half of 2007 and the first half of 2008, respectively. In January 2010, the OATi19 was issued via auction.

Issuance of inflation-indexed bonds has continuously increased since inception of the program. Prior to 2009, the AFT's issuance schedule allocated a minimum of ten percent of total bond issuance each year to indexed bonds. In 2004 and 2005, a total of €24 billion, and

€17 billion, respectively, were issued which represented 14% of gross bond supply in 2005. In 2006 and 2007, the amounts were €18 billion and €17 billion, respectively. After issuance of €15 billion in 2008, the amount dropped to €12 billion in 2009 as demand for indexed bonds fell in the aftermath of the financial crisis. Despite the decrease, the AFT tapped specific issues that were in demand. In 2010, the AFT was scheduled to issue about 10% of total bond issuance in index-linked bonds. As in the United States, the index-linked French bond market has seen significant growth. Between 2006 and 2012, the total notional amount of index-linked bonds outstanding doubled from €93 to €177 billion. As a share of total long-term debt, index-linked bonds stand at around 22%, compared to around 9 percent in Italy, 13 percent in Germany, and 5 percent in the United Kingdom. French inflation-linked OAT outstanding represent around 8–10 percent of the Euro HICPX market.

The data for France consist of twelve maturity-matched French inflation-linked–nominal OAT Bonds, STRIPS and Euro HICPx inflation swaps. All data is obtained from the Bloomberg terminal. Daily closing prices for OATi (OAT€i) and OAT bonds are adjusted for accrued interest following standard market conventions. The maturity mismatches of the twelve pairs is either 91 or 92 days. There are a total of seven bonds linked to the French CPI ex-tobacco (FRCPXTOB) and five bonds linked to the European HICPX index. The sample period is from July 23, 2004 until September 20, 2011. During most of the sample, there are fourteen inflation-linked OATs in the database. However, only the twelve pairs are included because the maturity mismatches for the 1.85% July 2007 and the 1.8% 2040 OATis are 456 and 274 days, respectively.

Italy Italy issued its first five-year inflation-linked BTP (Buono del Tesoro Poliennale) (BTPi) in the second half of 2003 via syndication with an initial size of €7 billion. A re-opening in October of that year brought the amount outstanding to over €10 billion. The indexation methodology of the BTPi 1.65% Sep 2008 was analogous to that of French Euro OATi bonds except that coupon payments were on a semi-annual basis like the corresponding nominal BTPs. The Italy Ministry of Economy and Finance next issued a ten-year maturity inflation-linked bond, the BTPi 2.15% September 2014. The initial size was €5 billion, but the notional size increased to €14.5 billion through multiple re-openings. The BTPi 2.35% 2035 was auctioned in the second half of 2004 with and initial size of €4 billion. The second five year issue, the BTPi10 was launched in January 2005. Towards the end of the first half of 2006, the ten-year BTPi 2.1% September 2017 was brought to market for €4 billion via syndication. The first index-linked bond to be auctioned was the BTPi 2012 during the first half of 2007. Later that year, however, the first fifteen-year index-linked bond the BTPi 2023 was brought to market via syndication. Lastly, two long-dated euro HICPx linkers maturing in September 2057 and September 2062 were issued. The ten-year BTPi 2019 was syndicated in May 2009 and in the second half of 2009 the BTPi 2041 was brought to market.

The total amount of inflation-linked BTP outstanding at the end of 2010 was on the order of $138 billion, down from $147 billion in 2009. In 2009 and 2010, inflation-indexed debt represented around 7% and 6.8% of Italy's total marketable debt outstanding, respectively. In 2010, gross issuance of inflation-linked BTPs made up around 3% of total gross debt issuance. Inflation-linked BTPs represented 14.53% of total long-term bonds outstanding at the end of 2010, down from 15.76% at the end of 2009. In 2010, issuance of indexed bonds

represented 16.93% of total long-term bond issuance, down from a peak of 30% in 2007. As a result of the Buono del Tesoro Poliennale's commitment to the inflation-linked market, Italy became the country with the largest notional amount linked to the Euro HICPx in 2005. However, as market conditions deteriorated during the financial crisis, the BTP starkly reduced its inflation-linked issuance from the third quarter of 2008 and in 2009.

The data for Italy consist of nine maturity-matched inflation-linked–nominal BTP Bonds, STRIPS, and Euro HICPx inflation swaps. Daily closing prices for BTP and BTPi bonds are adjusted for accrued interest following standard market conventions. The maturity mismatches of the nine pairs ranges from 0 days for the 1.65% September 2008 BTPi to 47 days for the 0.95% September 2010, the 2.6% September 2023, and the 3.1% September 2026 BTP. All inflation-linked bonds are indexed to the European HICPX index. The sample period is from July 23, 2004 until September 20, 2011. During most of the sample, there are eleven inflation-linked BTPs in the database. However, only the nine pairs are included because the maturity mismatches for the 2.15% July 2014 and the 2.35% 2035 BTPis are 139 and 410 days, respectively.

Germany   The Deutsche Finanzagentur (DFA) issued the first inflation-index bond in March 2006. Since then, all G7 countries have been issuers of inflation-indexed securities. The DFA brought the ten-year Bund€i 1.5% April 2016 to market via syndication with an initial size of €5.5 billion. It was re-opened in September 2006 for €3.5 billion. The inaugural issue was priced against the January 2016 nominal Bund bond. The second German inflation-indexed bond, the OBL€i 2.25% April 2013, was issued in October 2007 for €4 billion and

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the third German linker was launched in June 2009. During the aftermath of the 2008 financial crisis, fears of deflation were widespread and breakeven rates fell precipitously. However, the DFA did not suspend inflation linked issuance and the Bund€i 2020 was the first Euro linker launched after the crisis. The DFA announced in 2010 to issue €3–4 billion in inflation-linked bonds quarterly.

At inception of the program in 2006, inflation-linked issuance represented 3.86% of total marketable debt issuance and 15.76% of total long-term bonds issuance. At the end of 2010, indexed Bunds represented 3.52% of Germany's total marketable debt and around 6% of total long-term bonds outstanding. The face amount outstanding of inflation-linked Bunds has been increasing steadily since March of 2006. At the end of 2010, the total amount of indexed-debt outstanding was on the order of $50 billion, up from $39 billion at the end of 2009.

The data for Germany consist of four maturity-matched inflation-linked–nominal bonds, STRIPS, and Euro HICPx inflation swaps. The maturity mismatches of the four pairs range from 80 days for the 2.25% April 2013 and 1.5% April 2016 index-linked bonds to 101 and 102 days for the 0.75% April 2018, and the 1.75% April 2020 index-linked bonds. All inflation-linked bonds reference the European HICPX index. The sample period is from May 22, 2006 until September 20, 2011.

2.10.2   The G7 Inflation Swaps Markets

This section provides a brief introduction to the mechanics of inflation swaps and gives details on the country-specific indexation mechanisms. In particular, differences between the

indexation mechanism in the inflation swap markets and that in index-linked bond markets are addressed.

An inflation swap contract is executed between two counterparties at time zero and has only one cash flow which occurs at the maturity date of the swap and involves the exchange of a notional adjusted for inflation that has accrued over a specified time period against the notional capitalized with a fixed rate. The fixed rate, agreed at inception, reflects expected future inflation over the lifetime of the contract and is quoted as an annualized rate. The payoff on the inflation leg varies solely based on the final value of the reference inflation index at maturity of the swap. The cash flow on the fixed leg is predetermined by the quoted swap rate.[66]

The mechanics of inflation swaps are identical across the different markets. Hence, consider without loss of generality the United States. To describe the payoff structure of an inflation swap, imagine that at time zero, the five-year zero-coupon inflation swap rate is 200 basis points. As is standard with swaps, there are no cash flows at time zero when the swap is executed. At the maturity date of the swap in five years, the counterparties to the inflation swap exchange a cash flow of $(1 + .0200)^5 - I_t$, where I_t is again the inflation adjustment factor. Thus, if the realized inflation rate was 1.50 percent per year over the five year horizon of the swap, $I_t = 1.015^5 = 1.077284$. In this case, the net cash flow from the swap would be $(1 + .0200)^5 - 1.077284 = \$0.026797$ per dollar notional of the swap. The timing and index lag construction of the index I_t used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

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[66]For more details see Kerkhof (2005), Jarrow and Yildirim (2003) and Hinnerich (2008), and Pond and Mirani (2011).

Inflation swaps are quoted in terms of the constant rate on the contract's fixed leg. The traded maturities are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25, and 30 years for all countries in this study. Longer swap maturities are not included in this study since trading activity at the maturities exceeding 30 years is limited. In order to implement the arbitrage strategy described in the next section, swap rates for intermediate maturities are needed. These are obtained by cubic spline interpolation. For maturities that include fractional years (e.g. 2.3 years), seasonal patterns in inflation must be taken into account. To do this, first seasonal weightings for the CPI-U for each month of the year are estimated by regressing the CPI-U index values for each country for the period January 1980 to August 2011 on monthly indicator variables. The estimated seasonal weights are normalized to ensure than there is no seasonal effect for full-year swaps and then used to adjust the interpolated inflation swap curve. The algorithm is exactly the same for all countries in the sample. The details about the algorithm used to compute the seasonally-adjusted inflation swap curves are provided in the Appendix.

Analogous to inflation-linked bonds, the reference index on the inflation swap is subject to a lag. However, the lagging mechanism may differ from that used in the bond market. For standard U.S. CPI, FRCPIx (French CPI ex-tobacco), Canadian CACPI swaps, the inflation index reference value is determined on the same three-month lag and interpolated principle as in their corresponding bond markets. For standard U.K. RPI and Euro HICPx swaps, the lagging mechanisms are notably different from the corresponding inflation linked bond markets: the reference index for the inflation swaps are not interpolated, with a two-month lag in the U.K. and three months for Euro HICPx swaps. Consequently, a Euro HICPx swap

traded on any given day of a particular month is indexed to the same starting reference value published three months prior.

These differences must be taken into account when the interpolated inflation swap curve is used to match coupon payments on indexed bonds. For any given maturity date, Euro-Zone inflation swaps reference the HICPx value three months prior to that date. The corresponding reference index value for Euro-Zone inflation-indexed bonds is interpolated between the HICPx index values two and three months prior to that date. Hence, the inflation swap rate corresponding to a coupon payment at any date is obtained by applying exactly the same interpolation mechanism as for index-linked bonds to the inflation swap rates associated with the time of the coupon payment and the inflation swap rate for one month prior to that date. In the United Kingdom, there is an additional six month lag between the inflation swap and the eight-month index-linked gilts. Hence, the swap rate associated with a coupon payment at any date is the inflation swap rate six months prior to the coupon payment date. For index-linked gilts using the Canadian model there is an additional one month lag to the inflation swap. Hence, the swap rate associated with a coupon payment at any date is the inflation swap rate one month prior to that date. The interpolation is applied in exactly the same way as for European HICPx swaps.

2.10.3 Measuring TIPS-Treasury Mispricing

This section describes how the size of the ILB mispricing is computed. The algorithm is exactly the same for all countries in the sample. The adjustments to the inflation swap rates to account for differences in the indexation mechanism between inflation swaps and

inflation-linked bonds are as discussed in section 3.1.

Without loss of generality, consider the United States. In addition to the pricing data for TIPS, Treasury bonds, and STRIPS issues, I also download daily closing prices of inflation swaps with maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25 and 30 years for the period from July 23, 2004 to November 19, 2009 from the Bloomberg terminal. Inflation swaps are identified on the Bloomberg system by the ticker USSWITn, where n denotes the maturity of the swap. For a few of these swaps, inflation swap data are missing for several days. In these cases, I replace missing data points by the last available observation.

To implement the arbitrage strategy, the notional amount of each inflation swap is set to match the corresponding semiannual coupon payment (before inflation adjustment) on the TIPS issue which is designated s. At date t, the inflation swap pays a cash flow of $s\left(1+f_t\right)^t - sI_t$, where I_t is the indexed leg and f_t is the fixed inflation swap rate for maturity t.

Implementing the arbitrage strategy requires interpolating the quoted inflation swap rates for all maturities ranging from 0 to 30 years. Furthermore, seasonal patterns in inflation must be taken into account for swap maturities that include fractional years (e.g. 2.3 years). To interpolate the inflation swap rate curve, I first fit a standard cubic spline through the quoted maturities using a grid size of one month. Let the interpolated swap rates be denoted by $f_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$, where the first index refers to the year and the second to the month.

I then estimate seasonal components in inflation from the monthly non-season-ally adjusted U.S. CPI index (CPI-U NSA) series between January 1980 and October 2009 by estimating an OLS regression of monthly log changes in the CPI index on month dummies. More

specifically,

$$\Delta CPI_t \equiv \log\left(\frac{CPI_t}{CPI_{t-1}}\right) = \sum_{i=1}^{12} \beta_i \, d_i \; + \; \varepsilon_i, \qquad (A1)$$

where t is measured in months. The month dummies d_i, $i = 1, 2, \ldots, 12$ are defined as

$$d_i = \begin{cases} 1, & \text{for month i,} \\[2mm] 0, & \text{otherwise.} \end{cases} \qquad (A2)$$

and $d_1 =$ January, $d_2 =$ February, $\ldots$, $d_{12} =$ December. I obtain an estimate of the seasonal effect in month i by subtracting the average of the coefficients $\bar{\beta} = \frac{1}{12}\sum \hat{\beta}_i$ from the estimated coefficients $\hat{\beta}_i$, $i = 1, 2, \ldots, 12$. Let this estimate be denoted by $\hat{b}_i = \hat{\beta}_i - \bar{\beta}$, $i = 1, 2, \ldots, 12$.

Next, I construct monthly forward rates $H_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$ from the interpolated swap rates $f_{i,j}$. Then, I normalize the seasonal factors $\hat{b}_i$ so that their product is unity. Let the normalized monthly adjustment factors be denoted by $\hat{m}_i$, $i = 1, 2, \ldots, 12$, where $\prod_{i=1}^{12} \hat{m}_i = 1$. I then multiply the forward rates $H_{i,j}$ by the corresponding adjustment factor $\hat{m}_j$, $j = 1, 2, \ldots, 12$ to obtain seasonally adjusted forward rates $\tilde{H}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$. By construction, there will be no seasonal effects for full-year swaps. In the last step, I obtain the seasonally adjusted inflation swap curve by converting the forward rates $\tilde{H}_{i,j}$ into inflation swap rates $\tilde{f}_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 1, 2, \ldots, 12$. I do not interpolate or adjust maturities smaller than one year, but use the one-year swap rate instead, because the interpolated rates are sensitive to short-term inflation assumptions in that case. I set $\tilde{f}_{0j} = f_1$, $j = 1, 2, \ldots, 12$.

With the inflation swap curve, I implement the TIPS–Treasury arbitrage strategy and compute the size of the mispricing in the following way. First, I take a position in a TIPS issue with a semi-annual coupon rate of s and maturity T for a price of V. Each period, the TIPS issue pays coupons of sI_t and makes a principal payment of $100I_T$ at maturity.

Next, I enter into an inflation swap for each coupon payment date $t = 1, 2, \ldots, T$ with notional amount of s for $t < T$ and $s + 100$ for the final principal payment at time T. Let f_t denote the fixed rate on the inflation swap for date $t = 1, 2, \ldots, T$ obtained from the interpolated inflation swap curve. At each coupon payment date t, the inflation swap pays a cash flow of $s(1 + f_t)^t - sI_t$ and $(s + 100)(1 + f_T)^T - (s + 100)I_T$ at maturity T. The sum of the cash flows at date t from the TIPS issue and the inflation swap is constant, since $sI_t + s(1 + f_t)^t - sI_t = s(1 + f_t)^t$. Similarly, at maturity $(s + 100)I_T + (s + 100)(1 + f_T)^T - (s + 100)I_T = (s + 100)(1 + f_T)^T$. This converts all of the indexed cash flows from the TIPS bond into fixed cash flows.

Let P and c denote the price and the semiannual coupon payment for the Treasury bond, respectively. To match the cash flows c from the Treasury bond exactly, the replicating portfolio must include a small long or short position in Treasury STRIPS for each coupon payment date t and the maturity date T, such that $s(1 + f_t)^t + x_t = c$ and $(s + 100)(1 + f_T)^T + x_T = c + 100$, where x_t denotes the notional amount of STRIPS for date $t = 1, 2, \ldots, T$. This step converts the indexed bond into a synthetic security with fixed cash flows that exactly replicate the magnitude of the cash flows from the Treasury bond. Given the fixed cash flows and the value of the replicating portfolio, I then calculate the yield to maturity for the replicating portfolio.

In the last step, I use the yield to maturity for the replicating portfolio to determine the price of a synthetic Treasury bond with the same maturity, coupon rate, and cash flows as the matched Treasury bond. The difference between the prices of the synthetic Treasury bond and the matched Treasury bond represents the TIPS–Treasury mispricing.

The procedure is analogous for all countries in the sample.

Chapter 3: Deflation Risk

3.1   Introduction

Deflation has played a central role in the worst economic meltdowns experienced in U.S. history. Key examples include the deflations associated with the Panic of 1837, the Long Depression of 1874–1896, and the Great Depression of the 1930s. In light of this, it is not surprising that deflation is now one of the most-feared risks facing participants in the financial markets. In recent years, the financial press has increasingly raised concerns about a global deflationary spiral and has used terms such as "nightmare scenario" or "looming disaster" to describe the growing threat.[67] Furthermore, addressing the risk of deflation is one of the primary motivations behind a number of actions taken by the Federal Reserve in the past several years such as the quantitative easing programs.[68]

Despite the severe potential effects of deflation, relatively little is known about how large the risk of deflation actually is, or about the economic and financial factors that contribute to deflation risk. The primary reason for this may simply be that deflation risk has traditionally been very difficult to measure. For example, as shown by Ang, Bekaert, and Wei (2007) and others, econometric models based on the time series of historical inflation perform poorly

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[67]For example, see Coy (2012), "Five Charts that Show that Deflation is a Growing Threat," www.businessweek.com/articles/1012-06-05/five-charts-that-showdeflation-is-a-growing-threat, Lange (2011), "Nightmare Scenario: U.S. Deflation Risks Rising," www.reuters.com/article/2011/10/16/us-economy-deflationidUSTRE79P7FV201110126, and Carney (2010), "Deflation: Looming Disaster for Banks," www.cnbc.com/id/39648704/DeflationLoomingDisasterforBanks.

[68]For example, see Bernanke (2012), "Monetary Policy since the Onset of the Crisis," www.federalreserve.gov/newsevents/speech/bernanke20120831a.htm.

even in estimating the first moment of inflation. In addition, while surveys of inflation tend to do better, these surveys are limited to forecasts of expected inflation over shorter horizons and provide little or no information about the tail probability of deflation.

This paper presents a new market-based approach for measuring deflation risk. This approach allows us to solve directly for the market's assessment of the probability of deflation for horizons of up to 30 years using the prices of inflation swaps and options. In doing this, we first identify the risk-neutral density of inflation implied by inflation calls and puts. We then estimate the inflation risk premium embedded in the term structure of inflation swap rates. Finally, we solve for the actual or objective distribution of inflation by inverting the risk-premiumadjusted characteristic function of the risk-neutral density. A key advantage of this approach is that we recover the entire distribution of inflation rather than just the first moment or expected inflation. This is important since this allows us to measure the probability of tail events such as deflation.

The shape of the distribution of inflation varies significantly for shorter horizons, but is much more stable for longer horizons. Inflation risk premia are slightly negative for horizons of one to five years, but increase to about 30 basis points for a 30-year horizon. We find that the market expects inflation of close to 2.5 percent for horizons from 10 to 30 years. The volatility of inflation is roughly two percent for shorter horizons, but is about one percent or less for horizons of ten years or more. Thus, the market views inflation as having a strongly mean reverting nature. The distribution of inflation is skewed towards negative values and has longer tails than a normal distribution.

We solve for the probability of deflation over horizons ranging up to 30 years directly from

the distribution of inflation. The empirical results are very striking. We find that the market

places a significant amount of weight on the probability that deflation occurs over extended

horizons. Furthermore, the market-implied probability of deflation can be substantially

higher than that estimated by policy makers. For example, in a speech on August 27,

2010, Federal Reserve Chairman Ben S. Bernanke stated that "Falling into deflation is not a

significant risk for the United States at this time."[69] On the same date, the market-implied

probability of deflation was 15.11 percent for a two-year horizon, 5.36 percent for a five-

year horizon, and 2.84 percent for a ten-year horizon. These probabilities are clearly not

negligible. On average, the market-implied probability of deflation during the sample period

was 11.44 percent for a two-year horizon, 5.34 percent for a five-year horizon, 3.29 percent

for a ten-year horizon, and 2.33 percent for a 30-year horizon. The risk of deflation, however,

varies significantly and these probabilities have at times been substantially larger than the

averages. In particular, the probability of deflation exhibits jumps which tend to coincide

with major events in the financial markets such as the ratings downgrades of Spain in 2010

or the downgrade of U.S. Treasury debt by Standard and Poors in August 2011.

Deflation is clearly an economic tail risk and changes in deflation risk may reflect the market's

fears of a meltdown scenario.[70] Thus, a natural next step is to examine whether deflation risk

is related to other serious types of tail risk in the financial markets or in the macroeconomy

in general. Focusing first on the pricing of deflation risk, we find that the ratio of the risk-

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[69]See Bernanke (2010), "The Economic Outlook andMonetary Policy," www.federalreserve.gov/newsevents/speech/2010speech.htm.

[70]Note that we are interpreting tail risk as including more than just event risk or jump risk. Event or jump risks are adverse economic events that occur relatively suddenly. In contrast, tail risk can also include extreme scenarios with severe economic consequences which may unfold over extended periods. The modeling framework used in this paper is consistent with both types of risks.

neutral probability of deflation to the objective probability of deflation is on the order of three to one. This ratio is very similar to that of other types of tail risk. For example, Froot (2001) finds that the ratio of the price of catastrophic reinsurance to expected losses ranges from two to seven. Driessen (2005), Berndt, Duffie, Douglas, Ferguson, and Schranz (2005), Giesecke, Longstaff, Schaefer, and Strebulaev (2011) estimate that the ratio of the price of expected losses on corporate bonds to actual expected losses is on the order of two to three.

We next consider the relation between deflation risk and specific types of financial and macroeconomic tail risks that have been described in the literature. In particular, we consider a number of measures of systemic financial risk, collateral revaluation risk, sovereign default risk, and business cycle risk and investigate whether these are linked to deflation risk. We find that a number of systemic risk variables are significantly related to the probability of deflation. For example, the risk of deflation increases as the price of protection on super senior tranches increases. This is intuitive since the types of economic meltdown scenarios that would result in losses on super senior tranches would likely be associated with sharp declines in the level of prices. Similarly, we find that deflation risk increases as the credit and liquidity risks faced by the financial sector increase. Thus, economic tail risk increases as the financial sector becomes more stressed. We also find that the risk of deflation increases as the unemployment rate increases. This is consistent with a number of classical macroeconomic theories about the relation between prices and employment. Overall, these results provide support for the view that the risk of severe macroeconomic shocks in which deflation occurs is closely related to tail risks in financial markets. This is consistent with U.S. historical experience in which depressions/deflationary spirals have been associated with major collapses

in the financial system.

Finally, we also compute the probabilities of inflation exceeding various thresholds. The results indicate that while the probability of inflation in the near term is relatively modest, the long-term probabilities of inflation are much higher. Interestingly, we find that the ratio of the probability of inflation exceeding five percent under the risk-neutral measure is only about 1.4 times that under the actual measure. Thus, inflation tail risk is priced much more modestly than is deflation tail risk.

Our results also have important implications for Treasury debt management. In particular, whenever the Treasury issues Treasury Inflation Protected Securities (TIPS), the Treasury essentially writes an at-the-money deflation put and packages it together with a standard inflation-linked bond. The returns on writing these deflation puts are potentially large because of the substantial risk premium associated with deflation tail risk. If the Treasury is better suited to bear deflation tail risk than the marginal investor in the market for inflation protection, then providing a deflation put provides an extra source of revenue for the Treasury that is non-distortionary. There are good reasons to think that the Treasury is better equipped to bear deflation risk, not in the least because the Treasury and the Federal Reserve jointly control the price level.[71]

This paper contributes to the extensive literature on estimating the properties of inflation. Important papers on estimating inflation risk premia and expected inflation include Hamilton (1985), Barr and Campbell (1997), Evans (1998, 2003), Campbell and Viceira (2001), Bardong and Lehnert (2004), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007,

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[71]Since the ratio of risk-neutral to actual probabilities is much larger for deflation than for high-inflation scenarios, this same logic is not as applicable to the standard inflation protection built into TIPS.

2008), Adrian and Wu (2007), Bekaert and Wang (2010), Chen, Liu, and Cheng (2010), Christensen, Lopez, and Rudebusch (2010, 2011), Gurkaynak and Wright (2010), Gurkaynak, Sack, and Wright (2010), Pflueger and Viceira (2011a, 2011b), Chernov and Mueller (2012), Haubrich, Pennachi, and Ritchken (2012), Faust and Wright (2012), Grishchenko and Huang (2012), and many others. Key papers on deflation include Hamilton (1992), Fischer (1933), Cecchetti (1992), Atkeson and Kehoe (2004), and Kilian and Manganelli (2007).

Two important recent papers have parallels to our work. Christensen, Lopez, and Rudebusch (2011) fit an affine term structure model to the Treasury real and nominal term structures and estimate the value of the implicit deflation option embedded in TIPS prices. Our research significantly extends their results by estimating deflation probabilities for horizons out to 30 years directly using market inflation option prices. Kitsul and Wright (2012) also use inflation options to infer the risk-neutral density for inflation, but do not formally solve for the objective density of inflation. Our results corroborate and extend their innovative work.

The remainder of this paper is organized as follows. Section 2 briefly discusses the history of deflation in the United States. Section 3 provides an introduction to the inflation swap and options markets. Section 4 presents the inflation model used to value inflation derivatives. Section 5 discusses the maximum likelihood estimation of the inflation model. Section 6 describes the distribution of inflation. Section 7 considers the implications of the results for deflation probabilities and the pricing of deflation risk. Section 8 examines the relation between deflation risk and other types of financial and macroeconomic tail risks. Section 9 presents results for the probabilities of several inflation scenarios. Section 10 summarizes the results and makes concluding remarks.

3.2   Deflation in U.S. History

The literature on deflation in the U.S. is far too extensive for us to be able to review in this paper. Key references on the history of deflation in the U.S. include North (1961), Friedman and Schwartz (1963), and Atack and Passell (1994). We will simply observe that deflation was a relatively frequent event during the 19th Century, but has diminished in frequency since then. Bordo and Filardo (2005) report that the frequency of an annual deflation rate was 42.4 percent from 1801-1879, 23.5 percent from 18801913, 30.6 percent from 1914-1949, 5.0 percent from 1950-1969, and zero percent from 1970-2002. The financial crisis of 2008-2009 was accompanied by the first deflationary episode in the U.S. since 1955.

Economic historians have identified a number of major deflationary episodes. Key examples include the crisis of 1815-1821 in which agricultural prices fell by nearly 50 percent. The banking-related Panic of 1837 was followed by six years of deflation in which prices fell by nearly 30 percent. The post-Civil-War greenback period experienced a number of severe deflations and the 18731896 period has been called the Long Depression. This period experienced massive amounts of corporate bond defaults and Friedman and Schwartz (1963) estimate that the price level declined by 1.7 percent per year from 1875 to 1896. The U.S. suffered a severe deflationary spiral during the early stages of the Great Depression in 1929-1933 as prices rapidly fell by more than 40 percent.

Although Atkeson and Kehoe (2004), Bordo and Filardo (2005), and others show that not all deflations have been associated with severe declines in economic output, a common thread throughout U.S. history has been that deflationary episodes are typically associated with

turbulence or crisis in the financial system.

3.3   The Inflation Swaps and Options Markets

In this section, we begin by reviewing the inflation swaps market. We then provide a brief introduction to the relatively new inflation options market.

3.3.1   Inflation Swaps

As discussed by Fleckenstein, Longstaff, and Lustig (2012), U.S. inflation swaps were first introduced in the U.S. when the Treasury began auctioning TIPS in 1997 and have become increasingly popular among institutional investment managers. Pond and Mirani (2011) estimate the notional size of the inflation swap market to be on the order of hundreds of billions.

In this paper, we focus on the most widely-used type of inflation swap – the zero-coupon swap. This swap is executed between two counterparties at time zero and has only one cash flow which occurs at the maturity date of the swap. For example, imagine that at time zero, the ten-year zero-coupon inflation swap rate is 300 basis points. As is standard with swaps, there are no cash flows at time zero when the swap is executed. At the maturity date of the swap in ten years, the counterparties to the inflation swap exchange a cash flow of $(1 + .0300)^{10} I_T$, where I_T is the relative change in the price level between now and the maturity date of the swap. The timing and index lag construction of the inflation index used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

Table 3.1 – Summary Statistics for Inflation Swap Rates. This table reports summary statistics for the inflation swap rates for the indicated maturities. Swap maturity is expressed in years. Inflation swap rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to October 5, 2012.

Swap Maturity	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.758	1.369	−4.545	2.040	3.802	2141
2	1.930	1.087	−3.605	2.191	3.460	2141
3	2.088	0.853	−2.047	2.293	3.351	2141
4	2.219	0.688	−1.228	2.390	3.342	2141
5	2.324	0.575	−0.570	2.468	3.310	2141
6	2.403	0.492	−0.080	2.521	3.310	2141
7	2.472	0.424	0.402	2.575	3.229	2141
8	2.530	0.375	0.639	2.613	3.195	2141
9	2.577	0.331	0.904	2.651	3.135	2141
10	2.621	0.295	1.146	2.685	3.145	2141
12	2.677	0.278	1.280	2.738	3.160	2141
15	2.739	0.278	1.161	2.797	3.330	2141
20	2.801	0.290	1.069	2.865	3.360	2141
25	2.848	0.302	1.211	2.911	3.390	2141
30	2.903	0.300	1.454	2.959	3.500	2141
40	2.784	0.246	1.454	2.819	3.377	1016
50	2.781	0.261	1.465	2.830	3.500	842

The zero-coupon inflation swap rate data used in this study are collected from the Bloomberg system. Inflation swap data for maturities ranging from one to 30 years are available for the period from July 23, 2004 to October 5, 2012. Data for inflation swaps with maturities of 40 and 50 years are available beginning later in the sample. Recent research by Fleming and Sporn (2012) concludes that "the inflation swap market appears reasonably liquid and transparent despite the market's over-the-counter nature and modest activity." They estimate that realized bid-ask spreads for customers in the inflation swap market are on the order of three basis points. Conversations with inflation swap traders confirm that these instruments are fairly liquid with typical bid-ask spreads consistent with those reported by Fleming and Sporn. To guard against any possibility of using illiquid or stale prices in the sample, however, we only include an inflation swap rate when that rate has changed from the previous day. Table 3.1 presents summary statistics for the inflation swap rates.

As shown, average inflation swap rates range from 1.758 percent for one-year inflation swaps, to a high of 2.903 percent for 30-year inflation swaps. The volatility of inflation swap rates is generally declining in the maturity of the contracts. The dampened volatility of long-horizon inflation swap rates suggests that the market may view inflation as being strongly mean-reverting in nature. Table 3.1 also shows that there is evidence of deflationary concerns during the sample period. For example, the one-year swap rate reached a minimum of 4.545 percent during the height of the 2008 financial crisis amid serious fears about the U.S. economy sliding into a full-fledged depression/deflation scenario.

3.3.2 Inflation Options

The inflation options market had its inception in 2002 with the introduction of caps and floors on the realized inflation rate. While trading in inflation options was initially muted, the market gained considerable momentum as the financial crisis emerged and total interbank trading volume reached $100 billion.[72] While the inflation options market is not yet as liquid as, say, the stock index options market, the market is sufficiently liquid that active quotations for inflation cap and floor prices for a wide range of strikes have been readily available in the market since 2009.

In Europe and the United Kingdom, insurance companies are among the most active participants in the inflation derivatives market. In particular, much of the demand in ten-year and 30-year zero percent floors is due to pension funds trying to protect long inflation swaps positions. In contrast, insurance companies and financial institutions that need to hedge in-

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[72]For a discussion of the inflation derivatives markets, see Jarrow and Yildirim (2003), Mercurio (2005), Kerkoff (2005), and Barclay's Capital (2010).

flation risk are the most active participants on the demand side in the U.S. inflation options market.

The most actively traded inflation options are year-on-year and zero-coupon inflation options. Year-on-year inflation options are caps and floors that pay the difference between a strike rate and annual inflation on an annual basis. Zero-coupon options, in contrast, pay only one cash flow at the expiration date of the contract based on the cumulative inflation from inception to the expiration date. To illustrate, assume that the realized inflation rate over the next ten years was two percent. A ten-year zero-coupon cap struck at one percent would pay a cash flow of $\max(0, 1.0200^{10} - 1.0100^{10})$ at its expiration date. In this paper, we focus on zero-coupon inflation options since their cash flows parallel those of zero-coupon inflation swaps.

As with inflation swaps, we collect inflation cap and floor data from the Bloomberg system. Data are available for the period from October 5, 2009 to October 5, 2012 for strikes ranging from negative two percent to six percent in increments of 50 basis points. We check the quality of the data by insuring that the cap and floor prices included satisfy standard option pricing bounds such as those described in Merton (1973) including put-call parity, monotonicity, intrinsic value lower bounds, strike price monotonicity, slope, and convexity relations. To provide some perspective on the data, Table 3.2 provides summary statistics for call and put prices for selected strikes. As illustrated, inflation cap and floor prices are quoted in basis points, or equivalently, as cents per $100 notional. Interestingly, inflation option prices are not always monotonically increasing in maturity. This may seem coun-terintuitive given standard option pricing theory, but is it important to recognize that the

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Table 3.2 – **Summary Statistics for Inflation Caps and Floors**. This table reports the average values for inflation caps and floors for the indicated maturities and strikes. The average values are expressed in terms of basis points per \$100 notional. Option Maturity is expressed in years. Ave. denotes the average number of caps and floors available each day from which the risk-neutral density of inflation is estimated. N denotes the number of days for which the risk-neutral density of inflation is estimated. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Option Maturity	Average Floor Value by Strike								Average Cap Value by Strike								Ave.	N
	−2	−1	0	1	2	3	4	5	−1	0	1	2	3	4	5	6		
1	7	14	27	54	105	181	294	398	237	153	91	41	17	7	4	2	25.6	572
2	19	29	48	92	184	334	544	749	523	349	204	96	43	22	13	9	25.9	593
3	27	39	61	111	229	444	751	1050	831	572	342	162	72	38	24	17	26.5	500
5	36	49	76	134	282	599	1058	1565	1456	1045	658	325	154	70	42	29	25.9	577
7	32	46	76	144	311	697	1261	1954	2040	1518	1014	545	230	133	79	55	25.7	434
10	31	49	85	170	368	843	1561	2536	2752	2111	1478	846	413	198	120	76	25.7	566
12	28	44	80	164	376	900	1704	2862	3119	2424	1732	1005	482	236	126	75	26.4	531
15	24	39	73	156	376	967	1918	3332	3536	2793	2035	1191	560	274	142	84	26.2	569
20	21	36	69	157	408	1121	2278	3959	3985	3254	2467	1473	690	326	172	94	25.1	488
30	15	31	75	205	629	1807	2786	5812	4614	4117	3428	2163	1154	573	309	187	21.5	215

inflation rate is a macro variable rather than the price of a traded asset.[73] For most maturities, we have about 25 separate cap and floor prices with strikes varying from negative two percent to six percent from which to estimate the risk-neutral density of inflation.

3.4   Modeling Inflation

In this section, we present the continuous time model used to describe the dynamics of inflation under both the objective and risk-neutral measures. We also describe the application of the model to the valuation of inflation swaps and options.

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[73]We observe that similar nonmonotonic behavior occurs with interest rate options such as interest rate caps, floors, and swaptions; see Longstaff, Santa-Clara, and Schwartz (2001).

3.4.1 The Inflation Model

We begin with a few key items of notation. For notational simplicity, we will assume that all inflation contracts are valued as of time zero and that the initial price level at time zero is normalized to one.[74] Furthermore, time zero values of state variables are unsubscripted. Let I_t denote the relative change in the price level from time zero to time t.

Under the objective measure P, the dynamics of the price level are given by,

$$dI = IX dt + I\sqrt{V} dZ_I \tag{1}$$

$$dX = \kappa (Y - X) dt + \sigma dZ_X \tag{2}$$

$$dY = (\alpha - \beta Y) dt + \eta dZ_Y \tag{3}$$

$$dV = \mu dt + s dZ_V \tag{4}$$

In this specification, X_t represents the instantaneous expected inflation rate. The state variable Y_t represents the long-run trend in expected inflation towards which the process X_t reverts. The process V_t represents the stochastic volatility of realized inflation. An important implication of stochastic volatility is that extreme declines in the price level can occur during periods of high volatility, which may resemble large downward jumps. Thus, this specification is consistent with the intuition of deflation representing an economic tail risk or event risk. Clearly, the same argument also holds for inflation. Rather than fully parameterizing the dynamics for V at this stage, we leave the drift and diffusion terms μ and s unspecified and allow for the possibility that they may depend on a vector of additional

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[74]Since the initial price level equals one, we will further simplify notation by not showing the dependence of valuation expressions on the initial price level I.

state variables.[75] The processes Z_I, Z_X, Z_Y, and Z_V are Brownian motions. The correlation between dZ_X and dZ_Y is ρdt, the correlation between dZ_I and dZ_V is Θdt, and the remaining correlations are assumed to be zero. This primarily affine specification has parallels to the long-run risk model of Bansal and Yaron (2004) and allows for a wide range of possible time series properties for realized inflation.

Under the risk-neutral valuation measure Q, the dynamics of the price level are given by

$$dI = IX dt + I\sqrt{V} dZ_I \tag{5}$$

$$dX = \lambda (Y - X) dt + \sigma dZ_X \tag{6}$$

$$dY = (\phi - \gamma Y) dt + \eta dZ_Y \tag{7}$$

$$dV = \mu dt + s dZ_V \tag{8}$$

where the parameters λ, ϕ, and γ that now appear in the system of equations allow for the possibility that the market incorporates time-varying inflation-related risk premia into asset prices. In particular, the model allows the risk-neutral distributions of X, Y, and I to differ from the corresponding distributions under the objective measure. Thus, the model permits a fairly general structure for inflation risk premia. On the other hand, the model assumes that variation in the state variable V is not priced in the market. This assumption appears to be a modest one and has the important advantage of making the analysis much more tractable. We acknowledge, however, that more general types of risk premium specifications are possible.

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[75]Although we model V as being driven by a (possibly vector) Brownian motion, the model could easily be extended to allow for a jump-diffusion specification for the stochastic volatility of the inflation process. This specification would be completely consistent with our empirical approach.

Finally, let r_t denote the nominal instantaneous riskless interest rate. We can express this rate as $r_t = R_t + X_t$ where R_t is the real riskless interest rate and X_t is expected inflation. For tractability, we also assume that R_t is uncorrelated with the other state variables I_t, X_t, Y_t, and V_t.

3.4.2 Valuing Inflation Swaps

From the earlier discussion, an inflation swap pays a single cash flow of $I_T F$ at maturity date T, where F is the inflation swap price set at initiation of the contract at time zero. Note that $F = (1 + f)^T$ where f is the inflation swap rate. The Appendix shows that the inflation swap price can be expressed in closed form as

$$F(X,Y,T) = \exp\left(-A(T) - B(T)X - C(T)Y\right) \tag{9}$$

where

$$
\begin{aligned}
A(T) = {} & \frac{\sigma^2}{2\lambda^2}\left(T - \frac{2}{\lambda}\left(1 - \exp^{-\lambda T}\right) + \frac{1}{2\lambda}\left(1 - e^{-2\lambda T}\right)\right) \\
& - \frac{\sigma\eta\rho}{\gamma\lambda(\lambda-\gamma)}\left(\gamma\left(T - \frac{2}{\lambda}\left(1 - e^{-\lambda T}\right) + \frac{1}{2\lambda}\left(1 - e^{-2\lambda T}\right)\right)\right. \\
& \left. -\lambda\left(T - \frac{1}{\lambda}\left(1 - e^{-\lambda T}\right) - \frac{1}{\gamma}\left(1 - e^{-\gamma T}\right) + \frac{1}{\gamma+\lambda}\left(1 - e^{-(\gamma+\lambda)T}\right)\right)\right) \\
& + \frac{\eta^2}{2\gamma^2(\lambda-\gamma)^2}\left(\gamma^2\left(T - \frac{2}{\lambda}\left(1 - e^{-\lambda T}\right) + \frac{1}{2\lambda}\left(1 - e^{-2\lambda T}\right)\right)\right. \\
& \left. - 2\gamma\lambda\left(T - \frac{1}{\lambda}\left(1 - e^{-\gamma T}\right) + \frac{1}{2\gamma}\left(1 - e^{-2\gamma T}\right)\right)\right) \\
& + \frac{\phi}{\gamma(\lambda-\gamma)}\left((\gamma-\lambda)T - \frac{\gamma}{\lambda}\left(1 - e^{-\gamma T}\right) + \frac{\gamma}{\lambda}\left(1 - e^{-\gamma T}\right)\right), \tag{10}
\end{aligned}
$$

$$B\left(T\right) = \frac{-\left(1 - e^{\lambda T}\right)}{\lambda}, \tag{11}$$

$$C\left(T\right) = \frac{\gamma\left(1 - e^{-\lambda T}\right) - \lambda\left(1 - e^{-\gamma T}\right)}{\gamma\left(\lambda - \gamma\right)}. \tag{12}$$

3.5 Valuing Inflation Options

Let $C(X, Y, V, T)$ denote the time zero value of a European inflation cap or call option with strike K. The payoff on this option at expiration date T is $max(0, I_T - (1 + K)^T)$. The Appendix shows that the value of the call option at time zero can be expressed as

$$C\left(X, Y, V, T\right) = D\left(T\right) E^{Q^*}\left[max(0, I_T - (1 + K)^T\right], \tag{13}$$

where $D(T)$ is the price of a riskless discount bond with maturity T, and the expectation is taken with respect to the adjusted risk-neutral measure Q^* for inflation defined by the following dynamics,

$$dI = IX dt + I\sqrt{V} dZ_I \tag{14}$$

$$dX = \left(\lambda\left(Y - X\right) + \sigma^2 B\left(T - t\right) + \rho\sigma\eta C\left(T - t\right)\right) dt + \sigma dZ_X \tag{15}$$

$$dY = \left(\alpha - \beta Y + \eta^2 C\left(T - t\right) + \rho\sigma\eta B\left(T - t\right)\right) dt + \eta dZ_Y \tag{16}$$

$$dV = \mu dt + s dZ_V \tag{17}$$

The adjustment to the risk-neutral measure arises because the inflation rate is correlated with the riskless interest rate and allows us to discount the option cash flow outside of the

expectation.[76] This adjusted measure has been referred to variously as a certainty-equivalent measure or a forward measure in the literature. The Appendix also shows that under this measure, the expected value of I_T equals the inflation swap price F. In this paper, we focus primarily on the adjusted risk-neutral density which will be implied from inflation option prices. To streamline the discussion, however, we will typically refer to the implied density simply as the risk-neutral density. A similar representation holds for the value of an inflation floor or put option $P(X, Y, V, T)$ with payoff at expiration date T of $max(0, (1 + K)^T - I_T)$.

3.5.1 The Distribution of the Price Level

From the dynamics given above, an application of Itô's Lemma implies that the log of the relative price level can be expressed as,

$$\ln I_T = \int_0^T X_s ds$$
$$- \frac{1}{2} \int_0^T V_s ds + \int_0^T \sqrt{V_s} dZ_V \tag{18}$$

The Appendix shows that this can be expressed as

$$\ln I_T = u_T + w_T \tag{19}$$

$$\ln I_T = v_T + w_T \tag{20}$$

---

[76]See Jamshidian (1989) and Longstaff (1990) for a discussion of this adjustment to the risk-neutral measure.

under the (adjusted) risk-neutral and objective measures, respectively, where u_T and v_T are normally distributed random variates. The terms u_T and v_T are simply the value of the integral on the right hand side in the first line in Equation (18) under the respective measures, where the distribution of this integral is different under each of the two measures. It is important to observe that both u_T and v_T are independent of the value of w_T, where w_T represents the term on the second line in Equation (18). This latter feature, in conjunction with the explicit solutions for the densities of u_T and v_T provided in the Appendix, will allow us to solve directly for the objective density of $\ln I_T$ given the risk-neutral density.

3.6 Model Estimation

In identifying the distribution of inflation, we use a simple three-step approach. First, we solve for the risk-neutral distribution of inflation embedded in the prices of inflation caps and floors having the same maturity but differing in their strike prices. Second, we identify the inflation risk premia by maximum likelihood estimation of an affine model of the term structure of inflation swaps. Third, we make the transformation from the implied risk-neutral distribution to the objective distribution of inflation.

3.6.1 Solving for the Risk-Neutral Distribution

There is an extensive literature on the estimation of risk-neutral distributions from option prices. Key examples include Banz and Miller (1978), Breeden and Litzenberger (1978), Longstaff (1995), Aït-Sahalia and Lo (1998), and others.

In modeling the risk-neutral distribution, it is important to allow for very general types of densities while preserving sufficient structure for the results to be interpretable. Accordingly, we assume that the density $h(z)$ of the continuously compounded inflation rate $z = \ln(I_T)/T$ under the (adjusted) risk-neutral measure is a member of the five-parameter class of generalized hyperbolic densities. As shown by Ghysels and Wang (2011), this broad class of distributions nests many of the distributions that appear in the financial economics literature including the normal, gamma, Student t, Cauchy, variance gamma, normal inverse Gaussian, normal inverse chi-square, generalized skewed t, and hyperbolic distributions. The generalized hyperbolic density is given by

$$
h(z) = \frac{(a^2 - b^2)^{q/2} d^{-q} e^{b(z-c)}}{\sqrt{2\pi} a^{q-1/2} K_q\left(d\sqrt{a^2 - b^2}\right)} \frac{K_{q-1/2}\left(a\sqrt{d^2 + (z-c)^2}\right)}{\left(\sqrt{d^2 + (z-c)^2}\right)^{1/2-q}}
\tag{21}
$$

where a, b, c, d, and q are parameters, and $K_q(\cdot)$ denotes the modified Bessel function (see Abramowitz and Stegun (1965), Chapter 10).

We solve for the implied risk-neutral density in the following way. For each date and horizon, we collect prices for all available inflation caps and floors. Typically, we have prices for roughly 25 caps and floors with strike prices ranging from negative two percent to six percent in steps of 50 basis points. Next, we solve for the five parameter generalized hyperbolic density that results in the best fit to the set of cap and floor prices, while requiring that the model exactly match the corresponding inflation swap rate.[77] With this latter condition,

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[77]This latter condition implicitly requires that the moment generating function for the density be finite. This requirement places some mild restrictions on the parameters which are incorporated in the fitting algorithm.

there are essentially four free parameters that can be optimized to fit the cross-section of option prices. To value the options, we numerically integrate the product of the option payoff and the density. The optimization algorithm solves for the parameter vector that minimizes the sum of squared pricing errors, where each option receives equal weight. We then repeat this process for each day in the sample period and for each horizon of option expirations, one, two, three, five, seven, ten, 20, and 30 years.[78] Although not shown, the algorithm is able to fit the inflation cap and floor prices very accurately. In particular, the model prices are typically within several percent of the corresponding market prices and would likely be well within the actual bid-ask spreads for these options.

3.6.2   Maximum Likelihood Estimation

As shown in Equation (9), the closed-form solution for inflation swap prices depends only on the two state variables X and Y that drive expected inflation. An important advantage of this feature is that it allows us to use standard affine term structure modeling techniques to estimate X and Y and their parameters under both the objective and risk-neutral measures. In doing this, we apply the maximum likelihood approach of Duffie and Singleton (1997) to the term structure of inflation swaps for maturities ranging from one to 30 years (but not for the 40 and 50 year maturities).

Specifically, we assume that the two-year and 30-year inflation swap rates are measured without error. Thus, given a parameter vector Θ, substituting these maturities into the log

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[78]We solve for the density of each option expiration horizon separately since the model allows for a general inflation specification rather than a specific representation. Thus, we place no a priori restrictions on the term structure of risk-neutral densities possible at a specific date.

of the inflation swap expression in Equation (9) results in a system of two linear equations

$$\ln F\left(X,Y,2\right) = -A(2) - B(2)X - C(2)Y, \tag{22}$$

$$\ln F\left(X,Y,30\right) = -A(30) - B(30)X - C(30)Y, \tag{23}$$

in the two state variables X and Y. This means that X and Y can be expressed as explicit linear functions of the two inflation swap prices $F(X,Y,2)$ and $F(X,Y,30)$. Let J denote the Jacobian of the mapping from the two swap rates into X and Y.

At time t, we can now solve for the inflation swap rate implied by the model for any maturity from the values of X_t and Y_t and the parameter vector Θ. Let ϵ_t denote the vector of differences between the market value and the model value of the inflation swaps for the other maturities implied by X_t, Y_t, and the parameter vector Θ. Under the assumption that ϵ_t is conditionally multivariate normally distributed with mean vector zero and a diagonal covariance matrix Σ with main diagonal values v_j (where the subscripts denote the maturities of the corresponding inflation swaps), the log of the joint likelihood function LLK_t of the two-year and 30-year inflation swap prices and $t + \Delta t$ conditional on the inflation swap term structure at time t is given by

$$
\begin{aligned}
= &- \ln\left(2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}}\right) - \frac{1}{2\left(1-\rho_{XY}^2\right)}\left[\frac{\left(X_{t+\Delta t}-\mu_{X_t}\right)^2}{\sigma_X^2}\right. \\
&\left. - 2\rho_{XY}\left(\frac{X_{t+\Delta t}-\mu_{X_t}}{\sigma_X}\right)\left(\frac{Y_{t+\Delta t}-\mu_{Y_t}}{\sigma_Y}\right) + \frac{\left(Y_{t+\Delta t}-\mu_{Y_t}\right)^2}{\sigma_Y^2}\right]
\end{aligned}
\tag{24}
$$

where the conditional moments μ_{X_t}, μ_{Y_t}, σ_X, σ_Y, and $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}$ of $X_{t+\Delta t}$ and $Y_{t+\Delta t}$ are

given in the Appendix. The total log likelihood function is given by summing LLK_t over all values of t.

We maximize the log likelihood function over the 22-dimensional parameter vector

$$\Theta = (\kappa, \sigma, \alpha, \beta, \eta, \rho, \lambda, \phi, \gamma, v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{12}, v_{15}, v_{20}, v_{25})$$

using a standard quasi-Newton algorithm with a finite difference gradient. As a robustness check that the algorithm achieves the global maximum, we repeat the estimation using a variety of different starting values for the parameter vector. Table 3.3 reports the maximum likelihood estimates of the parameters and their asymptotic standard errors. The fitting errors from the estimation are all relatively small with the typical standard deviation ranging from roughly six to to ten basis points, depending on maturity.

3.6.3 Solving for the Objective Distribution

Let $\Phi(x; w)$ denote the characteristic function for the density function $h(x)$,

$$\Phi(x; w) = \int_{-\infty}^{\infty} e^{iwx} h(x) \, dx \tag{25}$$

Recall from the earlier discussion that the log of the relative price level can be expressed as $u_T + w_T$ under the risk-neutral measure, and as $v_T + w_T$ under the objective measure, where w_T is independent of u_T and v_T . Using the properties of characteristic functions, it is easily shown that

$$\Phi(v_T + w_T; w) = \frac{\Phi(u_T + w_T; w)\, \Phi(v_T; w)}{\Phi(u_T; w)} \tag{26}$$

Table 3.3 – **Maximum Likelihood Estimation of the Inflation Swap Model**. This table reports the maximum likelihood estimates of the parameters of the inflation swap model along with their asymptotic standard errors. The model is estimated using daily inflation swap prices for the period from July 23, 2004 to October 5, 2012.

Parameter	Value	Standard Error
κ	1.041346	0.477189
σ	0.037872	0.000544
α	0.089929	0.002182
β	3.540201	0.087404
η	0.006448	0.000007
λ	1.063634	0.005859
ϕ	0.000708	0.000001
γ	0.000001	0.000000
ρ	-0.1667560	0.004911
v_1	0.00001107	0.00000081
v_3	0.00000089	0.00000024
v_4	0.00000153	0.00000034
v_5	0.00000187	0.00000040
v_6	0.00000174	0.00000037
v_7	0.00000179	0.00000038
v_8	0.00000187	0.00000040
v_9	0.00000218	0.00000047
v_{10}	0.00000254	0.00000057
v_{12}	0.00000202	0.00000043
v_{15}	0.00000129	0.00000030
v_{20}	0.00000076	0.00000022
v_{25}	0.00000050	0.00000018

Thus, given the densities for u_T and v_T , once we can identify the characteristic function of the price $u_T + w_T$ under the risk-neutral measure, we can immediately solve for the characteristic function of the log of the relative price level $u_T + w_T$ under the objective measure. Given this characteristic function $\phi(v_T + w_T)$, we can recover the cumulative density function $\Psi(ln(I_T)/T)$ of the realized inflation rate using the Gil-Pelaez inversion integral,

$$\Psi(z) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty w^{-1} \mathrm{Im}\left[e^{-iwz} \phi(v_T + w_T; w) \right] dw \qquad (27)$$

where $Im[\cdot]$ represents the imaginary component of the complex-valued argument. Once the cumulative distribution function for the inflation rate $z = ln(I_T)/T$ is determined, the cumulative distribution function for the relative price level I_T is obtained by a simple change of variables.

3.7 The Distribution of Inflation

As a preliminary to the analysis of deflation risk, it is useful to first present the empirical results for inflation risk premia, expected inflation, inflation volatility, and the higher moments of inflation.

3.7.1 Inflation Densities

To provide some perspective on the nature of the inflation density under the objective measure, Figure 3.1 plots the time series of densities of inflation for several horizons. As shown, there is considerable variation in the shape of the inflation distribution for the shorter hori-

Table 3.4 – Summary Statistics for Inflation Risk Premia. This table reports summary statistics for the estimated inflation risk premia for the indicated horizons. Horizon is expressed in years. The inflation risk premia are measured in basis points. The inflation risk premia are estimated using the period from July 23, 2004 to October 5, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	−1.92	5.47	−25.66	−1.36	9.95	2141
2	−3.80	11.18	−54.46	−2.42	20.07	2141
3	−3.94	14.84	−72.08	−1.95	27.47	2141
4	−2.91	17.16	−82.27	−0.45	33.24	2141
5	−1.22	18.71	−88.07	1.50	38.05	2141
6	0.83	19.79	−91.26	3.78	42.27	2141
7	3.04	20.57	−92.86	6.11	46.05	2141
8	5.30	21.17	−93.48	8.44	49.50	2141
9	7.57	21.63	−93.46	10.77	52.70	2141
10	9.79	22.00	−93.04	13.09	55.67	2141
12	14.02	22.56	−91.04	17.44	61.10	2141
15	19.62	23.12	−88.63	23.11	67.72	2141
20	26.56	23.68	−84.40	30.20	75.78	2141
25	30.26	24.01	−82.32	33.95	80.15	2141
30	30.62	24.23	−83.04	34.37	80.96	2141

zons. In contrast, the distribution of inflation for longer horizons is more stable over time.

3.7.2 Inflation Risk Premia

We measure the inflation risk premium by simply taking the difference between the fitted inflation swap and expected inflation rates. This is the way in which many market participants define inflation risk premia. When the inflation swap rate is higher than expected inflation, the inflation risk premium is positive, and vice versa. There is no compelling theoretical reason why the inflation risk premium could not be negative in sign. In this case, the risk premium might well be viewed as a deflation risk premium.

Table 3.4 presents summary statistics for the average inflation risk premia for horizons ranging from one year to 30 years. Figure 3.2 plots the time series of inflation risk premia for a number of horizons. As shown, the average risk premia are slightly negative for horizons out
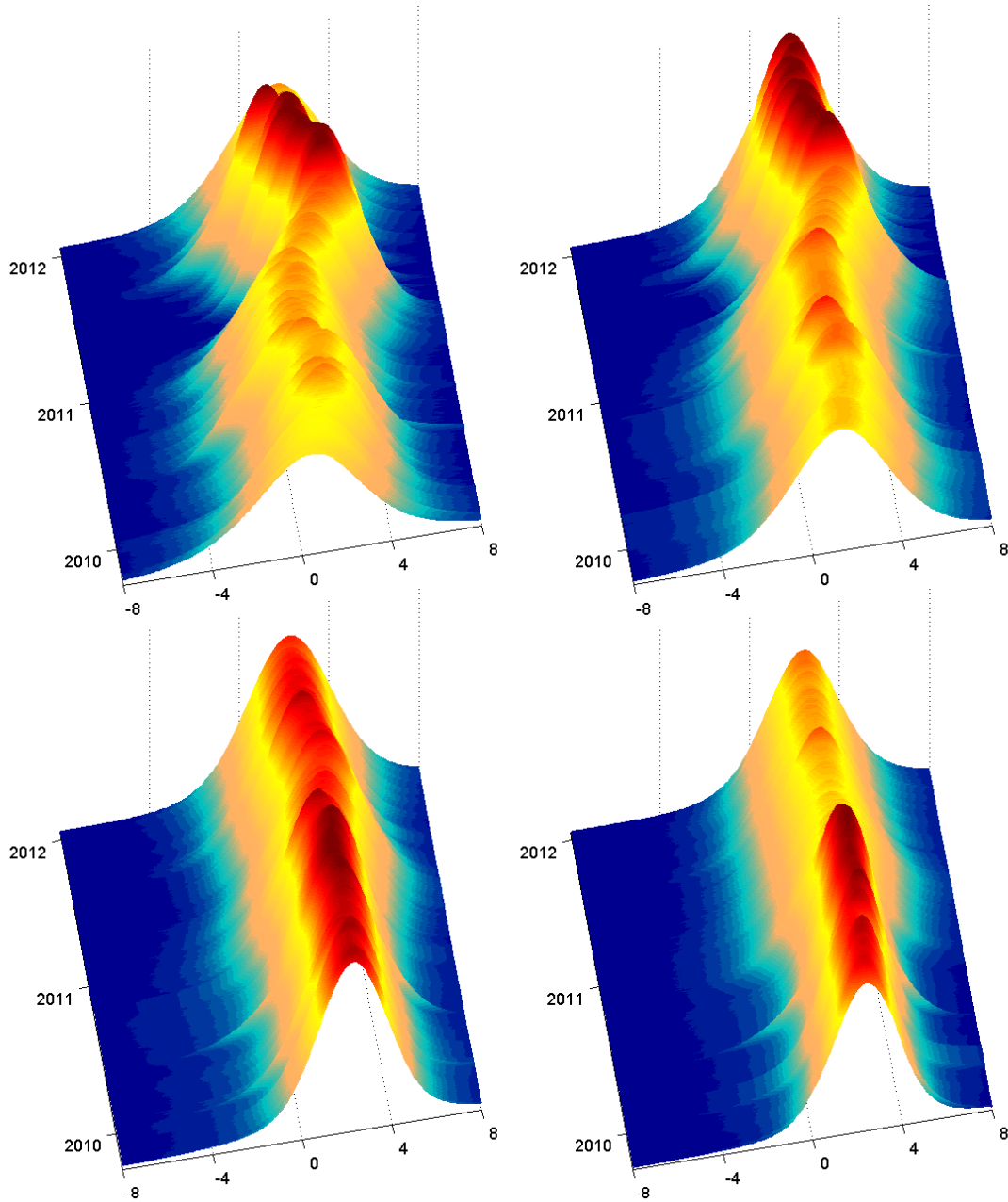
Figure 3.1 – **Inflation Densities**. This figure plots the time series of inflation densities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).
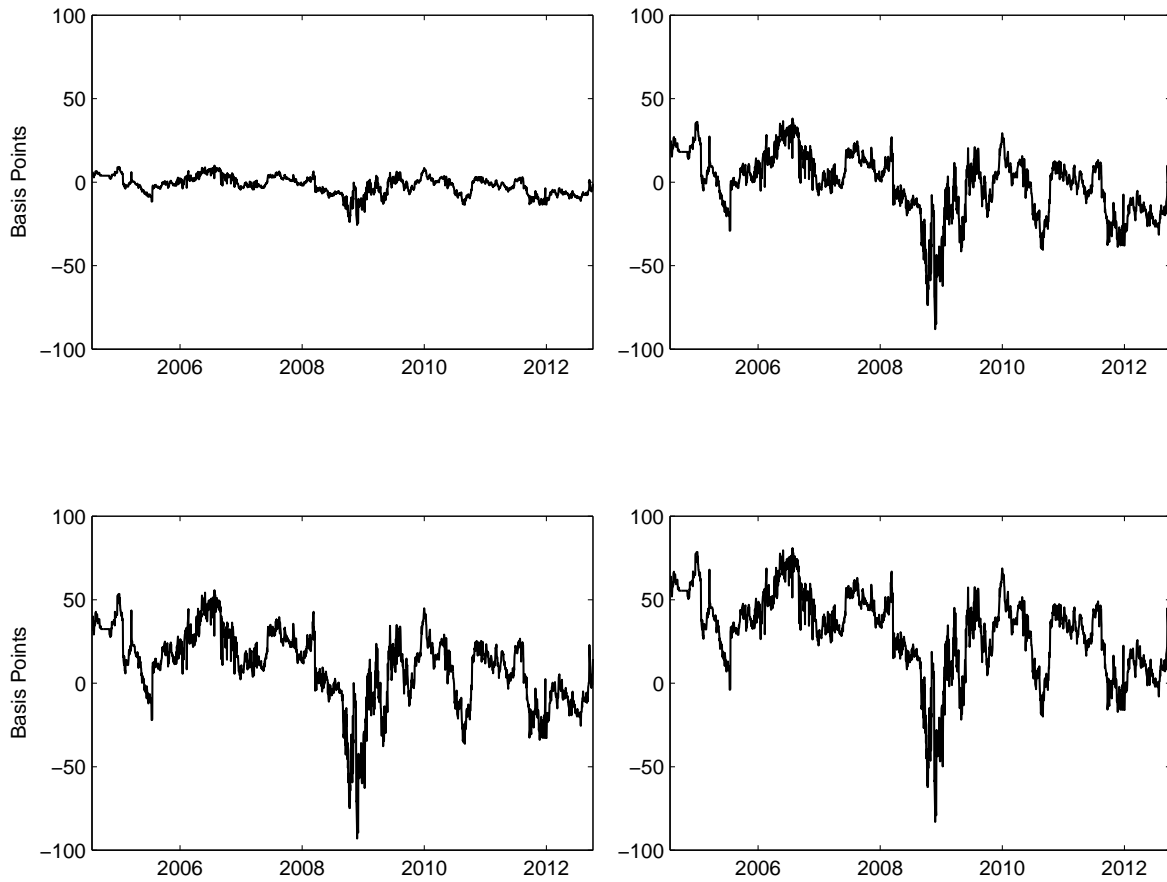
Figure 3.2 – **Inflation Risk Premia**. This figure plots the time series of inflation risk premia for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).

to five years, but are positive for longer horizons and reach a value of about 30 basis points at the 30-year horizon. The inflation risk premia vary significantly through time, although the volatility of inflation risk premia for longer horizons is slightly higher than for shorter horizons.

These inflation risk premia estimates are broadly consistent with previous estimates obtained using alternative approaches by other researchers. For example, Haubrich, Pennachi, and Ritchken (2012) estimate the ten-year and 30-year inflation risk premia to be 51 and 101

Table 3.5 – Summary Statistics for Expected Inflation. This table reports summary statistics for the expected inflation rate for the indicated horizons. Horizon is expressed in years. Expected inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to October 5, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.776	1.348	−4.399	2.051	3.882	2141
2	1.968	1.031	−3.317	2.281	3.588	2141
3	2.127	0.775	−1.650	2.284	3.497	2141
4	2.248	0.597	−0.746	2.351	3.496	2141
5	2.336	0.475	−0.054	2.422	3.452	2141
6	2.395	0.386	0.496	2.468	3.427	2141
7	2.442	0.314	0.908	2.492	3.327	2141
8	2.477	0.259	1.143	2.514	3.199	2141
9	2.501	0.213	1.441	2.525	3.130	2141
10	2.523	0.177	1.595	2.536	3.143	2141
12	2.537	0.151	1.702	2.540	3.035	2141
15	2.543	0.134	1.652	2.547	2.960	2141
20	2.536	0.124	1.609	2.560	2.896	2141
25	2.545	0.117	1.761	2.571	2.776	2141
30	2.597	0.084	2.165	2.616	2.716	2141

basis points, respectively. Buraschi and Jiltsov (2005) and Campbell and Viceira (2001) estimate the ten-year inflation risk premium to be 70 and 110 basis points, respectively. Ang, Bekaert, and Wei (2008) estimate the five-year inflation risk premium to be 114 basis points. In addition, the fact that all of the estimated risk premia take negative values at some point during the sample period is consistent with the findings of Campbell, Shiller, and Viceria (2009), Bekaert and Wang (2010), and others.

3.7.3   Expected Inflation

To solve for the expected inflation rate for each horizon, we use the inflation swap rates observed in the market and adjust them by the inflation risk premium implied by the fitted model. Table 3.5 presents summary statistics for the expected inflation rates for the various horizons.

The results indicate that the average term structure of inflation expectations is monotonically increasing during the 2004–2012 sample period. The average one-year expected inflation rate is 1.776 percent, while the average 30-year expected inflation rate is 2.597 percent. The table also shows that there is time variation in expected inflation, although the variation is surprisingly small for longer horizons. In particular, the standard deviation of expected inflation ranges from 1.348 percent for the one-year horizon to less than 0.20 percent for horizons of ten years or longer. To illustrate the time variation in expected inflation more clearly, Figure 3.3 plots the expected inflation estimates for the five-year, ten-year, and 30-year horizons.

It is also interesting to contrast these market-implied forecasts of inflation with forecasts provided by major inflation surveys. As discussed by Ang, Bekaert, and Wei (2007), these surveys of inflation tend to be more accurate than those based on standard econometric models and are widely used by market practitioners. Furthermore, these inflation surveys have also been incorporated into a number of important academic studies of inflation such as Fama and Gibbons (2004), Chernov and Mueller (2012) and Haubrich, Pennachi, and Ritchken (2012).

We obtain inflation expectations from four surveys: the University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters (SPF), the Livingston Survey, and the survey of market participants conducted by Bloomberg. The sample period for the forecasts matches that for the inflation swap data in the study. The Appendix provides the background information and details about how these surveys are conducted.
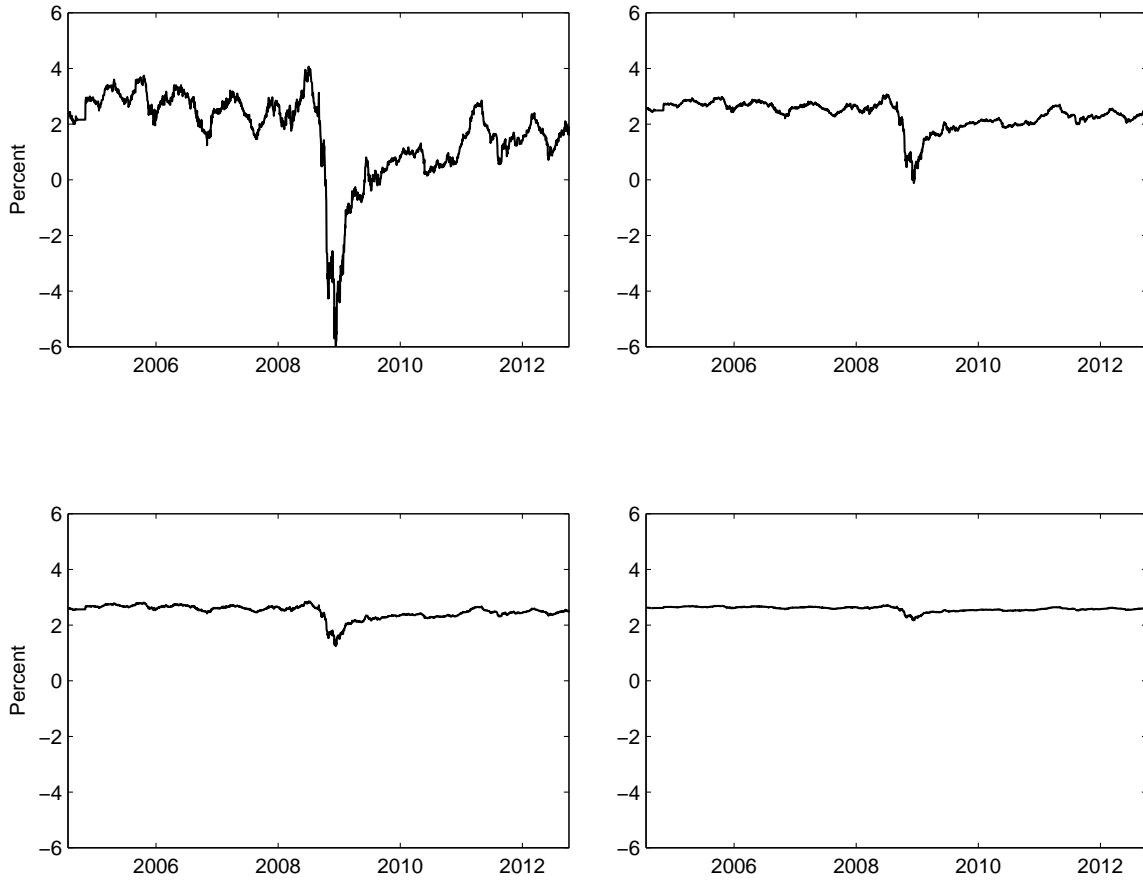
Figure 3.3 – **Expected Inflation**. This figure plots the time series of expected inflation for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).

Table 3.6 reports the average values of the various surveys during the sample period and the corresponding average values for the market-implied forecasts. These averages are computed using the month-end values for the months in which surveys are released. Thus, monthly averages are compared with monthly averages, quarterly averages with quarterly averages, etc. As shown, the average market-implied forecasts of inflation tend to be a little lower than the survey averages for shorter horizons. The market-implied forecasts, however, closely parallel those from the surveys for longer horizons. While it would be interesting to compare the relative accuracy of the market-implied and survey forecasts, our sample is too short to do this rigorously.

3.7.4 Inflation Volatility and Higher Moments

Table 3.7 reports the average values of the estimated volatility, skewness, and excess kurtosis of the continuously-compounded inflation rate for horizons ranging from one year to 30 years. The average inflation volatility estimates range from a high of about 2.258 percent at the two-year horizon to a low of about 0.693 percent at the 30-year horizon. The dampened volatility at the longer horizons is consistent with a scenario in which inflation is anticipated to follow a mean reverting process.

The distribution of inflation is typically negatively skewed for all horizons. The negative skewness is particularly pronounced for horizons of less than ten years, but is still evident in the distribution of inflation over a 30-year horizon. The median excess kurtosis coefficients are all positive (with the exception of the 30-year horizon), indicating that the distribution of inflation has heavier tails than a normal distribution.

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Table 3.6 – Comparison of Survey Forecasts with Market-Implied Forecasts. This table reports the average values of the survey forecasts for the indicated forecast horizon along with the corresponding average of the market-implied expected inflation for the same horizon. The averages of the market implied expected inflation estimates are taken using month-end values for months in which surveys are released. Inflation forecasts are expressed as percentages. The sample period is July 2004 to September 2012.

Forecast Horizon	Survey	Survey Forecast	Market-Implied Forecast	N
1 Year	Michigan	2.91	1.79	99
	Bloomberg	2.53	1.79	99
	SPF	2.22	1.80	33
	Livingston	4.71	1.60	16
2 Years	SPF	2.32	1.99	33
	Livingston	3.53	1.67	8
3 Years	SPF	2.30	2.08	29
5 Years	Michigan	2.39	2.31	29
10 Years	Michigan	3.24	2.53	99
	SPF	2.49	2.53	33
	Livingston	2.44	2.52	16

Table 3.7 – Summary Statistics for the Volatility, Skewness, and Kurtosis of the Inflation Distribution. This table reports the average values of the standard deviation and skewness coefficient, and the median excess kurtosis coefficient for the annualized inflation rate for the indicated horizons. The standard deviation is expressed as a percentage. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Average Volatility	Average Skewness	Median Excess Kurtosis	N
1	2.171	−2.376	3.095	572
2	2.258	−1.251	8.107	593
3	2.226	−1.115	13.390	500
5	2.165	−1.480	18.364	577
7	2.016	−1.230	9.789	434
10	1.831	−1.207	8.263	566
12	1.525	−0.913	6.165	531
15	1.241	−0.722	3.429	569
20	1.076	−0.513	1.873	488
30	0.693	−0.160	−0.006	215

3.8 Deflation Risk

We turn now to the central issue of measuring the risk of deflation implied by market prices and studying the properties of deflation risk. First, we present descriptive statistics for the implied deflation risk. We then examine how the market prices the tail risk of deflation and contrast the results with those found in other markets.

3.8.1 How Large is the Risk of Deflation?

Having solved for the characteristic function for the inflation distribution, we can apply standard inversion techniques to solve for the cumulative distribution function for inflation. In turn, we can then directly compute the probability that the average realized inflation rate over a specific horizon is less than zero, which represents the risk of deflation. Table

Table 3.8 – Summary Statistics for Deflation Probabilities. This table reports summary statistics for the probability of the average inflation rate being below zero for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	17.25	8.75	1.88	17.57	44.37	572
2	11.44	5.29	2.07	11.33	23.04	593
3	4.28	1.94	1.33	3.63	8.89	500
5	5.34	1.60	2.67	5.17	11.39	577
7	2.93	0.62	1.89	2.80	5.93	434
10	3.29	0.77	2.40	2.90	5.86	566
12	2.28	0.20	1.95	2.25	3.31	531
15	2.97	0.28	2.62	2.93	4.61	569
20	2.32	0.12	1.98	2.32	3.31	488
30	2.33	0.17	1.85	2.32	3.35	215

3.8 provides summary statistics for the estimated probabilities of deflation over the various horizons. To provide some additional perspective, Figure 3.4 graphs the time series of probabilities of a deflation over one-year, two-year, five-year, and ten-year horizons.

As shown, the market places a surprisingly large weight on the possibility that deflation may occur over extended horizons. In particular, the average probability that the realized inflation rate will be less than or equal to zero is 17.25 percent for a one-year horizon, 11.44 percent for a two-year horizon, 5.34 percent for a five-year horizon, 3.29 percent for a ten-year horizon, and ranges from two to three percent for longer horizons.

What is perhaps more striking is that the probability of deflation varies significantly over time and reaches relatively high levels during the sample period. For example, the probability of deflation reaches a value of 44.37 percent for a one-year horizon, 23.04 percent for a two-year horizon, and 11.39 for a five-year horizon. At other times, the market assesses the probability of deflation at any horizon to be only on the order of one to two percent. This variation in the probability of deflation is due not only to changes in expected inflation, but also to
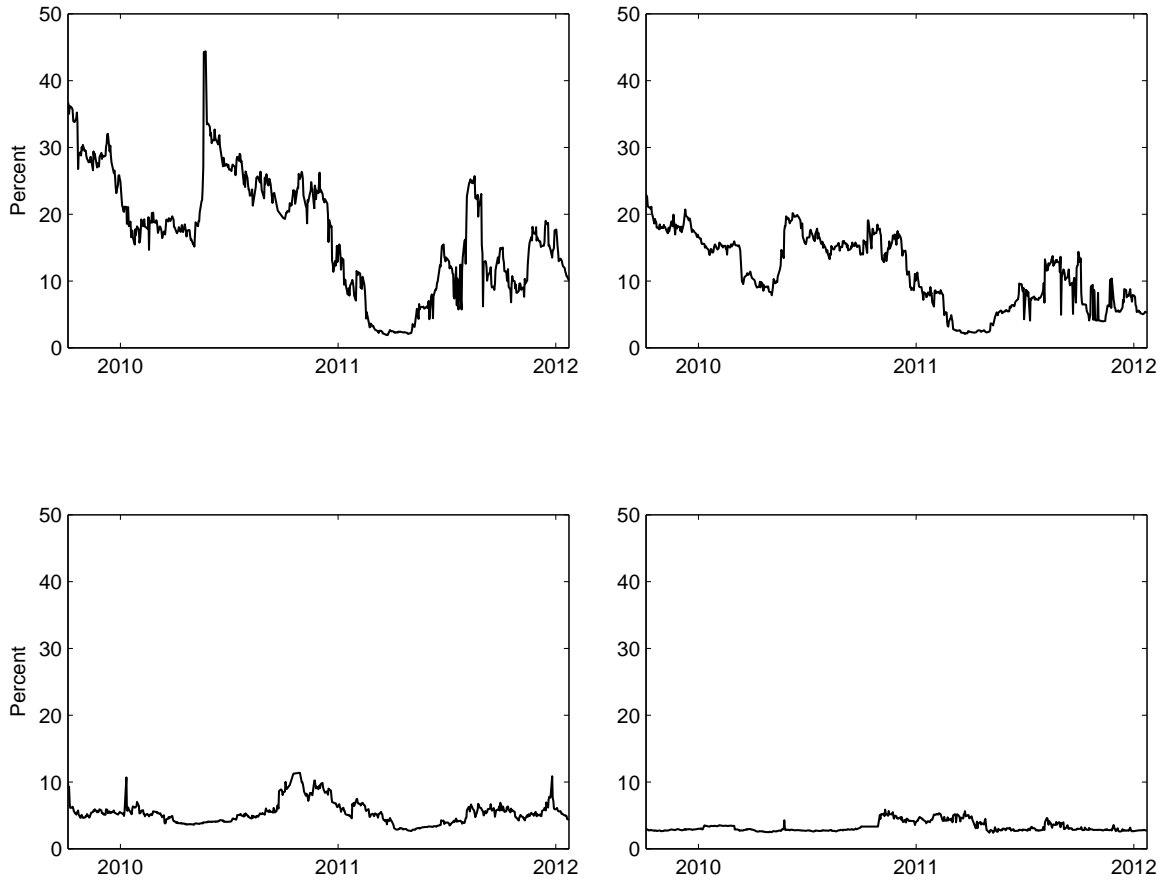
Figure 3.4 – **Deflation Probabilities**. This figure plots the time series of deflation probabilities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

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changes in the volatility of inflation.

These probabilities are broadly consistent with the historical record on inflation in the U.S. For example, based on the historical inflation rates from 1800 to 2012, the U.S. has experienced deflation over a one-year horizon 65 times, which represents a frequency of 30.5 percent. Considering only non-overlapping periods, the U.S. has experienced a two-year deflation 41 times, a five-year deflation 19 times, a ten-year deflation 11 times, and a 30-year deflation three times. These translate into frequencies of 24.0 percent, 14.4 percent, 10.6 percent, and 3.1 percent, respectively.[79]

Figure 3.4 also shows that the deflation probabilities for the shorter horizons have occasional jumps upward. These jumps tend to occur around major financial events such as those associated with the European Debt Crisis. For example, the Eurozone experienced major turmoil during April and May of 2010 as concerns about the ongoing solvency of Portugal, Italy, Ireland, Greece, and Spain become more urgent and a number of bailout plans were put into place. Spain's debt was first downgraded by Fitch on May 29, 2010. Similarly, the five-year deflation probability nearly doubles during the last week of September 2010 which coincides with the downgrade of Spain by Moodys. In addition, the one-year deflation probability spikes again in early August of 2011, coinciding with the downgrade of U.S. Treasury debt by Standard and Poors. We will explore the link between deflation risk and major financial risk more formally later in the paper.

Although not shown, we also calculate the partial moment in which we take the expected

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[79]Historical inflation rates are tabulated by Sahr (2012), www.oregonstate.edu/cla/polisci/faculty-research/sahr/infcf17742007.pdf. More recent inflation rates are reported by the Bureau of Labor Statistics.

value of inflation conditional on the inflation rate being less than or equal to zero. This partial moment provides a measure of the severity of a deflation, conditional on deflation occuring over some horizon. For example, finding that this partial moment was only slightly negative would argue that a deflationary episode was likely to be less severe, while the opposite would be true for a more negative value of this partial moment. The results indicate that the expected severity of a deflation is typically very substantial with these conditional moments increasing from about 1.60 percent for a one-year deflation, to 1.85 for a five-year horizon, and then decreasing to 1.15 percent for a 20-year horizon. On average, the expected value of deflation over all of the horizons is 1.56 percent. Note that a deflation of 1.56 percent per year would translate into a decline in the price level of 7.6 percent over a five-year period, 15.5 percent over a ten-year period, and 27.0 percent over a 20-year period. These would represent protracted deflationary episodes comparable in severity to many of those experienced historically in the U.S.

3.8.2 Pricing Deflation Tail Risk

Although we have solved for the inflation risk premium embedded in inflation swaps earlier in the paper, it is also interesting to examine how the market prices the risk that the tail event of a deflation occurs. This analysis can provide insight into how financial market participants view the risk of events that may happen infrequently, but which may have catastrophic implications.

A number of these types of tail risks have been previously studied in the literature. For example, researchers have investigated the pricing of catastrophic insurance losses such as

those caused by hurricanes or earthquakes. Froot (2001) finds that the ratio of insurance premia to expected losses in the market for catastrophic reinsurance ranges from about two to seven during the 1989 to 1998 period. Lane and Mahul (2008) estimate that the pricing of catastrophic risk in a sample of 250 catastrophe bonds is about 2.69 times the actual expected loss over the long term. Garmaise and Moskowitz (2009) and Ibragimov, Jaffee, and Walden (2009) offer both empirical and theoretical evidence that the extreme left tail catastrophic risk can be significantly priced in the market.

The default of a corporate bond is also an example of an event that is relatively rare for a specific firm, but which would result in an extremely negative outcome for bondholders of the defaulting firm. The pricing of default risk has been considered in many recent papers. For example, Giesecke, Longstaff, Schaefer, and Strebulaev (2011) study the pricing of corporate bond default risk and find that the ratio of corporate credit spreads to their actuarial expected loss is 2.04 over a 150-year period. Similarly, Driessen (2005) and Berndt, Duffie, Douglas, Ferguson, and Schranz (2005) estimate ratios using data for recent periods that range in value from about 1.8 to 2.8.

Following along the lines of this literature, we solve for the ratio of the riskneutral probability of deflation to the objective probability of deflation. This ratio provides a simple measure of how the market prices the tail risk of deflation and has the advantage of being directly comparable to the ratios discussed above.

Table 3.9 presents the means and medians for the ratios for the various horizons. As shown, the mean and median ratios range from between one and two to slightly higher than five. The overall average of the ratios is 3.321 and the overall median of the ratios is about 3.166. These

Table 3.9 – Summary Statistics for the Pricing of Deflation Tail Risk. This table reports the means and medians of the ratio of the probability of deflation under the risk-neutral Q measure divided by the probability of deflation under the actual P measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Mean Ratio	Median Ratio
1	1.632	1.345
2	2.163	1.797
3	5.165	4.393
5	3.057	3.068
7	4.477	4.374
10	3.167	3.011
12	3.437	3.360
15	3.089	3.094
20	4.022	3.707
30	4.246	4.257
All	3.321	3.166

values are in the same ballpark as those for the different types of tail risk discussed above. These ratios all indicate that the market is deeply concerned about financial and economic tail risks that may be difficult to diversify or may be strongly systematic in nature.

3.9   What Drives Deflation Risk?

A key advantage of our approach is that by extracting the market's assessment of the objective probability of deflation, we can then examine the relation between these probabilities and other financial and macroeconomic factors. In particular, we can study the relation between the tail risk of deflation and other types of tail risk that may be present in the markets.

In doing this, we will focus on four broad categories of tail risk that have been extensively

discussed in the literature. Specifically, we will consider the links between deflation risk and systemic financial system risk, collateral revaluation risk, sovereign default risk, and business cycle risk.

The link between systemic risk in the financial system and major economic crisis is well established in many important papers including Bernanke (1983), Bernanke, Gertler, and Gilchrist (1996), and others. Systemic risk in the financial system is widely viewed as having played a central role in the recent global financial crisis and represents a motivating force behind major regulatory reforms such as the Dodd-Frank Act. We use a number of measures of systemic risk in the analysis.

First, we use a measure of the flight-to-liquidity risk in the market which is computed as the spread between a one-year zero-coupon Refcorp bond and a corresponding maturity zero-coupon Treasury bond. This variable is introduced in Longstaff (2004) as a measure of the premium that market participants place on Treasuries because of their role as the "safest" asset in the financial markets during episodes when investors fear that massive losses will occur in less liquid markets. We obtain the data for the flight-to-liquidity spread from the Bloomberg system.

Second, we use data on the pricing of super senior tranches on a basket of corporate debt to measure the risk of a major systemic collapse in the credit markets. Specifically, we collect data on the points-up-front pricing on the five-year 10-15 percent tranche on the CDX IG index. This index is computed as an average of the five-year CDS spreads for 125 U.S. firms with investment grade ratings. The 10-15 percent tranche would only experience losses if the total credit losses on the index exceeded 10 percent of the total notional of

an equally weighted basket of the underlying debt obligations of these firms. As shown by Coval, Jurek, and Stafford (2009) and Giesecke, Longstaff, Schaefer, and Strebulaev (2011), a major meltdown that would produce losses of this magnitude in a portfolio of investment grade corporate debt would be a very rare tail event. The points-up-front price for this tranche represents the market price to compensate investors for taking the risk of this extreme scenario.[80] We obtain data on the pricing of the super senior tranche from the Bloomberg system.

Third, we use the spread between the one-year Libor rate and a one-year Treasury bond as a proxy for the systemic credit and liquidity risk embedded in the Libor rate. The data are from the Bloomberg system.

Fourth, we use the five-year swap spread as a measure of the systemic credit and liquidity stresses on the financial system. As discussed by Duffie and Singleton (1997), Liu, Longstaff, and Mandell (2006), and others, the swap spread reflects differences in the relative liquidity and credit risk of the financial sector and the Treasury. We obtain five-year swap spread data from the Bloomberg system.

We also considered a number of other measures of systemic risk such as the average CDS spread for both major U.S. and non-U.S. banks and financial firms. These measures, however, were highly correlated with the other measures such as swap spreads and provided little incremental information.

Recent economic theory has emphasized the role that the value of collateral plays in prop-

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[80]For a description of the CDX index tranche markets and the pricing of CDO tranches, see Longstaff and Rajan (2008) and Longstaff and Myers (2013).

agating economic downturns. Key examples include Kiyotaki and Moore (1997) who show that declines in asset values can lead to contractions in the amount of credit available in the market which, in turn, can lead to further rounds of declines in asset values. Bernanke and Gertler (1995) describe similar interactions between declines in the value of assets that serve as collateral and severe economic downturns. Collateral revaluation risk, or the risk of a broad decline in the market value of leveraged assets, played a major role in the Great Depression as the sharp declines in the values of stock and corporate bonds triggered waves of defaults among both speculators and banks. A similar mechanismwas present in the recent financial crisis as sharp declines in real estate values led to massive defaults by "underwater" mortgagors. In the context of this study, we explore the relation between deflation probabilities and valuations in several major asset classes that may represent important sources of collateral in the credit markets: stocks and bonds. In addition, we also include measures of the volatility of these asset classes since these measures provide information about the risk that large downward revaluations in these forms of collateral may occur.

The first of these proxies for collateral revaluation risk is the VIX index of implied volatility for options on the S&P 500. This well-known index is often termed the "fear index" in the financial press since it reflects the market's assessment of the risk of a large downward movement in the stock market. We collect VIX data from the Bloomberg system.

The second measure is the Merrill Lynch MOVE index of implied volatilities for options on Treasury bonds. This index is essentially the fixed income counterpart of the VIX index. This index also captures the market's views of the likelihood of a large change in the prices of Treasury bonds, which are widely used as collateral for a broad variety of credit transactions.

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This data is also obtained from the Bloomberg system.

The third measure is simply the time series of daily returns on the S&P 500 index (price changes only). This return series reflects changes in the value of one of the largest potential sources of collateral in the macroeconomy. We compute these returns from S&P 500 index values reported in the Bloomberg system.

The fourth measure is the spread between the yield for the Moody's Baa rated index of corporate bonds and the yield on five-year Treasury bonds. Variation in this credit spread over time reflects changes in the market's assessment of default risk in the economy as well as the pricing of credit risk. We collect data on the Baa-Treasury spread from the Bloomberg system.

Another major type of economic tail risk stems from the risk that a sovereign defaults on its debt. As documented by Reinhart and Rogoff (2009) and many others, sovereign defaults tend to be associated with severe economic crisis scenarios.

As a measure of the tail risk of a sovereign default by the U.S., we include in the analysis the time series of sovereign CDS spreads on the U.S. Treasury. Ang and Longstaff (2012) show that the U.S. CDS spread reflects variation in the valuation of major sources of tax revenue for the U.S. such as capital gains on stocks and bonds. This data is also obtained from the Bloomberg system.

Finally, to capture the effect of traditional types of business cycle risk or economic downturn risk, we also include a number of key macroeconomic variables that can be measured at a monthly frequency. In particular, we include the monthly percentage change in industrial

production as reported by the Bureau of Economic Analysis, the monthly change in the national unemployment rate as reported by the Bureau of Labor Statistics, and the change in the Consumer Confidence Index reported by the University of Michigan. The link between the business cycle and its effects on output and employment are well established in the macroeconomic literature and forms the basis of many classical theories including the Phillips curve.

Since these measures of systemic, collateral, and sovereign default risk are all available on a daily basis, we begin our analysis by regressing daily changes in the deflation probabilities on daily changes in these variables (the macroeconomic variables, which are only observed monthly, will be included in later regressions). In doing this, it is important to note that while these variables were chosen as a measure of a specific type of tail risk, most of these variables may actually reflect more than one type of tail risk. Thus, the effects of the variables in the regression should be interpreted carefully since the different types of tail risk need not be mutually exclusive.

Table 3.10 presents summary statistics for the regression results. The results indicate that the variables proxying for systemic risk are often statistically significant for a number of the horizons. In particular, the coefficient for the super senior tranche price is significant for four of the horizons. The sign of this coefficient is uniformly positive in sign for all but the longest horizons, which is clearly consistent with the intuition that an increase in the extreme type of tail risk reflected in the tranche price would be associated with an economic meltdown in which deflation occured. The Libor spread is positive and significant for the two shortest horizons. The positive sign of this effect is also consistent with our intuition about

Table 3.10 – Results from the Regression of Daily Changes in Deflation Probabilities on Financial Tail Risk Variables. This table reports the results from the regression of daily changes in the deflation probabilities for the indicated horizon on the daily changes in the following variables: the flight-to-liquidity spread (the one-year Refcorp-Treasury yield spread), the super senior tranche price (the points-up-front price for the 10-15 percent CDX IG index tranche), the Libor spread (the one-year Libor-Treasury spread), the five-year swap spread, the VIX index, the Merrill Lynch MOVE index of implied Treasury volatility, the return on the S&P 500 index, the Baa spread over the five-year Treasury rate, and the spread for a five-year CDS contract on the U.S. Treasury. Horizon is expressed in years. The t-statistics are based on the Newey-West estimator of the covariance matrix (five lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to January 2012.

Horizon	Flight to Liquidity	Super Senior	Libor Spread	Swap Spread	VIX	Trsy Vol	SP 500 Return	Baa Spread	Trsy CDS	R^2	N
1	−0.19	1.89*	2.32**	−1.96**	−1.14	−1.31	−3.45**	1.96**	−1.79*	0.088	550
2	−0.41	2.75**	1.91*	−1.33	−2.02**	0.03	−3.16**	2.70**	−0.66	0.063	585
3	−1.04	0.30	1.40	−0.18	−0.21	−0.69	−0.67	−0.08	1.63	0.018	439
5	−0.95	1.55	−0.16	1.82*	−0.97	0.66	0.10	−1.02	1.05	0.034	563
7	−0.49	1.77*	−1.18	1.76*	−1.62	1.12	−1.39	−1.03	−0.70	0.091	364
10	−0.74	1.28	−0.05	2.33**	−1.04	−0.21	−0.74	−1.01	0.69	0.027	551
12	−0.31	0.75	0.32	1.25	0.80	−0.83	1.19	−1.54	1.37	0.015	495
15	0.25	1.05	0.26	0.61	0.26	−1.46	1.02	−1.69*	2.03**	0.024	554
20	2.01**	−0.54	−1.35	0.28	−0.69	1.09	−1.06	−1.57	1.47	0.030	439
30	0.77	−2.02**	−0.69	3.34**	−1.30	−0.10	−1.65*	−2.69**	0.55	0.078	203

the effects of a systemic financial crisis on the macroeconomy. The five-year swap spread has some of the strongest effects in the regression. In particular, it is significant for five of the ten horizons. Interestingly, the signs of the significant coefficients are all positive, with the exception of the shortest horizon. This is again consistent with an economic scenario in which stress in the financial sector leads to an increase in the perceived risk of an adverse macroeconomic shock in which price levels decline.

Surprisingly, Table 3.10 shows that neither of the two volatility variables has much explanatory power for changes in deflation risk. In contrast, both S&P 500 index returns and changes in the Baa credit spread are often significant. For example, the stock market return is significant for the one-year, two-year, and 30-year horizons. In each of these three cases, the sign is negative, indicating that an increase in the stock market reduces fears about

deflation. This is again very intuitive and completely consistent with a simple collateral revaluation interpretation. The Baa credit spread is also significant for three of the horizons. The signs of the significant coefficients for the one-year and two-year horizons are both positive, indicating that deflation risk increases as credit fears in the economy increase. It is also interesting to note that the sign of the coefficients for this variable become negative for all longer horizons. Thus, the long-run effects of increased credit risk on deflation risk are somewhat counterintuitive.

Finally, Table 3.10 shows that the effect of an increase in U.S. CDS spreads on deflation risk is relatively limited. The coefficient for changes in U.S. CDS spreads is only significant for the one-year and 15-year horizons. In addition, the signs of these two significant coefficients differ from each other.

The overall R^2s from the regressions are fairly modest, ranging from just under two percent to more than nine percent. Note, however, that these results are based on daily changes in these variables. Thus, given the challenges in measuring tail risks, the explanatory power of these regressions is far from negligible.

Turning now to regressions in which we include the macroeconomic variables, we observe that since our sample period is relatively short, it is important to use a parsimonious specification. Accordingly, we regress monthly changes in the deflation probabilities (using the last deflation probability for each month) on a selected set of variables. Specifically, as proxies for systemic risk, we include only monthly changes in the super senior prices and the five-year swap spread. As the proxy for collateral revaluation risk, we include only the change in the Baa credit spread. We then include the monthly percentage change in industrial production, the change

Table 3.11 – Results from the Regression of Monthly Changes in Deflation Probabilities on Financial and Macroeconomic Variables. This table reports the results from the regression of monthly changes in the deflation probabilities for the indicated horizons on the monthly changes in the following variables: the super senior tranche price (the points-up-front price for the 10-15 percent CDX IG index tranche), the five-year swap spread, the Baa spread over the five-year Treasury rate, the percentage change in industrial production, the unemployment rate, and the Michigan consumer confidence index. Horizon is expressed in years. The t-statistics are based on the Newey-West estimator of the covariance matrix (two lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to January 2012.

Horizon	Super Senior	Swap Spread	Baa Spread	Indus Prod	Unempl	Cons Conf	R^2	N
1	2.24**	1.60	1.84*	0.76	1.63	0.70	0.540	28
2	2.10**	0.90	4.15**	0.65	1.88*	1.38	0.643	28
3	1.21	0.56	1.05	1.22	2.00*	1.04	0.432	26
5	−0.02	1.02	1.20	−0.90	−0.57	−0.95	0.336	28
7	1.07	1.49	0.67	0.02	1.10	0.99	0.239	26
10	−0.61	1.39	1.76*	0.64	0.27	−0.47	0.256	28
12	−0.71	1.64	2.03*	0.60	1.32	−1.02	0.423	26
15	−0.64	1.20	1.38	0.63	1.95*	0.88	0.385	28
20	1.95*	1.44	−0.71	−0.10	1.10	−0.06	0.388	26

in the unemployment rate, and the change in the Michigan Consumer Confidence Index. The regression results for horizons ranging from one to 20 years (there are too few observations for the 30-year horizon) are summarized in Table 3.11.

The results in Table 3.11 are consistent with those reported in Table 3.10. In particular, several of the proxies for systemic and collateral revaluation tail risk are again significant. The coefficient for the super senior tranche variable is significant for three of the horizons, with those for the one-year and two-year horizons again having a positive sign. The coefficient for the Baa credit spread is significant for four of the horizons. All four of these coefficients are positive in sign, indicating that increases in credit risk are associated with an increased risk of deflation.

The results for the macroeconomic variables are somewhat surprising. Far from being strongly related to deflation risk, none of the coefficients for changes in industrial production

or the consumer confidence index are significant. The only macroeconomic variable that is significantly related to changes in deflation risk is the change in the unemployment rate which is significant and positive for three of the horizons. The positive sign of these coefficients is intuitive since it indicates that deflation risk increases as the unemployment rate increases. This is also consistent with classical macroeconomic theory about the relation between price levels and unemployment such as the Phillips curve.

In summary, the empirical results indicate that there is a strong relation between tail risk in financial markets and the risk of deflation. In particular, a number of the proxies for systemic financial risk and the value of potentially collateralizable financial assets are significantly linked to deflation risk. These results underscore the importance of understanding the role that the financial sector plays in economic downturns such as the recent financial crisis that began in the subprime structured credit markets. In contrast, these results suggest that more traditional macroeconomic variables such as industrial production may play less of a role in the risk of economic tail events such as a deflationary spiral.

3.10   Inflation Risk

Although the focus of this paper is on deflation risk, it is straightforward to extend the analysis to other aspects of the distribution of inflation. As one last illustration of this, we compute the probabilities that the inflation rate exceeds values of four, five, and six percent using the techniques described earlier. Table 3.12 reports summary statistics for these probabilities.

Table 3.12 – Summary Statistics for the Probabilities of Inflationary Scenarios. This table reports summary statistics for the probability of the average inflation rate being above the indicated thresholds for the respective horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Probability Inflation > 4.00			Probability Inflation > 5.00			Probability Inflation > 6.00			N
	Mean	Min.	Max	Mean	Min.	Max	Mean	Min.	Max	
1	10.38	1.75	33.40	4.09	0.48	13.80	1.42	0.47	6.06	572
2	14.16	4.36	31.76	5.72	2.19	13.07	1.82	0.77	3.82	593
3	24.69	16.23	40.92	10.33	6.49	19.53	2.88	1.62	6.67	500
5	17.02	6.95	26.47	6.75	3.07	10.72	1.83	1.01	4.04	577
7	26.54	20.81	33.52	10.94	8.36	14.07	2.78	1.95	4.37	434
10	25.21	19.36	29.03	10.28	8.16	12.21	2.54	2.01	3.36	566
12	30.19	25.13	32.98	12.43	10.13	13.90	2.97	2.36	3.76	531
15	24.23	17.11	26.90	9.74	6.86	10.92	2.27	1.80	2.55	569
20	29.49	22.38	32.53	12.06	8.91	13.41	2.81	2.08	3.13	488
30	29.45	22.79	33.69	12.04	9.10	13.97	2.81	2.12	3.30	215

As shown, the market-implied probabilities of experiencing significant inflation are uncomfortably large. Specifically, the average probability of inflation exceeding four percent is less that 15 percent for the two shortest horizons, but increases rapidly to nearly 30 percent for horizons ranging from ten to 30 years. Figure 3.5 plots the time series of probabilities that inflation exceeds four percent for several horizons. These plots also show that the probability of inflation in the long run appears substantially higher than in the short turn. This is exactly the opposite from the risk of deflation which tends to be higher in the short run.

Table 3.12 also shows that the average probability of an inflation rate in excess of five percent is substantial. An average inflation rate of five percent or more over a period of decades would rival any inflationary scenario experienced by the U.S. during the past 200 years. Finally, the results show that the market anticipates that there is roughly a three percent probability of inflation averaging more than six percent over the next several decades.

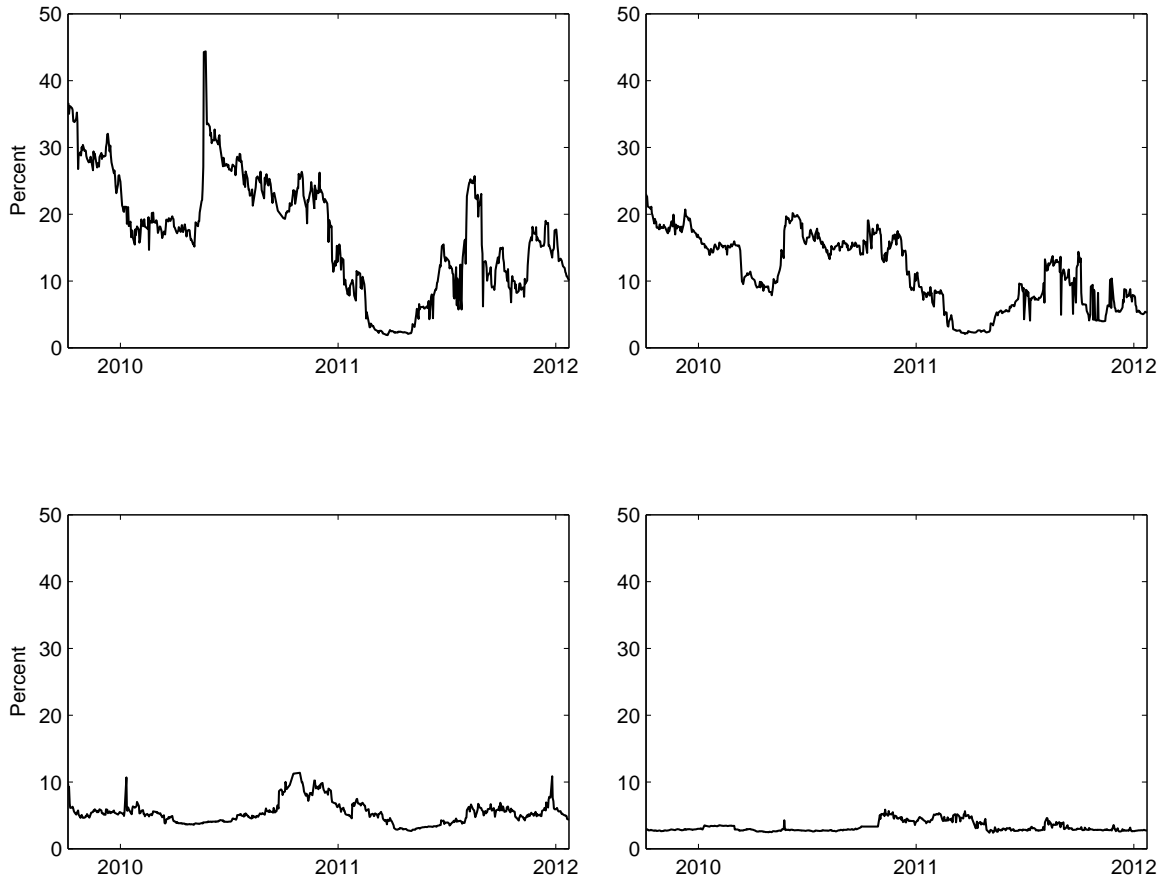As we did earlier for deflation tail risk, we can also examine the pricing of inflation tail risk

Figure 3.5 – **Inflation Probabilities**. This figure plots the time series of probabilities that inflation is greater than or equal to four percent for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

Table 3.13 – Summary Statistics for the Pricing of Inflation Tail Risk. This table reports the means and medians of the ratio of the probability of inflation exceeding five percent under the risk-neutral Q measure divided by the corresponding probability under the actual P measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

| Horizon | Probability Inflation > 4.00 | | | Probability Inflation > 5.00 | | | Probability Inflation > 6.00 | | | N |
	Mean	Min.	Max	Mean	Min.	Max	Mean	Min.	Max	
1	10.38	1.75	33.40	4.09	0.48	13.80	1.42	0.47	6.06	572
2	14.16	4.36	31.76	5.72	2.19	13.07	1.82	0.77	3.82	593
3	24.69	16.23	40.92	10.33	6.49	19.53	2.88	1.62	6.67	500
5	17.02	6.95	26.47	6.75	3.07	10.72	1.83	1.01	4.04	577
7	26.54	20.81	33.52	10.94	8.36	14.07	2.78	1.95	4.37	434
10	25.21	19.36	29.03	10.28	8.16	12.21	2.54	2.01	3.36	566
12	30.19	25.13	32.98	12.43	10.13	13.90	2.97	2.36	3.76	531
15	24.23	17.11	26.90	9.74	6.86	10.92	2.27	1.80	2.55	569
20	29.49	22.38	32.53	12.06	8.91	13.41	2.81	2.08	3.13	488
30	29.45	22.79	33.69	12.04	9.10	13.97	2.81	2.12	3.30	215

by computing the ratio of the probability of inflation (in excess of five percent) under the risk-neutral measure to the corresponding probability under the actual measure. Table 3.13 provides summary statistics for the ratios.

As illustrated, inflation tail risk is priced at all horizons. The magnitude of the inflation risk premium, however, is significantly smaller than is the case for deflation tail risk. In particular, the average ratio is only 1.463 and the median ratio is 1.441. These values are less than half of the corresponding value of 3.321 and 3.166 shown in Table 3.9 for deflation tail risk. Thus, these results suggest that the market requires far less compensation for the risk of inflation that it does for the risk of deflation. This is consistent with the view that deflations are associated with much more severe economic scenarios than are inflationary periods.

3.11   Conclusion

We solve for the objective distribution of inflation using the market prices of inflation swap and option contracts and study the nature of deflation risk. We find that the market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.5 percent for horizons of up to 30 years. We show that deflation risk is priced by the market in a manner similar to that of other major types of tail risk such as catastrophic insurance losses or corporate bond defaults. By embedding a deflation floor into newly issued TIPS, the Treasury insures bondholders against deflation. Our findings imply that the Treasury receives a generous insurance premium in return. In contrast, the market appears much less concerned about inflation tail risk.

In theory, economic tail risks such as deflation may be related to other financial and macroeconomic tail risks. We study the relation between deflation risk and a number of measures of systemic financial risk, collateral revaluation risk, sovereign credit risk, and business cycle risk. We find that there is a significant relation between deflation risk and measures capturing stress in the financial system and credit risk in the economy. These results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major systemic shocks in the financial markets.

3.12 Appendix

A.1. The Inflation Swap Rate.

From Equation (1), the relative price level index at time T can be expressed as

$$I_T = \exp\left(\int_0^T X_s \ ds\right) \exp\left(-\tfrac{1}{2}\int_0^T V_s \ ds + \int_0^T \sqrt{V_s} \ dZ_V(s)\right). \qquad (A1)$$

The cash flow associated with a zero-coupon inflation swap at time T is simply $I_T - F(X,Y,T)$ where $F(X,Y,T)$ is the inflation swap price at the initiation of the contract at time zero. Since the present value of the inflation swap is zero at inception we have,

$$E^Q\left[\exp\left(-\int_0^T r_s \ ds\right)(I_T - F(X,Y,T))\right] = 0. \qquad (A2)$$

Substituting in for r_t and I_T gives,

$$E^Q\left[\exp\left(-\int_0^T R_s \ ds\right)\right]$$

$$\left[E^Q\left[\exp\left(-\int_0^T X_s \ ds\right)\exp\left(\int_0^T X_s \ ds\right)\exp\left(-\tfrac{1}{2}\int_0^T V_s \ ds + \int_0^T \sqrt{V_s} \ dZ_V(s)\right)\right]\right.$$

$$\left. - E^Q\left[\exp\left(-\int_0^T X_s \ ds\right)F(X,Y,T)\right] \ \right] = 0, \qquad (A3)$$

which implies

$$F(X,Y,T) = \frac{E^Q\left[\exp\left(-\tfrac{1}{2}\int_0^T V_s \ ds + \int_0^T \sqrt{V_s} \ dZ_V(s)\right)\right]}{E^Q\left[\exp\left(-\int_0^T X_s \ ds\right)\right]}, \qquad (A4)$$

$$= \frac{1}{E^Q\left[\exp\left(-\int_0^T X_s \ ds\right)\right]}. \qquad (A5)$$

Let $H(X,Y,\tau)$ denote the value of the expectation $E^Q[\exp(\int_t^T X_s \ ds)]$, where $\tau = T - t$. Standard results imply that this expectation satisfies the partial differential equation

$$\tfrac{1}{2}\sigma^2 H_{XX} + \rho\sigma\eta H_{XY} + \tfrac{1}{2}\eta^2 H_{YY}$$
$$+ \lambda(Y - X)H_X + (\phi - \gamma Y)H_Y - XH = H_\tau, \qquad (A6)$$

subject to the terminal condition $H(X, Y, 0) = 1$. We conjecture a solution of the form $H(X, Y, \tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)$. Taking derivatives of this expression and substituting into Equation $(A6)$ results in a system of three linear first order ordinary differential equations for the horizon dependent functions $A(\tau)$, $B(\tau)$, and $C(\tau)$,

$$B' + \lambda B = -1, \qquad (A7)$$
$$C' + \gamma C = \lambda B, \qquad (A8)$$
$$A' = \tfrac{1}{2}\sigma^2 B^2 + \rho\sigma\eta BC + \tfrac{1}{2}\eta^2 C^2 + \phi C. \qquad (A9)$$

These three equations are readily solved by the use of an integrating factor and direct integration. Substituting the solutions into the expression for $H(X, Y, \tau)$, substituting $H(X, Y, \tau)$ into Equation $(A5)$, and then evaluating as of time zero $(\tau = T)$ gives the expression for the inflation swap price in Equation (9).

A.2 Inflation Option Prices

Let $C(X, Y, V, T)$ denote the price at time zero of a European call option on the price level at time T with strike K. The cash flow at the option expiration date is $\max(0, I_T - (1 + K)^T)$. The present value of this cash flow can be expressed as

$$E^Q \left[\exp\left(-\int_0^T r_s \, ds\right) \max(0, I_T - (1 + K)^T) \right], \qquad (A10)$$

which can be written as

$$E^Q \left[\exp\left(-\int_0^T R_s \, ds\right) \right] E^Q \left[\exp\left(-\int_0^T X_s \, ds\right) \max(0, I_T - (1 + K)^T) \right], \quad (A11)$$

after substituting in for r_t. Let $N(I, X, Y, \tau)$ denote the value of the expectations $E^Q[\exp(-\int_t^T X_s \, ds) \max(0, I_T - (1 + K)^T)]$. Note that we show the explicit functional dependence of $N(I, X, Y, \tau)$ on I since I_t need not equal one for $t > 0$. The value of $N(I, X, Y, \tau)$ satisfies the following partial differential equation,

$$\tfrac{1}{2}I^2 V N_{II} + \theta s I \sqrt{V} N_{IV} + \tfrac{1}{2}s^2 N_{VV} + \tfrac{1}{2}\sigma^2 N_{XX} + \rho\sigma\eta N_{XY} + \tfrac{1}{2}\eta^2 N_{YY}$$
$$+ I X N_I + \mu N_V + \lambda(Y - X)N_X + (\phi - \gamma Y)N_Y - X N = N_\tau,$$

$$(A12)$$

subject to the terminal condition $N(I, X, Y, V, 0) = \max(0, I_T - (1 + K)^T)$. We conjecture that the solution is of the form

$$N(I, X, Y, V, \tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)M(I, X, Y, V, \tau). \quad (A13)$$

Substituting in this expression into the partial differential equation in Equation (A12) and simplifying gives

$$\tfrac{1}{2}I^2 V M_{II} + \theta s I \sqrt{V} M_{IV} + \tfrac{1}{2}s^2 M_{VV} + \tfrac{1}{2}\sigma^2 M_{XX} + \rho\sigma\eta M_{XY} + \tfrac{1}{2}\eta^2 M_{YY}$$
$$+ I X M_I + \mu M_V + (\sigma^2 B(\tau)) + \rho\sigma\eta C(\tau) + \lambda(Y - X)M_X$$
$$+ (\eta^2 C(\tau) + \rho\sigma\eta B(\tau) + \phi - \gamma Y)M_Y + [\tfrac{1}{2}\sigma^2 B^2(\tau) + \rho\sigma\eta B(\tau)C(\tau) + \tfrac{1}{2}\eta^2 C^2(\tau)$$
$$+ \lambda(Y - X)B(\tau) + (\phi - \gamma Y)C(\tau) - X - A' - B'X - C'Y]M = M_\tau.$$

$$(A14)$$

From Equations (A7) through (A9) the term in brackets multiplying M is zero. Without the M term in the partial differential equation, however, the solution to the partial differential equation can be expressed as

$$M(I, X, Y, V, \tau) = E^{Q^*}[\max(0, I_T - (1 + K)^T)], \quad (A15)$$

where the expectation is taken with respect to the density of I_T implied by the dynamics,

$$dI = I \, X \, dt \, + \, I \, \sqrt{V} \, dZ_I, \quad (A16)$$
$$dX = (\lambda \, (Y \, - \, X) + \sigma^2 B(\tau) + \rho\sigma\eta C(\tau)) \, dt \, + \, \sigma \, dZ_X, \quad (A17)$$
$$dY = (\alpha \, - \, \beta \, Y + \eta^2 C(\tau) + \rho\sigma\eta B(\tau)) \, dt \, + \, \eta \, dZ_Y, \quad (A18)$$
$$dV = \mu \, dt \, + \, s \, dZ_V. \quad (A19)$$

Since

$$D(T) = E^Q \left[\exp\left(-\int_0^T r_s \ ds\right) \right], \tag{A20}$$

$$= E^Q \left[\exp\left(-\int_0^T R_s \ ds\right) \right] \exp(A(T) + B(T)X + C(T)Y). \tag{A21}$$

combining these results implies

$$C(X, Y, V, T) = D(T) \ E^{Q*}[\max(0, I_T - (1 + K)^T)]. \tag{A22}$$

Note that under this measure, the expected value of the price level equals the inflation swap price F. This follows since the cash flow from an inflation swap at time T is $I_T - F$. Under the Q^* measure, however, the present value of this cash flow is given by $D(T)E^{Q^*}[I_T - F]$. Since the initial value of the inflation swap contract is zero, this implies $E^{Q^*}[I_T] = F$.

A.3 The Distribution of the Price Level

From Equation (A1), $\ln I_T$ can be expressed as

$$\ln I_T = \int_0^T X_t \ dt + w_T, \tag{A23}$$

where w_T represents the terms that involve V_t. Let u_T denote the value of the integral of X_t is the above expression under the objective measure P. Under P, solving the stochastic differential equation for Y_t gives

$$Y_t = Ye^{-\beta t} + (\alpha/\beta)(1 - e^{-\beta t}) + \eta e^{-\beta t}\int_0^t e^{\beta s} \ dZ_Y(s). \tag{A24}$$

Likewise, solving for X_t gives

$$X_t = Xe^{-\kappa t} + \kappa e^{-\kappa t}\int_0^t e^{\kappa s} \ Y_s \ ds + \sigma e^{-\kappa t}\int_0^t e^{\kappa s} \ dZ_X(s). \tag{A25}$$

Substituting Equation (A24) into the above equation, interchanging the order of integration, and evaluating terms gives the following expression for X_t

$$X_t = Xe^{-\kappa t} + (\alpha/\beta)(1 - e^{-\kappa t}) + \frac{\kappa}{\kappa - \beta}(Y - \alpha/\beta)(e^{-\beta t} - e^{-\kappa t})$$

$$+ \kappa/(\kappa - \beta)\int_0^t e^{-\beta t} \ e^{\beta s} - e^{-\kappa t}e^{\kappa s} \ dZ_Y(s) + \sigma e^{-\kappa t}\int_0^t e^{\kappa s} \ dZ_X(s). \tag{A26}$$

Taking the integral of X_t, interchanging the order of integration, and evaluating terms gives

$$
\begin{aligned}
\int_0^T X_t\, dt = {} & X(1 - e^{-\kappa})/\kappa + (\alpha/\beta)(T - (1 - e^{-\kappa T})/\kappa) \\
& + \frac{\kappa}{\kappa - \beta}(Y - \alpha/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa) \\
& + \kappa\eta/(\kappa - \beta) \int_0^T \left((1 - e^{-\beta(T-t)})/\beta - (1 - e^{-\kappa(T-t)})/\kappa\right) dZ_Y(t) \\
& + \sigma \int_0^T (1 - e^{-\kappa(T-t)})/\kappa\, dZ_X(t).
\end{aligned} \tag{A27}
$$

Thus, u_T is a normally distributed random variable with mean

$$
\begin{aligned}
& X(1 - e^{-\kappa})/\kappa + (\alpha/\beta)(T - (1 - e^{-\kappa T})/\kappa) \\
& + (\kappa/(\kappa - \beta))(Y - \alpha/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa),
\end{aligned} \tag{A28}
$$

and variance

$$
\begin{aligned}
& \left(\frac{\kappa^2\eta^2}{(\kappa - \beta)^2}\left(\frac{1}{\beta^2} - \frac{2}{\beta\kappa} + \frac{1}{\kappa^2}\right) + \frac{\sigma^2}{\kappa^2} + \frac{2\kappa\rho\sigma\eta}{\kappa(\kappa - \beta)}\left(\frac{1}{\beta} - \frac{1}{\kappa}\right)\right) T \\
& + \left(\frac{2\kappa^2\eta^2}{\beta(\kappa - \beta)^2}\left(\frac{1}{\beta\kappa} - \frac{1}{\beta^2}\right) - \frac{2\kappa\rho\sigma\eta}{\beta^2\kappa(\kappa - \beta)}\right)(1 - e^{-\beta T}) \\
& + \left(\frac{\kappa^2\eta^2}{2\beta(\kappa - \beta)^2\beta^2}\right)(1 - e^{-2\beta T}) \\
& + \left(\frac{2\kappa\eta^2}{(\kappa - \beta)^2}\left(\frac{1}{\beta\kappa} - \frac{1}{\kappa^2}\right) - \frac{2\sigma^2}{\kappa^3} + \frac{2\rho\sigma\eta}{\kappa(\kappa - \beta)}\left(\frac{2}{\kappa} - \frac{1}{\beta}\right)\right)(1 - e^{-\kappa T}) \\
& + \left(\frac{\eta^2}{2(\kappa - \beta)^2\kappa} + \frac{\sigma^2}{2\kappa^3} - \frac{\rho\sigma\eta}{2\kappa^2(\kappa - \beta)}\right)(1 - e^{-2\kappa T}) \\
& + \left(\frac{-2\kappa^2\eta^2}{\beta\kappa(\beta + \kappa)(\kappa - \beta)^2} + \frac{2\kappa\rho\sigma\eta}{\kappa(\beta + \kappa)(\kappa - \beta)\beta}\right)(1 - e^{-(\beta + \kappa)T}).
\end{aligned} \tag{A29}
$$

A similar argument shows that v_T is a normally distributed random variable with mean

$$X(1 - e^{-\lambda T})/\lambda + \left(\frac{\phi}{\gamma} - \frac{\eta^2}{\gamma^2} - \frac{\rho\sigma\eta}{\lambda\gamma} - \frac{\sigma^2}{\lambda^2} - \frac{\rho\sigma\eta}{\gamma\lambda}\right)(T - (1 - e^{-\lambda T})/\lambda)$$

$$+ \frac{\lambda}{\lambda - \gamma}\left(Y - \frac{\phi}{\gamma} + \frac{\eta^2}{\gamma^2} + \frac{\rho\sigma\eta}{\lambda\gamma} - \frac{\eta^2 e^{-\lambda T}}{(\lambda - \gamma)(\gamma + \lambda)} + \frac{\rho\sigma\eta e^{-\lambda T}}{\lambda(\gamma + \lambda)} + \frac{\eta^2 \lambda e^{-\gamma T}}{2\gamma^2(\lambda - \gamma)}\right)$$

$$((1 - e^{-\lambda T})/\gamma - (1 - e^{-\lambda T})/\lambda)$$

$$+ \left(-\frac{\eta^2 e^{-\lambda T}}{2(\lambda - \gamma)(\gamma + \lambda)} + \frac{\rho\sigma\eta e^{-\lambda T}}{\lambda(\gamma + \lambda)} + \frac{\sigma^2 e^{-\lambda T}}{2\lambda^2} - \frac{\rho\sigma\eta e^{-\lambda T}}{2\lambda\gamma(\lambda - \gamma)}\right)$$

$$((e^{\lambda T} - 1)/\lambda - (1 - e^{-\lambda T})/\lambda)$$

$$+ \left(\frac{\eta^2 \lambda^2 e^{-\lambda T}}{2\gamma^2(\gamma + \lambda)(\lambda - \gamma)} + \frac{\rho\sigma\eta \lambda e^{-\lambda T}}{\gamma(\lambda - \gamma)(\gamma + \lambda)}\right)$$

$$((e^{\gamma T} - 1)/\gamma - (1 - e^{-\lambda T})/\lambda). \tag{A30}$$

and variance

$$\left(\frac{\lambda^2 \eta^2}{(\lambda - \gamma)^2}\left(\frac{1}{\gamma^2} - \frac{2}{\gamma\lambda} + \frac{1}{\lambda^2}\right) + \frac{\sigma^2}{\lambda^2} + \frac{2\lambda\rho\sigma\eta}{\lambda(\lambda - \gamma)}\left(\frac{1}{\gamma} - \frac{1}{\lambda}\right)\right)T$$

$$+ \left(\frac{2\lambda^2 \eta^2}{\gamma(\lambda - \gamma)^2}\left(\frac{1}{\gamma\lambda} - \frac{1}{\gamma^2}\right) - \frac{2\lambda\rho\sigma\eta}{\gamma^2\lambda(\lambda - \gamma)}\right)(1 - e^{-\gamma T})$$

$$+ \left(\frac{\lambda^2 \eta^2}{2\gamma(\lambda - \gamma)^2\gamma^2}\right)(1 - e^{-2\gamma T})$$

$$+ \left(\frac{2\lambda\eta^2}{(\lambda - \gamma)^2}\left(\frac{1}{\gamma\lambda} - \frac{1}{\lambda^2}\right) - \frac{2\sigma^2}{\lambda^3} + \frac{2\rho\sigma\eta}{\lambda(\lambda - \gamma)}\left(\frac{2}{\lambda} - \frac{1}{\gamma}\right)\right)(1 - e^{-\lambda T})$$

$$+ \left(\frac{\eta^2}{2(\lambda - \gamma)^2\lambda} + \frac{\sigma^2}{2\lambda^3} - \frac{\rho\sigma\eta}{2\lambda^2(\lambda - \gamma)}\right)(1 - e^{-2\lambda T})$$

$$+ \left(\frac{-2\lambda^2 \eta^2}{\gamma\lambda(\gamma + \lambda)(\lambda - \gamma)^2} + \frac{2\lambda\rho\sigma\eta}{\lambda(\gamma + \lambda)(\lambda - \gamma)\gamma}\right)(1 - e^{-(\gamma + \lambda)T}). \tag{A31}$$

A.4 The Conditional Moments

Integrating the dynamics for X and Y under the P measures gives the following expressions for the conditional means and variances

$$\mu_{Y_t} = Y_t e^{-\beta \Delta t} + (\alpha/\beta)(1 - e^{-\beta \Delta t}), \tag{A32}$$

$$\mu_{X_t} = X_t e^{-\kappa \Delta t} + (\alpha/\beta)(1 - e^{-\kappa \Delta t}) + \frac{\kappa}{\kappa - \beta}(Y_t - \alpha/\beta)(e^{-\beta \Delta t} - e^{-\kappa \Delta t}), \tag{A33}$$

$$\sigma_Y = \frac{\eta^2}{2\beta}(1 - e^{-2\beta \Delta t}), \tag{A34}$$

$$\sigma_X = \frac{\kappa^2 \eta^2}{(\kappa - \beta)^2}\left(\frac{1}{2\beta}(1 - e^{-2\beta \Delta t}) - \frac{2}{\beta + \kappa}(1 - e^{-(\beta + \kappa)\Delta t}) + \frac{1}{2\kappa}(1 - e^{-2\kappa \Delta t})\right)$$

$$+ \frac{2\kappa \rho \sigma \eta}{\kappa - \beta}\left(\frac{1}{\beta + \kappa}(1 - e^{-(\beta + \kappa)\Delta t}) - \frac{1}{2\kappa}(1 - e^{-2\kappa \Delta t})\right)$$

$$+ \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa \Delta t}), \tag{A35}$$

$$\sigma_{XY} = \frac{\kappa \eta^2}{\kappa - \beta}\left(\frac{1}{2\beta}(1 - e^{-2\beta \Delta t}) - \frac{1}{\beta + \kappa}(1 - e^{-(\beta + \kappa)\Delta t})\right)$$

$$+ \frac{\rho \sigma \eta}{\beta + \kappa}(1 - e^{-(\beta + \kappa)\Delta t}). \tag{A36}$$

A.5 The Inflation Surveys

The data from the University of Michigan Survey of Consumers consist of one and five year ahead inflation forecasts. The series is released at monthly frequency and reports the median expected price change over the next twelve months and the next five years, respectively. A detailed description of how the survey is conducted is available at http:// www.sca.isr.umich.edu/documents.php?c=i. In contrast to the Livingston survey and the Survey of Professional Forecasters, the participants in the University of Michigan Survey of Consumers are actual consumers (households) and not professionals. The time between the conduct of the survey and release is up to three weeks. The University of Michigan reports that a review of the estimates of inflation expectations indicated that for comparisons over time, the median, rather than the mean, may be a more reliable measure of the central tendency of the response distribution due to the changing influence of extreme responses. Therefore, we use the median survey forecasts throughout our analysis.

The Philadelphia Federal Reserve Bank Survey of Professional Forecasters is conducted on a quarterly basis. The questionnaires are sent to the participants at the end of January, at the end of April of the second quarter, at the end of July for the third quarter, and at the end of October for the fourth quarter. The survey results are published in the middle of February, May, August, and November, for the first, second, third, and fourth quarter, respectively. In contrast to the Livingston survey, participants in the SPF forecast changes in

the quarterly average CPI-U levels. A detailed description of how the survey is conducted is available at http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professionalforecasters/spf-documentation.pdf.

The Livingston survey is conducted twice a year, in June and in December, usually in the middle of the month. Participants include economists from industry, government, and academia. The surveys taken in June consist of two annual average CPI forecasts: for the current year, and for the following year. The December surveys include three annual average forecasts: for the current year, for the next year, and for the year after. The participants forecast non-seasonally-adjusted CPI level six and twelve months in the future. A detailed description of how the Livingston survey is conducted is available at: http://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey/livingston-documentation.pdf. For the Livingston surveys, there is a lag of up to four and three weeks between the time the survey and when the results are disseminated.

Finally, Bloomberg provides one-year-ahead forecasts at the monthly frequency compiled from more than 80 professionals. Participants include economists from Bank of America, BNP Paribas, JP Morgan, and many others. Detailed information on the composition of each forecast index can be found in the Bloomberg system under US CPI Forecast Index.

References

References

[1] Abramowitz, Milton, and Irene A. Stegun, 1965, *Handbook of Mathematical Functions*, Dover Publications, New York, NY.

[2] Acharya, Viral, and Lasse H. Pedersen, 2005, Asset Pricing with Liquidity Risk, *Journal of Financial Economics* 77, 375-410.

[3] Ackermann, C., R. McEnally, and D. Ravenscraft, 1999, The Performance of Hedge Funds: Risk, Return, and Incentives, *Journal of Finance*, 54, 833874.

[4] Adams, Lorna, Rob Warren and Alistair Kuechel, Research Exploring the Effect of Uprating by CPI on Occupational Pension Schemes, United Kingdom, Department for Work and Pensions, Available online at: `http://research.dwp.gov.uk/asd/asd5/WP102.pdf`.

[5] Adrian, Tobias, and Hao Z. Wu, 2009, The Term Structure of Inflation Expectations, Working paper, Federal Reserve Bank of New York.

[6] Agarwal, V., and N. Naik, 2004, Risks and Portfolio Decisions Involving Hedge Funds, *Review of Financial Studies*, 17, 6398.

[7] Aït-Sahalia, Yacine, Jochen Adnritzky, Andreas Jobst, Sylwia Nowak, and Natialia Tamirisa, 2010, Market Response to Policy Initiatives During the Global Financial Crisis, NBER Working Paper 15809.

[8] Aït-Sahalia, Yacine, and Andrew Lo, 1998, Nonparametric Estimation of State- Price Densities Implicit in Financial Asset Prices, *Journal of Finance* 53, 499-547.

[9] Amihud, Yakov, and Haim Mendelson, 1991, Liquidity, Maturity, and the Yields on U.S. Treasury Securities, *Journal of Finance* 46, 1411-1425.

[10] Amihud, Yakov, Haim Mendelson, and Lasse H. Pedersen, 2005, Liquidity and Asset Pricing, *Foundations and Trends in Finance* 1, 269-364.

[11] Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Clara Vega, 2003, Micro Effects of Macro Announcements: Realtime Price Discovery in Foreign Exchange. *American Economic Review* 93 (1), 38-62.

[12] Andonov, Aleksandar, Florian Bardong, and Thorsten Lehnert, 2010, TIPS, Inflation Expectations, and the Financial Crisis, *Financial Analysts Journal* 66, November/December, 1-13.

[13] Ang, Andrew, Geert Bekaert, and Min Wei, 2007, Do Macro Variables, Asset Markets, or Survey Forecast Inflation Better? *Journal of Monetary Economics* 54, 1163-1212.

[14] Ang, Andrew, Geert Bekaert, and Min Wei, 2008, The Term Structure of Real Rates and Expected Inflation, *Journal of Finance* 63, 797-849.

[15] Ang, Andrew, and Francis A. Longstaff, 2011, Systemic Sovereign Default Risk: Lessons from the U.S. and Europe, Working paper, UCLA.

[16] Angeletos, George-Marios, 2002, Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure, *The Quarterly Journal of Economics* 117, 1105-1131.

[17] Arora, Navneet, Priyank Gandhi, and Francis A. Longstaff, 2012, Counterparty Credit Risk and the Credit Default Swap Market, *Journal of Financial Economics*, 103, 280-293.

[18] Ashcraft, Adam, Nicolae Gârleanu, and Lasse H. Pedersen, 2010, Two Monetary Tools: Interest Rates and Haircuts, Working paper, New York University.

[19] Ashton, Michael J., 2006, Drivers of Inflation-Linked Corporate Bond Spreads: Are Corporate Linkers Indeed as Rich as They Look?, Working paper, IXIS Financial Products.

[20] Atack, Jeremy, and Peter Passell, 1994, *A New Economic View of American History*, New York: W. W. Norton and Co.

[21] Atkeson, Andrew, and Patrick J. Kehoe, 2004, Deflation and Depression: Is there an Empirical Link?, *American Economic Review, Papers and Proceedings* 94, 99-103.

[22] Baba, Naohiko, and Frank Packer, 2009, From Turmoil to Crisis: Dislocations in the FX Swap Market Before and after the Failure of Lehman Brothers, Bank for International Settlements Working Paper No. 285, *Journal of International Money and Finance*, forthcoming.

[23] Bai, Jennie, and Pierre Collin-Dufresne, 2011, The CDS-Bond Basis During the Financial Crisis of 2007–2009, Working paper, Columbia University.

[24] Banerjee, Snehal, and Jeremy Graveline, 2011, The cost of short-selling liquid securities, *Journal of Finance*, forthcoming.

[25] Bansal, Ravi, Wilbur John Coleman II, and Christian T. Lundblad, 2010, Endogenous Liquidity Supply, Working paper, Duke University.

[26] Bansal, Ravi, and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1481-1509.

[27] Banz, Rolf, and Merton H. Miller, 1978, Prices for State Contingent Claims: Some Estimates and Applications, *Journal of Business* 51, 653-672.

[28] Barclay's Capital, 2010, *Global Inflation-Linked Products, A Users Guide*.

[29] Bardong, Florian, and Thorsten Lehnert, 2004a, TIPS, Break-Even Inflation, and Inflation Forecasts, *The Journal of Fixed Income* 14, 15-35.

[30] Bardong, Florian, and Thorsten Lehnert, 2004b, European Inflation-Indexed Government Debt Security Markets, *Journal of Portfolio Management* 30, 22638.

[31] Bardong, Florian, and Thorsten Lehnert, 2008, TIPS and Inflation Expectations, *Applied Economics Letters*, Vol. 15, 513-518.

[32] Barnes, Michelle L., Zvi Bodie, Robert K. Triest, and J. Christina Wang, 2009, TIPS Scorecard: Are TIPS Accomplishing what They were Supposed to Accomplish? Can They be Improved?, *Financial Analysts Journal*

[33] Barr, David G., and John Y. Campbell, 1997, Inflation, Real Interest Rates, and the Bond Market, *Journal of Monetary Economics* 3, 361-383.

[34] Bauer, Michael D., and Glenn D. Rudebusch, 2011, The signaling Channel for Federal Reserve Bond Purchases, Working Paper Series 2011-21, Federal Reserve Bank of San

Francisco.

[35] Beechey, Meredith J., and Jonathan H. Wright, 2009, The High-Frequency Impact of News on Long-Term Yields and Forward Rates: Is it Real?, *Journal of Monetary Economics* 56, 535-544.

[36] Bekaert, Geert, and X. S. Wang, 2010, Inflation Risk and the Inflation Risk Premium, Working paper, Columbia University.

[37] Bernanke, Ben S., 2004, What Policymakers Can Learn from Asset Prices, Speech Before the Investment Analyst Society of Chicago.

[38] Bernanke, Ben S., and Kenneth N. Kuttner, 2005, What Explains the Stock Markets Reaction to Federal Reserve Policy? *Journal of Finance* 60 (3), 1221-1257.

[39] Bernardo, A., and O. Ledoit, 2000, Gain, Loss, and Asset Pricing, *Journal of Political Economy*, 108, 144172.

[40] Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz, 2005, Measuring Default Risk Premia from Default Swap Rates and EDFs, Working paper, Stanford University.

[41] Bohn, H., 1988, Why Do We Have Nominal Government Debt?, *Journal of Monetary Economics* 21, 127-140.

[42] Boudoukh, Jacob, and Robert F. Whitelaw, 1991, The Benchmark Effect in the Japanese Government Bond Market, *Journal of Fixed Income* 1, 52-59.

[43] Boudoukh, Jacob, and Robert F. Whitelaw, 1993, Liquidity as a Choice Variable: A

Lesson from the Japanese Government Bond Market, *Review of Financial Studies* 6, 265-292.

[44] Bordo, Michael D, and Andrew J. Filardo, 2005, Deflation in a Historical Perspective, Working paper No. 186, Bank for International Settlements

[45] Breeden, Douglas T., and Robert H. Litzenberger, 1978, Prices of State Contingent Claims Implicit in Options Prices, Journal of Business 51, 621-651.

[46] Brown, S., W. Goetzmann, and R. Ibbotson, 1999, Offshore Hedge Funds: Survival and Performance, 19891995, *Journal of Business*, 72, 91118.

[47] Brown, S., W. Goetzmann, and J. Park, 2000, Hedge Funds and the Asian Currency Crisis of 1997, *Journal of Portfolio Management*, 26, 95101.

[48] Brown, S., and W. Goetzmann, 2003, Hedge Funds with Style, *Journal of Portfolio Management*, 29, 101-112.

[49] Brunnermeier, Markus, and Lasse H. Pedersen, 2009, Market Liquidity and Funding Liquidity, *Review of Financial Studies* 22, 2201-2238.

[50] Buera, Francisco, and Juan Pablo Nicolini, 2004, Optimal Maturity or Government Debt Without State Contingent Bonds, *Journal of Monetary Economics* 51, 531-554.

[51] Buraschi, Andrea, and Alexei Jiltsov, 2005, Inflation Risk Premia and the Expectations Hypothesis, *Journal of Financial Economics* 75, 429-490.

[52] Buraschi, Andrea, Emrah Sener, and Murat Mengütürk, 2010, The Dynamics of Limits to Arbitrage: An Empirical Investigation of the Sovereign Bond Market, Working paper,

Imperial College.

[53] Cammack, Elizabeth B., 1991, Evidence on Bidding Strategies and the Information in Treasury Bill Auctions, *Journal of Political Economy* 99, 100-130.

[54] Campbell, John Y., 1987, Stock Returns and the Term Structure, *Journal of Financial Economics*, 18, 373 399.

[55] Campbell, John Y., and G. Taksler, 2002, Equity Volatility and Corporate Bond Yields, *Journal of Finance*, 58, 23212349.

[56] Campbell, John Y., and Luis M. Viceira, 2001, Who Should Buy Long Term Bonds?, *American Economic Review* 91, 99-127.

[57] Campbell, John Y., Robert Shiller, and Luis Viceira, 2009, Understanding Inflation-Indexed Bond Markets, *Brookings Papers on Economic Activity* (Spring), 79-120.

[58] Carlstrom, Charles T., Fuerst, Timothy S., 2004, Expected Inflation and TIPS. Economic Commentary, Federal Reserve Bank of Cleveland, issue Nov.

[59] Cecchetti, Stephen, 1992, Prices During the Great Depression: Was the Deflation of 19301932 Really Unanticipated?, *American Economic Review* 62, 141-156.

[60] Chan, Nicholas, Mila Getmansky, Shane M. Haas, and Andrew W. Lo, 2007, Systemic Risk and Hedge Funds, NBER Chapters, in: The Risks of Financial Institutions, pages 235-338. National Bureau of Economic Research, Inc.

[61] Chen, R., B. Liu, and X. Cheng, 2010, Pricing the Term Structure of Inflation Risk Premia: Theory and Evidence from TIPS, *Journal of Empirical Finance* 17, 702-721.

[62] Chen, Long, David A. Lesmond, and Jason Wei, 2007, Corporate Yield Spreads and Bond Liquidity, *Journal of Finance* 62, 119-149.

[63] Chernov, Mikhail, and Philippe Mueller, 2012, The Term Struture of Inflation Expectations, *Journal of Financial Economics* 106, 367-394.

[64] Chicago Mercantile Exchange, 2004, Break-Even Inflation - A Primer.

[65] Christensen, Jens H.E., Jose A. Lopez, and Glenn D. Rudebusch, 2009, Do Central Bank Liquidity Facilities Affect Interbank Interest Rates?, Federal Reserve Bank of San Francisco Working Paper 2009-13.

[66] Christensen, Jens H., Jose A. Lopez, and Glenn D. Rudebusch, 2010a, Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields, *Journal of Money, Credit, and Banking* 42, 143-178.

[67] Christensen, Jens H., Jose A. Lopez, and Glenn D. Rudebusch, 2010b, TIPS Deflation Protection: The Role of Stochastic Volatility, Working paper, Federal Reserve Bank of San Francisco.

[68] Christensen, Jens H., and James M. Gillan, 2011a, A Model-Independent Maximum Range for the Liquidity Correction of TIPS Yields, Working paper 2011-16, Federal Reserve Bank of San Francisco.

[69] Christensen, Jens H., and James M. Gillan, 2011b, Has the Treasury Benefited from Issuing TIPS?, *FRBSF Economic Letter 2011-12*.

[70] Christensen, Jens H., and James M. Gillan, 2011c, TIPS Liquidity, Breakeven Inflation, and Inflation Expectations, *FRBSF Economic Letter 2011-19*.

[71] Christensen, Jens H., and Glenn D. Rudebusch, 2012, The Response of Interest Rates to U.S. and U.K. Quantitative Easing, Federal Reserve Bank of San Francisco Working Paper 2012-06.

[72] Collin-Dufresne, Pierre, and Robert Goldstein, 2002, Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility, *Journal of Finance* 57, 1685-1730.

[73] Cornell, Bradford, and Alan C. Shapiro, 1990, The Mispricing of U.S. Treasury Bonds: A Case Study, *Review of Financial Studies* 2, 297-310.

[74] Coval, Joshua, Jakub W. Jurek, and Erik Stafford, 2009, Economic Catastrophe Bonds, *American Economic Review* 99, 628-666.

[75] D'Amico, Stefania, Don H. Kim, and Min Wei, 2010, Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices, Finance and Economics Discussion Series 2010-19, Federal Reserve Board.

[76] D'Amico, Stefania, and Thomas B. King, Flow and Stock Effects of Large-Scale Treasury Purchases, Working paper, Federal Reserve Board of Governors, February 2011.

[77] Dammon, Robert M., Kenneth B. Dunn, and Chester S. Spatt, 1993, The Relative Pricing of High-Yield Debt: The Case of RJR Nabisco Holding Capital Corporation, *American Economic Review* 83, 1090-1111.

[78] Daves, Philip R., and Michael C. Ehrhardt, 1993, Liquidity, Reconstitution, and the Value of U.S. Treasury Strips, *Journal of Finance* 48, 315-329.

[79] Deacon, Mark, Andrew Derry and Dariush Mirfendereski, *Inflation-Indexed Securities: Bonds, Swaps, and other Derivatives*, 2nd Edition, John Wiley & Sons Inc., Hoboken, NJ 07030, USA

[80] Doh, Taeyoung, 2010, The Efficacy of Large-Scale Asset Purchases at the Zero Lower Bound, *Economic Review*, Federal Reserve Bank of Kansas City, issue Q II, pp. 5-34.

[81] Dor, A., and R. Jagannathan, 2002, Understanding Mutual Fund and Hedge Fund Styles Using Return Based Style Analysis, working paper, Northwestern University.

[82] Duarte, Jefferson, Francis A. Longstaff, and Fan Yu, 2007, Risk and Return in Fixed Income Arbitrage: Nickels in Front of a Steamroller?, *Review of Financial Studies* 20, 769-811.

[83] Dudley, William C., Jennifer Roush, and Michelle Steinberg Ezer, 2009, The Case for TIPS: An Examination of the Costs and Benefits, *Federal Reserve Bank of New York Economic Policy Review* 15, (July) 1-17.

[84] Duffie, Darrell, 2010, Asset Price Dynamics with Slow-Moving Capital, *Journal of Finance* 65, 1238-1268.

[85] Duffie, Darrell, and Ming Huang, 1996, Swap Rates and Credit Quality, *Journal of Finance* 51, 921-950.

[86] Duffie, Darrell, Lasse H. Pedersen, and Kenneth J. Singleton, 2003, Modeling Sovereign Yield Spreads: A Case Study of Russian Debt, *Journal of Finance* 58, 119-159.

[87] Duffie, Darrell, and Kenneth J. Singleton, 1997, An Econometric Model of the Term Structure of Interest Rate Swap Yields, *Journal of Finance* 52, 1287-1321.

[88] Driessen, Joost, 2005, Is Default Event Risk Priced in Bonds?, *Review of Financial Studies* 18, 165-195.

[89] Dybvig, Philip H. and Stephen A. Ross, Arbitrage, The New Palgrave Dictionary of Economics, Second Edition, Eds. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan, 2008.

[90] Evans, Martin D., 1998, Real Rates, Expected Inflation, and Inflation Risk Premia, *Journal of Finance* 53, 187-218.

[91] Evans, Martin D., 2003, Real Risk, Inflation Risk, and the Term Structure, *The Economic Journal* 113, 345-389.

[92] Fama, E., and K. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics*, 33, 356.

[93] Fama, Eugene F., and Michael Gibbons, 1984, A Comparison of Inflation Forecasts, *Journal of Monetary Economics* 13, 327-348.

[94] Faust, Jon, John H. Rogers, Shing-Yi B. Wang, and Jonathan H. Wright, 2007, The High-Frequency Response of Exchange Rates and Interest Rates to Macroeconomic Announcements. *Journal of Monetary Economics* 54, 1051-1068.

[95] Faust, Jon, and Jonathan H. Wright, 2012, Forecasting Inflation, Working paper, Johns Hopkins University.

[96] Fischer, Irving, 1933, The Debt-Deflation Theory of the Great Depression, *Econometrica* 1, 337-357.

[97] Fisher, Mark, 2002, Special repo rates: An introduction, Federal Reserve Bank of Atlanta Economic Review 2nd Quarter, 27-43.

[98] Fleckenstein, Matthias, Francis A. Longstaff, and Hanno Lustig (2012), The TIPS-Treasury Bond Puzzle, *SSRN Working Paper Series*.

[99] Fleming, Michael J., 2003, Measuring Treasury Market Liquidity, *Federal Reserve Bank of New York Economic Policy Review (September)*, 83-108.

[100] Fleming, Michael J., 2007, Who Buys Treasury Securities at Auctions?, *Federal Reserve Bank of New York Current Issues in Economics and Finance* 13, Number 1, 1-7.

[101] Fleming, Michael J., and Neel Krishnan, 2009, The Microstructure of the TIPS Market, Working paper, Federal Reserve Bank of New York.

[102] Fleming, Michael, and John Sporn, 2012, An Analysis of OTC Interest Rate Derivatives Transactions: Implications for Public Reporting, Appendix: Trading Activity and Price Transparency in the Inflation Swap Market, Federal Reserve Bank of New York Staff Reports.

[103] Fostel, Ana, and John Geanakoplos, 2008, Collateral Restrictions and Liquidity Under-Supply: A Simple Model, *Economic Theory* 35, 441-467.

[104] Fung, W., and D. Hsieh, 1997, Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds, *Review of Financial Studies*, 10, 275302.

[105] Fung, W., and D. Hsieh, 2001, The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers, *Review of Financial Studies*, 14, 313341.

[106] Fung, W., and D. Hsieh, 2002, The Risk in Fixed-Income Hedge Fund Styles, *Journal of Fixed Income*, 12, 627.

[107] French, Kenneth R., 1983, A Comparison of Futures and Forward Prices, *Journal of Financial Economics* 12, 311-342.

[108] Friedman, Milton and Anna Schwartz, 1963, A Monetary History of the United States, 18671960. Princeton University Press, Princeton.

[109] Froot, Kenneth A., 2001, The Evolving Market for Catastrophe Event Risk, *Risk Management and Insurance Review* 2, 1-28.

[110] Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, 2010, Large Scale Asset Purchases by the Federal Reserve: Did They Work?, Federal Reserve Bank of New York Staff Report No. 441.

[111] Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases, *International Journal of Central Banking*, March 2011, 7 (1), 343.

[112] Garmaise, Mark J., and Tobias Moskowitz, 2009, Catastrophic Risk and Credit Markets, *Journal of Finance* 64, 657-707.

[113] Getmansky, M., A. Lo, and I. Makarov, 2004, An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns, *Journal of Financial Economics*, 74, 529609.

[114] Ghysels, Eric, and Fangfang Wang, 2011, Some Useful Densities for Risk Management and their Properties, Working paper, University of North Carolina at Chapel Hill.

[115] Giesecke, Kay, Francis A. Longstaff, Stephen Schaefer, and Ilya Strebulaev, 2011, Corporate Bond Default Risk: A 150-Year Perspective, *Journal of Financial Economics* 102, 233-250.

[116] Goldberg, Linda, Craig Kennedy, and Jason Miu, 2010, Central Bank Dollar Swap Lines and Overseas Dollar Funding Costs, National Bureau of Economic Research Working Paper 15763.

[117] Gongloff, Mark, 2010, TIPS Given the Cold Shoulder, *Wall Street Journal*, April 27, p. C8.

[118] Gorton, Gary B., and Andrew Metrick, 2009, Haircuts, Working paper, Yale University.

[119] Grinblatt, Mark, and Francis A. Longstaff, 2000, Financial Innovation and the Role of Derivative Securities: An Empirical Analysis of the U.S. Treasury's STRIPS Program, *Journal of Finance* 55, 1415-1436.

[120] Grishchenko, Olesya. V., and Jing-Zhi Huang, 2008, Inflation Risk Premium: Evidence from the Tips Market. Working Paper.

[121] Grishchenko, Olesya V., and Jing-zhi Huang, 2012, Inflation Risk Premium: Evidence from the TIPS Market, Finance and Economics Discussion Series, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board.

[122] Gromb, Denis, and Dimitri Vayanos, 2002, Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs, *Journal of Financial Economics* 66, 361-407.

[123] Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2008, The TIPS Yield Curve and Inflation Compensation, Working Paper, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington D.C.

[124] Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2010, The TIPS Yield Curve and Inflation Compensation, *American Economic Journal: Macroeconomics* 2, 70-92.

[125] Hamilton, James D., 1985, Uncovering Financial Market Expectations of Inflation, *Journal of Political Economy* 93, 1224-1241.

[126] Hamilton, James D., 1992, Was the Deflation During the Great Depression Anticipated? Evidence from the Commodity Futures Market, *American Economic Review* 82, 157-178.

[127] Han, Bing, Francis A. Longstaff, and Craig Merrill, 2007, The U.S. Treasury Buyback Auctions: The Cost of Retiring Illiquid Bonds, *Journal of Finance* 62, 2673-2693.

[128] Hancock, Diane, and Wayne Passmore, 2011, Did the Federal Reserve's MBS Purchase Program Lower Mortgage Rates?, Finance and Economics Discussion Series, 2011-01.

[129] Haubrich, Joseph, George Pennacchi, and Peter Ritchken, 2012, Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps, *Review of Financial Studies* 25, 1588-1629.

[130] Heider, Markus, Alex Li, and Vanshree Verma, 2012, DB Inflation Volatility Report – Inflation Volatility Update, Deutsche Bank Global Markets Research, Deutsche Bank AG/London, 11 July 2012

[131] Hinnerich, Mia, 2008, Inflation-Indexed Swaps and Swaptions, *Journal of Banking and Finance* 32, 2293-2306.

[132] Hördahl, Peter, and Oreste Tristani, 2007, Inflation Risk Premia in the Term Structure of Interest Rates, BIS Working Papers 228, Bank for International Settlements.

[133] Hördahl, Peter, 2008, The Inflation Risk Premium in the Term Structure of Interest Rates, Working Papers No 228, Bank for International Settlements.

[134] Hördahl, Peter, and Oreste Tristani, 2010, Inflation risk premia in the US and the Euro area, BIS Working Papers 325.

[135] Hu, Xing, Jun Pan, and Jiang Wang, 2011, Noise as Information for Illiquidity, Working paper, MIT.

[136] Huang, Jennifer, and Jiang Wang, 2010, Market Liquidity, Asset Prices, and Welfare, *Journal of Financial Economics* 95, 107-127.

[137] Ibragimov, Rustam, Dwight Jaffee, and JohanWalden, 2011, Diversification Disasters, *Journal of Financia Economics* 99, 333-348.

[138] Jamshidian, Farshid, 1989, An Exact Bond Option Formula, Journal of Finance 44, 205-209.

[139] Jarrow, Robert, and Y. Yildirim, 2003, Pricing Treasury Inflation Protected Securities and Related Derivatives Using an HJM Model, *Journal of Financial and Quantitative Analysis* 38, 409-430.

[140] Johannes, Michael, and Suresh Sundaresan, 2007, Pricing Collateralized Swaps, *Journal of Finance* 62, 383-410.

[141] Jones, Sam, and Izabella Kaminska, 2010, Bond Strategy Led to Big Win After Lehman, `http://www.ft.com/intl/cms/s/0/a9832c1e-c109-11df-99c4-00144feab49as01=1.html`

[142] Jones, Sam, 2012, ECB Liquidity Fuels High Stakes Hedging, `http://www.ft.com/intl/cms/s/0/cb74d63a-7e75-11e1-b009-00144feab49a.html`.

[143] Jordan, Bradford, Randy Jorgensen, and David Kuipers, 2000, The Relative Pricing of U.S. Treasury STRIPS, *Journal of Financial Economics* 56, 89-123.

[144] Joyce, Michael A. S., Ana Lasaosa, Ibrahim Stevens, and Matthew Tong, 2011, The Financial Market Impact of Quantitative Easing in the United Kingdom, *International Journal of Central Banking* 7 (3), 113-161.

[145] Kamara, Avi, 1988, Market Trading Structures and Asset Pricing: Evidence from the Treasury Bill Markets, *Review of Financial Studies* 1, 357-375.

[146] Kamara, Avi, 1994, Liquidity, Taxes, and Short-Term Treasury Yields, *Journal of Financial and Quantitative Analysis* 29, 403-417.

[147] Kaminska, Izabella, 2010, Who Played the Largest Ever Arbitrage?, `http://ftalphaville.ft.com/blog/tag/barnegat-fund-management`.

[148] Kerkhof, Jeroen, 2005, Inflation Derivatives Explained, *Fixed Income Quantitative Research, Lehman Brothers* (July), 1-80.

[149] Kilian, Lutz, and Simone Manganelli, 2007, Quantifying the Risk of Deflation, *Journal of Money, Credit, and Banking* 39, 561-590.

[150] Kitsul, Yuriy, and Jonathan H. Wright, 2012, The Economics of Options-Implied Inflation Probability Density Functions, Working paper, Johns Hopkins University.

[151] Kitamura, Yukinobu, 2006, Interim Evaluation of Inflation Indexed Bonds in Japan: Based on the JGB Trading Information, mimeo, Japan Bond Trading, Co. Ltd. (in Japanese).

[152] Krishnamurthy, Arvind, 2002, The Bond/Old Bond Spread, *Journal of Financial Economics* 66, 463-506.

[153] Krishnamurthy, Arvind, 2010, How Debt Markets Have Malfunctioned in the Crisis, *Journal of Economic Perspectives* 24, 3-28.

[154] Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2010a, The Aggregate Demand for Treasury Debt, Working paper, Northwestern University.

[155] Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2011, The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy, NBER Working paper 17555.

[156] Kuttner, Kenneth N., 2001, Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Markets, *Journal of Monetary Economics* 47, 523-544.

[157] Kuttner, Kenneth N., 2004, Comment on 'Price Stability and Japanese Monetary Policy' by Robert Hetzel, *Bank of Japan Institute for Monetary and Economic Studies*, 37-46.

[158] Lamont, Owen A., and Richard H. Thaler, 2003, Can the Market Add and Subtract? Mispricing in Tech Stock Carve-outs, *Journal of Political Economy* 111, 227-268.

[159] Lane, Morton, and Olivier Mahul, 2008, Catastrophe Risk Pricing, An Empirical Analysis, Policy Research Working Paper 4765, The World Bank.

[160] Lesmond, David A., Joseph P. Ogden, and Charles A. Trzcinka, 1999, A New Estimate of Transaction Costs, *Review of Financial Studies* 12, 1113-1141.

[161] Liu, Jun, and Francis A. Longstaff, 2005, Losing Money on Arbitrages: Optimal Dynamic Portfolio Choice in Markets with Arbitrage Opportunities, *Review of Financial Studies* 17, 611-641.

[162] Liu, Jun, Francis A. Longstaff, and Ravit Mandell, 2006, The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks, *Journal of Business* 79, 2337-2360.

[163] Longstaff, Francis A., 1990, The Valuation of Options on Yields, *Journal of Financial Economics* 26, 97-121.

[164] Longstaff, Francis. A., 1992, Are Negative Option Prices Possible? The Callable U.S. Treasury Bond Puzzle, *Journal of Business* 65, 571-592.

[165] Longstaff, Francis A., 1995, Option Pricing and the Martingale Restriction, *Review of Financial Studies* 8, 1091-1124.

[166] Longstaff, Francis A., 2004, The Flight to Liquidity Premium in U.S. Treasury Bond Prices, *The Journal of Business* 77, 511-526.

[167] Longstaff, Francis A., 2009, Portfolio Claustrophobia: Asset Pricing in Markets with Illiquid Assets, *American Economic Review* 99, 1119-1144.

[168] Longstaff, Francis A., and Andrew Ang, 2012, Systemic Sovereign Default Risk: Lessons from the U.S. and Europe, Working paper, UCLA.

[169] Longstaff, Francis A., and Brett Myers, 2013, How Does the Market Value Toxic Assets?, *Journal of Financial and Quantitative Analysis*, forthcoming.

[170] Longstaff, Francis A., and Arvind Rajan, 2008, An Empirical Analysis of the Pricing of Collateralized Debt Obligations, *Journal of Finance* 63, 509-563.

[171] Longstaff, Francis A., Sanjay Mithal, and Eric Neis, 2005, Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market, *Journal of Finance* 60, 2213-2253.

[172] Longstaff, Francis A., Jun Pan, Lasse H. Pedersen, and Kenneth J. Singleton, 2010, How Sovereign is Sovereign Credit Risk?, *American Economic Journal: Macroeconomics*, forthcoming.

[173] Longstaff, Francis A., Pedro Santa-Clara, and Eduardo S. Schwartz, 2001, The Relative Valuation of Caps and Swaptions: Theory and Empirical Evidence, *The Journal of Finance* 56, 2067.

[174] Longstaff, Francis A., Jefferson Duarte, and Fan Yu, 2007, Risk and Return in Fixed Income Arbitrage: Nickels in Front of a Steamroller?, *Review of Financial Studies*, 769-811.

[175] Lucas, Robert E., and Nancy L. Stokey, 1983, Optimal Fiscal and Monetary Policy in an Economy Without Capital, *Journal of Monetary Economics* 12, 55-93.

[176] Lustig, Hanno, Christopher Sleet, and Sevin Yeltekin, 2008, Fiscal Hedging with Nominal Assets, *Journal of Monetary Economics* 55, 710-727.

[177] Malkiel, B., Saha, A., 2005, Hedge funds: Risk and Return. *Financial Analysts Journal* 61, 80-88.

[178] McAndrews, James, Asani Sarkar, and Zhenyu Wang, 2008, The Effect of the Long-Term Auction Facility of the London Inter-Bank Offered Rate, *Federal Reserve Bank of New York Staff Report* No. 335, July.

[179] McCauley, R., and K. Ueda, 2009, Government Debt Management at Low Interest Rates. BIS Quarterly Review (June): 3551.

[180] Mercurio, Fabio, 2005, Pricing Inflation-Indexed Derivatives, *Quantitative Finance* 5, 289-302.

[181] Merton, Robert C., 1973, Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science* 4, no. 1., 141-183.

[182] Mishkin, Frederic S., 2011, Monetary Policy Strategy - Lessons from the Crisis, NBER Working Papers 16755, National Bureau of Economic Research

[183] Mitchell, M., and T. Pulvino, 2001, Characteristics of Risk and Return in Risk Arbitrage, *Journal of Finance*, 56, 21352175.

[184] Mitchell, M., T. Pulvino, and E. Stafford, 2002, Limited Arbitrage in Equity Markets, *Journal of Finance*, 57, 551584.

[185] Mitchell, Mark, Lasse H. Pedersen, and Todd Pulvino, 2007, Slow Moving Capital, *American Economic Review, Papers and Proceedings* 97, 215-220.

[186] Moulton, Pamela C., 2004, Relative repo specialness in U.S. Treasuries, *Journal of Fixed Income*, June, 40-47.

[187] North, Douglass C., 1961, *The Economic Growth of the United States*, 17901860, Prentice Hall.

[188] Pan, Jun, and Kenneth J. Singleton, 2008, Default and Recovery Implicit in Options: Evidence from an Integrated Time-Series Study, *Journal of Finance* 63, 2345-2384.

[189] Pflueger, Carolin E., and Luis M. Viceira, 2011a, Inflation Indexed Bonds and the Expectations Hypothesis, *Annual Review of Financial Economics*, forthcoming.

[190] Pflueger, Carolin E., and Luis M. Viceira, 2011b, An Empirical Decomposition of Risk and Liquidity in Nominal and Inflation-Indexed Government Bonds, Working paper, Harvard University.

[191] Pond, Michael, and Chirag Mirani, 2011, Barclays Capital, Market Strategy Americas, June 3, 12-14.

[192] Roll, Richard, 1996, U.S. Treasury Inflation-Indexed Bonds: The Design of a New Security, *Journal of Fixed Income* (December), 9-28.

[193] Roll, Richard, 2004, Empirical TIPS, *Financial Analysts Journal* 60 (Jan-Feb), 31-53.

[194] Rosa, Carlo, 2011a, Words That Shake Traders, *Journal of Empirical Finance* 18 (5), 915-934.

[195] Rosa, Carlo, 2011b, The High-Frequency Response of Exchange Rates to Monetary Policy Actions and Statements. *Journal of Banking and Finance* 35, 478-489.

[196] Rosa, Carlo, 2012, How Unconventional Are Large-Scale Asset Purchases, Federal Reserve Bank of New York Staff Reports, 560.

[197] Sack, Brian, and Robert Elsasser, 2004, Treasury Inflation-Indexed Debt: A Review of the U.S. Experience, *Economic Policy Review FRBNY*, Vol. 10(1), 47-63.

[198] Sarkar, Asani, and Jeffrey Shrader, 2010, Financial Amplification Mechanisms and the Federal Reserve's Supply of Liquidity During the Crisis, Federal Reserve Bank of New York Staff Report No. 431.

[199] Schulz, Alexander, and Jelena Stapf, 2011, Price Discovery on Traded Inflation Expectations: Does the Financial Crisis Matter?, BIS IFC Bulletin 34, 202-234.

[200] Seppälä, Juha, 2004, The Term Structure of Real Interest Rates: Theory and Evidence from UK Index-Linked Bonds, *Journal of Monetary Economics* 51, 1509-1549.

[201] Shleifer, Andrei, and Robert W. Vishny, 1997, The Limits of Arbitrage, *Journal of Finance* 52, 35-55.

[202] Stanton, Richard, and Nancy Wallace, 2011, The Bears Lair: Indexed Credit Default Swaps and the Subprime Mortgage Crisis, *Review of Financial Studies*, forthcoming.

[203] Shen P., 1998, How Important Is the Inflation Risk Premium? *Economic Review, Federal Reserve Bank of Kansas City*, issue Q IV, 35-47.

[204] Shen P., 2006, Liquidity Risk Premia and Breakeven Inflation Rates. *Economic Review, Federal Reserve Bank of Kansas City*, issue Q II, 29-54.

[205] Taylor, John, B., and John Williams, 2009, A Black Swan in the Money Market, *American Economic Journal: Macroeconomics* 1(1): 58-83.

[206] Trehan, B., 2010. Survey Measures of Expected Inflation and the Inflation Process. Working Paper Series 2009-10, Federal Reserve Bank of San Francisco.

[207] Tucker, Matthew and Stephen Laipply, 2010, Understanding Bond ETF Premiums and Discounts: A Conceptual Framework, *Investment Insights: The Investment Research Journal from Blackrock* 13 (April), 1-20.

[208] United Kingdom Debt Management Office, CPI-Linked Gilts: A Consultation Document, June 2011, Available online at: http://www.dmo.gov.uk/documentview.aspx?docname=publications/giltsmarket/consultationpapers/cons20110629.pdf&page=Gilts/Consultation

[209] United States Government Accountability Office, 2009, Treasury Inflation Protected Securities Should Play a Heightened Role in Addressing Debt Management Challenges, GAO-09-932.

[210] Vayanos, Dimitri, 2004, Flight to Quality, Flight to Liquidity, and the Pricing of Risk, Working paper, London School of Economics.

[211] Vayanos, Dimitri, and Jean-Luc Vila, 1999, Equilibrium Interest Rate and Liquidity Premium with Transaction Costs, *Economic Theory* 509-539.

[212] Vayanos, Dimitri, and Pierre-Oliver Weill, 2008, A Search-Based Theory of the On-the-Run Phenomenon, *Journal of Finance* 63, 1361-1398.

[213] Wright, Jonathan, 2011, What does Monetary Policy do to Long-Term Interest Rates at the Zero Lower Bound?, Working Paper, Johns Hopkins University.

[214] Wu, Tao, 2008, On the Effectiveness of the Federal Reserve's New Liquidity Facilities, Federal Reserve Bank of Dallas, Working Paper No. 2008-08.

[215] Yu, Fan, 2006, How Profitable is Capital Structure Arbitrage?, *Financial Analysts Journal* 62 (Sep-Oct), 47-62.