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Essays on Search and Matching Theory

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Zheng Wei

To my parents, Guangqiang Wei and Ying Wang,
for their unconditional love and support

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EDUCATION

2021 (expected)	Ph.D. in Economics, University of California, Santa Barbara
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WORKING PAPERS

Sorting, Search Frictions, and Shocks
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FELLOWSHIPS

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Abstract

Essays on Search and Matching Theory

by

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The first chapter studies the role of search frictions and preference shocks and how they lead to sorting heterogeneous agents in the labor market. I develop a stochastic sorting framework where the equilibrium exhibits sorting across types. This feature helps to understand empirical labor market trends, such as mismatch and wage inequality. In particular, I find that differences in productivity, entry cost, market noise, and unemployment benefits have contributed to changes in observed sorting patterns and income inequality in the U.S. labor market.

In the second chapter, I examine the role of various skill levels in a frictional labor market with heterogeneous workers and firms. The economy consists of workers with different skill levels and firms with diverse skill requirements for job vacancies. By assumption, workers qualify for jobs with lower skill requirements, though productivity is lower due to mismatch. In the model, workers may or may not choose to match firms with lower job skill requirements. This has implications on equilibrium unemployment, wage inequality, and the optimal mix of job openings.

The third chapter studies the decision-making behavior of agents who are simultaneously searching in dual markets, namely the labor market and the marriage market. When considering the marriage decision in the single agent labor search model, I find that individual searchers have similar reservation wage behavior that affects wage dynamics and marriage outcomes under risk-neutral preferences.

Contents

Chapter 1

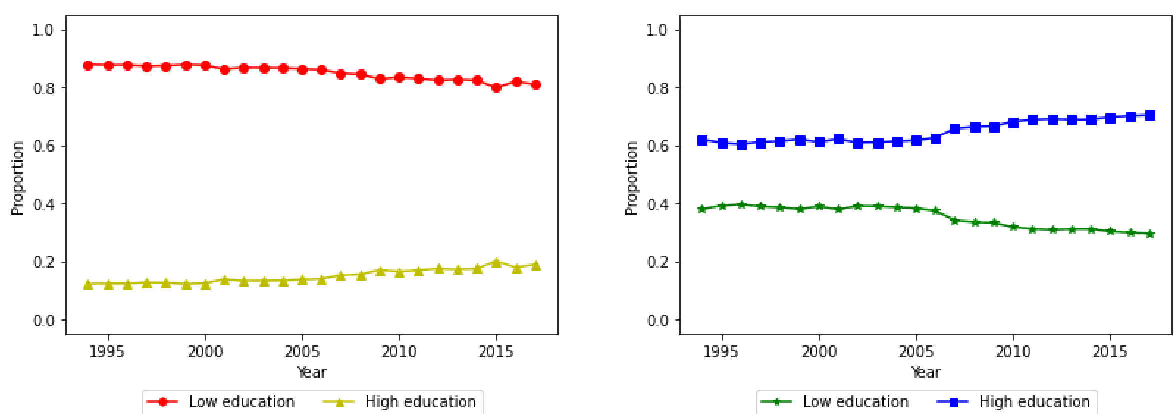
Sorting, Search Frictions, and Shocks

1.1 Introduction

This paper examines an assignment problem in an environment with search frictions and preference shocks. I study a large labor market where heterogeneous workers and firms trade pairwise. Match formation takes time, and agents weigh match surplus against trade probability when finding partners. Compared to the deterministic sorting literature (Becker (1973), Shimer and Smith (2000), and Eeckhout and Kircher (2010)). where the main focus is supermodularity conditions that lead to positive assortative matching (PAM) or submodularity conditions that induce negative assortative matching (NAM), I consider preference shocks that allows a mix of PAM and NAM.

I develop a stochastic sorting framework in this paper to characterize cross-type assortative matching as an equilibrium feature in the labor market, identify the driving forces of sorting patterns, and study the channels through which the underlying factors affect mismatch and wage inequality. I calibrate my model to the U.S. economy and find that a wider gap in productivity between jobs, changes in entry cost for workers, increases in unobserved heterogeneity, and higher unemployment benefits lead to recent labor market changes in sorting patterns and wage inequality.

Empirically, we often observe both positive and negative sorting. Using data from the Current Population Survey (CPS) and the Occupational Information Network (O*NET), I show in Figure 1.1 the sorting patterns between workers' education attainment and jobs' skill requirements in the U.S. labor market for full-time, full-year workers in recent decades.[1] High education indicates a person has obtained a bachelor's degree or above and low education otherwise. Simple jobs are occupations in O*NET job zone 1 to 3 such as baristas or actors, requiring medium preparation or less. These occupations usually need less education, training, and experience compared to complex jobs. Complex jobs correspond to O*NET job zone 4 to 5 such as accountants or chief executives, demanding considerable preparation or more. A majority of those occupations requires a four-year bachelor's degree or above. The figure shows workers with and without a college degree employed in both simple and complex jobs. This provides empirical evidence for stochastic sorting in the labor market.[2]



(a) Employment shares of workers in simple jobs

(b) Employment shares of workers in complex jobs

Figure 1.1: Sorting patterns in the U.S. labor market from 1994 to 2017

[1] In Figure 1, I use education as a proxy for workers' skill and skill requirements as a proxy for firms' job type. For a detailed treatment of the data, see Appendix A.1.

[2] See Appendix A.2 for summary statistics of employment shares of workers by job types.

Understanding the sorting mechanism has important implications on skill mismatch and wage inequality in the labor market. Match surplus relies on who matches with whom. Thus studying how workers sort themselves into jobs gives better insights into why employees with identical skills work in different positions, as well as why workers in the same job get paid differently. Figure 1.2 shows the real log weekly labor income for full-time, full-year workers from 1994 to 2017.[3] Accompanied by developments in sorting patterns, wage inequality grows in the U.S. during this period. The most notable change comes from college graduates performing complex jobs, with a significant increase in earnings from the mid-1990s to early-2000s. Though less significant, other groups follow a similar time trend.

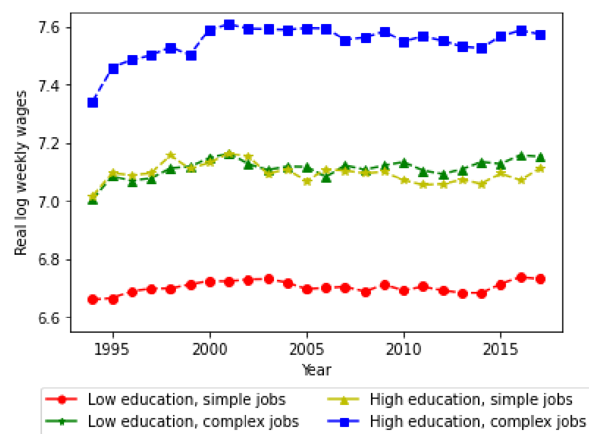


Figure 1.2: Real log weekly earnings for full-time, full-year workers from 1994 to 2017

The economy has a frictional market with two-sided heterogeneity. There are two types of workers (with low or high education) and firms (with simple or complex jobs) in a frictional labor market with full information on prices and types. Matching is pairwise: each firm offers one job, and each job seeker applies to one employer with an entry cost. Search is competitive. Firms decide the type of worker to match with

[3]See Appendix A.3 for summary statistics of real weekly wages for full-time, full-year workers from 1994 to 2017.

and post wages accordingly. After observing the distribution of firms and wage offers, workers simultaneously choose where to apply. Firms' decision on the worker type to attract is a discrete choice problem: there exists unobservable heterogeneity that affects employers' payoff through match-specific shocks, which leads to random utility. Therefore, firms' choice on workers depends on the difference between expected payoffs of specific matches.

I consider four driving forces of labor market sorting. The first channel is a productivity channel. An increase in productivity output yields a higher match surplus, widening firms' profit gap between matching different worker types. If this increase happens between workers and firms of the same kind, we see more positive sorting. Similarly, if such a change occurs between workers and firms of the opposite type, more negative sorting arises. Second, I explain a channel of entry cost and frictions. As the entry cost for one worker group (with low or high education) goes up, their share of match value increases. Consequently, firms' profit shrinks for matching the same kind of workers. Employers respond to this change by setting up relatively more vacancies targeting the other worker group. For example, if high-educated workers' entry cost increases, more firms choose to match with low-educated workers instead. The third channel is a market noise channel. If agent types capture a lot of heterogeneity between workers and firms, the model converges to deterministic sorting with either PAM or NAM. At the other extreme, if there are sufficiently large heterogeneity unaccounted for by differences in agent types, we observe random sorting. Finally, the fourth channel is through unemployment benefits. Changes in unemployment benefits have an ambiguous effect on sorting patterns, as an increase in benefits lowers the match value but increases the trading probability for all pairs.

This paper contributes to the search and matching literature on sorting with transferable utility. I adopt the basic structure of Eeckhout and Kircher (2010) with decentralized price competition. There are two distinctions: my paper considers discrete agent types and introduces preference shocks. Discretizing type space allows me to connect my model with data better and study empirical trends of mismatch and wage dispersion. Introducing shocks make the equilibrium display a mix of PAM and NAM, a novel property that makes sorting stochastic. In particular, Eeckhout and Kircher (2010) show root-supermodularity of the match value function is sufficient to attain PAM, a condition stronger than supermodularity in Becker (1973) for the frictionless benchmark but weaker than log-supermodularity in Shimer and Smith (2000) for markets with random search.[4] Other influential work on sorting includes Shi (2001), who studies frictional assignment with directed search and finds efficient sorting maybe negative sometimes under certain conditions. In other words, PAM requires a stronger supermodularity condition. Recently, Chade and Eeckhout (2016) study CEO performance in a matching model with stochastic types where agents match based on *ex ante* observable types, but relevant payoff attributes are revealed *ex post*, and they conclude the role of random components are large in affecting sorting outcomes. Despite having stochastic parameters in the model, their conditions for deterministic sorting are more general than previous papers mentioned.

This paper extends the literature on skill mismatch and wage inequality in frictional labor markets with heterogeneous agents. Albrecht and Vroman (2002) study a model with endogenous skill requirements and show skill mismatch is an equilibrium phenomenon under what the authors refer to as a cross-skill matching equilibrium. Dolado et al. (2009) allow on-the-job search in Albrecht and Vroman (2002)'s model

[4] A function $f : \mathbb{R}^n \to \mathbb{R}$ is *supermodular* if $f(x \uparrow y) + f(x \downarrow y) \geq f(x) + f(y)$ for all x and y in $\mathbb{R}^n$, where $f(x \uparrow y) = max\{x, y\}$ and $f(x \downarrow y) = min\{x, y\}$. If f is twice continuously differentiable, this is equivalent to $\partial^2 f(x,y)/\partial x \partial y \geq 0$ for all $x \neq y$. A function f is *submodular* if $-f$ is *supermodular*. *Log-supermodularity* is equivalent to $\partial^2 log f(x,y)/\partial x \partial y \geq 0$ for all $x \neq y$ if f is twice continuously differentiable. *Root-supermodularity* is equivalent to $\partial^2 \sqrt[n]{f(x,y)}/\partial x \partial y \geq 0$ for all $x \neq y$ if f is twice continuously differentiable.

and concludes that temporary mismatch can be optimal for highly-educated workers. In my model, skill mismatch arises naturally due to price competition. The tradeoff between wages and the probability of finding a job makes workers employed in both job types. Skill mismatch is a topic that also links to other relevant issues. Shimer (2007) develops a dynamic mismatch model that has distinctive business cycle properties. Şahin et al. (2014) find mismatched workers can explain up to one-third of the increase in recent U.S. unemployment.

This paper is related to the changing role of firms in wage inequality in the labor market. Song et al. (2019) analyzed the Social Security Administration (SSA) earnings data and concluded that a majority of the observed wage dispersion comes from between firm instead of within firm variations. Furthermore, the between firm variation is tied to changes in the composition of workers. In Card et al. (2018), high-wage workers have a higher elasticity of labor supply or occupational mobility, hence they are more likely to match with firms offering high-pay jobs. Abowd et al. (2018) have a similar finding that workers in high-pay firms see faster earnings growth, which leads to increased sorting of high-wage workers into such firms. These conclusions links between firm earnings inequality with composition changes in workers: sorting improves if individual firms have a high concentration of high-pay jobs, deepening the gap for income inequality.

The rest of this paper is organized as follows. In section 1.2, I introduce the model environment. Section 1.3 derives and defines the equilibrium. Section 1.4 discusses sorting. Section 1.5 provides a calibrated example. Section 1.6 demonstrates the model's numerical feature. Section 1.7 compares changes in parameter values over different time periods. Finally, Section 1.8 concludes.

1.2 A Heterogeneous Agent Model of the Labor Market

1.2.1 Players and Preferences

Consider a static model in which a frictional labor market consists of N workers and M firms. Both sides are very large ($i.e.$ $M, N \to \infty$), but neither side is infinitely larger than the other side ($i.e.$ $0 < M/N < \infty$). Workers are different in skill attributes such as education attainment, while firms are heterogeneous in job openings such as wage offer. There are two types of workers and firms in this economy, low type and high type that are observable. Define the set of types as $\mathcal{T} = \{L, H\}$. To distinguish the difference in types, a worker's type is indexed by $x \in \mathcal{T}$ and a firm's type is indexed by $y \in \mathcal{T}$. The distribution of types across agents is exogenous. Specifically, a fraction ϕ of workers and a fraction ψ of firms are low type, whereas the remaining are high type. All agents are risk-neutral and maximize their expected utility.

Each worker has one unit of labor for sale and can apply to only one job. To participate in market activity, workers pay an entry cost κ_x based on their types, assuming $\kappa_L < \kappa_H$. If employed, a worker produces an output $f(x, y)$ and earns wages $w(x, y)$, where both f and w depend on types x and y. If unemployed, a worker receives benefits $b > 0$, which includes the value of leisure or home production. Likewise, each firm has one job opening that can be filled or vacant. If a job is filled, the firm consumes the output $f(x, y)$, pays its employee $w(x, y)$, and makes a profit of $f(x, y) - w(x, y)$. If a job remains vacant, the firm's payoff is 0. Production is assumed to be increasing in agent's type. That is, when paired with the same type of workers / firms, high type firms / workers yield greater output value than their low type counterparts:

Assumption 1 $f(x, L) < f(x, H)$ *and* $f(L, y) < f(H, y)$.

No worker is willing to accept wages lower than b and no firm wants to pay wages above $f(x,y)$. Therefore, conditional on x and y, the set of feasible wages is $\mathcal{W} = [b, f(x,y)]$.

Firms have idiosyncratic tastes on workers. Beyond systematic preference on skill attributes indicated by workers' type, employers have different appraisals on other factors such as gender, race, appearance, etc. Hence the utility derived from hiring type x workers is not the same for all type y firms. To capture these unobservable differences in firms' individual taste that is unaccounted for by workers' type, firms' utility includes a stochastic component based on the specification of random utility/discrete choice models.[5] The preference shocks, z_{xy}, are assumed to be drawn independently and identically from normalized type I extreme value ($a.k.a.$ standard Gumbel) distribution with cumulative distribution function $F(z_{xy}) = e^{-e^{-z_{xy}}}$ and variance of $\pi^2/6$. Furthermore, there is no correlation between shocks and other variables in the model.

1.2.2 Labor Market Structure

The labor market is frictional with competitive search. Since a worker can only apply to one job opening regardless of how many employers may have posted, firms are not guaranteed to match with workers of their choices. The degree of frictions relies on the competition for workers among firms. Firms offering higher wages are more likely to hire workers at the cost of lower profits, and vice versa. Consider the ratio of vacancy to unemployment ($i.e.$ firms to workers), $\lambda \in [0, \infty)$, which is also called the queue length or market tightness in *directed search models*. One can expect this ratio corresponds to variations in agent types and wage offers. Given any queue length λ, a worker finds a job with probability $m(\lambda)$. To reflect the idea that more firms make it easier for a worker to land a job, $m(\lambda)$ is strictly increasing and concave by

[5] For random utility/discrete choice models, see McFadden (1973, 2001), and Train (2009).

assumption: $m'(\lambda) > 0, m''(\lambda) < 0$. Similarly, a firm hires a worker with probability $m(\lambda)/\lambda$ when trading pairwise. Given the restriction $m(0) = 0$, a strictly increasing and concave $m(\lambda)$ implies $m(\lambda)/\lambda$ is a strictly decreasing function because more firms decrease the chance of each firm in queue to fill its vacancy. In a one-shot model, both $m(\lambda), m(\lambda)/\lambda$ are probabilities between 0 and 1.

The labor market is also decentralized. A submarket consists of a set of firms posting the same wage w and workers searching, with queue length λ. In general, we do not see workers or firms of both types in the same submarket, while it is possible for agents of the same kind to participate in different submarkets. In this economy with two-sided heterogeneity, we have a total of four submarkets $(x, y) \in \mathcal{T}^2 = \{(L, L), (L, H), (H, L), (H, H)\}$ that can be characterized by their wages $w(x, y)$ and queue lengths $\lambda(x, y)$. Every worker or firm chooses one submarket to participate according to its type.

1.2.3 The Extensive Form Game

Trade takes place in a two-stage extensive form game. In stage 1, all firms make job opening decisions on x and post wages $w(x, y)$ at the same time. By choosing $x \in \mathcal{T}$, firms indicate the type of worker they want to meet. In stage 2, workers simultaneously decide where to apply after observing all employer-wage combination $(y, w(x, y))$. Workers can apply to any job opening that matches their type, or take benefits b if they find no wages attractive.[6]

Firms form beliefs about the queue length when they make wage offers. To maximize their expected payoffs, firms only post wages at which they expect positive queue

[6] I discuss the case where workers post and firms search in Appendix B.1. Because of the duality of this problem, it does not matter who posts and who searches. One can verify that solving the workers' problem have the same solution since the number of meetings do not depend on who searches.

length. Wright et al. (2019) show that in a submarket with homogenous workers and firms, all firms make identical wage offers and any attempt to deviate is not profitable. Therefore, all firms choose the same wage as well as the queue length within each submarket.

1.2.4 The Bellman Equations

Let $W(x,y)$ denote a worker's utility and $U(x,y)$ denote a firm's expected payoff in the corresponding submarket. A firm chooses $\{x, w(x,y), \lambda(x,y)\}$ to maximize $U(x,y)$, subject to offering its employee $W(x,y)$. The firm's problem is:

$$U(x,y) = \max_{x \in \{L,H\}} \big\{ V(L,y) + \mu z_{Ly}, V(H,y) + \mu z_{Hy} \big\}, \tag{1.1}$$

where $U(x,y)$ is decomposed into two parts $V(x,y)$ and μz_{xy}.

The first part labeled $V(x,y)$ is known to a firm, defined as the product of trade probability and match surplus:

$$\begin{aligned} V(x,y) = \max_{w(x,y),\lambda(x,y)} & \left\{ \frac{m(\lambda(x,y))}{\lambda(x,y)} \big[f(x,y) - w(x,y) \big] \right\} \\ s.t. \quad & W(x,y) = m(\lambda(x,y)) \big[w(x,y) - b \big]. \end{aligned} \tag{1.2}$$

Given the choice of x, a firm chooses wages $w(x,y)$ and queue length $\lambda(x,y)$ to maximize its profit. Note that x is a choice of a discrete random variable, while $w(x,y)$ and $\lambda(x,y)$ are a choice of continuous random variables.

The second part μz_{xy} is stochastic. A shock multiplier $\mu > 0$ determines the weight of firms' idiosyncratic taste on their utility to compare different scales between shocks and search frictions[7] As μ increases, firms' individual preference plays a more signifi-

[7] See Anderson and De Palma (1992).

cant role in affecting utility, compared to systematic preference on workers' type. It is convenient to think μ as the price of random shocks relative to firms' profits.

1.3 Equilibrium

In this model, the equilibrium consists of a set of optimal queue lengths $\lambda(x, y)$, wage offers $w(x, y)$, utility for workers $W(x, y)$, and payoffs for firms $U(x, y)$. To solve for equilibrium, I follow the *market utility approach*: firms take the worker's utility $W(x, y)$ as given, and this value will be determined later in the equilibrium.[8] In the spirit of backward induction, I first solve $w(x, y)$ and $\lambda(x, y)$ and then discuss the firm's choice of x. To begin, rewrite the firm's problem by substituting wages $w(x, y)$:

$$V(x,y) = \max_{\lambda(x,y)} \left\{ \frac{m(\lambda(x,y))\big[f(x,y) - b\big] - W(x,y)}{\lambda(x,y)} \right\}. \tag{1.3}$$

For firms trading at an interior queue length, first-order condition with respect to $\lambda(x, y)$ yields the following expression for workers:

$$W(x,y) = \big[m(\lambda(x,y)) - \lambda(x,y)m'(\lambda(x,y))\big]\big[f(x,y) - b\big]. \tag{1.4}$$

In equilibrium, workers of the same type x must be indifferent between employment by any type y firms ($i.e.$ $W(x, L) = W(x, H)$). Free entry of workers pins down the optimal queue length $\lambda^*(x, y)$, and the number of workers and firms sums up to their respective fractions of types:

[8] See papers by Peters (1991, 2000) and Moen (1997).

$$\kappa_x = \big[m(\lambda^*(x,L)) - \lambda^*(x,L)m'(\lambda^*(x,L))\big]\big[f(x,L) - b\big] \tag{1.5}$$

$$= \big[m(\lambda^*(x,H)) - \lambda^*(x,H)m'(\lambda^*(x,H))\big]\big[f(x,H) - b\big]. \tag{1.6}$$

Define $\varepsilon(\lambda(x,y)) \equiv \lambda(x,y)m'(\lambda(x,y))/m(\lambda(x,y))$ as the elasticity of $m(\lambda(x,y))$ with respect to queue length $\lambda(x,y)$. Given equation (5) and (6), one can obtain unique solutions for $\{w(x,y), W(x,y), V(x,y)\}$:

$$w^*(x,y) = \varepsilon(\lambda^*(x,y))b + \big[1 - \varepsilon(\lambda^*(x,y))\big]f(x,y), \tag{1.7}$$

$$W^*(x,y) = \big[m(\lambda^*(x,y)) - \lambda^*(x,y)m'(\lambda^*(x,y))\big]\big[f(x,y) - b\big], \tag{1.8}$$

$$V^*(x,y) = m'(\lambda^*(x,y))\big[f(x,y) - b\big]. \tag{1.9}$$

Proposition 1 *Holding a worker's type x constant, low type firms have longer queue, pays a lower wage, and receives less payoff compared to high type firms in equilibrium:*

$$\lambda^*(x,L) > \lambda^*(x,H), w^*(x,L) < w^*(x,H), V^*(x,L) < V^*(x,H).$$

Proof: See Appendix B.2. ■

Regardless of her type, a worker matches with a low type employer faster but paid lower wages as a result of impatience.

1.4 Sorting

Firms' choice of workers depends on the price of shocks μ and the value of shocks z_{xy}. This is where sorting arises in the model with probabilities. Define $P(x,y) \equiv$

$Pr(U(x,y) \geq U(x',y))$ as the probability of a type y firm choose to match with a type x worker. The difference between two i.i.d. random variables of type I extreme value distribution follows a logistic distribution. Thus equation (1) implies the following:

$$\begin{aligned} P(x,y) &= Pr\big[U(x,y) \geq U(x',y)\big] && (1.10)\\ &= Pr\big[V^*(x,y) + \mu z_{xy} \geq V^*(x',y) + \mu z_{x'y}\big] && (1.11)\\ &= Pr\left[z_{x'y} - z_{xy} \leq \frac{V^*(x,y)}{\mu} - \frac{V^*(x',y)}{\mu}\right] && (1.12)\\ &= \frac{e^{\frac{V^*(x,y)}{\mu}}}{e^{\frac{V^*(x,y)}{\mu}} + e^{\frac{V^*(x',y)}{\mu}}} && (1.13)\\ &= \frac{1}{1 + e^{\frac{V^*(x',y) - V^*(x,y)}{\mu}}}, && (1.14) \end{aligned}$$

where $x' = L$ if $x = H$ and $x' = H$ if $x = L$.[9] PAM requires firms to match workers of the same type ($i.e.$ $y = x$), while the opposite is true for NAM ($i.e.$ $y = x'$). As the degree of heterogeneity in firms' idiosyncratic tastes approaches zero ($i.e.$ $\mu \to 0$), firms' choice of workers x is deterministic ($i.e.$ $P(x,y) = 0$ or $P(x,y) = 1$). On the other hand, if the degree of heterogeneity in firms' idiosyncratic tastes is large ($i.e.$ $\mu \to \infty$), then firms' choice of workers x is random ($i.e.$ $P(x,y) = 1/2$).

Proposition 2 *Holding all else constant, the probability of acquiring PAM or NAM increases with higher output levels on the corresponding diagonal and declines with higher output levels on the other diagonal. A higher entry cost for one worker group decrease sorting probabilities for the same group of workers but increase for the other worker group. Additionally, the relationship between such a probability and unemployment benefits is ambiguous:*

$$\frac{\partial P(x,y)}{\partial f(x,y)} > 0, \frac{\partial P(x,y)}{\partial f(x',y)} < 0, \frac{\partial P(x,y)}{\partial \kappa_x} < 0, \frac{\partial P(x,y)}{\partial \kappa_{x'}} > 0, \frac{\partial P(x,y)}{\partial b} \lesseqqgtr 0.$$

Proof: See Appendix B.4. ■

[9] See Appendix B.3 for derivation.

An increase in b reduces expected payoff values to firms of the same type y. Therefore the change in probability of PAM or NAM depends on which payoff ($V(x, y)$ and $V(x', y)$) decreases more.

1.5 Calibration

The model is calibrated to the U.S. economy from 1994 to 2017. I assume a telephone-line matching technology: $m(U, V) = UV/(U + V)$ with constant returns to scale. This implies the matching probabilities are $m(\lambda) = \lambda/(1 + \lambda)$ for unemployed workers and $m(\lambda)/\lambda = 1/(1 + \lambda)$ for unfilled vacancies, respectively.

The unemployment benefits are set to be 30% of the mean weekly income, $b = 0.428$.[10] Workers' entry costs are chosen to match 30% of the difference between mean wages based on the worker's type and the value of unemployment insurance. This yields $\kappa_L = 0.201$ and $\kappa_H = 0.498$. To construct a measure for sorting probabilities, I match worker's occupation using the 4-digit Census code in the CPS with the job zone using the Standard Occupational Classification (SOC) code in O*NET.[11] The employment shares of full-time, full-year workers in each job category are: $P(L, L) = 0.850$, $P(H, L) = 0.150$, $P(L, H) = 0.354$, and $P(H, H) = 0.646$.

I normalize all wages in the data by mean weekly labor income for workers in the CPS. The real average weekly earnings for workers with low education in simple jobs is 815.79 between 1994 to 2017 (base year), and $w(L, L)$ is normalized to 1. The wages of other corresponding groups are $w(L, H) = 1.504$, $w(H, L) = 1.479$, and $w(H, H) = 2.326$. Table 1.1 recapitulates the parameters used in the calibration.

[10] For alternative choices of the value for b, see Shimer (2005b) and Hagedorn and Manovskii (2008).

[11] For a detailed comparison of occupation classification between the Census code and the SOC code, see https://www.census.gov/programs-surveys/cps/technical-documentation/methodology/industry-and-occupation-classification.html

Parameter	Definition	Value
$f(L,L)$	Production for low-educated workers in simple jobs	2.056
$f(H,L)$	Production for high-educated workers in simple jobs	2.646
$f(L,H)$	Production for low-educated workers in complex jobs	6.188
$f(H,H)$	Production for high-educated workers in complex jobs	7.662
κ_L	Entry cost for low-educated workers	0.201
κ_H	Entry cost for high-educated workers	0.498
μ	Market noise	0.126
b	Unemployment benefits	0.428

Table 1.1: Parameter values

1.6 Comparative Statics

This section examines changes in several economic factors and highlights the channels through which these underlying determinants affect sorting patterns in the labor market. Figure 1.3 to Figure 1.6 summarizes the effects of changes in production, entry cost, market noise, and unemployment benefits. See Appendix A.4 for detailed comparative statics tables.

1.6.1 Production

Proposition 3 *An increase in $f(L,L)$: reduces $\lambda(L,L)$, raises $w(L,L)$, and induces more PAM.*

An increase in the output $f(L,L)$ raises the match value for low-educated workers in simple jobs. Holding the entry cost κ_L constant for workers with low education, $m(\lambda(L,L)) - \lambda(L,L)m'(\lambda(L,L))$ decreases to accommodate the increase in match surplus. This implies a decline in the ratio of simple job firms to low-educated workers, $\lambda(L,L)$, hence a higher $m'(\lambda(L,L))$. For firms with simple job vacancies, the expected payoff of matching with low-educated workers, $V^*(L,L)$, goes up since more low-educated workers are likely to apply. For workers with low education in simple jobs, the increase in match value, $f(L,L) - b$, is greater than the decrease in their share of wages, $1 - \epsilon(\lambda(L,L))$, resulting in higher wages $w(L,L)$. In equilibrium, simple job firms are more likely to match with workers with low education. Quantitatively, a 0.2 increase (approximately 10%) in $f(L,L)$ from the baseline value 2.056 changes sorting probability of $P(L,L)$ from 85% to 99.3%, while a 0.2 decrease in $f(L,L)$ brings down $P(L,L)$ to 19.9%. Given the magnitude of production at different levels, these numerical changes are rather significant. An increase in $f(L,L)$ ameliorates skill mismatch, narrows the gap for both between and within group wage inequality, and improves sorting in simple jobs. Table A.4 summarizes the effects of changing $f(L,L)$.

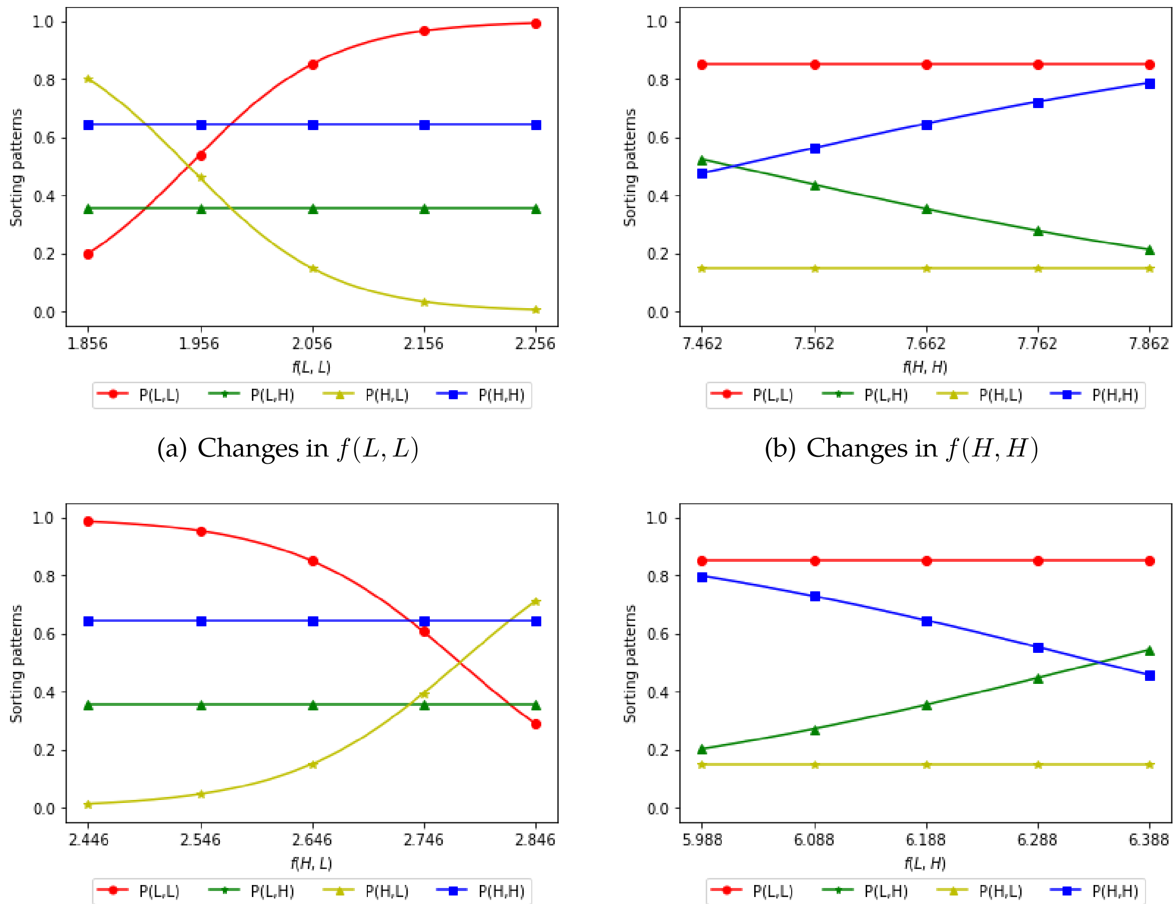
Proposition 4 *An increase in $f(H,H)$: reduces $\lambda(H,H)$, raises $w(H,H)$, and induces more PAM.*

In terms of sorting, changes in $f(H,H)$ have a similar effect to changes in $f(L,L)$. An rise in production of $f(H,H)$ increases the match value for workers with high education in complex jobs. $m(\lambda(H,H)) - \lambda(H,H)m'(\lambda(H,H))$ decreases to offset the increase in match surplus, holding the participation cost κ_H constant for high-educated workers. This suggests a decrease in market tightness $\lambda(H,H)$, thus a higher $m'(\lambda(H,H))$. More workers with high education are likely to direct their search for complex jobs. For firms with such job openings, the expected payoff for targeting high-educated workers $V^*(H,H)$ goes up. For high-educated workers in complex jobs, the decrease in their share of wages, $1 - \epsilon(\lambda(H,H))$, is less than the increase in match value, $f(H,H) - b$. Hence they earn higher wages $w(H,H)$. In equilibrium, firms with complex jobs are more likely to match with high-educated workers. Quantitatively, a 0.2 increase (approximately 2.6%) in $f(H,H)$ from the baseline value 7.762 adjusts the sorting probability of $P(H,H)$ from 64.6% to 78.6%, while a 0.2 decrease in $f(L,L)$ brings down $P(L,L)$ to 47.6%. An increase in $f(H,H)$ reduces skill mismatch, widens the gap for both between and within group wages dispersion, and improves sorting in complex jobs. Table A.5 sums up the effects of changing $f(H,H)$.

Proposition 5 *An increase in $f(H,L)$: reduces $\lambda(H,L)$, raises $w(H,L)$, and induces more NAM.*

On the other hand, changes in $f(H,L)$ have an opposite effect of $f(H,H)$. This is not surprising, given that $f(L,L)$, $f(H,H)$ and $f(H,L)$, $f(L,H)$ are on different diagonals for assortative matching. An increase in the output $f(H,L)$ raises the match value for high-educated workers in simple jobs. Holding the entry cost κ_H constant for workers with high education, $m(\lambda(H,L)) - \lambda(H,L)m'(\lambda(H,L))$ decreases to accommodate the increase in match surplus. This implies a fall in the ratio of firms with simple jobs to workers with high education, $\lambda(H,L)$, therefore a higher $m'(\lambda(H,L))$. The expected payoff for firms with simple job vacancies $V^*(H,L)$ increases since more high-educated workers are likely to apply for simple jobs. For workers with high education in simple jobs, the increase in match value, $f(H,L)-b$, overpowers the decrease in their share of wages, $1-\epsilon(\lambda(H,L))$, resulting in higher wages $w(H,L)$. In equilibrium, simple job firms are more likely to match with workers with high education. Quantitatively, a 0.2 increase (approximately 7.6%) in $f(H,L)$ from the baseline value 2.646 reduces the sorting probability of $P(L,L)$ from 85% to 29%, while a 0.2 decrease in $f(H,L)$ boosts $P(L,L)$ to 98.6%. An increase in $P(H,L)$. An increase in $f(H,L)$ exacerbates skill mismatch, expands within-group but reduces between-group wage inequality, and induces more negative sorting. Table A.6 summarizes the effects of changing $f(H,L)$.

Proposition 6 *An increase in $f(L,H)$: reduces $\lambda(L,H)$, raises $w(L,H)$, and induces more NAM.*



(a) Changes in $f(L, L)$

(b) Changes in $f(H, H)$

(c) Changes in $f(H, L)$

(d) Changes in $f(L, H)$

Figure 1.3: Effects of changing production $f(x, y)$

Likewise, changes in $f(L, H)$ are comparable with changes in $f(H, L)$. A rise in production of $f(L, H)$ increases the match value for workers with low education in simple jobs. $m(\lambda(L, H)) - \lambda(L, H)m'(\lambda(L, H))$ comes down to cancel out the increase in match surplus, holding the participation cost κ_L constant for low-educated workers. This suggests a shorter queue $\lambda(L, H)$, hence a higher $m'(\lambda(L, H))$. More workers with low education are likely to direct their search for complex jobs. For firms with such job openings, the expected payoff for targeting low-educated workers $V^*(L, H)$ increases. For low-educated workers in complex jobs, the decrease in their share of wages, $1 -$

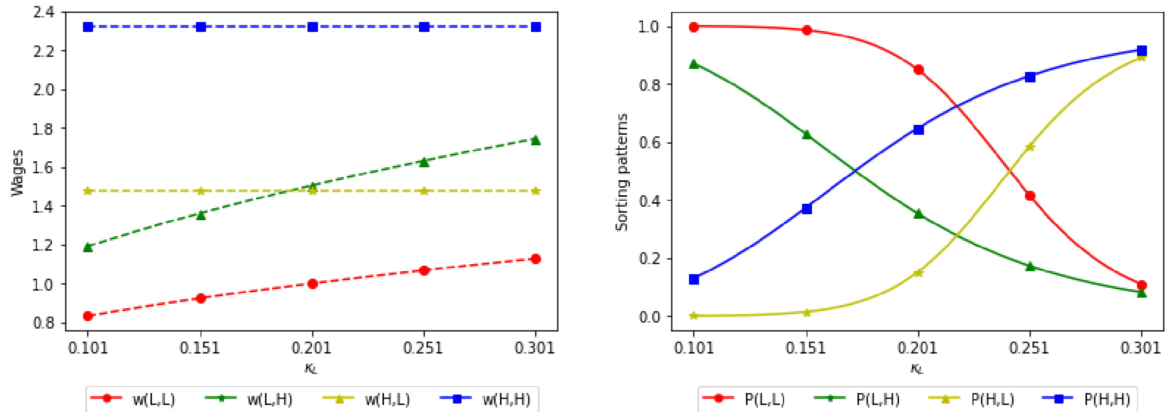
$\epsilon(\lambda(L,H))$, is less than the increase in match value, $f(L,H) - b$. Hence they earn higher wages $w(L,H)$. In equilibrium, firms with complex jobs are more likely to match with low-educated workers. Quantitatively, a 0.2 increase (approximately 3.2%) in $f(L,H)$ from the baseline value 6.188 alternates the sorting probability of $P(H,H)$ from 64.6% to 45.7%, while a 0.2 decrease in $f(L,H)$ amplifies $P(H,H)$ to 79.8%. An increase in $f(L,H)$ aggravates skill mismatch, enlarges within-group but shrinks between-group wage inequality, and induces more negative sorting. Table A.7 sums up the effects of changing $f(L,H)$.

1.6.2 Entry Cost and Frictions

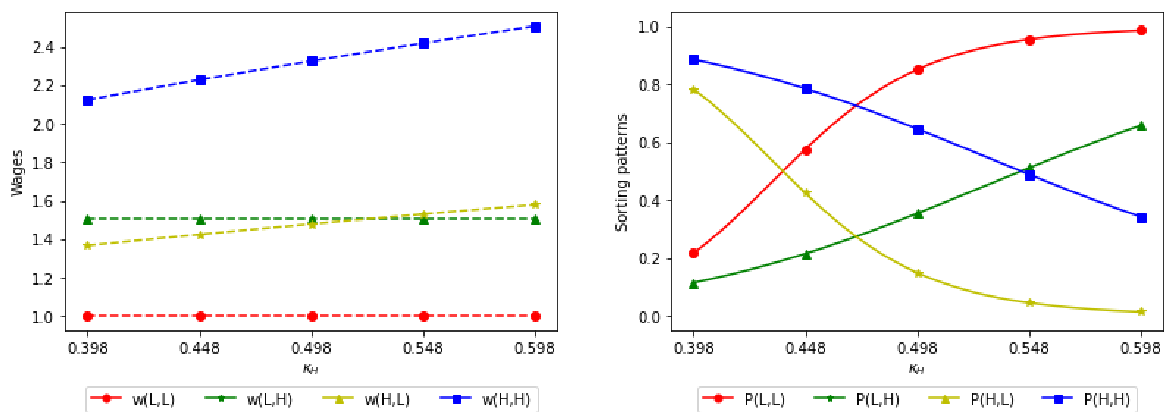
Proposition 7 *An increase in κ_L: raises $\lambda(L,L), \lambda(L,H), w(L,L), w(L,H), P(H,L), P(H,H)$ and reduces $P(L,L), P(L,H)$.*

Given the match surplus for workers with low education, an increase in the entry cost κ_L raises the value for $m(\lambda(L,L)) - \lambda(L,L)m'(\lambda(L,L))$ and $m(\lambda(L,H)) - \lambda(L,H)m'(\lambda(L,H))$. As a result, both $\lambda(L,L)$ and $\lambda(L,H)$ increase, followed by higher wages $w(L,L)$ and $w(L,H)$. A higher entry cost suggests low-educated worker's share of match value goes up, giving firms more incentives to match with workers with high education. Therefore, $P(H,L)$ and $P(H,H)$ increase while $P(L,L)$ and $P(L,H)$ decrease. Quantitatively, a 0.1 increase (approximately 50%) in κ_L changes from the baseline value 0.201 changes the sorting probability of $P(L,L)$ from 85% to 11% and $P(H,H)$ from 64.6% to 91.8%, while a 0.1 decrease in κ_L adjusts $P(L,L)$ from 85% to 99.9% and $P(H,H)$ from 64.6% to 13%. Table A.8 summarizes the effects of changing κ_L.

Proposition 8 *An increase in κ_H: raises $\lambda(H,L), \lambda(H,H), w(H,L), w(H,H), P(L,H), P(L,L)$ and reduces $P(H,L), P(H,H)$.*



(a) Changes in κ_L



(b) Changes in κ_H

Figure 1.4: Effects of changing entry cost κ_x

Given the production output for workers with high education, a higher the entry cost κ_H increases the value for $m(\lambda(H,L)) - \lambda(H,L)m'(\lambda(H,L))$ and $m(\lambda(H,H)) -$ $\lambda(H,H)$ $m'(\lambda(H,H))$. Consequently, both $\lambda(H,L)$ and $\lambda(H,H)$ increase, followed by higher earnings $w(H,L)$ and $w(H,H)$. A higher entry cost implies high-educated worker's share of match value goes up, giving firms more incentives to match with workers with low education. Therefore, $P(L,L)$ and $P(L,H)$ goes up while $P(H,L)$ and $P(H,H)$ goes down. Quantitatively, a 0.1 increase (approximately 20%) in κ_H changes from the baseline value 0.498 changes the sorting probability of $P(L, L)$ from 85% to 98.6% and $P(H, H)$ from 64.6% to 34.3%, while a 0.1 decrease in κ_H shifts $P(L, L)$ from 85% to 21.8% and $P(H, H)$ from 64.6% to 88.5%. Table A.9 sums up the effects of changing κ_H.

1.6.3 Market Noise

Proposition 9 *An increase in μ makes sorting less deterministic.*

μ reflects the extent of heterogeneity captured by types. If types contain the majority of employer-employee differences, then μ is close to zero. μ gets large if types pick up little heterogeneity between workers and firms. Holding all other parameters constant, a higher μ lessens the difference in firms' expected payoff between matching with different types of workers, making sorting less deterministic. Similarly, a lower μ makes sorting more deterministic. Table A.10 summarizes the impact of μ on sorting outcomes.

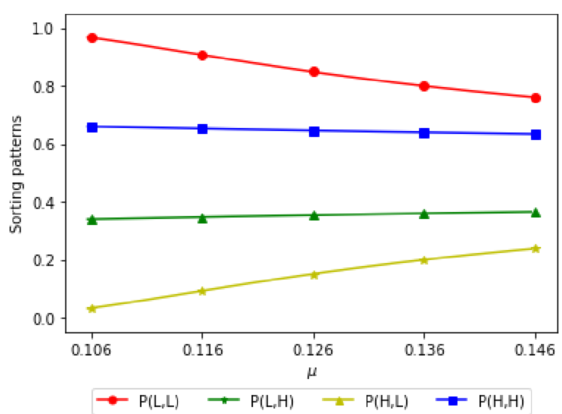


Figure 1.5: Effects of changing market noise μ

1.6.4 Unemployment Benefits

Proposition 10 *An increase in b: raises $\lambda(L, L), \lambda(L, H), \lambda(H, L), \lambda(H, H)$ and $w(L, L)$, $w(L, H), w(H, L), w(H, H)$. The effect on sorting is ambiguous.*

A rise in unemployment benefits b lowers match values between workers and firms. To compensate for this change, market tightness in all submarkets is higher because workers have less incentive to apply for jobs. Therefore, firms post higher wage offers to attract workers. For the given set of parameter configuration, an increase in benefits b induces more PAM. In general, this is not the case as for other parameter values, an increase in b leads to more NAM. Table A.11 sums up the impact of b on sorting outcomes.

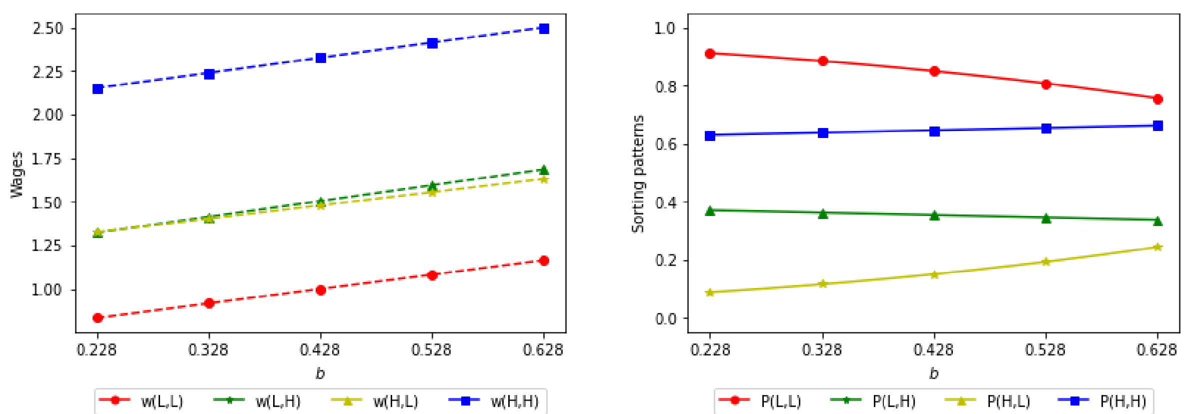


Figure 1.6: Effects of changing unemployment benefits b

1.7 Decompose Income Inequality

In this section, I divide the CPS data into two 12-year periods and discusses the underlying parameter changes. Table 1.2 provides summary statistics for 1994 to 2005 and 2006 to 2017. Over the two time periods, output values for simple jobs decline, yet those for complex jobs increase. Two probable causes for these output changes are skill-biased technical change (SBTC) and offshoring. The former increases the production for complex jobs while the latter decreases the value for simple jobs. This is also reflected in labor income changes for workers: mean wage falls for those who perform simple jobs but rises for those who do complex jobs. As a result, within group wage inequality expands for both low- and high-educated workers. Average weekly earnings rise from 1.384 ($1,130) to 1.471 ($1,200), causing an increase in unemployment benefits (set at 30% of the mean wage). Following the change in b, the entry cost for low-educated workers decreases while that for high-educated workers increases. This suggests between group wage inequality grows as well. In addition, market noise expands from 0.127 to 0.132 during these periods, indicating more heterogeneity unaccounted for by agent types. Indeed, we see more negative sorting for simple jobs: the share of workers with low education drops from 87.1% to 82.9%. Table 1.3 compares parameter values over the two periods.

Parameter	Definition	1994-2005	2006-2017
$w(L, L)$	Mean wage for low-educated workers in simple jobs	1.000	1.000
$w(H, L)$	Mean wage for high-educated workers in simple jobs	1.497	1.461
$w(L, H)$	Mean wage for low-educated workers in complex jobs	1.493	1.516
$w(H, H)$	Mean wage for high-educated workers in complex jobs	2.294	2.358
$P(L, L)$	Share of low-educated workers in simple jobs	0.871	0.829
$P(H, L)$	Share of high-educated workers in simple jobs	0.129	0.171
$P(L, H)$	Share of low-educated workers in complex jobs	0.387	0.320
$P(H, H)$	Share of high-educated workers in complex jobs	0.613	0.680

Table 1.2: Summary statistics for 1994 to 2005 and 2006 to 2017

To quantify the effect of each channel on growing earnings inequality, Table 1.4 estimates changes in production alone explain 93.59% of wage dispersion between 1994 to 2005 and 2006 to 2017, while changes in entry costs and unemployment benefits count for 6.81% and 25.11%, respectively. Since wages are independent of market noise, they

Parameter	Definition	1994-2005	2006-2017
$f(L,L)$	Production for low-educated workers in simple jobs	2.093	2.019
$f(H,L)$	Production for high-educated workers in simple jobs	2.775	2.526
$f(L,H)$	Production for low-educated workers in complex jobs	6.111	6.277
$f(H,H)$	Production for high-educated workers in complex jobs	7.533	7.806
κ_L	Entry cost for low-educated workers	0.204	0.198
κ_H	Entry cost for high-educated workers	0.496	0.499
μ	Market noise	0.127	0.132
b	Unemployment benefits	0.415	0.441

Table 1.3: Changes of parameter values

are unaffected by changes in μ. Because entry costs κ_x are positively correlated with output value $f(x,y)$ and negatively correlated with unemployment insurance value b, together the three channels tend to overexplain changes in wages. In other words, κ_x increases when f goes up since entry costs depend on wages, which follow the sign of change in production. Likewise, κ_x decreases when b increases because wages are decreasing in the value of unemployment benefits.

Parameter	1994-2005	$\Delta f(x,y)$	$\Delta\kappa_x$	Δb	2006-2017
$w(L,L)$	1.000	0.987	0.991	1.021	1.000
$w(H,L)$	1.497	1.438	1.500	1.517	1.516
$w(L,H)$	1.493	1.509	1.477	1.517	1.461
$w(H,H)$	2.294	2.330	2.300	2.317	2.358
Variation		0.0051	0.0004	0.0019	0.0055
Percent		93.59%	6.81%	25.11%	100%

Table 1.4: Quantitative effect of each channel for wage inequality

1.8 Conclusion

This paper develops a sorting framework that highlights the role of search frictions and preference shocks. Workers can perform simple or complex jobs, though output values are different. Search frictions make trade imperfect. In lacking coordination, workers are not guaranteed to match with firms they had directed their search to. Due to preference shocks, firms have random utility and face the "trembling hand" problem: employers intend to match with a specific type of worker but may get distracted along the way and end up matching the other type. The model exhibits stochastic sorting patterns in equilibrium: both PAM and NAM coexist with probabilities between 0 and 1. This equilibrium feature is close to empirical observations.

This paper could be extended in several ways. It would better describe reality if there were more than two discrete types or even with multidimensional types.[12] Another possible extension is to make the model dynamic and enable on-the-job search. However, these extensions would take us beyond the purpose of using a simple model to understanding how frictions and shocks affect sorting outcomes and its implications on skill mismatch and wage inequality.

[12]See Lindenlaub (2017) and Guvenen et al. (2020).

Chapter 2

Multiple Skill Levels in a Matching Model with Heterogeneous Workers and Firms

2.1 Introduction

In this paper, I highlight the role of skill in the labor market and study how multiple skill levels affect equilibrium matching patterns. Specifically, I consider a matching model with workers that have a range of skill levels and firms that have a range of skill requirements. Since worker's skill levels are positively correlated with their wages, studying how skills affect wage profiles will shed light on understanding issues such as skill-biased technical change, optimal job shares in the labor market, and growing inequality as well.

My analysis is based on the matching framework with endogenous skill requirements proposed by Albrecht and Vroman (2002). In their paper, there are two types of workers with different skill levels and two types of firms with different job skill requirements. High-skilled workers can do both high-skilled and low-skilled jobs, yet low-skilled workers can only do low-skilled jobs. On the other hand, high-skilled jobs are more productive than low-skilled jobs, yet when employed in low-skilled jobs, high-skilled workers are equally productive as low-skilled workers.

Depending on parameter configurations, their model has two types of equilibria. In a *cross-skill matching* equilibrium, low-skilled workers work in low-skilled jobs, while high-skilled workers work in both high-skilled and low-skilled jobs. In an *ex post segmentation* equilibrium, low-skilled workers work in low-skilled jobs, while high-skilled workers work in only high-skilled jobs. These equilibria are practical, but perhaps a more realistic model would also include workers and jobs at intermediate levels. That is, under certain circumstances, a high-skilled worker is willing to consider some but not all jobs with lower skill requirements. For example, an MBA graduate may temporarily be employed as a business consultant when unable to find a chief executive job but unlikely to work at a fast-food restaurant.

To generalize this phenomenon, I consider workers with multiple skill levels and firms with various skill requirements in the model. The rest of the setup has a similar structure compared to Albrecht and Vroman (2002): workers qualify for jobs with equal or lower skill requirements, though job productivity is the same for a particular firm, regardless of the type of worker that the firm establishes a match. The distribution of worker skills is exogenous, while the distribution of job skill requirements is endogenous. The matching technology is imperfect, and workers may have to fill jobs with lower skill requirements. Consequently, higher-skilled individuals have higher chances of meeting with potential employers.

Firms can freely enter or exit the labor market. Thus, the cost of posting a vacancy is zero. Together with a set of flow equations, a set of free entry conditions pin down the equilibrium. There are three types of equilibria: if workers choose to match with all firms with identical or lower job skill requirements, we have a*full cross-skill matching* equilibrium. If workers choose to match with some but not all firms with identical or lower job skill requirements, we have a *partial cross-skill matching* equilibrium. Lastly, if workers choose to match with only firms with identical job skill requirements, we have an *ex post segmentation*equilibrium.

This paper contributes to the search and matching literature with heterogeneous agents in the labor market. In particular, my paper extends Albrecht and Vroman (2002)'s model to a more general setting with multiple skill levels. In their paper, workers have binary skill levels that result in two types of equilibria. Because workers are endowed with multiple skill levels, I present a new type of equilibrium, in which workers find it optimal to fill only a subset of jobs with lower skill requirements, in addition to the *cross-skill matching* equilibrium and *ex post segmentation* equilibrium in Albrecht and Vroman (2002). Dolado et al. (2009) introduce on-the-job search as an additional source of between and within-group wage inequality while having the same equilibrium structure as Albrecht and Vroman (2002). It is demonstrated in Dolado et al. (2009) that compared to permanent mismatch, temporary mismatch benefits high-type workers at the cost of causing more harm to low-type workers. Allowing on-the-job search would complicate the model greatly, but the main conclusion is going to follow.[1]

This paper is related to the efficiency of the equilibrium allocation in a matching model with heterogeneous workers and firms. Blázquez and Jansen (2008) study the role of bargaining and prove that under the combination of random search and ex post wage bargaining, the equilibrium is hardly efficient due to wage distribution compressed by bargaining unless firms are able to sort workers into homogenous submarkets for different jobs. Shi (2002) and Shimer (2005a) develop directed search models

[1] See also Acemoglu (1999, 2001), Mortensen and Pissarides (1999), Shimer (1999), Shimer and Smith (2000)

with heterogeneous workers and firms in the labor market that have naturally efficient equilibrium since firms can commit to their posted wage offers. In my paper, the search and matching environment is similar to that in Blázquez and Jansen (2008). Therefore, the equilibrium allocation is inefficient.

The rest of the paper is organized as follows. 2.2 introduces the model environment. 2.3 derives and defines the different types of equilibrium. 2.4 provides a numerical example of the model. 2.5 demonstrates the comparative statics property. Finally, 2.6 concludes.

2.2 The Model

This section presents an $n \times n$ search and matching model with heterogeneous workers and firms in the labor market. Search is $undirected$, and I focus my analysis on steady-states.

2.2.1 Environment

Consider an economy that is populated by a large continuum of workers and firms. Time is continuous. All agents are risk-neutral, live forever, and have a common discount rate r. Each worker is endowed with a skill level $x_i \in \{x_1, x_2, ..., x_n\}$. The distribution of skills across workers is exogenous: a fraction μ_i of the workers in the labor force has the skill level x_i. Workers have measure normalized to one ($i.e.$ $\sum_{i=1}^{n} \mu_i = 1$). Production of the final good requires a worker-firm pair, which I call a job. All firms can freely enter the labor market. Each firm posts a job vacancy with skill requirement y_j that denotes the minimum skill levels required from a job seeker to perform the job. To avoid capital-labor substitution within firms, each firm employs up to one worker. A job can be filled or unfilled. A firm pays a flow cost k for an unfilled vacancy. A filled job is subject to turnover shocks with a constant arrival rate δ that destroys the job. When a worker and a firm form a matching pair, the output $f(x_i, y_j)$ is defined as the following:

$$f(x_i, y_j) = \begin{cases} y_j & if \quad i \geq j \\ 0 & if \quad i < j \end{cases}$$

where x_i is the worker's skill level and y_j is the job's skill requirement. A firm pays a flow cost of k for a vacancy until it is filled. If a vacancy is filled, a firm pays wages $w(x_i, y_j)$ to a worker. Unemployed workers receive benefits with a flow value b. Given the production function and the skill distribution across workers, firms endogenously choose $y_j = x_i$ with $j = i$ to maximize their profit. That is, depending on the vacancy type a firm wants to create, we have $y_1 = x_1, y_2 = x_2, ..., y_n = x_n$. Without loss of generality, one can assume $y_1 < y_2 < ... < y_n$. In total, there are n types of workers and n types of jobs in this economy.

Essentially, the production technology suggests that workers can fill any jobs with skill requirements below or equal to their skill levels, but not above. Furthermore, it also indicates that workers with a higher level of skills are not necessarily more productive than their lower-skilled counterparts if they are performing the same job. For example, an architect can work as a contractor and has equal productivity, but not vice versa, generally speaking.

In the presence of search frictions, workers may accept or reject specific jobs with skill requirements lower than their skill levels. This represents what Albrecht and Vroman (2002) call a *cross-skill matching* equilibrium and a *ex post segmentation* equilibrium. Since the distribution of skills across worker is n-point instead of 2-point, the model will have a similar equilibrium structure like Albrecht and Vroman (2002), with the difference that the *cross-skill matching* equilibrium can be a *full* one, which corresponds to the case where workers find it optimal to accept any jobs with skill requirements below or equal to their skill levels, or a *partial* one, which corresponds to the case where workers find it optimal to only accept some jobs with skill requirements below or equal to their own skill levels. The *ex post segmentation* equilibrium corresponds to the case where workers are only willing to accept jobs with skill requirements equal to their skill levels.

2.2.2 Matching

The labor market is frictional without on-the-job search. Unemployed workers and firms with job vacancies meet in pairs randomly according to a constant returns to scale matching function $m(u, v)$, where u_i is the mass of unemployed workers with skill level i and v_j is the mass of job vacancies with skill requirement j. u denotes the aggregate unemployment rate, and v denotes the total measure of job vacancies:

$$m(u, v) = m\left(\sum_{i=1}^{n} u_i, \sum_{j=1}^{n} v_j\right),$$

with labor market tightness $\theta \equiv v/u = \sum_{j=1}^{n} v_j / \sum_{i=1}^{n} u_i$. The job arrival rate for unemployed workers is $\theta q(\theta) \equiv m(\theta, 1)$, while the worker contact rate for job vacancies is $q(\theta) \equiv m(1, 1/\theta)$. However, job seekers may meet with vacancies they are not qualified for. Let γ_i denote the share of unemployed workers with skill level i and ϕ_j denote the share of job vacancies with skill requirement j. Obviously, $\sum_{i=1}^{n} \gamma_i = 1$ and $\sum_{j=1}^{n} \phi_j = 1$.The effective matching rate for workers with skill level i is $\theta q(\theta) \sum_{j=1}^{i} \phi_j$, indicating that unemployed workers can fill job vacancies with skill requirements up to their skill levels. On the other hand, the effective matching rate for firms with skill requirement j is $q(\theta) \sum_{i=j}^{n} \gamma_i$, suggesting that employers can hire workers with skill levels equal or above their skill requirements. By assumption, $\lim_{\theta\to 0} q(\theta) = \lim_{\theta\to\infty} \theta q(\theta) = \infty$ and $\lim_{\theta\to\infty} q(\theta) = \lim_{\theta\to 0} \theta q(\theta) = 0$.

2.2.3 Wages

Wage determination is based on Nash bargaining solutions. I adopt a linear rent-sharing rule that empowers workers a share $\beta \in [0, 1]$ of the total match surplus created between a pair of employer and employee. Let $U(x_i)$ be the unemployment value for a worker of type i and $V(y_j)$ be the vacancy value for a firm of type j. If a type i worker and a type j firm form a match, let $W(x_i, y_j)$ be the employment value for the worker and $J(x_i, y_j)$ be the filled job value for the firm. The joint surplus of a match formation is:

$$S(x_i, y_j) \equiv W(x_i, y_j) + J(x_i, y_j) - U(x_i) - V(y_j). \tag{2.1}$$

Wages $w(x_i, y_j)$ satisfy the following bargaining solution:

$$(1 - \beta)\left[W(x_i, y_j) - U(x_i)\right] = \beta\left[J(x_i, y_j) - V(y_j)\right]. \tag{2.2}$$

2.2.4 Asset Values

In this section, I define the value functions for workers and firms. To begin with, a job seeker with skill level i has a flow value of unemployment benefits b, plus an outside option upon successful job search:

$$rU(x_i) = b + \theta q(\theta)\sum_{j=1}^{i}\{\phi_j \max[W(x_i, y_j) - U(x_i), 0]\}. \tag{2.3}$$

The value function for unemployed workers undertakes two assumptions. First, it is practical to assume that workers with higher skill levels are qualified to do more jobs. Consequently, this means that a highly skilled individual can potentially suffer from a greater extent of mismatch. Second, it may not be optimal for workers to take jobs with lower skill requirements since the value of an outside option of waiting for a better match could dominate that of taking a less appealing job, hence the term $\max[W(x_i, y_j) - U(x_i), 0]$. An employed worker gets paid wages $w(x_i, y_j)$, keeps the job forever until exogenous separation between a worker and a firm:

$$rW(x_i, y_j) = w(x_i, y_j) + \delta[U(x_i) - W(x_i, y_j)]. \tag{2.4}$$

Next, an unfilled vacancy with skill requirement j has the value of a potentially filled job, minus the posting cost k to a firm:

$$rV(y_j) = -k + q(\theta)\sum_{i=j}^{n}\{\gamma_i \max[J(x_i, y_j) - V(y_j), 0]\}. \tag{2.5}$$

Likewise, the value function for filled jobs also incorporates two assumptions. First, firms can hire workers with skill levels above or equal to their job requirements. Second, conditional on finding qualified candidates, employers may hire lower-skilled workers to generate greater profits due to lower outside option value for those workers. The term $\max[J(x_i, y_j) - V(y_j), 0]$ reflects the latter assumption. In the long-run, free entry of firms yields zero value for all types of job vacancies: $V(y_j) = 0 \; \forall j \in [1, n]$. For a filled job, an employer earns a profit of $y_j - w(x_i, y_j)$, subject to exogenous job destruction:

$$rJ(x_i, y_j) = y_j - w(x_i, y_j) + \delta[V(y_j) - J(x_i, y_j)]. \tag{2.6}$$

Before proceeding to equilibrium analysis, it is helpful to derive a few mathematical expressions. Using equation (2), I simplify the expression for the match surplus in (1):

$$S(x_i, y_j) = W(x_i, y_j) + J(x_i, y_j) - U(x_i) - V(y_j) = \frac{y_j - rU(x_i)}{r + \delta}. \tag{2.7}$$

Thus, a particular match formation $S(x_i, y_j) \geq 0$ requires $y_j \geq rU(x_i)$, which reduces to $(r+\delta)(y_j - b) + \beta\theta q(\theta)\left(y_j \sum_{j=1}^{i} \phi_j - \sum_{j=1}^{i} \phi_j y_j\right) \geq 0$. Following equations (2) and (7), one can work out the following:

$$W(x_i, y_j) - U(x_i) = \beta S(x_i, y_j), J(x_i, y_j) - V(y_j) = (1-\beta) S(x_i, y_j), \tag{2.8}$$

which are standard in random search models with wage bargaining. Finally, wages can be expressed as a weighted average of the output y_j and the worker's unemployment value:

$$w(x_i, y_j) = \beta y_j + (1-\beta) rU(x_i). \tag{2.9}$$

2.3 Equilibria

In this section, I solve for equilibrium flows for workers and free entry conditions for firms. Depending on parameter configurations, there are three scenarios to consider. The first scenario is a *full cross-skill matching* equilibrium, in which workers are willing to fill any job with skill requirements below or equal to their skill levels. The second scenario is a *partial cross-skill matching* equilibrium, in which workers are only willing to fill some (but not all) jobs with skill requirements below or equal to their skill levels. The third scenario is an *ex post segmentation* equilibrium, in which workers are only willing to fill jobs with skill requirements precisely equal to their skill levels. Additionally, with particular parameter values, multiple equilibria are also possible and therefore worth discussing.

In any of the three cases I consider, a steady-state equilibrium consists of a vector of $\{u, \theta, \gamma_i, \phi_j\}$ with $2n+2$ endogenous variables that bring a non-negative match surplus for all match formations. Meanwhile, workers flow into and out of unemployment must equal in equilibrium. Furthermore, the value of creating a job vacancy is zero because of the free entry of firms in the labor market. In addition, it is required that $\theta > 0$ and $u, \gamma_i, \phi_j \in [0, 1]$.

2.3.1 Full Cross-Skill Matching

In this case, it is optimal for workers to accept all jobs with skill requirements below or equal to their skill levels. For convenience, I define the following adjacency matrix to facilitate my equilibrium analysis that follows. Let

$$A_{ij} = \begin{cases} 0 & if \quad i < j \\ 1 & if \quad i \geq j \end{cases},$$

the corresponding adjacency matrix A is:

$$A = \begin{bmatrix} 1 & 0 & ... & ... & 0 \\ ... & 1 & 0 & ... & ... \\ ... & ... & 1 & 0 & ... \\ ... & ... & ... & 1 & 0 \\ 1 & ... & ... & ... & 1 \end{bmatrix}.$$

In a *full cross-skill matching* equilibrium, $S(x_i, y_j) \geq 0 \; \forall A_{ij} = \mathbb{1}$. Because job productivity y_j is monotonically increasing, a necessary condition to for this kind of equilibrium to hold is $S(x_i, y_1) \geq 0 \; \forall i$. In steady-state, the flows of workers into and out of jobs follow Figure 2.1. The arrival rate of jobs for a job seeker with skill level i is

$\theta q(\theta) \sum_{j=1}^{i} \phi_j$, and the share of unemployed individuals with skill level i is $\gamma_i u$. For employed workers with skill level i, the share is the difference between the faction of the labor force with skill level i and those unemployed with skill level i, $\mu_i - \gamma_i u$. These employed individuals face a constant job separation rate δ. In total, I have a system of n equations for all skill levels $i \in \{1, 2, ..., n\}$:

$$\theta q(\theta) \left(\sum_{j=1}^{i} \phi_j \right) \gamma_i u = \delta(\mu_i - \gamma_i u). \tag{2.10}$$

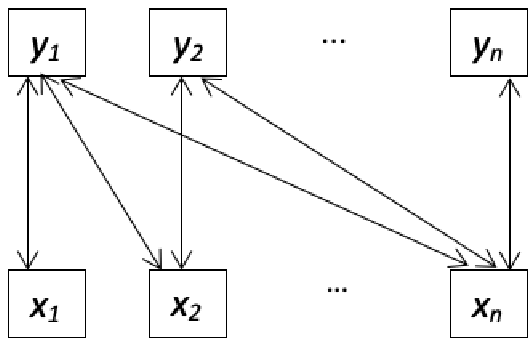


Figure 2.1: Flow chart for the *full cross-skill matching* case

Since the share of job vacancies with skill requirement j sums up to 1, I solve the equilibrium unemployment rate u and share of job vacancies ϕ_j as a function of θ, γ_i, μ_i, and δ, where δ and γ_i are endogenous and will be pinned down by the free entry conditions:

$$u(\theta, \gamma_i; \mu_i, \delta) = \frac{\delta \mu_n}{[\theta q(\theta) + \delta]\gamma_n}, \tag{2.11}$$

$$\phi_j(\theta, \gamma_i; \mu_i, \delta) = \begin{cases} \frac{[\theta q(\theta)+\delta]\gamma_n}{\theta q(\theta)\mu_n} \left(\frac{\mu_1}{\gamma_1} \right) - \frac{\delta}{\theta q(\theta)} & for \quad j = 1 \\ \frac{[\theta q(\theta)+\delta]\gamma_n}{\theta q(\theta)\mu_n} \left(\frac{\mu_i}{\gamma_i} - \frac{\mu_{i-1}}{\gamma_{i-1}} \right) & for \quad 2 \leq j \leq n. \end{cases} \tag{2.12}$$

In order to solve for free entry conditions $V(y_j) = 0 \, \forall j \in \{1, 2, ..., n\}$, I first solve the unemployment values for workers with skill levels x_i. Combining equations (3) and (8) yield the following expression:

$$rU(x_i) = b + \theta q(\theta) \left(\sum_{j=1}^{i} \phi_j \right) \beta S(x_i, y_j), \tag{2.13}$$

which can be used to plug into equation (7) to solve for the equilibrium value of unemployment for workers with skill level i:

$$rU(x_i) = \frac{(r+\delta)b + \beta\theta q(\theta) \sum_{j=1}^{i} \phi_j y_j}{r + \delta + \beta\theta q(\theta) \sum_{j=1}^{i} \phi_j}. \tag{2.14}$$

Note that $\partial rU(x_i)/\partial\theta > 0, \partial rU(x_i)/\partial\phi_j > 0$ and $\partial rU(x_i)/\partial\phi_{j'} < 0$. All workers are better off as the ratio of job vacancies to unemployment increases, holding the share of job vacancies ϕ_j constant. On the other hand, workers with skill level i are better off with an increase in the share of job vacancies with corresponding skill requirements j and worse off with an increase in the share of job vacancies with skill requirements $j' \neq j$, holding labor market tightness θ constant.

Now, equation (5) and (8) give the zero value condition for firms creating job vacancies:

$$k = \left(\frac{1-\beta}{r+\delta} \right) q(\theta) \sum_{i=j}^{n} \gamma_i \left[y_j - rU(x_i) \right]. \tag{2.15}$$

Plugging in equation (14), I get:

$$k = \left(\frac{1-\beta}{r+\delta} \right) q(\theta) \sum_{i=j}^{n} \gamma_i \left[\frac{(r+\delta)(y_j - b) + \beta\theta q(\theta) \left(y_j \sum_{j'=1}^{i} \phi_{j'} - \sum_{j'=1}^{i} \phi_{j'} y_{j'} \right)}{r + \delta + \beta\theta q(\theta) \sum_{j'=1}^{i} \phi_{j'}} \right], \tag{2.16}$$

which given the value of exogenous parameters $\{\beta, \delta, b, r, y_j\}$, one can obtain unique solutions for labor market tightness θ and the share of unemployed workers γ_i. Therefore, the equilibrium values for unemployment rate u and the share of job vacancies ϕ_j can also be acquired using equations (11) and (12).

2.3.2 Ex Post Segmentation

In this case, it is optimal for workers to accept only jobs with skill requirements identical to their skill levels. Let

$$B_{ij} = \begin{cases} 0 & if \quad i \neq j \\ 1 & if \quad i = j \end{cases},$$

the corresponding adjacency matrix B is:

$$B = \begin{bmatrix} 1 & 0 & ... & ... & 0 \\ 0 & 1 & 0 & ... & ... \\ ... & 0 & 1 & 0 & ... \\ ... & ... & 0 & 1 & 0 \\ 0 & ... & ... & 0 & 1 \end{bmatrix}.$$

In an *ex post segmentation* equilibrium, $S(x_i, y_j) \geq 0 \, \forall B_{ij} = \mathbb{1}$. In steady-state, workers' flow into jobs balances the flow out of jobs, as shown in figure Figure 2.2:

$$\theta q(\theta)\phi_j\gamma_i u = \delta(\mu_i - \gamma_i u). \tag{2.17}$$

Again, I write the equilibrium unemployment rate u and share of job vacancies ϕ_j as a function of θ, γ_i, μ_i, and δ, where δ and γ_i are endogenous and will be derived later using free entry conditions:

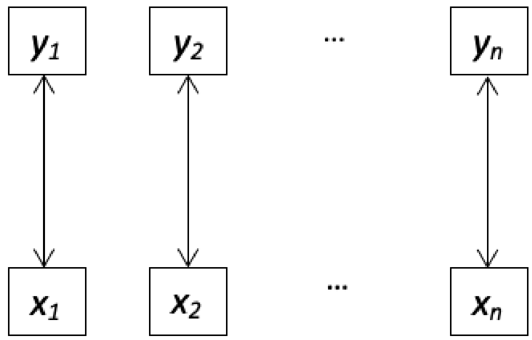


Figure 2.2: Flow chart for the *ex post segmentation* case

$$u(\theta, \gamma_i; \mu_i, \delta) = \frac{\delta \sum_{i=1}^{n} \left(\frac{\mu_i}{\gamma_i}\right)}{\theta q(\theta) + n\delta}, \tag{2.18}$$

$$\phi_j(\theta, \gamma_i; \mu_i, \delta) = \frac{[\theta q(\theta) + n\delta]\mu_i}{\theta q(\theta)\gamma_i \sum_{i'=1}^{n} \left(\frac{\mu_{i'}}{\gamma_{i'}}\right)} - \frac{\delta}{\theta q(\theta)}. \tag{2.19}$$

Next, free entry of firms requires $V(y_j) = 0\, \forall j \in \{1, 2, ..., n\}$. Once more, I combine equations (3) and (8):

$$rU(x_i) = b + \theta q(\theta)\phi_j \beta S(x_i, y_j), \tag{2.20}$$

which gives equilibrium value of unemployment for workers with skill level i, using equation (7):

$$rU(x_i) = \frac{(r+\delta)b + \beta\theta q(\theta)\phi_j y_j}{r + \delta + \beta\theta q(\theta)\phi_j}. \tag{2.21}$$

It follows that $\partial rU(x_i)/\partial\theta > 0, \partial rU(x_i)/\partial\phi_j > 0$ and $\partial rU(x_i)/\partial\phi_{j'} < 0$. The intuition is the same behind the *full cross-skill matching* case, though the quantitative effects vary.

Now, equation (5) and (8) give the zero value condition:

$$k = \left(\frac{1-\beta}{r+\delta}\right) q(\theta)\gamma_i \left[y_j - rU(x_i)\right]. \tag{2.22}$$

Incorporating equation (22), the free entry conditions in the *ex post segmentation* case is:

$$k = \left(\frac{1-\beta}{r+\delta}\right) q(\theta) \frac{(r+\delta)(y_j - b) - (1-\beta)\theta q(\theta)\phi_j y_j}{r+\delta+\beta\theta q(\theta)\phi_j}. \tag{2.23}$$

Applying equations (18) and (19), the approach is similar to the *full cross-skill matching* case to solve for equilibrium objects $\{u, \theta, \gamma_i, \phi_j\}$.

2.3.3 Partial Cross-Skill Matching

This is the intermediate scenario between *full cross-skill matching* and *ex post segmentation*. In this case, it is optimal for workers to take a fraction of jobs with skill requirements below or equal to their skill levels. Let

$$C_{ij} = \begin{cases} 0 & if \quad i < j \\ 0 \; or \; 1 & if \quad i > j \\ 1 & if \quad i = j \end{cases},$$

It is worth noting that there are multiple ways to express the corresponding adjacency matrix C. One example is:

$$C = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ \dots & 1 & 0 & \dots & \dots \\ 1 & \dots & 1 & 0 & \dots \\ \dots & 0 & \dots & 1 & 0 \\ 0 & \dots & 1 & \dots & 1 \end{bmatrix}.$$

In a *partial cross-skill matching* equilibrium, $S(x_i, y_j) \geq 0 \; \forall C_{ij} = \mathbb{1}$. The flow chart in steady-state is demonstrated by Figure 2.3. To obtain equilibrium solutions, the key equations are identical to equations (11), (12), and (16), except in this case, the restrictions on (i, j) differ.

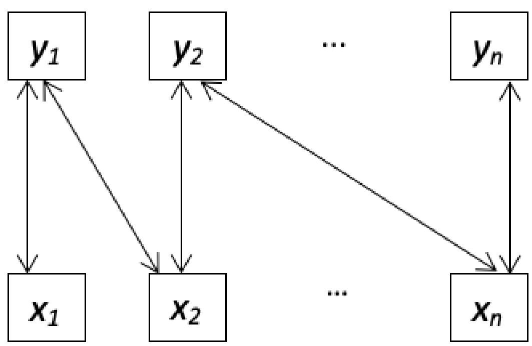


Figure 2.3: Flow chart for the *partial cross-skill matching* case

2.3.4 Multiple Equilibria

Multiple equilibria arise due to an externality of worker's choice of jobs. If mismatch rarely occurs, that is, most workers choose to match with vacancies requiring the same level of skills, the value of unemployment increases. As a result, a *full cross-skill matching* equilibrium takes place more frequently. Similarly, if mismatch happens often, that is, many workers choose to take on jobs with lower skill requirements, the value of unemployment falls. Consequently, an *ex post segmentation* equilibrium is more likely to occur. Therefore, for an individual worker with skill level i, the unemployment value also depends on other workers with similar skills. For an intermediate range of parameter values, it is possible to have multiple equilibria.

2.4 A Numerical Example

I provide a horizontal comparison of different types of equilibrium in this section. To simply my model and facilitate discussion, I assume three types of workers: low-skilled, medium-skilled, and high-skilled. There are also three types of jobs with similar skill requirements. By assumption, the matching function is in the form of $m(u, v) = \sqrt{uv}$, which indicates the arrival rates for workers and firms are $\sqrt{\theta}$ and $1/\sqrt{\theta}$, respectively.[2] Table 2.1 presents the parameter values I used in this example. Under this setup, the model demonstrates multiple equilibria. Applying the equilibrium conditions derived in section 3, I solve for equilibrium labor market outcomes in Table 2.2 for all three cases.

In *ex post segmentation* equilibria, the unemployment rate (42.5%) is significantly higher than other equilibria (13.0% and 21.1%, respectively). Given how matches are formed in the model, this is instead not as striking as it seems since workers are not able to fill jobs with lower skill requirements. This hurts the value of unemployment for medium- and high-skilled workers. As a result, the arrival rate of jobs for those workers is lower, leading to a higher overall unemployment rate and lower job market tightness than other *cross-skill matching* equilibria.

Analogously, *full cross-skill matching* equilibria has the highest concentration of low-skilled job vacancies (79.6%). Because workers are capable of matching with vacancies that require less skills, high-skilled workers creates a spillover effect on jobs with

[2] See Petrongolo and Pissarides (2001).

medium and low skill requirements. Likewise, some medium-skilled workers flow into jobs with low skill requirements as well. On the other hand, firms respond to these changes by posting more low skilled job vacancies and fewer medium and high skilled job vacancies, leading to a higher value of γ_1 and lower values of γ_2 and γ_3.

Parameter	Definition	Value
y_1	Output value for low skilled jobs	1
y_2	Output value for medium skilled jobs	1.25
y_3	Output value for high skilled jobs	1.75
n	Types of skill	3
b	Unemployment benefits	0.1
β	Bargaining power for workers	0.5
δ	Job destruction rate	0.2
k	Vacancy cost for firms	0.3
r	Discount factor	0.05
μ_1	Share of low skilled workers	$1/3$
μ_2	Share of medium skilled workers	$1/3$
μ_3	Share of high skilled workers	$1/3$

Table 2.1: Parameter values

Variable	Definition	Full CSM	Partial CSM	EPS
θ	Job market tightness	2.250	2.263	0.694
u	Unemployment rate	0.130	0.211	0.425
γ_1	Share of unemployed workers with low skill	0.369	0.360	0.288
γ_2	Share of unemployed workers with medium skill	0.328	0.333	0.326
γ_3	Share of unemployed workers with high skill	0.303	0.307	0.386
ϕ_1	Share for vacancies with low skill requirements	0.796	0.450	0.414
ϕ_2	Share for vacancies with medium skill requirements	0.117	0.047	0.338
ϕ_3	Share for vacancies with high skill requirements	0.087	0.504	0.248
$w(x_1, y_1)$	Wages for low-low worker-firm pair	0.787	0.783	0.696
$w(x_2, y_1)$	Wages for medium-low worker-firm pair	0.894	0.894	-
$w(x_2, y_2)$	Wages for medium-medium worker-firm pair	1.019	1.020	0.877
$w(x_3, y_1)$	Wages for high-low worker-firm pair	1.003	-	-
$w(x_3, y_2)$	Wages for high-medium worker-firm pair	1.128	1.132	-
$w(x_3, y_3)$	Wages for high-high worker-firm pair	1.378	1.382	1.247

Table 2.2: Baseline model

As for wage differentials, there are two things worth pointing out. First, for jobs with identical skill requirements, those workers with higher skill levels earn higher wages than their lower-skilled counterparts. In *full cross-skill matching* equilibria, wages for low-low worker-firm pair are 0.787, while wages for medium-low and high-low worker-firm pairs are 0.894 and 1.003, respectively. Wages for medium-medium worker-firm pair are 1.019, yet wages for high-medium worker-firm pair are 1.128. A similar pattern is detected in *partial cross-skill matching* equilibria as well. Wages for low-low worker-firm pair are lower than those for medium-low worker-firm pair (0.783 and 0.894). Wages for medium-medium worker-firm pair are 1.020 and wages for medium-medium worker-firm pair are 1.132. The intuition behind is that higher-skilled workers have the better outside option of matching with a greater set of employers, thus making higher labor income. As the equilibrium wage is determined as a weighted average of jobs' output value and workers' unemployment value, higher-skilled workers have higher value of unemployment, conditional on having equal values of production with lower-skilled individuals.

Second, for workers with identical skill levels, firms that have higher skill requirements pay more than those with lower-skilled requirements. In *full cross-skill matching* equilibria, medium-skilled workers earn wages of 0.894 in jobs with low skill requirements and 1.019 in jobs with medium skill requirements. High-skilled workers receive an income of 1.003 in low type jobs, 1.128 in medium type jobs, and 1.378 in high type jobs. In *partial cross-skill matching* equilibria, medium-skilled workers earn wages of 0.894 in jobs that require low skills and 1.019 in jobs that require medium skills. High-skilled workers receive 1.132 when matching with medium-type firms and 1.382 when matching with high-type firms. This is because jobs that require higher skills yield greater match values.

2.5 Comparative Statics

Using parameter and variable values in the baseline model as an example, I demonstrate the effect of productivity changes in each type of equilibria. To begin with, I show in Table 2.3 the effect of increasing the relative output value of jobs with high skill requirements in *full cross-skill matching* equilibria. This increase can be interpreted with skill-biased technical change, where technology advancements tend to favor different worker groups unequally. In particular, skilled workers are known to benefit more from technology improvements than unskilled workers. The most noticeable change is increasing wages for high skilled workers across all types of jobs, as the total surplus to share increases for all these jobs. For instance, wages for high-low worker-firm pair change from 1.003 when $y_3 = 1.75$ to 1.007 when $y_3 = 1.85$. Wages for high-medium worker-firm pair change from 1.128 when $y_3 = 1.75$ to 1.132 when $y_3 = 1.85$. Wages for high-high worker-firm pair change from 1.378 when $y_3 = 1.75$ to 1.432 when $y_3 = 1.85$. Associated with this change are the changes in wages for medium-skilled workers. Wages for medium-low worker-firm pair increase from 0.894 when $y_3 = 1.75$ to 0.901 when $y_3 = 1.85$. Wages for medium-medium worker-firm pair increase from 1.019 when $y_3 = 1.75$ to 1.026 when $y_3 = 1.85$. The intuition is when the productivity for high skilled jobs increases, high-skilled workers are now creating a smaller spillover effect into low and medium-skilled jobs, therefore indirectly increasing the unemployment value for medium-skilled workers. However, wages for low-skilled workers stay the same (0.787) because they are only allowed to match with jobs with low skill requirements, resulting in the same value of unemployment throughout changes in y_3. Apart from differences in wages, firms post more high skilled job vacancies in response to increases in productivity for jobs with high skill requirements. Consequently, ϕ_3 increases significantly from 8.7% when $y_3 = 1.75$ to 28.9% when $y_3 = 1.85$, lowering ϕ_1

and ϕ_2, the shares of vacancies with low and medium skill requirements.

Variable	Definition	$y_3 = 1.75$	$y_3 = 1.80$	$y_3 = 1.85$
θ	Job market tightness	2.250	2.250	2.250
u	Unemployment rate	0.130	0.138	0.146
γ_1	Share of unemployed workers with low skill	0.369	0.369	0.369
γ_2	Share of unemployed workers with medium skill	0.328	0.346	0.361
γ_3	Share of unemployed workers with high skill	0.303	0.285	0.269
ϕ_1	Share for vacancies with low skill requirements	0.796	0.742	0.693
ϕ_2	Share for vacancies with medium skill requirements	0.117	0.060	0.018
ϕ_3	Share for vacancies with high skill requirements	0.087	0.198	0.289
$w(x_1, y_1)$	Wages for low-low worker-firm pair	0.787	0.787	0.787
$w(x_2, y_1)$	Wages for medium-low worker-firm pair	0.894	0.898	0.901
$w(x_2, y_2)$	Wages for medium-medium worker-firm pair	1.019	1.023	1.026
$w(x_3, y_1)$	Wages for high-low worker-firm pair	1.003	1.005	1.007
$w(x_3, y_2)$	Wages for high-medium worker-firm pair	1.128	1.130	1.132
$w(x_3, y_3)$	Wages for high-high worker-firm pair	1.378	1.405	1.432

Table 2.3: Effects of changing y_3 in *full cross-skill matching* equilibria

Next, I show in Table 2.4 the effect of increasing the relative output value of jobs with medium skill requirements in *partial cross-skill matching* equilibria. This increase has more complicated effects on equilibrium labor market outcomes for two reasons: first, this will create a 2-way spillover effect on both low- and high-skilled job vacancies instead of a 1-way spillover effect, like in the previous example. Second, in the current setup, high skilled workers cannot fill jobs with low skill requirements. Again, the most noticeable change is increasing wages for workers in medium-skilled jobs, as a direct effect of changes in y_2. For instance, wages for medium-medium worker-firm pair change from 1.020 when $y_2 = 1.25$ to 1.065 when $y_2 = 1.35$. Wages for high-medium worker-firm pair change from 1.132 when $y_2 = 1.25$ to 1.173 when $y_2 = 1.35$. Associated with this change are the changes in wages for other worker groups. Wages for low-low worker-firm pair increase from 0.783 when $y_2 = 1.25$ to 0.818 when $y_2 = 1.35$. Wages for medium-low worker-firm pair decrease from 0.894 when $y_2 = 1.25$ to 0.890 when $y_2 = 1.35$. Wages for high-high worker-firm pair also shrinks from 1.382

when $y_2 = 1.25$ to 1.373 when $y_2 = 1.35$. Other than wage differentials, firms post more medium-skilled job vacancies in response to increases in productivity for jobs with medium skill requirements. Consequently, ϕ_2 increases dramatically from 4.7% when $y_2 = 1.25$ to 64.4% when $y_2 = 1.35$, lowering ϕ_1 and ϕ_3, the shares of low- and medium-skilled vacancies.

Variable	Definition	$y_2 = 1.25$	$y_2 = 1.30$	$y_2 = 1.35$
θ	Job market tightness	2.263	2.259	2.247
u	Unemployment rate	0.211	0.197	0.189
γ_1	Share of unemployed workers with low skill	0.360	0.428	0.492
γ_2	Share of unemployed workers with medium skill	0.333	0.267	0.210
γ_3	Share of unemployed workers with high skill	0.307	0.305	0.298
ϕ_1	Share for vacancies with low skill requirements	0.450	0.394	0.345
ϕ_2	Share for vacancies with medium skill requirements	0.047	0.317	0.644
ϕ_3	Share for vacancies with high skill requirements	0.504	0.290	0.011
$w(x_1, y_1)$	Wages for low-low worker-firm pair	0.783	0.803	0.818
$w(x_2, y_1)$	Wages for medium-low worker-firm pair	0.894	0.893	0.890
$w(x_2, y_2)$	Wages for medium-medium worker-firm pair	1.020	1.043	1.065
$w(x_3, y_2)$	Wages for high-medium worker-firm pair	1.132	1.155	1.173
$w(x_3, y_3)$	Wages for high-high worker-firm pair	1.382	1.380	1.373

Table 2.4: Effects of changing y_2 in *partial cross-skill matching* equilibria

Finally, I discuss in Table 2.5 the effect of increasing the relative output value of jobs with low skill requirements in *ex post segmentation* equilibria. This is a reasonable scenario for discussion under the current theme of de-globalization, given that countries like the US have decided to reduce offshoring of domestic jobs and protect industries such as manufacturing. The direct change in wages is for low-low worker-firm pair, increasing from 0.696 when $y_1 = 1$ to 0.774 when $y_1 = 1.10$. On the other hand, wages for medium-medium worker-firm pair change from 0.877 when $y_1 = 1$ to 0.884 when $y_1 = 1.10$ and wages for high-high worker-firm pair change from 1.247 when $y_1 = 1$ to 1.260 when $y_1 = 1.10$. The response by firms is of the opposite direction compared to two *cross-skill matching* equilibria, that is, firms post fewer jobs with low skill requirements, decreasing the value for ϕ_1, and more jobs with medium and high skill requirements, increasing the values for ϕ_2 and ϕ_3. In *ex post segmentation* equilibria, firms are only able to hire workers with skill levels matching their vacancies. Because the spillover effects no longer exist in this case, a higher value of y_1 reduces the profit of maintaining job vacancies with low skill requirements. Therefore, firms end up setting more of other types of jobs.

Variable	Definition	$y_1 = 1$	$y_1 = 1.05$	$y_1 = 1.10$
θ	Job market tightness	0.694	0.746	0.798
u	Unemployment rate	0.425	0.415	0.406
γ_1	Share of unemployed workers with low skill	0.288	0.293	0.298
γ_2	Share of unemployed workers with medium skill	0.326	0.323	0.320
γ_3	Share of unemployed workers with high skill	0.386	0.384	0.382
ϕ_1	Share for vacancies with low skill requirements	0.414	0.403	0.393
ϕ_2	Share for vacancies with medium skill requirements	0.338	0.344	0.350
ϕ_3	Share for vacancies with high skill requirements	0.248	0.253	0.257
$w(x_1, y_1)$	Wages for low-low worker-firm pair	0.696	0.735	0.774
$w(x_2, y_2)$	Wages for medium-medium worker-firm pair	0.877	0.881	0.884
$w(x_3, y_3)$	Wages for high-high worker-firm pair	1.247	1.254	1.260

Table 2.5: Effects of changing y_1 in *ex post segmentation* equilibria

2.6 Conclusion

This paper highlights the role of multiple skill levels in a frictional labor market with heterogeneous workers and firms. I have built a one-dimensional $n \times n$ search and matching model where job seekers with particular skill levels are qualified for jobs with equal or lower skill requirements. This captures the mismatch phenomenon, an important empirical feature in modern labor markets such as the US and Europe. Comparative statics of the model provides insights into the effect of productivity change on labor market structure, such as the optimal share of vacancies, as well as unemployment and wage inequality.

I have introduced three different types of equilibria in the model, depending on the value of model parameters. In a *full cross-skill matching* equilibrium, it is optimal for workers in the labor force to fill any job vacancies with skill requirements equal to or below. The second case is a *partial cross-skill matching* equilibrium, an intermediate case between a *full cross-skill matching* equilibrium and an *ex post segmentation* equilibrium, where workers take only a subset of jobs requiring skills equal or below. The third case is an *ex post segmentation* equilibrium where no mismatch exists. In other words, workers match with firms with identical skill requirements.

There are several ways for future work. First, one can incorporate shocks into the model to have more empirical features. In order to this, one wants to create a data generating process that allows stochastic skills for workers that can improve matching outcomes. Second, one could extend the model to be n-dimensional instead of one-dimensional, incorporating various skills with multiple skill levels for each individual skill in an $n \times n \times n$ space. The third extension is to consider a continuous level of skills, matching the distribution of skills on the worker side with that of skill requirements on the firm side. This extension would be technically challenging but theoretically intriguing to pursue.

Chapter 3

Labor Market and Marriage: A Joint-Search Model

3.1 Introduction

In the year 2016, 50% of U.S. adults who are 18 or older are married, a share that has been relatively stable in recent years but dramatically different from several decades ago, when more than 70% of the adult population in the U.S. was married in 1960 (U.S. Census, 1960-2000; American Community Survey 1-year estimates).

Part of the decline in the share of married adults can be explained by the fact that Americans are marrying later in life nowadays. In 2016, the median age for a first marriage was 29.5 for men and 27.4 for women, roughly seven years more than the median ages in 1960 (22.8 for men and 20.3 for women). One major driving force behind this phenomenon is the overall increase in the labor force participation rate. As more people (especially women) choose to work full-time instead of part-time, the value of market production has gradually exceeded that of home production for most agents in the economy. A person that is more financially independent may also be "pickier" when it comes to marriage. Hence, participation in labor market activities provides a possible explanation for the downturn of marriage trends.

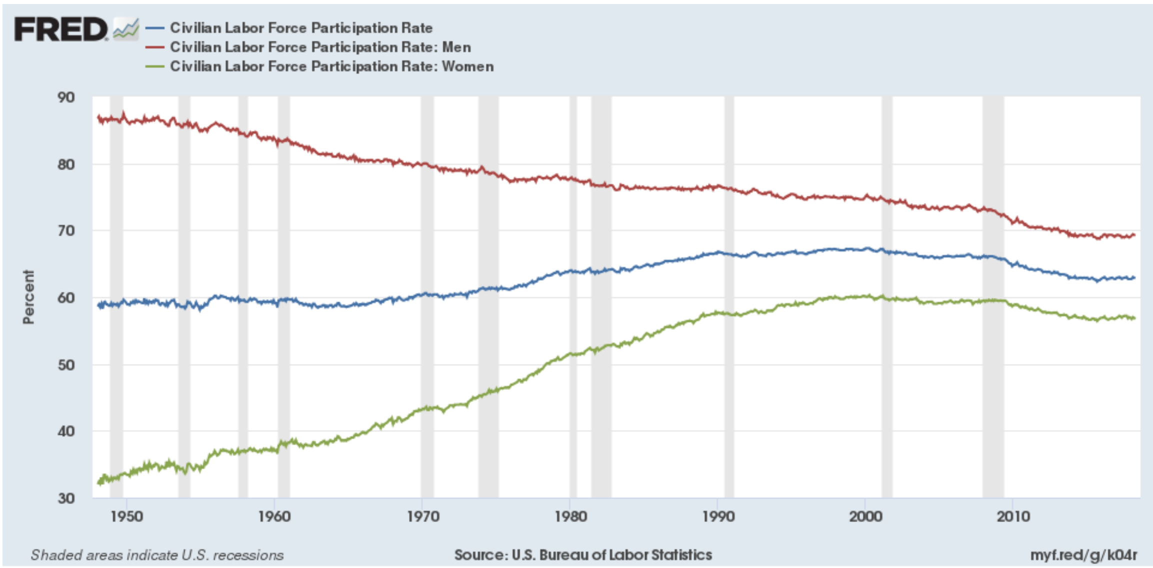


Figure 3.1: U.S. labor force participation rate

According to Becker (1974), positive assortative matching is generally optimal in the marriage market. The competition for spouses leads to sorting of mates by traits such as attractiveness, education, income, intelligence, and other characteristics. Suppose we shrink all market and non-market characteristics to a one-dimensional figure, which I refer to as an agent's type, and assume that agents in the economy have the same preferences for higher types over lower types. In that case, the highest type man marries the highest type woman, and the second-highest man forms a match with the highest woman, and so on. As shown in Becker (1974), one exception to this positive mating strategy, however, is sorting by earning power of men and women. The author suggests that high-income males should marry low-income females rather than marring their female counterparts since low-income females are more specialized in home production such as cooking, knitting, cleaning, etc. Unfortunately, this prediction seems to contradict what we observe in the marriage market today. Obviously, technological advances in the past few decades have made housework less burdensome. Consequently, more women choose to participate in the labor force, and fewer women choose to stay at home all day these days.

In this paper, I study the search problem of an agent where labor and marriage market outcomes are jointly determined. Standard search models such as McCall (1970) and Mortensen (1970) without on-the-job search, and Burdett (1978) with on-the-job-search concentrate primarily on the labor market. Generally speaking, the timing for the majority of the population getting a first full-time job is similar to that of getting married. To cope with this fact, it is useful to develop a model where agents search in both markets simultaneously. From an empirical perspective, an agent's labor market decision often affects his or her marriage decision and vice versa. If we think of marriage as an insurance policy that protects agents from various types of shock, such as job loss or disease, then changes in labor income could affect the decision to get married. Thus extending canonical labor market search models to a joint-search framework with both labor and marriage markets helps our understanding of the interaction between the two.

A building block for my framework is Guler et al. (2012). Their paper analyzes the optimal labor supply in a joint-search model and finds that married couples can pool their income and provide insurance to each other when unemployed. Depending on the agent's risk preferences, the authors conclude the individual searchers may have a different reservation wage behavior that affects search efforts and wage dynamics. This is mathematically similar to the question I consider. However, in my paper, marriage has additional values than just pooling income. Therefore, the equilibrium search behavior of individuals varies slightly. In essence, my model is an asymmetrical version of the model presented in Guler et al. (2012), with marriage values drawn from a distribution that is different from the wage distribution. Nevertheless, I find that under the assumption of risk-neutral agents, the equilibrium search behavior for dual markets searchers is comparable to that of single-market searchers.

To my best knowledge, several relevant papers have made some stellar findings on the question I investigated. Cornwell and Rupert (1997) investigate the source of the wage premium for married men. Using the National Longitudinal Survey of Young Men (NLSYM) data, they find that unobservable individual effects are positively correlated with marital status and wages. Lundberg and Pollak (1993, 1996) demonstrate how husbands and wives bargain in marriages and introduce the ”separate spheres” bargaining model, which differs from divorce threat bargaining models such as Manser and Brown (1980), McElroy and Horney (1981), etc. Cornelius (2003) builds a search model of marriage and divorce where individuals are able to search while matched. The author creates a good/bad companion framework to characterize equilibrium distributions.

The rest of my paper is organized as follows. 3.2 provides a benchmark search model with the focus on only the labor market for comparison. 3.3 extends the canonical search model to both the labor and the marriage market. 3.4 analyzes the equilibrium search behavior. 3.5 concludes and discusses possible future work.

3.2 Standard Labor Market Search Problem

I provide a canonical labor market search model as shown in McCall (1970) and Mortensen (1970). The McCall-Mortensen model makes comparison easier when I develop my dual markets search model in 3.3.

Consider an economy consists of identical agents who are either employed or unemployed (all agents participate in the labor force). Time is continuous. There is no uncertainty on the aggregate level. Individuals are risk-neutral and choose consumption to maximize their expected lifetime utility,

$$E_0 \int_0^{\infty} u(c(t))dt,$$

where r is the rate of time preference, $c(t)$ is the consumption at time t, and $u(\cdot)$ is the instantaneous utility function that is assumed to be strictly increasing and concave. If a worker is employed, he or she receives wage offers, w, and there is no exogenous job separation risk. If unemployed, a worker is entitled to a flow utility of b, and receives wage offers, w, at the rate α from a distribution of $F(w)$ with support $[0, \infty)$. $F(w)$ is assumed to be exogenous. When an unemployed worker gets a draw from the wage offer distribution, he or she decides whether to accept or reject the job offer. The worker becomes employed at wage w if he or she accepts the offer. If a wage offer is rejected, the agent continues to search until a future offer is accepted.

Furthermore, agents cannot recall past job offers. All individuals have the same arrival rate of job offers α, and face the same wage offer distribution $F(w)$. Since there is an absence of exogenous job separation, the wage-earning profile is nondecreasing over life cycle for all individuals in the economy.

Under risk neutrality, the problem of classic labor market search can be represented in the following equations using Bellman equations with continuous time:

$$rU = b + \alpha \int max\{W(w) - U, 0\} dF(w), \tag{3.1}$$

$$rW(w) = w. \tag{3.2}$$

U and $W(w)$ respectively denote the value functions of an unemployed and employed agent. Together, the two bellman equations above yield a well-known result of reservation wage, w_R, where an agent chooses to work for any wage offers above or equal to w_R, and stay unemployed for any wage offers below w_R. In other words, the agent is indifferent between working or not at w_R. The following equation characterizes the reservation wage:

$$w_R = b + \frac{\alpha}{r} \int_{w_R} (w - w_R) dF(w). \tag{3.3}$$

The left-hand side (LHS) of equation (3) is the value of accepting a job offer paying the reservation wage w_R. The right-hand side (RHS) of equation (3) equals the flow value of continue searching, hoping to get a better draw from the wage offer distribution, $F(w)$, in the future. Since the LHS increases in w_R and the RHS decreases in w_R, the solution of w_R is unique. A common algebra trick is to integrate by parts and apply Leibniz's rule to rewrite equation (3) as:

$$w_R = b + \frac{\alpha}{r} \int_{w_R} [1 - F(w)] dw. \tag{3.4}$$

3.3 Dual Markets Search Problem

Now I study the problem which an agent search in labor and marriage markets simultaneously. The economic environment agents facing is similar to the joint search framework of Guler et al. (2012). The labor market is the same as the situation described above. On the employment side, individual searchers can be either employed or unemployed. On the marriage market side, individuals can be either single or married. There are no exogenous separation shocks to employment or marriage. However, under certain circumstances, individuals can choose to endogenously quit the labor or the marriage market. All agents enter the economy as unemployed bachelors at time 0 and search for jobs or partners. There are four possible states in this economy: employed and married (EM), employed and single (ES), unemployed and married (UM), and unemployed and single (US). The state space, therefore, is $\{EM, ES, UM, US\}$.

If single, an agent receives no flow utility from the marriage market. This single worker receives flow utility from the labor market, depending on his or her employment status. If married, an agent enjoys utility from his or her income, plus the utility generated by having a partner. The value of a spouse, z, is primarily based on income but also contains information on traits such as attractiveness, companionship, health etc.

Similar to an income draw from the wage distribution, an individual samples a potential partner from a non-degenerate distribution, $G(z)$ with support $[0, \infty)$ and contacts at rate β. Let $\Theta(w, z)$ denote the value function for individuals in state EM, S be the value function for agents in state US, $V(w)$ be the value function for individuals in state ES, and $M(z)$ be the value function for individuals in state UM. Then the Bellman equations are the following:

$$r\Theta(w,z) = w + z, \tag{3.5}$$

$$rS = b + \alpha \int max\{V(w) - U, 0\}dF(w) + \beta \int max\{M(z) - U, 0\}dG(z), \tag{3.6}$$

$$rV(w) = w + \beta \int max\{\Theta(w,z) - V(w), M(z) - V(w), 0\}dG(z), \tag{3.7}$$

$$rM(z) = b + z + \alpha \int max\{\Theta(w,z) - M(z), V(w) - M(z), 0\}dF(w). \tag{3.8}$$

For each married and employed individual, he or she gets a utility of $w + z$ forever. For an unemployed single individual, he or she can either get a job, changing state from US to ES, or get married, switching from US to UM. For an employed single person, a marriage proposal arrives at rate β. An agent has three choices upon being proposed. First, he or she can keep the job and become married, which increases value by $\Theta(w,z)-V(w)$ for the rest of his or her life. Second, he or she can endogenously quit the job and get married, in which case the value changes from $V(w)$ to $M(z)$. Third, he or she can simply reject the marriage offer and continue to search for a partner. In the last case, an agent will never quit his or her job since quitting make things worse off.

Similarly, for an unemployed married person, a job offer arrives at a rate α. Again, he or she will have three choices upon being contacted. To begin with, he or she can become employed and married, and the value changes from $M(z)$ to $\Theta(w,z)$. What is more, an agent can quit marriage (divorce) and become employed, which increases value by $\Theta(w,z) - M(z)$. In addition, a person can stay status quo and turn down the job offer he or she receives. Once more, if an individual rejects the job offer, he or she will never quit the marriage.

3.4 Results

Before I present my findings, the following lemma helps to characterize reservation values later in this section.

Lemma 1 $V(w)$ and $M(z)$ are strictly increasing in w and z, respectively.

$$\frac{\partial V(w)}{\partial w} > 0, \frac{\partial M(z)}{\partial z} > 0.$$

Proof: See appendix B.5. ■

3.4.1 The Decision For A Unemployed Single Individual

For individuals searching in both the labor and the marriage market, the decision to become employed or married is simple: enter the labor market if the wage offer is higher than the job seeker's reservation wage or enter the marriage market if the spouse's value exceeds the partner seeker's reservation value. Let w^* and z^* denote the reservation values for employment and marriage, respectively. The above condition can be demonstrated by the following:

$$V(w^*) = M(z^*) = S. \tag{3.9}$$

3.4.2 The Marriage Decision For A Worker

When a marriage opportunity arrives for an employed worker, the decision rule depends on the relative magnitude between $M(z)$ and $\Theta(w, z)$. There are two cases to consider. First, I assume $M(z) < \Theta(w, z)$. That is, the value of becoming employed and married exceeds that of just married but unemployed. In this case, if $\Theta(w, z) \geq V(w)$, then it is optimal for the worker to accept the marriage proposal and change state from ES to EM. However, if $\Theta(w, z) < V(w)$, then the agent is better off by rejecting the marriage offer. He or she stays in state ES and continues to search for a partner. The reservation value for marriage $\phi(w)$, when $M(z) < \Theta(w, z)$, is be obtained from the following equation:

$$\Theta(w, \phi(w)) = V(w). \tag{3.10}$$

Second, assume that $M(z) \geq \Theta(w, z)$. That is, the value of becoming employed and married is less than that of just married but unemployed. In this scenario, if $M(z) \geq V(w)$, then the agent is willing to substitute work with marriage. He or she will get married but quit working, switching from ES to UM. If $M(z) < V(w)$, the agent will reject the marriage proposal. He or she keeps working while remaining single. The reservation value for marriage $\phi(w)$, when $M(z) \geq \Theta(w,z)$, is written as the following:

$$M(\phi(w)) = V(w). \tag{3.11}$$

3.4.3 The Labor Market Decision For A Married Person

On the other hand, When a job offer arrives for a married person, the decision rule depends on the relative magnitude between $V(w)$ and $\Theta(w,z)$. Again, I consider two different scenarios. Let me begin by assuming that $V(w) < \Theta(w,z)$. That is, the value of becoming employed and married exceeds that of just employed but remain single. In this case, if $\Theta(w,z) \geq M(z)$, then it is optimal for the worker to accept the job offer and changes state from UM to EM. However, if $\Theta(w,z) < M(z)$, then the agent is better off by rejecting the wage offer. He or she stays in state UM and continues to search for a better job. The reservation wage $\psi(z)$, when $V(w) < \Theta(w,z)$, is obtained from the following equation:

$$\Theta(\psi(z), z) = M(z). \tag{3.12}$$

Now, I instead assume $V(w) \geq \Theta(w,z)$. That is, the value of becoming employed and married is less than that of just employed but remain single. In this case, if $V(w) \geq M(z)$, then the agent is willing to substitute marriage with work. He or she will become employed but quit marriage, changing state from UM to ES. If $V(w) < M(z)$, the agent will reject w. He or she keeps looking for a job while remaining married. The reservation wage $\psi(z)$, when $V(w) \geq \Theta(w,z)$, is characterized by:

$$V(\psi(z)) = M(z). \tag{3.13}$$

Proposition 11 *Under risk-neutral preferences, the reservation wage of an individual searching in dual markets is identical to that of a person searching only in the labor market,* $i.e. w^* = w_R$.

Proof: See appendix B.6. ■

Figure 3.2 shows the equilibrium search behavior for individuals simultaneously searching in the labor and marriage market in the (w, z) space. For unemployed single individuals, they will change state from US to ES once they get a wage offer with $w \geq w^*$, or switch to UM when $z \geq z^*$. For employed single individuals, they will change state from ES to UM or EM when $z \geq \phi(w)$, depending on the value of $\Theta(w, z)$ and $M(z)$. On the other hand, for unemployed married individuals, they will switch from UM to ES or EM if $w \geq \psi(z)$, depending on the value of $\Theta(w, z)$ and $V(w)$.

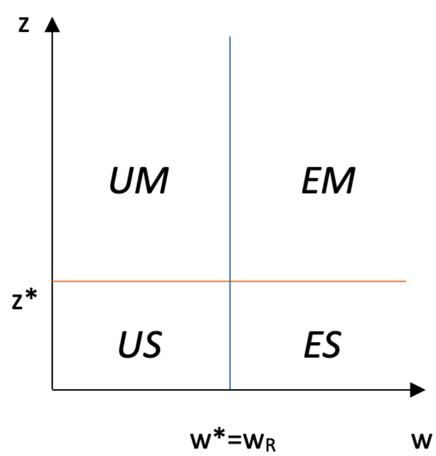


Figure 3.2: Reservation wage for dual markets searchers

3.5 Conclusion

In this preliminary work, I study the decision-making problem for individuals searching in both the labor market and the marriage market at the same time. Compared to the canonical labor search model, the reservation wage have the same characteristics in the dual markets search model under risk neutrality.

There are several directions for future work. First and foremost, it is worth examining the equilibrium search behavior for risk-averse agents or risk-loving agents. As demonstrated in Guler et al. (2012), different types of risk preferences will alter the optimal joint-search behavior for couples.

Another extension would be adding exogenous separation into the model. This can be exogenous job destruction on the labor market side, or constant divorce risk on the marriage market side. Allowing on-the-job search would even extend my model further. Additionally, it would be interesting to consider spatial mismatch. That is, whether the agent's search behavior changes if the ideal job or spouse is in a different location.

Appendix A

Empirical Appendix

A.1 Data source and construction

Data on sorting patterns in the U.S. labor market from 1994 to 2017 comes from: (i) the March Current Population Survey (March CPS), (ii) the Occupational Information Network (O*NET) from the U.S. Department of Labor.[1] High education indicates the highest degree obtained by a person is bachelor's or above. Low education suggests the highest degree received by a person is associate degree or below. People with some college experience are categorized as low education, as long as they have not been granted a college degree. Full-time means working time exceeds 35 hours weekly, and full-year indicates working time surpasses 50 weeks annually. The O*NET job zones group occupations into one of five categories based on levels of education, experience, and training necessary to perform the occupation. Table A.1 summarizes information on job zones.[2]

[1] The March CPS data is downloaded from the Center for Economic and Policy Research at https://ceprdata.org.

[2] For a detailed description of each occupation with requirements on education, related experience, job training, job zone samples, and SVP range, see https://www.onetonline.org/find/zone.

Simple jobs are occupations in job zones 1,2, and 3, while complex jobs correspond to job zones 4 and 5. I match worker's occupation using the 4-digit Census code in the March CPS data with the job zone using the Standard Occupational Classification (SOC) code in O*NET.

A.2 Employment Shares of Workers by Job Types

Table A.2 summarizes the employment shares of workers by simple and complex jobs from 1994 to 2017. For notational convenience, $P(L, L)$ denotes the share of low-educated workers in simple jobs, $P(H, L)$ denotes the share of high-educated workers in simple jobs, $P(L, H)$ denotes the share of low-educated workers in complex jobs, and $P(H, H)$ denotes the share of high-educated workers in complex jobs.

A.3 Real Weekly Wages

Using 2017 as the base year, Table A.3 summarizes the average real weekly labor income of each worker-job group. The notations follow a similar fashion as seen in Appendix A.2: $w(L, L)$ denotes the mean weekly labor income of low-educated workers in simple jobs, $w(H, L)$ denotes the mean weekly labor income of high-educated workers in simple jobs, $w(L, H)$ denotes the mean weekly labor income of low-educated workers in complex jobs, and $w(H, H)$ denotes the mean weekly labor income of high-educated workers in complex jobs.

Job Zone	Description
1	Little or no preparation needed
2	Some preparation needed
3	Medium preparation needed
4	Considerable preparation needed
5	Extensive preparation needed

Table A.1: O*NET job zones

Year	$P(L,L)$	$P(H,L)$	$P(L,H)$	$P(H,H)$
1994	0.878	0.122	0.380	0.620
1995	0.876	0.124	0.392	0.608
1996	0.876	0.124	0.396	0.604
1997	0.873	0.127	0.389	0.611
1998	0.874	0.126	0.386	0.614
1999	0.877	0.123	0.380	0.620
2000	0.875	0.125	0.389	0.611
2001	0.862	0.138	0.379	0.621
2002	0.867	0.133	0.391	0.609
2003	0.867	0.133	0.390	0.610
2004	0.866	0.134	0.386	0.614
2005	0.863	0.137	0.383	0.617
2006	0.860	0.140	0.374	0.626
2007	0.848	0.152	0.342	0.658
2008	0.844	0.156	0.336	0.664
2009	0.829	0.171	0.334	0.666
2010	0.835	0.165	0.318	0.682
2011	0.830	0.170	0.312	0.688
2012	0.824	0.176	0.309	0.691
2013	0.827	0.173	0.311	0.689
2014	0.824	0.176	0.312	0.688
2015	0.799	0.201	0.303	0.697
2016	0.820	0.180	0.299	0.701
2017	0.810	0.190	0.296	0.704

Table A.2: Summary statistics for sorting patterns

Year	$w(L,L)$	$w(H,L)$	$w(L,H)$	$w(H,H)$
1994	780.41	1115.52	1103.81	1544.42
1995	784.26	1207.22	1194.00	1734.99
1996	803.76	1198.66	1174.35	1780.66
1997	811.38	1206.96	1185.35	1810.87
1998	811.49	1281.16	1227.19	1860.49
1999	822.95	1225.09	1234.17	1814.23
2000	832.15	1250.25	1269.70	1975.87
2001	831.18	1290.80	1291.95	2013.92
2002	836.77	1278.41	1244.81	1983.94
2003	837.89	1204.27	1220.17	1980.21
2004	828.19	1222.99	1234.32	1976.48
2005	810.05	1175.30	1232.90	1987.64
2006	813.29	1222.32	1192.12	1985.16
2007	816.19	1217.67	1238.88	1908.99
2008	803.40	1206.12	1221.07	1922.99
2009	822.08	1212.72	1238.88	1966.76
2010	806.26	1177.42	1253.26	1895.91
2011	816.74	1158.02	1217.88	1931.59
2012	806.12	1161.84	1203.83	1901.53
2013	799.94	1178.17	1222.73	1864.79
2014	798.91	1162.80	1253.61	1853.81
2015	824.24	1205.88	1245.18	1932.71
2016	843.50	1177.24	1282.97	1972.11
2017	837.92	1226.15	1276.07	1947.73

Table A.3: Summary statistics for real weekly wages

A.4 Comparative Statics

	$f(L,L)$				
	1.856	1.956	2.056	2.156	2.256
	market tightness				
$\lambda(L,L)$	**0.601**	**0.569**	**0.542**	**0.518**	**0.496**
$\lambda(H,L)$	0.901	0.901	0.901	0.901	0.901
$\lambda(L,H)$	0.230	0.230	0.230	0.230	0.230
$\lambda(H,H)$	0.356	0.356	0.356	0.356	0.356
	wages				
$w(L,L)$	**0.964**	**0.982**	**1.000**	**1.017**	**1.034**
$w(H,L)$	1.479	1.479	1.479	1.479	1.479
$w(L,H)$	1.504	1.504	1.504	1.504	1.504
$w(H,H)$	2.326	2.326	2.326	2.326	2.326
	sorting patterns				
$P(L,L)$	**0.199**	**0.539**	**0.850**	**0.966**	**0.993**
$P(H,L)$	**0.801**	**0.461**	**0.150**	**0.034**	**0.007**
$P(L,H)$	0.354	0.354	0.354	0.354	0.354
$P(H,H)$	0.646	0.646	0.646	0.646	0.646

Table A.4: Effects of changing production $f(L,L)$

	$f(H,H)$				
	7.462	7.562	7.662	7.762	7.862
	market tightness				
$\lambda(L,L)$	0.542	0.542	0.542	0.542	0.542
$\lambda(H,L)$	0.901	0.901	0.901	0.901	0.901
$\lambda(L,H)$	0.230	0.230	0.230	0.230	0.230
$\lambda(H,H)$	**0.363**	**0.359**	**0.356**	**0.352**	**0.349**
	wages				
$w(L,L)$	1.000	1.000	1.000	1.000	1.000
$w(H,L)$	1.479	1.479	1.479	1.479	1.479
$w(L,H)$	1.504	1.504	1.504	1.504	1.504
$w(H,H)$	**2.300**	**2.313**	**2.326**	**2.339**	**2.352**
	sorting patterns				
$P(L,L)$	0.850	0.850	0.850	0.850	0.850
$P(H,L)$	0.150	0.150	0.150	0.150	0.150
$P(L,H)$	**0.524**	**0.437**	**0.354**	**0.278**	**0.214**
$P(H,H)$	**0.476**	**0.563**	**0.646**	**0.722**	**0.786**

Table A.5: Effects of changing production $f(H,H)$

	$f(H,L)$				
	2.446	2.546	2.646	2.746	2.846
	market tightness				
$\lambda(L,L)$	0.542	0.542	0.542	0.542	0.542
$\lambda(H,L)$	**0.987**	**0.941**	**0.901**	**0.864**	**0.831**
$\lambda(L,H)$	0.230	0.230	0.230	0.230	0.230
$\lambda(H,H)$	0.356	0.356	0.356	0.356	0.356
	wages				
$w(L,L)$	1.000	1.000	1.000	1.000	1.000
$w(H,L)$	**1.430**	**1.455**	**1.479**	**1.502**	**1.525**
$w(L,H)$	1.504	1.504	1.504	1.504	1.504
$w(H,H)$	2.326	2.326	2.326	2.326	2.326
	sorting patterns				
$P(L,L)$	**0.986**	**0.953**	**0.850**	**0.606**	**0.290**
$P(H,L)$	**0.014**	**0.047**	**0.150**	**0.394**	**0.710**
$P(L,H)$	0.354	0.354	0.354	0.354	0.354
$P(H,H)$	0.646	0.646	0.646	0.646	0.646

Table A.6: Effects of changing production $f(H,L)$

	$f(L,H)$				
	5.988	6.088	6.188	6.288	6.388
	market tightness				
$\lambda(L,L)$	0.542	0.542	0.542	0.542	0.542
$\lambda(H,L)$	0.901	0.901	0.901	0.901	0.901
$\lambda(L,H)$	**0.235**	**0.232**	**0.230**	**0.227**	**0.225**
$\lambda(H,H)$	0.356	0.356	0.356	0.356	0.356
	wages				
$w(L,L)$	1.000	1.000	1.000	1.000	1.000
$w(H,L)$	1.479	1.479	1.479	1.479	1.479
$w(L,H)$	**1.485**	**1.495**	**1.504**	**1.513**	**1.523**
$w(H,H)$	2.326	2.326	2.326	2.326	2.326
	sorting patterns				
$P(L,L)$	0.850	0.850	0.850	0.850	0.850
$P(H,L)$	0.150	0.150	0.150	0.150	0.150
$P(L,H)$	**0.202**	**0.271**	**0.354**	**0.446**	**0.543**
$P(H,H)$	**0.798**	**0.729**	**0.646**	**0.554**	**0.457**

Table A.7: Effects of changing production $f(L,H)$

	κ_L				
	0.101	0.151	0.201	0.251	0.301
	market tightness				
$\lambda(L,L)$	**0.332**	**0.438**	**0.542**	**0.647**	**0.754**
$\lambda(H,L)$	0.901	0.901	0.901	0.901	0.901
$\lambda(L,H)$	**0.153**	**0.193**	**0.230**	**0.264**	**0.296**
$\lambda(H,H)$	0.356	0.356	0.356	0.356	0.356
	wages				
$w(L,L)$	**0.833**	**0.924**	**1.000**	**1.067**	**1.128**
$w(H,L)$	1.479	1.479	1.479	1.479	1.479
$w(L,H)$	**1.191**	**1.361**	**1.504**	**1.630**	**1.745**
$w(H,H)$	2.326	2.326	2.326	2.326	2.326
	sorting patterns				
$P(L,L)$	**0.999**	**0.986**	**0.850**	**0.417**	**0.110**
$P(H,L)$	**0.001**	**0.014**	**0.150**	**0.583**	**0.890**
$P(L,H)$	**0.870**	**0.628**	**0.354**	**0.173**	**0.082**
$P(H,H)$	**0.130**	**0.372**	**0.646**	**0.827**	**0.918**

Table A.8: Effects of changing entry cost κ_L

	κ_H				
	0.398	0.448	0.498	0.548	0.598
	market tightness				
$\lambda(L,L)$	0.542	0.542	0.542	0.542	0.542
$\lambda(H,L)$	**0.735**	**0.816**	**0.901**	**0.988**	**1.080**
$\lambda(L,H)$	0.230	0.230	0.230	0.230	0.230
$\lambda(H,H)$	**0.306**	**0.331**	**0.356**	**0.380**	**0.404**
	wages				
$w(L,L)$	1.000	1.000	1.000	1.000	1.000
$w(H,L)$	**1.368**	**1.425**	**1.479**	**1.530**	**1.580**
$w(L,H)$	1.504	1.504	1.504	1.504	1.504
$w(H,H)$	**2.125**	**2.228**	**2.326**	**2.419**	**2.508**
	sorting patterns				
$P(L,L)$	**0.218**	**0.575**	**0.850**	**0.954**	**0.986**
$P(H,L)$	**0.782**	**0.425**	**0.150**	**0.046**	**0.014**
$P(L,H)$	**0.115**	**0.215**	**0.354**	**0.511**	**0.657**
$P(H,H)$	**0.885**	**0.785**	**0.646**	**0.489**	**0.343**

Table A.9: Effects of changing entry cost κ_H

	μ				
	0.106	0.116	0.126	0.136	0.146
	market tightness				
$\lambda(L,L)$	0.542	0.542	0.542	0.542	0.542
$\lambda(H,L)$	0.901	0.901	0.901	0.901	0.901
$\lambda(L,H)$	0.230	0.230	0.230	0.230	0.230
$\lambda(H,H)$	0.356	0.356	0.356	0.356	0.356
	wages				
$w(L,L)$	1.000	1.000	1.000	1.000	1.000
$w(H,L)$	1.479	1.479	1.479	1.479	1.479
$w(L,H)$	1.504	1.504	1.504	1.504	1.504
$w(H,H)$	2.326	2.326	2.326	2.326	2.326
	sorting patterns				
$P(L,L)$	**0.968**	**0.909**	**0.850**	**0.801**	**0.762**
$P(H,L)$	**0.032**	**0.091**	**0.150**	**0.199**	**0.238**
$P(L,H)$	**0.340**	**0.347**	**0.354**	**0.360**	**0.366**
$P(H,H)$	**0.660**	**0.653**	**0.646**	**0.640**	**0.634**

Table A.10: Effects of changing market noise μ

	b				
	0.228	0.328	0.428	0.528	0.628
	market tightness				
$\lambda(L,L)$	**0.496**	**0.518**	**0.542**	**0.569**	**0.601**
$\lambda(H,L)$	**0.831**	**0.864**	**0.901**	**0.941**	**0.987**
$\lambda(L,H)$	**0.225**	**0.227**	**0.230**	**0.232**	**0.235**
$\lambda(H,H)$	**0.349**	**0.352**	**0.356**	**0.359**	**0.363**
	wages				
$w(L,L)$	**0.834**	**0.917**	**1.000**	**1.082**	**1.164**
$w(H,L)$	**1.325**	**1.402**	**1.479**	**1.555**	**1.630**
$w(L,H)$	**1.323**	**1.413**	**1.504**	**1.595**	**1.685**
$w(H,H)$	**2.152**	**2.239**	**2.326**	**2.413**	**2.500**
	sorting patterns				
$P(L,L)$	**0.912**	**0.884**	**0.850**	**0.808**	**0.757**
$P(H,L)$	**0.088**	**0.116**	**0.150**	**0.192**	**0.243**
$P(L,H)$	**0.371**	**0.362**	**0.354**	**0.346**	**0.338**
$P(H,H)$	**0.629**	**0.638**	**0.646**	**0.654**	**0.662**

Table A.11: Effects of changing benefit b

Appendix B

Derivations and Proofs

B.1 Alternative Model

In this section, I discuss the case where workers post terms of trade and firms search. Let $V(x,y)$ denote a firm's utility and $T(x,y)$ denote a worker's expected payoff in the corresponding submarket. A worker chooses $\{y, w(x,y), \lambda(x,y)\}$ to maximize $T(x,y)$, subject to offering its employer $V(x,y)$. The worker's problem is:

$$T(x,y) = \max_{y \in \{L,H\}} \big\{W(x,L) + \mu z_{xL}, V(x,H) + \mu z_{xH}\big\},$$

where μz_{xy} denotes workers' match-specific shock. $W(x,y)$ is the product of trade probability and match surplus:

$$\begin{aligned} W(x,y) &= \max_{w(x,y),\lambda(x,y)} \Big\{ m(\lambda(x,y))\big[w(x,y) - b\big] \Big\} \\ s.t. \quad V(x,y) &= \frac{m(\lambda(x,y))}{\lambda(x,y)}\big[f(x,y) - w(x,y)\big]. \end{aligned}$$

In this model, the equilibrium consists of a set of optimal queue lengths $\lambda(x,y)$, wage offers $w(x,y)$, utility for firms $V(x,y)$, and payoffs for workers $T(x,y)$. To solve for equilibrium, I follow again the *market utility approach*. Rewrite the worker's problem by substituting wages $w(x,y)$:

$$W(x,y) = \max_{\lambda(x,y)} \Big\{ m(\lambda(x,y))\big[f(x,y) - b\big] - \lambda(x,y)V(x,y) \Big\}.$$

First-order condition with respect to $\lambda(x,y)$ yields the following expression for firms:

$$V(x,y) = m'(\lambda(x,y))\big[f(x,y) - b\big].$$

Now, imposing entry cost κ_y on firms to get the equilibrium condition and pins down the optimal queue length $\lambda^*(x,y)$:

$$\kappa_y = m'(\lambda^*(L,y)\big[f(L,y) - b\big] = m'(\lambda^*(H,y))\big[f(H,y) - b\big].$$

Define $\varepsilon(\lambda(x,y)) \equiv \lambda(x,y)m'(\lambda(x,y))/m(\lambda(x,y))$ as the elasticity of $m(\lambda(x,y))$ with respect to queue length $\lambda(x,y)$. Given the indifference condition above, one can obtain unique solutions for $\{w(x,y), W(x,y), V(x,y)\}$:

$$\begin{aligned}
w^*(x,y) &= \varepsilon(\lambda^*(x,y))b + \big[1 - \varepsilon(\lambda^*(x,y))\big]f(x,y), \\
W^*(x,y) &= \big[m(\lambda^*(x,y)) - \lambda^*(x,y)m'(\lambda^*(x,y))\big]\big[f(x,y) - b\big], \\
V^*(x,y) &= m'(\lambda^*(x,y))\big[f(x,y) - b\big].
\end{aligned}$$

Note the equilibrium solution is identical to the model where firms post wages and workers search.

B.2 Proof to Proposition 1

From equation (5) and (6), $f(x, L) < f(x, H)$ implies:

$$\big[m(\lambda^*(x, L)) - \lambda^*(x, L)m'(\lambda^*(x, L))\big] > \big[m(\lambda^*(x, H)) - \lambda^*(x, H)m'(\lambda^*(x, H))\big].$$

Note that $m(\lambda) - \lambda m'(\lambda)$ is increasing in λ:

$$\frac{d[m(\lambda) - \lambda m'(\lambda)]}{d\lambda} = -\lambda m''(\lambda) > 0,$$

since $\lambda > 0$ under any interior solution and $m''(\lambda) < 0$. Therefore, $\lambda^*(x, L) > \lambda^*(x, H)$. Because $m(\lambda)$ is increasing an concave, $m(\lambda^*(x, L)) > m(\lambda^*(x, H)), m'(\lambda^*(x, L)) < m'(\lambda^*(x, H))$. Given the worker's indifference condition $W(x, L) = W(x, H), m(\lambda^*(x, L)) > m(\lambda^*(x, H))$ suggests $w^*(x, L) < w^*(x, H)$ using equation (2). Lastly, $V^*(x, L) < V^*(x, H)$ is implied by $m'(\lambda^*(x, L)) < m'(\lambda^*(x, H))$ and $f(x, L) < f(x, H)$.

B.3 Derivation for $P(x, y)$

Following Train (2009), the probability that a type y firm choose to match with a type x worker is:

$$\begin{aligned} P(x, y) &= Pr\big[U(x, y) \geq U(x', y)\big] \\ &= Pr\big[V^*(x, y) + \mu z_{xy} \geq V^*(x', y) + \mu z_{x'y}\big] \\ &= Pr\left[z_{x'y} \leq z_{xy} + \frac{V^*(x, y)}{\mu} - \frac{V^*(x', y)}{\mu}\right]. \end{aligned}$$

If z_{xy} is given, the expression equals to the cumulative distribution function of $z_{x'y}$ evaluated at $\left(z_{xy} + \frac{V^*(x,y)}{\mu} - \frac{V^*(x',y)}{\mu}\right)$:

$$P(x, y)|z_{xy} = e^{-e^{-\left(z_{xy}+\frac{V^*(x,y)}{\mu}-\frac{V^*(x',y)}{\mu}\right)}}.$$

However, z_{xy} is not given. So the choice probability is the integral of $P(x, y)|z_{xy}$ over all values of z_{xy} weighted by its density:

$$P(x, y) = \int_{-\infty}^{\infty} \left(e^{-e^{-\left(z_{xy}+\frac{V^*(x,y)}{\mu}-\frac{V^*(x',y)}{\mu}\right)}}\right) e^{-z_{xy}} e^{-e^{-z_{xy}}}\, dz_{xy}.$$

Rewrite $P(x, y)$ as:

$$P(x, y) = \int_{s=-\infty}^{\infty} \left(e^{-e^{-\left(s+\frac{V^*(x,y)}{\mu}-\frac{V^*(x',y)}{\mu}\right)}}\right) e^{-s} e^{-e^{-s}}\, ds,$$

where $s = z_{xy}$. To evaluate the integral, we have:

$$\begin{aligned}
P(x, y) &= \int_{s=-\infty}^{\infty} \left(e^{-e^{-\left(s+\frac{V^*(x,y)}{\mu}-\frac{V^*(x',y)}{\mu}\right)}-e^{-s}}\right) e^{-s}\, ds \\
&= \int_{s=-\infty}^{\infty} \left(e^{-e^{-s}\left[1-\left(\frac{V^*(x,y)}{\mu}-\frac{V^*(x',y)}{\mu}\right)\right]}\right) e^{-s}\, ds \\
&= \int_{s=-\infty}^{\infty} \left(e^{-e^{-s}\left[1+\frac{V^*(x',y)}{\mu}-\frac{V^*(x,y)}{\mu}\right]}\right) e^{-s}\, ds.
\end{aligned}$$

Define $t = e^{-s}$ such that $-e^{-s}ds = dt$. Note that $t \to 0$ as $s \to \infty$ and $t \to \infty$ as $s \to -\infty$. Therefore,

$$
\begin{aligned}
P(x,y) &= \int_{t=\infty}^{0} e^{-t\left[1+\frac{V^*(x',y)}{\mu}-\frac{V^*(x,y)}{\mu}\right]}(-dt) \\
&= \int_{t=0}^{\infty} e^{-t\left[1+\frac{V^*(x',y)}{\mu}-\frac{V^*(x,y)}{\mu}\right]}dt \\
&= \left.\frac{e^{-t\left[1+\frac{V^*(x',y)}{\mu}-\frac{V^*(x,y)}{\mu}\right]}}{-\left[1+\frac{V^*(x',y)}{\mu}-\frac{V^*(x,y)}{\mu}\right]}\right|_{t=0}^{\infty} \\
&= \frac{1}{1+e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}}.
\end{aligned}
$$

B.4 Proof to Proposition 2

Given that $m'(\lambda) > 0, m''(\lambda) < 0, \mu > 0$ and $f(x,y) > b$, one can verify the following:

$$
\begin{aligned}
\frac{\partial P(x,y)}{\partial f(x,y)} &= \frac{e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}}{\left(1+e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}\right)^2} \cdot \frac{m'(\lambda^*(x,y))}{\mu} > 0, \\
\frac{\partial P(x,y)}{\partial f(x',y)} &= \frac{-e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}}{\left(1+e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}\right)^2} \cdot \frac{m'(\lambda^*(x',y))}{\mu} < 0, \\
\frac{\partial P(x,y)}{\partial b} &= \frac{e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}}{\left(1+e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}\right)^2} \cdot \frac{m'(\lambda^*(x,y)) - m'(\lambda^*(x',y))}{\mu} \lesseqgtr 0.
\end{aligned}
$$

For $\frac{\partial P(x,y)}{\partial \kappa_x}$ and $\frac{\partial P(x,y)}{\partial \kappa_{x'}}$, first note that:

$$
\frac{\partial P(x,y)}{\partial \lambda(x,y)} = \frac{e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}}{\left(1+e^{\frac{V^*(x',y)-V^*(x,y)}{\mu}}\right)^2} \cdot \frac{m''(\lambda^*(x,y))}{\mu} \cdot \left[f(x,y) - b\right] < 0.
$$

From equation (5) and (6), we have:

$$\frac{\partial \lambda(x,y)}{\partial \kappa_x} > 0, \frac{\partial \lambda(x,y)}{\partial \kappa_{x'}} < 0,$$

as $m(\lambda) - \lambda m'(\lambda)$ is increasing in λ. Holding match value constant, $m(\lambda) - \lambda m'(\lambda)$ increases when κ goes up. Therefore,

$$\frac{\partial P(x,y)}{\partial \kappa_x} = \frac{\partial P(x,y)}{\partial \lambda(x,y)} \cdot \frac{\partial \lambda(x,y)}{\partial \kappa_x} < 0,$$
$$\frac{\partial P(x,y)}{\partial \kappa_{x'}} = \frac{\partial P(x,y)}{\partial \lambda(x,y)} \cdot \frac{\partial \lambda(x,y)}{\partial \kappa_{x'}} > 0.$$

B.5 Proof for Lemma 1

Rewrite equations (7) and (8) as the following:

$$rV(w) = w + \beta\gamma(w),$$

$$rM(z) = b + z + \alpha\zeta(z),$$

where

$$\gamma(w) = \int max\left\{\frac{\Theta(w,z)}{r} - V(w), M(z) - V(w), 0\right\} dG(z),$$
$$\zeta(z) = \int max\left\{\frac{\Theta(w,z)}{r} - M(z), V(w) - M(z), 0\right\} dF(w).$$

Assume $V(w)$ is non-increasing in w, then $\gamma(w)$ is also non-decreasing in w. Therefore, $w + \beta\gamma(w)$ becomes strictly increasing in w, which contradicts my assumption. Hence, $V(w)$ must be strictly increasing in w. The proof for $M(z)$ is similar.

B.6 Proof for Proposition 11

From equation (10), we have:

$$\begin{aligned} V(w^*) &= b + \alpha \int max\{V(w) - U, 0\} dF(w) + \beta \int max\{M(z) - U, 0\} dG(z) \\ &= b + \frac{\alpha}{r} \int_{w^*} [1 - F(w)] dw + \frac{\beta}{r} \int_{w^*} [1 - G(z)] dz. \end{aligned}$$

At $w = w_R$, equation (7) can be rewritten as:

$$V(w_R) = w_R + \frac{\beta}{r} \int_{w_R} [1 - G(z)] dz.$$

Combining the two equations above, we have:

$$\begin{aligned} V(w^*) - V(w_R) &= w_R + \frac{\beta}{r} \int_{w_R} [1 - G(z)] dz - b + \frac{\alpha}{r} \int_{w^*} [1 - F(w)] dw - \frac{\beta}{r} \int_{w^*} [1 - G(z)] dz \\ &= w_R - b - \frac{\alpha}{r} \int_{w^*} [1 - F(w)] dw + \frac{\beta}{r} \int_{w_R}^{w^*} [1 - G(z)] dz. \end{aligned}$$

At $w^* = w_R$, both sides of the equation above equal to zero since:

$$w_R = b + \frac{\alpha}{r} \int_{w^*} [1 - F(w)] dw.$$

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