UNIVERSITY OF CALIFORNIA, SAN DIEGO

Topics in Supersymmetry and Supersymmetry Breaking

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Physics

by

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2009

The dissertation of Matthew C. Sudano is approved, and it is acceptable in quality and form for publication on microfilm:

Chair

University of California, San Diego

2009

EPIGRAPH

In order to understand the world,
one has to turn away from it on occasion.
—*Albert Camus*

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ACKNOWLEDGEMENTS

There are many people who I would like to acknowledge for contributing in various direct and indirect ways to this dissertation. My committee members[1] actually had responsibilities in this process, so I'll start with them. To Mark Gross, Julius Kuti, Justin Roberts, and Frank Würthwein, thank you for your time and effort.

The chair of my committee, my advisor, has had considerably more responsibility. For about five years now he has been trying to turn me into a physicist. With countless quick suggestions and long conversations, he has improved my skills and expanded my knowledge, he has steered me to and through projects, and he has taught me about the nuances of the field in which we work. It takes a lot to succeed in academia, and while it may not prove to be the course for me, if I do get to stay in the game, it will be in no small part due to the advice and example that he has provided. Thanks, Ken.

I have also benefitted tremendously from interactions with Ben Grinstein, Julius Kuti, and Aneesh Manohar. They have taught me a huge amount of physics, both formally and informally. This is a group of brilliant guys, who gladly share their knowledge and their confusion, and also happen to be a lot of fun to hang out with. Thanks for helping to make my time at UCSD pleasant and productive.

There are several other UCSD faculty members who I'd like to mention. First I'd like to thank the HEP-EXers, Jim Branson, Vivek Sharma, Frank Würthwein, and Avi Yagil, for many interesting discussions, kind hospitality, and for being (for the most part) good to my wife. I would also like to thank my teachers: George Fuller, Mark Gross, Terry Hwa, Hans Lindblad, Tom O'Neil, Justin Roberts, Nolan Wallach, and the particle theory guys. The faculty at UC Davis also deserve thanks for teaching me the basics and for fueling my curiosity. Here's an incomplete list of memorable (in a good way) professors: Steven Carlip, Linton Corruccini, C.Y. Fong, Roderick Reid, Richard Scalettar, and Rajiv Singh. Ling-Lie Chau deserves to be singled out, not just as an exceptional teacher, but also for seeing potential

[1]Nytu Kitchloo deserves to be acknowledged for serving on my advancement committee. He was unable to attend the defense.

in me and for going out of her way to encourage my development.

Thanks to the particle theory post-docs and graduate students who have let me ask them stupid questions: Eddy Barnes, Jui-yu "Mannie" Chiu, Andy Fuhrer, Randy Kelley, Dani Nogradi, Chris Schroeder, Andy Stergiou, Zevik Surujon, Mike Trott, Jason Wright, and especially Elie Gorbatov who taught me a lot of physics that I didn't even know how to ask questions about, and Patipan Uttayarat who has to work nights and weekends because I waste so much of his time. Come to think of it, I asked stupid questions in Princeton, too. Here are some names: Marcus Benna, Matt Buican, Diego Hofman, Josh Friess, Dima Malyshev, Arvind Murugan, David Shih. While I'm at it, let me thank the people of the IAS and Princeton University for the hospitality they showed my wife and me over the 05-06 academic year. Important names include Nati Seiberg and Chris Tully. Thanks also to Igor Klebanov and Sasha Polyakov for the fascinating courses and discussions.

Finally, I want to thank several people with whom I do not discuss physics, but who have contributed indirectly to this dissertation. I'll first offer a broad thank you to all of my friends, past and present, for helping to keep me sufficiently sane. The few who I will name explicitly have also helped to keep my wife happy – the married people: Andrew and Emily, Chris and Cari, Evan and Hollie, and Sean and Eva.

I also want to acknowledge my family, the people who most shaped who I am. In two sentences, I can't begin to thank the woman who gave birth to me. She was my first teacher and I'm still reaping the benefits of the lessons that I learned. Thanks for giving me a firm foundation and a large head start, and then allowing me the freedom to find my course in life. I am grateful to my father for serving as a model of discipline and ambition, and for always encouraging curiosity and research. It was a discussion with my father about the origin and nature of the universe that inspired me to study physics. And it was my stepfather, Doug, who gave me my first physics books. Thanks, Doug, for always being an enthusiastic supporter of my studies and for encouraging me to be more precise.

That leaves my siblings, the people responsible for most of the joy and misery

that I've experienced. Julia helped me to mature, to learn self-control, and to be a better teacher; I hope I didn't ruin math for her. Brian trained me to be competitive and had me performing nearly three years ahead of my peers; we have to find more opportunities to play on the same team. And Anne showed me how to be a good student while still having fun; it turns out that all good physicists are great procrastinators.

My wife, Elizabeth, gets the last paragraph. I owe her far more than that. She has helped me at every step along the way, in every way she could. Thanks for fixing my computer, for editing my drafts, and for picking up the slack at home when I get too busy with work. Thanks for letting me work long hours, for following me around the world, and for letting me play for a living instead of starting a real life. You see evidence every day of how much I love physics; I hope you never doubt that my love for you is immeasurably greater.

Technical acknowledgements: Chapter 2 is a reprint of the material as it appears in "The Exact Superconformal R-symmetry Minimizes τ_{RR}," E. Barnes, E. Gorbatov, K. Intriligator, M. Sudano, J. Wright, Nucl. Phys. B **730**, 210 (2005), arXiv:hep-th/0507137. Chapter 3 and Appendix A comprise a reprint of the material as it appears in "Sparticle Masses in Higgsed Gauge Mediation," E. Gorbatov, M. Sudano, JHEP **0810**, 066 (2008), arXiv:0802.0555. Chapter 4 is a reprint of the material as it appears in "Comments on General Gauge Mediation," K. Intriligator, M. Sudano, JHEP **0811**, 008 (2008), arXiv:0807.3942. Chapter 5 and Appendix B comprise a reprint of the material as it appears in "Surveying Pseudomoduli: the Good, the Bad, and the Incalculable," K. Intriligator, D. Shih, M. Sudano, JHEP **0903**, 106 (2009), arXiv:0809.3981. I was a co-author of all of these papers. This work was supported in part by UCSD grant DOE-FG03-97ER40546.

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Kenneth Intriligator, David Shih, M.C.S., "Surveying Pseudomoduli: the Good, the Bad, and the Incalculable," JHEP **0903**, 106 (2009) [arXiv:0809.3981].

Kenneth Intriligator, M.C.S., "Comments on General Gauge Mediation," JHEP **0811**, 008 (2008) [arXiv:0807.3942].

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ABSTRACT OF THE DISSERTATION

Topics in Supersymmetry and Supersymmetry Breaking

by

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Doctor of Philosophy in Physics
University of California San Diego, 2009
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There are two basic topics investigated is this dissertation. The first has to do with the symmetries of superconformal field theories (SCFTs). A general constraint that, in principle, determines the superconformal $U(1)_R$ symmetry of 4d $\mathcal{N} = 1$ SCFTs and 3d $\mathcal{N} = 2$ SCFTs is identified. Among all possibilities, the superconformal $U(1)_R$ is that which minimizes the coefficient, τ_{RR}, of its two-point function. For 4d $\mathcal{N} = 1$ SCFTs, τ_{RR}-minimization gives an alternative to a-maximization. τ_{RR}-minimization also applies in 3d, where no condition for determining the superconformal $U(1)_R$ had been known.

The second general topic discussed is supersymmetry breaking. Three chapters are devoted to this topic. In the first, the gauge sector of ordinary gauge mediation is generalized. The two-loop calculation of sfermion masses is generalized to allow for an arbitrary gauge group with an arbitrary supersymmetric Higgsing. The generic effect on the MSSM spectrum from additional Higgsed gauge structure is to increase the sfermion masses relative to the gaugino masses.

The subsequent chapter deals with "general gauge mediation," in which the hidden-sector effects are expressed in terms of current two-point functions. The previously discussed generalization of the gauge sector to allow for Higgsing is computed in this formalism. The effective potential for squark pseudo-D-flat directions is also given. This reduces to the sfermion soft masses near the origin, and the full potential, away from the origin, can be useful for cosmological applications. The

results are then analyzed in the limit of small supersymmetry breaking.

In the final chapter, pseudomoduli, which determine whether or not supersymmetry is broken, are studied. Types of pseudomoduli that arise when supersymmetry is dynamically broken in infrared-free low-energy theories are classified. It is shown that, even if the pseudomoduli potential is generated at higher loops, there is a regime where the potential can be determined from one-loop running data. In this regime, we compute whether the potential for various types of pseudomoduli is safe, has a dangerous runaway to the UV cutoff, or is incalculable.

Chapter 1

Introduction

In this chapter, I attempt to provide context for the work contained in the following chapters and background material for the lay-reader. Those who already know basic particle physics should skip to Section 1.2. Specialists should skip this introduction altogether.

1.1 The Basics

We do not fully understand how the universe works, and it is entirely possible that we never will. I find it encouraging, however, to think of the extraordinary range of phenomena that are described by a simple set of ideas. Before we discuss some of the mysteries that remain, and how the work in this dissertation fits in with the attempts to resolve them, it's worth taking stock of our successes.

A convenient way to summarize our current understanding is in terms of a set of basic building blocks (particles) and a set of rules for how they interact (forces). Depending on how you count, we know of about ten types of fundamental particles and four fundamental forces. That's it! Not surprisingly, gravity was the first force given a mathematical formulation. It's easy to take for granted, but of all the humans who have walked the Earth, only a tiny fraction have known that a common impetus is at work in the falling of objects, the ocean tides, and the relative motion of all celestial bodies. With Einstein's improvements upon the original theory of Newton, every gravitational experiment has been found to

agree with the theoretical predictions. Despite its successes, gravity remains the most mysterious of the forces. It is not known how gravity works when quantum mechanics is taken into account. The good news is that gravity is so weak that we've never needed to know how it works beyond the classical level. The bad news is that gravity is so weak that it's so far been impossible to probe beyond the classical level. Gravity doesn't play a central role in this dissertation, so I won't discuss it further here.

The next most familiar force is electromagnetism. Of course, it is not remotely obvious that there is a common origin for static cling and the affinity between a magnet and your refrigerator. That was a major breakthrough, which I'll touch on later. Perhaps more surprising is that *light* is also an electromagnetic phenomenon. And light gets us more than you might think because it looks quite different depending on its energy. This is true both literally and figuratively. Our eyes are able to detect a range of light energies, and it interprets them as colors. More-extreme energy ranges give us x-rays, microwaves, radiowaves, infrared radiation, and much more. Almost all technology can be given a precise mathematical formulation that is derived from the *classical* theory of electromagnetism and more basic physics. Armed with our theory of gravity and the *quantum* theory of electromagnetism, there are very few things that you or I will ever experience that can't be addressed. This is because quantum mechanics and electromagnetism are all we need to have an excellent quantitative understanding of atoms and how they interact. So, for example, we can explain how atoms stick together to form molecules, which stick together to form things like the chair I'm sitting in as I write this. Remarkably, we don't need anything new to explain all sorts of seemingly unrelated and exotic chemical reactions.

I want to conclude by listing a few things that we do *not* get from gravity and electromagnetism. One major piece of our world due to novel phenomena is the nucleus of the atom. For most practical applications, the details of the nucleus are completely irrelevant. It's only upon peering deep into the atom, that one finds a fascinating substructure that requires an explanation. It proves to be a new force and new fundamental particles at work. This nuclear force, known as the

strong force, is our third. The effects of the fourth force, known as the weak force, were first seen in a particular form of radioactivity. We will discuss some physics associated with weak interactions in more detail shortly. But there are still a few basic ingredients that have been neglected.

I mentioned quantum mechanics as a necessary element in describing atoms without explaining what quantum mechanics is. It stands out from the discussion so far as the only *principle* that I have named. Unlike forces, which act differently between different particles, quantum mechanics is a set of rules that apply universally[1]. The conservation of energy and momentum are more-familiar principles. The fact that the total energy and momentum of any isolated system doesn't change are integral in understanding a plethora of both macroscopic and microscopic phenomena. What's exciting about the conservation principles is that their origin is understood in terms of something more basic: symmetry. In fact, as we will discuss in the next subsection, the forces and particles also arise naturally from symmetries.

1.1.1 Introduction to Symmetries

Within the force paradigm, the particles can be organized into a bunch of "matter" particles, and a few special particles that "mediate" the force between them. There's a nice analogy for how this works. Imagine floating in space[2] with a fellow astronaut; let's call her Jane. You and Jane are both stationary, only a few yards apart. You might think that you could swim toward or away from her, but without any air molecules to push off of, there's no hope. What you can do is take the wrench off of your tool belt and throw it to your comrade. When you release it, you sail backwards (think of the recoil from a gun), and when Jane catches it, she gets pushed away from you. By exchanging this wrench, you and your friend have been repelled from one another. Similarly people talk about the repulsive force

[1] As I mentioned before, we still have the embarrassing issue of gravity, which we do not yet know how to reconcile with quantum mechanics.

[2] Older versions of this analogy tend to involve skidding on ice, but you need more qualifying statements to make that work. I typically instruct students to, "Assume everything necessary for this to make sense," but that doesn't seem appropriate here, so I'm going to ask the reader to imagine being in space, where things are simpler.

between electrons as arising though an exchange of photons (particles of light).

There are several problems with this analogy. One obvious shortcoming is that it doesn't seem to have anything to say about attractive forces. If you're clever, you can patch up some of the issues. For example, you can put the astronauts in a non-trivial space so that you can throw the wrench away from Jane, have it wrap abound the universe, and hit her in the back. To get at the more fundamental question of what particles and forces really are, however, a new paradigm is needed. This brings us to symmetries. In the analogy, the important piece of physics that we implicitly used was the conservation of momentum. So why is momentum conserved? Surprisingly, this turns out to be a consequence of the obvious fact that none of our conclusions would have changed if we had placed our astronauts a couple of parsecs to the left or right. Symmetry under translations implies the conservation of momentum.

Symmetry is the cornerstone of modern theoretical physics. Though we don't know exactly why we have the symmetries that we have, we know that we can basically get all of physics by proclaiming them to be true; what we see in nature is then everything that is allowed by the symmetries. In fact, you need some rules besides "what is is what can be", and there are still some things that aren't fixed by the rules and symmetries that we know about, but it seems to be a pretty powerful concept, so it's worth spending some time to understand more about symmetry. Intuitively we have some sense of what a symmetry is, but it's useful to start with a simple example to give a sense for how physicists deal with symmetries.

Consider a reflection symmetry. Lots of things are approximately symmetric under reflection. Most people, for example, don't look very different from their mirror images. When considering symmetries, it's useful to identify the *transformations* associated with them and the *objects* on which these transformations act. In our example, we say that the object (a person) is reflection symmetric if it looks the same after applying the reflection transformation (exchanging left and right). We can also understand the math associated with this example. Given a function of x, we can say that it is *invariant under reflection* if it doesn't change when we make the transformation, $x \to -x$. An example of such a function is

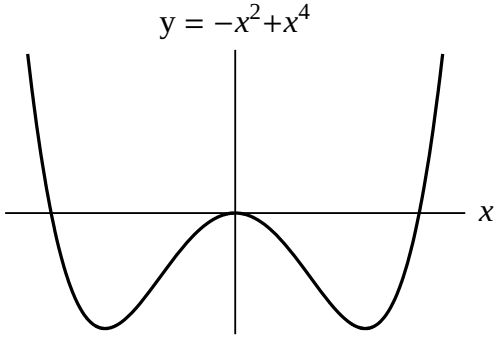


Figure 1.1: This function has the property that it is invariant under reflection through the vertical axis.

$y = -x^2 + x^4 = -(-x)^2 + (-x)^4$, see Figure 1.1. This is a physically relevant example that I'll discuss more later, but this sort of symmetry is among the least interesting.

There are two types of symmetries that are more interesting and important. The first is a gauge symmetry. The gauge symmetry determines the form of the interactions among particles – basically the gauge symmetry defines the force. There's much more that I could write, but it's the other sort of symmetry, one of space-time, that is central to what follows, so I'll leave it at that for now. The basic space-time symmetries are associated with transformations of where something is, how it's moving, or how it's oriented. I mentioned earlier that you get conservation of momentum when it doesn't matter where you are. Similarly, if it doesn't matter *when* something happens (symmetry under *time* translations) you get conservation of energy. The equivalence of electric and magnetic phenomena is also a consequence of a space-time symmetry.

Space-time symmetries also tell us what sorts of particles we can have. It turns out that we can make sense of five types of particles[3], which are described as having spin 0, 1/2, 1, 3/2, or 2. The reasons for calling it "spin" and for the choice of numbering scheme aren't important. What's important is that these particles have very different properties. In fact, the particles with integer spin (0, 1, ...) and

[3]Earlier I referred to ten types of particles. There I was grouping them by how they transform under gauge transformations. Here I'm grouping them by how they transform under Lorentz (space-time) transformations.

those with non-integer spin (1/2, 3/2, ...) are so different from each other that these two groups have special names; the former are called bosons and the latter are called fermions. So far, the particles that we have detected and believe to be fundamental all have either spin 1/2 (the matter particles from above) or spin 1 (the force mediators)[4].

1.1.2 Introduction to Supersymmetry

Supersymmetry is a peculiar form of space-time symmetry. A convenient way to think of it is in terms of an expansion of space-time to include a set of "fermionic" coordinates $(\theta_1, \theta_2, \bar{\theta}_1, \bar{\theta}_2)$ to go along with the ordinary "bosonic" ones (t, x, y, z). Mathematically, being fermionic means that there is a sign change when one of these things is moved passed another, $\theta_1\theta_2 = -\theta_2\theta_1$. For ordinary numbers, this is only true if both sides of this equation are zero. These are *not* ordinary numbers. While hard to interpret physically, at the end of the day, the theories that we build out of them look essentially like ordinary non-supersymmetric theories. The basic difference is that there are tighter constraints on the parameters in the theory, like particle masses, and on what particles are present. We shouldn't be surprised that the particle types are affected because we already know that space-time symmetries govern what sorts of particles there can be. In fact, this is manifest if we think about supersymmetry in another way, which I'll describe now.

It's important to understand what the transformation associated with this symmetry is. I went to the trouble of introducing the terminology, "boson" and "fermion," just for this purpose. The remarkable thing about supersymmetry is that the transformation defining it takes bosons into fermions and vice versa. This predicts, for example, that in addition to the electron (spin 1/2), there should be a "selectron" (spin 0) that transforms in the same way under gauge transformations. We have not observed this partner particle. In fact, we have not observed the supersymmetric partner of any of the known particles, so we have to ask why we think this might be a symmetry of the real world.

[4]The mediator of the gravitational force, the graviton, is a spin 2 particle, but this is not yet an experimental fact, and I'm not supposed to be discussing gravity, so let's not worry about this.

In fact, if we stick to the paradigm that nature is simply what is allowed, we should be asking why we can't have supersymmetry. There's actually a theorem [1] that makes some conservative assumptions[5] and concludes that the only sorts of symmetries that we can have are the ones we know about and supersymmetry. This is enough to warrant careful study of supersymmetric field theories, but it does not give a reason to expect any sign of supersymmetry at an accessible energy scale. This is the subject of the next subsection.

1.1.3 Low-energy Physics and Supersymmetry

I doubt we would have made any progress in physics if it weren't for the simple fact that we don't need to know everything to know something. We've already discussed an example of this: the atom. The details of the nucleus aren't important for ordinary processes, that is, processes that don't penetrate deep into the atom[6]. This is why we were taught in chemistry just to look at the outermost electrons; the issue of exactly what the extremely tiny lump of matter at the center of the atom is, while important and fascinating, can be set aside while you're trying to figure out why salt forms pretty crystals. Similarly, it doesn't matter that billiard balls are made of atoms if you just want to know how they will bounce off of each other. It's good enough just to know the net masses and that the balls are pretty close to being spherical and rigid.

The Standard Model of particle physics, the most fundamental and successful physical theory in history, is a lot like this. We have a fantastic description of the world up to the energies so far accessed in particle colliders, but we know it fails at some scale. I already told you that it doesn't do gravity. That's a serious issue, but it's probably not the first one we'll have to deal with. Keep in mind that the only reason we feel the effects of gravity is because we're stuck to a giant rock made up

[5]The assumptions have to do with large scale scattering properties, the mass spectrum, and a bit of group theory. It's healthy to be skeptical of theorems written by physicists. The original version, which predates supersymmetry, did not include supersymmetry (graded Lie algebras were not considered). In any case, if there's still a loophole, it will only expand the possibilities. Supersymmetry is definitely OK.

[6]This might be intuitively obvious if the schematic drawing of the atom that we saw in elementary school had been drawn to scale. It's understandable why it wasn't, though; the atom would be roughly the size of the whole school if the nucleus were the size of the tip of a pencil.

of something like 10^{50} atoms all pulling on us; it's actually an extremely weak force, so in interactions among small numbers of particles, gravity is negligible. To see its effects in a particle physics experiment, we would have to be able to differentiate length scales on the order of about 10^{-35} meters. To get a sense for what this means, suppose I were to stretch all of my length scales, so the gravity scale is just visible, about 1 millimeter. In these rescaled units, our best experiments to date would be on the order of the diameter of the solar system. We aren't close, so it would be surprising if there were nothing interesting in the interval.

One reason to suspect that we need some new degrees of freedom before getting to very high energy scales (or small length scales) is for the sake of unifying the (non-gravitational) forces. The "constants" that parameterize the strength of these forces vary with energy, and our measurements to date indicate that, though they are quite different at low energies, extrapolating to higher energies shows that they *nearly* meet. If we redo the calculation, putting in supersymmetry partner particles like the selectron, we can do much better. We should make clear that there isn't a lot of freedom in how you do this, it could easily have turned out that supersymmetry makes things worse.

Another unappealing feature of the Standard Model is the so-called "hierarchy problem". The basic issue is that we have experimentally detected an important energy scale and we don't know what sets it. This is known as the weak scale because it's related to the weak force, which I mentioned earlier. We understand the origin of the nuclear scale, and we have a much higher scale associated with gravity that is fundamental, but the weak scale seems to be arbitrary. Another disturbing aspect of this problem is that the input scale seems to be tremendously different from the physically relevant output scale. It's as if someone had won \$65,798,970 in the lottery to bring their net worth to \$100. It's easy to explain; they were \$65,798,870 in debt. But it makes you wonder if there's actually some sort of conspiracy here. The Standard Model seems to exhibit a coincidence of this sort. It could be that there really is a cancellation like this, but since it is unknown small-scale physics that determines the size of the debt (and thus the jackpot), there is a lot of effort being put into developing extensions of the

Standard Model that do not have this aesthetic deficiency. Supersymmetry is a popular component of such attempts because in supersymmetry, you're born poor and you die poor. The theory doesn't produce large corrections to the input value. The issue of initially setting the scale (why $100?) requires some more effort, but supersymmetry seems well-suited to address this issue as well.

I'll conclude this subsection by reminding the reader that experiment has the last word. The aesthetic issues are annoying and they may be important clues, but they don't demand new physics. Until the Standard Model fails to accurately predict the result of an experiment, it reigns as king. What makes this such an exciting time to be a physicist is that there appears to be strong evidence of something new. Astrophysical observations indicate that most of the mass in the universe belongs to something we call dark matter. For several years, evidence has been mounting that dark matter is real, it is made up of particles, and it is largely comprised of stuff not included in the Standard Model. Supersymmetry naturally contains particles with the strange properties inferred about dark matter.

The goal of this subsection was not to convince you that supersymmetry is real and is going to solve all of the mysteries of the universe. The goals were to point out that we don't yet know everything, to give a sample of some of the problems that people are worried about, and to provide a taste of why supersymmetry is so prevalent among models of new physics. In the work to be presented, you will not find attention paid to any particular model. Nor will you find any detailed phenomenological analysis. Instead, an effort is made to understand some of the more general aspects of the dynamics of supersymmetry, supersymmetry breaking, and how it might affect low-energy physics. In the next section, some more details of the specific issues relevant to this dissertation are discussed, and the basic results obtained are summarized.

1.2 Outline

1.2.1 Exact Results in Superconformal Theories

Chapter 2 has to do with the dynamics of supersymmetric field theories. I mentioned previously the curious fact that the coupling constants of a theory vary with energy scale. You can measure the charge of the electron, for example, by shooting electrons and positrons together and measuring the properties of the electrons and positrons that come out. It has been found (see [2], for example, and references therein) that the result varies – exactly as predicted by the theory – depending on how fast the beams of particles are going relative to one another when they collide. In fact, in a general theory, several things vary as the scale is varied. Understanding this evolution is necessary for extracting accurate predictions from theories, but it has also consistently played a central role in illuminating important conceptual issues. The stand-out example is the discovery of the last major theoretical piece of the Standard Model. A particular model attempting to explain the nature of the strong force was brought to the forefront with the discovery that its coupling constant grows large at short distances and weak at large distances [3, 4]. This model explains the strong force, the tremendously strong force holding the nucleus together that is utterly negligible far from the nucleus. Supersymmetry has proved to be a rich and fascinating playground for studying dynamical phenomena like this.

One of the advantages of studying supersymmetric theories is that they can, on occasion, provide non-trivial exact results. In the following chapter, we further specialize our study to superconformal theories; in these theories, all evolution has ceased. This may seem contrary to the goal of understanding the dynamics of theories, but much can be learned because a general theory can be viewed as a perturbed conformal theory, which evolves into a different conformal theory as we go low energies. The precise problem addressed here is that of determining the exact superconformal R-symmetry. This symmetry is particularly interesting because it gives exact formulas for some of the most basic and useful data of a theory, the dimensions of operators.

You might wonder how it is that a symmetry could hide. The problem arises when there are additional symmetries with which the R-symmetry can mix. Another bad analogy is in order. Imagine a croquet ball. In case you aren't familiar with croquet balls, I'll describe the features vital to the analogy. First, they are spherical. Less well-known but equally important for us is the fact that they tend to be striped so that there is exactly one axis of rotational symmetry. If we ignore the stripes, we can rotate about any axis through the origin of the ball without changing things. If we choose a set of three independent axes without looking at the stripes, odds are we won't have chosen the magical axis but it will be given by a linear combination of our axes. Until recently, it was only known that the stripes existed; a method for determining where they are on the ball was not known.

In the next chapter, a new constraint that determines this symmetry will be presented. This new constraint has the advantages that it is not restricted to four dimensions, as the original method [5] is, and it gives a dual description of a method developed in a supergravity theory [6]. Unfortunately, it would take us too far afield to give a proper discussion of this duality (see [7] for a review). In a sentence, it has been found that certain manifestations of string theory in a certain limit are exactly equivalent to certain manifestations of ordinary field theory in a certain limit. It is highly nontrivial that you can calculate quantities in a theory involving gravity and get the same answer that you get from a calculation in a theory in fewer dimensions that does not include gravity. It is in the context of this duality that this new method comes into its own. In practice, the method of [5] is usually the more powerful alternative because it yields the exact result with a minimal effort. The new method that will be presented here generally only gives an approximation.

1.2.2 Exploring Gauge Mediation

The second part of the thesis has to do with broken supersymmetry. Supersymmetry is known not to be a symmetry of the vacuum in which we live. If it were, there would be a selectron, for example, with the same mass as the electron, which we surely would have detected by now. This does not mean, however, that

our universe does not descend from a supersymmetric theory. The idea is simple. Take another look at Figure 1.1. Viewed in a mirror, it would look the same; it is symmetric under reflection. Now imagine placing a ball on the peak in the center. It's intuitively clear that this is an unstable configuration; it's still reflection symmetric, but it's unstable. The ball is going to roll down to one of the valleys, and once it does, the symmetry is going to be broken; left is now distinct from right. This is an example of spontaneous symmetry breaking, and it is known to occur in nature in a variety of contexts. It is believed that the state of our universe was determined by essentially the same mechanism that determined the location of the ball in our analogy.

Non-supersymmetric vacua arise naturally in supersymmetric theories. Unfortunately, no one has ever found a very compelling supersymmetric model that leads to the observed low-energy physics. It's not difficult to write down a supersymmetric extension of the Standard Model, but it's very difficult to come up with one that doesn't have significant aesthetic deficiencies, which, in the absence of data, has been the guiding principle. With or without aesthetic deficiencies, understanding how such models would appear in data is an important problem to study. This is the general topic of chapters 3 and 4. In particular, we'll consider a broad class of models characterized by how supersymmetry breaking is communicated to the experimentally accessible particles, dubbed "the visible sector".

Recall that when supersymmetry is unbroken, every particle has a partner with the exact same mass. When supersymmetry is broken, the masses of some particles change and this ceases to be true. These particles whose masses change are called messengers because by interacting with other particles, they change the masses of those particles, leading to further adulteration of these partnerships. The nature of this process by which supersymmetry breaking is communicated to the visible sector is a useful way of categorizing different classes of models. The class that we will discuss is known as "gauge mediation". In gauge mediation, the messengers interact with the observable sector only through gauge interactions.

Much of the appeal of gauge mediation is due to the fact that experimental constraints are rather easily evaded in this scenario. For example, there are certain

meson mixing experiments that have given extremely precise results. A meson is not a fundamental particle; it's made out of quarks, which are the same things that make up nuclei. The experiments of interest involve mesons with a certain quark composition going in and mesons with a very different quark composition coming out. These are known as "flavor changing" processes.[7] In the standard model, the probability of some of these rare events is found to be roughly proportional to a difference of masses of two flavors of quarks divided by the weak scale; this gives a small number. The problem that can arise in supersymmetric theories is that in an analogous calculation with squarks, the partner particles of the quarks, the weak scale in the denominator is no longer large compared to the masses in the numerator. In other words, the squarks are much heavier than the quarks, so the probability of these processes can be large, which would directly conflict with experiment. In gauge mediation, we have a way out because the squarks get their masses through gauge interactions; the ones that differ only by flavor have very nearly equal masses. Since the probability of flavor changing processes depends crucially on a difference of flavor masses, we don't expect these new particles to significantly change the (experimentally confirmed!) Standard Model prediction.

This isn't to say that gauge mediation is without its shortcomings. One has to invent a reason for the messengers not to communicate directly with the observable sector, for example. And there are mass scales in the theory that are not automatically what we would want them to be. A different sort of problem – the one that is the focus of some of the work in this dissertation – is the lack of a full parameterization of its potential low-energy effects. An important step toward filling in the gaps was made in "General Gauge Mediation" [8]. It was pointed out that, with a few assumptions, the low-energy physics could be expressed in terms of a few pieces of data from the supersymmetry-breaking "hidden sector" of the theory. Some of the work in this dissertation is devoted to whittling down the set of assumptions. In Chapter 3, the masses of visible-sector particles, like the squarks, are explicitly computed for a large class of hidden-sector models and with

[7] One of the great mysteries is why we have three copies of all of the observed matter particles, including quarks. These different "flavors" have different masses, but otherwise have the same basic properties.

a generalized set of allowed gauge interactions. In Chapter 4, this generalization of the gauge sector is applied within the formalism of General Gauge Mediation. Note that there is a trade-off here in that the hidden sector is allowed to be general, but we have less-explicit predictions; the result now has more free parameters. We will see how a more constrained result emerges for a typical form of supersymmetry breaking.

Finally, in Chapter 5, we will study the other side of the problem of supersymmetry breaking. In Chapters 3 and 4, we take advantage of the fact that nature permits simple effective descriptions of complicated phenomena to study the potential impact of supersymmetry on low-energy physics without concerning ourselves with the details of the high-energy supersymmetric theory or how it is broken. In the final chapter, the issue of supersymmetry breaking takes center stage. A set of tools is developed to address a practical obstacle encountered in studying models of supersymmetry breaking. The basic problem arises in trying to demonstrate the existence of a stable non-supersymmetric vacuum. In our example of spontaneous symmetry breaking (see Figure 1.1), the symmetric point in the center was unstable, but in each valley, where the symmetry is broken, we have stability. In general, the analogous function is not so simple. First of all, we know longer have just one variable to worry about. If we think again of the geometric analogy with the ball, in that case the ball was either going to go left or right. If we go up a dimension, it now has the options of going into the page or out of the page. In practice, one has several directions to worry about, which is not a problem on its own. The problem is that in some of these directions, the surface is flat in the first approximation. To see if they curve up like you want them to, you need to do a harder calculation. Sometimes the shape of the surface still isn't reveal in the next easiest calculation or the one after that. One doesn't need to look far to find examples in which the required calculation is entirely impractical to perform. Our methods reveal the fate of these nearly flat directions in a region of this space. No tedious calculations are required, and the result, though not applicable everywhere, is often sufficient to answer the question at hand.

Chapter 2

The Exact Superconformal R-symmetry Minimizes τ_{RR}

2.1 Introduction

Our interest here will be in the coefficients τ_{IJ} of two-point functions of globally conserved currents J_I^μ (I labels the various currents) in d-dimensional CFTs:

$$\langle J_I^\mu(x) J_J^\nu(y)\rangle = \frac{\tau_{IJ}}{(2\pi)^d}(\partial^2\delta^{\mu\nu} - \partial^\mu\partial^\nu)\frac{1}{(x-y)^{2(d-2)}}. \tag{2.1}$$

The general form (2.1) of the correlator is completely fixed by conformal invariance, with the specific dynamics of the theory entering only in the coefficients τ_{IJ}. Unitarity restricts τ_{IJ} to be a positive matrix (positive eigenvalues). For 4d CFTs, τ_{IJ} give [9, 10] the violation of scale invariance, $\langle T^\mu_\mu\rangle = \frac{1}{4}\tau_{IJ}(F^I)_{\mu\nu}(F^J)^{\mu\nu}$, when the global currents are coupled to background gauge fields.

We'll here consider field theories with four supercharges: $\mathcal{N} = 1$ in 4d, and $\mathcal{N} = 2$ in 3d (one could also consider $\mathcal{N} = (2,2)$ in 2d), and their renormalization group fixed point SCFTs (where there are an additional four superconformal supercharges). The stress tensor of these theories lives in a supermultiplet $T_{\alpha\dot\beta}(x,\theta,\overline\theta)$ (in 4d Lorentz spinor notation; for $d < 4$ the dot on $\dot\beta$ is unnecessary), which also contains a $U(1)_R$ current – this is "the superconformal $U(1)_R$ symmetry". Supersymmetry relates this current and its divergence to the dilitation current and its divergence. The scaling dimension of chiral operators are related to their

superconformal $U(1)_R$ charge by

$$\Delta = \frac{d-1}{2} R. \tag{2.2}$$

For a chiral superfield, writing $\Delta = \frac{1}{2}d - 1 + \frac{1}{2}\gamma$, with γ the anomalous dimension, (2.2) yields

$$R = \frac{d-2}{d-1} + \frac{1}{d-1}\gamma. \tag{2.3}$$

There are often additional non-R flavor currents, whose charges we'll write as F_i, with i labeling the flavor symmetries. In superspace, these currents reside in a different kind of supermultiplet, which we'll write as $J_i(x, \theta, \overline{\theta})$. When there are such additional flavor symmetries, the superconformal $U(1)_R$ of RG fixed point SCFTs can not be determined by the symmetries alone, as the R-symmetry can mix with the flavor symmetries. Some additional dynamical information is then needed to determine precisely which, among all possible R-symmetries, is the superconformal one, in the $T_{\alpha\dot{\beta}}$ supermultiplet.

We will here present a new condition that, in principle, completely determines which is the superconformal $U(1)_R$. We write the most general possible trial R-symmetry as

$$R_t = R_0 + \sum_i s_i F_i, \tag{2.4}$$

where R_0 is any initial R-symmetry, and F_i are the non-R flavor symmetries. The subscript "t" is for "trial", with the s_i arbitrary real parameters. The superconformal R-symmetry, which we'll write as R without the subscript, corresponds to some special values s_i^* of the coefficients in (2.4), that we'd like to determine, $R = R_t|_{s_j = s_j^*}$.

As we'll discuss, the fact that the superconformal R-symmetry and the non-R flavor symmetries reside in different kinds of supermultiplets, implies that their current-current two-point function necessarily vanishes, $\langle J_R^\mu(x) J_{F_i}^\nu(y)\rangle = 0$, i.e.

$$\tau_{Ri} = 0 \qquad \text{for all non-R symmetries } F_i. \tag{2.5}$$

This condition uniquely characterizes the superconformal R-symmetry among all possibilities (2.4). To see this, use (2.4) to write (2.5) as

$$0 = \tau_{Ri} = \tau_{R_t i}|_{s_j = s_j^*} = \tau_{R_0 i} + \sum_j s_j^* \tau_{ij} \quad \text{for all i.} \tag{2.6}$$

Here $\tau_{R_0 i}$ is the coefficient of the $\langle J^\mu_{R_0}(x) J^\nu_{F_i}(y)\rangle$ current-current two-point function of the currents for R_0 and F_i, and τ_{ij} is the coefficient of the $\langle J^\mu_{F_i}(x) J^\nu_{F_j}(y)\rangle$ of the current-current two-point function for the non-R flavor symmetries F_i and F_j. The conditions (2.6) is a set of linear equations which uniquely determines the s_j^*, if the coefficients $\tau_{R_0 i}$ and τ_{ij} are known. Unitarity implies that the matrix τ_{ij} is necessarily positive, with non-zero eigenvalues, so it can be inverted, and the solution of (2.6) is

$$s_j^* = -\sum_i (\tau^{-1})_{ij} \tau_{R_0 i}. \tag{2.7}$$

The conditions (2.6) can be phrased as a minimization principle: *the exact superconformal R-symmetry is that which minimizes the coefficient $\tau_{R_t R_t}$ of its two-point function among all trial possibilities (2.4).* Using (2.4), the coefficient of the trial R-current R_t two-point function is a quadratic function of the parameters s_j:

$$\tau_{R_t R_t}(s) = \tau_{R_0 R_0} + 2\sum_i s_i \tau_{R_0 i} + \sum_{ij} s_i s_j \tau_{ij}. \tag{2.8}$$

Our result (2.5) implies that the exact superconformal R-symmetry extremizes this function,

$$\frac{\partial}{\partial s_i} \tau_{R_t R_t}(s)|_{s_j = s_j^*} = 2\tau_{Ri} = 0. \tag{2.9}$$

The unique solution of (2.9) is a global minimum of the function (2.8) since

$$\frac{\partial^2}{\partial s_i \partial s_j} \tau(s) = 2\tau_{ij} > 0, \tag{2.10}$$

with the last inequality following from unitarity.

The value of $\tau_{R_t R_t}$ at its unique minimum is the coefficient τ_{RR} of the superconformal R-current two-point function. As is well known, supersymmetry relates this to the coefficient, "c", of the stress tensor two-point function, $\tau_{RR} \propto c$; as we'll discuss, the proportionality factor is

$$\tau_{RR} = \frac{(2\pi)^d}{d(d^2-1)(d-2)} C_T \qquad \text{or, for } d=4, \quad \tau_{RR} = \frac{16}{3} c. \tag{2.11}$$

τ_{RR} minimization immediately implies some expected results. For non-Abelian flavor symmetry, (2.5) is automatically satisfied for all flavor currents with traceless generators, if the superconformal R-symmetry is taken to commute with these

generators. This shows, as expected, that the superconformal R-symmetry does not mix with such non-Abelian flavor symmetries. Similarly, (2.5) is automatically satisfied by any baryonic flavor currents which are odd under a charge conjugation symmetry, taking the superconformal $U(1)_R$ to be even under charge conjugation. So, as expected, the superconformal $U(1)_R$ does not mix with baryonic symmetries which are odd under a charge conjugation symmetry.

As a simple example of τ_{RR} minimization, consider a single, free, chiral superfield Φ in d spacetime dimensions. The R-symmetry can mix with a non-R $U(1)_F$ flavor current, under which Φ has charge 1 (the "Konishi current"). Write the general trial R-charges for the scalar and fermion components as $R(\phi) = R_t$, $R(\psi) = R_t - 1$. As we'll review, the free field two-point function of this R-current is

$$\tau_{R_t R_t} = \frac{\Gamma(\frac{d}{2})^2 2^{d-2}}{(d-1)(d-2)} \left(\frac{1}{d-2} R_t^2 + (R_t - 1)^2 \right) \tag{2.12}$$

with the two terms the scalar and fermion contributions. Taking the derivative w.r.t. R_t,

$$\tau_{R_t F} = \frac{1}{2} \frac{d}{dR_t} \tau_{R_t R_t} = \frac{\Gamma(\frac{d}{2})^2 2^{d-2}}{(d-1)(d-2)} \left(\frac{R_t}{d-2} + R_t - 1 \right). \tag{2.13}$$

Requiring $\tau_{RF} = 0$ then gives the correct result (2.3), with anomalous dimension $\gamma = 0$, for a free chiral superfield in d spacetime dimensions.

The above considerations all apply independent of space-time dimension; they are equally applicable for 4d $\mathcal{N} = 1$ SCFTs as with 3d $\mathcal{N} = 2$ SCFTs. For 4d $\mathcal{N} = 1$ SCFTs, there is already a known method for determining the superconformal R-symmetry: a-maximization [5]. It was shown in [5] that the s_i^* can be determined by a-maximization, maximizing w.r.t. the s_i in (2.4) the combination of 't Hooft anomalies

$$a_{trial}(R_t) = \frac{3}{32}(3\mathrm{Tr}R_t^3 - \mathrm{Tr}R_t), \tag{2.14}$$

(where we decided here to include the conventional normalization prefactor). For example, for a free 4d chiral superfield we locally maximize the function

$$a_{trial}(R_t) = \frac{3}{32}(3(R_t - 1)^3 - (R_t - 1)). \tag{2.15}$$

The local maximum of (2.15) is at $R = 2/3$, which indeed coincides with the global minimum of (2.12), but it's illustrative to see how the functions themselves differ.

a-maximization in 4d is much more powerful than $\tau_{R_t R_t}$ minimization, because one can use the power of 't Hooft anomaly matching to exactly compute $a_{trial}(R_t)$ (2.14), whereas the current two-point functions $\tau_{R_0 i}$ and τ_{ij} needed for $\tau_{R_t R_t}$ minimization receive quantum corrections. Actually, once the exact superconformal R-symmetry is known, there is a nice way to evaluate τ_{ij} in terms of 't Hooft anomalies [11]:

$$\tau_{ij} = -3\mathrm{Tr} R F_i F_j, \tag{2.16}$$

as we'll review in what follows. (The result (2.16) generally can not be turned around, and used as a way to determine the superconformal $U(1)_R$, because plugging (2.4) in (2.16) can not always be inverted to solve for the s^*.)

In the context of the AdS/CFT correspondence, the criterion (2.6) for determining the superconformal R-symmetry becomes more useful and tractable, because the AdS duality gives a weakly coupled dual description of $\tau_{R_0 i}$ and τ_{ij}: these quantities become the coefficients of gauge field kinetic terms in the AdS bulk [12]. As discussed in a separate paper [13], these coefficients are computable by reducing SUGRA on the corresponding Sasaki-Einstein space. It is shown in [13] that the conditions (2.6) are in fact equivalent to the "geometric dual of a-maximization" of Martelli, Sparks, and Yau [6].

There is no known analog of a-maximization for 3d $\mathcal{N} = 1$ SCFTs, and in 3d there is no useful analog of 't Hooft anomalies and matching (aside from a Z_2 parity anomaly matching [14]). $\tau_{R_t R_t}$ minimization gives an alternative to a-maximization in 4d, which applies equally well to 3d $\mathcal{N} = 2$ SCFTs.

a-maximization in 4d ties the problem of finding the superconformal $U(1)_R$ together with Cardy's conjecture [15], that the conformal anomaly a counts the degrees of freedom of a quantum field theory, with $a_{UV} > a_{IR}$ and $a_{CFT} > 0$. The result that a is maximized over its possibilities implies that relevant deformations decrease a [5], in agreement with Cardy's conjecture. Unfortunately, we have not gained any new insights here into general RG inequalities from our τ_{RR} minimization result. Indeed, τ_{RR} is related to the conformal anomaly c in 4d,

which is known to not have any general behavior, neither generally increasing nor generally decreasing, in RG flows to the IR. And there is no analogous argument to that of [5], to conclude that τ_{RR} generally increases in RG flows in the IR, from the fact that τ_{RR} is minimized among all possibilities: the quantum corrections to τ_{RR}, coming from the relevant interactions, can generally have either sign. (The difference is that the argument of [5] was based on 't Hooft anomalies, which do not get any quantum corrections for conserved currents).

Our τ_{RR} minimization result applies for SCFTs at their RG fixed point. It would be interesting to extend τ_{RR} minimization to study RG flows away from the RG fixed point. Perhaps this can be done by using Lagrange multipliers, as in [16], to impose the constraint that one minimize only over currents that are conserved by the relevant interactions.

2.2 Current two point functions; free fields and normalization conventions

Two point functions of currents and stress tensors for free bosons and fermions in d-spacetime dimensions were worked out, e.g. in [17]. To compare with [17], rewrite (2.1) as

$$\langle J_I^\mu(x) J_J^\nu(y)\rangle = \tau_{IJ} \frac{2(d-1)(d-2)}{(2\pi)^d} \frac{I_{\mu\nu}(x-y)}{(x-y)^{2(d-1)}}, \tag{2.17}$$

with $I_{\mu\nu}(x) \equiv \delta_{\mu\nu} - 2x_\mu x_\nu (x^2)^{-1}$. The normalization conventions of [17] is

$$\langle J_\mu(x) J_\nu(0)\rangle = \frac{C_V}{x^{2(d-1)}} I_{\mu\nu}(x), \qquad \langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \frac{C_T}{x^{2d}} I_{\mu\nu,\sigma\rho}(x), \tag{2.18}$$

with $I_{\mu\nu,\sigma\rho}(x) = \frac{1}{2}(I_{\mu\sigma}(x) I_{\nu\rho}(x) + I_{\mu\rho}(x) I_{\nu\sigma}(x)) - d^{-1}\delta_{\mu\nu}\delta_{\sigma\rho}$. Thus $C_V = 2\tau(d-1)(d-2)/(2\pi)^d$. With these normalizations, the coefficients (2.18) for a single complex scalar are

$$C_V = \frac{2}{d-2}\frac{1}{S_d^2}, \qquad C_T = \frac{2d}{d-1}\frac{1}{S_d^2}, \tag{2.19}$$

where $S_d \equiv 2\pi^{\frac{1}{2}d}/\Gamma(\frac{1}{2}d)$ and the current was normalized to give ϕ and ϕ^* charges ± 1. The coefficients for a free fermion having the same number of components as a

4d complex chiral fermion (half the components of a Dirac fermion) the coefficients are

$$C_V = 2\frac{1}{S_d^2}, \qquad C_T = d\frac{1}{S_d^2} \tag{2.20}$$

(we don't have the factors of $2^{d/2}$ of [17], because we're here considering a fermion with the same number of components as the dimensional reduction of a 4d chiral fermion for all d).

More generally, let current $J_I(x)$ give charges $q_{I,b}$ to the complex bosons and charges $q_{I,f}$ to the chiral fermions. Using (2.19) and (2.20), we have

$$\tau_{IJ}^{\text{free field}} = \frac{\Gamma(\frac{d}{2})^2 2^{d-2}}{(d-1)(d-2)} \left(\frac{1}{d-2} \sum_{\text{bosons } b} q_{I,b} q_{J,b} + \sum_{\text{fermions } f} q_{I,f} q_{J,f} \right). \tag{2.21}$$

In particular, for a $U(1)_R$ symmetry, this gives (2.12). For $d = 4$, $\Gamma(d/2)^2 2^{d-2}/(d-1)(d-2) = 2/3$, so e.g. a 4d $U(1)_F$ non-R symmetry which assigns charge q to a single chiral superfield has $\tau_{FF}^{free field} = q^2$.

2.3 Supersymmetric field theories

Supersymmetry relates the superconformal R-symmetry to the stress tensor: both reside in the supercurrent supermultiplet

$$T_{\alpha\dot\alpha}(x,\theta,\overline\theta) \sim J_{R,\alpha\dot\alpha}(x) + S_{\alpha\dot\alpha\beta}(x)\theta^\beta + \overline S_{\alpha\dot\alpha\dot\beta}(x)\overline\theta^{\dot\beta} + T_{\alpha\dot\alpha\beta\dot\beta}(x)\theta^\beta\overline\theta\dot\beta + \dots, \tag{2.22}$$

whose first component is the superconforal $U(1)_R$ current and whose $\theta\overline\theta$ component is the stress energy tensor (we're omitting numerical coefficients here). Our notation is for the 4d case; similar results hold for 3d $\mathcal{N} = 2$ theories, with $\overline\theta^{\dot\alpha}$ replaced with a second flavor of θ^α. For superconformal theories, the stress tensor is traceless, and the superconformal R-current is conserved. As discussed in [18], the supercurrent two-point function is then of a completely determined form, with the only dependence on the theory contained in a single overall coefficient C:

$$\langle T_{\alpha\dot\alpha}(z_1) T_{\beta\dot\beta}(z_2)\rangle = C\frac{(x_{1\bar2})_{\alpha\dot\beta}(x_{2\bar1})_{\beta\dot\alpha}}{(x_{2\bar1}^2 x_{1\bar2}^2)^{d/2}}; \tag{2.23}$$

see [18] for an explanation of the superspace notation in (2.23).

Expanding out (2.23) in superspace, the LHS includes both the R-current two-point function and the stress-tensor two-point function. So (2.23) shows that the coefficient $C \propto \tau_{RR}$, and also $C \propto C_T$, and so it follows that $\tau_{RR} \propto C_T$. We could determine the precise coefficients in these relations by being careful with the coefficients in (2.22) and in expanding both sides of (2.23); instead we will fix these universal proportionality factors by considering the particular example of a free chiral superfield. Using (2.19) and (2.20) to get C_T, and comparing with the free-field value of τ_{RR} computed from (2.21), gives the general proportionality factor that we quoted in (2.11); e.g. for $d = 3$ it's $\tau_{RR} = \pi^3 C_T/3$. In 4d, $C_T \propto c$, one of the conformal anomaly coefficients, and the proportionality can again be fixed by considering the case of a free 4d $\mathcal{N} = 1$ chiral superfield, for which $c = 1/24$ and (2.21) gives $\tau_{RR} = 2/9$ (or a free 4d $\mathcal{N} = 1$ vector superfield, for which $c = 1/8$ and (2.21) gives $\tau_{RR} = 2/3$); this gives the relation quoted in (2.11).

The non-R global flavor currents $J_i^\mu(x)$ are the $\theta^\alpha \overline{\theta}^{\dot{\alpha}}$ components of superfields $J_i(x, \theta, \overline{\theta})$, whose first component is a scalar. We can write their two-point functions in superspace [18], with the coefficients given by that of the flavor current correlators, τ_{ij}:

$$\langle J_i(z_1) J_j(z_2) \rangle = \frac{\tau_{ij}}{(2\pi)^d} \frac{1}{(x_{\overline{2}1}^2 x_{\overline{1}2}^2)^{(d-2)/2}}. \tag{2.24}$$

In general d dimensional CFTs, two-point functions of primary operators vanish unless the operators have conjugate Lorentz spin and the same operator dimension. Noting that the first component of the supermultiplet (2.22) has dimension $\Delta(T_{\alpha\dot{\beta}}) = d - 1$, and the first component of the current J_i has dimension $\Delta(J_i) = d - 2$ (since the $\theta^\alpha \overline{\theta}^{\dot{\alpha}}$ component is the current, with dimension $d - 1$), the two-point function of the first components of these two different supermultiplets must vanish. Because there is no non-trivial nilpotent invariant for two-point functions [18], this implies that two-point function of the entire supermultiplets must vanish:

$$\langle T_{\alpha\dot{\alpha}}(z_1) J_i(z_2) \rangle = 0. \tag{2.25}$$

I.e. the two-point function of any operator in the $T_{\alpha\dot{\alpha}}$ supermultiplet and any operator in the J_i supermultiplet vanishes; in particular, this implies that the two-

point function of the superconformal $U(1)_R$ current and all non-R flavor currents necessarily vanish, $\tau_{RF_i} = 0$. We thus have the general result (2.5), and this same argument applies equally for $d = 4$ $\mathcal{N} = 1$ as well as lower dimensional SCFTs with the same number of supersymmetries.

The superspace version of an anomaly in the dilatation current is

$$\overline{\nabla}^{\dot{\alpha}} T_{\alpha\dot{\alpha}} = \nabla_\alpha L_T, \tag{2.26}$$

with L_T the trace anomaly, which is the variation of the effective action with respect to the chiral compensator chiral superfield [19].

On a curved spacetime, there is the conformal anomaly

$$\langle T^\mu_\mu \rangle = \frac{1}{120} \frac{1}{(4\pi)^2} \left(c(\text{Weyl})^2 - \frac{a}{4}(\text{Euler}) \right), \tag{2.27}$$

(there can also be an $a'\partial^2 R$ term, whose coefficient a' is ambiguous, which was discussed in detail in [20]). The coefficient "c" is that of the stress tensor two-point function in flat space, whereas the coefficient "a" can be related to a stress tensor 3-point function in flat space. The superspace version of this anomaly, including also background gauge fields coupled to the superconformal R-current, is as in (2.26), with $L_T = (c\mathcal{W}^2 - a\Xi_c)/24\pi^2$ [11]. Taking components of this superspace anomaly equation relates the conformal anomaly coefficients a and c to the 't Hooft anomalies of the superconformal $U(1)_R$ symmetry [11]:

$$a = \frac{3}{32}(3\text{Tr}R^3 - \text{Tr}R) \qquad c = \frac{1}{32}(9\text{Tr}R^3 - 5\text{Tr}R). \tag{2.28}$$

2.3.1 4d $\mathcal{N} = 1$ SCFTs: relating current correlators to 't Hooft anomalies

An alternate derivation [18] of these relations follows from the fact that, in flat space, the 3-point function $\langle T_{\alpha\dot{\alpha}}(z_1) T_{\beta\dot{\beta}}(z_2) T_{\gamma\dot{\gamma}}(z_3) \rangle$ is of a form that's completely determined by the symmetries and Ward identities, up to two overall normalization coefficients, with one linear combination of these coefficients proportional to the coefficient (2.23) of the $T_{\alpha\beta}$ two-point function. In components, this relates the stress tensor three-point functions, and hence a and c, and to the R-current 3-point

functions, and hence the $\mathrm{Tr}U(1)_R$ and $\mathrm{Tr}U(1)_R^3$ 't Hooft anomalies, to these two coefficients. It follows that a and c can be expressed as linear combinations of $\mathrm{Tr}U(1)_R$ and $\mathrm{Tr}U(1)_R^3$, and the coefficients in (2.28) can easily be determined by considering the special cases of free chiral and vector superfields.

Combining (2.11) and (2.28), we have

$$\tau_{RR} = \frac{3}{2}\mathrm{Tr}R^3 - \frac{5}{6}\mathrm{Tr}R. \tag{2.29}$$

It was also argued in [11] that the two-point functions τ_{ij} of non-R flavor currents are related to 't Hooft anomalies, as

$$\tau_{ij} = -3\mathrm{Tr}RF_iF_j. \tag{2.30}$$

Again, this can be argued for either by turning on background fields, or by considering correlation functions in flat space. In the former method, one uses the fact that coupling background field strengths to the non-R currents leads to $\Delta L_T = C_{ij}W_{\alpha i}W_j^\alpha$, in (2.26), for some coefficients C_{ij}. In components, (2.26) then gives $\delta\langle T_\mu^\mu\rangle \sim C_{ij}F_{\mu\nu,i}F_j^{\mu\nu}$ and $\delta\langle\partial_\mu J_R^\mu\rangle \sim C_{ij}F_{\mu\nu i}\widetilde{F}_j^{\mu\nu}$. The former gives $C_{ij} \sim \tau_{ij}$ and the latter gives $C_{ij} \sim \mathrm{Tr}RF_iF_j$, so $\tau_{ij} \propto \mathrm{Tr}RF_iF_j$. The coefficient in (2.30) is again easily determined by considering the special case of free field theory.

The alternate derivation would be to consider the flat space 3-point function of the stress tensor and two flavor currents, $\langle T_{\alpha\dot\alpha}(z_1)J_i(z_2)J_j(z_3)\rangle$. It was shown in [9] that such 3-point functions are completely determined by the symmetries and Ward identities, up to two overall coefficients, and that one linear combination of these coefficients is proportional to the current-current two point functions, and hence τ_{ij}. In our supersymmetric context, that same linear combination should be related by supersymmetry to $\langle\partial_\mu J_R^\mu(x_1)J_{F_i}^\rho(x_2)J_{F_j}^\sigma(x_3)\rangle$, and hence to the $\mathrm{Tr}RF_iF_j$ 't Hooft anomaly.

The a-maximization [5] constraint on the superconformal R-symmetry follows from the fact that supersymmetry relates the $\mathrm{Tr}R^2F_i$ and $\mathrm{Tr}F_i$ 't Hooft anomalies:

$$9\mathrm{Tr}R^2F_i - \mathrm{Tr}F_i = 0, \tag{2.31}$$

which again can be argued for either by considering again an anomaly with background fields, or by considering current correlation functions in flat space [5]. In

the former method, one considers the anomaly of the flavor current coming from a curved background metric and background gauge field coupled to the superconformal R-current, $\overline{\nabla}^2 J \propto \mathcal{W}^2$. With the latter method, one uses the result of [18] that the flat space 3-point function $\langle T_{\alpha\dot{\alpha}}(z_1) T_{\beta\dot{\beta}}(z_2) J_i(z_3)\rangle$ is completely determined by the symmetries and superconformal Ward identities, up to a single overall normalization constant.

We note that supersymmetry does not relate τ_{Ri} to the 't Hooft anomaly $\mathrm{Tr} R^2 F_i$. Naively, one might have expected some such relation, in analogy with the above arguments, for example by trying to use (2.26) to relate a term $\delta\langle T^\mu_\mu\rangle \sim \tau_{Ri} F_{R,\mu\nu} F_i^{\mu\nu}$ to a term $\delta\langle \partial_\mu J^\mu_R\rangle \sim (\mathrm{Tr} R^2 F_i) F_{R,\mu\nu} \widetilde{F}_i^{\mu\nu}$, when background fields are coupled to both $U(1)_R$ and $U(1)_{F_i}$ currents. But there is actually no way to write such combined contributions of the $U(1)_R$ and $U(1)_{F_i}$ background fields to (2.26), because the former resides in the spin 3/2 chiral super field strength $\mathcal{W}_{\alpha\beta\gamma}$, and the latter resides in the spin 1/2 chiral super field strength $W_{\alpha i}$, and there is no way to combine the two of them into the spin zero chiral object L_T. Likewise, in flat space, a relation between τ_{Ri} and $\mathrm{Tr} R^2 F_i$ would occur if the 3-point function $\langle T_{\alpha\dot{\alpha}}(z_1) T_{\beta\dot{\beta}}(z_2) J_i(z_3)\rangle$, which includes a term proportional to $\mathrm{Tr} R^2 F_i$, were related to the two-point function $\langle T_{\beta\dot{\beta}}(z_2) J_i(z_3)\rangle$, which is proportional to τ_{Ri} (and, as we have argued above, vanishes). It sometimes happens that 3-point functions with a stress tensor are simply proportional to the 2-point function without the stress tensor, e.g. this is the case when the other two operators are chiral and anti-chiral primary [18]. But the the $\langle T_{\alpha\dot{\alpha}}(z_1) T_{\beta\dot{\beta}}(z_2) J_i(z_3)\rangle$ 3-point function in [18] is not related to the $\langle T_{\beta\dot{\beta}}(z_2) J_i(z_3)\rangle$ two-point function. Indeed, the free field example discussed in the introduction illustrates that $\mathrm{Tr} R^2 F_i$ and τ_{Ri} are not related by supersymmetry, as $\mathrm{Tr} R^2 F_i \neq 0$ for this example but, as always, $\tau_{Ri} = 0$.

2.3.2 Using $\tau_{Ri} = 0$ to determine the superconformal R-symmetry

As discussed in the introduction, using (2.4), we have for a general trial R-symmetry

$$\tau_{R_t i} = \tau_{R_0 i} + \sum_j s_j \tau_{ij}. \tag{2.32}$$

Imposing $\tau_{R_i i} = 0$ gives a set of linear equations, which determines the particular values s_j^* of the parameters for which the trial R-symmetry is the superconformal R-symmetry. As discussed in the introduction, this can equivalently be expressed as "the exact superconformal R-symmetry minimizes its two-point function coefficient $\tau_{R_t R_t}(s)$, which is given by (2.8), and which we can re-write using $\tau_{Ri} = 0$ for the superconformal R-symmetry as

$$\tau_{R_t R_t}(s) = \tau_{RR} + \sum_{ij} (s_i - s_i^*)(s_j - s_j^*)\tau_{ij}, \tag{2.33}$$

making it manifest that $\tau_{R_t R_t}$ has a unique global minimum, when the s_j are set to the particular value s_j^*. At $s_j = s_j^*$, the general R-symmetry R_t in (2.4) becomes the superconformal R-symmetry, in the supermultiplet stress tensor $T_{\alpha\dot{\alpha}}$.

The function $\tau_{R_t R_t}(s)$ to minimize and the function $a_{trial}(s)$ to locally maximize in 4d are different. Let us compare the values of them and their derivatives at the extremal point $s_i = s_i^*$. For (2.32), we have:

$$\begin{aligned} \tau_{R_t R_t}|_{s^*} &= \tau_{RR} = \frac{16}{3}c = \frac{3}{2}\mathrm{Tr}R^3 - \frac{5}{6}\mathrm{Tr}R, \\ \frac{\partial}{\partial s_i}\tau_{R_t R_t}|_{s^*} &= 0, \\ \frac{\partial^2}{\partial s_i \partial s_j}\tau_{R_t R_t} &= 2\tau_{ij}, \end{aligned} \tag{2.34}$$

whereas for $\frac{16}{3}a_{trial}(R_t) \equiv \frac{1}{2}(3\mathrm{Tr}R_t^3 - \mathrm{Tr}R_t)$ we have:

$$\begin{aligned} \frac{16}{3}a_{trial}(R_t)|_{s^*} &= \frac{16}{3}a = \frac{3}{2}\mathrm{Tr}R^3 - \frac{1}{2}\mathrm{Tr}R, \\ \frac{\partial}{\partial s_i}\frac{16}{3}a_{trial}(R_t)|_{s^*} &= \frac{9}{2}\mathrm{Tr}R^2 F_i - \frac{1}{2}\mathrm{Tr}F_i = 0, \\ \frac{\partial^2}{\partial s_i \partial s_j}\frac{16}{3}a_{trial}(R_t)|_{s^*} &= 9\mathrm{Tr}RF_iF_j = -3\tau_{ij}. \end{aligned} \tag{2.35}$$

The derivatives of both functions of s vanish at the same values s^*. The values of the two functions in (2.34) and (2.35) differ, except for SCFTs with $a = c$, i.e. $\mathrm{Tr}R = 0$, as is the case for SCFTs with AdS duals [1] The second derivatives of the functions in (2.34) and (2.35) are proportional, though with opposite sign, reflecting the fact that the exact superconformal R-symmetry minimizes $\tau_{R_t R_t}$ and maximizes $a_{trial}(R_t)$.

For the sake of comparison, let's also consider the function $\frac{16}{3}c_{trial}(R_t) \equiv \frac{3}{2}R_t^3 - \frac{5}{6}R_t$; the value of this function and its first two derivatives at $R_t = R$, i.e. $s_i = s_i^*$, are

$$\begin{aligned} \frac{16}{3}c_{trial}(R_t)|_{s^*} &= \frac{16}{3}c = \frac{3}{2}\mathrm{Tr}R^3 - \frac{5}{6}\mathrm{Tr}R, \\ \frac{\partial}{\partial s_i}\frac{16}{3}c_{trial}(R_t)|_{s^*} &= \frac{9}{2}\mathrm{Tr}R^2F_i - \frac{5}{6}\mathrm{Tr}F_i = -\frac{1}{3}\mathrm{Tr}F_i, \\ \frac{\partial^2}{\partial s_i \partial s_j}\frac{16}{3}c_{trial}(R_t)|_{s^*} &= 9\mathrm{Tr}RF_iF_j = -3\tau_{ij}. \end{aligned} \tag{2.36}$$

The value of $\tau_{R_t R_t}$ and $c_{trial}(R_t)$ coincide at $R_t = R$. The value of their first derivatives differ for any flavor symmetries with $\mathrm{Tr}F_i \neq 0$. General SCFTs can have flavor symmetries with $\mathrm{Tr}F_i = 0$, but SCFTs with AdS duals always have $\mathrm{Tr}F_i = 0$, and $\mathrm{Tr}F_i = 0$ for general superconformal quivers with only bifundamental matter [21, 22]. The second derivatives in (2.36) differ from those of (2.34) by a factor of $-3/2$, coinciding with those of (2.35).

As a further comparison of a-maximization in 4d with τ_{RR} minimization, let's consider the equations for the case where the superconformal $U(1)_R$ can mix with a single non-R flavor symmetry, $R_t = R_0 + sF$. a-maximization gives the value s^* for the superconformal $U(1)_R$ as a solution of the quadratic equation

$$s^2\mathrm{Tr}F^3 + 2s\mathrm{Tr}R_0F^2 + \mathrm{Tr}R_0^2F - \frac{1}{9}\mathrm{Tr}F = 0. \tag{2.37}$$

τ_{RR} minimization gives s^* as (2.7)

$$s^* = -\tau_{R_0F}/\tau_{FF}. \tag{2.38}$$

[1] Quite generally, quiver gauge theories with only bi-fundamental matter have $\mathrm{Tr}R = 0$, and hence $a = c$ [21, 22].

If $\mathrm{Tr}F^3$ is non-zero, s^* can also be obtained from (2.16), which here gives

$$s^* = -\left[\mathrm{Tr}R_0F^2 + \frac{1}{3}\tau_{FF}\right]/\mathrm{Tr}F^3. \tag{2.39}$$

For any given choice of R_0 and F, the value of s^* obtained in these three different ways must agree. It would be nice to have a direct proof of the relations that this implies. E.g. comparing (2.39) with (2.38) gives the identity $\tau_{R_0F}\mathrm{Tr}F^3 = \tau_{FF}\left(\frac{1}{3}\tau_{FF} + \mathrm{Tr}R_0F^2\right)$ which, evidently, must hold for any choice of the R-symmetry R_0 (taking R_0 to equal the superconformal $U(1)_R$, both sides vanish).

2.4 SQCD Example

4d $\mathcal{N} = 1$ SCQD, with gauge group $SU(N_c)$ and N_f fundamental and anti-fundmantal flavors, Q and $\widetilde{Q}$, has been argued to flow to a SCFT in the IR for the flavor range $\frac{3}{2}N_c < N_c < 3N_c$ [23]. Taking the superconformal $U(1)_R$ to be the anomaly free R-symmetry, the superconformal R-charges are $R(Q) = R(\widetilde{Q}) = 1 - (N_c/N_f)$. Let's also consider the baryonic $U(1)_B$ symmetry, with $B(Q) = -B(\widetilde{Q}) = 1/N_c$. Using the 't Hooft anomaly relations,

$$\tau_{RR} = \frac{3}{2}\mathrm{Tr}R^3 - \frac{5}{6}\mathrm{Tr}R = \frac{3}{2}\left[N_c^2 - 1 - 2\frac{N_c^4}{N_f^2}\right] + \frac{5}{6}\left[N_c^2 + 1\right], \tag{2.40}$$

$$\tau_{BB} = -3\mathrm{Tr}RBB = 6. \tag{2.41}$$

For $N_f \approx 3N_c$, where the RG fixed point is at weak coupling as in [24, 25], these expressions reduce to the free field values.

There is a unique, anomaly free $U(1)_R$ symmetry that commutes with charge conjugation and the $SU(N_f)$ global symmetries. Our $\tau_{R_tR_t}$ minimization condition immediately leads to the same conclusion. $\tau_{R_tR_t}$ is minimized by having $\tau_{RB} = 0$ and $\tau_{RF_i} = 0$ for the $U(1)_B$ and $SU(N_f)$ global symmetries. Taking the $U(1)_R$ to be even under charge conjugation ensures that $\tau_{RB} = 0$, because the $U(1)_B$ current is odd, so charge conjugation symmetry gives $\tau_{RB} = -\tau_{RB}$. Likewise $\tau_{RF_i} = 0$ for the $SU(N_f)$ flavor currents, simply by the tracelessness of the generators, if $U(1)_R$ is taken to commute with $SU(N_f)$.

2.5 Perturbative analysis

Consider a general 4d $\mathcal{N}=1$ SCFT with gauge group G and matter chiral superfields Q_f in representations r_f (of dimension $|r_f|$) of G, with no superpotential, $W=0$. If the theory is just barely asymptotically free, there can be a RG fixed point at weak gauge coupling, where perturbative results can be valid. We will verify that the leading order pertubative expression for the anomalous dimension for fields,

$$\gamma_f(g) = -\frac{g^2}{4\pi^2}C(r_f) + \mathcal{O}(g^4), \qquad \text{i.e} \qquad R_f = \frac{2}{3} - \frac{g^2}{12\pi^2}C(r_f) + O(g^4). \quad (2.42)$$

agrees with τ_{RR} minimization. As standard, we define group theory factors as

$$\mathrm{Tr}_{r_f}(T^A T^B) = T(r_f)\delta^{AB}, \quad \sum_{A=1}^{|G|} T^A_{r_f} T^A_{r_f} = C(r_f)\mathbf{1}_{|r_f|\times|r_f|}, \quad \text{so} \quad C(r_f) = \frac{|G|T(r_f)}{|r_f|}. \quad (2.43)$$

The RG fixed point value g_* of the coupling is determined by the constraint that the R-symmetry be anomaly free, $T(G) + \sum_f T(r_f)(R_f - 1) = 0$.

For the free UV theory, we minimize τ_{RR} over all possible R charges R_f of the matter chiral superfields, which are unconstrained for $g = 0$. As we discussed in the introduction, this gives the free-field term $R_f^{(0)} = 2/3$. For $g \neq 0$, we write $R_f = R_f^{(0)} + R_f^{(1)} + \ldots$, with $R_f^{(1)}$ the $O(g^2)$ term that we'd like to find via τ_{RR} minimization. For $g \neq 0$, τ_{RR} should be minimized subject to the constraint that the symmetries be anomaly free, i.e. we impose $\tau_{Ri} = 0$ over all anomaly free $U(1)_R$ and $U(1)_{F_i}$ symmetries, with R charges R_f, and flavor F_i charges $q_i(r_f)$ constrained to satisfy

$$T(G) + \sum_f T(r_f)(R_f - 1) = 0, \qquad \text{and} \qquad \sum_f T(r_f) q_i(r_f) = 0. \quad (2.44)$$

The $U(1)_R$ current assigns charges R_f to the squark and $R_f - 1$ to the quarks components of Q_f. The $U(1)_{F_i}$ non-R current assigns charges $q_i(r_f)$ to both the quark and squark components of Q_f. To compute τ_{RF_i}, we consider the diagrams for the two point function $\langle J_R^\mu(x_1) J_{F_i}^\nu(x_2)\rangle$. Because we take the currents to be conserved, they have vanishing anomalous dimension, so we anticipate that the

various diagrams sum such that all apparent divergences cancel, and we're left with only finite contributions to τ_{RF_i}. The $O(g^2)$ contributions can be written as

$$\tau_{Ri}^{(1)} = \sum_f q_i(r_f)\left[(\frac{1}{3}R_f^{(1)} + \frac{2}{3}R_f^{(1)})|r_f| + R_f^{(0)}(A_f^{(1)} + C_f^{(1)}) + (R_f^{(0)} - 1)(B_f^{(1)} + C_f^{(1)})\right] \tag{2.45}$$

The first two terms come from the leading diagrams, without interactions, exactly as in the free-field result (2.13), but weighted by the $O(g^2)$ R-charges $R_f^{(1)}$. The first term is from connecting the currents at x_1 and x_2, with squark ϕ_f propagators, and the second from connecting them with quark ψ_f propagators. The remaining contributions in (2.45) are $O(g^2)$ because they involve $O(g^2)$ interaction diagrams, and the R-charge weighting is thus taken as $R^{(0)} = 2/3$. Here $A_f^{(1)}$ is the contribution of all $O(g^2)$ 1PI diagrams connecting squark ϕ_f, at x_1, to squark ϕ_f at x_2. $B_f^{(1)}$ is similarly the contribution from all $O(g^2)$ diagrams connecting quark ψ_f at x_1 to quark ψ_f at x_2. $C_f^{(1)}$ is the contributions of diagrams connecting squark ϕ_f at x_1 to quark ψ_f at x_2 (or vice-versa). We note that the group theory factors in all of these diagrams with $O(g^2)$ interactions is the same: $\mathrm{Tr}_{r_f}\sum_{A=1}^{|G|} T_{r_f}^A T_{r_f}^A = |r_f|C(r_f) = |G|T(r_f)$, i.e. $A_f^{(1)} = |G|T(r_f)A^{(1)}$, $B_f^{(1)} = |G|T(r_f)B^{(1)}$, and $C_f^{(1)} = |G|T(r_f)C^{(1)}$, where $A^{(1)}$, $B^{(1)}$, and $C^{(1)}$ are independent of the gauge group and representation, e.g. they could be computed in $U(1)$ SQED.

Using the second constraint in (2.44), $\sum_f T(r_f)q_i(r_f) = 0$, it immediately follows, without even having to compute $A^{(1)}$, $B^{(1)}$, and $C^{(1)}$, that their contributions to $\tau_{Ri}^{(1)}$ in (2.45) all vanish, for all anomaly free flavor symmetries F_i. The only contributions remaining in (2.45) are the $R_f^{(1)}$ ones, $\tau_{Ri}^{(1)} = \sum_f q_i(r_f)R_f^{(1)}|r_f|$. Our τ_{RR} minimization result implies that this must vanish, for any $q_i(r_f)$ satisfying the anomaly free constraint in (2.44). This implies that $R_f^{(1)} = \alpha C(r_f)$ for some constant α that's independent of the rep. r_f.

We have thus used $\tau_{R_tR_t}$ minimization to re-derive the group theory dependence of the $O(g^2)$ term in the anomalous dimension (2.42). The coefficient is also fixed to agree with (2.42), at the fixed point g_*, by using the condition in (2.44) that the R-symmetry be anomaly free to solve for α (which is appropriately small when the matter content is such that the theory is barely asymptotically free). This

reproduces the $O(g^2)$ contribution to the R-charges in (2.42) at the RG fixed point.

In principle, one could extend this analysis, and use τ_{RR} minimization to compute the anomalous dimensions to all orders. Using a-maximization [5] (assuming that the RG fixed point has no accidental symmetries), the general result can be written as [16]

$$R_f = \frac{2}{3}(1+\frac{1}{2}\gamma_f(g_*)) = 1 - \frac{1}{3}\sqrt{1+\frac{\lambda_* T(r_f)}{|r_f|}} = 1 - \frac{1}{3}\sqrt{1+\frac{\lambda_* C(r_f)}{|G|}}, \qquad (2.46)$$

where λ_* is a Lagrange multiplier [16], which is determined by the constraint that the R-symmetry be anomaly free, $T(G)+\sum_f T(r_f)(R_f-1)=0$. The result (2.46) was successfully compared [26, 27] with the results for the anomalous dimensions to 3-loops of [28, 29]. But, because current two-point functions get quantum corrections, τ_{RR} minimization does not seem to be a very efficient way to compute anomalous dimensions. Indeed, the higher order quantum corrections to τ_{Ri} include diagrams where the currents at x_1 and x_2 are connected by renormalized propagators, with all quantum corrections from the interactions, and computing such τ_{Ri} contributions is already tantamount to directly computing the anomalous dimensions $\gamma_f(g)$.

This chapter is a reprint of the material as it appears in "The Exact Superconformal R-symmetry Minimizes τ_{RR}," E. Barnes, E. Gorbatov, K. Intriligator, M. Sudano, J. Wright, Nucl. Phys. B **730**, 210 (2005), arXiv:hep-th/0507137, of which I was a co-author.

Chapter 3

Sparticle Masses in Higgsed Gauge Mediation

3.1 Introduction

There are countless ways in which the standard model could fit into a supersymmetric framework. For the sake of phenomenology, a useful categorization is in terms of how supersymmetry breaking is communicated to the observable particles. Gauge-mediated supersymmetry breaking (GMSB) [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41] assumes that SUSY breaking is communicated via the standard model gauge group. This mechanism has several attractive features. In particular, it makes calculable predictions for the soft parameters of the MSSM in terms of a few parameters while naturally evading the tight constraints from flavor physics. It also accommodates radiative electroweak symmetry breaking [42] and offers a solution to the CP problem [43].

It is important to remember, however, that this is not a complete theory. It is only meant to apply below the scale of SUSY-breaking. The hope is that it provides a successful parameterization of our ignorance of physics at the higher scale, but this has recently been called into question. In [44], for example, it was pointed out that the standard approach omits a set of renormalizable interactions that are allowed by the symmetries, are consistent with experiment, and lead to novel

phenomenology. In this note, we explore another generalization of ordinary gauge mediation that has previously been ignored. Specifically, we consider a general, supersymmetric Higgsing of the mediating gauge group.

In a sense, this is not new at all because it is generally assumed that there is a supersymmetric Higgsing of the mediating gauge group, both at the GUT scale and at the weak scale. For a messenger scale much larger than the weak scale, the masses of the $SU(2)_W$ gauge fields can be neglected. And the gauge fields with GUT-scale masses can be ignored if the messenger scale is sufficiently small, but for models with a messenger scale near the GUT scale, as in [45], they can be very important. Of course, gauge symmetry breaking may also occur at an intermediate scale. For example, additional $U(1)$'s [46, 47, 48, 49, 50, 51], which arise naturally in SUSY-GUTs with large gauge groups and in string theory.

In what follows, we will briefly review the standard treatment of GMSB and the associated sparticle spectrum. We will then discuss how one can approximate these results for much of parameter space, and we will show that these techniques fail to capture the effects of interest here. Finally, we will present the leading-order sparticle spectrum in standard GMSB for an arbitrary, supersymmetric Higgsing of the gauge group, and comment on the results. The messier details of the calculation are left for Appendix A.

3.2 Standard Gauge Mediation

In the basic scenario (see [41] for a review), a set of chiral superfields, Φ_i and $\tilde{\Phi}_i$, are added to a GUT-extended MSSM. They can all be taken to be $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, for example. Note that this choice preserves gauge-coupling unification, is anomaly-free, and allows for the superpotential term,

$$\Delta W = \sum_i \lambda_i X \tilde{\Phi}_i \Phi_i, \tag{3.1}$$

where $X = M + F\theta^2$ is a gauge-singlet background field. The index, i, is a flavor index. Gauge indices are supressed. The non-zero F-component of the "spurion", X, results in a non-supersymmetric mass spectrum for our chiral superfields, which

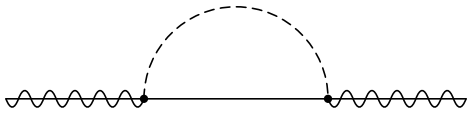


Figure 3.1: The sole one-loop diagram contributing to gaugino masses

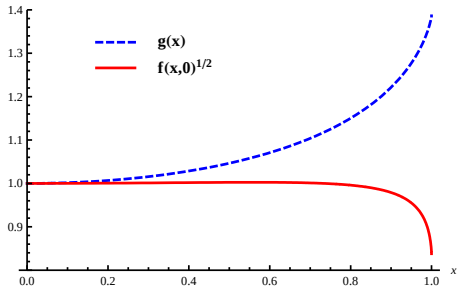


Figure 3.2: $f(0,0) = 1 = g(0)$, but $g(1)/f(1,0)^{1/2} \approx 5/3$

then act as messengers, splitting masses of MSSM superfields through loops. Their contributions to MSSM gaugino and scalar masses have been computed for the scenario described above and for some generalizations [52, 53, 54][1]. The gauginos get masses, from the diagram in Figure 3.1,

$$\Delta m^a_{1/2} = \frac{\alpha_a}{2\pi}\frac{F}{M}\sum_i n_a(r_i) g(x_i), \qquad x_i = \left|\frac{F}{\lambda_i M^2}\right|, \tag{3.2}$$

and the first eight diagrams of Figure 3.5 give the scalar masses,

$$\Delta m_0^2 = \left|\frac{F}{M}\right|^2 \sum_a \left(\frac{\alpha_a}{2\pi}\right)^2 C_a(r_Q) \sum_i n_a(r_i) f(x_i, 0). \tag{3.3}$$

Much of the notation is the same as that of [53]. The index, $a = 1, 2, 3$, labels the gauge group, $\alpha_a = g_a^2/4\pi$, $C_a(r_Q)$ is the quadratic Casimir of the scalar field that is getting mass, and $n_a(r_i)$ is the Dynkin index of the messenger representation. The extra argument in the function, $f(x_i, 0)$, will be explained shortly. Note that (3.2) is a mass, and (3.3) is a mass squared, so in the ratio of a gaugino and a

[1] In principle, the two-loop effective potential of [55] contains these scalar masses and those presented later in this paper. In practice, however, extracting such results is difficult with basic computational resources in anything but the simplest theories.

scalar mass, the factors of F/M cancel. And as one can see in Figure 3.2, the functions, $g(x)$ and $f(x,0)^{1/2}$, deviate little from one over most of parameter space.[2] This means that $m^a_{1/2}/m_0$ primarily depends on the "effective messenger number", $N_a \equiv 2\sum_i n_a(r_i)$. With the conventional normalization of the generators, $n_a = 1/2$ for fundamentals, so in the simple case of $SU(5)$ with fundamental messengers, the effective messenger number is the number of messengers. This is one way in which measuring only a couple of soft parameters of the MSSM could reveal something about the messenger sector. We will see, however, that this simple picture can be modified when the mediating gauge group is Higgsed.

3.3 Analytic Continuation to Superspace

In the limit of small supersymmetry breaking, the results of the previous section can be obtained in an entirely different way [56, 57, 58]. Consider a massless chiral superfield, Q, that only couples to the messengers through gauge fields. The Lagrangian will have a term,

$$\mathcal{L} \supset \int d^4\theta Z_Q(\mu) Q^\dagger Q, \tag{3.4}$$

where $Z_Q(\mu)$ is the wave-function renormalization of Q at the scale μ. If this scale is below the messenger scale, then $Z_Q(\mu) = Z_Q(\mu, M^\dagger, M)$. Now comes the interesting part. The idea is to replace M with the superfield, $X = M + F\theta^2$. This new object, call it $\tilde{Z}_Q(\mu)$, has an expansion in powers of θ, which yields a mass for the scalars,

$$\Delta m_0^2(\mu) = -\left|\frac{F}{M}\right|^2 \frac{\partial^2 \ln \tilde{Z}_Q(\mu)}{\partial \ln X \partial \ln X^\dagger}\bigg|_{X=M}. \tag{3.5}$$

Performing the derivatives, one finds agreement with (3.3) to $\mathcal{O}(x^2)$ for $x = F/M^2 \ll 1$.

What we are interested in is the spectrum when we have chiral messengers and

[2]Note that x_i cannot exceed one. This would give a tachyonic messenger, and there is nothing to stabilize the field. In general, however, the UV completion can accommodate $x > 1$. The negative mass-squared simply indicates that the true vacuum is elsewhere, and in that vacuum, there is a massive gauge field.

a supersymmetric Higgsing. If we take $\Lambda_{UV} > M > m_W > \mu$, then we have

$$\begin{aligned} Z_Q(\mu) &= Z_Q(\Lambda_{UV})\Big(\frac{\alpha(\Lambda_{UV})}{\alpha(M)}\Big)^{2C(r_Q)/b}\Big(\frac{\alpha(M)}{\alpha(m_W)}\Big)^{2C(r_Q)/b'}\Big(\frac{\alpha(m_W)}{\alpha(\mu)}\Big)^{2C'(r_Q)/b''} \\ \alpha^{-1}(M) &= \alpha^{-1}(\Lambda_{UV}) + \frac{b}{4\pi}\ln\frac{M^\dagger M}{\Lambda_{UV}^2}, \\ \alpha^{-1}(m_W) &= \alpha^{-1}(M) + \frac{b'}{4\pi}\ln\frac{m_W^2}{M^\dagger M} \\ \alpha^{-1}(\mu) &= \alpha^{-1}(m_W) + \frac{b''}{4\pi}\ln\frac{\mu^2}{m_W^2}. \end{aligned} \tag{3.6}$$

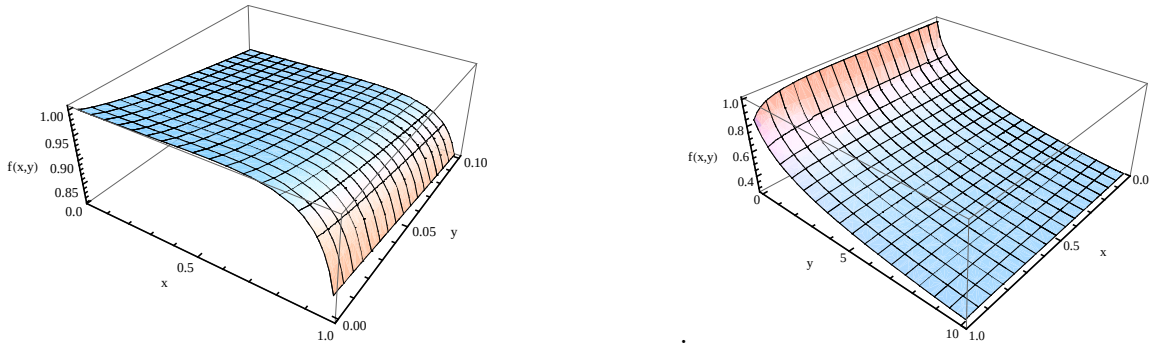
Making the substitution, $M \to X$, and plugging into (3.5) gives a mass that depends on m_W, but only trivially. It only acts to give the correct running of the coupling to the scale, μ. This should not be surprising since the method takes the gauge fields to be massless above m_W and infinitely massive below. In a sense, what we are interested in is a threshold effect. The gauge messenger case, in which the spurion breaks the gauge group, is different because the scale, $m_W = M$, enters in the Grassman-parameter expansion. Perhaps there is a clever way of approximating a non-trivial effect of a supersymmetric Higgsing, but we will not pursue this further here. Instead, we perform the Feynman-diagram calculation.

3.4 Higgsed Gauge Mediation

We are interested in the effects of modifying the gauge sector of gauge mediation. In particular, we allow for massive gauge fields coupled to both messengers and MSSM fields, but do not study the gauge messenger scenario in which a gauge superfield has split masses.

3.4.1 Case 1 – $G \times U(1)'$, A Toy Model

Starting with the simplest extension, consider the set of messenger fields, Φ_i and $\tilde{\Phi}_i$, as in the introduction. Now let them be charged under an additional $U(1)'$ gauge symmetry that is spontaneously broken. The desired spectrum is obtained if we add fields, Ψ and $\tilde{\Psi}$, that are only charged under this $U(1)'$, and take the



a. **b.**

Figure 3.3: $f(x, y)$ is plotted for small (a.) and large (b.) values of y.

superpotential to be

$$\Delta W = \sum_i \lambda_i X \tilde{\Phi}_i \Phi_i + hT(\tilde{\Psi}\Psi - v^2), \qquad X = M + F\theta^2, \tag{3.7}$$

where the field, T, is a singlet dynamical field, which, for our purposes, plays no role except to give vevs to the added superfields. Suppressing all indices, this produces a massive vector multiplet, (A, C, λ, χ), with supersymmetric mass $m_W = 2gv$, where A is a gauge boson, C is a real scalar field, λ is a gaugino, and χ is another Weyl fermion.

Turning to the radiative spectrum, the gauginos of the unbroken gauge group get the standard one-loop masses (3.2) computed in [53], which we reproduce here:

$$\Delta m^a_{1/2} = \frac{\alpha_a}{2\pi}\frac{F}{M}\sum_i n_a(r_i) g(x_i), \tag{3.8}$$

where

$$g(x) = \frac{1}{x^2}(1+x)\ln(1+x) + (x \to -x). \tag{3.9}$$

The notation is discussed after (3.3). To leading order, the effect of the $U(1)'$ on the gaugino spectrum is simply to add a gaugino of mass m_W. The generalization to more interesting gauge structure is trivial and will not be discussed further. Now if we couple some set of chiral superfields, Q, that transform under the given gauge symmetry, their scalar components will acquire radiative masses at two-loop

order. The contribution from G is as before [52, 53],

$$\Delta m_0^2 = \left|\frac{F}{M}\right|^2 \sum_a \left(\frac{\alpha_a}{2\pi}\right)^2 C_a(r_Q) \sum_i n_a(r_i) f(x_i, 0), \tag{3.10}$$

where

$$f(x,0) = \frac{1+x}{x^2}\left[\ln(1+x) - 2\mathrm{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{2x}{1+x}\right)\right] + (x \to -x). \tag{3.11}$$

With a Higgsed mediating gauge group, there are ten relevant diagrams, which are shown in Figure 3.5. For our toy model, the contribution from the $U(1)'$ vector multiplet is

$$\Delta m_0^2 = \left|\frac{F}{M}\right|^2 \left(\frac{\alpha}{2\pi}\right)^2 C(r_Q) \sum_i n(r_i) f(x_i, y), \qquad y = \left|\frac{m_W}{M}\right|^2, \tag{3.12}$$

and $C(r_Q)$ and $n(r_i)$ are respectively the squared charges of Q and Φ_i under the $U(1)'$. The function, $f(x,y)$, is given in Appendix A along with more details of the computation. In Figure 3.3, this function is plotted in the limits of large and small y. At small y, it is seen to agree with the known result of Figure 3.2. At large y, the kinematic suppression of the amplitude is evident. More explicitly, we find

$$\begin{aligned} f(x, y \ll 1) &= f(x,0) + \left(\frac{1}{3} + \frac{x^2}{30} + \mathcal{O}(x^4)\right) y \ln y + \mathcal{O}(y) \\ f(x, y \gg 1) &= \frac{2}{y}\ln y + \mathcal{O}\left(\frac{1}{y}\right) \end{aligned} \tag{3.13}$$

3.4.2 Case 2 – Products of Simple Groups with Degeneracy

When each factor of the mediating gauge group is simple and has a single supersymmetric mass for all of its gauge superfields, the result is a simple extension of what was done in the previous subsection. In fact, it is simply (3.10) with the substitution,

$$f(x_i, 0) \to f(x_i, y_a). \tag{3.14}$$

3.4.3 Case 3 – Products of Simple Groups without Degeneracy

In generalizing to an arbitrary supersymmetric Higgsing, the most obvious obstacle is that the gauge field associated with a given generator need not be a

mass eigenstate. This is familiar from the standard model, in which the $U(1)_Y$ generator mixes with the diagonal generator of $SU(2)_W$ to form the massive Z and the massless photon. It is typically most convenient to calculate in the mass eigenbasis, working with "effective generators" that are linear combinations of the original generators. With this strategy, one can quickly work out the result for a general Higgsing of a product of simple Lie groups,

$$\Delta m_0^2 = \left|\frac{F}{M}\right|^2 \sum_a \left(\frac{\alpha_a}{2\pi}\right)^2 \sum_j \mathcal{T}^j_{a,Q}\mathcal{T}^j_{a,Q} \sum_i n_a(r_i) f(x_i, y_{a,j}), \qquad y_{a,j} = \left|\frac{\mathcal{M}_a^{jj}}{M}\right|^2. \tag{3.15}$$

The effective generators of each group are given by $\mathcal{T}^j = \mathcal{O}^{jk}T^k$, where $\mathcal{O}$ is the orthogonal matrix that diagonalizes the mass matrix of the gauge fields. In matrix notation,

$$\frac{1}{2}W^T M W = \frac{1}{2}W^T\mathcal{O}^T\mathcal{O}M\mathcal{O}^T\mathcal{O}W = \frac{1}{2}(\mathcal{O}W)^T\mathcal{M}(\mathcal{O}W) \equiv \frac{1}{2}\mathcal{W}^T\mathcal{M}\mathcal{W}, \tag{3.16}$$

where $\mathcal{M}$ is diagonal, and $\mathcal{W}$ is the vector of mass eigenstates. The effective generators emerge when the covariant derivative is written in this basis. Note that in the case of full degeneracy, $\mathcal{M}^{jk} = m_W\delta^{jk}$, summing over j in (3.15) reproduces the familiar quadratic Casimir, $\mathcal{O}^{jk}T^k\mathcal{O}^{jl}T^l = T^kT^l\delta^{kl} = C(r_q)$.

3.4.4 Case 4 – Allowing for $U(1)$'s

If the gauge group includes a $U(1)$, the potential for a new complication emerges. Fortunately, the problem and its solution are found in the simple case of a product of two $U(1)$'s. The result for an arbitrary Higgsing of an arbitrary gauge group is easily obtained from this case; though we will not attempt to write a formula for the general case.

Letting the gauge superfields have masses m_W and $\widetilde{m}_W$ and couplings g and $\tilde{g}$, one expects in general to have a contribution proportional to $g^2\tilde{g}^2$. The presence of different gauge fields within a diagram stems from the fact that the effective generators need not be traceless, so the trace that usually produces the Dynkin index no longer has to vanish for generators of different groups. The sum of diagrams proportional to $g^2\tilde{g}^2$ yields a new function, $h(x, y, \tilde{y})$, where $x = F/M^2$,

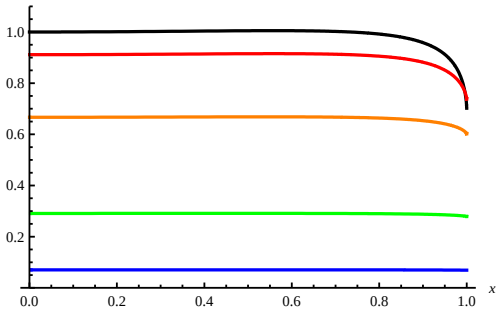


Figure 3.4: From top to bottom, $f(x,0)$, $f(x,.1)$, $f(x,1)$, $f(x,10)$, and $f(x,100)$

$y = m_W^2/M^2$, $z = \widetilde{m}_W^2/M^2$, and $h(x,y,y) = f(x,y)$. This function is given in Appendix A. The full result for the case of two $U(1)$'s is

$$\Delta m_0^2 = \left|\frac{F}{M}\right|^2 \sum_i \left[\left(\frac{\alpha}{2\pi}\right)^2 q_i^2 q_Q^2 f(x_i, y) \;+\; \left(\frac{\tilde{\alpha}}{2\pi}\right)^2 \tilde{q}_i^2 \tilde{q}_Q^2 f(x_i, \tilde{y}) \right. \\ \left. +\; 2\frac{\alpha\tilde{\alpha}}{(2\pi)^2} q_i \tilde{q}_i q_Q \tilde{q}_Q h(x_i, y, \tilde{y}) \right], \quad (3.17)$$

where the q's are the various charges of the fields in what is hopefully an obvious notation. In general, one simply needs to transform to the mass eigenbasis and identify all of the $U(1)'s$ that result. Each pair will have a contribution of this form.

3.5 Implications

The mass spectrum provides some of the key predictions for the potential discovery of GMSB. The masses calculated here (renormalized to the scale of MSSM sparticle masses [59]) predict relationships among particle masses given by simple group theory factors and known functions of scales. Our results reproduce those of standard gauge mediation [41] in the appropriate limit, but provide a new set of predictions if the mediating group is Higgsed. For example, if the messenger scale were low enough to make the mass of the $SU(2)_W$ fields non-negligible, one would find sfermions with lower than expected masses. Additional mediating gauge fields, however, lead to higher masses.

In ordinary gauge mediation the ratios of gaugino and sfermion masses depend primarily on the matter content of the messenger sector, but they are also highly sensitive to the gauge structure. The modification of the spectrum can be particularly interesting if the messenger scale is near a Higgsing scale. In that case, the massive gauge fields give significant contributions and cannot be approximated as massless (see frefxsec1). In this scenario, the ratio of gaugino and scalar masses would not readily yield the effective messenger number. Assuming ordinary gauge mediation, one would find that it is not an integer.

Of course, a proximity of scales need not be an accident. In [60], for example, the breaking-scale of a gauged Peccei-Quinn symmetry and the supersymmetry-breaking scale coincide. And in the ISS model [61], all scales are set by a single dimensionful parameter. Recall that our analysis applies to the spectrum and interactions that result from (3.7). The ISS model is of this form. It has messengers with masses, $m_{\pm}^2 = |h\mu|^2 \pm |h\mu|^2$ and a supersymmetric Higgsing of the gauge group with $m_W = g\mu$. This gives $y = g/h$, which is naturally of order one. The model also has $x = 1$, so a small-x approximation cannot be trusted.

This chapter is a reprint of material as it appears in "Sparticle Masses in Higgsed Gauge Mediation," E. Gorbatov, M. Sudano, JHEP **0810**, 066 (2008), arXiv:0802.0555, of which I was a co-author.

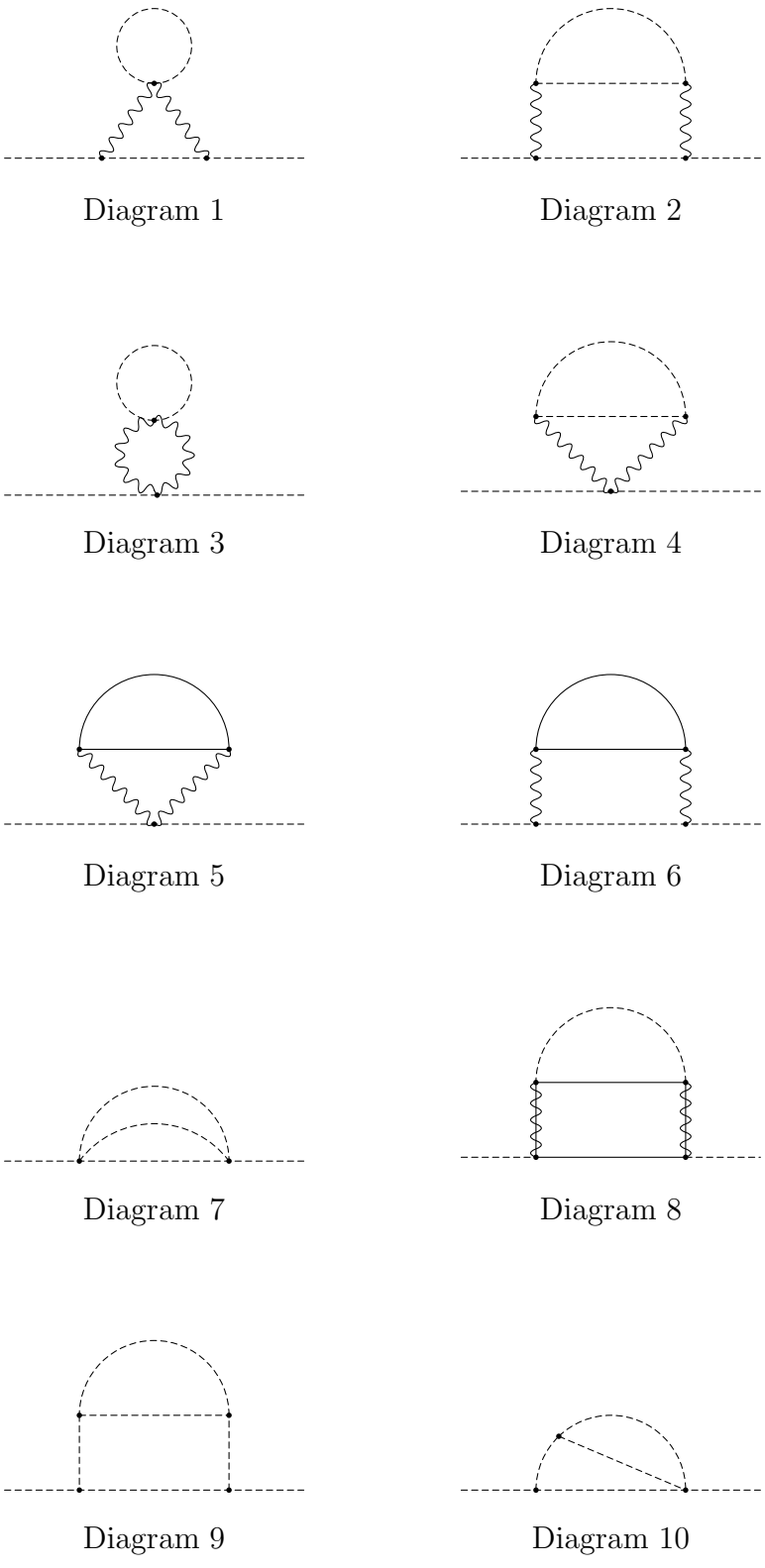


Figure 3.5: The two-loop diagrams contributing to MSSM scalar masses

Chapter 4

Comments on General Gauge Mediation

4.1 Introduction

A standard framework for building potentially realistic supersymmetric models is based on theories of the form

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{int}, \tag{4.1}$$

where $\mathcal{L}_1$ is the MSSM or some extension, $\mathcal{L}_2$ is the hidden sector with broken supersymmetry, and the $\mathcal{L}_{int}$ interactions couple them. In gauge mediation [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41], the gauge interactions are the most important part of $\mathcal{L}_{int}$. This scenario has been extensively studied for simple, weakly coupled, hidden sectors $\mathcal{L}_2$. It is potentially interesting to extend such results to more complicated hidden sectors, including those which are not necessarily weakly coupled, see e.g. [35, 62, 63, 64, 65]. A general framework that can accommodate this scenario was considered in [8], where it was shown that the soft masses of MSSM gauginos and sfermions, to leading order in the $\mathcal{L}_{int}$ gauge interactions, can be expressed in terms of the $\mathcal{L}_2$ current correlation functions. Related following works include [66, 67, 68, 69, 70, 71].

In this short note, we extend the results of [8] to compute the full effective potential for the sfermion fields. Expanding the effective potential around the

origin gives the sfermion masses. The form of the effective potential far from the origin can be of interest for cosmological models, as in [72]. When the susy-breaking/messenger sector $\mathcal{L}_2$ is weakly coupled and expanded for small susy-breaking F-terms, our general effective potential reduces to that obtained in [72]. We also express the full effective potential, generalized to allow for the possibility that the messenger gauge group is Higgsed. Expanding around the origin, this gives the gaugino and sfermion masses for general Higgsed gauge mediation. When the susy-breaking sector $\mathcal{L}_2$ is weakly coupled, these results reduce to those recently obtained in [73] for Higgsed gauge mediation. Finally, we discuss a relation between these current-correlator results and the discussion in [58] of the 1PI effective action and RG running. We discuss the results of [8] in terms of superspace and show how a spurion analysis reproduces the results obtained by analytic continuation in superspace [57, 58] in the limit of small supersymmetry breaking.

The organization of this paper is as follows. In Section 4.2, we give a brief review of General Gauge Mediation [8]. In Section 4.3, the full effective potential is presented in this formalism, and the generalization to Higgsed gauge groups is discussed. In Section 4.4, superspace techniques [57, 58] are used to extract results for small F-term breaking. The main observations of Section 4.4 were independently obtained in the recent work [71].

4.2 Review of General Gauge Mediation

In supersymmetric gauge theories, the gauge supermultiplet $\mathcal{V}$ couples to the current supermultiplet, $\mathcal{J}$, which is a real linear superfield satisfying $D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$. In components,

$$\mathcal{J} = J + i\theta j - i\bar{\theta}\bar{j} - \theta\sigma^\mu\bar{\theta}j_\mu + \frac{1}{2}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu j - \frac{1}{2}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{j} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box J, \qquad (4.2)$$

with $\partial^\mu j_\mu = 0$ and the other components unconstrained. The gauge interactions couple to $\mathcal{J}$ as

$$\mathcal{L}_{int} \supset 2g\int d^4\theta\mathcal{J}\mathcal{V} + \cdots = g(JD - \lambda j - \bar{\lambda}\bar{j} - j^\mu V_\mu) + \ldots, \qquad (4.3)$$

where the component expansion is in Wess-Zumino gauge. As shown in [8], the diagrams of Figure 4.1, which give the soft supersymmetry breaking masses of the visible sector, can be expressed in terms of the hidden-sector current-current two-point functions. Lorentz invariance and current conservation fix the form of the Euclidean momentum-space two-point functions of these fields as (dropping a $(2\pi)^4\delta^{(4)}(0)$):

$$\begin{aligned}
\langle J(p)J(-p)\rangle &= \widetilde{C}_0(p^2/M^2) && (4.4)\\
\langle j_\alpha(p)\bar{j}_{\dot\alpha}(-p)\rangle &= -\sigma^\mu_{\alpha\dot\alpha}p_\mu\widetilde{C}_{1/2}(p^2/M^2) && (4.5)\\
\langle j_\mu(p)j_\nu(-p)\rangle &= -(p^2\eta_{\mu\nu}-p_\mu p_\nu)\widetilde{C}_1(p^2/M^2) && (4.6)\\
\langle j_\alpha(p)j_\beta(-p)\rangle &= \epsilon_{\alpha\beta}M\widetilde{B}_{1/2}(p^2/M^2) && (4.7)
\end{aligned}$$

for some functions, $\widetilde{C}_0$, $\widetilde{C}_{1/2}$, $\widetilde{C}_1$, and $\widetilde{B}_{1/2}$. If supersymmetry were unbroken, $\widetilde{C}_0=\widetilde{C}_{1/2}=\widetilde{C}_1$, and $\widetilde{B}_{1/2}=0$. Here M is a mass scale in the problem. We are interested in these two-point functions in the hidden sector, where supersymmetry is broken. The $\widetilde{C}_{j=0,1/2,1}$ also depend on a UV cutoff, which is needed to regulate the Fourier transform from position space to momentum space, as

$$\widetilde{C}_j(p^2/M^2)=2\pi^2 c\log(\Lambda/M)+\widetilde{C}_j^{finite}(p^2/M^2) \qquad (4.8)$$

where only the finite terms $\widetilde{C}_j^{finite}$ depend on j when supersymmetry is spontaneously broken. Also, in this case, $\widetilde{B}_{1/2}$ is independent of Λ [8].

To $\mathcal{O}(g^2)$, the hidden sector then contributes to the effective action for the gauge supermultiplet fields as [8]

$$\begin{aligned}
\delta\mathcal{L}_{eff} = &\frac{1}{2}g^2\widetilde{C}_0(0)D^2-g^2\widetilde{C}_{1/2}(0)i\lambda\sigma^\mu\partial_\mu\bar\lambda-\frac{1}{4}g^2\widetilde{C}_1(0)F_{\mu\nu}F^{\mu\nu}\\
&-\frac{1}{2}g^2(M\widetilde{B}_{1/2}(0)\lambda\lambda+c.c.)+\ldots. \qquad (4.9)
\end{aligned}$$

The divergent part of (4.8) is the hidden-sector contribution to the gauge beta function:

$$\delta\frac{dg}{d\ln\mu}=\frac{g^3}{16\pi^2}(2\pi)^4 c, \qquad (4.10)$$

with $c>0$.

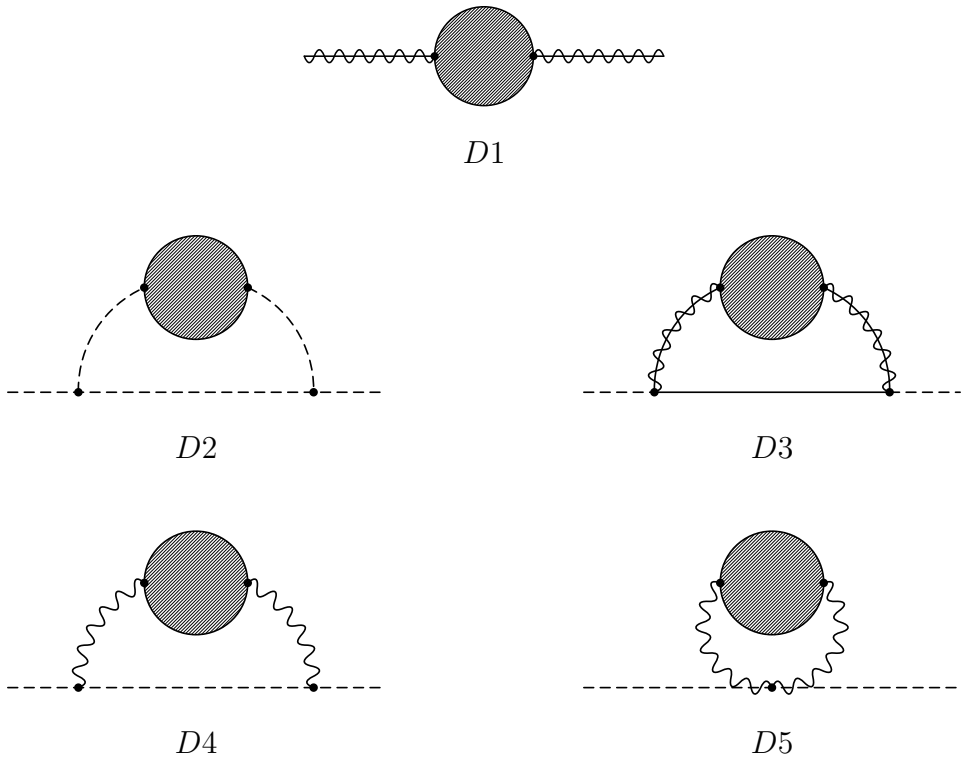


Figure 4.1: Diagram $D1$ gives mass to gauginos and is expressible in terms of the function $\widetilde{B}_{1/2}$. Diagrams $D2$-$D5$ contribute to the masses of sfermions and involve the functions $\widetilde{C}_0$, $\widetilde{C}_{1/2}$, and $\widetilde{C}_1$, respectively.

The diagrams of Figure 4.1, which give masses to the visible sector gauginos and sfermions were evaluated in [8] in terms of the current correlator functions as

$$\begin{aligned} M_a &= g_a^2 M \widetilde{B}_{1/2}^{(a)}(0), \qquad m_{\tilde{f}}^2 = g_1^2 Y_f \xi + \sum_a g_a^4 c_2(a_f) A_a \\ A_a &\equiv -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left(3\widetilde{C}_1^{(a)}(p^2/M^2) - 4\widetilde{C}_{1/2}^{(a)}(p^2/M^2) + \widetilde{C}_0^{(a)}(p^2/M^2)\right) \end{aligned} \tag{4.11}$$

The index a runs over the gauge groups, f runs over the sfermions, Y is the hypercharge, and ξ is an FI parameter. Note that the integrand of A_a has the form of a super-trace[1] and, without additional information or constraints, it looks like it can have either sign.

[1]A related expression appears in [72] for the messenger m_{mess}^2, in the context of models with a separate messenger sector (where it was argued that perturbative estimates based on naive dimensional analysis should be essentially reliable even for strongly coupled susy-breaking sectors).

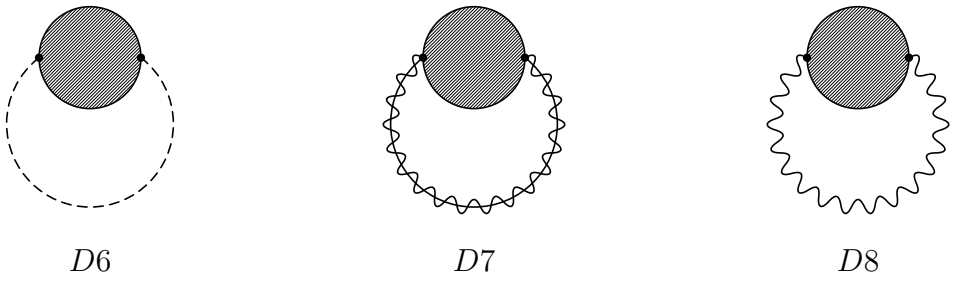


Figure 4.2: Diagrams $D6$, $D7$, and $D8$ give contributions to the effective potential involving the functions $\widetilde{C}_0$, $\widetilde{C}_{1/2}$, and $\widetilde{C}_1$, respectively.

4.3 The effective potential and Higgsed gauge mediation

The sfermions of the visible sector generally have tree-level D-flat directions. With supersymmetry breaking, these directions are lifted first by the two-loop effective potential. Near the origin, this effective potential reduces to the sfermion mass terms. Far from the origin, the effects of susy breaking shut off, and the effective potential becomes very flat. The full effective potential can be of interest for cosmological models and was computed in [72], to leading order in small F-terms, for the case of a weakly coupled messenger sector. Here we give a simple expression for the full effective potential, for arbitrary F-terms, in general gauge mediation.

The effective potential is computed from the diagrams of Figure 4.2. For simplicity, we quote the result for a single $U(1)$ gauge group – the more general case is similar. We find that the effective potential is simply

$$V_{eff}(m_W^2) = \frac{g^2}{2}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+m_W^2}\left(3\widetilde{C}_1(p^2/M^2) - 4\widetilde{C}_{1/2}(p^2/M^2) + \widetilde{C}_0(p^2/M^2)\right). \tag{4.12}$$

Here $m_W = 2g|\langle Q\rangle|$ is the mass of the vector multiplet, where $\langle Q\rangle$ is along a direction which would have been D-flat if not for the supersymmetry breaking. The terms in (4.12) simply come from contracting the massive vector multiplet propagators with the appropriate current-current correlator, e.g. from diagram

$D8$ we have (in Euclidean space)

$$\frac{g^{\mu\nu}}{p^2+m_W^2}(p^2\eta_{\mu\nu}-p_\mu p_\nu)\widetilde{C}_1(p^2/M^2)=\frac{p^2}{p^2+m_W^2}3\widetilde{C}_1(p^2/M^2). \tag{4.13}$$

Diagram $D7$ is similar, with the massive gaugino propagator contracted with (4.5). Diagram $D6$, with the auxiliary field D, requires a bit more attention because it mixes with a real scalar, C, of the massive gauge multiplet:

$$\Delta_{CD}=\begin{pmatrix}1 & m_W\\ m_W & -p^2\end{pmatrix}^{-1}=\frac{1}{p^2+m_W^2}\begin{pmatrix}p^2 & m_W\\ m_W & -1\end{pmatrix}, \tag{4.14}$$

so the D-field propagator is $p^2/(p^2+m_W^2)$, which then yields the $\widetilde{C}_0$ contribution to (4.12).

Let us now verify that our general effective potential (4.12) reduces to the sfermion $m_{\widetilde{f}}^2$ in (4.11), when expanded around the origin. Consider

$$\begin{aligned} m_{\widetilde{f}}^2 &= \frac{\partial V_{eff}}{\partial|\langle Q\rangle|^2} \qquad (4.15)\\ &= -2g^4\int\frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2+m_W^2)^2}\left(3\widetilde{C}_1(p^2/M^2)-4\widetilde{C}_{1/2}(p^2/M^2)+\widetilde{C}_0(p^2/M^2)\right). \end{aligned}$$

Evaluating this for $m_W^2=0$ indeed reduces to the expression of [8].

One can also verify that the general result (4.12) reduces to the effective potential obtained in [72], for the special case of weakly coupled hidden sector (using the expressions for $\widetilde{C}_j$ in the appendix of [8]), when evaluated to leading order in small F-terms. The case of weakly coupled $\mathcal{L}_2$ sector, for general (not necessarily small) F-terms, can also be compared and verified with the result obtained using the formulae of [55].

With no additional work, we can extend our result to allow for the possibility of Higgsed gauge mediation. We simply replace $m_W^2\to m_W^2+4g^2|\delta Q|^2$ in (4.12), and keep the $m_W\neq 0$. Expanding to $\mathcal{O}(|\delta Q|^2)$, the result (4.15), with $m_W\neq 0$, gives the sfermion $m_{\widetilde{f}}^2$ in general Higgsed gauge mediation. For the case of weakly coupled messengers, it can be verified that the result (4.15) indeed reduces to the results obtained in [73] for Higgsed gauge mediation.

4.4 Superspace techniques and analytic continuation in superspace

In the context of weakly coupled gauge mediation, there are nice methods [57, 58] which allow multi-loop quantities, including sfermion masses, to be reduced to one-loop quantities at leading order in small supersymmetry breaking F-terms. Fields φ and $\widetilde{\varphi}$ of the susy-breaking sector get susy-split masses via $W = X\varphi\widetilde{\varphi}$, where X is a spurion (background) chiral superfield $X = M + \theta^2 F$. The results follow from imposing the constraints of holomorphy in X on the effective action. The results are limited to leading order in small F, because terms higher order in F arise from higher super-derivative terms in superspace, which are not considered. Taking $x \equiv |F/M^2| \ll 1$, the methods determine the soft masses to $\mathcal{O}(x)$.

The methods of [57, 58] extend immediately to general gauge mediation. The gaugino masses come from the holomorphic gauge coupling $\tau = \theta/2\pi + 4\pi i/g^2$, which is a holomorphic function $\tau(X)$ below the scale X thanks to the threshold matching and the contribution (4.10) of the hidden sector to the beta function there. The sfermion masses come from the one-loop $\mathcal{O}(g^2)$ contribution to $Z_Q(X, \bar{X})$, which depends of X again via the gauge coupling. This gives $\beta^{(1)}_{g_a}$, and $m^2_{\widetilde{f}} \sim \gamma^{(1)}_{\widetilde{f}} \Delta\beta^{(1)}_{g_a}$, in terms of the beta-function coefficient c in (4.10) and (4.8):

$$M_a \approx \frac{g_a^2}{16\pi^2}(2\pi)^4 c_a \frac{F}{M}, \qquad m^2_{\widetilde{f}} \approx \sum_a 2c_2(a_f)\frac{g_a^4}{(16\pi^2)^2}(2\pi)^4 c_a \Big|\frac{F}{M}\Big|^2. \tag{4.16}$$

In this small F limit, the masses $m^2_{\widetilde{f}}$ are manifestly positive.

The simple expressions (4.16) motivate a parallel spurion analysis of the current correlation functions, to connect with the results of [8], quoted above in (4.11), when expanded in small F. In the small-F limit, we have $\widetilde{C}_j \approx \widetilde{C}_{susy}$, independent of $j = 0, 1/2, 1$. To leading order in small F, we find that the susy-breaking quantities appearing in the soft masses (4.11) can be expressed, for all p^2/M^2, as

$$-M\widetilde{B}_{1/2}(p^2/M^2) = F\frac{\partial}{\partial M}\widetilde{C}_{susy}(p^2/M^2) + \mathcal{O}(F|F|^2/M^6), \tag{4.17}$$

and

$$3\widetilde{C}_1(p^2/M^2)-4\widetilde{C}_{1/2}(p^2/M^2)+\widetilde{C}_0(p^2/M^2) = 2\frac{|F|^2}{p^2}\frac{\partial^2}{|\partial M|^2}\widetilde{C}_{susy}(p^2/M^2)+\mathcal{O}\left(\frac{|F|^4}{M^8}\right), \tag{4.18}$$

It is easily verified that these identities are indeed satisfied for the particular case of weakly coupled messengers, by expanding for small F/M the explicit expressions for $\widetilde{C}_j$ and $\widetilde{B}_{1/2}$ in the appendix of [8]. The identities (4.17) and (4.18) were independently derived, with the same motivation, in a recent paper of Distler and Robbins [71].

One approach is to prove (4.17) and (4.18) directly in terms of the current correlation functions, first enforcing the supersymmetric and current conservation Ward identities, and introducing the supersymmetry breaking spurion via $M \to X = M + \theta^2 F$. The first step is simplified by writing the current supercorrelators in superspace. In particular, the current 2-point functions for unbroken supersymmetry are given by an immediate generalization of the conformal result in [18] to the nonconformal case:

$$\langle \mathcal{J}(z_1)\mathcal{J}(z_2)\rangle = \frac{C(M^4 x_{\bar{2}1}^2 x_{\bar{1}2}^2)}{x_{\bar{2}1}^2 x_{\bar{1}2}^2}, \qquad x_{\bar{1}2}^\mu = x_1^\mu - x_2^\mu - i\theta_1\sigma^\mu\bar{\theta}_1 - i\theta_2\sigma^\mu\bar{\theta}_2 + 2i\theta_2\sigma^\mu\bar{\theta}_1. \tag{4.19}$$

Instead of introducing the spurions in (4.19), we will now do it in terms of the 1PI effective action, since that is anyway more directly relevant for extracting the implications for gauge mediation.

Consider first the 1PI effective action to $\mathcal{O}(F^0)$, neglecting supersymmetry breaking effects. Following the discussion and notation of [58], there is the term involving the gauge fields

$$\begin{aligned}\Gamma_{1PI} &\supset \int d^4p \int d^4\theta\ \gamma(p^2) W^\alpha \frac{D^2}{-8p^2} W_\alpha + h.c. \\ &= \int d^4p \int d^2\theta \frac{1}{2}\gamma(p^2) W^\alpha W_\alpha + h.c.\end{aligned} \tag{4.20}$$

There are also terms involving the visible sector matter, e.g.

$$\Gamma_{1PI} \supset \int d^4p \int d^4\theta \zeta(p^2)\big(Q^\dagger e^V Q + \widetilde{Q}^\dagger e^{-V}\widetilde{Q}\big). \tag{4.21}$$

Classically, $\gamma(p^2) = 1/g^2$, and one can define the quantum running couplings and Z-factors as $\gamma(p^2)|_{p^2=-\mu^2} = 1/g^2(\mu^2)$, and $\zeta_f(p^2)|_{p^2=-\mu^2} = Z_f(\mu^2)$. To $\mathcal{O}(g^2)$, the hidden sector current-current two-point functions contribute

$$\delta\gamma(p^2/M^2) = g^2\widetilde{C}_{susy}(p^2/M^2). \tag{4.22}$$

The leading term in the low-momentum expansion of (4.20) then gives the terms (4.9) in the effective Lagrangian (in the susy limit). We are here interested in the relation (4.22) for general p^2.

We now introduce the supersymmetry breaking spurion via $M \to X = M+\theta^2 F$. To $\mathcal{O}(|F|^2)$, the relation (4.22) is simply preserved, and both sides pick up θ components. The 1PI action then includes

$$\begin{aligned}\Gamma_{1PI} \;\supset\; & \frac{1}{2}\int d^4p \int d^2\theta\, \left(\gamma(p^2)|_0 + \theta^2\gamma(p^2)|_{\theta^2}\right)W^\alpha W_\alpha + h.c. \\ & + \int d^4p\, \gamma(p^2)|_{\theta^2\bar\theta^2}\frac{\lambda\sigma^\mu p_\mu\overline{\lambda}}{-p^2}\end{aligned} \tag{4.23}$$

where, according to (4.22) with the spurions, we have

$$\begin{aligned}\gamma(p^2)|_{\theta^2} &= g^2F\frac{\partial}{\partial X}\widetilde{C}_{susy}(p^2/|X|^2)\Big|_{X=M}, \\ \gamma(p^2)_{\theta^4} &= g^2|F|^2\frac{\partial^2}{|\partial X|^2}\widetilde{C}_{susy}(p^2/|X|^2)\Big|_{X=M}.\end{aligned} \tag{4.24}$$

The $\gamma(p^2)|_{\theta^2}$ term in (4.22) corresponds to the non-supersymmetric analog of (4.22), generated in the effective action by the non-supersymmetric current two-point functions:

$$\gamma(p^2)|_{\theta^2} = -g^2M\widetilde{B}_{1/2}(p^2/M^2) - \mathcal{O}(F|F|^2), \tag{4.25}$$

where on the RHS we keep only the $\mathcal{O}(F)$ term, and the factor of $-M$ is as in (4.9). Comparing (4.24) and (4.25) gives the relation (4.17).

Likewise, the $\gamma(p^2)|_{\theta^4}$ term in (4.23) is generated by the non-supersymmetric analog of (4.22), from supersymmetry breaking contributions of the current-current two-point functions to the effective action. Indeed, it is easily seen, as in (4.9), that a supersymmetry breaking shift of $\widetilde{C}_{1/2}(p^2/M^2)$ will generate the $\gamma(p^2)|_{\theta^4}$ term in (4.23). Since this supersymmetry breaking term comes from such a $\widetilde{C}_{1/2}$ shift

relative to $\widetilde{C}_0$ and $\widetilde{C}_1$, it is generated only by the supertrace:

$$\frac{1}{p^2}\gamma(p^2)|_{\theta^4} = \frac{g^2}{2}\left(\widetilde{C}_0(p^2/M^2) - 4\widetilde{C}_{1/2}(p^2/M^2) + 3\widetilde{C}_1(p^2/M^2)\right) - \mathcal{O}(|F|^4)), \quad (4.26)$$

where only terms to $\mathcal{O}(|F|^2)$ are kept on the RHS. This relation essentially appears already in [58] (in terms of the gauge field effective propagators), where it was noted to follow from considering all the possible contributing terms involving the spurion and supercovariant derivatives acting on it. Comparing (4.26) with (4.24) yields the relation (4.18).

It is evident that the θ^2 term in (4.23) yields the gaugino mass, and (4.25) agrees with the result of [8], quoted above in (4.11). This result for M_a indeed agrees with (4.16), as seen from (4.17) and the contribution of the $\log\Lambda$ term in (4.8) to $\widetilde{C}_{susy}$. The expression for $m^2_{\tilde{f}}$ quoted above in (4.16) follows from (4.18) and the result (4.11).

This chapter is a reprint of the material as it appears in "Comments on General Gauge Mediation," K. Intriligator, M. Sudano, JHEP **0811**, 008 (2008), arXiv:0807.3942, of which I was a co-author.

Chapter 5

Surveying Pseudomoduli: the Good, the Bad and the Incalculable

5.1 Introduction

5.1.1 Motivation

Dynamical supersymmetry breaking (DSB) is a promising scenario [74] for explaining the huge hierarchy between the weak scale and the Planck scale. Only very special examples seem to exhibit complete DSB at weak electric coupling (see e.g. [35] and references cited therein). Another framework for DSB, which leads to many new classes of examples, is via theories with IR-free magnetic duals[1], with SUSY broken at tree-level in the dual. Accepting long-lived metastable vacua further expands the classes of theories with DSB, among them massive SQCD, which suggests that metastable DSB can be common, even generic, in field theory and string theory [61]. See [76] for a recent review and references.

In analyzing such theories, one must always pay attention to the tree-level flat directions in the potential. Such "pseudomoduli" fields – which we will collectively denote throughout by Φ – are always present in the low-energy F-term SUSY

[1]The IR phase must be under control, as seen in the original, still inconclusive, example [75].

breaking models.[2] One is the superpartner of the Goldstino, and typically there are many others, corresponding to (at least some of) the moduli of the IR-free theory before turning on the supersymmetry-breaking perturbation. To definitively determine whether or not supersymmetry is broken requires determining what happens to all of the pseudomoduli in the quantum theory. In the context of DSB in IR-free duals, as we will discuss, pseudomoduli are either "good," "bad," or "incalculable," depending on their quantum effective potential and how it is generated.

We will distinguish *calculable* DSB models, where the demonstration of DSB is under full control, from models where incalculable quantum effects could be important. In the original models of DSB [35], calculability required that the fields of the electric theory be far from the origin, i.e. $|Q_{elec}| \gg |\Lambda|$, where Λ is the strong-coupling scale. On the other hand, calculable DSB in a low-energy IR-free dual requires the dual fields to be close to the origin, $|q_{mag}| \ll |\Lambda|$, in order for unknown higher-dimension operators, which are suppressed by powers of $|\Lambda|$, to be unimportant.

The condition $|q_{mag}| \ll |\Lambda|$ can be non-trivial to check for the pseudomoduli fields Φ as it entails computing their quantum effective potential. A model has calculable DSB only if $V_{eff}(\Phi)$ stabilizes all pseudomoduli below the cutoff scale, $|\Phi| \ll |\Lambda|$. All bets are off if any pseudomodulus has a potential with a runaway[3] to the cutoff of the low energy theory, $\langle\Phi\rangle \sim \Lambda$. In the context of metastable DSB, one must also ensure that no pseudomodulus gives a sliding direction down to the SUSY vacuum. Because the low-energy theory is IR free, the lowest non-trivial loop order of $V_{eff}(\Phi)$ suffices. In the SQCD example, all pseudomoduli are safely stabilized at one loop in the low-energy theory [61]. But in many other potentially interesting generalizations, e.g. [78, 79, 80, 81, 82], some pseudomoduli are unlifted at one loop, so a higher-loop analysis is then required to determine if they have dangerous runaways to $\langle\Phi\rangle \sim \Lambda$.

[2]This was proven in [77] for renormalizable Käher potentials. Additional non-renormalizable Käher potential terms, which are present (with unknown coefficients) in the IR-free low-energy duals, contribute to lifting the pseudomoduli. As we will discuss further in what follows, such contributions are negligible in "calculable" models of DSB.

[3]For calculable electric DSB [35], one instead checks that V_{tree} prevents $Q_{elec} \to \infty$ runaways.

We will here consider general aspects of pseudomoduli, and their dynamical lifting by $V_{eff}(\Phi)$. This will serve to determine whether pseudomoduli are "good," "bad," or "incalculable." Briefly put, we refer to pseudomoduli as "good" if their quantum effective potential is calculable and robust, stabilizing them within the regime of validity of the IR-free low-energy theory. The "bad" pseudomoduli, on the other hand, have a calculable, robust potential, but with a runaway to the cutoff of the low-energy theory. Finally, the "incalculable" pseudomoduli are inconclusive, because their quantum effective potential is not robust against incalculable effects from modes outside of the low-energy theory.

We should note that the "bad" and "incalculable" cases can be salvaged by a simple fix, which has already been implemented in models in the literature: one can modify the ultraviolet theory under consideration to give any dangerous pseudomoduli Φ_d a supersymmetric mass "by hand." This can be done by introducing additional gauge singlets Σ, coupled to pseudomoduli Φ_d via $W_{tree} \supset m\Sigma\Phi_d$, as was implemented for the examples in [78, 79]. (This generally introduces additional pseudomoduli, which need to be examined.) Alternatively, one can add the term $W_{tree} \supset m\Phi_d^2$, as in [81, 82]. (This can introduce new supersymmetric vacua, so the lifetime of any DSB vacua must be re-checked.) From the perspective of the original UV theory, these are modifications of W_{tree} by some particular higher-dimension operators. In fact, the recent work [82] illustrates, in the context of a particular example, how such a modification can turn "bad" pseudomoduli into a model-building virtue, providing a meta-stable vacuum where R-symmetry is broken spontaneously.

5.1.2 Methods and connection to gauge mediation

In our general analysis, we will find it useful to adapt the language of gauge mediation in order to characterize the coupling of the pseudomoduli to the SUSY-breaking sector. Recall the idea of gauge mediation (see e.g. [41] for a review and references): to communicate "hidden" sector SUSY-breaking to the "visible" sector (the MSSM or some extension) via loops of "messenger" fields which couple directly to the SUSY-breaking fields and which are charged under the SM gauge

groups. In a wide class of gauge mediated models, the details of the SUSY-breaking sector are irrelevant and the dynamics can be described in terms of a spurion field X that breaks supersymmetry spontaneously through its F-component expectation value, $\langle X \rangle = M + \theta^2 f$.[4] The spurion then couples to messengers φ, $\widetilde{\varphi}$ via

$$W \supset h_X X \varphi \widetilde{\varphi}, \tag{5.2}$$

which gives the messenger scalars SUSY-split masses at tree-level (and the MSSM sfermions soft masses at two loops [36, 38, 39, 40]).

As we will see, the key point is that in order for the pseudomoduli potential to be calculable, the theory necessarily has some "messenger" fields φ with SUSY-split tree-level masses. Then the pseudomoduli are analogous to the sfermions of the visible sector, and they will feel the effects of SUSY-breaking through weakly-coupled "messengers." Using this language of messengers, we will apply and extend various results and techniques developed for gauge mediation to this new context.

Each calculable pseudomodulus Φ is lifted at the loop order given by the number of relevant interactions needed to couple it to the messengers φ. (As we will discuss, the pseudomodulus is "incalculable" if any of these interactions are power-law irrelevant.) For some classes of models we will consider, the pseudomoduli are first lifted at three or more loops. Since it would be difficult (to say the least) to directly compute the needed multi-loop effective potential for such cases, we will instead develop here a simpler method, which is related to those of [56, 57, 58], and which allows us to determine multi-loop effective potentials in terms of one-loop

[4] We consider F-term breaking, where φ is a chiral multiplet. As a concrete example, the SUSY-breaking sector can be a generalized O'Raifeartaigh model, e.g. one like

$$W_{low} \supset X\varphi^2 + \phi\varphi^2 + fX, \tag{5.1}$$

where ϕ and φ contain multiple fields, in representations of a group. This wide class of supersymmetry breaking models, with only cubic and linear terms (called O'R$_2$ in [83]), includes the original inverse hierarchy model of [84], and also the rank-condition supersymmetry breaking of [61]. Accounting for $\langle \varphi \rangle \neq 0$ in these models, they are of type 1 in the classification of [44]. If ϕ and φ are charged under a sufficiently strong (but still perturbative) gauge group, some pseudomoduli can have a minimum away from the origin, spontaneously breaking the (accidental) $U(1)_R$ symmetry of this O'Raifeartaigh sector; see e.g. [84, 85, 83]. These examples are given only for illustrative purposes.

RG running data. Our method will apply in the regime

$$m_0 \ll |\Phi| \ll |\Lambda|, \tag{5.3}$$

where the pseudomodulus is relatively far from the origin, but still below the cutoff of the low-energy theory. Using our method, one can easily determine whether calculable pseudomoduli are "good" or "bad" in this regime. Examples suggest that the behavior of the potential in the range (5.3) is indeed a reliable indicator of whether the pseudomodulus is good or bad.

The analogy between our analysis of pseudomoduli and the standard analysis of gauge mediation is only an analogy, and it is important to stress some differences between the two scenarios:

1. The most obvious difference is that in actual gauge mediation the sfermions only acquire their soft masses through loops involving SM gauge fields, but here the messenger couplings to pseudomoduli are not restricted by flavor considerations. So as we will see in various examples, the pseudomoduli potential can involve both Yukawa and gauge interactions, and it can start at any loop order.

2. The messengers φ can have large SUSY-breaking mass splittings (as in [61] where some messengers have $x \equiv |F_X/M^2| = 1$). Then the beautiful methods [56, 57, 58] which have been developed for $x \ll 1$, to extract multi-loop effects from one-loop data, are not directly applicable. Nevertheless, we will here discuss a different limit, where similar methods can be employed.

3. Something that generally does not happen in gauge mediation, but which can easily happen here, is that some messengers can have $\langle\varphi\rangle \neq 0$ (again, as is the case in [61]), and this can partly or fully Higgs a hidden-sector gauge group. (See [73] for additional discussion of mediation by Higgsed gauge groups.)

4. Because the low energy theory is assumed to be IR free, all interactions between pseudomoduli Φ and messengers φ are either marginally irrelevant – Yukawa couplings or IR free gauge couplings – or power-law irrelevant.

5. Being SUSY-breaking mediation in an effective theory, there are unknown higher dimension operators, suppressed by powers of $1/|\Lambda|$, which could be potentially dangerous; in the "good" cases, such operators are unimportant.

5.1.3 Summary of the survey of pseudomoduli

Let us now survey various types of pseudomoduli, and their dynamical lifting. While some of the pseudomoduli may seem rather contrived, all of the ones in this list occur "naturally" in the effective magnetic description of some strongly-coupled gauge theory. We will study these theories in more detail in Section 5.5, and with examples in Section 5.6.

- **Gauge singlet pseudomoduli, with cubic direct couplings to messengers.**

$$W_{low} \supset \Phi_1 \varphi^2 + \tilde{\Phi}_1 \varphi \chi, \tag{5.4}$$

  where Φ_1 and $\tilde{\Phi}_1$ are gauge-singlet pseudomoduli, φ have SUSY-split tree-level masses, whereas χ do not. For light messengers φ, the Φ_1, $\tilde{\Phi}_1$ fields enter into the one-loop Coleman-Weinberg [86] potential $V_{eff}^{(1)}$, which safely stabilizes such pseudomoduli near the origin. Such pseudomoduli are "good," as they have a calculable potential which prevents their vev from sliding to the cutoff.

- **Gauge singlet pseudomoduli, with cubic indirect couplings to messengers.** Add to (5.4) the term

$$W_{low} \supset \Phi_2 \chi^2. \tag{5.5}$$

  Pseudomoduli like Φ_2 are first lifted at two loops, since they couple to the messengers via $\Phi_2 \leftrightarrow \chi \leftrightarrow \varphi$, where each $\leftrightarrow$ costs a loop via a Yukawa interaction. An example realizing such pseudomoduli is SQCD with both massive and massless flavors [78]; the full two-loop effective potential for the analogue of Φ_2 in this example was recently explicitly computed in [87], and was shown to have monotonically decreasing runaway behavior. We will here

use a simpler analytic method to determine the potential, using only one-loop data, in the regime of relatively large $\langle\Phi_2\rangle$. The potential in this regime reveals that such pseudomoduli are "bad," as they have a calculable runaway potential pushing their vev to the cutoff of the low-energy theory.

- **Higgsing pseudomoduli, gauge-coupled to messengers.** Charged matter fields can lead to pseudomoduli Φ_q, corresponding to their D-flat expectation values[5]. For lack of a better name, we call these "Higgsing" pseudomoduli. If the messengers φ are charged under the same gauge group as the matter Φ_q, the coupling $\Phi_q \leftrightarrow$ gauge $\leftrightarrow \varphi$, leads to a two-loop effective potential (again, each $\leftrightarrow$ coupling costs a loop). In this sense, this type of pseudomodulus is most directly analogous to that of sfermions in ordinary gauge mediation. The generalization to determine the sfermion soft mass-squared in the here-relevant case of Higgsed gauge groups was recently considered in [73]. As we discuss, these pseudomoduli are safely stabilized: their two-loop effective potential pushes them to the origin. All pseudomoduli from gauge non-singlets have this good, two-loop, stabilizing effect.

- **Saxion-type pseudomoduli.** These are gauge-singlet fields Φ_3 that couple to the messengers φ only via superpotential interactions

$$W_{low} \supset \Phi_3\ p^2, \tag{5.6}$$

to charged matter fields p, which in turn couple to the φ via gauge interactions. Since Φ_3 pseudomoduli couple to the messengers φ via $\Phi_3 \leftrightarrow p \leftrightarrow$ gauge $\leftrightarrow \varphi$, they are lifted first by a *three-loop* effective potential. Φ_3 is referred to as "saxion-type" because of how it enters into the low-energy theory when it gets a large expectation value. We will here argue that such saxion pseudomoduli are bad, with a destabilized runaway to the cutoff. This is qualitatively similar (though differing in the details) to the behavior found in [58] for the saxions in the context of the usual, heavy-messenger scenario of gauge mediation of supersymmetry breaking.

[5] If there are non-zero D-term expectation values, such fields can be lifted by tree level or one-loop supersymmetry breaking effects, and thus not be pseudomoduli after all. We thank N. Seiberg for this comment.

- **Gauge messengers.** In the previous examples, we have assumed implicitly that the SUSY-breaking spurion X in (5.2) is a gauge singlet. If instead it is charged under some gauge group, then the massive gauge fields themselves become messengers. A classic example is the theory of [84], where gauge messengers arise from the F-term of an adjoint of $SU(5)_{GUT}$. See also [57] for a discussion of some general aspects of gauge messengers. In such cases, the Higgsing and saxion pseudomoduli couple more directly to the messengers and their potentials are generated at one lower loop order. As we will discuss, the sign of the pseudo-moduli effective potential is reversed in both these cases, meaning that the Higgsing pseudomoduli are destabilized while the saxion pseudomoduli are stabilized by the gauge messengers.

- **Irrelevantly coupled pseudomoduli:** are gauge-singlet pseudomoduli which couple to the SUSY-breaking sector only via power-law irrelevant interactions. For example, pseudomoduli Φ_4 with direct coupling to messengers and some other fields,

$$W_{low} \supset \frac{1}{\Lambda^{n+m-2}} \Phi_4 \varphi^n p^m, \tag{5.7}$$

  for $n + m \geq 3$. (As we discuss, some of these interactions can become relevant, if $m < 3$ and $\langle \varphi \rangle \neq 0$.) Such pseudomoduli are *not* reliably lifted by quantum effects in the low energy theory: the calculable effective potential in the low energy theory is not parametrically larger than incalculable effects of the unknown irrelevant terms in the effective Kahler potential. All such models are thus inconclusive: whether or not their pseudomoduli are dynamically stabilized depends on the sign of terms which cannot, in principle, be calculated with currently known methods.

5.1.4 Outline

The outline of this paper is as follows. The next section discusses general aspects of DSB in IR-free duals. In Section 5.3, we note that models with power-law irrelevantly coupled pseudomoduli are always inconclusive. In Section 5.4, we note that there is a limit where even multi-loop pseudomoduli effective potentials can

be easily computed from one-loop quantities: this is the limit where the pseudomodulus is far from the origin, as compared with the tree-level mass scale, but still below the cutoff. In Section 5.5, we use these results to survey which of the above pseudomoduli types are safe, and which have a dangerous runaway to the cutoff. In Section 5.6, we apply these results to comment on a number of examples.

5.2 Generalities of DSB in IR free duals

The low-energy theory is assumed to be an IR-free effective theory with a cutoff scale given by Λ. The low-energy theory will in general have a variety of mass scales, including the SUSY-breaking scale set by the parameter f in $W_{low} \supset fX$. This mass scale must be well below the cutoff of the low-energy theory, so we define a small parameter ϵ, given by

$$\epsilon \equiv \frac{f}{\Lambda^2} \sim \left(\frac{m_0}{\Lambda}\right)^2 \qquad \text{with} \quad |\epsilon| \ll 1. \tag{5.8}$$

Calculable IR-free DSB requires such a small parameter ϵ.[6] The mass m_0 in (5.8) sets the scale of the tree-level masses in the low-energy theory, as well as the supersymmetry-breaking scale. The IR-free low-energy theory has unknown corrections from higher dimension operators, in particular, Kähler potential corrections, suppressed by powers of $1/|\Lambda|$. Such incalculable terms contribute to the pseudomoduli potentials – for example $K_{incalc} \supset cX\bar{X}\Phi\bar{\Phi}/|\Lambda|^2$ leads to $V_{incalc} \supset -c|m_0|^4|\Phi|^2/|\Lambda|^2$ with unknown $\mathcal{O}(1)$ coefficient c. In general, there are incalculable contributions to pseudomoduli potentials of the form

$$V_{incalc} \sim |m_0|^4 f_{incalc}\left(\frac{|\Phi|^2}{\Lambda^2}\right) \sim |\epsilon|^2\Lambda^4 f_{incalc}\left(|\epsilon|\frac{|\Phi|^2}{m_0^2}\right) \tag{5.9}$$

where the real analytic function f_{incalc} has a regular Taylor expansion around the origin. On the other hand, the calculable effective potential in the low-energy theory can depend only on m_0 and not on Λ, so

$$V_{calc} \sim |m_0|^4 f_{calc}\left(\frac{|\Phi|^2}{m_0^2}\right) \sim |\epsilon|^2\Lambda^4 f_{calc}\left(\frac{|\Phi|^2}{m_0^2}\right) \tag{5.10}$$

[6]The parameter ϵ is related to a superpotential coupling λ of a dual, UV description by $\epsilon = \lambda\Lambda^{\Delta_{UV}-3}$, where $\Delta_{UV} > 1$ is the UV dimension of the composite operator X (and $\Delta_{IR} = 1$). Thus $\epsilon \ll 1$ is natural if $\Delta_{UV} \geq 3$, so the UV coupling is irrelevant or marginally irrelevant. If $\Delta_{UV} < 3$, the small parameter ϵ could still be naturalized by additional dynamics [88].

for some real function f_{calc}. The calculable potential is robust against the unknown effects provided that $|\epsilon| \ll 1$ and $|\Phi| \ll |\Lambda|$.

The IR-free low-energy theory can have marginally irrelevant coupling constants, like the Yukawa coupling h_X in $W_{low} \supset h_X X \varphi^2$, or the gauge coupling g of an IR-free gauge group. Such couplings take some fixed, but unknown $\mathcal{O}(1)$ values at the UV cutoff $|\Lambda|$ of the low-energy theory, $g(|\Lambda|) \sim h(|\Lambda|) \sim \mathcal{O}(1)$, and then run down to smaller values in the IR. The running is over a large energy range, from $|\Lambda|$ down to the much lower scale m_0 of the tree-level masses, below which the running essentially stops. The couplings thus run down to small IR values. However, it is important that they are not *too* small:

$$\frac{g^2}{16\pi^2} \sim \frac{h^2}{16\pi^2} \gtrsim (-\ln|\epsilon|)^{-1} \qquad \text{so} \qquad |\epsilon| \ll \frac{|g|^2}{16\pi^2},\ \frac{|h|^2}{16\pi^2} \ll 1. \tag{5.11}$$

This ensures that perturbation theory in the low-energy theory is reliable, with higher-order perturbative effects suppressed as compared with leading-order effects. A calculable ℓ-loop mass-term contribution coming from gauge or Yukawa interactions generally has $m^2_{calc,\ell} \sim |\epsilon \Lambda^2 h^{2\ell+1}|$, so pseudomoduli are parametrically lighter, by appropriate powers of the loop-factor (5.11), than the fields which get tree-level masses. Nevertheless, for any ℓ, it follows from (5.11) that their calculable mass and potential can be robust, $m^2_{calc,\ell} \gg m^2_{incalc}$.

Finally, let us remark that non-perturbative effects are insignificant as long as pseudomoduli are not too far from the origin. Non-perturbative effects can only become significant if the low-energy theory is driven interacting by a sufficiently large pseudomodulus expectation value, $|\Phi| > |\Phi_{n.p.}|$. Since the low-energy theory is IR free, the scale $\Phi_{n.p.}$ where non-pertubative effects could become important is generally above the mass scale of the light fields, $|\Phi_{n.p.}| \gg m_0$. For example, non-perturbative effects are irrelevant for the metastable DSB vacua of [61], but the $W_{n.p.}$ eventually becomes important, and leads to the SUSY vacua, at the scale $\Phi_{n.p.} \sim \Lambda_{low} \sim \epsilon^{(N_f - N_c)/N_c} \Lambda$. In the regime where pseudomoduli are not too far from the origin, $|\Phi| \ll |\Phi_{n.p.}|$, perturbative effects are the most important, and non-pertubative effects are negligible. For example, even if non-perturbative effects happen to generate a runaway for a pseudomodulus, the pseudomodulus could still be safely stabilized in the regime $|\Phi| \ll |\Phi_{n.p.}|$ by the larger perturbative effects

there (see [89] for an example of this).

5.3 Irrelevantly coupled pseudomoduli are always inconclusive

In this section, we discuss power-law irrelevantly coupled pseudomoduli, like Φ_4 in (5.7). Consider the least irrelevant example,

$$W_{low} \supset \frac{1}{\Lambda}\Phi_4\widetilde{\Phi}_4\varphi^2. \tag{5.12}$$

This term can potentially become relevant, if the DSB vacuum has $\langle\varphi\rangle \neq 0$. For example, we could have $\langle\varphi^2\rangle \sim \epsilon\Lambda^2/h$ as in [61], where we have included an IR-free coupling constant h (5.11). (We will illustrate this with a concrete example in Subsection 5.6.5.) In this case, (some components of) Φ_4 are not pseudomoduli after all – they get a tree-level supersymmetric mass $m_{calc} \sim \langle\varphi^2\rangle/\Lambda \sim \epsilon\Lambda/h$. Comparing this m_{calc} with the unknown mass contribution (5.9), $m^2_{incalc} \sim |\epsilon\Lambda|^2$, we see that the calculable tree-level masses are here just barely larger, and thus just barely robust, thanks to the $h^{-2} \sim -\ln|\epsilon|$ enhancement of m^2_{calc}.

When some fields like Φ_4 in (5.12) are pseudomoduli, however, their m^2_{calc} comes with additional loop factors of the IR-free couplings, so $m^2_{calc} \sim |h^{\ell-1}\epsilon\Lambda|^2$, which for any loop order $\ell \geq 1$ is not robust against the incalculable contributions (5.9). Pseudomoduli Φ_4 with couplings which are more irrelevant than (5.12) have even smaller m^2_{calc}. The conclusion is that the effective potentials for power-law irrelevantly coupled pseudomoduli can never be reliably computed in the low-energy effective field theory. It is impossible to determine whether or not such pseudomoduli are safely stabilized at a vacuum within the regime of validity of the low-energy theory, with expectation values properly below its cutoff Λ. Even if the low-energy theory appears to break supersymmetry at tree-level, supersymmetry might not be broken after all, if there is no static DSB vacuum within the regime of validity of the low-energy theory. We refer to such pseudomoduli and theories as *incalculable*.

This point afflicts and renders as inconclusive many potential examples of (per-

haps metastable) DSB via IR-free duality dynamics; for example, all of the duality examples of [90, 91, 92], and the many other similar generalizations. All such examples have incalculable pseudomoduli. Thus *none* of these examples can have reliably calculable metastable DSB – they are all inconclusive. As discussed in the introduction, one can still modify the UV theory by hand to give tree-level masses to the incalculable pseudomoduli, as in [79, 81], for example.

5.4 A regime where the pseudomodulus' potential follows simply from running

The effective potential for pseudomoduli which are lifted at one loop is easily computed from the expression for $V_{eff}^{(1)}$ of [86]. For pseudomoduli which are first lifted at two loops, one can, in principle, use the expression for $V_{eff}^{(2)}$ in [55], though in practice this can be quite technically involved – see [87] for an example and some methods. And pseudomoduli like the saxion, which is lifted first at three loops, would require extensive work in order to evaluate $V_{eff}^{(3)}$.

Here we note that there is a useful regime where all of the general higher-loop effective potentials can be easily determined by one-loop quantities, through a generalization of the wavefunction renormalization methods of [57, 58]. The regime of interest is where the pseudomodulus Φ is relatively far from the origin but still within the validity of the low-energy effective theory, i.e.[7]

$$m_0 \ll |\Phi| \ll |\Lambda|, \tag{5.13}$$

where m_0 is the typical mass scale of the light fields in the low-energy theory, or equivalently the scale at which SUSY is broken. (We also assume that $|\Phi| \ll |\Phi_{n.p.}|$, so non-perturbative effects are negligible.) It can be useful to know the potential in this regime, since if it increases with $|\Phi|$, then we can be sure that the pseudomodulus must be stabilized *somewhere* along its flat direction. On the other hand, if the potential decreases in the range (5.13), then this is evidence for

[7] We ignore any factors of coupling constants which could be multiplying Φ in this section, since they will be irrelevant to the discussion.

runaway behavior – although, from this computation alone, one cannot rule out the possibility that there is a local minimum of the effective potential near the origin.

To compute the effective potential in the regime (5.13), we use the fact that the pseudomoduli only couple to the other fields in the theory linearly. Thus at large Φ, all that happens is some fields of the low-energy theory get masses $\sim \Phi$. Moreover, since $|\Phi| \gg m_0$, these massive fields are approximately supersymmetric. So to a good approximation, integrating them out yields an approximately supersymmetric effective theory below the Φ scale, where the only dependence on Φ comes from threshold effects in the effective Kähler potential. If we assume for simplicity that a single field X has nonzero F-term vev, $F_X = f \neq 0$, then the Wilsonian effective action below the scale Φ takes the form

$$K_{eff} = Z_X(Q; |\Phi|) X^\dagger X + \ldots, \qquad W_{eff} = fX + \ldots \tag{5.14}$$

where Q is the RG scale and Z_X is X's wavefunction renormalization. In the regime (5.13), the leading-log-enhanced dependence of Z_X on Φ is determined using only one-loop supersymmetric RGEs. Then using this in computing the *tree-level* vacuum energy in the effective theory gives the leading approximation to the effective potential for Φ:

$$V_{eff}(\Phi) \approx |f|^2 Z_X(m_0; |\Phi|)^{-1} \tag{5.15}$$

in the regime (5.13).

We will find it convenient to introduce the notation $\Omega_X = -\frac{1}{2} \log Z_X$ so that the anomalous dimension of X is given by

$$\gamma_X = \frac{d\Omega_X}{dt} \tag{5.16}$$

where $t = \log \frac{Q}{m_0}$ is the RG time. Then (5.15) becomes

$$V_{eff}(\Phi) \approx |f|^2 e^{2\Omega_X(m_0; |\Phi|)} \tag{5.17}$$

The details of the calculation of Ω_X are contained in Appendix B. The upshot is that the lowest-order leading-log contribution to Ω_X is given by

$$\Omega_X(m_0; |\Phi|) = const. - \frac{1}{n!} \Delta\Omega_X^{(n)} (-t_\Phi)^n + \mathcal{O}(\kappa^{n+1}) \tag{5.18}$$

where $t_\Phi \equiv \log \frac{|\Phi|}{m_0}$ and *const.* refers to the $|\Phi|$ independent part of the wavefunction; κ is the loop-counting parameter (like $\kappa_h = h^2/16\pi^2$ or $\kappa_g = g^2/16\pi^2$); and $\Delta\Omega_X^{(n)}$ is short for

$$\Delta\Omega_X^{(n)} \equiv \frac{d^n\Omega_X}{dt^n}\bigg|_{t_\Phi^-}^{t_\Phi^+} \tag{5.19}$$

i.e. the discontinuity at $t = t_\Phi$ in the nth derivative of Ω_X with respect to RG time. Each derivative of Ω_X with respect to RG time brings down a factor of the loop-counting parameter, so $\Delta\Omega_X^{(n)} \sim \mathcal{O}(\kappa^n)$ if one uses the one-loop anomalous dimension in (5.16). Higher-loop corrections to the anomalous dimension add additional factors of κ and do not contribute at lowest leading-log order, so in the following all anomalous dimensions and beta functions will implicitly be one-loop quantities in order to simplify the notation.

Explicitly, we have for the first few values of n:

$$\Delta\Omega_X^{(1)} = \Delta\gamma_X, \qquad \Delta\Omega_X^{(2)} = \sum_I \frac{\partial\gamma_X}{\partial g^I}\Delta\beta^I, \qquad \Delta\Omega_X^{(3)} = \sum_{I,J} \frac{\partial\gamma_X}{\partial g^I}\frac{\partial\beta^I}{\partial g^J}\Delta\beta^J, \tag{5.20}$$

where $\beta^I = dg^I/dt$ and Δ refers to the discontinuity across the Φ threshold as in (5.19). Note that these formulas for $\Delta\Omega_X^{(n)}$ assume that the lower order $\Delta\Omega_X^{(m<n)}$ vanish. This is why, for example, we have not written a contribution to $\Delta\Omega_X^{(3)}$ from $\frac{\partial^2\gamma_X}{\partial g^I \partial g^J}\beta^I\Delta\beta^J$, since if such a term were non-zero it would've already contributed to $\Delta\Omega_X^{(2)}$ as well.

In any event, substituting (5.18) into (5.17), we obtain at the lowest leading-log order:

$$V_{eff}(\Phi) \approx const. - \frac{2}{n!}V_0\Delta\Omega_X^{(n)}\left(-\log\frac{|\Phi|}{m_0}\right)^n \tag{5.21}$$

where $V_0 = |f|^2$ is the tree-level vacuum energy. In this way, the n-loop leading-log potential is completely determined by one-loop quantities.[8] The sign of the coefficient of the leading-log term then determines whether the pseudomodulus Φ

[8]As will be clear in the examples, the order n of the leading-log effective potential approximation (5.21) indeed agrees with the expected loop order, given by the number of interactions needed to couple the pseudomodulus Φ to some messengers with SUSY-split masses. Thus we can be confident that the approximations used to obtain (5.21) are indeed capturing the dominant term in the effective potential (in the regime (5.13)).

is "good" or "bad" – in short, Φ is

$$\text{"good" if } \ (-1)^{n+1}\Delta\Omega_X^{(n)} > 0 \tag{5.22}$$

$$\text{"bad" if } \ (-1)^{n+1}\Delta\Omega_X^{(n)} < 0, \tag{5.23}$$

where the loop order n is the lowest number for which $\Delta\Omega_X^{(n)} \neq 0$.

It is trivial to generalize to the case where multiple fields X_i have non-zero F-terms. Then the leading effective potential for a pseudomodulius Φ in the range (5.13) is given by:

$$V_{eff} \approx \sum_i |F_{X_i}|^2 (Z_{X_i}(m_0;\Phi))^{-1} \equiv \sum_i |F_{X_i}|^2 e^{2\Omega_{X_i}(m_0;|\Phi|)} \ , \tag{5.24}$$

and each term in the sum can be approximated as in (5.21),

$$|F_{X_i}|^2 (Z_{X_i}(m_0;\Phi))^{-1} \approx const. - \frac{2}{n_i!}|F_{X_i}|^2 \Delta\Omega_{X_i}^{(n_i)} \left(-\log\frac{|\Phi|}{m_0}\right)^{n_i}, \tag{5.25}$$

where $\Delta\Omega_{X_i}^{(n_i)}$ is defined as in (5.19). Then the potential (5.24) is approximated by keeping only those terms i with the lowest loop order, i.e. the smallest value of n_i. In the next section, we will apply (5.21) and (5.25) to the cases of interest.

Finally, let us make a few comments on the various corrections to (5.15).

1. Finite effects cannot be captured by the one-loop RGEs, but they are clearly subleading compared to the large logarithms.

2. Loop effects in the effective theory below the scale Φ only depend on Φ through the wavefunctions, so they are clearly subleading as well.

3. In order for (5.21) to be the dominant term in the effective potential at large Φ, all of the one-loop anomalous dimensions and beta functions must be nonzero. In all the examples we study, this will indeed be the case. If this condition is not satisfied, and some anomalous dimensions or beta functions vanish, then subleading logarithms or even finite effects at a lower loop order could be larger than the effect shown in (5.21).

4. The mistake we are making in assuming that the theory is a supersymmetric effective theory described by (5.14) comes in the form of terms that are higher

order in $f/|\Phi|^2$. In the regime (5.13) these are clearly negligible compared to the log-enhanced effects.

5.5 Surveying the pseudomoduli

We here discuss each of the pseudomoduli types mentioned in the introduction. In each case, we will apply our general result (5.21) for the leading-log effective potential in the regime (5.13).

5.5.1 Gauge singlet pseudomoduli, with Yukawa coupling to messengers

Let us rewrite (5.4) and (5.5) with Yukawa couplings reintroduced,

$$W_{low} \supset h(\Phi_1\varphi^2 + \tilde{\Phi}_1\varphi\chi + \Phi_2\chi^2) \tag{5.26}$$

Here the φ have SUSY-split tree-level masses, whereas χ do not. We take all the Yukawa couplings equal just for simplicity, so the loop-counting parameter is uniformly given by

$$\kappa_h = \frac{h^2}{16\pi^2}. \tag{5.27}$$

The φ couple to fields X (which can differ from the Φ_1 fields) with $F_X \neq 0$ and, as in (5.2), we distinguish that Yukawa coupling as h_X. When $\Phi = \Phi_1$ or $\Phi = \tilde{\Phi}_1$ is large, the anomalous dimension of X is discontinuous at the scale $|h\Phi|$ since the number of messengers is reduced below this scale:

$$\gamma_X = n_\varphi \kappa_{h_X}, \qquad \text{so} \qquad \Delta\gamma_X = \Delta n_\varphi \kappa_{h_X}, \tag{5.28}$$

where $\Delta n_\varphi = n_\varphi - n'_\varphi > 0$ is the discontinuity in the number of messenger fields above and below $|h\Phi|$. Thus the effective potential can be determined from the $n = 1$ case of (5.21), leading to the well-known result

$$V_{eff} \approx 2V_0 \Delta\gamma_X \ln\frac{|h\Phi|}{m_0}. \tag{5.29}$$

Since $V_0 > 0$ and $\Delta\gamma_X > 0$, such pseudomoduli are safely stabilized below the cutoff of the IR-free dual by the one-loop effective potential. (Depending on the

specifics of the model, they could be stabilized either at the origin or away from it. For instance, the second term in (5.26) is analogous to the messenger-matter mixing considered in [40]; as shown there, it can lead to a negative mass-squared for $\tilde{\Phi}_1$ at the origin, so the minimum is elsewhere.)

On the other hand, pseudomoduli like Φ_2 in (5.26) require two loops to couple to the messengers, via $\Phi_2 \leftrightarrow \chi \leftrightarrow \varphi$. In principle, the two-loop effective potential for Φ_2 can be obtained from the general expressions in [55]; this was recently carried out in the context of SQCD with massive and massless flavors in [87]. Here we simply consider the potential for $\Phi = \Phi_2$ in the range (5.13). The χ fields in (5.26) then get a large mass $\sim |\Phi_2|$, and can be integrated out at lower scales. This does not cause a discontinuity in γ_X but it does affect its first derivative with respect to RG time. In more detail, we have

$$\Delta\Omega_X^{(2)} = \Delta\left(\frac{d\gamma_X}{dt}\right) = \frac{\partial\gamma_X}{\partial h_X}\Delta\beta_{h_X} = 4n_\varphi\kappa_{h_X}\Delta\gamma_\varphi, \tag{5.30}$$

where in the last equality we have used $\gamma_X = n_\varphi\kappa_{h_X}$ and $\beta_{h_X} = h_X(\gamma_X + 2\gamma_\varphi)$. Finally, taking $\Delta\gamma_\varphi = n_\chi\kappa_h$ with n_χ being the number of χ fields which got a mass from Φ_2, this becomes

$$\Delta\Omega_X^{(2)} = 4n_\varphi n_\chi\kappa_{h_X}\kappa_h \tag{5.31}$$

Substituting into (5.21), we obtain

$$V_{eff}(\Phi_2) \approx -V_0\kappa_h\kappa_{h_X}n_\varphi n_\chi\left(\ln\frac{|h\Phi_2|^2}{m_0^2}\right)^2, \tag{5.32}$$

Therefore, the two-loop potential for Φ_2 reveals a destabilized runaway.

5.5.2 Higgsing pseudomoduli

As described in the introduction, these pseudomoduli come from expectation values of matter fields Φ_q charged under a gauge group in which the messengers φ also transform. The physics is quite different depending on whether the field(s) X with $F_X \neq 0$ are neutral or charged under the gauge group. For charged F_X, the gauge fields are "gauge messengers" and can lift Higgsing pseudomoduli at one loop. We will discuss this case more in Subsection 5.5.4 and present an example of

it in Subsection 5.6.3. In this subsection we will focus on the case of neutral F_X, where Higgsing pseudomoduli are instead lifted at two loops.

We consider the limit of large $\Phi = \Phi_q$, in the range (5.13). Suppose the messengers φ decompose into fields φ_i transforming in irreducible representations r_{φ_i} of G. Above the scale Φ_q, the gauge coupling contribution to the one-loop anomalous dimension of the messengers is given by

$$\gamma_{\varphi_i} \supset -2c(r_{\varphi_i})\kappa_g \tag{5.33}$$

with $\kappa_g = g^2/16\pi^2$ being the loop-counting parameter for gauge coupling g and $c(r_{\varphi_i})$ being the quadratic Casimir invariant. (In general $c(r) = T(r)|G|/|r|$; so $c(\text{fund}) = \frac{N^2-1}{2N}$ and $c(\text{adj}) = N$ for $SU(N)$.)

The gauge group is (partially or fully) Higgsed down to $G' \subset G$ at the mass scale $|\Phi_q|$, and that affects the anomalous dimensions of the messengers. Below the Higgsing scale we can decompose the messengers into fields φ'_i transforming as G' irreps $r_{\varphi'_i}$. Each field φ'_i then has an anomalous dimension given by (5.33) with $\varphi_i \to \varphi'_i$.

As in the previous subsection, (5.30) gives the leading contribution to the effective potential, but instead of (5.31) we have

$$\Delta\Omega_X^{(2)} = 4\kappa_{h_X}\Delta\left(\sum_i |r_{\varphi_i}|\gamma_{\varphi_i}\right) = -8\kappa_{h_X}\kappa_g\Delta\left(\sum_i |r_{\varphi_i}|c(r_{\varphi_i})\right) \tag{5.34}$$

Since $|r_{\varphi_i}|c(r_{\varphi_i}) = T(r_{\varphi_i})|G|$ and the index T is additive, (5.34) becomes simply

$$\Delta\Omega_X^{(2)} = -8\kappa_{h_X}\kappa_g T(r_\varphi)(|G| - |G|') \tag{5.35}$$

Finally, substituting into (5.21), we have

$$V_{eff}(\Phi_q) \approx 2V_0\kappa_{h_X}\kappa_g T(r_\varphi)(|G| - |G|')\left(\ln\frac{|\Phi_q|^2}{m_0^2}\right)^2. \tag{5.36}$$

which implies (since $|G|' < |G|$) that the potential always has a stabilizing effect on the pseudomodulus field. This is analogous to the two-loop potentials for D-flat directions found in [72, 58][9].

[9]Indeed the general gauge mediation [8] effective potential for Higgsing pseudomoduli is [93]

$$V_{eff}(m_W^2) = \frac{g^2}{2}\int\frac{d^4p}{(2\pi)^4}\text{Tr}\left(\frac{p^2}{p^2+m_W^2}\right)(3\widetilde{C}_1 - 4\widetilde{C}_{1/2} + \widetilde{C}_0), \tag{5.37}$$

5.5.3 Saxion-type pseudomoduli

Consider gauge singlet "saxion" pseudomoduli Φ_3, coupling to charged matter p via

$$W_{low} \supset h\Phi_3 p^2, \tag{5.38}$$

The Φ_3 potential is first generated at three loops, since they couple to the messengers only via $\Phi_3 \leftrightarrow p \leftrightarrow \text{gauge} \leftrightarrow \varphi$. In the range (5.13) of large Φ_3, the saxion effective potential can be determined using (5.21) with $n = 3$:

$$\Delta\Omega_X^{(3)} = \Delta\left(\frac{d^2\gamma_X}{dt^2}\right) = \frac{\partial\gamma_X}{\partial h_X}\frac{\partial\beta_{h_X}}{\partial g}\Delta\beta_g. \tag{5.39}$$

This is nonzero, since the p fields are massive and can be integrated out at the scale $h\Phi_3$, which affects the beta function of the gauge coupling below the $h\Phi_3$ threshold:

$$\Delta\beta_g = -\frac{g^3}{16\pi^2}(b - b') = -g\kappa_g(b - b'), \tag{5.40}$$

where e.g. $b = 3N - N_f$ for an $SU(N)$ gauge theory with N_f flavors. Substituting this into (5.39) and using $\gamma_X = n_\varphi\kappa_{h_X}$, $\beta_{h_X} = h_X(\gamma_X + 2\gamma_\varphi)$ and $\gamma_\varphi \supset -2c(r_\varphi)\kappa_g$, this becomes

$$\Delta\Omega_X^{(3)} = 16\kappa_{h_X}\kappa_g^2 n_\varphi c(r_\varphi)(b - b') \tag{5.41}$$

Then it follows from (5.21) that

$$V_{eff}(\Phi_3) \approx \frac{2}{3}V_0\kappa_{h_X}\kappa_g^2 n_\varphi c(r_\varphi)(b - b')\left(\ln\frac{|h\Phi_3|^2}{m_0^2}\right)^3. \tag{5.42}$$

Because giving mass to some matter makes the IR group more strongly interacting, we here have $b - b' = -T_2(r_p) < 0$. With this sign in (5.42), we conclude that saxion-type pseudomoduli Φ_3 always have a destabilizing runaway potential, generated at three loops, at least in the range (5.13).

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where Tr sums over vector bosons, with mass matrix m_W^2. For sfermions q_f in reps r_{q_f} we have $(m_W^2)^{AB} = \sum_f \mathrm{Tr}(T^{(A}_{r_{q_f}} q_f^\dagger T^{B)}_{r_{q_f}} q_f)$. We are here interested in the case of weakly coupled messengers φ, so $C_a(p^2/M^2)$ is $T(r_\varphi)$ times the expressions quoted in [8]. Expanding (5.37) for q_f near the origin gives the sfermion soft masses $m_{q_f}^2 \sim m_0^2 g^4 > 0$, including the group theory factor $c(r_{q_f})$. As was recently analyzed in [73], there can be some numerical differences in the coefficient of $m_{q_f}^2$, as compared with the usual gauge mediation scenario, because the messengers can have $\langle\varphi\rangle \sim m_0 \neq 0$, Higgsing the gauge group (as in the SQCD example of [61]). Approximating (5.37) for some $Q_f \sim \Phi_q$ far from the origin also indeed yields (5.36).

Let us make two comments on this result. First, note that the change in the beta function (5.40) is accounted for by an added term in the low-energy theory,

$$W_{low} \supset -\frac{1}{64\pi^2}(b-b')\int d^2\theta \ln(\Phi_3)W_\alpha^2, \tag{5.43}$$

which is the only way that Φ_3 enters into the low-energy theory in this limit. This is the reason for calling such pseudomoduli saxions: the phase of Φ_3 enters the low-energy theory like an axion, and $\ln|\Phi_3|$ is its saxion superpartner.

Second, in the standard gauge-mediation setup, it can be shown [58] that saxion pseudomoduli Φ_3 also have destabilizing, tachyonic $m_3^2 < 0$ near the origin, in analogy with how the top Yukawa and $m_t^2 = \mathcal{O}(\alpha_s^2)$ can lead to electroweak breaking $m_H^2 < 0$. This argument, however, relies on the large separation of scales of a high messenger scale m_φ: one starts at the high scale m_φ with the two-loop supersymmetry breaking $m_p^2|_{\mu=m_\varphi} > 0$, and $m_3^2|_{\mu=m_\varphi} \approx 0$. Then $m_3^2|_{\mu\ll m_\varphi} < 0$ follows from RG running the one-loop h_3^2 Yukawa contribution to the running m_3^2 and m_p^2. But in our examples of interest, there are light messengers, no such large range of running, and the finite terms in the effective potential cannot be neglected in computing the three-loop potential for Φ_3 near the origin. So here a full-fledged explicit calculation of the three-loop effective potential for Φ_3 would generally be required to determine if there is any (metastable) local minimum near the origin. In any case, we have argued for the destabilized runaway farther from the origin.

5.5.4 Modifications when there are gauge messengers

Finally, let us discuss models with gauge messengers, i.e. models where the gauge multiplets have tree-level, SUSY-split masses. Gauge messengers occur if any charged matter field has a non-zero, tree-level F-term. The methods discussed in Section 5.4 can be applied, with only trivial modification, to determine the effective potential for pseudomoduli when there are gauge messengers. Though the methods are the same, the physics with gauge messengers is quite different. Higgsing and saxion-type pseudomoduli are lifted at one fewer loop order by gauge messengers. Correspondingly, the sign of the leading-log effective potential for these pseudomoduli can be opposite from cases without gauge messengers. So Higgsing

pseudomoduli can become destabilized at one loop, and saxion pseudomoduli can become stabilized at two loops.

Let us first consider Higgsing-type pseudomoduli. With gauge messengers, Higgsing pseudomoduli Φ_q are lifted at 1-loop. This can be seen simply from (5.24) and (5.25) (we are allowing for the possibility that there are multiple fields X_i with F-terms, transforming in different representations of the gauge group). That is, charged fields X_i have a discontinunity in their 1-loop anomalous dimensions when the gauge group is Higgsed at the scale $|g\Phi_q|$,

$$\Delta\Omega^{(1)}_{X_i} = \Delta\gamma_{X_i} = -2\kappa_g \Delta c(r_{X_i}). \tag{5.44}$$

Here $\Delta c(r_{X_i}) = c(r_{X_i})|^G_{G'}$, where G denotes the group above the Higgsing scale, and G' denotes that below, and in general $\Delta c(r_{X_i}) > 0$. As in (5.29), the potential in the large $|\Phi|$ regime is then approximated by

$$V_{eff}(\Phi) \approx -2\sum_i |F_{X_i}|^2 \kappa_g \Delta c(r_{X_i}) \ln \frac{|g\Phi|}{m_0}. \tag{5.45}$$

Because $\Delta c(r_{X_i}) > 0$, we see that Higgsing pseudomoduli are generally unstable at 1-loop in theories with gauge messengers. (One exception is if the pseudomodulus Φ is itself one of the fields X_i with non-zero F-term, which also couples to matter messengers φ; in this case, there can be also a positive contribution to $\Delta\gamma_{X_i}$ as in [84].)

Let us now consider saxion-type pseudomoduli, like Φ_3 in (5.38). When such pseudomoduli are in the range (5.3), the change at the scale Φ_3 is

$$\Delta\Omega^{(2)}_{X_i} = \Delta\left(\frac{d}{dt}(-2c(r_i)\kappa_g)\right) = -\frac{4c(r_i)\kappa_g}{g}\Delta\beta_g = +4\kappa_g^2 c(r_i)(b-b'). \tag{5.46}$$

where in the last equation we have used (5.40). Then (5.25) gives

$$V_{eff}(\Phi_3) \approx \sum_i 4|F_{X_i}|^2 \kappa_g^2 c(r_i)(b'-b)\left(\log\frac{|\Phi|}{m_0}\right)^2. \tag{5.47}$$

Because $b' - b > 0$ in this case (as discussed after (5.42)), the 2-loop potential stabilizes the saxion. This has the opposite sign of (5.42), and appears at one fewer loop order. So gauge messengers can stabilize saxions.

5.6 Examples

5.6.1 Warmup: SQCD with massive flavors

To illustrate our method, and set up the notation for following examples, let us briefly review the metastable DSB theory based on SQCD with massive flavors [61]. The UV theory is $SU(N_c)$ SQCD, with N_f in the IR free-magnetic range, $N_c < N_f < \frac{3}{2}N_c$. The N_f flavors have a small mass m_Q, which for simplicity we take to be the same for all flavors, so there is a global $SU(N_f) \times U(1)_B \cong U(N_f)$ symmetry. The low-energy theory is given by the IR free $SU(N = N_f - N_c)$ dual, with

$$W = h\mathrm{Tr}\Phi\varphi\widetilde{\varphi} - h\mu^2\mathrm{Tr}\Phi. \tag{5.48}$$

The mass scale is given in terms of UV data as $\mu^2 \equiv (-1)^{N_c/N_f} m\Lambda$ (see [61, 76] for discussion of these factors). This theory has metastable SUSY breaking vacua, given at tree level by

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \widehat{\Phi} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \widehat{\varphi} \\ 0 \end{pmatrix}, \quad \widetilde{\varphi} = \begin{pmatrix} \widetilde{\widehat{\varphi}} & 0 \end{pmatrix}, \quad \text{with} \quad \widetilde{\widehat{\varphi}}\widehat{\varphi} = \mu^2 \mathbf{1}_N, \tag{5.49}$$

with $\widehat{\Phi}$ an $(N_f - N)^2 = N_c^2$ matrix of pseudomoduli. SUSY is broken by

$$F_\Phi = \begin{pmatrix} 0 & 0 \\ 0 & f\mathbf{1}_{N_f - N} \end{pmatrix}, \qquad \text{with} \qquad f^\dagger = h\mu^2 \neq 0. \tag{5.50}$$

The vacuum energy is $V_0 = (N_f - N)|h^2\mu^4|$, and the tree-level mass scale is $m_0 = h\mu$.

The pseudomoduli are all lifted at one loop, as is evident from the fact that they have direct superpotential coupling to the messengers φ. The 1-loop effective potential for all values of the pseudomoduli was computed in [61], and it was noted that the metastable DSB vacua are at $\widehat{\Phi} = 0$, $\widehat{\varphi} = \widetilde{\widehat{\varphi}} = \mu\mathbf{1}_N$. The gauge and global symmetry group is broken in these vacua as $SU(N) \times U(N_f) \to SU(N)_D \times U(N_f - N)$.

When the pseudomoduli $\widehat{\Phi}$ in (5.49) are in the range (5.3), we can alternatively use the result (5.29) to easily determine the form of the effective potential. Taking

e.g. $\widehat{\Phi}$ in (5.49) to have $N_f - N$ independent and large diagonal entries, each of which has $\gamma_\Phi = N\kappa_h$, and then accounting for the flavor structure, immediately yields

$$V_{eff}(\widehat{\Phi}) \approx |f|^2 N\kappa_h \mathrm{Tr}\ln\frac{\widehat{\Phi}^\dagger\widehat{\Phi}}{|\mu|^2}. \tag{5.51}$$

The rising potential (5.51) gives a quick and indeed reliable indication that the $\widehat{\Phi}$ pseudomoduli are good. (As discussed in [61], for sufficiently large Φ the non-pertubative W_{dyn} becomes important and the potential eventually slopes down to the supersymmetric vacua. This happens past the far, but still perturbative regime of (5.51).)

5.6.2 Example with a two-loop runaway: SQCD with both massive and massless flavors

We again take the UV theory to be $SU(N_c)$ SQCD with N_f flavors in the free-magnetic phase. But here we take only N_{f1} of the flavors to be massive, leaving N_{f0} massless flavors. For simplicity, we will give the N_{f1} flavors equal mass, m. In the IR, the superpotential is then of the general form (5.4). In particular, for magnetic fields (we use the notation of [82])

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{10} \\ \Phi_{01} & \Phi_{00} \end{pmatrix}, \qquad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_0 \end{pmatrix}, \qquad \widetilde{\varphi} = \begin{pmatrix} \widetilde{\varphi}_1 & \widetilde{\varphi}_0 \end{pmatrix}, \tag{5.52}$$

we have

$$W = h\mathrm{Tr}\Phi_{ij}\varphi_j\widetilde{\varphi}_i - h\mu^2\mathrm{Tr}\Phi_{11}, \tag{5.53}$$

where Φ_{ij} is an $N_{fi} \times N_{fj}$ matrix and φ_i and $\widetilde{\varphi}_i^T$ are $N_{fi} \times N$ matrices.

The first two terms in (5.53) lead to rank-condition supersymmetry breaking when $N_{f1} - N = N_c - N_{f0} > 0$, with $V_0 = (N_c - N_{f0})|h\mu^2|^2$. All fields other than Φ_{00} get masses at tree-level or one-loop level. This was observed in [78], where it was also shown that Φ_{00} has a non-perturbative runaway coming from a dynamically generated superpotential. This is not yet fatal, as there is a range of the pseudomoduli space where the non-perturbative effects are negligible in comparison with higher-loop perturbative effects. But it was recently shown in

[87] that higher-loop perturbative effects also lead to a potential with a runaway to large Φ_{00}.

Let us now use the formalism developed in the previous two sections to demonstrate this two-loop runaway behavior in the range (5.3). We identify Φ_{00} with the Φ_2 pseudomodulus of Subsection 5.5.1. The fields Φ_{11} have F-terms given by (5.50), replacing N_f with N_{f1}, so they play the role of X. The fields φ_1, $\tilde{\varphi}_1$ then play the role of the messengers φ, while the fields φ_0, $\tilde{\varphi}_0$ are analogues of χ in (5.4).

To simplify the flavor index structure, let us take $\langle\Phi_{00}\rangle \propto \mathbf{1}_a \oplus 0_{N_{f0}-a}$, for an integer a between 1 and N_{f0}. Below the $\langle\Phi_{00}\rangle$ threshold, the a flavor components of φ_0 get a mass and no longer contribute to γ_φ, leading to $\Delta\gamma_\varphi = ah^2/16\pi^2 \equiv a\kappa_h$. As in (5.30), we then find $\Delta\Omega^{(2)}_{\Phi_{11}} = 4Na\kappa_h$. Accounting for the $SU(N_{f0})_L \times SU(N_{f0})_R$ flavor symmetry, the effective potential in this regime is found to be

$$V_{eff}(\Phi_{00}) \approx -V_0 N \kappa_h^2 \mathrm{Tr}\left(\ln\frac{|\Phi_{00}|^2}{|\mu|^2}\right)^2. \tag{5.54}$$

The sign indicates that the Φ_{00} pseudomoduli have a two-loop runaway to the cutoff of the low-energy theory, and this model thus does not have a calculable metastable DSB vacuum.

This runaway can be lifted in a variety of ways. As noted in [78], one way is to add singlets, Σ, to the UV theory with a marginal superpotential coupling to Φ_{00}, $W_{tree} \supset m\mathrm{Tr}\Phi_{00}\Sigma$. In this modified theory, all pseudomoduli are stabilized at the origin, and there is metastable DSB. Another modification – which is potentially more suitable for model building – is to deform the theory by $W_{tree} \supset m\mathrm{Tr}\,\Phi_{00}^2$ [82]. Since this is a nonrenormalizable interaction in the UV, m is naturally small; as shown in [82], this makes it possible to balance the two-loop runaway potential against this tree-level stabilizing potential and obtain a meta-stable vacuum at $\Phi_{00} \neq 0$ where the R-symmetry is completely broken.

5.6.3 SQCD with weakly gauged flavor symmetry – an example with gauge messengers

We start with the SQCD theory considered in [61], and reviewed in Subsection 5.6.1, where all N_f flavors are give the same mass $m = m_Q$. There is a global $SU(N_f) \times U(1)_B \cong U(N_f)$ symmetry, and we here consider gauging some subgroup $G \subseteq SU(N_f)$ of the flavor symmetry. We will very weakly gauge this subgroup, $g \ll 1$, so that $\Lambda_{SU(N_c)} \gg \Lambda_G$. For energies below $\Lambda_{SU(N_c)}$, we dualize $SU(N_c) \to SU(N = N_f - N_c)$ and, as in [61], there are metastable SUSY-breaking vacua, given at tree level by (5.49).

Let us first consider the case that $G = SU(N_f)$. (In the next subsection we will analyze the case of proper subgroups.) Some preliminary analysis of this theory appeared in [61, 80]. The classical vacua are still given by (5.49) for $g \neq 0$, though the $SU(N_f)$ D-terms lift some of the $g = 0$ pseudomoduli at tree level. The SUSY-breaking F-terms are still given by (5.50). Our interest here will be in the theory with k added $SU(N_f)$ flavors, $\rho^i \in (1, N_f)$, $\widetilde{\rho}_i \in (1, \overline{N_f})$, $i = 1, \ldots, k$, with k in the range $N_f + N_c < k < 3N_f - N_c$, so that the $SU(N_f)$ gauge coupling is asymptotically free in the UV $SU(N_c) \times SU(N_f)$ theory, and IR free in the $SU(N) \times SU(N_f)$ low-energy dual [80]. The dual theory has

$$W = h\mathrm{Tr}\Phi\varphi\widetilde{\varphi} + m_Q\Lambda\mathrm{Tr}\Phi + m_\rho\rho\widetilde{\rho}. \tag{5.55}$$

We are interested in the case where $m_\rho = 0$. The added $SU(N_f)$ matter fields in this case lead to additional pseudomoduli, given by the expectation values of ρ and $\widetilde{\rho}$ along the tree-level D-flat directions, Higgsing $SU(N_f)$ (or $U(N_f)$). The effective potential for these pseudomoduli had not yet been computed in the literature.

We here highlight a key point about this theory: because $\hat{\Phi}$ in (5.49) is charged under $SU(N_f)$, the non-zero F-term for this charged field implies that the model has gauge messengers, as discussed in Subsection 5.5.4. Thus the ρ, $\tilde{\rho}$ pseudomoduli are lifted at one instead of two loops. Indeed, a direct computation of the one-loop Coleman-Weinberg potential exhibits the dependence on the ρ and $\widetilde{\rho}$ expectation values. Alternatively, we can use apply the discussion in Subsection 5.5.4 to see that the one-loop potential is non-vanishing at large ρ, $\tilde{\rho}$.

Consider the pseudomodulus direction where ρ and $\widetilde{\rho}$ each have a single large entry, ρ_1. This Higgses $SU(N_f)$ to $SU(N_f-1)$ at the threshold scale $g\rho_1$. Under $SU(N_f-1)$, Φ decomposes into an adjoint ϕ_A, $A=1,\ldots,\dim(SU(N_f-1))$; two singlets $\phi_{0'}$ and ϕ_0; and a fundamental plus anti-fundamental. The latter two do not participate in the SUSY breaking so we will set them to zero henceforth. The decomposition of Φ into the remaining fields is:

$$\Phi = \phi_A T^A + \phi_{0'} T^{0'} + \phi_0 N_f^{-1/2} \mathbf{1}_{N_f} \tag{5.56}$$

where T^A are the generators of $SU(N_f-1)$ (as $N_f \times N_f$ matrices), and $T^{0'}$ is proportional to the $SU(N_f-1)$ (but not the $SU(N_f)$) identity. These generators are all normalized to have $\mathrm{Tr}\,(T^A)^2 = \mathrm{Tr}\,(T^{0'})^2 = 1$ so that ϕ_A, $\phi_{0'}$ and ϕ_0 are canonically normalized fields. The corresponding F-terms are then given by decomposing (5.50) as

$$F_\Phi = F_{\phi_A} T^A + F_{\phi_0'} T^0 + F_{\phi_0} N_f^{-1/2} \mathbf{1}_{N_f}. \tag{5.57}$$

In general these F-terms will all be non-zero. Then according to (5.45), the effective potential for ρ_1 in the regime (5.13) is

$$V_{eff} \approx -2\Big(|F_{\phi_A}|^2 \Delta c(\phi_A) + |F_{\phi_{0'}}|^2 \Delta c(\phi_{0'})\Big) \kappa_g \log \frac{|\rho_1|}{m_0} \tag{5.58}$$

Notice that F_{ϕ_0} does not contribute since it is neutral under $SU(N_f)$ and $SU(N_f-1)$. The change in the quadratic Casimir invariants is simply

$$\begin{aligned} \Delta c(\phi_A) &= c(SU(N_f)\ \mathrm{adj}) - c(SU(N_f-1)\ \mathrm{adj}) = 1 \\ \Delta c(\phi_{0'}) &= c(SU(N_f)\ \mathrm{adj}) - c(SU(N_f-1)\ \mathrm{sing}) = N_f. \end{aligned} \tag{5.59}$$

Substituting back into (5.58), we conclude that V_{eff} is indeed a downward-sloping function of the pseudomodulus field ρ_1, indicating "bad" runaway behavior.

Let us make two comments on this result. First, we have been intentionally vague about the precise forms of F_{ϕ_0} and F_{ϕ_A}, since these will depend on how the ρ vev is aligned with the SUSY-breaking pattern (5.49), (5.50). However, we see from this analysis that the conclusion of runaway behavior is robust and does not depend on the detailed form of the F-term vevs. Second, let us mention

that the coefficient in (5.58), and in particular its sign, can also be obtained from the fact that the coefficient of $\ln m_0$ is the same as the coefficient of $\ln M_{cutoff}$ in the Coleman-Weinberg potential: $\frac{\partial V}{\partial \ln m_0} = \frac{\partial V_{CW}^{(1)}}{\partial \ln M_{cutoff}} = -\frac{1}{32\pi^2}\mathrm{Str}\mathcal{M}^4$. Indeed the result (5.58) is reproduced upon evaluating the leading $\mathcal{O}(|F|^2)$ contribution to $\mathrm{Str}\mathcal{M}^4$ in the large $|\rho_1|$ limit, using the appropriate gauge messenger spectrum (which can be found as in [57]).

5.6.4 SQCD with weakly gauged flavor symmetry – no gauge messengers

Let us now consider the same model as the previous subsection except that instead of gauging the entire $SU(N_f)$ flavor symmetry, we gauge an $SU(K)$ subgroup which is sufficiently small, and aligned, such that such that F_Φ in (5.50) is gauge neutral. This of course leads to qualitatively different behavior, since now there are no longer any gauge messengers present to lift the pseudomoduli at one loop.

There are now two qualitatively different possibilities for how the $SU(K)$ gauge group is aligned. The expectation values (5.49) break $SU(N) \times SU(N_f) \to SU(N)_D \times SU(N_f - N)$, and the two possibilities are that the $SU(K)$ can align inside either $SU(N)_D$ or inside $SU(N_f - N)$ (assuming that $K \leq N$, or $K \leq N_f - N$, respectively). The qualitative difference is because (5.50) leads to tree-level SUSY-split masses for only the last $N_f - N$ flavors. So if $SU(K)$ aligns with the first N entries in (5.50), then the messenger fields are neutral under $SU(K)$, whereas if the $SU(K)$ aligns with the last $N_f - N$ entries in (5.50) the messengers φ are charged under $SU(K)$. In either case, we are here considering the case where the F-terms (5.50) are $SU(K)$ gauge singlets, so that there are no gauge messengers.

Let us first consider the case where $SU(K)$ aligns within the $SU(N_f - N)$, i.e. the last $N_f - N$ flavors in (5.50). Since there are then messengers φ with SUSY-split masses, and charged under $SU(K)$, the ρ pseudomoduli in this case couple as the Higgsing pseudomoduli of Subsection 5.5.2. As discussed there, such pseudomoduli are lifted at two loops, and such pseudomoduli are good – their potential safely stabilizes them. Indeed, consider the range (5.13), where their

potential can be read off from (5.36). Consider again the pseudo-D-flat direction where ρ and $\widetilde{\rho}$ each have a single large entry, ρ_1, Higgsing $SU(K)$ to $SU(K-1)$.

We then have from (5.35)

$$\Delta\Omega_X^{(2)} = -8\kappa_{h_X}\kappa_g T(r_\varphi)(|G| - |G'|) = -4\frac{\kappa_h}{N_f - N}\kappa_g N(2K-1), \tag{5.60}$$

where h is as in (5.48) and $h_X = h/\sqrt{N_f - N}$ comes from writing the F-terms (5.50) in terms of a canonically normalized $SU(N_f - N)$ singlet field X. As in (5.36), the potential is then

$$V_{eff}(\rho_1) \approx |f|^2\kappa_h\kappa_g N(2K-1)\ln^2\frac{|g\rho_1|^2}{m_0^2}, \tag{5.61}$$

which is an increasing function of $|\rho_1|$; the potential safely stabilizes these pseudomoduli. Allowing for several flavors of ρ and $\widetilde{\rho}$ with widely separated large expectation values yields a sum of terms like (5.61), safely lifting all of these Higgsing pseudomoduli.

We now briefly summarize the case where $SU(K)$ instead aligns inside the unbroken $SU(N)_D$, i.e. within the first N entries in (5.50). The SUSY-split messenger components of φ are now $SU(K)$ neutral, and the $SU(K)$ charged components of φ have SUSY masses. These components interact via the superpotential terms coming from (5.48), with coupling h. The upshot is that the ρ pseudomoduli in this case are first lifted at *three* loops. The three-loop effective potential in the range (5.13) is approximated by (5.21) with $\Delta\Omega_X^{(3)} = \frac{\partial\gamma_X}{\partial h_X}\frac{\partial\beta_{h_X}}{\partial h}\Delta\beta_h < 0$, where the sign comes from the $SU(K)$ gauge contribution to $\Delta\beta_h$. It then follows from (5.22) that the ρ pseudomoduli in this case are bad.

To summarize, if the gauged flavor group is the entire $SU(N_f)$, or more generally an $SU(K)$ subgroup which is not aligned within either the first N entries or the remaining $N_f - N$ entries in (5.50), then there are gauge messengers, and the ρ pseudomoduli are lifted at one loop and are bad, with a perturbative runaway. If the gauged $SU(K)$ flavor subgroup is entirely within the last $N_f - N$ entries in (5.50), then the ρ pseudomoduli are lifted at two loops and are good. Finally, if the gauged $SU(K)$ is entirely within the first N entries in (5.50), then the ρ pseudomoduli are lifted at three loops and are bad.

5.6.5 Example with a saxion-type pseudomodulus: $SU(N_c)$ with symmetric tensor and antifundamentals

We now take the UV theory to be an $SU(N_c)$ gauge theory with symmetric tensor S and $N_f = N_c + 4$ antifundamentals $\widetilde{Q}_i$, and we attempt to break supersymmetry by turning on the tree-level superpotential $W_{tree} = \mathrm{Tr}\lambda^{ij} S\widetilde{Q}_i\widetilde{Q}_j$. For simplicity, we take $\lambda^{ij} = \lambda\delta^{ij}$, preserving an $SO(N_f) \subset SU(N_f)$ flavor symmetry, along with a $U(1)_R$ symmetry with $R(S) = -2 + \frac{4}{N_c}$ and $R(\widetilde{Q}) = 2 - \frac{2}{N_c}$. This theory was originally considered long ago [35], where it was noted to have an interesting pseudo-flat direction, labeled by $\langle \det S \rangle$, along which $\langle \widetilde{Q} \rangle = 0$ and $\langle S \rangle = a\mathbf{1}_{N_c}$. Far from the origin in this direction, $\langle \det S \rangle \gg \Lambda^{N_c}$, there is a non-perturbative runaway superpotential which pushes $\det S \to \infty$ [35]:

$$W_{dyn} = c\left(\frac{(\Lambda^{2N-3})^2 \det\lambda}{\det S}\right)^{1/(N_c-2)} \tag{5.62}$$

(To obtain (5.62), note that $\langle S \rangle = a\mathbf{1}_{N_c}$ Higgses $SU(N_c)$ to $SO(N_c)$, and gives the $\widetilde{Q}$ mass $m_{\widetilde{Q}} = \lambda a$. Then (5.62) is generated by gaugino condensation in the low-energy $SO(N_c)$ Yang-Mills theory.) It was speculated in [35] that there might be a metastable minimum at smaller values of $\det S$, perhaps either in the $\langle S \rangle \gg \Lambda$ regime, or for S nearer the origin.

The theory near the origin can now be analyzed using its known magnetic dual [94]: an $SO(8)$ gauge theory with N_f matter fields $\varphi \in \mathbf{8}_v$, one matter field $p \in \mathbf{8}_s$, $\frac{1}{2}N_f(N_f+1)$ singlets Φ (with $\Phi = \Phi^T$), and one more singlet Z, with superpotential

$$W_{dual} = h(\mathrm{Tr}\Phi\varphi\varphi - \mu^2\mathrm{Tr}\Phi + Zp^2), \tag{5.63}$$

where, as before, we take the couplings to be the same for simplicity. In terms of the UV theory, $\Phi = \Lambda^{-2}S\widetilde{Q}\widetilde{Q}$, $Z = \Lambda^{1-N_c}\det S$, and $f = -h\mu^2 = \lambda\Lambda^2$. The $SO(8)$ magnetic dual theory is IR free for $N_f \geq 17$. The small parameter (5.8) is $\epsilon = \lambda$, so we need to take $|\lambda| \ll 1$. The first two terms in (5.63) lead to $F_\Phi \neq 0$ via the rank-condition supersymmetry breaking [61], with tree-level vacuum $V_0 = (N_f - 8)|h\mu^2|^2 \sim (N_f - 8)|\lambda\Lambda^2|^2$. Indeed, this sector of the theory is the IR dual of an $SO(N_c)$ gauge theory with $N_f = N_c + 4$ massive fundamentals which, along with the field Z, is the low-energy electric theory obtained for large Z. Ignoring the Z

pseudomodulus, this low-energy theory would have metastable DSB vacua at $\langle\Phi\rangle = 0$, with $\langle\varphi\rangle \neq 0$ (breaking $SO(8)_{gauge} \times SO(N_f)_{flavor} \to SO(8)_D \times SO(N_f - 8)$), just as in [61].

However, the additional Z pseudomodulus of the low-energy theory (5.63) spoils the metastable DSB minimum in the other pseudomoduli. The field Z is the same interesting pseudomodulus, with the non-perturbative runaway for $|Z| \gg |\Lambda|$, found in [35]. For $|Z| \ll |\Lambda|$, the IR free magnetic dual $SO(8)$ theory reveals a perturbative runaway, as the field Z is of the "saxion" type, like Φ_3 in (5.38). As we have argued, such pseudomoduli develop a perturbative runaway potential at three loops, and eventually Z slides to the UV cutoff of the low-energy theory, $Z \sim \Lambda$, where all bets are off. In the regime (5.13) of $|\mu| \ll |X| \ll |\Lambda|$, the effective potential is given by (5.42) (where g is the dual $SO(8)$ gauge coupling):

$$V_{eff}(X) \approx -\frac{56}{3} V_0 \kappa_h \kappa_g^2 \left(\ln \frac{|Z|^2}{|\mu^2|}\right)^3 \tag{5.64}$$

since $b = 17 - N_f$ above the Z threshold, and below p gets a mass, so $b' = 18 - N_f$.

Because of this perturbative runaway, we expect that this theory does *not* have a metastable dynamical supersymmetry breaking near the origin[10]. All evidence points toward this theory having everywhere the runaway to $\langle S\rangle \to \infty$, starting with the perturbative magnetic runaway for $\langle S\rangle \ll \Lambda$, and ending with the nonperturbative electric potential from (5.62) for $\langle S\rangle \gg \Lambda$, rather than any metastable DSB vacuum.

5.6.6 Modifications of the above example, which do have metastable DSB near the origin

We can still modify the electric $SU(N_c)$ theory to remove the runaway by hand, and obtain a model of DSB. We add to the electric theory a gauge singlet field Σ, and take

$$W_{tree} = \lambda \mathrm{Tr} S \widetilde{Q} \widetilde{Q} + \frac{c}{M_p^{N_c-2}} \Sigma \ \det S, \tag{5.65}$$

[10]We have not computed the three-loop $V_{eff}^{(3)}$ in the range $|X| \sim |\mu|$, and in principle there could be a metastable minimum in this range very close to the origin. Even if that were the case, such a hypothetical minimum would likely not be sufficiently long-lived to be viable.

where M_p is the scale of some UV completion or other dynamics (suppose $M_p \gg |\Lambda|$) and c is a dimensionless constant[11]. In the IR free magnetic dual, the superpotential is

$$W_{dual} = h(\mathrm{Tr}\Phi\varphi\varphi - \mu^2\mathrm{Tr}\Phi + Zp^2) + m_Z\Sigma Z, \tag{5.66}$$

where $m_Z = c\Lambda(\Lambda/M_p)^{N_c-2}$. As before, we take $N_f > 17$, so the magnetic theory is IR free. The first two terms then lead to rank-condition supersymmetry breaking, with $V_0 = (N_f - 8)|h\mu^2|^2$. The last term gives Z a tree-level supersymmetric mass m_Z, so Z is no longer a pseudomodulus; the runaway direction of the previous subsection has been eliminated. The new field Σ leads to a new pseudomodulus, but this one is dynamically stabilized. Indeed, integrating out the massive field Z, its equation of motion sets $\Sigma \sim p^2$, so the new pseudomodulus is of the Higgsing type; it is a pseudo-D-flat direction along which the spinor $\langle p\rangle$ gets an expectation value, Higgsing $SO(8)$ to $SO(7)$. This pseudo-D-flat direction is lifted at two loops, with $V^{(2)}_{eff}$ minimized at the origin, $\langle\Sigma\rangle = \langle p\rangle = 0$. Near the origin, $V_{eff} \supset m_p^2 p^\dagger p \sim m_p^2\sqrt{\Sigma^\dagger\Sigma}$, with $m_p^2 > 0$. In the range (5.13) of the pseudomodulus $Y \sim g_{SO(8)}\sqrt{c\Lambda\Sigma}$, the effective potential is given by (5.36)

$$V_{eff}(\Sigma) \approx 16V_0\kappa_h\kappa_g\Delta c(r_\varphi)\left(\ln\frac{|Y|^2}{|\mu|^2}\right)^2, \tag{5.67}$$

with $\Delta c(r_\varphi) = (\frac{7}{2} - \frac{21}{8})$. So in this limit too the potential is an increasing function of $|\Sigma|$.

A different modification of the example of the previous subsection is to weakly gauge the $SO(N_f)$ symmetry, with gauge coupling g'. The Φ of the dual theory (5.66) decompose into an $SO(N_f)$ adjoint and singlet, $\Phi = \phi_A T^A + \phi_0 N_f^{-1/2}\mathbf{1}_{N_f}$, as does F_Φ, which is given as in (5.50) (with $N = 8$). Since $F_{\phi_A} \neq 0$, there are gauge messengers, which lifts the saxion Z at two loops. For Z in the range (5.13), its potential is given as in (5.47)

$$V_{eff}(Z) \approx 32\frac{(N_f-8)(N_f-2)}{N_f}|f|^2\kappa_{g'}^2\left(\log\frac{|Z|}{|\mu|}\right)^2, \tag{5.68}$$

[11]Replacing the last term in (5.65) with $c\ (\det S)^2/M_p^{2N_c-3}$ is qualitatively similar to the case described above. An alternative is to replace the last term in (5.65) with $c\ \det S/M_p^{N_c-3}$, which also halts the $\langle Z\rangle \to \infty$ runaway. This theory does admit metastable DSB vacua, related to those of $SO(N_c)$ with $N_f = N_c + 4$, but the necessary condition (5.8) becomes the requirement that λ be unnaturally small: $\lambda \ll (\Lambda/M_p)^n$ where $n > 0$ depends on N_c ($n \to 2$ for large N_c).

where we used $\sum_A |F_{\phi_A}|^2 = 8(N_f - 8)|f|^2/N_f$. This potential safely stabilizes the saxion, so the theory of the previous subsection can have viable DSB upon gauging the $SO(N_f)$ flavor symmetry. (The $SO(N_f)$ can run to strong coupling in the IR, unless additional $SO(N_f)$ charged matter is added. Such matter can lead to bad Higgsing pseudomoduli, as in Subsection 5.6.3.)

5.6.7 Examples with incalculable pseudomoduli potentials: Kutasov-type dualities

As in [90, 91, 92] there are many duality examples based on matter fields in multiple representations, with an added tree-level superpotential for some of the representations; see e.g. [95] for additional examples. In all of the duals, some moduli enter only via power-law irrelevant terms. If supersymmetry is broken, these become irrelevantly coupled pseudomoduli, of the type Φ_4 in (5.7). Thus *none* of these examples can have calculable metastable DSB – they are all inconclusive.

Consider, for example, the original example of [90]. The electric theory is $SU(N_c)$ SQCD, with N_f fundamental flavors and an added adjoint X, with $W_{tree} = \mathrm{Tr}X^3 + \lambda \mathrm{Tr}QX\widetilde{Q}$. The term with coupling λ has been added to try to dynamically break supersymmetry. The dual theory [90] has gauge group $SU(N = 2N_f - N_c)$, with adjoint Y, N_f fundamental flavors φ and $\widetilde{\varphi}$, and gauge singlets $\Phi_0 = Q\widetilde{Q}/\Lambda$ and $\Phi_1 = QX\widetilde{Q}/\Lambda^2$, with

$$W_{dual} = hTr\Phi_1\varphi\widetilde{\varphi} + f\mathrm{Tr}\Phi_1 + \mathrm{Tr}Y^3 + \frac{a}{\Lambda}\mathrm{Tr}\Phi_0\varphi Y\widetilde{\varphi}, \tag{5.69}$$

where the dimensionless couplings h and a are $\mathcal{O}(1)$ at the cutoff and $f = \lambda\Lambda^2$. The theory is IR free for $N_f < 2N_c/3$ and the small parameter ϵ in (5.8) is here given by $\epsilon = \lambda$.

The first two terms in (5.69) give the rank-condition supersymmetry breaking sector [61], with $F_{\Phi_1} \neq 0$, and $\langle\varphi\rangle = \langle\widetilde{\varphi}\rangle^T \neq 0$. The mass spectrum and pseudomoduli potential for the components of Φ_1 and φ are identical to that of the SQCD example of [61]. The additional fields Y in (5.69), and also N^2 components of the Φ_0 fields, get calculable tree-level supersymmetric masses, $m^2_{calc} \sim |a\epsilon\Lambda/h|^2 \sim |a^2\epsilon^2 \ln|\epsilon|\Lambda^2|$ from the $\widetilde{\varphi}\varphi \neq 0$. These calculable tree-level masses just barely

robust against the unknown $m^2_{incalculable} \sim |\epsilon\Lambda|^2$, thanks to the $|h|^{-2} \sim -\ln|\epsilon|$ enhancement.

But there are remaining pseudomoduli components of Φ_0, which can be first lifted at one loop. Because they enter into the low-energy theory only via the power-law irrelevant last term in (5.69), it follows from our general discussion in Section 5.3 that the effective potential for these pseudomoduli is incalculable, $m^2_{calc} \leq m^2_{incalculable}$. For example, the one-loop calculable contribution $m^2_{calc} \sim |a\epsilon\Lambda|^2$ is of the same order as the incalculable contributions, from terms like $K_{eff} \supset c|\Lambda|^{-2}\Phi_1^\dagger\Phi_1\Phi_0^\dagger\Phi_0$. So we cannot determine whether the Φ_0 pseudomoduli are stabilized in the region $|\Phi_0| < |\Lambda|$, or if they instead develop a dangerous runaway to larger values of Φ_0, where the low-energy analysis is inapplicable. It is thus inconclusive whether or not this theory dynamically breaks supersymmetry.

One can still modify the UV theory to eliminate, by hand, the dangerous pseudomoduli by adding mass terms for them. In the examples at hand, this fix has already been implemented in the literature, with Φ_0 given mass via superpotential term $\mathrm{Tr}\Sigma Q\widetilde{Q} \to \mathrm{Tr}\Sigma\Phi_0$ (with added gauge singlets Σ) [79] or alternatively $Tr(Q\widetilde{Q})^2 \to \mathrm{Tr}\Phi_0^2$ [81] .

5.6.8 Analogs of SQCD with $N_f = N_c + 1$: IR free theories without gauge fields

While there is no general classification of which supersymmetric gauge theories have IR free low-energy duals (as opposed to an interacting SCFT), some classes of theories have been well mapped out. For example, there is a classification of the "s-confining" $\mathcal{N} = 1$ theories with simple gauge group and $W_{tree} = 0$. These are the theories analogous to SQCD with $N_f = N_c + 1$ [96]: the low energy IR free fields have only superpotential, and no (dual) gauge interactions. Another example is $Sp(N_c)$ with $N_f = N_c + 2$ flavors [97]. Many analogous theories were summarized in [98]. The basic fields of the IR free theory are all gauge invariant composites of the UV matter, ignoring the classical relations, and there is a W_{dyn} whose F-term equations give the classical relations. Adding a linear term in one of the IR free fields can potentially break supersymmetry.

As shown in [61], the $SU(N_c)$ theory with $N_f = N_c+1$ has *calculable* metastable dynamical supersymmetry breaking, whereas the $Sp(N_c)$ theory with $N_f = N_c + 2$ does not. The difference is that all pseudomoduli of the $SU(N_c)$ theory enter into cubic terms in W_{dyn}, whereas pseudomoduli of the Sp theory couple via power-law irrelevant terms in W, so their effective potential is incalculable. Again, if there are *any* incalculable pseudomoduli, one cannot determine whether or not the theory has DSB – it depends on the sign of the incalculable higher-order Kahler potential terms.

A scan of the other examples in [98] reveals that the $SU(N_c)$ SQCD is a rather special example. The other examples more generically have many fields which appear in W_{dyn} via terms which are power-law irrelevant, which will become incalculable pseudomoduli if a sector of the theory breaks supersymmetry.

As an example, consider $SU(N_c)$ with one flavor of antisymmetric tensor, A and $\widetilde{A}$, and $N_f = 3$ fundamental flavors, Q and $\widetilde{Q}$. The IR free theory is discussed in [98]. If we add $W_{tree} = m_A A\widetilde{A} + m_Q \mathrm{Tr} Q\widetilde{Q}$, the m_Q term leads to a rank-condition supersymmetry breaking sector (with $\Phi_1 = Q\widetilde{Q}/\Lambda$ and $\varphi = \widetilde{A}(A\widetilde{A})^{\frac{1}{2}N_c-1}Q^2$), so it is possible that this theory has a metastable DSB vacuum near the origin. But many pseudomoduli, e.g. $T_1 = A\widetilde{A}$, couple only via superpotential terms of quartic and higher order. Thus they are not reliably stabilized within the low-energy effective theory, and the DSB vacuum requires an assumption about the sign of non-calculable terms in the Kahler potential. Such potentially dangerous pseudomoduli can still can be stabilized by hand, by modifying the UV theory to give them masses, to obtain a theory with (metastable) DSB vacua.

The scan of these classes suggest that calculability is perhaps not generic.

This chapter is a reprint of material as it appears in "Surveying Pseudomoduli: the Good, the Bad, and the Incalculable," K. Intriligator, D. Shih, M. Sudano, JHEP **0903**, 106 (2009), arXiv:0809.3981, of which I was a co-author.

Appendix A

Two-Loop Calculation of Sfermion Masses in Higgsed Gauge Mediation

There are ten diagrams relevant to the computation of the lowest-order scalar mass correction. They are shown in Figure 3.5. The first eight are the standard contributions. The final two arise from interactions with the scalar, C, of the massive vector multiplet. For $U(1) \times U(1)'$, we have

$$\mathcal{L} \supset -gm_W C(i\phi_+^*\phi_- - i\phi_+\phi_-^* + |q|^2), \tag{A.1}$$

where $\phi_\pm$ has mass-squared $M^2 \pm F$, and q is a scalar that will get a radiative mass. The charge assignments should be clear. Of course, there are many more diagrams that do not involve the messengers, but their contributions sum to zero. In fact, Diagram 5 is independent of supersymmetry breaking, but we prefer to compute with the complete messenger multiplet. This gives a finite result and thus a check on the calculation. There are no IR divergences. Dimensional reduction [99] is used to regulate the UV divergences. In this context, dimensional reduction simply amounts to performing all Lorentz algebra in four dimensions and then evaluating the resulting scalar integral in $4 - 2\epsilon$ dimensions. In evaluating the integrals, we expressed each integral as a sum of "master integrals" – integrals with momentum-independent numerators. This method is discussed in more detail in

[100]. In their notation, the most general two-loop master integral is

$$\begin{aligned}&\langle m_{11}, m_{12}, \ldots | m_{21}, m_{22}, \ldots | m_{31}, m_{32}, \ldots \rangle \\ &\qquad \equiv \prod_{i,j,k} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{1}{(k^2 + m_{1i}^2)(q^2 + m_{2j}^2)[(k+q)^2 + m_{3k}^2]}. \quad \text{(A.2)}\end{aligned}$$

In our calculation, only the following two integrals are needed,

$$\langle m_1 | m_2 | m_3 \rangle = \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{1}{(k^2 + m_1^2)(q^2 + m_2^2)[(k+q)^2 + m_3^2]}, \quad \text{(A.3)}$$

$$\langle m_1, m_1 | m_2 | m_3 \rangle = \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{1}{(k^2 + m_1^2)^2(q^2 + m_2^2)[(k+q)^2 + m_3^2]}. \quad \text{(A.4)}$$

Clearly these integrals are not independent; the second is a derivative of the first. In turns out, however, that the dimensionless integral (A.4) is the easier integral to evaluate, so it is useful to have the inverse identity,

$$\langle m_1 | m_2 | m_3 \rangle = \frac{m_1^2 \langle m_1, m_1 | m_2 | m_3 \rangle + m_2^2 \langle m_2, m_2 | m_3 | m_1 \rangle + m_3^2 \langle m_3, m_3 | m_1 | m_2 \rangle}{3 - d}. \quad \text{(A.5)}$$

The single integral that we need is

$$\begin{aligned}&\langle m_1, m_1 | m_2 | m_3 \rangle \\ &= \frac{1}{2(4\pi)^4} \left[\frac{1}{\epsilon^2} + \frac{1 - 2\ln \bar{m}_1^2}{\epsilon} + 1 + \frac{\pi^2}{6} - 2\ln \bar{m}_1^2 + 2\ln^2 \bar{m}_1^2 + 2F\Big(\frac{m_2^2}{m_1^2}, \frac{m_3^2}{m_1^2}\Big) \right], \quad \text{(A.6)}\end{aligned}$$

where $\bar{m}^2 = m^2 e^\gamma / 4\pi$, and the function of the mass ratios is[1]

$$\begin{aligned}&F(a,b) \quad \text{(A.7)} \\ &= -\frac{1}{2}\ln^2 a - \mathrm{Li}_2\Big(\frac{a-b}{a}\Big) \\ &+ \Big(\frac{a+b-1}{2r} - \frac{1}{2}\Big)\Big[\mathrm{Li}_2\Big(\frac{b-a}{x_+}\Big) - \mathrm{Li}_2\Big(\frac{a-b}{1-x_+}\Big) - \mathrm{Li}_2\Big(\frac{1-x_+}{-x_+}\Big) + \mathrm{Li}_2\Big(\frac{-x_+}{1-x_+}\Big)\Big] \\ &+ \Big(\frac{a+b-1}{2r} + \frac{1}{2}\Big)\Big[\mathrm{Li}_2\Big(\frac{b-a}{x_-}\Big) - \mathrm{Li}_2\Big(\frac{a-b}{1-x_-}\Big) - \mathrm{Li}_2\Big(\frac{1-x_-}{-x_-}\Big) + \mathrm{Li}_2\Big(\frac{-x_-}{1-x_-}\Big)\Big],\end{aligned}$$

having defined the parameters,

$$r = \sqrt{1 - 2(a+b) + (a-b)^2}, \qquad x_+ = \frac{1}{2}(1+b-a+r), \qquad x_- = \frac{1}{2}(1+b-a-r),$$

[1]This corrects a typo in [100]. Their simplified form of this function is correct. We prefer the unsimplified form because it presents fewer numerical complications.

and having made use of the dilogarithm,

$$\mathrm{Li}_2(z) = \int_0^1 dt \frac{\ln(1-zt)}{t}. \tag{A.8}$$

Finally, we have all the ingredients we need. For the case a single supersymmetric vector superfield with mass m_W, the decomposition of each diagram into master integrals is shown at the end of Appendix A. The parameter, ξ, determines the gauge. The absence of dependence on ξ in the sum of diagrams provides another check on the computation. The more general case with different vector superfields with masses m_W and $\tilde{m}_W$ follows. In evaluating this mixed contribution, one finds that the expressions for individual diagrams can be unwieldy when expressed in terms of two gauge-fixing parameters. It is worth calculating with the parameters for the sake of checking the calculation, but a lot of work can be saved by adding diagrams at intermediate stages. This has been done. The gauge-invariant combinations are shown.

We would like to note that the vanishing of Diagram 6 (see Figure 3.5) is a rather robust result, though the authors know of no principle requiring it to be zero. In particular, one can allow each gauge bosons to have arbitrary mass and to be in an arbitrary gauge. Using four-component spinors, the diagram is found to be proportional to

$$\int \frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\mathrm{Tr}[k^\mu \Delta_{\mu\nu}(k)\gamma^\nu \Delta_{1/2}(k+q)\gamma^\rho \Delta_{1/2}(q)\Delta_{\rho\sigma}(k)k^\sigma \Delta_0(k)] \tag{A.9}$$

where

$$\Delta_{\mu\nu}(k) = \frac{-i}{k^2-m_W^2}\Big[g_{\mu\nu} - \frac{(1-\xi)k_\mu k_\nu}{k^2-\xi m_W^2}\Big],\ \widetilde{\Delta}_{\mu\nu}(k) = \frac{-i}{k^2-\widetilde{m}_W^2}\Big[g_{\mu\nu} - \frac{(1-\tilde{\xi})k_\mu k_\nu}{k^2-\tilde{\xi}\widetilde{m}_W^2}\Big],$$
$$\Delta_{1/2}(k) = \frac{i(\not{k}+m_f)}{k^2-m_f^2},\ \Delta_0(k) = \frac{i}{k^2}. \tag{A.10}$$

A little algebra shows that (A.9) can be written as

$$\int \frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\frac{\mathrm{Tr}[\not{k}(\not{k}+\not{q}+m_f)\not{k}(\not{q}+m_f)]f(k^2)}{[(k+q)^2-m_f^2](q^2-m_f^2)}, \tag{A.11}$$

for a function, $f(k^2)$, which contains all of the information about the gauge bosons. The rest of the integral is simplified with the use of the identity,

$$\mathrm{Tr}[\not{k}(\not{k}+\not{q}+m_f^2)\not{k}(\not{q}+m_f^2)] = 4(k\cdot q)[(k+q)^2-m_f^2] - 4(k\cdot q)(q^2-m_f^2) - 4k^2(q^2-m_f^2) \tag{A.12}$$

If this is put back into the integral, and the change of variables, $q \to k+q$, is made in the first term, one finds that the second and third terms are exactly canceled, and the integral vanishes.

Finally, the sum of unmixed diagrams normalized so that $f(0,0) = 1$ gives the function in (3.3):

$$
\begin{aligned}
&f(x,y) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A.13)\\
&= \frac{1}{x^2}\Bigg[F(1,y) + (1+y)F\Big(\frac{1}{y},\frac{1}{y}\Big) - F(1+x,y) + \frac{1}{2}(1+x)F\Big(1,\frac{y}{1+x}\Big)\\
&-(1+x)F\Big(\frac{1}{1+x},\frac{y}{1+x}\Big) + \frac{1}{2}(1+x)F\Big(\frac{1-x}{1+x},\frac{y}{1+x}\Big) + (x-2y)F\Big(\frac{1+x}{y},\frac{1}{y}\Big)\\
&-(1+x-2y)F\Big(\frac{1+x}{y},\frac{1+x}{y}\Big) + \frac{y}{2}F\Big(\frac{1+x}{y},\frac{1-x}{y}\Big)\Bigg] + (x \to -x),
\end{aligned}
$$

and the sum of the mixed diagrams gives,

$$
\begin{aligned}
&h(x,y,z) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A.14)\\
&= \Bigg\{\frac{1}{2x^2(y-z)}\Bigg[2(2+y)F(1,y) + (2+y)yF\Big(\frac{1}{y},\frac{1}{y}\Big) + 2(x-y)F(1+x,y)\\
&-(1+x)(4+4x-y)F\Big(1,\frac{y}{1+x}\Big) + 2(1+x)(x-y)F\Big(\frac{1}{1+x},\frac{y}{1+x}\Big)\\
&+(1+x)yF\Big(\frac{1-x}{1+x},\frac{y}{1+x}\Big) + 2(x-y)yF\Big(\frac{1+x}{y},\frac{1}{y}\Big)\\
&-(4+4x-y)\frac{y}{2}F\Big(\frac{1+x}{y},\frac{1+x}{y}\Big) + \frac{y^2}{2}F\Big(\frac{1+x}{y},\frac{1-x}{y}\Big)\Bigg] + (x \to -x)\Bigg\}\\
&+(y \leftrightarrow z),
\end{aligned}
$$

This appendix is a reprint of material as it appears in "Sparticle Masses in Higgsed Gauge Mediation," E. Gorbatov, M. Sudano, JHEP **0810**, 066 (2008), arXiv:0802.0555, of which I was a co-author.

Unmixed Diagrams

$$\textbf{Diagram 1} = 2\xi^2\langle m_+\rangle\langle m_W, m_W\rangle + 2\xi^2\langle m_-\rangle\langle m_W, m_W\rangle$$

$$\textbf{Diagram 2} = -2\xi^2\langle m_+\rangle\langle m_W, m_W\rangle - 2\xi^2\langle m_-\rangle\langle m_W, m_W\rangle$$

$$\textbf{Diagram 3} = -2(3+\xi^2)\langle m_+\rangle\langle m_W, m_W\rangle - 2(3+\xi^2)\langle m_-\rangle\langle m_W, m_W\rangle$$

$$\textbf{Diagram 4} = 2(1+\xi^2)\langle m_+\rangle\langle m_W, m_W\rangle + 2(1+\xi^2)\langle m_-\rangle\langle m_W, m_W\rangle$$

$$-\langle m_+|m_+|m_W\rangle - \langle m_-|m_-|m_W\rangle$$

$$-(4m_+^2 - m_W^2)\langle m_+|m_+|m_W, m_W\rangle - (4m_-^2 - m_W^2)\langle m_-|m_-|m_W, m_W\rangle$$

$$\textbf{Diagram 5} = 8\langle m_f\rangle\langle m_W, m_W\rangle - 4\langle m_f|m_f|m_W\rangle + (8m_f^2 + 4m_W^2)\langle m_f|m_f|m_W, m_W\rangle$$

$$\textbf{Diagram 6} = 0$$

$$\textbf{Diagram 7} = -2\langle m_+|m_-|0\rangle$$

$$\textbf{Diagram 8} = -8\langle m_f\rangle\langle m_W, m_W\rangle + 4\langle m_+\rangle\langle m_W, m_W\rangle + 4\langle m_-\rangle\langle m_W, m_W\rangle$$

$$+4\langle m_+|m_f|m_W\rangle + 4\langle m_-|m_f|m_W\rangle$$

$$+(4m_+^2 - 4m_f^2 - 4m_W^2)\langle m_+|m_f|m_W, m_W\rangle + (4m_-^2 - 4m_f^2 - 4m_W^2)\langle m_-|m_f|m_W, m_W\rangle$$

$$\textbf{Diagram 9} = 4\langle m_+|m_-|0\rangle - 4\langle m_+|m_-|m_W\rangle$$

$$\textbf{Diagram 10} = -2\langle m_+|m_-|0\rangle + 2\langle m_+|m_-|m_W\rangle + 2m_W^2\langle m_+|m_-|m_W, m_W\rangle$$

Mixed Diagrams

$$\textbf{Diagram 1} + \textbf{Diagram 3} = -6\langle m_+\rangle\langle m_W, \widetilde{m}_W\rangle - 6\langle m_-\rangle\langle m_W, \widetilde{m}_W\rangle$$

$$\textbf{Diagram 2} + \textbf{Diagram 4} = 2\langle m_+\rangle\langle m_W, \widetilde{m}_W\rangle + 2\langle m_-\rangle\langle m_W, \widetilde{m}_W\rangle$$
$$-\frac{1}{2}\langle m_+|m_+|m_W\rangle - \frac{1}{2}\langle m_+|m_+|\widetilde{m}_W\rangle - \frac{1}{2}\langle m_-|m_-|m_W\rangle - \frac{1}{2}\langle m_-|m_-|\widetilde{m}_W\rangle$$
$$-\Big(4m_+^2 - \frac{1}{2}m_W^2 - \frac{1}{2}\widetilde{m}_W^2\Big)\langle m_+|m_+|m_W, \widetilde{m}_W\rangle$$
$$-\Big(4m_-^2 - \frac{1}{2}m_W^2 - \frac{1}{2}\widetilde{m}_W^2\Big)\langle m_-|m_-|m_W, \widetilde{m}_W\rangle$$

$$\textbf{Diagram 5} = 8\langle m_f\rangle\langle m_W, \widetilde{m}_W\rangle - 2\langle m_f|m_f|m_W\rangle - 2\langle m_f|m_f|\widetilde{m}_W\rangle$$
$$+\Big(8m_f^2 + 2m_W^2 + 2\widetilde{m}_W^2\Big)\langle m_f|m_f|m_W, \widetilde{m}_W\rangle$$

$$\textbf{Diagram 6} = 0$$

$$\textbf{Diagram 7} = -2\langle m_+|m_-|0\rangle$$

$$\textbf{Diagram 8} = -8\langle m_f\rangle\langle m_W, \widetilde{m}_W\rangle + 4\langle m_+\rangle\langle m_W, \widetilde{m}_W\rangle + 4\langle m_-\rangle\langle m_W, \widetilde{m}_W\rangle$$
$$+2\langle m_+|m_f|m_W\rangle + 2\langle m_+|m_f|\widetilde{m}_W\rangle + 2\langle m_-|m_f|m_W\rangle + 2\langle m_-|m_f|\widetilde{m}_W\rangle$$
$$+(4m_+^2 - 4m_f^2 - 2m_W^2 - 2\widetilde{m}_W^2)\langle m_+|m_f|m_W, \widetilde{m}_W\rangle$$
$$+(4m_-^2 - 4m_f^2 - 2m_W^2 - 2\widetilde{m}_W^2)\langle m_-|m_f|m_W, \widetilde{m}_W\rangle$$

$$\textbf{Diagram 9} = 4\langle m_+|m_-|0\rangle - 2\langle m_+|m_-|m_W\rangle - 2\langle m_+|m_-|\widetilde{m}_W\rangle$$

$$\textbf{Diagram 10} = -2\langle m_+|m_-|0\rangle + \langle m_+|m_-|m_W\rangle + \langle m_+|m_-|\widetilde{m}_W\rangle$$
$$+m_W^2\langle m_+|m_-|m_W, \widetilde{m}_W\rangle + \widetilde{m}_W^2\langle m_+|m_-|m_W, \widetilde{m}_W\rangle$$

Appendix B

Deriving the Leading-Log Effective Potential

In this appendix, we derive the formula (5.21) for the leading-log effective potential in the large field regime (5.13).

The statement that we can approximate Ω_X with leading logs is the statement that we have a good power-series expansion of the form

$$\Omega_X(t) \approx C_0 + C_1\kappa(t-t_\Lambda) + \frac{1}{2!}C_2(\kappa(t-t_\Lambda))^2 + \dots \tag{B.1}$$

with $\kappa \ll \kappa(t-t_\Lambda) \ll 1$ and $t = \log Q/m_0$. Note that we are expanding Ω_X around the UV scale Λ but we are defining the RG time with respect to the IR scale m_0; the reasons for this will be apparent in a moment. Because the effect we are after is, in fact, a leading log effect, we drop terms of $\mathcal{O}(\kappa^n(t-t_\Lambda)^m)$ with $m < n$ and (since there are not terms with $m > n$) we only keep the terms with $n = m$.

Now suppose that at $t = t_\Phi$, the nth derivative of Ω_X is discontinuous with all lower-order derivatives still continuous. Then for $t < t_\Phi$, the expansion (B.1) becomes

$$\Omega_X(t) = C_0' + C_1'\kappa(t-t_\Lambda) + \frac{1}{2!}C_2'(\kappa(t-t_\Lambda))^2 + \dots \qquad (t < t_\Phi) \tag{B.2}$$

with the new coefficients C_i' satisfying

$$\sum_{k=i}^{\infty} 1(k-i)!C_k(\kappa(t_\Phi - t_\Lambda))^{k-i} = \sum_{k=i}^{\infty} \frac{1}{(k-i)!}C_k'(\kappa(t_\Phi - t_\Lambda))^{k-i}, \quad i = 0, \dots, n-1. \tag{B.3}$$

The Taylor expansion coefficients C_i of Ω_X around the UV scale are independent of t_Φ. Then (B.3) yields a system of equations which determine the t_Φ dependence of the IR Taylor coefficients $C'_0, \ldots, C'_{n-1}$. It is straightforward to check that (B.3) are solved by

$$C'_i = C_i - \frac{1}{(n-i)!}(C_n - C'_n)(-\kappa(t_\Phi - t_\Lambda))^{n-i} + \ldots \qquad (i = 0, \ldots, n-1) \quad \text{(B.4)}$$

where $\ldots$ denote terms that are higher order in κ. Plugging this into (B.2), we see that

$$\begin{aligned} \Omega_X(0) &= const. - \sum_{i=0}^{n} \frac{1}{i!(n-i)!}\kappa^n(-1)^n(C_n - C'_n)(t_\Phi - t_\Lambda)^{n-i}t_\Lambda^i + \ldots \\ &= const. - \frac{1}{n!}(C_n - C'_n)(-\kappa t_\Phi)^n + \ldots . \end{aligned} \quad \text{(B.5)}$$

This reproduces (5.18) after we identify $\Delta\Omega_X^{(n)} \equiv \frac{d^n\Omega_X}{dt^n}\big|_{t_\Phi^-}^{t_\Phi^+} = \kappa^n(C_n - C'_n)$.

This appendix is a reprint of material as it appears in "Surveying Pseudomoduli: the Good, the Bad, and the Incalculable," K. Intriligator, D. Shih, M. Sudano, JHEP **0903**, 106 (2009), arXiv:0809.3981, of which I was a co-author.

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