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Rethinking Transparency: Constructing Meaning in a Physical and Digital Design for Algebra

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ABSTRACT
In the course of developing an experimental algebra unit, the researchers noted variability in their design's instructional potential across a set of implementation media. In an effort to explain this variability, we revisited the classical theoretical construct of transparency. Transparency is the perceptual and conceptual accessibility of the mechanism, logic, and application of a tool. Corroborating earlier literature, it appears that participants saw only what they had built—transparency is a subjective achievement of a learner rather than an inherent feature of a device. Our first design prompted students with an algebraic proposition, for example "3*x*+2=4*x*-1". The two equivalent expressions were to be interpreted by students as alternative quantifications of a single linear spatial interval; namely, the path that a giant took on two separate occasions to bury and recover treasure. Problem solving required manually adjusting the modeling media to coordinate two types of equivalence: (a) the total length represented by each expression; and (b) the length represented by the variable and known units. The researchers found that successful coordination was predicated on the subjective transparency of the models' perceptual. Therefore, in redesigning the activity as a computer-based application we will have learners first construct tools and only then automatize them.

Categories and Subject Descriptors
D.2.10 [Design]: Representations and Methodologies

General Terms
Design, Experimentation, Theory.

Keywords
Transparency, Modeling, Algebra, Representations, Mathematics, Learning.

1. INTRODUCTION
The structural and interactional properties of a specific tool affect the forms of manual engagement it enables. The same holds for learning tools, so that particular instantiations of a disciplinary idea afford different interactions and, ultimately, learning [16]. With respect to mathematical phenomena that we want students to mathematize, learning materials should not only elicit learners' intuitive understandings, but also enable them to calibrate these intuitions towards formal perspectives [5]. Our central concern for this study is whether students' calibration is conscious or tacit. In particular, the child's agency in building a representation has been shown to contribute to learning [5]. In a similar vein, Papert has explained his pedagogical philosophy thus: "constructionism boils down to demanding that everything be understood by being constructed" [12, p. 2].

We are interested in the relation between the types of problem-modeling media we offer students and their success in overcoming the content's learning issues. Here we argue that variations in modeling media that affect its transparency will engender differences in learning. Lastly, we infer implications for crossing the "low-tech" to "high tech" divide: working with technological media, students could first struggle to build transparent tools in accord with the design's conceptual objectives, and then transition to subtly improved versions of these tools. Moving from manual to automatic tools, students could avail of automatization without forsaking understanding.

1.1 Background and Objectives
The initial impetus for this design was a conjecture associating students' poor understanding of algebra content with the pervasive metaphor underlying their conceptualization of algebraic equations [17]. For example, in arithmetic the = sign is most often conceptualized operationally: given "2 + 3 =" the student is expected to use a procedure so as to generate a solution [1, 13]. This implicit conceptualization of the equal sign is absent of a relational sense [9], by which two sides of an equation are commensurate [8]. Consequentially, the arithmetic-to-algebra transition commonly presents many learners with cognitive challenges [7, 14].

More generally, we conjectured that the quality of students' progression from arithmetic to algebra is related to the particular conceptualizations of algebraic propositions used by teachers and textbooks to present algebraic equality. Algebraic propositions such as "$3x + 14 = 5x + 6$" are traditionally conceptualized as two equivalent quantities balanced across a scale (see Figure 1, next page). The "balance" metaphor offers a mechanistic logic for the algebraic algorithm. In particular, the balance of the scale is maintained by applying identical arithmetic operations to the expressions on each side of the scale, (e.g., "-6"). However, although the balance metaphor powerfully explicates the algebraic *procedure*, we were concerned that it may not optimally introduce the a priori algebraic *rationale*, that is, the fundamental idea that two superficially different expressions are nevertheless equivalent. We thus sought an alternative conceptualization of algebraic equivalence. As it turned out, such an alternative already existed.

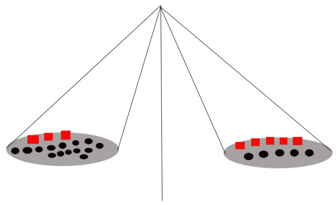

Figure 1. “3x + 14 = 5x + 6” on a balance scale

Dickinson and Eade used the number-line to diagram the equivalence of algebraic expressions. Figure 2 illustrates the number-line instantiation of “3x + 14 = 5x + 6” [3]. To the trained eye, this diagram “discloses” that 2x + 6 = 14 and, more specifically, that 2x = 8, so that x = 4. This form of algebraic reasoning is different from moving symbols across the equal sign (e.g., [17]) and therefore bears different pedagogical affordances. In particular, the number-line metaphor of algebraic equivalence conceptualizes the two equivalent expressions as alternative denotations *of one and the same linear extension*. This aligns the mechanism implied by the representation with the mechanism that actually motivates the problem. We no longer seek to “maintain balance”, but rather to determine an indeterminate quantity.

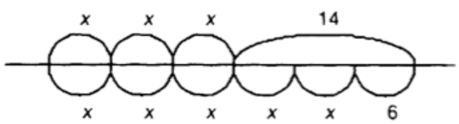

Figure 2. Number-line representation reproduced from [3]

The objective of our study was to further explore the pedagoical potential of the number-line algebra model. Our goal was to assign students greater agency in building their algebraic schemas, and our rationale was to draw on their intuitive spatial reasoning through engaging narratives that involve motion along the line.

2. DESIGN: GIANT STEPS FOR ALGEBRA

Our design draws on the familiar context of pirate stories and the related practice of treasure hunts. A problem narrative depicts a quasi-realistic situation in which a giant performs two consecutive journeys that begin and end at the same location yet differ in their respective progression sequences.

The journeys were modeled as occurring in a scaled down “desert”—a 12" by 12" sand tray—and the treasures were small glass marbles buried in the sand. Six short stories described the how these “treasures” were buried, for example:

> A giant has a treasure he wants to bury. The giant takes 3 giant steps forward. Then he goes 2 inches forward and buries his treasure. The next day he wants to bury more treasure but can’t remember exactly where to go. The giant goes 4 giant steps forward. Then he realizes that he has gone too far and goes back 1 inch, finds the right spot, and buries more treasure. Can you figure out where the giant buried his treasure?

Figure 3 depicts our diagrammatic interpretation of this narrative. Under three different conditions, the participants used one of the following tools to model their problem solving: (a) paper and pencil; (b) push-pins, a cork board and a ruler; and (c) a string of elastic bands knotted together (the elastic ruler) and pieces of wood measured and marked at different inch intervals (see Fig. 4). A total of 10 participants participated in the study, ranging in age from 8 – 10 years. Three participants worked with paper and pencil, 4 with the pushpins, and 3 with the elastic ruler.

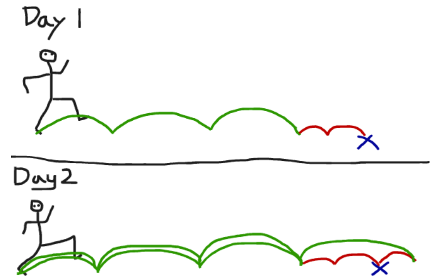

Figure 3. Diagram of the “3x + 2 = 4x – 1” story

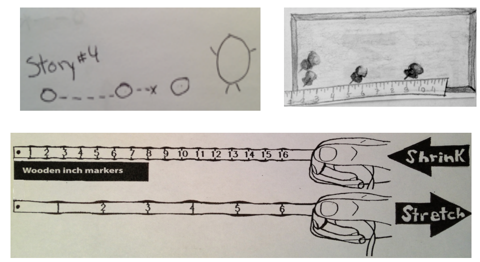

Figure 4. Three sets of instruments and media used for modeling. (1) paper and pencil; (2) pushpins, corkboard, and ruler; and (3) elastic ruler and inch strips.

All sessions were videotaped and later transcribed for the research team’s collaborative micro-ethnographic analysis.

3. TRANSPARENCY

When we say that a tool is *transparent*, we mean that the user can access, perceive, and understand its mechanism, logic, and application. As noted earlier, in the case of pedagogical tools, transparency is important for learning. For example, Meira considers the “intrinsic qualities of material displays and…how those qualities might promote individual cognitive efficiency by enabling users to see underlying principles and relations through them” [11, pp. 123-124]. From a distributed-cognition perspective, the pedagogical utility of learning media is that they re-organize and externalize aspects of cognitive process. Consequently, “physical actions [upon media] enable people to query the environment to test their ideas” [10, p. 589], which may result in new insights and hence learning. In a comparative study of two designs for modeling probabilistic phenomena, Garuti, Dapueto, and Boero found that learning tools that exposed covert mechanisms enabled students to create more sophisticated schematizations [4]. On the other hand “blackbox” tools—that is, tools that are not transparent—impede learning. Our study created opportunities to further discuss the detrimental cognitive consequences of opaque tools and the significance of user-constructed transparency.

4. RESULTS

Several annotated transcriptions were selected to depict an apparent relation between transparency and learning, illustrate the subjectivity of transparency, and emphasize students’ agency in constructing transparency. To facilitate juxtaposition, we selected events that all revolve around students’ actions related to the same conceptually critical coordination. Namely, our analyses focused on students’ attempts to spatially align the two journeys such that the relationship between the variable (giant steps) and integer (inches) could be identified and deciphered. Analyses focus on the three experimental conditions (see above) and the extent to which the tools obfuscated or illuminated this central coordination.

4.1 Building Transparency for Algebraic Equivalence via Coordinating Journeys

In this excerpt the participant is modeling with paper.

Res.: What happens in the second trip? Let's start there…He [Giant] gets to two steps [Mary is drawing the Day 2 trip over her Day 1 drawing, respecting common milestones.]

Mary: [simultaneously "stepping" her left-hand index (for Day 1) and her right-hand index (for Day 2) toward each other from two adjacent giant-step marks—each hand enacts the inch information in its respective Day 1 or Day 2 journey, and Mary is careful to apportion the "inch" steps across the single giant step so that the fingers meet at the same location]. It has to be there.

Res.: [mimicking Mary's hand movements] Why are you doing like that in the middle with your fingers?

Mary: Cause maybe there could be a way that it could be 2 giant steps and 2 inches here [indicates to the left] and 3 giant steps and 3 inches back here [indicates to the right].

Res.: and then they meet.

Mary: Yeah, and that's the right spot.

A joint review of the solution leads Mary to confirm its validity. Thus Mary created a model that enabled her to infer a solution by superimposing the two separate journeys. Mary could thus visualize both journeys simultaneously and draw inferences with respect to their quantitative relations.

Mary had to construct her solution from scratch, because she was given modeling media—paper and pencil—that bore no latent information relevant to the problem. She constructed all aspects of the story drawings herself, and so all of the inscribed elements in her model were meaningful residues of her own reasoning and fully transparent to her.

4.2 Media Impeding the Construction of Transparency: Concrete Manipulatives

The following example is taken from a group using the push-pins as their problem-solving medium. The participants are attempting to align the two journeys by coordinating the start and end points. They are using a 'guess and check' method and have estimated that a giant step is equal to one inch. Here they grapple with the realization that Day 1 and Day 2 end points are not co-located.

[Bob and York model the Day 2 trip.]

Res.: So what do we have to do?

York: We have to add.

Res.: Where do we have to add?

[York indicates a spot between the two end locations.]

We surmise that the concreteness of these modeling media "over-situated" them at the expense of bearing symbolic meanings—the model became a phenomenon onto itself, which therefore invited estimation operations that turned out to be counterproductive [15]. That is, the available medium caused "ontological slippage" between phenomenon and model: users attended to physical and topical aspects of the concrete modeling objects—the pins' actual situated locations and the measured intervals between them—at the expense of constructing a diagrammatic relational system whose numerical values are not pinned down to the measured lengths but instead are taken to stand in loosely for the real locations and may be determined via deduction and calculation.

4.3 Media Impeding the Construction of Transparency: Elastic Manipulatives

In this example we examine the process of using the variable marker (elastic bands) and fixed markers (wooden inch dowels). Kat has just modeled the Day 1 trip by placing the unstretched elastic ruler (steps) and fixed integer (inches) into the sand tray.

Kat: And then the next day he goes 2 steps forward—2 giant steps forward—which is here [touches ruler]. So…and then 2 inches forward. So that's like...But it's not exactly 3 giant steps…. [Kate sees that the Day 1 and Day 2 wood pieces do not end in the same spot and infers that her giant step sizes cannot be correct]

Res.: So it looks like he doesn't end up in exactly the same spot. [Kat nods to concur.] So what can we do to make it so that it is the same spot?

Kat: Add one. [She grabs a new inch marker and places it in the gap.]

Thus Kat attempts to resolve the contradiction by modifying elements of the "given" narrative. In this scenario, qualities of the modeling tools and their entailments are never constructed by the participant, and thus remain opaque, as follows.

First, there is only one elastic ruler, and so the participant must perforce model the two journeys using a single object. Recall that in the previous examples *the student had to build* the coordination of the two journeys either diagrammatically (on paper) or concretely (sets of pushpins). The absence of an opportunity to build this coordination in the case of the elastic band "robbed" the child of a conceptually critical coordination, and consequently the invariance of the variable units *across* journeys remained opaque.

Second, the designers anticipated that the ruler's elasticity would help the child determine the inch value of a giant step, because the child could easily adjust the step sizes uniformly. Yet this usage did not occur. By building an efficient affordance into the instrument, we propose, this property was opaque to the learner.

Lastly, we note that a phenomenon and its mathematical model, which are both epistemologically and ontologically distinct, can be laid out either one next to the other or one on top of the other. In the paper-and-pencil and pushpin conditions, the phenomenon (sand tray) and model (paper, corkboard) were spatially separate, yet in the elastic-band condition they were superimposed. In a related note, in the paper-and-pencil condition participants created diagrams that were not drawn to scale, but in the elastic-band condition they expected the model to index the actual physical location of the treasure.

4.4 Summary: Manual Beats Automatic

By varying the modeling tools across participants, we were able to demonstrate a relation between transparency and problem solving. The paper-and-pencil appeared to license imprecise yet effective spatial alignments, as though the model was never taken to be the phenomenon itself. The push-pins appeared to constrain the modeling process, as though these concrete manipulatives were ontologically too near to the phenomenon. The elastic ruler automatically scaled all the variable units uniformly, so that the students did not need to attend to making variable units equivalent *within* each expression (journey) and, moreover, the ruler rendered superfluous any need to coordinate the length of the two expression-journeys, and the student did not need to ensure that variable units were equivalent *between* the two expressions (journeys). Unsurprisingly, therefore, where the tools did work behind the scenes, their work remained unseen, un-understood.

5. DISCUSSION

It has been argued that transparency is a critical attribute of effective pedagogical devices. Notwithstanding, an objectively transparent aspect of a device—a feature or behavior that is ostensibly salient for all to see—may nevertheless remain covert to learners. What the device constructs mechanically, the learner may never construct psychologically. Our analyses of learners' actions and utterances suggested a positive relation between student agency in linking up a design's mechanical elements and their insight into the mathematical significance of these alignments. It is only through agency, struggle, and resolution that conceptually critical elements of pedagogical devices *become* transparent. Sometimes, transparency needs to be earned [13, 6].

6. COMPUTER-BASED RE-DESIGN

As we consider improvements on the Giant Steps design for learning, we have begun to develop a technology-based interface analogous to the concrete instantiations. Design considerations include previous research indicating that "computer manipulatives can help students build on their physical experiences, tying them tightly to symbolic representations" [2, p. 148].

Digital media afford programmable display elements that could include stretch/shrink equipartitioned rulers. However, our findings suggest that automatized transformation might remain opaque to the learner. Figure 5 presents a screenshot of a solution to the problem "$3x = 4x - 2$." The Giant's Day 1 journey is represented by the faded red, whereas Day 2 journey is superimposed upon Day 1 in bold red and green). Note a certain imprecision in the uniform sizes of the variable and, respectively, the known units, both within and between days. However, these consistencies were *intended*. As such, even though the interface is a digital arena with computational affordances, in this mode the actions can be compatible with the paper-and-pencil medium.

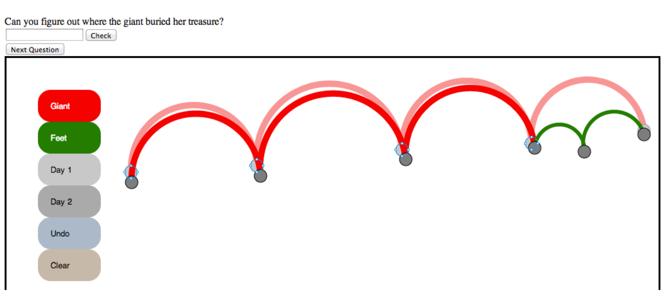

Figure 5. Computer-based solution to "$4x = 3x + 2$."

As we continue to develop the computer-based version of Giant Steps of Algebra, we will carefully determine criteria for assessing whether the learner is ready to shift gears from manual to automatic with respect to each programmable element. For example, once the learner has demonstrated proficiency in building journeys with equal steps, the interaction mode will change so as to enable scaling with uniform units.

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