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A Supersymmetric Theory of Flavor and R Parity*

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Abstract

We construct a renormalizable, supersymmetric theory of flavor and R parity based on the discrete flavor group $(S_3)^3$. The model can account for all the masses and mixing angles of the Standard Model, while maintaining sufficient squark degeneracy to circumvent the supersymmetric flavor problem. By starting with a simpler set of flavor symmetry breaking fields than we have suggested previously, we construct an economical Froggatt-Nielsen sector that generates the desired elements of the fermion Yukawa matrices. With the particle content above the flavor scale completely specified, we show that all renormalizable R-parity-violating interactions involving the ordinary matter fields are forbidden by the flavor symmetry. Thus, R parity arises as an accidental symmetry in our model. Planck-suppressed operators that violate R parity, if present, can be rendered harmless by taking the flavor scale to be $\lesssim 8 \times 10^{10}$ GeV.

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1 Introduction

In supersymmetric models of particle physics there are two aspects to the flavor problem. The first is the problem of quark and lepton mass and mixing hierarchies: why are there a set of small dimensionless Yukawa couplings in the theory? The second aspect of the problem is why the superpartner gauge interactions do not violate flavor at too large a rate. This requires that the squark and slepton mass matrices not be arbitrary. Rather, these matrices must also possess a set of small parameters which suppresses flavor-changing effects, even though all the eigenvalues are large. What is the origin of this second set of small dimensionless parameters?

An extremely attractive hypothesis is to assume that the two sets of small parameters, those in the fermion mass matrices and those in the scalar mass matrices, have a common origin: they are the small symmetry breaking parameters of an approximate flavor symmetry group G_f. This provides a link between the fermion mass and flavor-changing problems; both are addressed by the same symmetry. Such an approach was first advocated using a flavor group $U(3)^5$, broken only by the three Yukawa matrices $\lambda_{U,D,E}$ in the up, down and lepton sectors [1]. This not only solved the flavor-changing problem, but suggested a boundary condition on the soft operators which has a more secure theoretical foundation than that of universality. However, this framework did not provide a model for the origin of the Yukawa matrices themselves, and left open the possibility that G_f was more economical than the maximal flavor group allowed by the standard model gauge interactions.

The first explicit models in which spontaneously broken flavor groups were used to constrain both fermion and scalar mass matrices were based on $G_f = SU(2)$ [2] and $G_f = U(1)^3$ [3]. In the first case the approximate degeneracy of scalars of the first two generations was guaranteed by $SU(2)$ in the symmetry limit. In retrospect it seems astonishing that the flavor-changing problem of supersymmetry was not solved by such a flavor group earlier. The well known supersymmetric contributions to the K_L-K_S mass difference can be rendered harmless by making the $\tilde{d}$ and $\tilde{s}$ squarks degenerate [4]. Why not guarantee this degeneracy by placing these squarks in a doublet of a non-Abelian flavor group $(\tilde{d}, \tilde{s})$? In the case of Abelian G_f, the squarks are far from degenerate, however it was discovered that the flavor-changing

problem could be solved by arranging for the Kobayashi-Maskawa mixing matrix to have an origin in the up sector rather than the down sector.

A variety of supersymmetric theories of flavor have followed, including ones based on $G_f = O(2)$ [5], $G_f = U(1)^3$ [6], $G_f = \Delta(75)$ [7], $G_f = (S_3)^3$ [8, 9] and $G_f = U(2)$ [10]. Progress has also been made on relating the small parameters of fermion and scalar mass matrices using a gauged $U(1)$ flavor symmetry in a $N = 1$ supergravity theory, taken as the low energy limit of superstring models [11]. Development of these and other theories of flavor is of great interest because they offer the hope that an understanding of the quark and lepton masses, and the masses of their scalar superpartners, may be obtained at scales well beneath the Planck scale, using simple arguments about fundamental symmetries and how they are broken. The theories, to varying degrees, give understanding to the patterns of the mass matrices, and may, in certain cases, also lead to very definite mass predictions. Furthermore, flavor symmetries may be of use to understand a variety of other important aspects of the theory.

The general class of theories which address both aspects of the supersymmetric flavor problem have two crucial ingredients: the flavor group G_f and the flavon fields F, which have a hierarchical set of vacuum expectation values (vevs) allowing a sequential breaking of G_f[†]. These theories can be specified in two very different forms. In the first form, the only fields in the theory beyond F are the light matter and Higgs fields. An effective theory is constructed in which all gauge and G_f invariant interactions are written down, including non-renormalizable operators scaled by some mass scale of flavor physics, M_f. The power of this approach is that considerable progress is apparently possible without having to make detailed assumptions about the physics at the scale M_f which generates the non-renormalizable operators. Much, if not all, of the flavor structure of fermion and scalar masses comes from such non-renormalizable interactions, and it is interesting to study how their form depends only on the choice of G_f, how G_f is broken, and the light field content.

A second, more ambitious, approach is to write a complete, renormaliz-

[†]We assume that the scalar mass squared matrices are constrained by the flavor symmetry, i.e., that the messenger scale of supersymmetry breaking is higher than the flavor scale.

able theory of flavor at the scale M_f. Such a theory possesses a set of heavy fields which, when integrated out of the theory, lead to the effective theory discussed above [12]. However, it is reasonable to question whether the effort required to construct such full theories is warranted. Clearly these complete theories involve further assumptions beyond those of the effective theories, namely the G_f properties of the fields of mass M_f, and it would seem that the low energy physics of flavor is independent of this, depending only on the properties of the effective theory. In non-supersymmetric theories such a criticism may have some validity, but in supersymmetric theories it does not. This is because in supersymmetric theories, on integrating out the states of mass M_f, the low energy theory is not the most general effective theory based on the flavor group G_f. Several operators which are G_f invariant, and could be present in the effective theory, are typically not generated when the heavy states of mass M_f are integrated out. Which operators are missing depends on what the complete theory at G_f looks like. This phenomena is well known, and is illustrated, for example, in references [13, 14, 7, 10], and it casts doubt on the effective theory approach to building supersymmetric theories of flavor. Finally, one might hope that a complete renormalizable theory of flavor at scale M_f might possess a simplicity which is partly hidden at the level of the effective theory.

We have previously discussed an effective theory of flavor based on the gauged flavor group $G_f = (S_3)^3$ [8, 9]. In this paper we find a simple, complete, renormalizable theory with $G_f = (S_3)^3$, and we demonstrate, that acceptable fermion and scalar mass matrices result from integrating out the heavy states. In addition, we discover an origin for R parity in the G_f properties of the renormalizable interactions of the complete theory. In the effective theory approach there are R-parity-violating operators which are G_f allowed and must be forbidden by hand to avoid phenomenological difficulties. However, such operators are not generated from our full theory: we can understand R parity to be an unavoidable consequence of the G_f structure of the Higgs and matter representations of the complete theory.

Our choice of a gauged $(S_3)^3$ as the flavor group is motivated by a number of considerations. First, we choose a gauged flavor symmetry over a global one to avoid the criticism that global symmetries are not respected by quantum gravitational effects. If the gauged flavor symmetry is a continuous one

[2], then there will be D-term contributions to the scalar potential that couple ordinary squarks to the flavon fields. In this case, flavon expectation values may generate substantial nonuniversal contributions to the squark masses, and hence, dangerous flavor changing neutral current effects [15]. We therefore choose to work with a discrete gauged flavor symmetry, for which there are no associated D-terms. We then choose a discrete group that has both **2** and **1** dimensional representations. With this representation structure, we can embed the chiral superfields of the first two generations into the doublet, to maintain the near degeneracy of the corresponding squarks. The smallest discrete flavor group with these representations is S_3, which has a **2**, $\mathbf{1}_S$, and $\mathbf{1}_A$. The latter is a one-dimensional representation that transforms nontrivially under the group. We assign the third generation fields to the $\mathbf{1}_A$ rather than $\mathbf{1}_S$ so that the model is free of discrete gauge anomalies. The three generations of the standard model therefore correspond to the representation structure $\mathbf{2}+\mathbf{1}_A$. If we tried to build a model in which G_f involved only a single S_3 factor, we would find that it is impossible to explain the hierarchy between, for example, the down and strange quark masses, which both would be invariant under the flavor group. A simple way around this problem is to replicate S_3 factors, so that the left-handed doublet fields Q, and the right-handed singlet fields U and D each transform under a different S_3. In addition, if the Higgs fields are chosen to transform as $\mathbf{1}_A$'s under both S_3^Q and S_3^U simultaneously, only the top quark Yukawa coupling is left invariant under the flavor symmetry. The remaining quark Yukawa couplings can be treated as small symmetry-breaking spurions, and the deviation from squark degeneracy easily estimated. This analysis was carried out in Ref. [8], where it was shown that the forms of the squark mass-squared matrices were phenomenologically viable. In addition, the model can be extended to the lepton sector by assigning the doublet chiral superfield L and the singlet E to $\mathbf{2}+\mathbf{1}_A$'s of S_3^D and S_3^Q, respectively [9]. This leads to acceptable slepton mass-squared matrices and a distinctive proton decay signature that may be within the reach of SuperKamiokande [9].

It is the point of our current work to explain how an acceptable pattern of $(S_3)^3$ breaking originates at a fundamental level, and to show how R parity emerges from the flavor structure of the full theory. Unlike Refs. [8, 9], we will allow the flavor scale M_f to be considerably lower than the Planck scale

M_{Pl}. In this case, the constraints from proton decay on the acceptable flavon quantum number assignments [9] are considerably weakened. This in turn allows us to construct a much more elegant model. The paper is organized as follows. In the Section 2 we review the known mechanisms of suppressing baryon- and lepton-number-violating interactions in supersymmetric models. In Section 3, we present the quantum number assignments for the flavor symmetry breaking fields F in our model. We show that the most general set of higher dimension operators involving the F fields generate viable fermion Yukawa matrices when the flavons acquire vevs. In addition, we show that the pattern of flavor symmetry breaking in our model leads to squark and slepton mass-squared matrices that are phenomenologically acceptable. In Section 4, we present a renormalizable model that generates the necessary operators involving the F fields when a set of vector-like fields are integrated out beneath the flavor scale M_f. Given the field content above the scale M_f, we show that all renormalizable R-parity-violating operators are forbidden by the flavor symmetry. We also take into account the possibility of non-renormalizable R-parity-violating operators generated at the Planck scale. In the final section, we summarize our conclusions. In an appendix we provide an example of a workable potential that generates the pattern of vevs assumed in the main body of the paper.

2 The suppression of baryon and lepton number violation.

The standard model, for all its shortcomings, does provide an understanding for the absence of baryon (B) and lepton (L) number violation: the field content simply does not allow any renormalizable interactions which violate these symmetries. This is no longer true when the field content is extended to become supersymmetric; squark and slepton exchange mediate baryon and lepton number violation at unacceptable rates, unless an extra symmetry, such as R parity, is imposed on the theory. The need for a new symmetry, which in general we label X, was first realised in the context of a supersymmetric $SU(5)$ grand unified theory [16]. As will become clear, there

are a wide variety of possibilities for the X symmetry. Matter parity[‡] [4], Z_N symmetries other than matter parity [17, 18, 19] and baryon or lepton numbers [20, 21, 22] provide well known examples, each giving a distinctive phenomenology. One of the most fundamental questions in constructing supersymmetric models is [23, 24] *What is the origin of this extra symmetry needed to suppress baryon and lepton number violating processes?*

The X symmetry must have its origin in one of the three categories of symmetries which occur in field theory models of particle physics: spacetime symmetries, gauge (or vertical) symmetries and flavor (or horizontal) symmetries. The X symmetry is most frequently referred to as R parity[§], R_p, which is a Z_2 parity acting on the anti-commuting coordinate of superspace and on the chiral superfields, such that $\theta \rightarrow -\theta$, matter fields$\rightarrow$ $-$matter fields and higgs fields $\rightarrow$higgs fields. We view this as unfortunate, since it suggests that the reason for the suppression of baryon and lepton number violation is to be found in spacetime symmetries, which certainly need not be the case. R_p can be viewed as a superspace analogue of the familiar discrete spacetime symmetries, such as P and CP. In the case of P and CP we know that they can appear as accidental symmetries in gauge models which are sufficiently simple. For example P is an accidental symmetry of QED and QCD, while CP is an accidental symmetry of the two generation standard model. Nevertheless, in the real world P and CP are broken. This suggests to us that discrete spacetime symmetries are not fundamental and should not be imposed on a theory, so that if R_p is a good symmetry, it should be understood as being an accidental symmetry resulting from some other symmetry. These arguments can also be applied to alternative spacetime origins for X, such as a Z_4 symmetry on the coordinate θ [17]. [¶] Hence, while the symmetry X could have a spacetime origin, we find it more plausible that it arises from gauge or flavor symmetries.

In this case what should we make of R_p? If it is a symmetry at all, it would

[‡]Matter parity is equivalent to R parity, up to a 2π rotation.

[§]R_p was first introduced in a completely different context [25].

[¶]Clearly these arguments need not be correct: for example, it could be that both P and CP are fundamental symmetries, but they have both been spontaneously broken. However, in this case the analogy would suggest that R_p is also likely to be spontaneously broken.

be an accidental symmetry, either exact or approximate. If R_p is broken by operators of dimension 3, 4 or 5, then a weak-scale, lightest superpartner (LSP) would not be the astrophysical dark matter. The form of the R_p breaking interactions will determine whether the LSP will decay in particle detectors or whether it will escape leaving a missing energy signature. The realization that X may well have an origin in gauge or flavor symmetries, has decoupled the two issues of the suppression of B and L violation, due to X, and the lifetime of the LSP, governed by R_p [18, 26].

At first sight, the most appealing origin for X is an extension of the standard model gauge group, either at the weak scale [23], or at the grand unified scale [24]. An interesting example is provided by the crucial observation that adding $U(1)_{B-L}$ [24], or equivalently $U(1)_{T_{3R}}$, is sufficient to remove all renormalizable B and L violation from the low energy theory: matter parity is a discrete subgroup of $U(1)_{B-L}$. This is clearly seen in SO(10) [27], where the requirement that all interactions have an even number of spinor representations immediately leads to matter parity.

However, this example has a gauge group with rank larger than that of the standard model, and the simplest way to spontaneously reduce the rank, for example via the vev of a spinor **16**-plet in $SO(10)$, leads to a large spontaneous breaking of the discrete matter parity subgroup of $SO(10)$ [28, 29]. Thus theories based on $SO(10)$ need a further ingredient to ensure sufficient suppression of B and L violation of the low energy theory. One possibility is that the spinor vev does not introduce the dangerous couplings, which typically requires a discrete symmetry beyond $SO(10)$. Alternatively the rank may be broken by a larger Higgs multiplets [28], for example the **126** representation of $SO(10)$. Finally, if the reduction of rank occurs at low energies, the resulting R_p violating phenomenology may be acceptable [29], however, the weak mixing angle prediction is then lost (For exceptions, see Refs. [30]). The flipped $SU(5)$ gauge group allows for models with renormalizable L violation, but highly suppressed B violation [31]; however, these theories also lose the weak mixing angle prediction.

There are other possibilities for X to be a discrete subgroup of an enlarged gauge symmetry. Several Z_N examples from E_6 are possible [18]. Such a symmetry will be an anomaly free discrete gauge symmetry, and it has been argued that if X is discrete it should be anomaly free in order not to be

violated by Planck scale physics [32]. With the minimal low energy field content, there are only two such possibilities which commute with flavor: the familiar case of matter parity, and a Z_3 baryon parity [19], which also prohibits baryon number violation from dimension 5 operators. While the gauge origin of X remains a likely possibility, we are not aware of explicit compelling models which achieve this.

Another possible mechanism of suppressing R-parity violation, which is not discussed in the literature, is a Peccei–Quinn symmetry. This anomalous global symmetry was proposed in Ref. [33] to solve the strong CP problem in QCD. In the context of supersymmetric models, we assign the same charge $+1$ to all the matter chiral superfields, Q, U, D, L, and E, and a charge -2 to the Higgs chiral superfields H_u and H_d. This symmetry forbids all R-parity violating interactions. If we break the Peccei–Quinn symmetry using a field with even charges, it leaves an unbroken Z_2 symmetry which is nothing but the matter parity that we have discussed. The same Peccei–Quinn symmetry forbids the B-violating dimension-five operators in the symmetry limit, but they are induced by its breaking in general. The extent of suppression depends on the details of the models [34, 35, 36].

Finally we discuss the possibility that the X symmetry is a flavor symmetry: the symmetry which is ultimately responsible for the small parameters of the quark and lepton mass matrices, and also of the squark and slepton mass matrices, might provide sufficient suppression for B and L violation. Indeed, this is an extremely plausible solution for the suppression of L violation since the experimental constraints on the coefficients of the L violating interactions are quite weak, and would be satisfied by having amplitudes suppressed by powers of small lepton masses. However, the experimental constraints involving B violation are so strong, that suppression by small quark mass factors are insufficient [37]. Hence the real challenge for these theories is to understand the suppression of B violation.

Some of the earliest models involving matter parity violation had a discrete spacetime [17] or gauge [31] origin for B conservation, but had L violation at a rate governed by the small fermion masses. This distinction between B and L arises because left-handed leptons and Higgs doublets are not distinguished by the standard model gauge group, whereas quarks are clearly distinguished by their color. This provides a considerable motivation

to search for supersymmetric theories with matter parity broken only by the L violating interactions.

It is not difficult to understand how flavor symmetries could lead to exact matter parity. Consider a supersymmetric theory, with minimal field content and gauge group, which has the flavor group $U(3)^5$ broken only by parameters which transform like the usual three Yukawa coupling matrices. The Yukawa couplings and soft interactions of the most general such effective theory can be written as a power series in these breaking parameters, leading to a theory known as weak scale effective supersymmetry [1]. The flavor group and transformation properties of the breaking parameters are sufficient to forbid matter parity violating interactions to all orders: each breaking parameter has an even number of $U(3)$ tensor indices, guaranteeing that all interactions must have an even number of matter fields.[‖] To construct an explicit model along these lines it is perhaps simplest to start with a $U(3)$ flavor group, with all quarks and leptons transforming as triplets, but Higgs doublets as trivial singlets. An exact matter parity will result if the spontaneous breaking of this flavor group occurs only via fields with an even triality. A similar idea has recently been used in the construction of a four generation theory with gauged flavor $SU(4)$ symmetry [38, 22].

In view of the recent activity in constructing explicit supersymmetric theories of flavor [2, 3, 5, 6, 7, 8, 9, 10], an interesting question is whether the X symmetry is contained in a flavor group [39]. With Abelian flavor groups, the suppression of L violation is quite natural [40], while sufficient suppression of B violation is much harder to obtain [41]. In this paper we construct a theory of flavor based on the non-Abelian discrete group $(S_3)^3$. It is found to provide an explanation for the suppression of B and L violation that is analogous to the matter parity found in $SO(10)$ theories, with the difference, however, that B and L are not exact.

[‖]This point was missed in [1] where R_p was imposed unnecessarily as an additional assumption. We believe that the automatic conservation of R_p makes this scheme an even more attractive framework as a model independent low energy effective theory of supersymmetry.

3 The Model

As we described earlier, the three generations of Q, U, and D fields transform as $\mathbf{2}+\mathbf{1}_A$'s under the corresponding S_3 group. The ordinary Higgs fields transform as $(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S)$'s under $S_3^Q \times S_3^U \times S_3^D$. Given these assignments, the quark Yukawa matrices have well defined transformation properties under $(S_3)^3$:

$$Y_u \sim \left(\begin{array}{c|c} (\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S) & (\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S) \\ \hline (\mathbf{1}_S, \tilde{\mathbf{2}}, \mathbf{1}_S) & (\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_S) \end{array} \right) \ , \ Y_d \sim \left(\begin{array}{c|c} (\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2}) & (\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A) \\ \hline (\mathbf{1}_S, \mathbf{1}_A, \mathbf{2}) & (\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A) \end{array} \right) \tag{1}$$

where we use the notation $\tilde{\mathbf{2}} \equiv \mathbf{2} \otimes \mathbf{1}_A$,**. In the lepton sector, the fields L and E transform in the same way as D and Q under the flavor symmetry, so that the lepton Yukawa matrix transforms in the same way as Y_d^T.

We first specify the quantum number assignments for the fields that acquire flavor symmetry breaking vevs. Products of these fields must have the proper transformation properties to generate (at least some of) the various blocks of the fermion Yukawa matrices shown in eq. (1). The flavon fields F in our model are

$$\Phi_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_A, \mathbf{1}_S) \ , \ \ \Phi_D^{(i)} \sim (\mathbf{1}_A, \mathbf{1}_S, \mathbf{2}) \ , \ \ \Phi_U^{(i)} \sim (\mathbf{1}_A, \mathbf{2}, \mathbf{1}_S) \ ,$$

$$\chi_1 \sim (\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A) \ , \ \ \chi_2 \sim (\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_A) \ , \tag{2}$$

where $i = 1, 2$. Note that these are simpler representations for the flavon fields than those presented in Refs. [8, 9]. While we argued in Ref. [9] that some of the flavon representations shown above were excluded by their contribution to proton decay via Planck-suppressed dimension-five operators, we will see in Section 4 that these operators are easily suppressed by taking the flavor scale to be somewhat below M_{Pl}.

Let us now explicitly construct the fermion Yukawa matrices that follow from (2). The two-by-two down-strange and up-charm Yukawa matrices involve products of the form

$$\Phi_Q^{(i)} \Phi_D^{(j)} \sim (\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2}) \quad \text{and} \quad \Phi_Q^{(i)} \Phi_U^{(j)} \sim (\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S) \ . \tag{3}$$

**$\tilde{\mathbf{2}} = (a, b)$ is equivalent to $\mathbf{2} = (b, -a)$.

Each of the eight combinations of Φ fields shown above can form a flavor-invariant dimension-six operator that contributes to the usual Yukawa coupling matrices when the flavon fields acquire vevs. For example, the down-strange block originates from the operators

$$\frac{1}{M_f^2}\sum_{ij} c_{ij}^d Q H_d \Phi_Q^{(i)} \Phi_D^{(j)} D \tag{4}$$

where M_f is the flavor-physics scale, and the c_{ij}^d are order one coefficients. Note that we have introduced two Φ_Q doublets in order to assure a nonvanishing Cabibbo angle. In addition, we require two Φ_U and Φ_D fields so that the up and down quark masses are both nonvanishing. This would not be possible if the Yukawa matrices in (3) were each formed from the product of exactly two doublets; any matrix constructed in this way has a vanishing determinant. In our discussion below, we will let each Φ_a field (with $a = Q$, U, or D) represent some linear combination of $\Phi_a^{(1)}$ and $\Phi_a^{(2)}$, leaving it implicit that different occurrences of Φ_a may indicate different linear combinations.

Let us denote the ratio of the vevs of the Φ and χ fields to the flavor-physics scale M_f by the parameters ϵ and δ. If we choose the Φ field vevs

$$\frac{1}{M_f}\langle\Phi_Q\rangle \sim \epsilon_Q \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \quad \frac{1}{M_f}\langle\Phi_D\rangle \sim \epsilon_D \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \quad \frac{1}{M_f}\langle\Phi_U\rangle \sim \epsilon_U \begin{bmatrix} \lambda^3 \\ 1 \end{bmatrix} \tag{5}$$

then the down-strange and up-charm Yukawa matrices will take the form

$$\epsilon_Q\epsilon_D \begin{bmatrix} \lambda^2 & \lambda \\ \lambda & 1 \end{bmatrix} \quad \text{and} \quad \epsilon_Q\epsilon_U \begin{bmatrix} \lambda^4 & \lambda \\ \lambda^3 & 1 \end{bmatrix} \tag{6}$$

respectively, where $\lambda \approx 0.22$ is the Cabibbo angle. We set $\epsilon_Q\epsilon_D \sim \lambda^5$ and $\epsilon_Q\epsilon_U \sim \lambda^4$ so that the up, down, charm, and strange quark Yukawa couplings are of the correct order in λ (assuming $\tan\beta \sim 1$).

The lepton Yukawa matrix transforms in the same way as the down Yukawa matrix transposed. Therefore, the two-by-two block of the lepton Yukawa matrix is also determined by the vevs of the flavon product $\Phi_Q\Phi_D$. If this product represented a single matrix, then we would obtain the undesirable relation $m_e/m_\mu = m_d/m_s$. However, we have seen that there are in fact four contributions to the Yukawa matrices, each multiplied by an unknown coefficient of order one. This gives us enough degrees of freedom to suppress

the electron mass relative to that of the down quark. For concreteness, let us assume that $\Phi_Q^{(1)}$ and $\Phi_D^{(1)}$ have vevs proportional to $(0,1)$, while $\Phi_Q^{(2)}$ and $\Phi_D^{(2)}$ have vevs proportional to (λ,λ). If we take the coefficients $c_{11}^l = 3$ and $c_{22}^l = 1/3$ (where the c^l are the coefficients for the leptons that are analogous to the c^d in eq. (4)), and take all other coefficients to be 1, then we obtain $9m_e/m_\mu = m_d/m_s \sim \lambda^2$, which is an acceptable result. Had we required coefficients much larger than 3 (or much smaller than 1/3), then one might object that the choice of parameters is not consistent with naive dimensional analysis.

The remaining diagonal elements of the quark Yukawa matrices consist of the bottom and top Yukawa couplings. The bottom Yukawa coupling transforms exactly like χ_1, so we require $\delta_1 \sim \lambda^3$. The top Yukawa coupling is invariant under $(S_3)^3$, and is therefore of order 1 relative to the other elements.

Finally, we must evaluate the other off-diagonal elements of the up and down Yukawa matrices. In the down sector, the two-by-one off-diagonal block transforms as a $(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A) \sim \Phi_Q \chi_2$, and is therefore of the form

$$\epsilon_Q \delta_2 \begin{bmatrix} \lambda \\ 1 \end{bmatrix} . \tag{7}$$

If we choose $\epsilon_Q \delta_2$ to be of order λ^5, then these elements will generate the Cabibbo-Kobayashi-Maskawa (CKM) elements V_{ub} and V_{cb}. The one-by-two block of the down Yukawa matrix, which transforms as a $(\mathbf{1}_S, \mathbf{1}_A, \mathbf{2})$, is generated by the product $\Phi_D \chi_1 \chi_2$ and is therefore of the form

$$\epsilon_D \delta_1 \delta_2 \begin{bmatrix} \lambda & 1 \end{bmatrix} . \tag{8}$$

In the up sector, the off-diagonal block transforming as a $(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$ is given by the doublet component of $(\Phi_Q)^2$. When taking the product of two doublets, we will let $\times$ represent the projection onto the doublet component, $\wedge$ the $\mathbf{1}_A$ component, and $\cdot$ the $\mathbf{1}_S$. In this case, we want $\Phi_Q \times \Phi_Q$:

$$\epsilon_Q^2 \begin{bmatrix} \lambda \\ 1 \end{bmatrix} . \tag{9}$$

Similarly, the off-diagonal block transforming as a $(\mathbf{1}_S, \tilde{\mathbf{2}}, \mathbf{1}_S)$ is given by $\Phi_U \times \Phi_U$ and is of the form

$$\epsilon_U^2 \begin{bmatrix} \lambda & 1 \end{bmatrix} . \tag{10}$$

Given the constraints described above ($\epsilon_Q \epsilon_D \sim \lambda^5$ from the strange mass, $\epsilon_Q \epsilon_U \sim \lambda^4$ from the charm mass, $\delta_1 \sim \lambda^3$ from the bottom mass, and $\epsilon_Q \delta_2 \sim \lambda^5$ to generate adequate V_{ub} and V_{cb}) there is only one set of symmetry breaking parameters in which no ϵ or δ is larger than order λ^2:

$$\epsilon_Q \sim \lambda^2 \ , \ \epsilon_U \sim \lambda^2$$

$$\epsilon_D \sim \lambda^3 \ , \ \delta_1 \sim \lambda^3 \ , \ \delta_2 \sim \lambda^3 \tag{11}$$

With this choice, flavor changing neutral current effects will not be especially large in any one sector of our model. Given this choice, we can write down the down and up quark Yukawa matrices:

$$Y_d \sim \left[\begin{array}{cc|c} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \hline \lambda^{10} & \lambda^9 & \lambda^3 \end{array}\right] , \tag{12}$$

$$Y_u \sim \left[\begin{array}{cc|c} \lambda^8 & \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^4 & \lambda^4 \\ \hline \lambda^5 & \lambda^4 & 1 \end{array}\right] . \tag{13}$$

These results are consistent with the masses and mixing angles of the Standard Model.

Finally we consider the form of the squark and slepton mass matrices. Spurions transforming as either a **2** or $\mathbf{1}_A$ under a single S_3 group contribute to the off-diagonal entries of the corresponding squark mass matrix. These representations can be formed at lowest order by the products $\Phi_a \times \Phi_a$, $\Phi_a^{(1)} \wedge \Phi_a^{(2)}$ or $\Phi_D \chi_2$. The analysis is analogous to the one we presented in detail for the quark Yukawa matrices, so here we will simply quote our results. The left-handed squark mass matrices are of the form

$$m_Q^2 = \left[\begin{array}{cc|c} M_1^2 + m^2\lambda^4 & m^2\lambda^5 & m^2\lambda^5 \\ m^2\lambda^5 & M_1^2 - m^2\lambda^4 & m^2\lambda^4 \\ \hline m^2\lambda^5 & m^2\lambda^4 & M_3^2 \end{array}\right] . \tag{14}$$

The right-handed squark mass matrices are given by

$$m_U^2 = \left[\begin{array}{cc|c} M_1^2 + m^2\lambda^4 & m^2\lambda^5 & m^2\lambda^5 \\ m^2\lambda^5 & M_1^2 - m^2\lambda^4 & m^2\lambda^4 \\ \hline m^2\lambda^5 & m^2\lambda^4 & M_3^2 \end{array}\right] \tag{15}$$

and

$$m_D^2 = \left[\begin{array}{cc|c} M_1^2 + m^2\lambda^6 & m^2\lambda^7 & m^2\lambda^7 \\ m^2\lambda^7 & M_1^2 - m^2\lambda^6 & m^2\lambda^6 \\ \hline m^2\lambda^7 & m^2\lambda^6 & M_3^2 \end{array}\right] . \tag{16}$$

All of the off-diagonal elements are consistent with the flavor changing neutral current bounds given in Ref. [42]. The slepton mass matrices m_L^2 and m_E^2 are of the same form as m_D^2 and m_Q^2, respectively.

Finally, we should point out that the supersymmetry breaking trilinear interactions have the same flavor structure as the fermion Yukawa matrices, but generally involve different order one coefficients. Thus, the trilinear interactions are not simultaneously diagonalizable with the Yukawa matrices in general (unlike the situation in Ref. [9]). An important constraint on the form of these couplings comes from the bounds on $\mu \to e\gamma$. The (12) entry of the left-right slepton mass mixing in our model is given by

$$(m_{LR}^2)_{21} \sim m_s \lambda A \tag{17}$$

This is approximately 20 times larger than the result obtained in Ref. [9]. If we choose the slepton masses to be of order 300 GeV, the bino mass and the A parameter to be $\sim$ 100 GeV, then our model saturates the experimental bound $\mathrm{Br}(\mu \to e\gamma) < 4.9 \times 10^{-11}$. Here we use the formulae presented in Ref. [9].

4 The Froggatt-Nielsen Model

In the previous section we constructed a low-energy effective theory in which the lowest-dimension nonrenormalizable operators involving the flavon fields generate acceptable fermion Yukawa matrices when the flavons acquire vevs, without significantly affecting the degeneracy of the squarks (or sleptons) of the first two generations. If the effective theory below M_f is generated by integrating out heavy states in a renormalizable theory, then we will generally obtain some subset of the operators described in the previous section. All operators that are consistent with the symmetries of the low-energy theory may not necessarily be present. In building a renormalizable theory of flavor, we need only to verify that the operators we need for generating the elements of the fermion Yukawa matrices are present; our general operator analysis

tells us *a priori* that the full theory will otherwise be phenomenologically acceptable.

In this section, we will construct a renormalizable version of our $(S_3)^3$ model incorporating the mechanism of Froggatt and Nielsen [12]. We will show that the operators we need to account for the fermion masses and mixing angles are generated assuming that there is a relatively economical set of heavy, vector-like particles present at the scale M_f. We will then show that our choice of quantum numbers for these fields has an added bonus: all the possible renormalizable interactions that violate R parity are forbidden by the flavor symmetry. This implies that no R-parity-violating nonrenormalizable operators (suppressed by powers of M_f only) are generated when the heavy states are integrated out. While there may be Planck-scale-suppressed operators that violate R parity and are invariant under the flavor group, these may be rendered harmless by taking the flavor scale to be sufficiently low. We discuss the implications of this scenario at the end of this section.

The flavor quantum number assignments of the vector-like chiral superfields are given in the first column of Table 1. The electroweak quantum numbers of the heavy, unbarred fields are the same as those of the corresponding MSSM field (i.e. Q^H is a color triplet, weak doublet with hypercharge $\frac{1}{6}$, etc.) While we have displayed only one generation of the vector-like fields in Table 1, we assume the existence of two generations, for reasons detailed below. In addition to the two heavy generations, there are also the 'extra' heavy fields L'^H, $\overline{L'}^H$, D'^H, and $\overline{D'}^H$, also shown in the table. In SU(5) language, the heavy particle content consists of two generations, two antigenerations, and an additional $\mathbf{5}+\overline{\mathbf{5}}$. Note that R parity assignments are also displayed in Table 1.

Given the particle content in Table 1, it is straightforward to construct the operators that generate the fermion Yukawa matrices. Consider the two-by-two block of the down Yukawa matrix. The relevant couplings in the superpotential are of the form

$$W = \sum_{ij} (Q \cdot \Phi_Q^{(i)}) \overline{Q}_j^H + Q_i^H H_d D_j^H + (D \cdot \Phi_D^{(i)}) \overline{D}_j^H \tag{18}$$

where the subscript on the heavy fields indicates the heavy generation or antigeneration. By integrating out the heavy fields in (18), we are left with the

R-parity odd		R-parity even	
$Q^H, \overline{Q}^H$	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$	$\Phi_Q^{(i)}$	$(\mathbf{2}, \mathbf{1}_A, \mathbf{1}_S)$
$U^H, \overline{U}^H$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$	$\Phi_D^{(i)}$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{2})$
$D^H, \overline{D}^H$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$	$\Phi_U^{(i)}$	$(\mathbf{1}_A, \mathbf{2}, \mathbf{1}_S)$
$L^H, \overline{L}^H$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$	χ_1	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$
$E^H, \overline{E}^H$	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$	χ_2	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_A)$
$L'^H, \overline{L'}^H$	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$	H_u	$(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S)$
$D'^H, \overline{D'}^H$	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$	H_d	$(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S)$
+ matter			

Table 1: Field content of the theory above the flavor scale. Only one generation of the vector-like fields is shown.

four operators presented in equation (4). This result is represented graphically in Figure 1. Notice that the coupling $Q\Phi_Q^{(i)}\overline{Q}^H$ is involved in generating both the two-by-two up and down quark Yukawa matrices. If only one generation of heavy fields were present, then a single linear combination of $\Phi_Q^{(1)}$ and $\Phi_Q^{(2)}$ would enter in these diagrams, and we would be left with no Cabibbo angle. We require two heavy generations so that two linearly independent combinations of the $\Phi_a^{(i)}$ contribute to the operators in the effective theory described in the previous section. Note that the couplings $\overline{D}_j^H \chi_2 b$, $\overline{D'}^H \chi_1 b$, and $Q_3 H_d D'^H$ in the superpotential are necessary for generating the other elements of Y_d.

Notice that the Yukawa matrices are simpler in this model than we would have expected from our general operator analysis. With the particle content specified in Table 1, we find that the (3,1) and (3,2) entries of the up and down Yukawa matrices as well as the (1,3) and (2,3) entries of the up matrix are not generated by heavy particle exchange. While sparse, the Yukawa matrices are nonetheless phenomenologically acceptable.

One of the interesting features of the quantum number assignments in this model is that it is not possible to write down any R-parity-violating renormalizable interactions that are invariant under the flavor group. Consider first the R-parity-violating operators that involve three heavy R-odd fields. Since each heavy field transforms as a $\mathbf{1}_A$ under a single S_3 group, the

Operator	Transformation	Operator	Transformation
UDD	$(\mathbf{1}_S, \mathbf{2}+\mathbf{1}_A, \mathbf{1}_A)$	$L^H LE$	$(\mathbf{2}+\mathbf{1}_S, \mathbf{1}_S, \mathbf{2}+\mathbf{1}_A)$
QLD	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_S, \mathbf{2}+\mathbf{1}_A+\mathbf{1}_S)$	$L'^H LE$	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_A, \mathbf{2}+\mathbf{1}_A)$
LLE	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_A)$	LLE^H	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$
$U^H DD$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_A)$	$QQ\overline{D}^H$	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$
$UD^H D$	$(\mathbf{1}_A, \mathbf{2}+\mathbf{1}_A, \mathbf{2}+\mathbf{1}_A)$	$QQ\overline{D}'^H$	$(\mathbf{2}+\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$
$UD'^H D$	$(\mathbf{1}_S, \mathbf{2}+\mathbf{1}_S, \mathbf{2}+\mathbf{1}_A)$	$Q\overline{L}^H U$	$(\mathbf{2}+\mathbf{1}_S, \mathbf{2}+\mathbf{1}_A, \mathbf{1}_S)$
$Q^H LD$	$(\mathbf{1}_S, \mathbf{1}_A, \mathbf{2}+\mathbf{1}_A+\mathbf{1}_S)$	$Q\overline{L}'^H U$	$(\mathbf{2}+\mathbf{1}_A, \mathbf{2}+\mathbf{1}_S, \mathbf{1}_S)$
$QL^H D$	$(\mathbf{2}+\mathbf{1}_S, \mathbf{1}_S, \mathbf{2}+\mathbf{1}_A)$	$U\overline{D}^H E$	$(\mathbf{2}+\mathbf{1}_S, \mathbf{2}+\mathbf{1}_A, \mathbf{1}_S)$
$QL'^H D$	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_A, \mathbf{2}+\mathbf{1}_A)$	$U\overline{D}'^H E$	$(\mathbf{2}+\mathbf{1}_A, \mathbf{2}+\mathbf{1}_S, \mathbf{1}_S)$
QLD^H	$(\mathbf{2}+\mathbf{1}_S, \mathbf{1}_S, \mathbf{2}+\mathbf{1}_A)$		
QLD'^H	$(\mathbf{2}+\mathbf{1}_A, \mathbf{1}_A, \mathbf{2}+\mathbf{1}_A)$		

Table 2: Trilinear operators involving three R-odd fields, with zero or one heavy field.

product of three can never form an invariant. Next consider the operators that involve two heavy R-odd fields and one light matter field. The product of the two heavy fields either forms a singlet or transforms as $(\mathbf{1}_A, \mathbf{1}_A)$ under exactly two of the S_3 groups. Since the light field transforms nontrivially under a single S_3 group, the heavy-heavy-light combination can never form an invariant. The remaining interactions involving three R-odd fields are those with zero or one heavy field. These are cataloged in Table 2.

In almost every interaction shown in Table 2, at least one of the three fields involved transforms under a different S_3 group than that of the other two, so that there is no possibility of forming an invariant. The only exception is the operator $QQ\overline{D}^H$, which involves three fields that each transform under S_3^Q. In this case, however, the operator is symmetric under interchange of the two Q fields, so we can never form the $\mathbf{1}_A$ that we would need to produce an invariant.

The remaining trilinear operators that we need to consider are those that involve one R-odd and two even fields. Since the R-odd fields all carry electroweak quantum numbers, these operators must be of the following form to preserve electroweak gauge invariance: LH_dF, $L^H H_dF$, $L'^H H_dF$, $\overline{L}^H H_uF$

or $\overline{L'}^H H_u F$, where F is a flavon field (either Φ or χ). The product of the first two fields in each of these interactions transform as a $(\mathbf{1}_A, \mathbf{1}_A, \mathbf{2}+\mathbf{1}_A)$, $(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$, $(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$ $(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S)$ and $(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_S)$ respectively. Since the flavon fields transform under exactly two S_3 groups, while the representations above involve either one or three S_3 groups, no invariants are possible. As a corollary, we have shown that all the dimension-2 R-odd operators in the superpotential transform nontrivially under the flavor group, and are forbidden as well.

R parity is an accidental symmetry in our $(S_3)^3$ model, a consequence of both the flavor symmetry and the particle content given in Table 1. Our preceding discussion, however, has two limitations. First, we may need to enlarge the particle content of the model to construct a renormalizable potential for the flavon fields that yields the pattern of expectation values assumed in Section 2. We show in the Appendix that the additional fields required to construct a suitable potential do not have interactions that spoil the accidental R parity described in this section. Secondly, we have restricted ourselves to a renormalizable Lagrangian. There may be non-renormalizable interactions induced at the Planck scale, and some of these may violate R parity. Of course, Planck-suppressed R-parity violating operators simply may not be present; it is known, for example, that superstring compactification usually does not lead to the most general Lagrangian consistent with the symmetries of the low-energy theory. However, it is interesting to consider the constraints on our model if such R-parity-violating operators are indeed generated at the Planck scale.

The most stringent constraint on R-parity violation comes from non-observation of nucleon decay. The most dangerous combination of operators is uds and $Q_1 s L_{1,2}$, where the subscript is the generation index. Since we must combine each of these with at least two flavon fields to form an $(S_3)^3$ invariant at the Planck scale, both trilinears are suppressed by $(M_f/M_*)^2$ in the low-energy theory, where $M_* = M_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass. There are operators involving third generation fields and/or heavy Froggatt–Nielsen fields, however, that can be constructed using only one flavon field, yielding trilinear operators that are suppressed by one power of (M_f/M_*). Since the third generation and the heavy fields mix with the first generation fields, dangerous operators may result [43, 44]. There are two UDD-type

operators allowed at linear order in the flavor symmetry breaking and also linear order in either third generation or heavy fields: $\chi_1 U_3(D \wedge D)/M_*$ and $\chi_2 U^H(D \wedge D)/M_*$. Given the structure of the Yukawa matrices, U_3 does not mix with the first generation fields (recall that the (3,1) and (3,2) entries of Y_u were not generated in the full theory) while U^H mixes at order $\epsilon_U \lambda^3 \simeq \lambda^5$. Similarly, there are three QDL-type operators at linear order in spurion and also linear in either third generation or heavy fields: $\chi_2 Q_3(D \wedge L)/M_*$, $Q_3(\Phi_D \cdot (D \times L))/M_*$ and $\chi_1 Q^H(D \wedge L)/M_*$. The last one dominates among these three. Assuming that these operators are present, they are tightly constrained from proton decay [37]:

$$\frac{\delta_2 \epsilon_U \lambda^3 M_f}{M_*} \frac{\delta_1 \epsilon_Q \lambda M_f}{M_*} \lesssim 10^{-24} . \tag{19}$$

With our previous choice $\epsilon_U \simeq \epsilon_Q \simeq \lambda^2$ and $\delta_1 \simeq \delta_2 \simeq \lambda^3$, we obtain an upper bound on the flavor scale

$$M_f \lesssim 8 \times 10^{10} \text{ GeV} . \tag{20}$$

Given this bound, the coefficients h of the R-parity-violating operators are always smaller than $\lambda^2 M_f/M_* \lesssim 2 \times 10^{-9}$, and all existing experimental bounds are satisfied (for a comprehensive discussion of these bounds, see *e.g.*, Refs. [45] or [46, 43]); the tightest bound on the h comes from n-$\bar{n}$ oscillation with $h \lesssim 10^{-7}$. Note that the bound from sphaleron erasure of the cosmic baryon asymmetry $h \lesssim 10^{-8}$ [47] is also satisfied. *

There is a potentially strong constraint from cosmology if the R-parity violation is very weak. The lightest neutralino may decay after big bang nucleosynthesis and spoil its successful predictions [39]. For instance, we can estimate the lifetime of a bino-like neutralino assuming it decays via squark exchange and an R-parity-violating trilinear coupling:

$$\Gamma_{\tilde{\chi}_1^0} \sim \frac{1}{64\pi^2} \frac{\alpha}{\cos^2 \theta_W} \left(\frac{h}{m_{\tilde{q}}^2} \right)^2 m_{\tilde{\chi}_1^0}^5 . \tag{21}$$

If we take $h = \lambda^2 M_f/M_*$, $m_{\tilde{\chi}_1^0} \sim 100$ GeV, $m_{\tilde{q}} \sim 1$ TeV, and $M_f \sim 10^{10}$ GeV, we obtain the lifetime $\tau_{\tilde{\chi}_1^0} \sim 20$ sec. This satisfies the constraint from nucleosynthesis on a long-lived particle decaying into jets $\tau \lesssim 10^3$ sec [49]. The

*This bound may be even weaker in some cases [48].

constraint is weaker ($\tau \lesssim 10^6$ sec) if $\tilde{\chi}_1^0$ decays primarily into photons or leptons [50].† The constraint from the distortion in the cosmic microwave background spectrum is weaker than the one from nucleosynthesis [51].

For completeness, it is important to consider the proton decay constraints on Planck-suppressed dimension-five operators as well. Recall that in Ref. [9], we used these bounds to restrict the transformation properties of the flavon fields, assuming that the flavor scale was identical to the Planck scale. However, when $M_f < M_*$, the dimension-five operators are significantly suppressed. The largest dimension-five operators in our model are generated from the following flavor-invariant dimension-6 operators: $(Q \cdot Q)(Q_3 \Phi_D \cdot L)/M_*^2$ and $(Q \cdot Q)(Q_3 \chi_2 L_3)/M_*^2$. When the flavon fields acquire vevs, these operators generate dimension-five operators with coefficients $(M_f/M_*)(\lambda^3/M_*)$. The third generation doublet field mixes with the second generation at order λ^2. Thus, the coefficient of the operator that directly contributes to the decay is $(M_f/M_*)(\lambda^5/M_*)$. If we compare this to the experimental bound, which requires the coefficient to be smaller than $\mathcal{O}(\lambda^8/M_*)$ [9], then we obtain

$$M_f \lesssim 10^{16} \text{ GeV} \tag{22}$$

This bound is much weaker than the one we obtained from the R-parity-violating operators in eq. (20).

Finally, we should mention that the gauge coupling constants become non-perturbative below the Planck scale in our model, assuming that the vector-like particles are integrated out at a scale M_f satisfying Eq. (20). If we require perturbativity of the gauge couplings up to the scale M_*, then we obtain the lower bound $M_f \gtrsim 3 \times 10^{12}$ GeV. However, we do not consider this as a serious problem of the model since this scale is rather close to the upper bound given in Eq. (20). The particle content or gauge group may be altered close to the Planck scale, or one may go over to the dual description of the theory which remains weakly coupled.

†If the neutralino is too abundant, corresponding to $\Omega_\chi \gtrsim 10^2$ in the stable limit, and has a lifetime longer than 1 sec, it contributes to the energy density of the Universe and affects the expansion rate when the neutron abundance freezes out, and spoils the standard big bang nucleosynthesis predictions. Recall, however, the neutralino abundance is typically between $\Omega_\chi \sim 10^{-3}$ to 10^2.

5 Conclusions

We have presented a supersymmetric theory of flavor and R parity based on the discrete flavor group $(S_3)^3$. After specifying the flavor symmetry breaking fields, we showed that the most general low-energy effective theory consistent with the flavor and gauge symmetries does not lead to large flavor changing neutral current effects. The hierarchical pattern of the fermion Yukawa matrices and the near degeneracy of the squarks (or sleptons) of the first two generations are both guaranteed in our model by the flavor symmetry. In addition, we showed that an acceptable effective theory could originate from a renormalizable model via the Froggatt-Nielsen mechanism, and we presented an economical set of heavy vector-like fields responsible for generating the necessary operators. After specifying the particle content of the theory above the flavor scale M_f, we showed that all renormalizable operators that violate R parity were forbidden by the flavor symmetry. Thus, at the renormalizable level, R parity arose as an accidental symmetry in our model, a consequence of the flavor group and particle content. Furthermore, we showed that R-parity-violating nonrenormalizable operators generated at the Planck scale could be sufficiently suppressed by taking the flavor scale to be less than 10^{11} GeV. Our model demonstrates that it is possible to explain simultaneously the hierarchical form of the fermion Yukawa matrices, the suppression of flavor changing neutral current processes, and the absence of renormalizable baryon and lepton number violating couplings in supersymmetric models by introducing a flavor group and a specific mechanism of flavor symmetry breaking.

In section 2 we stressed that supersymmetric theories require some new symmetry, which we called X, to suppress B and L violation, and that there are many candidates for X. It is interesting to compare the X symmetry introduced in this paper with other elegant possibilities.

It is possible for X to be a discrete gauge symmetry, the most compelling of which is the Z_2 subgroup of $SO(10)$ generated by the element

$$X(SO(10)) = e^{i\pi(2T_{3_L}+2T_{3_R})} = e^{i\pi N_s} \qquad (I)$$

where N_s is 1 for spinorial representations and zero otherwise. When the rank of $SO(10)$ is broken, a special choice of representation or further discrete

symmetry is required to ensure that this X symmetry is left unbroken.

An elegant flavor group origin for X is possible with a flavor group $U(3)$, which contains a Z_2 with element

$$X(U(3)) = e^{i\pi N_T} \qquad (II)$$

where N_T is the triality of the representation. X conservation of the low energy theory follows if all flavor violation, in particular that which generates the quark and lepton masses, is generated by vevs of flavon fields with N_T even.

In the $(S_3)^3$ model of this paper, the X symmetry can similarly be defined as a Z_2 generated by an element which depends on representation type:

$$X(S_3^3) = e^{i\pi(N_{\mathbf{1}_A}+N_2)} \qquad (III)$$

where $N_{\mathbf{1}_A}, N_2$ count the number of $\mathbf{1}_A, \mathbf{2}$ representations of a given field. (For example, the representation $(\mathbf{2}, \mathbf{1}_A, \mathbf{1}_S)$ has $N_{\mathbf{1}_A} + N_2 = 2$.) This X will not be spontaneously broken if all Higgs and flavon fields have $N_{\mathbf{1}_A} + N_2$ even, as occurs in the model of this paper[‡].

From equations (I,II,III), one sees that these three examples of X symmetry have a comparable elegance. However, there is an important distinction. In cases I,II the symmetry group SO(10), U(3) is sufficient to ensure that X is an exact symmetry of the Lagrangian; indeed, X is a discrete subgroup of the gauge or flavor symmetry. This is not true in the case III: X is explicitly broken by any $\mathbf{2}^3$ or $\mathbf{2}^2\mathbf{1}_A$ invariant allowed by the gauge symmetry. Hence in case III, explicit violations of B and L are expected at some level, and the LSP is not expected to be absolutely stable.

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[‡]To forbid all the phenomenologically dangerous operators, it is necessary only for X to be a symmetry of the matter fields.

A Flavon Potential

In this appendix we present a possible form of the potential for the flavon fields. We discuss this issue for the following reasons. First, it is not possible to generate flavon vevs via a renormalizable potential using the flavon fields presented in the main body of the paper alone. If we rely only on the minimal flavon content, we must rely on higher dimension operators to obtain the desired form of the expectation values. If the higher dimension operators arise at the Planck scale, we obtain typical flavon masses of order $m_\phi \sim (\lambda^2 M_f)^2/M_*$. Furthermore, if we require that M_f satisfy the upper bound given in Eq. (20), then the flavon fields turn out to be rather light, $m_\phi \lesssim 400$ MeV. Unless one arranges the scales such that $m_\phi > m_K - m_\pi$, we will have the dangerous flavor-changing decays $K^+ \to \pi^+\phi$ or $\mu^- \to e^-\phi$ at rates beyond the experimental bounds.[§] The simplest way to avoid this potential phenomenological disaster is to arrange for renormalizable couplings among the flavon fields themselves to generate flavon masses of order M_f. Second, if we extend the particle content of flavons in a way that allows us to write down an explicit renormalizable potential, we may find that R parity is no longer an accidental consequence of the flavor symmetry and particle content, as emphasized in Section 3. The danger is that the new flavons may couple directly to the ordinary matter fields, and generate flavor-invariant, renormalizable R-odd couplings. The purpose of this section is to show that an extension of the particle content that allows us to write down a suitable potential for the flavon fields still preserves the accidental R parity of the minimal theory.

Writing down a potential for $\chi_{1,2}$ fields is easy. One needs to introduce fields ξ which transforms as a $(\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_S)$. The most general renormalizable potential is then

$$W = \frac{1}{2} m_\chi \chi^2 + \frac{1}{2} m_\xi \xi^2 - g_\chi \chi^2 \xi - g_\xi \xi^3 \,. \tag{23}$$

[§]For instance, the effective operator generated by Froggatt–Nielsen fields $W = (H_Q Q)(H_D D) H_d / M_f^2$ gives us an operator $W = (\epsilon_Q \langle H_d \rangle / M_f) \overline{d_R} s_L \varphi$, where φ is the physical field corresponding to the upper component of H_D. On the other hand, $K^+ \to \pi^+\phi$ with a massless ϕ constrains the coupling $(1/F)\partial_\mu \phi \bar{d}\gamma^\mu s$ such that $F \gtrsim 10^{11}$ GeV. If φ is light, we obtain $M_f \gtrsim 10^{13}$ GeV.

This potential has a stationary configuration,

$$\xi = \frac{m_\chi}{2g_\chi}, \tag{24}$$

$$\chi = \sqrt{(m_\xi \xi + 3g_\xi \xi^2)/g_\chi}. \tag{25}$$

Since ξ does not carry any flavor quantum number, none of our previous conclusions are affected by its existence.

Constructing a potential for $\Phi_{Q,U,D}$ is slightly more difficult. Since all Φ's have one doublet and one $\mathbf{1}_A$ factor, different types of Φ's cannot couple to each other in the renormalizable superpotential. Therefore, we consider potentials for different types of Φ's separately and discuss a Φ field generically transforming as a $(\mathbf{2}, \mathbf{1}_A)$ under $(S_3)^2$ without worrying which two S_3 groups are involved. Let us introduce another doublet field $K \sim (\mathbf{2}, \mathbf{1}_S)$. The most general renormalizable potential is[¶]

$$W = \frac{1}{2} m_\Phi \Phi^2 + \frac{1}{2} m_K K^2 - g_\Phi (\Phi \times \Phi) \cdot K - g_K (K \times K) \cdot K. \tag{26}$$

The reader should not worry that the third and fourth terms are X-violating couplings. Since K does not couple directly to any of the fields in the first column of Table 1, X remains conserved on the matter fields. This potential (26) allows a stationary configuration

$$\Phi = \begin{pmatrix} 0 \\ \sqrt{(m_K K_1 + 3g_K K_1^2)/g_\Phi} \end{pmatrix}, \tag{27}$$

$$K = \begin{pmatrix} m_\Phi / 2g_\Phi \\ 0 \end{pmatrix}. \tag{28}$$

Note that this configuration leaves a non-trivial S_3 subgroup unbroken

$$S_3 = \{(e,e),\ (e,(123)),\ (e,(132)),\ ((12),(12)),\ ((12),(23)),\ ((12),(31))\}$$

and hence the existence of this extremum is guaranteed by the symmetry. By having another independent set of Φ' and K', one may have the same

[¶]There may be couplings of the type $\Phi^2 \xi$ or $K^2 \xi$. However, these coupling do not affect the stationary configurations we discuss, and can be absorbed into m_Φ and m_K by a redefinition.

type of extremum but with a Z_3 rotation,

$$\Phi' = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{(m'_K K'_1 + 3g'_K K'^2_1)/g'_\Phi} \end{pmatrix} \tag{29}$$

$$K' = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} m'_\Phi/2g'_\Phi \\ 0 \end{pmatrix}. \tag{30}$$

If the overall scale of Φ', K' is lower than Φ and K by a factor of λ, we obtain the desired form of the expectation values of Φ and Φ'. ‖

The important point is that K fields do not contribute to the mixing between light and Froggatt–Nielsen fields because they lack the $\mathbf{1}_A$ factor. It is easy to check that none of our conclusions regarding the form of the Yukawa matrices, scalar matrices, and the accidental R parity present at the renormalizable level are modified by the existence of the K fields. Our discussion of nonrenormalizable R-parity-violating operators is only slightly modified, by the existence of the operator $W = (K_Q \cdot Q)(d \cdot L)/M_*$. If the expectation value of K_Q is similar to that of Φ_Q, this operator gives an R-parity violating $Q_1 s L_2$ operator with a coupling of $\epsilon_Q \lambda M_f/M_*$, which is larger than that discussed in section 3 by λ^3. The upper bound on M_f in Eq. (20) is strengthened by $\lambda^{3/2}$, or $M_f \lesssim 8 \times 10^9$ GeV. Note, however, that the expectation value of K can be made different from Φ by varying m_K from m_Φ. Hence the bound given in Eq. (20) is the only one that is parameter-independent.

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‖If a coupling between Φ, K sector and Φ', K' sector is present, such as $(\Phi \times \Phi') \cdot K$, the minima are shifted due to mixing between Φ and Φ'. Such a mixing makes both components of Φ and Φ' non-vanishing, and does not lead to any problem.

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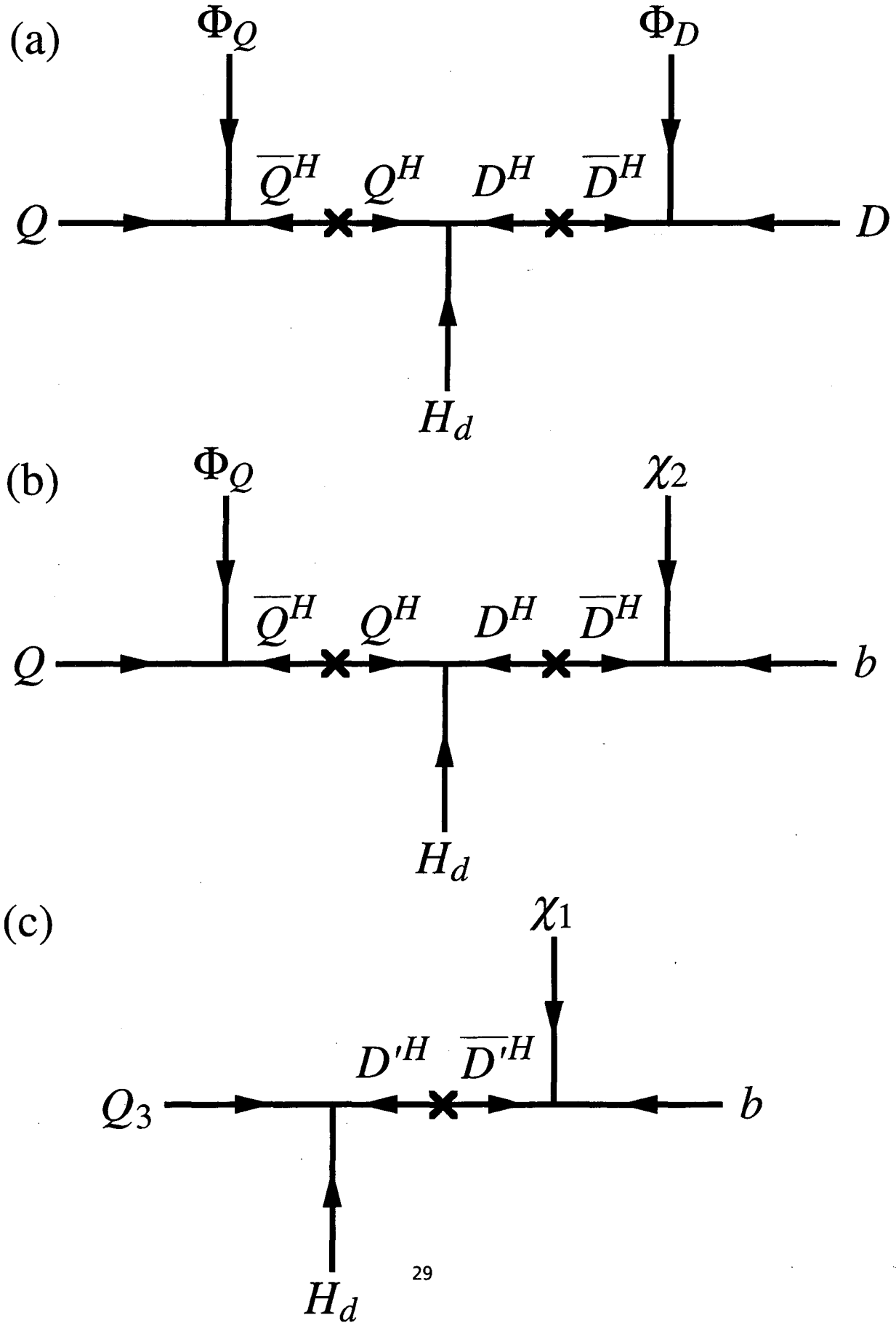
Figure Captions

Fig. 1 Diagrammatic representation of the operators generated by heavy particle exchange. The operators shown contribute to the up and down quark Yukawa matrices when the flavons acquire vacuum expectation values.


(a)
Φ_Q
Φ_D
$\overline{Q}^H$
Q^H
D^H
$\overline{D}^H$
Q
D
H_d
(b)
Φ_Q
χ_2
$\overline{Q}^H$
Q^H
D^H
$\overline{D}^H$
Q
b
H_d
(c)
χ_1
D'^H
$\overline{D'}^H$
Q_3
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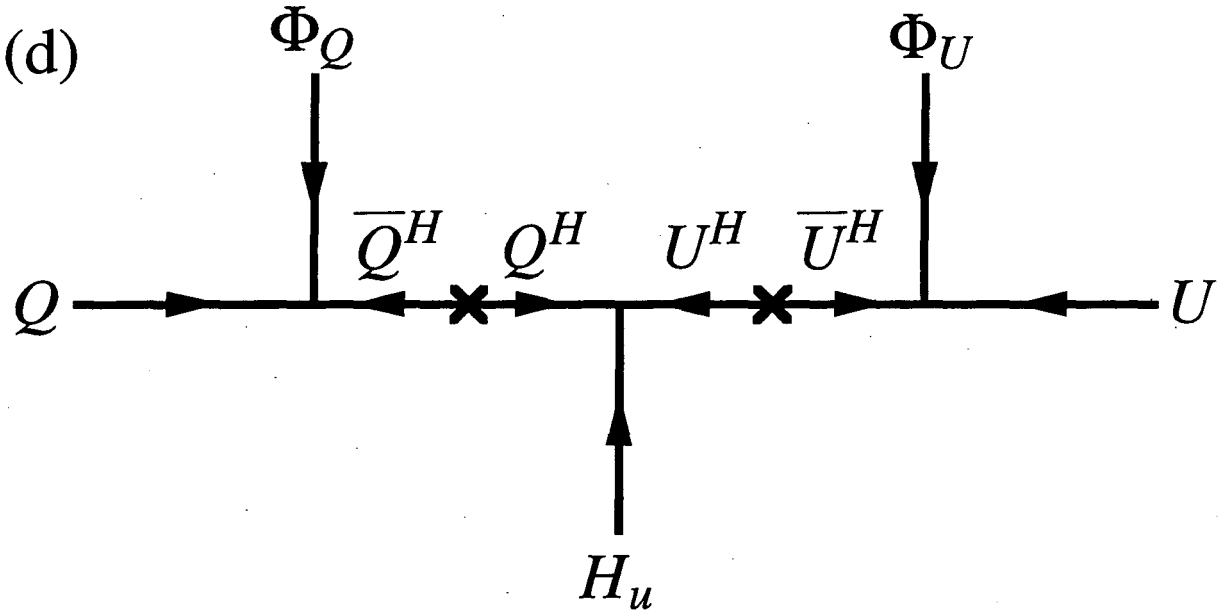
H_d

(d)



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