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Stress-Ratio-Based Interpretation of Modulus Reduction and Damping Curves

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Abstract

Modulus reduction and damping values are commonly plotted against cyclic shear strain amplitude (γ_c), and the resulting curves are known to depend on mean effective stress (p'), plasticity characteristics, strain rate, and number of loading cycles. The dependence on p' is potentially problematic for undrained effective stress analysis where excess pore pressure may develop during loading. This paper presents a new concept in which normalized modulus reduction (G/G_{max}) and damping (D) values are plotted against stress ratio (η) rather than γ_c. Relations developed for sand, clay, and peat are found to be essentially pressure-independent when G/G_{max} and D-D_{min} are plotted vs. η, whereas all three are pressure-dependent when plotted vs. γ_c. This finding is potentially useful for undrained effective stress analysis where p' may change during loading, and provides a new approach for interpreting laboratory tests in future development of G/G_{max} and D curves.

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Motivation

The cyclic stress-strain behavior of soil is commonly characterized using modulus reduction and damping (MRD) curves in which secant shear modulus and percent damping are expressed as functions of cyclic shear strain amplitude (γ_c). Curves have been derived from cyclic laboratory testing equipment capable of measuring small-strain behavior, including specialized simple shear devices (e.g., Vucetic and Dobry 1991, Doroudian and Vucetic 1995), specialized triaxial compression devices (e.g., Wehling et al. 2003, Kishida et al. 2009), and resonant column / torsional shear devices (e.g., Menq 2003, Darendeli 2001). Research studies have found that modulus reduction and damping curves depend on the following factors: soil type, effective stress (e.g., Darendeli 2001, Menq 2003, Kishida et al. 2009, EPRI 1993), plasticity index (e.g., Vucetic and Dobry 1991, Darendeli 2001), number of loading cycles (e.g., Matasovic and Vucetic 1995), and strain rate (e.g., Matesic and Vucetic 2003). This paper focuses on the dependence of MRD curves on effective stress. In this paper, the term "modulus reduction curve" denotes the normalized secant modulus G/G_{max}, and the term "normalized" is omitted for brevity and consistency with established convention.

Pressure-dependence of MRD curves is often evaluated in the laboratory by consolidating soils to different pressures, and shearing them either in drained or undrained loading. When sheared in undrained loading, specimens may develop excess pore pressure that alters the effective stress from its initial condition. Effective stress ground response analysis codes often explicitly model excess pore pressures, in which case the MRD curves evolve during loading due to their pressure-dependence [e.g., Deepsoil (Hashash et al. 2015) and D-MOD (Matasovic 2006)]. Formulating modulus reduction and damping curves in a manner that does not depend on effective stress would therefore be beneficial for implementation in effective stress ground response analysis codes, and potentially for plasticity formulations.

Hardin and Drnevich (1972a and b) proposed plotting G/G_{max} versus γ_c/γ_r, where γ_r is the reference shear strain defined as the ratio of shear strength to G_{max}. Curves plotted in this manner were found to be independent of effective stress, but γ_r itself depends on effective stress because shear strength and G_{max} scale differently with effective stress. Therefore the reference strain must be adjusted as effective stress changes during an effective stress ground response analysis. This can be done using the equations provided in Hardin and Drnevich (1972), or Hashash and Park (2001).

This paper presents a new concept in which modulus reduction and damping curves are plotted versus stress ratio (η) instead of shear strain, which provides an alternative to the reference shear strain approach that is better suited to implementation in plasticity models formulated in stress-ratio space. First, the calculation of the stress ratio η is described. The concept is then demonstrated by applying it to relationships formulated by Darendeli (2001) for clay, Menq (2003) for sand, and Kishida *et al.* (2009) for peat. The resulting relationships for G/G_{max} and $D - D_{min}$ vs η are shown to be pressure-independent. Finally, implications and potential uses of the new approach are discussed

Calculation of the Stress Ratio

The stress ratio η is defined as the deviator stress q divided by mean effective stress p', though it is also sometimes defined for a simple shear stress path as the shear stress on a vertical-horizontal plane τ divided by p'. Existing MRD curves are developed for a simple shear stress path, hence the latter definition is adopted for analyses presented in this paper, whereas the former is more suitable for application in a multi-dimensional plasticity model. For a given G/G_{max} versus γ_c curve and G_{max} value, a corresponding G/G_{max} versus η curve can be computed using Eq. 1. Note that Eq. 1 is formulated based on the assumption that users are beginning with a G/G_{max} curve and value of G_{max} based on an empirical relationship. However, the procedure may be applied more directly to laboratory measurements by simply computing τ/p' without involving γ_c. Note that G/G_{max} and G depend on variables in addition to γ_c and p', such as plasticity characteristics, overconsolidation ratio, organic content, strain rate, frequency, number of cycles, etc.

$$\eta = \frac{G_{max}}{p'} \cdot \frac{G}{G_{max}}(\gamma_c, p', \ldots) \cdot \gamma_c \tag{1}$$

G/G_{max} and D-D_{min} vs η for Commonly Used Relations

The proposed concept is demonstrated for published equations defining the modulus reduction and damping behavior of sand (Menq 2003), clay (Darendeli 2001), and peat (Kishida et al. 2009). The models were selected because they are widely used in practice and cover a wide range of material types. The equations are too lengthy to reproduce herein, but the input parameters are provided so that readers can reproduce the results after consulting relevant sections of the references associated with each model. This section presents the input parameters selected to generate the modulus reduction and damping curves for each soil type selected for the example. In each case, the input parameters are consistent with the database from which the relations were derived. For each relationship, MRD curves are computed at four different values of p'. Those curves are first plotted versus shear strain to illustrate their dependence on confining pressure, and then versus stress ratio to validate the concept presented herein.

Relationship for Sand

Menq (2003) constructed a large-scale, multi-mode, free-free resonant column device and studied the dynamic properties of non-plastic sandy and gravelly soils. Based on his tests, Menq developed regression equations for G_{max} and modulus reduction and damping curves. The modulus reduction and damping curves depend on the mean effective stress (p'), the coefficient of uniformity (C_u), the mean grain size (D_{50}), and the number of cycles (N_c). G_{max} also depends on the void ratio (e). Input properties utilized herein are provided in Table 1.

Relationship for Clay

Darendeli (2001) developed regression equations defining the modulus reduction and damping behavior measured in resonant column / torsional shear tests of clayey soils. The equations depend on the plasticity index (*PI*), the overconsolidation ratio (*OCR*), the number of cycles, and frequency. Darendeli did not provide recommendations for computing G_{max}, so we adopt Eq. 2 developed by Hardin and Drnevich (1972b) and normalized by Schneider et al. (1999), where e is void ratio, *OCR* is overconsolidation ratio, M and N depend on soil type, and p_a is atmospheric pressure. M and N were selected based on the recommendations of Hardin and Drnevich (1972b) and Schneider et al. (1999). Other models exist for computing G_{max}, and the exponent on the effective stress term is the most important contributor to pressure-dependence of the G/G_{max} versus η relationship. The pressure-dependence of G_{max}

counterbalances the pressure-dependence of G/G_{max} versus γ_c to render a pressure-independent G/G_{max} versus η relationship.

$$\frac{G_{max}}{p_a} = 321\frac{(2.973-e)^2}{1+e}OCR^M\left(\frac{p'}{p_a}\right)^N \quad (2)$$

Eq. 2 depends on e, which is a function of consolidation condition for clays according to Eq. 3:

$$e = e_N - \lambda \ln\left(\frac{p'_c}{p_1}\right) + \kappa \ln\left(\frac{p'_c}{p'}\right) \quad (3)$$

where e_N is the void ratio at reference pressure p_1, λ is the slope of the virgin compression line and κ is the slope of the recompression line in e-ln p' space, and p_c is the maximum past pressure computed as p'_c=OCR·p'. Input parameters utilized herein are provided in Table 2. The modulus reduction and damping curves were computed using the regression constants from Table 8-12 in Darendeli (2001).

<u>Relationship for Peat</u>

Kishida et al. (2006, 2009) developed a regression model for MRD curves and G_{max} for peat based on p', organic content (*OC*), and the laboratory consolidation ratio (*LCR*). The *LCR* is defined as the laboratory consolidation stress divided by the in-situ vertical effective stress. Soil properties input to the Kishida et al. (2009) model are summarized in Table 3.

Example of Modulus Reduction and Damping Curves Plotted against η

Modulus reduction curves computed for sand, clay, and peat are plotted in Fig. 1a,b,c versus γ_c and in Fig. 1d,e,f versus η . Different MRD curves arise for different values of p' when G/G_{max} is plotted versus γ_c. The influence of p' on MRD is significant for all three soil types, and is highest for sand, and lower for clay and peat. However, the G/G_{max} and D-D_{min} curves are essentially pressure-independent when plotted versus η for each soil type. The reason why this occurs is that the overburden scaling for G_{max} combines with the overburden scaling for G/G_{max} versus γ_c in a manner that renders G/G_{max} versus η insensitive to p'.

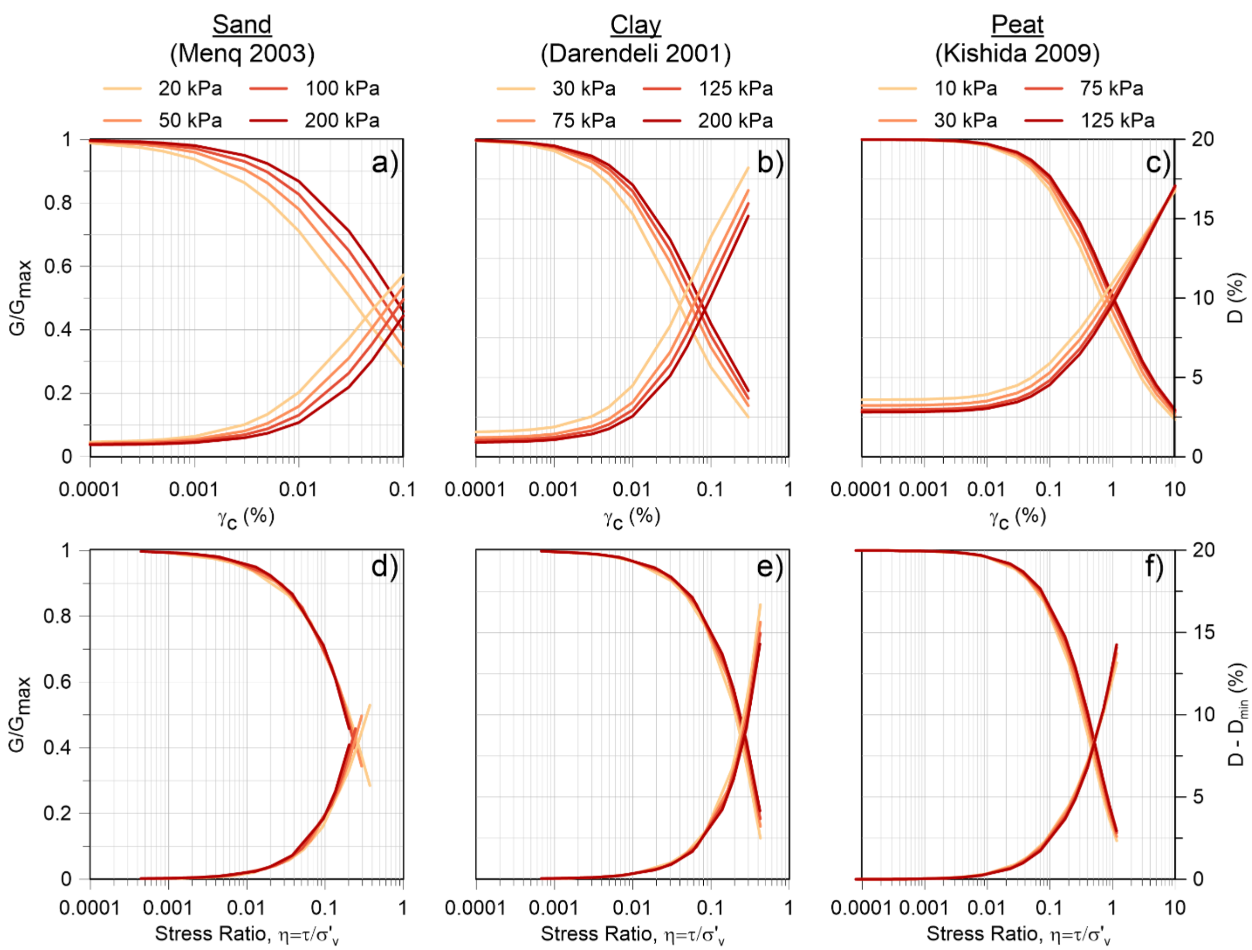

Fig. 1. Modulus reduction and damping curves versus γ_c and η for (a and d) sand; (b and e) clay; and (c and f) peat.

The small-strain damping value, D_{min}, is subtracted from the strain-dependent damping relationship when plotting versus η. This procedure was adopted because $D - D_{min}$ versus η is pressure-independent for each soil type, whereas D versus η is not. Hysteretic damping formulations typically do not capture small-strain damping, relying instead on Rayleigh damping formulations. Subtracting D_{min} is therefore reasonable and convenient for typical implementations.

This approach was repeated for the three relationships presented here with different sets of input parameters in order to verify the concept. The results from this parameter study are not presented for brevity, but the concept proved to be true for any set of input parameters consistent with each model's database.

Benefits of Proposed Approach

The proposed approach provides four distinct benefits compared with the traditional approach. First, the MRD curves studied herein are independent of p' when plotted versus η. Effective stress ground response analysis for undrained loading conditions is much simpler when the soil behavior is independent of p' because excess pore pressure development does not necessitate changes to the backbone curve or damping relationship. Hardin and Drnevich (1972a,b) utilized a reference strain concept to render pressure-independent curves, but the reference strain is itself pressure-dependent. Hence, the value of γ_r must be updated during analyses for which the effective stress changes during loading, meaning that different G/G_{max} versus γ_c curves must be utilized. Hashash and Park (2001) describe implementation of such a procedure.

Second, modulus reduction and damping curves are often extrapolated to large strains beyond the range of empirical validation, which can result in significant errors in the implied shear strength (e.g., Yee et al. 2013, Afacan et al. 2014). The relationships presented herein were constrained within the range of experimental validation, but ground response analysis often exceeds this range, requiring extrapolation. Very small differences in the large-strain tail of the G/G_{max} vs. γ_c curve can result in significant differences in the implied shear strength. By contrast, representing G/G_{max} as a function of η instead of γ_c provides direct control over the mobilized shear strength because a single peak stress ratio can easily be specified.

Third, advanced constitutive models are often formulated such that the plastic modulus is defined in stress-ratio space. For example, Dafalias and Manzari (2004) and Boulanger and Ziotopoulou (2015) developed stress-ratio based bounding surface plasticity models in which the plastic modulus is a function of the distance in stress space between the current stress ratio and the stress ratio at an image point on the bounding surface. Adjusting bounding surface model parameters to provide desired G/G_{max} versus γ_c behavior is a complex and difficult task. The proposed approach could conceivably be utilized to simplify this task, or possibly to directly define plastic modulus based on position in stress-ratio space.

Fourth, the proposed framework provides a new approach for interpreting laboratory test data in a manner that may eliminate *p′* as an influential variable, which can reduce uncertainties when developing regression models.

Discussion

The G/G_{max} and D-D_{min} versus η curves are independent of *p′* for the particular relationships utilized in this manuscript, which is valuable because these relationships are commonly utilized in engineering practice. However, independence may not be achieved for other relationships.

Identifying the relationships for which the proposed procedure provides pressure-independence lies beyond the scope of this paper. Readers are encouraged to verify pressure independence of the G/G_{max} and D-D_{min} relationships if they wish to use this concept for relations not presented herein.

Furthermore the effect of *OCR* is not included in this formulation. G/G_{max} versus η curves are independent of *p′* but not *OCR*. Since *OCR* can change during loading as effective stress changes, this is a limitation of the concept. However, G/G_{max} versus γ_c curves present the same limitation.

Conclusions

Modulus reduction and damping curves have traditionally been plotted versus cyclic shear strain. However, this approach has several drawbacks: (1) G/G_{max} and *D* versus γ_c depend on *p′*, which can cause problems for undrained loading where effective stress may change due to development of excess pore pressure, and (2) advanced constitutive models typically represent shear modulus as a function of η rather than γ_c, requiring sometimes complex calibration procedures to achieve desired modulus reduction and damping behavior. This paper demonstrates that plotting G/G_{max} versus η results in pressure-independent modulus reduction and damping curves for three commonly-used relationships. This finding is potentially useful for implementation in one-dimensional effective stress codes for undrained loading conditions, and in advanced plasticity models.

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