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Publication Date

1972-04-01

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PHYSICS AND
ASTRONOMY SECTION

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April 1972

Prepared for the U.S. Atomic Energy Commission
under Contract W-7405-ENG-48

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LBL-1766
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A VERSATILE, HIGH-RESOLUTION PARTICLE IDENTIFIER - THEORY*

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Received 4 April 1972

UNIVERSITY OF CALIFORNIA

1972 402 F

REPRINT NUMBER

LAWRENCE BERKELEY LABORATORY

LBL 1766

A method of measurement and analysis is presented which makes maximum use of the dependence of energy loss dE/dx upon range to identify isotopically particles which stop in an identifier consisting of several thick, solid state detectors. Theoretical identification efficiency, mass resolution, charge resolution, and energy resolution are determined. A following paper provides experimen-

tal resolutions and describes two identifiers; one used to provide particle identification during heavy ion fragmentation studies at the Bevatron, Lawrence Berkeley Laboratory; the other, a rocket-borne instrument used to measure the isotopic abundances in the inner trapped radiation belt.

1. Introduction

It is convenient to begin this paper by discussing the situation which required the development of this particle identifier, for the motivation leads in a natural way to the method developed.

One consequence of the Apollo Command Module fire was the cancellation of many of the scientific experiments planned for the earth-orbiting Apollo missions. One such experiment (proposed by H. H. Heckman) would have used nuclear emulsions to measure the flux, energy spectrum, and isotopic abundances of particles trapped in the inner radiation belt. Transfer of the experiment to one of the later, longer duration missions was precluded due to the accumulative detrimental effects of background. Thus a need arose for an electronic particle identifier which could assume the tasks originally assigned to the nuclear emulsion experiment.

Earlier the author had considered the possibility of achieving particle identification using an identifier consisting of solid state detectors separated by layers of absorber. This configuration represented an attempt to obtain information useful for particle identification by observing the change in the rate of energy loss (dE/dx) with range. It was found that such a detector absorber design could provide identification, however, only over a limited range of energies. Other configurations utilizing the same approach were investigated in order to meet the scientific requirements to measure proton energies from 20 to 1000 MeV and to identify reliably geomagnetically trapped deuterons and tritons estimated to be less than 1% of the proton component.

As a result of Monte Carlo calculations and prototype evaluation, we found that isotopic identification

over a wide range of energies is possible provided dE/dx is measured *continuously* along the entire range of a particle. This requirement can be achieved by an identifier design which uses enough detectors to stop the isotopes of interest and has a minimal amount of inactive material between dE/dx sensitive regions.

The maximum energy at which isotopes can be identified is limited by the nuclear interaction length. Chemical separation extends to relativistic energies and does not require the particle to stop in the instrument.

2. Qualitative description of method and limitations

In this section we develop approximate equations for the dependence of the energy deposited in a detector upon the mass m , charge z , and energy E of the incident particle. Using these equations, we then examine in a *qualitative* fashion this method of particle identification and its inherent limitations.

2.1. ENERGY LOSS IN A DETECTOR

For our purposes here we express the energy loss rate of a particle moving through material as

$$\frac{dE}{dx} = \frac{az^2}{\beta^2} \ln \left(\frac{b\beta^2}{1-\beta^2} \right), \quad (1)$$

where β is the velocity of the incident particle in units of the speed of light; constants a and b are determined only by the stopping material.

In this qualitative approach we will assume a solid state detector produces a pulse of charge proportional to the energy deposited by the incident particle and will refer to this energy deposited as the pulse produced in the detector.

In order to evaluate the pulse, P , when the velocity is changing, we first integrate eq. (1) obtaining a

* Work was supported by the National Aeronautics and Space Administration NASA Grants NGR 05-003-405 and NGR 05-003-256, and Contract NAS 9-5249.

relationship for the total range, R , of the particle

$$R = \int_0^E \frac{dx}{dE} dE = \frac{m}{z^2} \int_0^\beta \frac{\beta^3 d\beta}{\ln \left[\frac{\beta^2}{(1-\beta^2)} \right]}.$$

For $\beta < 0.7$ the logarithmic term is slowly varying and we approximate the integral by a power law,

$$R = \frac{m}{cz^2} \left(\frac{E}{m} \right)^{1/d}.$$

In this equation the constants c and d are derived from a and b and thus depend only on the properties of the matter through which the particle passes. Solving for E in terms of R ,

$$E = m \left(\frac{cRz^2}{m} \right)^d.$$

When a particle traverses a detector of thickness t , its range is shortened by an amount t . The pulse that is produced is therefore

$$P = c^d z^{2d} m^{1-d} [R^d - (R-t)^d].$$

Evaluating the constants for silicon and expressing P in MeV, we have

$$P = 0.244 z^{1.12} m^{0.44} [R^{0.56} - (R-t)^{0.56}], \quad (2)$$

where R and t are expressed in μm , z in units of electron charge, and m in units of proton mass.

Now we consider particles penetrating or coming to rest in a particle identifier. For convenience, let us assume all detectors are of equal thickness and that there is no inactive material between the energy-sensitive regions. (In practice it is possible to reduce this inactive material to less than 1% of the thickness of the active region for 3 to 5 mm lithium drifted silicon detectors by lapping the lithium dead layer.) For the moment let us also assume the particles enter the identifier perpendicular to the surface of the detectors. Definitions of the variables which will be used follow:

- n = number of detectors in the identifier,
- t = detector thickness,
- x = range of the incident particle in units of t ,
- P_i = pulse produced in the i th detector,
- k = number of the last detector with non-zero pulse for stopping particle events.

Shown at the top of fig. 1 is a five-detector identifier with lines drawn to represent three event configurations: ^4He , which penetrates all five detectors; ^3He , which stops in detector 5; and a particle of range $x = 2.3$.

Using eq. (2), the pulse in the i th detector is:

$$P_i = \begin{cases} 0.244 z^{1.12} m^{0.44} t^{0.56} \times \\ \quad \times [(x-i+1)^{0.56} - (x-i)^{0.56}], & i < x, \\ 0.244 z^{1.12} m^{0.44} t^{0.56} (x-i+1)^{0.56}, & i-1 \leq x \leq i, \\ 0, & x < i-1. \end{cases}$$

The range dependence can be seen more clearly if we define

$$C(m, z, t) = 0.244 z^{1.12} m^{0.44} t^{0.56},$$

$$f_1(x) = \begin{cases} 0, & x < 0, \\ x^{0.56}, & 0 \leq x \leq 1, \\ x^{0.56} - (x-1)^{0.56}, & x > 1, \end{cases}$$

$$f_i(x) = f_1(x-i+1).$$

The pulse in the i th detector can now be expressed as

$$P_i = C(m, z, t) f_i(x). \quad (3)$$

In fig. 1 the functions $f_i(x)$ are shown for a five-detector identifier. Each f_i rises from zero when the range is such that the particle enters the i th detector, reaches a maximum when $x = i$, and falls off at longer ranges. The set of pulses produced by a particle, which we shall refer to as the "signature" of the particle, is calculated by multiplying each $f_i(x)$ by the single *range-indepen-*

Fig. 1. Conceptual drawing of a five-detector identifier. Pulse predicting functions f_i are shown for each detector as a function of particle range x in units of detector thickness.

dent constant, $C(m, z, t)$. For example, the signature of the ^3He event pictured in fig. 1 is obtained by multiplying the circled f_i values for the appropriate range by the constant $0.224 (2)^{1.12} (3)^{0.44} t^{0.56}$.

2.2. STOPPING PARTICLES

Let us consider when different particles produce identical signatures. Using eq. (3) this can be expressed as

$$C(m, z, t) f_i(x) = C(m', z', t) f_i(x'),$$

where $i = 1, n$ and the primed variables refer to the second particle.

In the region $1 < x < n-1$, identical signatures can be obtained only if the second particle stops in the same detector since the next detector must have zero pulse. If $P_k \neq 0$, and $P_{k+1} = 0$, then we require $k-1 < x' < k$. In fig. 1, $k = 3$, and the dashed lines show the interval in which x' is confined.

Examination of the variation of f_{1-3} in this region reveals that proper ratios of the pulses can only be obtained by requiring $x' = x$. Thus, to produce identical signatures we require,

$$C(m, z, t) = C(m', z', t), \quad \text{and} \quad x' = x. \quad (4)$$

In other words, only particles with the same range energy relationship can be confused.

The smallest difference in signatures is when $x' = x$, and in this worst case the percentage difference in pulses produced in each detector is

$$PD = \frac{C(m, z, t) - C(m', z', t)}{C(m, z, t)} \times 100.$$

As a specific example, the percentage difference in pulses for ^3He and ^4He of the same range is

$$PD = \frac{4^{0.44} - 3^{0.44}}{4^{0.44}} \times 100 = 12\%.$$

Fig. 2 shows the dE/dx as a function of range for these isotopes and also the pulses in each detector, assuming 3000 μm detectors and 11 000 μm range. The differences between $P_i(^4\text{He}) - P_i(^3\text{He})$ are from 3.5 MeV for $i = 1$ to 8.3 MeV for $i = 4$, giving a total of 21.8 MeV separation in the four pulses combined. Because the energy resolution of a solid state detector is generally better than 0.1 MeV, the difference between the signatures of ^3He and ^4He is unambiguous. In section 6 we shall evaluate the separation efficiency in detail.

Particle identification using several *thick* detectors is an alternative to the $\Delta E-E$ method^{1,2} where the dE/dx of a particle is measured by a thin detector followed by a single thick detector that measures the

Fig. 2. dE/dx vs range (μm) for ^3He and ^4He of equal range. The dotted lines indicate detector boundaries. The accompanying table lists the pulses produced in each detector.

Fig. 3. dE/dx vs range (μm) for ^3He and ^4He of equal energy (160 MeV). The dotted lines indicate detector boundaries. The accompanying table lists the pulses produced in each detector.

energy E . At the energies considered here, the isotopic resolution of the ΔE - E method is limited because the straggle in the energy deposited in the thin dE/dx detector allows different isotopes of the same energy to have the same signature. The identification information obtained by observing dE/dx along the entire range can be clearly seen by comparing the two methods in this limiting case. Fig. 3 shows the dE/dx versus range and lists the pulses produced in each 3000 μm detector when ^3He and ^4He enter an identifier at an energy of 160 MeV. The difference in the signatures is drastic; in fact, the ^3He deposits 49.2 MeV in detector 4, while the ^4He does not reach this detector. If the ΔE - E method were used in this case, the identification would depend solely on the difference in dE/dx measured by a thin detector which precedes a thick total energy detector. At this energy the straggle in the energy deposited in a thin dE/dx detector can cause a 10% uncertainty in the separation of ^3He and ^4He . Utilizing two ΔE detectors, Goulding et al.⁴⁾ have achieved an uncertainty of 0.06% separating these isotopes at slightly lower energies.

The separation achieved when *thick* detectors are used to observe dE/dx over a larger portion of the range can be estimated by calculating what percentage of the ^4He particles reach detector 4 due to straggle in range. In this case, the range straggle distribution has a standard deviation of 1% of the range. The range of the 160 MeV ^3He is $8300 \pm 83 \mu\text{m}$; therefore, a range straggle of eight standard deviations is required for the ^4He to reach a range of 9000 μm and enter detector 4. Thus, the number of ^4He reaching detector 4 is less than 10^{-10} %. In this case we conclude that there is an increase of at least a factor of 10^6 in resolution when several *thick* detectors are used to more fully utilize the information contained in the change of dE/dx with range. However, while the ΔE - E method can be applied electronically²⁾, this thick detector technique requires computer analysis to extract the identification information.

2.3. NON-STOPPING EVENTS

When particle ranges reach the second half of the last detector, the mass resolution decreases quickly. The reason for the drop is the absence of an f_{n+1} curve, which makes it possible to achieve similar pulse amplitudes and ratios by proper choice of x' . The events labeled ^3He and ^4He in fig. 1 demonstrate how the range of the ^4He can be chosen so that it simulates the ^3He signature. If there were a sixth detector, the ^4He would be forced to have a range less than 5.0 and could not be confused with the ^3He .

The charge resolution likewise decreases for events which traverse the identifier. However, useable charge resolution can be achieved at relativistic velocities.

2.4. IDENTIFICATION LIMITATIONS

For stopping particles to have the same signature, eq. (4) requires

$$C(m', z', t) = C(m, z, t),$$

or

$$z'^2 m'^{0.79} = z^2 m^{0.79}. \quad (5)$$

This relationship is useful to examine when neutron-rich or neutron-deficient isotopes can be simulated by the more abundant isotopes of an element of higher or lower charge. If the signature of a neutron-rich isotope with mass m' , charge z' is to be simulated by a particle of mass m , charge $z = z' + 1$, eq. (5) requires

$$m' = m \left(\frac{z' + 1}{z'} \right)^{2.54}.$$

As an example, if $z' = 7$, $m = 16$, then $m' = 22.3$, which implies that ^{16}O gives the same signature as a hypothetical^{22,3}N.

The mass of a neutron-deficient isotope with charge z' , which can be confused with a particle of mass m , charge $z = z' - 1$, is

$$m' = m \left(\frac{z' - 1}{z'} \right)^{2.54}.$$

Letting the simulating particle be ^{14}N , we see that $m' = 14(7/8)^{2.54} = 10$ or ^{10}O is the neutron-deficient isotope mimicked by ^{14}N .

Limitations on the acceptance solid angle can also be investigated using eq. (5). A particle which enters an identifier at an angle θ relative to normal incidence can be described by replacing t with $t \sec \theta$. The angle at which a particle of mass m' , charge z' produces the same signature as that produced by a particle of mass m , charge z is

$$\theta = \cos^{-1} \left(\frac{z'}{z} \right)^2 \left(\frac{m'}{m} \right)^{0.79}. \quad (6)$$

From this relation we see that higher mass isotopes entering at an angle cannot be confused with isotopes of lower mass. However, ^{14}N produces the same signature as ^{15}N if the ^{14}N enters the identifier at an angle

$$\theta = \cos^{-1} \left(\frac{14}{15} \right)^{0.79} = 18.7^\circ.$$

Eq. (5) also shows that higher charge isobars may be simulated by those of lower charge entering at an angle; for instance, the angle at which ^{32}S must enter in order to simulate the isobar ^{32}Cl is

$$\theta = \cos^{-1} \left(\frac{16}{17} \right)^2 = 28.0^\circ.$$

The conclusions reached in this section are qualitative and intended to provide insight into the identification method and possible ambiguities. To determine actual identification efficiencies, we develop in the following sections a chi-square identification scheme that incorporates a realistic range-energy relation and detailed error analysis.

3. Identification technique

The method of identifying particles from their signatures presented here assumes the pulses $P_{i=1,n}$ are available for off-line analysis by a large computer. A small computer used on-line can give approximate identification while recording the signatures for later detailed analysis. A system of this type consisting of a nine-element particle identifier interfaced to a PDP-8 computer (described in the following experimental paper) was programmed to provide charge spectra on-line during a heavy-ion fragmentation experiment. We shall not discuss the on-line analysis methods further, as they depend strongly upon the particular experiment and are all simplified versions of the detailed analysis developed here.

The basic identification problem is to determine which particle has the highest probability of producing a given pulse signature. This is quite similar to the analysis of interaction vertices in a bubble chamber⁵ and we shall use essentially the same method of chi-square fitting. This approach consists of calculating a chi-square fit to the observed signature for each hypothetical particle choice. The particle with the lowest chi-square then has the highest probability of having produced the observed signature.

Let $P_{i=1,n}$ be the set of observed pulses and $P'_{i=1,n}$ be the signature of the hypothesized particle of energy E . The chi-square function is defined by the expression

$$\chi^2 = \sum_{i=1}^n \frac{(P_i - P'_i)^2}{\sigma_i^2},$$

where σ_i is the normal error in $P_i - P'_i$ (ref. 6). The fit to the observed signature is obtained by minimizing χ^2 values with respect to the energy E .

Use of the χ^2 formulation allows straightforward evaluation of mass resolution, charge resolution, and

energy resolution. However, we must first develop a method of calculating the predicted signatures and the errors, σ_i . This is done in the next two sections.

4. Pulse calculation

The power law range-energy relationship used earlier to qualitatively predict the pulse signatures of various particles is not suitable for practical use due to several drawbacks. The most serious of these is that its accuracy is limited to a small energy range. Particles which stop in an identifier pass through detectors over a wide range of energies. The pulse, predicted for a particle which leaves a detector with a low velocity, is quite dependent upon the range-energy relation at low energies because of the large rate of energy loss in this region. Thus, to accurately predict the signature of a stopping event, a range energy relation must be accurate from very low energies up to the energy at which particles enter the first detector - which may be several hundred MeV/nucleon for identifiers with total silicon thicknesses the order of 10 g/cm². Another difficulty is that the power law range-energy function fails to reflect the fact that the rate of energy loss decreases and passes through a minimum near 2 GeV/nucleon before rising at higher relativistic energies. This high-energy behavior is important in relation to the proper charge identification of particles which pass through an identifier.

The pulse predictions should be at least as accurate as the pulse measurements in order to efficiently use the information provided. Possible pulses range from almost zero energy deposited, for a particle which enters a detector at low energy, to hundreds of MeV when a multiple charge particle stops in the rear of a detector. Under these conditions, the resolution over most of the energy range is limited primarily by the calibration and digitization accuracy.

A reasonable assumption is that resolution the order of 0.1-1% of full scale can be achieved over the entire observed energy range. Thus, the range energy relation for a given particle should provide pulses accurate to 0.1-1% of the maximum pulse observed (which is produced when the particle stops at the rear of the detector).

4.1. RANGE-ENERGY RELATIONS

Because there are no empirical range-energy data available over wide energy ranges for many of the particles of interest, we have generated range-energy functions for the heavier ions using the semi-empirical proton energy loss rate in silicon as a starting point.

The purpose of this calculation is twofold: one, to provide a set of range energy tables of reasonable accuracy for use in the identification efficiency calculations; two, to achieve an understanding of how the various refinements to the range energy relation affect the predicted pulse.

The simple scaling of dE/dx by z^2 does not apply at velocities low enough that the particle can capture electrons. The electron capture lowers the energy loss rate and this causes the range to increase⁷). The semi-empirical relationship presented by Pierce and Blann⁸) for the effective charge, z^* , of a particle of velocity v and charge z is:

$$z^* = z(1 - e^{-0.95 v_r}),$$

where $v_r = v/z^{2.3} v_0$ and v_0 is the Thomas-Fermi electron velocity. The electronic stopping power for a particle of charge z and velocity v becomes

$$\left. \frac{dE}{dx} \right|_z = \left. \frac{dE}{dx} \right|_p \frac{z^{*2}}{\gamma_p^2},$$

where dE^e/dx_p is the empirical proton energy loss rate at the same velocity, and γ_p^2 is the effective charge of a proton of velocity v . The proton effective charge measurements of Hall⁹) as parameterized by Booth and Grant¹⁰) were used for γ_p .

The energy lost to the nuclei of the absorbing medium must be determined for two reasons: first, because such energy loss decreases the range of a particle; second, this nuclear stopping energy loss does not appear in the pulse produced by the detectors. The energy loss due to nuclear stopping can be determined using an equation developed by Haines and Whitehead¹¹) from LSS stopping theory¹²) and empirically adjusted for silicon absorbers by Wilkens et al.¹³). The equation for the energy lost in nuclear stopping, E_n , when a particle of energy E (MeV) is brought to rest is

$$E_n = 4.464 E / (6.20 + 0.72 c E).$$

Here the constant c converts to LSS units and has the value for silicon,

$$c = \frac{3.252 \times 10^4}{14z} (z^{\frac{1}{2}} + 5.809)^{-\frac{1}{2}} \frac{28.09}{M + 28.09},$$

where M is the mass of the particle in amu and z the charge. Like charge neutralization, nuclear stopping is a low velocity effect, which reaches an asymptotic value of $6.2/c$ at an energy of $100/c$. Expressing the total energy loss rate as a sum of nuclear and electronic

terms,

$$\frac{dE}{dx} = \left. \frac{dE}{dx} \right|_z^e + \left. \frac{dE}{dx} \right|_z^n = \left. \frac{dE}{dx} \right|_z^e + \frac{dE_n}{dE} \frac{dE}{dx},$$

we can solve for the total stopping power,

$$\frac{dE}{dx} = \frac{\left. \frac{dE}{dx} \right|_z^e}{1 - dE_n/dE} = \frac{(z^{*2}/\gamma_p^2) \left. \frac{dE}{dx} \right|_p^e}{1 - dE_n/dE}.$$

This expression must then be integrated to obtain the range, or the actual path length, as a function of energy. This path length is always greater than the projected distance traversed due to multiple scattering. For a given angle of entry, a solid state detector measures the energy deposited along a path which is defined by its projection on the original direction. Thus, what is required for our purposes is the projected range as a function of energy. If ϕ is the angle of projection, then

$$R_p(E) = R_p(E_0) + \int_{E_0}^E \frac{\langle \cos \phi \rangle}{dE/dx} dE.$$

Here E_0 is some small starting energy chosen where empirical range-energy data¹⁴) are available. The

Fig. 4. Pulses in MeV produced in a 3000 μm Si(Li) detector by particles ^1H , ^4He , ^{14}N , and ^{32}S as a function of particle energy in MeV/nucleon.

average of the angle of projection $\langle \cos \phi \rangle$ was calculated using the screened Rutherford cross section of Nigam et al.¹⁵⁾ in the general format of Scott¹⁶⁾. The proton dE/dx function was taken from the compilation of Janni¹⁷⁾ for energies below 1 GeV; above 1 GeV, from the calculations of Barkas and Berger¹⁸⁾.

4.2. PULSE AND EFFECT OF VARIOUS CORRECTIONS

Given a projected range-energy relation as described above, the energy E_t , which a particle has at exit from a detector of thickness t , is related to the energy at entrance E_e by the equation

$$E_t = R_p^{-1}[R_p(E_e) - t],$$

where R_p^{-1} is the inverse of R_p .

The energy recorded by the detector, however, is not $E_e - E_t$ because some energy is lost due to nuclear stopping. This energy is

$$E_n(E_e) - E_n(E_t) = \Delta E_n.$$

The pulse seen is then

$$P = E_e - R_p^{-1}[R_p(E_e) - t] - \Delta E_n. \quad (7)$$

In fig. 4, pulses calculated in this fashion for ^1H , ^4He , ^{14}N , and ^{32}S are shown as a function of energy per nucleon for a 3 mm detector. In general, the pulse rises in proportion to the total energy until exiting the detector. Then there is an abrupt slope change, and the pulse decreases with increasing energy. The energy at

Fig. 5. Error in calculated pulse expressed as a percentage of the maximum pulse expected for a particle if the charge neutralization effect is neglected. The energy scale is $E - E_t$ in MeV/nucleon, where E is the energy of the particle and E_t is the energy at which the particle just penetrates the 3000 μm detector.

which the hypothesized particle enters the i th detector is determined by subtracting the measured value of the energy lost in the preceeding detectors ($\sum_{j=1}^{i-1} P_j$) from the energy E assumed at entrance to detector 1. The predicted pulse for the i th detector is then calculated using eq. (7).

The initial energy, E_i , such that a particle just traverses a detector is affected by the refinements to the range-energy relation made above. From fig. 4 we see that the pulse is rapidly changing at this point, due to the large ionization rate at low velocities. This fact indicates that a small change in range will produce a relatively large change in the predicted pulse. In fig. 5 we have plotted the difference between pulses predicted for a 3000 μm detector, with and without the effects of charge neutralization. Because the charge neutralization and multiple scattering effects are largest in the energy region when the particles are leaving the back of the detector, the energy variable is $E - E_t$ in units of MeV/nucleon. Also in keeping with our viewpoint that the resolution is a fixed percentage of full scale, we have plotted the pulse differences for the various particles as a percentage of the maximum pulse observed for each particle. Charge neutralization is a dominant effect for $z > 1$, ranging from a minimum of 1% for $z = 2$ to 30% for $z = 92$.

If the multiple scattering effect is neglected, predicted pulses are too low, as shown in fig. 6. As expected, this effect again reaches a maximum near E_t , then decreases. Above mass 4, the effect of multiple scattering is negligible.

Fig. 6. Error in calculated pulse expressed as a percentage of the maximum pulse expected for a particle if the multiple scattering effect is neglected. The energy scale is $E - E_t$ in MeV/nucleon, where E is the energy of the particle and E_t is the energy at which the particle just penetrates the 3000 μm detector.

The nuclear stopping effect can be estimated from the asymptotic value of $6.2/c$ at energy $100/c$. For instance, the maximum energy a U ion can lose in nuclear stopping is 12 MeV, when it is brought to rest from an energy greater than 192 MeV. If the detector thickness is $3000 \mu\text{m}$ the maximum pulse is 3.64×10^4 MeV and the effect of neglecting nuclear stopping is less than 0.04%. The nuclear stopping effect must be included when detector thickness becomes small; for instance, for ^{235}U the nuclear stopping is 0.1% of the maximum pulse at a detector thickness of $600 \mu\text{m}$.

4.3. RESIDUAL PULSE HEIGHT DEFECT

At high-energy loss rates in silicon-surface barrier detectors, in addition to the nuclear stopping defect, a residual pulse defect has been observed^{13,19}. The residual defect has been theoretically attributed by Hansen²⁰ to reduced charge collection efficiency due to the lowered mobility in a thin p-type inversion layer at the surface. This qualitative explanation described the defect as a loss of signal from a region at the barrier of thickness

$$T = 0.081 \left(\frac{dE}{dx} \right)^{\frac{1}{2}},$$

where T is in μm and dE/dx expressed in $\text{MeV}/\mu\text{m}$. The maximum dE/dx for U is $27.2 \text{ MeV}/\mu\text{m}$, for which $T = 1.73 \mu\text{m}$; all lower charged particles have a smaller value of T . This relation for T agrees with the revised measurements of Wilkins et al.¹³, which were reported by Hansen, to within a factor of 2. The disagreement implies that the identity of the ion is an additional variable.

It is clearly premature to attempt a quantitative description of the corresponding phenomena in lithium drift silicon detectors at this time. However, an order of magnitude estimate can be made if one projects the empirical data to higher energy loss rates. If this is done for U, a maximum defect of 33 MeV is predicted. The maximum pulse for a U ion in a $3000 \mu\text{m}$ detector is 3.64×10^4 MeV; therefore, the defect is the order of 0.1% of the maximum pulse.

We conclude that the range-energy relation used should include connections for charge neutralization, multiple scattering for $z = 1, 2$ particles and nuclear stopping for heavy particles in detectors of thickness such that the maximum pulse observed is less than $6200/c$. The residual pulse height defect requires further experimental study; however, present knowledge indicates this defect will cause no significant errors if detectors $\geq 3000 \mu\text{m}$ are used. We have seen in section 2 that this method of identification is based

on the relative difference in energy loss for different particles. Efficiency calculations, therefore, do not depend on the absolute values of the individual ranges. Ideally, empirical range energy relations should be used for data analysis. At present, these data are available only at low energies for the higher z particles²¹.

5. Error calculation

The χ^2 method of identification achieves its mathematical simplicity and computational utility by assuming the errors involved are distributed in a gaussian fashion. Making this assumption, the variance, σ_i^2 , of the difference between predicted and observed pulses $P'_i - P_i$ can be expressed as

$$\sigma_i^2 = \sigma^2(P_i) + \sigma^2(P'_i),$$

where $\sigma(P_i)$ and $\sigma(P'_i)$ represent the standard deviations of the observed and predicted pulse spectra.

5.1. ERROR IN THE OBSERVED PULSE

There are three contributions to the error in the observed pulse. The first of these is the energy resolution of the system, σ_r , which we have estimated to range from 1.0% to 0.1% of full scale. The second contribution is the straggle in the actual energy deposited when a particle passes through a detector. The variations in detector thickness produce the third contribution.

The straggle in the energy deposited is caused by the statistical nature of the energy loss process. This straggling process has been investigated by many authors. In terms of increasing energy loss relative to the initial energy they are: Landau²² in the region of very small relative energy loss, Vavilov²³ and Symon²⁴ in the region up to 10% energy loss, Tschalär^{25,26} and Payne²⁷ in the region of high percentage energy loss. The straggling distribution is characterized by a wide region of relative energy loss for which the shape is gaussian. There is increasing asymmetry for very low and very high relative energy losses. The region of gaussian behavior increases with the mass of the particle; for protons, the distribution is approximately gaussian for relative energy losses ranging from 5 to 75%²⁵. An equation governing the moments of the energy loss distribution was derived by Symon. Using Payne's formulation for the second moment, we can express the standard deviation as

$$\sigma_{sg} = \left[2 \left(\frac{dE}{dx} \right)_{E_f}^2 \int_{E_f}^{E_c} \frac{M_2(E)}{(dE/dx)^3} dE \right]^{\frac{1}{2}} \quad (8)$$

In this expression, E_c is the initial particle energy, E_f is

the energy at exit from the detector, and $M_2(E)$ is the second moment of the collision spectrum. $M_2(E)$ is calculated using the Born approximation by Livingston and Bethe²⁸.

The contribution from the thickness variations depends on the energy loss rate when the particle leaves the detector. If we let E_t^i be the energy of a particle leaving detector i , the error in the observed pulse is

$$\sigma_{th} = \left. \frac{dE}{dx} \right|_{E_t^i} \sigma_t,$$

where σ_t is the variation of the active region thickness.

Combining the errors due to instrumental resolution (σ_r), straggle, and thickness variations, the error in the observed pulse is

$$\sigma(P_i) = (\sigma_r^2 + \sigma_{sg}^2 + \sigma_{th}^2)^{1/2}.$$

Because σ_{th} and σ_{sg} are zero for stopping particles, there is a discontinuity in $\sigma(P_i)$ when a particle has energy E_t , sufficient to just penetrate the detector. This nonphysical discontinuity is caused by the assumption that the particle range is uniquely defined by the initial energy. For a given energy there is a distribution of possible ranges. This range straggle distribution is generally gaussian with a width the order of $(m)^{-1/2}$

of the total range²⁹. The discontinuity in $\sigma(P_i)$ is removed by averaging over the range straggle distribution. The resultant error in the observed pulse, $\sigma(P_i)$, is shown in fig. 7 for ^1H , ^4He , and ^{14}N as a function of $E - E_t$ in a 3000 μm detector. Again, we have plotted the error as a percentage of the maximum pulse for each particle. The value chosen for the resolution was 1.0% of the maximum pulse produced by each particle. The error is constant at the value σ_r until the range is close to a detector thickness. The maximum at ranges close to t is due to the large values of dE/dx_{E_t} in this region. The error $\sigma(P_i)$ has the same form for all the detectors in an identifier. The contribution due to straggling σ_{sg} cannot be reduced by increased instrument resolution and defines the minimum error achievable.

5.2. ERROR IN THE PREDICTED PULSE

Pulse prediction accuracy is limited by the knowledge of the energy E_e^i with which the particle enters the i th detector. The energy at entrance to the i th detector was expressed in section 4.2 as

$$E_e^i = E - \sum_{j=1}^{i-1} P_j,$$

where E is the energy of the *hypothesized* particle when it enters the identifier, and P_j is the *observed* pulse in

Fig. 7. Error in observed pulse, $\sigma(P_i)$, for any 3000 μm detector. $\sigma(P_i)$ is expressed as a percentage of the maximum pulse expected for the particle. Resolution (σ_r) is assumed to be 1.0% of the maximum pulse. The energy scale is $E - E_t$ in MeV/nucleon, where E is the energy at which the particle just exits the detector.

detector j . Again, assuming the errors are gaussian, the uncertainty in E_c^i is $(i-1)^{\frac{1}{2}} \sigma_r$. The contribution to the error in the predicted pulse is then

$$\sigma(P'_i) = \left. \frac{\partial P'_i}{\partial E} \right|_{E_c^i} (i-1)^{\frac{1}{2}} \sigma_r. \quad (9)$$

Like the error $\sigma(P_i)$, this expression must also be integrated over the range straggle distribution in order to give physical results.

Fig. 8. Error in the predicted pulse $\sigma(P'_i)$ of ^{14}N for 3000 μm detectors in positions $i=2, 4$ and 6 of an identifier. $\sigma(P'_i)$ is expressed as a percentage of the maximum pulse expected for ^{14}N . Resolution (σ_r) assumed was 1.0% of the maximum pulse. The energy scale is $E-E_t$ in MeV/nucleon where E is the energy of the particle and E_t is the energy at which the particle just penetrates the detector.

Fig. 9. Peak values of total error σ , and the energy loss straggle contribution σ_{sg} for ^{14}N in detectors of 3000 μm thickness. σ_i and σ_{sg} are expressed as a percentage of the maximum pulse expected for ^{14}N . Curves are plotted for resolution values $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

Because $\partial P_i / \partial E \approx 1$ when $E < E_t$, the error in the predicted pulse is constant at a value of $(i-1)^{\frac{1}{2}} \sigma_r$ in this region. There is a maximum in the region of $E \approx E_t$ caused by the rapidly changing $\partial P'_i / \partial E$. The error then decreases to zero at minimum ionization energy as P_i becomes a weak function of E . The form of $\sigma(P'_i)$ for ^{14}N in a 3000 μm detector can be seen in fig. 8. The value used for σ_r is 1% of the maximum pulse. Curves are shown for detectors 2, 4, and 6. A comparison with $\sigma(P_i)$ from fig. 7 indicates that for $\sigma_r = 1\%$, the increase in $\sigma(P'_i)$ with i causes $\sigma(P'_i)$ to be the dominant error for $i \geq 2$.

Fig. 10. Peak values of total error in detector 1 (σ_1) and energy loss straggle contribution (σ_{sg}) for particles ^1H , ^{14}N , and ^{235}U expressed as a percentage of the maximum pulse expected for each particle vs detector thickness t in microns. Resolution is 0.1% except for ^1H where a value of 100 keV was used.

Fig. 11. Peak value of total error in detector 1 (σ_1) expressed as a percentage of the maximum expected pulse versus the mass of the particle in amu. Curves are shown for resolution (σ_r) values of 1.0 and 0.1%.

5.3. TOTAL ERROR, σ_i

Reduction of the total error is limited by the straggle contribution σ_{sg} mentioned above. The increase with i can be reduced to the point where σ_{sg} is the dominant contribution by reducing σ_r to values the order of 0.1%. This is shown in fig. 9 for ^{14}N in a ten-element identifier. In fig. 9 the maximums of σ_i and σ_{sg} are plotted for several choices of σ_r . Thus further reduction of σ_r is not necessary.

The advantage of using thick detectors can be seen by consulting fig. 10, which plots the maximum values of σ_i and σ_{sg} as a function of detector thickness. Resolution used was 0.1% except where this resulted in $\sigma_r < 100$ keV. In such cases, a value of 100 keV was used. Detectors become more useful as the thickness increases because of the decrease in σ_{sg} with t .

The mass dependence of σ_i is shown in fig. 11. The decrease with increasing mass is due to σ_{sg} which decreases at high values of dE/dx .

In general, σ_i can be reduced to a point which is determined by σ_{sg} . We have seen that detector thickness, $t \geq 3000 \mu\text{m}$, and instrumental resolution $\sigma_r \approx 0.1\%$ of maximum pulse achieve a situation where σ_i is dominated by σ_{sg} . The resolution of such an identifier would be close to the theoretical maximum assuming no errors present.

6. Identifier resolution

In this section the methods of error and pulse calculation developed in the preceding sections are used to examine the identification capabilities of an identifier. In section 5 we saw that an identifier composed of $3000 \mu\text{m}$ detectors and achieving an instrumental resolution of 0.1% of full scale would provide errors dominated only by the effect of straggle in the energy deposited in the detectors. This configuration will be used in the following calculations because it reflects, generally, the capabilities of identifiers that are operating the region where errors are due to physical and not instrumental limitations. Unless specifically noted, the identifier considered below consists of five lithium drifted silicon detectors $3000 \mu\text{m}$ thick with a $12 \mu\text{m}$ variation (σ_t). The resolution of the system is assumed to be 0.1% of the maximum pulse for the particle under consideration. When this results in a resolution less than 100 keV, a value of 100 keV is used.

6.1. ENERGY DETERMINATION AND RESOLUTION

Whenever a set of observed pulses is compared to the predicted signature of a hypothesized particle, the energy assumed must be varied in order to minimize χ^2 . For a given event, the accuracy of the energy determined in this fashion is related to the curvature of the

Fig. 12. χ^2+1 is plotted vs range and energy for a ^{14}N signature of 5700 range. Curves for dead layer thickness of 0.0 and $25 \mu\text{m}$ are shown. Resolution (σ_r) assumed was 0.1%.

χ^2 curve as follows:

$$\sigma_E = \left(\frac{1}{2} \frac{\partial^2 \chi^2}{\partial E^2} \right)^{-1/2} \quad (10)$$

Eq. (10) can be used to determine the energy resolution of the identifier if the observed pulses used in the χ^2 calculation are set equal to those of an "average" event; i.e., an event where the pulse in each detector assumes its average value for the given particle and energy.

Fig. 12 shows χ^2 as a function of assumed energy where the observed pulses are those of an average ^{14}N event with a range of 5700 μm . Dashed and solid curves refer to detectors with zero and 25 μm dead layers, respectively. Eq. (10), applied at the minimum point, E' , gives a value of $\sigma_E = 0.065$ MeV/nucleon for σ_E . Consulting fig. 12, we see that the χ^2 function has increased to a value of approximately 1.0, when $E = E' \pm \sigma_E$. This illustrates the fact that the 68.3% confidence level contour of χ^2 encloses 2σ in the gaussian case.

The existence of other minima in χ^2 as a function of energy is also shown in fig. 12. The minimum at $x = 2.0$ can be understood by observing from fig. 1 that the pulse required in detector 2 can be obtained by hypothesizing an event stopping in detector 2 or in detector 3. Because the pulse decreases so rapidly when a particle leaves a detector, the existence of a dead layer can greatly accentuate these secondary minima. This is achieved because the particle loses energy in the dead layer instead of in the succeeding detector. The second curve in fig. 12 illustrates this fact for 25 μm dead layers. It is clear that care must be taken in locating the best fit energies because of these secondary minima. This is especially true for $z = 1$, where it is good practice to restrict the fitted spectrum to ranges that stop in the active regions.

The dead layers affect the energy resolution in a rather complex fashion. If the energy of the ^{14}N event is increased in fig. 12, the secondary minimum becomes more pronounced. When it becomes possible to produce the same pulse in detector 2 with an event which enters the dead layer of detector 2 but does not penetrate detector 3, the signatures become almost identical. This can be seen by displacing the f_3 curve in fig. 1 by the thickness of the dead layer. The effect on the energy resolution is shown in fig. 13 where the 2σ contour about the energy is plotted versus range. The ordinate scale is $E - E_\sigma$ in MeV/nucleon where E_σ is the energy at which the χ^2 function reaches the 68.2% confidence level. The shape shown there is produced when the

secondary minimum sweeps through the region of the primary minimum. The case of no dead layer is also plotted.

If such a degradation of resolution cannot be tolerated, the region (corresponding in this case to $<3\%$ of the detector thickness) can be removed from the spectrum of analyzed events.

Energy resolution decreases when a particle has energy sufficient to pass entirely through an identifier. To illustrate this, we plot in fig. 14 the energy resolution for ^{14}N throughout the identifier as well as at higher energies. The same general behavior is found for other particles. The mass dependence can be seen in fig. 15, where the range is fixed at $x = 3.5$ and σ_E is plotted as a function of mass.

6.2. MASS AND CHARGE RESOLUTION

In section 2 it was pointed out that particles with the same energy loss function could produce identical signatures. In fact there are an infinite number of (m, z) pairs that give the same signature. Fortunately, most of these pairs are non-physical or are far from stability.

The identification capability as a function of m and z is demonstrated by considering a single isotope in detail. The particle we consider is ^{56}Fe with a range such that it stops in detector 4. This defines the average signature for ^{56}Fe , which is taken as the *observed* pulse. The mass and charge of the *hypothesized* particle are then allowed to vary. The minimum χ^2 for each mass and charge choice is determined, and a contour corresponding to 2σ confidence level is drawn in the m, z plane. These contours are shown for three resolution values in fig. 16.

Any particle whose mass and charge lie on a 2σ contour produces an average signature separated from that of ^{56}Fe by one standard deviation. The efficiency of particle separation can be determined by the use of σ_m and σ_z shown in fig. 16A. The percentage of ^{55}Fe and ^{57}Fe events that would be mis-identified as ^{56}Fe events is

$$200 \int_{0.5/\sigma_m}^{\infty} \frac{e^{-\frac{1}{2}x^2}}{(2\pi)^{1/2}} dx.$$

For the case of 0.1% resolution, $\sigma_m = 0.25$, the percentage of incorrectly identified ^{56}Fe is 4.5%. The value of σ_m in the limit of $\sigma_r = 0$ and no thickness uncertainty is $\sigma_m = 0.117$, corresponding to about 0.01% mis-identified events. This is the limit set by the energy straggle, σ_{str} .

The variation of σ_z is shown in fig. 17 as a function

Fig. 13. Energy difference $E - E_\sigma$ in MeV/nucleon vs range x for a ^{14}N event near the interface between detectors 2 and 3. E is the energy at which the χ^2 fit reaches the 1- σ confidence level. Detector thicknesses are $3000\ \mu\text{m}$, assumed resolution is 0.1%. The zero dead layer curve is drawn assuming the labeled dead layer of detector 2 was active.

Fig. 14. Energy resolution σ_E in MeV/nucleon for ^{14}N is a five-element identifier. Detector thicknesses are $3000\ \mu\text{m}$, curves are drawn for assumed resolutions $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

Fig. 15. Energy resolutions σ_E in MeV/nucleon vs mass in amu for particles of range $x = 3.5$ in an identifier consisting of five $3000\ \mu\text{m}$ detectors. Resolutions assumed are $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

Fig. 16. One standard deviation confidence level contours in z (units of proton charge) and in amu. Curves enclose mass and charge values of particles whose average signature is close to that of ^{56}Fe . Known isotopes of Mn, Fe, and Co are indicated by solid circles. Insert 16A shows detail near the ^{56}Fe region and defines charge resolution σ_z and mass resolution σ_m . Range was fixed at $x = 3.5$ and resolutions plotted are $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

Fig. 17. Charge z at which the χ^2 function reaches the 68.2% confidence level for ^{56}Fe of range x . A five-element identifier is used with assumed resolutions $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

Fig. 18. Mass resolution σ_m (amu) for $m = 56$ amu vs range x throughout a five-element identifier. Resolution values used are $\sigma_r = 1.0\%$, 0.5% , and 0.1% .

of range, x . The 68% confidence level contours are used in this case to demonstrate the asymmetry in resolution near the exit range. Curves are shown for three values of σ_r . The magnitude of the mass resolution, σ_m , plotted as a function of range is shown in fig. 18. The loss of resolution when particles exit the last detector is readily apparent. The qualitative arguments in section 2 indicated that the resolution should increase with the penetration of more detectors. This increase is evident in figs. 17 and 18 when the resolution approaches 0.1% and is dominated by σ_{sg} . Mass resolution decreases as expected when particles leave the telescope. Charge resolution decreases but not as

Fig. 19. Charge z at which the χ^2 function reaches the 68.2% confidence level versus energy E in MeV/nucleon for a ^{56}Fe particle which completely traverses a five-element identifier. Detector thicknesses are $3000\text{ }\mu\text{m}$; assumed resolution $\sigma_r = 0.1\%$.

drastically, because there is still the z^2 dependence in the dE/dx function.

At high energies where particles penetrate the entire identifier, the z resolution becomes asymmetric. This is caused by the fact that near minimum ionization energies a particle of higher z cannot produce lower signals, while a particle of lower z can produce a higher signal if it has a lower energy. The $2\sigma_z$ contour for high energy Fe is plotted in fig. 19, this behavior is characteristic for other particles. If the energy of the particles is known, the charge resolution is considerably higher. This fact is of importance in accelerator work.

The isotopic resolution (σ_m) decreases with increasing mass. The mass dependence for several choices of σ_r for particles which have range $x = 3.5$ is shown in fig. 20. The limit imposed by σ_{sg} is also shown. The important fact is that this limiting case is quite rapidly approached as σ_r decreases.

The variation of σ_z with mass is shown in fig. 21. The use of figs. 20 and 21 allows one to approximate the contour curves of fig. 16A for arbitrary mass and charge. Thus separation efficiencies for neutron-rich or deficient nuclei can be estimated without detailed calculation.

6.3. SOLID ANGLE LIMITATIONS

The effects of particles entering at other than normal

Fig. 20. Mass resolution σ_m (amu) vs mass m (amu) for particles of range $x = 3.5$. Assumed resolutions are $\sigma_r = 1.0\%$, 0.5% , and 0.1% . Theoretical limit is determined by the straggle in energy loss.

Fig. 21. Charge resolution σ_z (units of proton charge) for particles of range $x = 3.5$. Assumed resolutions are $\sigma_r = 1.0\%$, 0.5% , and 0.1% . Theoretical limit is determined by the straggle in energy loss.

Fig. 22. One standard deviation confidence level contours in z (units of proton charge) and m (amu) of particles entering an identifier at angles 0° , 5° , and 10° fit to a ^{56}Fe hypothesis. Range was fixed at $x = 3.5$ and resolution $\sigma_r = 0.1\%$.

incidence behave qualitatively, as discussed in section 2. One approach to this problem is to consider which particles entering an identifier at an angle can simulate the signal of the particle of interest. Again considering the case of ^{56}Fe , we allow particles of arbitrary m and z to enter the identifier at angles of 5° and 10° , respectively. The signal produced is compared to a ^{56}Fe , 0° incidence hypothesis. The range was fixed at $x = 3.5$ and the resulting 2σ confidence level contours in m and z are shown in fig. 22. The expected result indicating lighter mass particles entering at an angle can produce the signatures of heavier particles at normal incidence is quantitatively verified. It is clear that particles entering at angles greater than 5° greatly reduce the resolution of Fe isotopes. The allowable angle increases at lower masses, and resolution is preserved up to 30° for ^1H , ^2H separation.

An identifier designed to separate heavy ions with a large acceptance cone could incorporate devices such as wire proportional chambers or position sensitive detectors to provide angular information.

6.4. MONTE CARLO CALCULATIONS

Another approach to the calculation of separation efficiencies is to generate events using Monte Carlo techniques and then analyze the resulting pulses. We choose to present the resolutions via the χ^2 approach because the physical understanding is clearer.

The resolutions presented here have been confirmed using Monte Carlo analysis. In particular, the mass resolution for ^{13}N at $\sigma_r = 0.5\%$ predicts nine events per thousand will favor a ^{14}N signature. The analysis of 1000 Monte Carlo generated ^{13}N events yielded ten events that had a lower χ^2 for the ^{14}N hypothesis.

7. Conclusion

We have seen that the change in dE/dx with x is quite sensitive to particle identity. A particle identifier design based on this fact consists of several thick solid state detectors. The χ -square method outlined earlier allows particles to be distinguished using the pulse signatures they produce in the identifier.

Calculations show that the maximum possible resolution is limited by the straggle in the energy deposited in the detectors. This limiting case is approached rapidly as instrumental resolution improves. With presently available electronics and detectors, the predicted mass resolution is $\sigma_m \approx 0.016 + 0.0041 m$ for particles of mass m stopping in the third detector. This gives a 5% resolution between the isotopes of iron;

charge resolution extends throughout the periodic table.

Without the guidance, encouragement, and support of H. H. Heckman, this work would have been impossible. F. Bieser and F. Goulding have been valuable sources of information concerning the capabilities of solid state detector systems. Conversations with P. Lindstrom helped settle points concerning the form of the χ^2 function. This work was supported by NASA Grants NGR 05-003-405 and NGR 05-003-256, and Contract NAS 9-5249.

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