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Essays on Contracting: Explicit Managerial Contracts and Implict Relational Influence Contracts

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Essays on Contracting: Explicit Managerial Contracts and Implict Relational Influence Contracts

by

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Abstract

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Contracts are an economic tool used to arrange transactions which are not tradable in simple spot markets. This thesis focuses on the implications of different kinds of contracts to understand behavior in complicated interactions. The first part of this thesis focuses on explicit formal contracts that provide non-linear payoffs and examines theoretically and empirically the implications for effort and risk-taking. The second part of this thesis focuses in contrast on implicit contracts. Starting from a theoretical perspective about how implicit contracts for influence buying might work in a setting that precludes explicit contracts. This helps explains empirical puzzles as well as has new predictions. I then show empirical evidence consistent with the predictions.

In the first part, I explore managerial incentive contracts. Managerial incentives induce risk-taking as well as effort. Theoretical research has long considered risk-taking a potential side effect of incentives, but empirical investigation is limited. This thesis first develops nuanced predictions about how contracts in use in many industries induce risk-taking and effort. The contracts considered match closely those used in real-world contracts. The thesis then uses exogenous variation in hedge fund manager's incentives, one of the settings where these contracts are used, to examine both performance and risk-taking. I find that, consistent with theory, being farther below a key incentive threshold increases risk-taking and decreases performance. On average, a manager's risk-taking increases 50 percent and their performance falls 2.1 percentage points when he is below the incentive threshold. I also show, consistent with the theoretical predictions, risk-taking behavior is non-monotonic; very distant managers take less risk and perform better than less distant managers. Further, I examine the role of organizational features in impacting the responsiveness to explicit incentives and the mechanisms managers use to increase risk. My results highlight the importance of risk-taking in response to incentives designed to induce effort and inform empirical research, contract design, practitioners, and policy makers. The results also show that moral hazard, not just selection, is an important determination of manager performance.

In the second part, I explore contracts for influence buying. Existing empirical evidence that finds very high actual or potential return to some campaign contributions and wonders, if contributions buy influence, why more exchange does not occur. Other empirical work has found consistent long-term relationships of contributions from interest groups to politicians. Yet, models of influence buying have treated the exchange as a simple spot transaction. This paper develops a formal model of relational influence buying between a firm and a politician where campaign contributions are exchanged for policy favors in a self-enforcing contract. This contract provides several insights. First, not all favors that have positive joint surplus to the firm and politician are contractible. Second, the model predicts that horizons of politicians will reduce the ability to raise funds. Third, the model provides empirical predictions for when firms should lobby themselves or outsource and on the structure of legislation. The first can explain why more, apparently valuable, trade does not occur. I find evidence consistent with the horizon effects from US Congress people's age and term limits in US state legislatures. The third insight speaks both to potential regulatory implications and implications for managers' influence activities. Finally, the insights from the model suggest empirical tools to detect influence buying without directly observing the favors.

To my loving and supportive wife and adoring daughters.

Contents

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Introduction

Contracts are an economic tool used to arrange transactions which are not tradable in simple spot markets. The economic notion of contracts is broader than the legal one - contracts need not be explicit, written, or even negotiated. They may simply capture the understandings each party has of expected behavior and consequences. On the other hand, contracts may be formal, explicit, legal contracts that govern exchanges. This thesis explores the implications of both types of contracts in specific settings. The first part of the thesis focuses on a positive question, not about the contracts themselves, but about the behavior they induce. This research depends on having information about the incentives provided by the contracts and measures of the behaviors incented. To do this, I focus on a setting with rich data about explicit contracts and resulting actions. This work strives to inform the design and understanding of contracts rather than test a normative view of contract terms. The second part of the thesis focuses on implicit contracts that are unobserved and some of the incented activities are difficult to measure. With those empirical limitations, the focus is on understanding what contracts would be feasible and the implications of the ability or inability to contract on behavior.

Managerial incentives for risk-taking are crucial to understanding how compensation schemes affect the performance and behavior of firms. Understanding managerial risk-taking, and how to manage it, is also more broadly important. As the run up to the deepest recession since the Great Depression demonstrated, risk-taking by managers can have drastic consequences not just for their own firms but also for the global economy. Practically speaking the growth of risk management institutions within firms suggests that risk-taking is a fundamental issue of which firms of all kinds are increasingly aware.

Managerial incentives for risk-taking are widespread. Theoretical work has long noted that performance incentives may induce risk-taking (at least since Jensen and Meckling, 1976). Practically, performance incentives are pervasive. Performance pay is the majority of executive compensation.[1] 9 million workers have stock options as part of their compensation scheme. And millions more have non-stock incentive schemes that can induce risk-taking.[2]Many of these incentives are threshold based. A common feature to these

[1]Anderson and Muslu (2011) estimate that half of executive compensation is from options, and an additional 30% is from bonuses and long term incentive plans.

[2]"Taking Stock: Are Employee Options Good for Business?"
http://knowledge.wpcarey.asu.edu/article.cfm?cid=8&aid=26

incentive schemes is that compensation may vary little, if it all, with outcomes in the region below a threshold, but is much more responsive to changes in outcomes above this threshold.

In the first chapter, I develop a model of risk-taking and performance in response to these sorts of threshold-incentives, focusing specifically on the role of distance to the threshold in changing. A manager facing these incentives who can influence outcomes through effort and the distributions of outcomes through risk-taking will change their responsiveness on both of these tasks depending on their distance to the threshold. The general idea of this model has been understood in the literature, but explicitly developing the predictions for this relative narrow, though oft used, set of contracts plays an important role in the third chapter where I use variation in distance to identify changes in incentives.

Empirically, the role of incentives in driving risk-taking has been difficult to identify. In the second chapter, I use as my setting the Hedge Fund industry where I have rich data about contracts and outcomes so that I can measure performance and risk-taking. This basis allows me to examine the role of incentives in driving risk-taking. Importantly, I can use strategy level factors as instruments to identify the variation in contracts that is not driven by the choices managers make and I can observe the same manager over time so that the variation I use is not driven by static differences between managers. Further, I leverage the predictions of the model to disentangle the impacts of effort and risk-taking on performance. This research empirically identifies the role of incentives on risk-taking and shows that risk-taking is an important negative consequence of incentives.

The third chapter focuses on a setting where explicit contracts are naught and exchange is only possible under implicit contracts: the market for political influence buying. This is a setting in which formal contracts do not exist. Yet, existing literature largely ignores the contracting problem and puzzles over some empirical regularities. I develop a formal model of political influence buying which is self-enforcing. Formal, or externally enforced contracts are not necessary. This contract help rationalize the existing empirical puzzles as well as provides additional implications on the time path of contributions and the structure of lobbying and legislation. These additional implications provide an opportunity to empirically detect influence buying using transparent contribution data instead of difficult to observe favors. Finally, I show empirical evidence consistent with the empirical predictions on time paths of contributions.

Chapter 1

Risk-Taking and Performance under Incentive Thresholds

1.1 Introduction

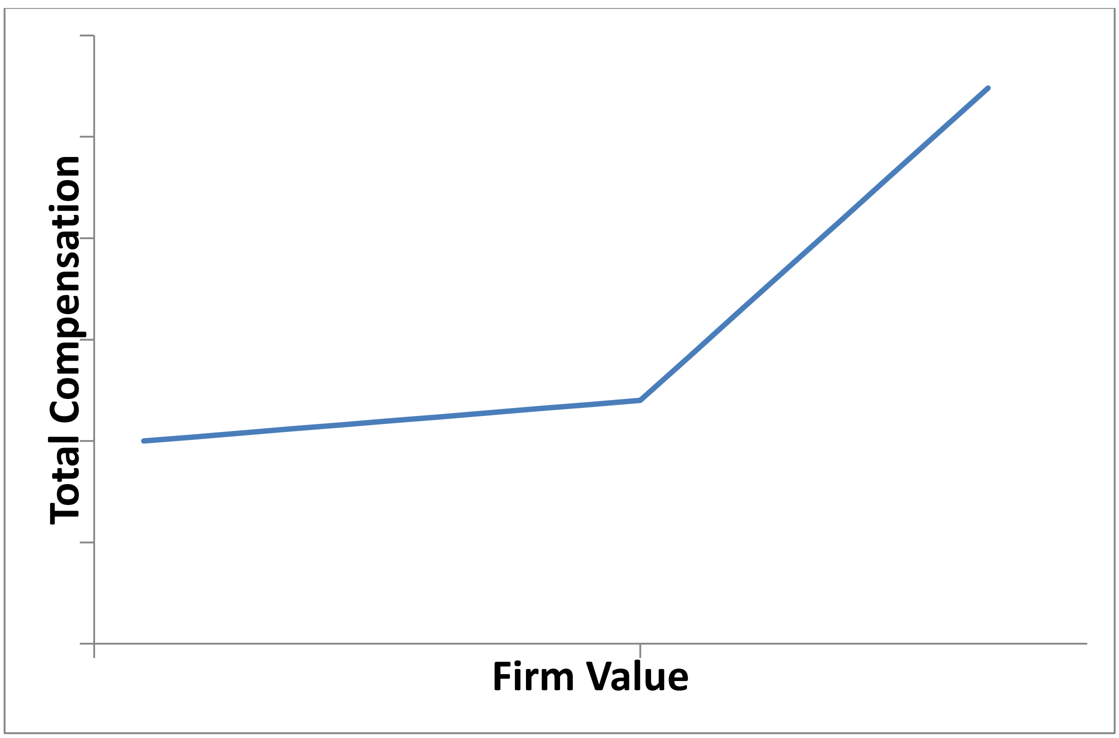
The general conclusion that convex incentive schemes influence managerial risk-taking is not new. Classic incentive theory has linked options as an effective way to align managers and principals (e.g. Haugen and Senbet 1981) but at the cost of inefficient risk allocation (Jensen and Meckling 1976). More recently Vereshchagina and Hopenhayn (2009) argue that entrepreneurs facing a similarly shaped incentive scheme will choose higher risk projects. Hall and Murphy (2000 and 2002) directly consider the question of at what price options should be granted, but do not consider risk-taking as a consequence. Other research has looked at risk-taking incentives but does not consider the placement of the incentive threshold. Carpenter (2000) finds that additional options may reduce risk-taking, because risk aversion may dominate the additional convex incentives. Panageas and Westerfield (2009) focus on the dynamic ratcheting of thresholds in hedge funds and show that the value of future periods reduces risk-taking. The focus of this work is on how incentives lead to risk-taking, and not, as in Hermalin and Katz (2001) risk-taking is used to influence incentives. In this section I develop some predictions on the consequence of the distance to the threshold in threshold incentives. I focus specifically on how distance to the threshold changes the manager's risk-taking and performance.

This research focuses specifically on threshold incentive schemes. These are compensation schemes in which total compensation varies little, if at all, with performance when below a performance threshold but varies widely with performance above. Figure 1.1 shows an example of a compensation scheme of a firm manager. In this example, the intercept of the compensation scheme is the manager's base compensation including fringe benefits. The low initial slope represents the impact of equity holdings on total compensation as firm value increases. The steeper region represents the realized value of option holdings where the exact threshold is determined by the exercise price of those options.

Such threshold incentives are pervasive. While we might imagine that only executives' compensation has this structure, in reality, the same incentive scheme characterizes the compensation of any employee holding stock options in their employer. Entrepreneurs also face a similar incentive scheme from limited liability if they have any debt or debt-like terms that are common in venture capital financing terms. Downs and Rocke (1994) argue that political leaders face similar limited liability where the threshold reflects the approval necessary to remain in office. Sales people who face an increasing commission schedule also face threshold incentives, though the relevant horizontal axis would be sales rather than firm value (Larkin 2012). Profit-sharing contracts, such as those used in the movie industry, have a similar shape where the horizontal axis is revenue (Weinstein 1998). Many employment contracts in the asset management industry also have a similar incentive scheme in regards to return; an asset management fee that moves with returns and a performance fee that grows quickly above a threshold.

To do this, I first formalize a stylized model of the decision a manager makes between projects with threshold incentives. This formal model is helpful to fix ideas as well as formally develop empirical predictions. The manager simultaneously chooses an effort

Figure 1.1: Threshold Incentive Schemes



level where higher effort increases the mean of the distribution of outcomes at a private cost and a risk level. The risk level the manager selects a position on a risk-return frontier. The risk-return frontier that the manager chooses from is a generalization of the standard assumption from the CAPM, following Palomino and Prat (2003) that the risk-return frontier is single peaked. That is, there is a level of risk-taking above which additional risk-taking reduces return. As Palomino and Prat argue, standard depictions of the risk-return frontier simply stop at this peak because no risk-neutral or risk-averse agent with linear incentives (including those normally facing those making investment decisions for themselves) would choose assets beyond this frontier. Indeed, investment opportunities that have high risk and return below the frontier certainly exist (gambling, dominated trading strategies, overpaying managers to be active traders, etc...). However, in financial settings where opportunities for leverage exist the peak of the frontier would not represent the peak of the underlying investment opportunities. Instead it would reflect the point where market frictions (e.g. liquidity, borrowing costs and limitations) make the cost of additional risk lower than the return to that underlying risk. Effort in this setting simply raises the frontier. For any given level of risk more effort yields higher return. In some settings there might be low return to effort (for example, if market prices are perfectly efficient), but in others managers may be able to change the performance of their firm separately from its risk exposure. To formalize this stylization, the agent chooses effort level $e \in [0, \infty)$ at cost $c(e)$ where $c(0) = 0, c'(\cdot) > 0, c''(\cdot) > 0$. The agent also chooses risk level r that yields return $-q(r)$. So that $-q$ is single peaked I assume that $q'(r) \leq 0$ on range $[0, r_1)$, $q'(r) = 0$ on range $[r_1, r_2]$, $q'(r) \geq 0$ on range (r_2, ∞). Further, to avoid degenerate normal distributions, I assume that $r_2 > 0$. Finally, I assume that $q'' \geq 0$ on range (r_2, ∞). This would hold if the manager were able to mix between underlying positions. A decision is a pair (e, r), and yields outcome x that is normally distributed with mean $e - q(r)$ and variance r, $x \sim \mathcal{N}(e - q(r), r)$.

The manager in my model is opportunistic and risk-neutral, and the manager's compensation scheme is an exogenous, convex, two part linear contract. The contract pays the manager a share of the performance of the project (base rate) and a share of the performance of the project above a threshold (performance rate). For an executive the base rate would reflect his equity holdings and the performance rate would reflect options.[1] For a fund manager these reflect the base or management fee and the performance or incentive fee respectively. Formally, the compensation of the manager for realized outcome x is:

$$\pi(x) = bx + \max\{0, p(x - d)\}$$

Where $b > 0$ is the base rate, $p > 0$ is the performance rate, and d is the manager's distance below the threshold. As such the expected welfare of the manager is:

[1]In a one period game the level of fixed compensation provides no incentives so I ignore it. The base rate can equally be thought of as expected (linear) increases in future compensation conditional on this period's performance.

$$\begin{aligned}\Pi(e,r) &= b\left(e-q\left(r\right)\right)\\ &+p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right)\left(-d+e-q\left(r\right)+\frac{\sqrt{r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}{1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}\right)-c\left(e\right)\end{aligned}$$

Where the first term is the base payment for the expected mean, the second term is the expectation of the truncated distribution of outcomes being above the threshold, and the final term is the private cost of effort.

1.2 Solution approach

The manager then chooses e, r to maximize her welfare. The first order conditions yield the first insight:

Lemma 1.1. *Risk taking is on the strictly downward sloping part of the curve. That is* $q'\left(r^*\right) > 0$.

Proof. See appendix. □

This result follows because the risk neutral agent facing a linear increasing incentive curve would be at the peak of the risk-return frontier. However, the convexity of the incentive scheme implies that the manager would always take some amount of risk above the peak. This, plus convexity of the cost of effort, $b > 0$,and $p > 0$ is enough to ensure that the solution is interior in both effort and risk-taking. While the set-up of the problem does not assume that risk-taking is inherently bad for return, these managers always choose risk levels that are high enough so that the marginal return of risk is negative. Additional risk-taking will be costly to returns.

Assumption b is high enough or c is sufficient convex so that $c''\left(c'^{-1}\left(b\right)\right) = \frac{p}{\sqrt{2\pi r_2}}$.

This technical assumption is a sufficient condition to ensure that the maximization problem we have is concave over all possible d,$r \geq r_2$and $e \geq c'^{-1}\left(b\right)$. It is straight forward that effort is above this threshold $e \geq c'^{-1}\left(b\right)$, because this is the effort level that the manager would exert with no performance fee.

Lemma 1.2. *There exists a single solution to the maximization problem for any set of parameters* d, b, p. *And further, it is the unique local maxima as well.*

Proof. See appendix. □

This result allows use of the implicit function theorem to characterize comparative statics everywhere. Since the implicit function describes changes in the local maximum around a solution knowing that there is always one local maximum ensures that these comparative statics are meaningful beyond local results. That is, the global maximum does not jump between one local maximum to another as parameters change.

1.3 Effects of distance on effort and risk-taking

With the above results, the empirical predictions of this model on effort and risk-taking as distance below the thresholds follow from the implicit function theorem.

Proposition 1.1. *Effort is decreasing with distance to the threshold.*

Proof. See appendix. □

The intuition behind this is increasing the distance decreases the probability that outcomes will be above the threshold. The effort level undertaken is driven by the marginal return to improving outcomes, which is the average slope of the incentive curve that the manager expects outcomes to reach. Increasing the distance decreases the probability the manager will be in the high marginal incentive region, so the manager reduces effort.

Proposition 1.2. *Risk-taking is increasing with distance to the threshold when the threshold is near.*

Proof. See appendix. □

Proposition 1.3. *Risk-taking is decreasing with distance to the threshold when the threshold is far.*

Proof. See appendix. □

The intuition behind these is more nuanced. Increasing the distance below the threshold decreases the marginal return to risk-taking because the convexity in the compensation scheme is farther away. However, increasing the distance below the threshold also decreases the cost of the risk-taking, because it is more likely that the marginal movement of outcomes is in the low marginal incentive region than the high incentive region. At first, the second effect dominates. However, when the manager is very far from the threshold, increasing the risk-taking only has second order effect because the compensation scheme the manager faces is nearly linear. But the manager still faces first order costs of risk. Further, an assumption of sufficiently costly effort ensures that these two regions meet.

Proposition 1.4. *If effort is sufficiently costly risk-taking is single peaked. A sufficient condition for effort to be sufficiently costly is* $c'(b) > \frac{\exp(-1/2)}{\sqrt{2\pi}}$.

Proof. See appendix. □

However, the above propositions describe effort and risk-taking. However, empirically, we do not observe effort directly. Instead, we observe performance, $e - q(r)$. How performance changes with effort and risk-taking depends on the relative cost of risk and value of effort. We have

Corollary 1.1. *Regardless of the cost of risk-taking or value of effort, when the threshold is near, increasing distance reduces performance.*

and

Corollary 1.2. *If the threshold is far, increasing distance improves performance if the cost of risk-taking is high and reduces performance if the cost of risk-taking is low relative to the impact of effort.*

Effectively, this implies that far managers can be used to tease out the importance of effort and risk-taking on performance.

1.3.1 Interactive effects of fees

The interactive effects of fees are also an important predictions of the model. The intuition behind these effects are simple. Consider a manager with a very small performance fee. This manager is going to respond to distance very little because her compensation scheme is nearly linear. On the other hand, a manager with a very small base fee will respond quite sharply to distance because she faces no other incentives.

Proposition 1.5. *If the threshold is near, the rate of decrease in effort with distance is increasing in the performance fee.*

Proof. See appendix. □

Proposition 1.6. *If the threshold is near, the rate of decrease in effort with distance is decreasing in the base fee.*

Proof. See appendix. □

Proposition 1.7. *If the threshold is near, the rate of increase in risk-taking with distance is increasing in the performance fee.*

Proof. See appendix. □

Proposition 1.8. *If the threshold is near, the rate of increase in risk-taking with distance is decreasing in the base fee.*

Proof. See appendix. □

1.3.2 Organizational Moderators

The organizational economics literature provides several additional factors that influence the responsiveness of managers. Here I explore predictions of how direct ownership, reputations, and task-design each affect the responsiveness of managers to their thresholds. The organizational economics literature often considers these factors separately from explicit incentives, but in this context I explore how the organizational features interact with the explicit incentives. The results suggest that some forms of organizations might be more resilient to the misalignment of incentive thresholds, as well as some cautionary factors that may magnify responsiveness.

The first factor I consider is direct ownership. The principal-agent literature has long acknowledged that, absent risk aversion, direct ownership by the agent is a first-best solution to the agency problem. Indeed, the literature has developed to explain contexts where direct ownership is infeasible or subject to some other negative consequence. Managers, however, often own significant shares in the firms they manage. Entrepreneurs and founders generally retain large shares in their ventures. Other managers may amass large holdings through equity compensation. In the hedge fund context, hedge fund managers are often large investors into the hedge funds they manage, either from initial investment much like firm founders or from subsequent investment of earnings. Different investment levels would lead to differing responses. From the conceptual framework these investments function like the base rate and the predictions of propositions 1.6 and 1.8. The manager receives directly all the gains and losses to the investments, without regard to the threshold. To the extent that their own investment returns drive their behavior rather than the potential fees, we should see funds with larger manager's capital respond less to being below their thresholds. That is, risk increases and average performance drops should be smaller.

Reputation is well developed as an important incentive mechanism in organizations. Conceptually, we would expect managers and firms with more valuable reputations to protect these intangible assets and not to increase risk as much when below their incentive threshold as those with less valuable reputations, while the differential effect on effort would be small. This logic follows from the same argument regarding the base rate in Proposition 1.8. A valuable reputation makes riskier, even negative expected valued, choices less attractive because of the harm to reputation. However the value of marginal effort on increasing the average performance is not as clearly affected. Indeed, following the argument above about the base rate, the interactive effects on performance are driven by the direct effect on performance of risk-taking.

Task allocation is also an important organizational consideration. One result of the classic multitasking literature (Holmstrom and Milgrom 1991) is that when incentives are reduced for one task then an agent with a second incentivized task will reduce effort more on the first task than and agent without a second incentivized task. In this context, the implication on average performance is straight forward: managers with more incentivized tasks will have bigger drops in performance when their incentives are reduced. On the

Table 1.1: Summary of Predictions

	Risk-Taking	Performance
Distance to Threshold	+	-
Distance to Threshold X Far	-	+
Explicit interactions		
Performance Fee	+	-
Base Fee	-	+
Organizational Interactions		
Direct Ownership	-	+
Reputation	-	
Multiple Tasks		-

other hand, implications for risk-taking are less clear. To the extent that changing the level of risk is a decision, rather than something that takes effort, we should expect task allocation to have little direct impact on risk-taking. Indirectly, lower average performance effectively means a greater distance to the threshold so the second order effect would be more risk-taking. Table 1 provides a summary of the empirical predictions.

1.4 Conclusion

This chapter focuses specifically on threshold incentive schemes and develops predictions about performance and risk-taking depending on the distance to the threshold. The general idea that these sorts of incentives might induce risk-taking is not new. However, the prediction that this risk-taking may come at the cost of performance is not developed in the literature. Importantly, for the empirical chapter that follows, the model developed in this chapter provides predictions that allow the performance implications of effort and risk-taking to be separated. Beyond these fundamental predictions the model in this chapter develops a number of extensions that will be utilized in the following chapter to understand the heterogeneous response to the same shocks as well as validate the empirical approach.

Chapter 2

Evidence from Incentive Thresholds in Hedge Funds

2.1 Introduction

There are four main limitations of the existing empirical research on incentives for risk-taking that I build from. First, because a manager's compensation structure is set based on the specific manager's skills, risk attitudes, characteristics as well as the firm's risk exposure, opportunities and desired risk-taking there can be an issue of endogeneity. Typical cross-sectional comparisons of executives' compensation, such as Wright et al. (2007) or Carpenter et al. (2003), do not distinguish between the effects of incentives and the decision to award option based compensation. It might be, for example, that riskier firms give more option compensation. Second, measures of risk-taking incentives are limited. Many studies use measures such as option counts which are difficult to compare across firms, or local measures of incentives such as option delta and vega.[1] Further, more options may not imply more incentive to take risk because they may induce more risk-aversion than the additional risk incentives they provide (e.g. Carpenter, 2000) and compensation for extreme outcomes can provide significant risk-taking incentives. Others, such as Chevalier and Ellison (1997), estimate imputed implicit incentives but lack the richness and foundation of examining explicit incentives. Third, measuring risk-taking is difficult in many contexts. Common measures in the literature such as merger and acquisition behavior and financing decisions (e.g. Devers et al., 2009, Eisenmann, 2002, Sanders and Hambrick 2007) are hard to interpret from the framework of an agency problem because they are measures over which the principal (boards) have direct control. Finally, few studies examine both risk-taking and performance. However, without understanding the performance consequences of risk-taking it is difficult to evaluate its importance.

The two trillion dollar hedge fund industry is a fertile setting in which to empirically investigate impacts of incentive contracts on risk-taking and performance for four major reasons. First, incentive contracts in hedge funds are fixed ex-ante, so a fixed-effects approach can control for endogenous contracts. Second, market movements and industry level asset flows provide exogenous variation in the effective incentives of the fixed contract. Thus, using this exogenous variation in effective incentives and a fixed-effects approach allows examination of risk-taking holding the endogenous contract fixed. Third, risk-taking is a standard metric of hedge fund outcomes. Further, risk-taking measures in this setting are not subject to veto or review by the principals who set incentives, in contrast to many measures used to examine risk-taking of executives. Moreover, unlike in other settings, agents have similar opportunity sets of risk choices.[2] Fourth, performance measures are straightforward and driven by the same contracts that incent risk-taking. In addition to the empirical features of this setting, incentive contracts, performance out-

[1] Option delta and vega are the derivatives of the value of an option with respect to price and volatility respectively. They are a subset of option "Greeks" or sensitivities of option value to marginal changes in parameters.

[2] For example, two managers at different firms contemplating an acquisition of a third firm face different outcomes because of the different synergies with their firms. In contrast, two different funds making the same investment realize the same returns on that investment.

comes, and risk-taking are particularly relevant in this context. In fact, unlike other investment vehicles, risk management is a first-order concern for hedge funds as the particular appeal of hedge funds is often not the prospect of outsized returns, but rather the promise of steady returns.

Hedge fund fee contracts have a threshold – known as a "high-water mark" – in the determination of fees paid to the investment manager. These contracts specify a management fee which is a fixed percentage of all assets. In addition, the contracts specify a performance fee which is a fixed percentage of the investment profits and which is only paid when the returns are above a high-water mark for the investments. The high-water mark is the highest value for which performance fees have previously been paid or the initial value of the investments if none have been paid, and so is adjusted up each time the performance fee is paid. As a simplified example,[3] if a fund starts with \$100 million in assets and earns \$10 million in the first year, the management fee would be a percentage of \$110 million, the performance fee would be a percentage of \$10 million, the new high-water mark would be \$110 million, and the fund would be at its high-water mark for the following year's calculation. If the fund instead losses \$10 million, the management fee would be a percentage of \$90 million, the performance fee would be zero, the high-water mark for next year's calculation would remain \$100 million, and the fund would be \$10 million below its high-water mark. The threshold from which profits are measured – the high-water mark – is adjusted up each time the performance fee is paid. If in the second year the fund earned \$15 million the size of the performance fee would depend on the distance to the high-water mark which is different in the two cases above. If the fund was at its high-water mark (i.e. had not lost money its first year), the performance fee would be a percentage of \$15 million, but if the fund was \$10 million below, the performance fee would be a percentage of \$5 million.

The fund's distance to the high-water mark, and thus effective incentives, depends on past performance. However, market movements, particularly downturns, provide an exogenous movement of funds away from their threshold and the fixed contracts mean that there is no discretion in resetting incentives. My data set contains hedge funds that self-categorize into one of 34 strategies, which reflect the types of markets the funds intend to participate in. I use the return of each of the funds in each strategy to estimate the exposure of the strategy to a set of market indexes used to explain performance of hedge funds and other financial assets (Fama and French 1993, Carhart 1994, and Fung and Hsieh 2004). This approach provides a measure of how exposed a strategy is to a unique composite of the market indices. Since downturns in the indexes affect strategies differently this provides within time period variation in the exogenous distance to the threshold. Further, I use the panel nature of my data set to control for cross-sectional differences between managers, contracts, incentives, performance, and risk-taking with fixed effects. Thus I use within fund variation in distance to the threshold caused by exposure to the strategy specific market to examine the effect of these incentive contracts

[3] See Section 4 for more details.

within funds on outcomes, both in terms of performance and risk-taking.

My results provide causal evidence that managers respond to being farther below their incentive thresholds by increasing risk and reducing performance. The results show sizable effects: the average treatment effect, equivalent to moving a fund just 15% below its threshold, reduces returns over the next year by 2.1 percentage points and increases the riskiness of the fund by about 50%.

Beyond this initial result, this chapter tests the predictions of the model in the previous chapter of a manager's decision making when facing a threshold incentive. In the model the manager chooses both how much costly effort to exert, where effort improves outcomes on average, and a risk level, where higher risk spreads the distribution of outcomes but also may have a performance cost. In addition to the prediction that risk-taking increases and performance falls when managers are farther below their thresholds, the model tested here yields predictions about what happens when managers are very far from the threshold and how different management and performance fee rates would affect responsiveness to distance from the threshold.

With respect to the former, the model predicts that when managers are very far below their threshold they stop taking additional risks, but their incentives for effort continue to decrease monotonically. Empirically, the results are consistent with this prediction, as I find that managers that are very far below their thresholds take less risk and perform better than managers who are moderate distances below their thresholds. The results further suggest that the performance costs of risk-taking are large. Given my baseline assumption on the functional forms, the data suggest that 83% of the performance drop observed by managers that are not very far below their thresholds is due to the performance costs of risk-taking and 17% of the performance declines are due to effort reduction. The next set of results is that managers with higher performance fees or lower management fees should respond more to being below their thresholds. Again I find evidence consistent with these predictions. These add additional causal evidence that the performance and risk-taking effects I estimate are being driven by the contracts themselves rather than implicit incentives from aspirations, performance targets (as in March and Shapira, 1987), reference point behavior, loss aversion (as in Wiseman and Gomez-Mejia, 1998), or relative performance contests.

Applying these findings to executives, the results suggest that guaranteeing compensation for members of the top management teams, which reduces the importance of performance pay, and granting them equity compensation and holdings in firms, which act like the management fee for fund managers in that managers compensation varies with both failure and success, would temper risk-taking when managers' option holdings are out of the money.

The organizational economics literature suggests additional reasons for heterogeneous responses (e.g. Gibbons 1995 & 2005). Reputational value is often an important implicit incentive for managers. Direct incentives provided by increasing a manager's ownership stake are also an often suggested solution to agency problems in firms. In the model, both of these incentives should have the same types of heterogeneous responses as the management fee. This is because these incentives do not vary depending on the manager's threshold. This leads to the predictions that reputation and ownership stakes should decrease risk-taking and mitigate performance declines when managers are below their thresholds. Multi-tasking arguments (Kerr 1975, Holmstrom and Milgrom 1991) suggest that managers with more incented choices of where to expend effort will reduce effort and thus performance more when incentives are reduced on the focal task. Put simply if a manager paid for both A and B has incentives for A reduced they will reduce effort on A more than if they were paid only for A. Using proxies for each of these predictions, I find variation in responses consistent with each of these theoretical predictions.

These results suggest that thresholds are a critical feature of incentive contracts and have important effects on a manager's behavior. I show that when these incentives are misaligned they can lead to meaningful and undesirable increases in risk-taking behavior. When thresholds are more distance managers perform worse. By exploiting a non-monotonic prediction, I also find evidence that suggests that risk-taking, not effort, may be the source of the majority of the performance effects I find. Finally, organizational features also contribute to the impact of contractual incentives. Reputational value and direct incentives moderate the risk-taking induced, while the decision to allocate effort among different tasks can magnify performance declines.

This research makes several contributions to different streams of work. First, this research provides strong casual empirical evidence that managers do take meaningfully more risk when they have incentives to do so. Second, the combination of findings on both risk and performance reinforces the importance of contracting research to examine multidimensional tasks and contracts. Third, I show that incentives for effort are important in complex jobs and that these incentives serve to induce managers to improve performance. Fourth, these findings plausibly extend to other contexts with similar contracts such as corporate executives. Fifth, these findings have significant policy implications towards risk-taking. Finally, the effects I estimate are economically large and suggest the importance of improving contracts in this context.

As discussed above, existing empirical research into risk taking faces many limitations. This research addresses them in providing strong causal evidence that incentives do change behavior and lead to more risk-taking. The fixed-effect approach with fixed contracts and the exogenous movement of the effective incentives for risk-taking together provide a causal foundation for the findings. Explicit incentives allow examination of not only non-monotonic incentives, but also evidence that these responses are being driven by the terms of the contracts. I use direct and clear measures of risk-taking which are under the control of the manager. Finally, I present evidence that this extra risk is associated with worse performance.

This work also informs research on contract design. The high powered incentives I study were designed to induce effort and reward success. However, I show that while they do impact effort they also induce undesirable risk-taking. Indeed, my results suggest that the standard incentive answer to how to induce more productivity: increase incentives, has undesirable consequences because not only can managers work harder they can also take risk. The magnitudes of my findings suggest that ignoring such multidimensional response is a significant loss. These multidimensional agency problems deserve more attention by both empirical and theoretical approaches.

Practitioners have already begun experimenting with the contract design. Following the financial crisis in 2008 many hedge funds began experimenting with alternative contracts that allowed investors and managers to agree to move the fund closer to its threshold in exchange for a lower performance fees. Other funds instituted longer and rolling high-water marks so that managers would effectively remain closer to their thresholds. In other contexts, publicly traded firms regularly reprice employee stock options following stock market declines by replacing an option for which the employee was far below the threshold to one in which the employee is at the threshold. For example, Google spent \$460 million in 2009 resetting employee stock options.[4]

The performance effects I measure are also significant on their own. Murphy (1999) and Bloom and Van Reenen (2011) in surveys of the empirical incentive literature ask whether the strong causal evidence that incentives matter in simple jobs (e.g. Lazear, 2000; Hamilton et al. 2003; Shearer, 2004) translate to more complicated jobs such as managers. Some argue that incentive compensation may just be ways managers have to pay themselves more (e.g. Bertrand and Mullainathan, 2001). The surveys note that the literature has not answered this question with causal evidence. This research provides that causal evidence. Even fund managers with complex jobs perform better when they have higher incentives. Oyer and Shafer (2011) question the relative importance of incentives to lead managers to improve their output, or just to find the right managers. The scale of the performance effect I find addresses this question: explicit contractual incentives matter.[5]

The empirical context of this research is hedge fund management, but the implications are broader. Compensation for hedge fund managers and executives share a similar structure. I estimate that hedge funds realize 46% percent of their fees from option-like performance fees. By comparison, CEOs of public firms earn 51% percent of their total compensation from option pay. Indeed, CEO's compensation schemes may even be more "convex", because an additional 30% of total pay is in other incentive pay such as bonuses, long term incentive plans and equity (Anderson and Muslu, 2011). While executive compensation contracts may reflect executives power in setting their own compensation and "incentive" pay may not actually reward managers for shareholder performance, the incentives under the compensation schemes are quite similar. This similarity in the share of CEO option pay suggests that the thresholds provided by options are significant features

[4]http://www.nytimes.com/2009/03/27/business/27options.html;
http://www.cbsnews.com/2100-500395_162-4750463.html

[5]One way to compare the scale of these two effects is to compare the performance effect of incentives that I measure with the variation in performance. Fama and French (2010) provide a measure of the distribution of abilities for mutual fund managers. While these measures are in slightly different industries, the magnitudes of their results that moving a manager from 15% below their threshold to their threshold is equivalent to replacing an average manager with one in the 95th percentile of managers.

of executive compensation schemes. More broadly, 80 percent of employee stock options are issues to non-executives and 9 million workers have stock options as part of their compensation scheme.[6] While the magnitudes may differ, all of these employees face the same incentives. Further, the job of a hedge fund manager though focused on financial transactions, involves many of the same tasks of a CEO. They review information, make decisions under uncertainty, and organize, motivate, manage and develop people and organizations. Though the discretion to directly take risk and the ability to influence performance may vary among managers, all managers have the ability to use both avenues in response to their incentive schemes.

These findings are important not only for the individual firms involved, but also inform policy. The recent financial crisis makes clear that risk-taking by firms is not only a private concern; it can have significant externalities on the economy as a whole. Compensation contracts have been the object of much regulatory attention, and this research reinforces its potential significance. Indeed, the structure of these incentive schemes has the potential to transform a transient negative shock into a persistent increase in risk-taking. Once the shock moves managers below their thresholds, they then take more risk and perform worse, which can perpetuate the process. Reshaping contracts has the potential not only to reduce total incentives for risk, but also arrest the propagation of negative shocks.

Finally, the cost of these imperfect incentives is high. To get a sense of the magnitude of the impacts I perform the following partial equilibrium hypothetical calculation. Suppose that the contracts were redesigned so that the threshold would reset following a loss so that the manager began each period always at their threshold, but in a way that did not impact incentives for managers who did not have a loss and thus were already at their thresholds, these results imply that performance would be higher by an average 1 percentage point per year. This is an annual cost of \$20 billion to hedge funds' investors. If I assume that investors have a coefficient of risk aversion equal to one, the cost of the extra risk is \$11 billion a year. These results also only evaluate the cost to the investors in the funds, not to society as a whole. To do that, I would need to know more about the nature of the transactions that have changed and their trading partners. Of course, simply resetting the threshold would presumably affect both how the managers behave in other periods and the selection of managers into these roles so this calculation should be thought of as only suggestive. However, this calculation suggests that if contracts could be designed to minimize the general equilibrium effects there is plenty of potential value for improved incentives.

The next section describes the data and institutional context. Section 2.3 describes the empirical approach used to estimate the risk and average return consequences of being below the incentive threshold. Section 2.4 provides the primary results. Section 2.5 extends these results examining managers very far from their thresholds, testing the predictions on the differential effects of fees, testing predictions from the organizational economics literature, discussing mechanisms of risk-taking, and finally discussing robustness concerns

[6] "Taking Stock: Are Employee Options Good for Business?"

and considering several additional pathways for these results. The last section concludes.

2.2 Industry and Context

The setting for this study is the hedge-fund industry. In this industry hedge fund managers are paid fees to make investments with investor's assets. Each hedge fund is a standalone private investment vehicle with hedge fund management firms as general partners and high net worth individuals and institutional investors as limited partners. Hedge funds face minimal regulatory constraints and managers are free, unlike other asset managers such as those who manage mutual funds, to make almost any investments, including derivatives, short sales, leveraging and private transactions. Hedge funds identify an investment strategy that broadly identifies the sort of assets the fund will invest in, the sort of profit opportunities that the manager will pursue, and the risk exposure that the fund will accept. In this research I view these categorizations as much like industry classifications; they identify that within strategy firms face similar exogenous factors that influence performance.

Hedge fund management firms earn revenue from fees paid from the assets of investors. These fees are composed of a management fee and a performance fee. The management fee pays the manager a percentage of fund assets each year. Management fees are usually between 1 and 2%. On average, the performance fee pays the manager a substantially larger share of the profits the fund makes than the management fee. The most common performance fee rate is 20%. Because the performance fee is calculated on profits, sometimes above a benchmark rate, it provides the threshold in the incentive structure. The details of this performance fee are central to my analysis and I discuss it in detail in the context of my empirical approach. While the internal organization of the management firms vary, all are known for high powered incentives that tie compensation of the individuals in the firm quite closely to fees and performance. Each hedge fund generally has a single individual within the management firm known as the portfolio manager responsible for ultimate investment decisions. These fund managers are usually owners or partners in the management firm. [7]

Much of the existing research on hedge fund risk-taking and performance is descriptive and cross-sectional in nature. Ackermann, McEnally, and Ravenscraft (1999) describe hedge fund risk, return and fee profiles. Agarwal and Naik (2004) focus on identifying market factors that are relevant to explaining hedge fund performance and risk exposures. Agarwal, Daniel, and Naik (2009) correlate a measure of marginal fees with expected outcomes. Kouwenberg and Ziemba (2007) correlate fee rates and various measures of risk-

[7]Because the data I use does not include information about the decisions of the manager separate from the ultimate actions of the management firm I cannot distinguish between actions of the individual or of the firm. However, the strong internal incentives suggest that in this context the two are closely aligned. These results can be fairly interpreted as either about the individuals' decisions or the firms' response.

taking. Like Chevalier and Ellison (1997), Brown, Goestzmann, and Park (2001) focus on intra year changes in risk-taking following good absolute and relative performance, but do not consider the contracts explicitly. Similarly, Holland, Kazemi, and Li (2010) correlate performance in the first half of the year and changes in risk-taking. Smith (2011) looks at how investors respond to idiosyncratic risk-taking by managers.

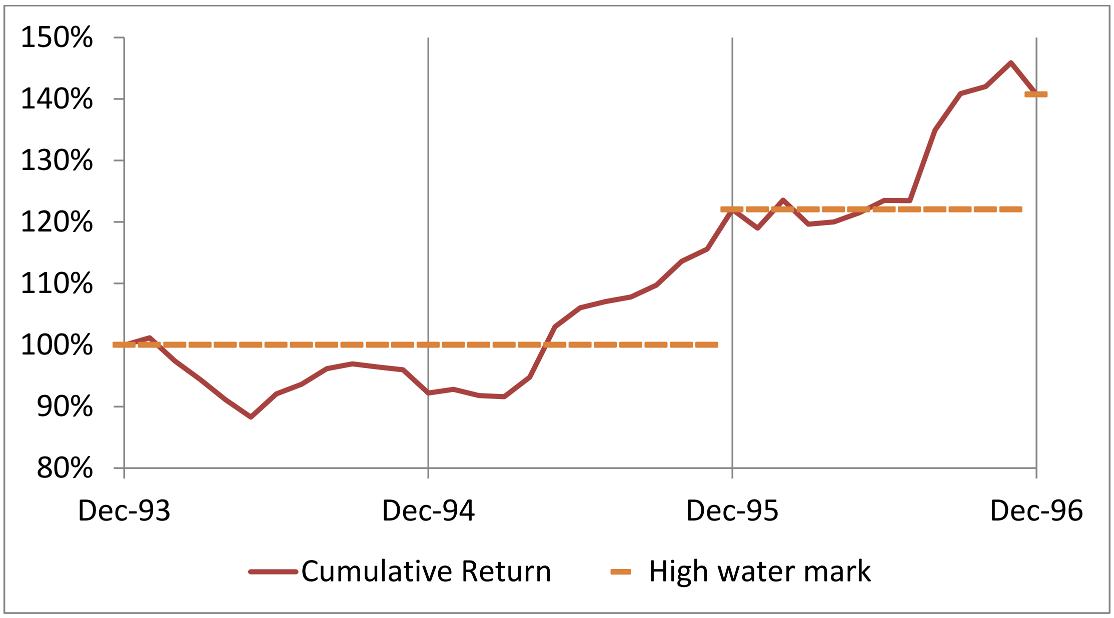
I use a dataset of month assets and returns of about 9,000 hedge funds from 1994 through 2006. This dataset was compiled by merging data on hedge funds from Lipper-TASS and Hedge Fund Research. Each of these datasets retain data on funds even once they stop reporting. While exact measures do not exist, these datasets are together estimated to include about a quarter of the entire hedge fund industry. In addition to the monthly assets and returns in these datasets, I use data about the fee structures of the funds and self-classification of the funds into 34 categories that reflect their investment strategies and exposure. One limitation of these sources of data is that the data are self-reported, presumably for self-interest. This leads to several potential selection concerns. These results, however, are robust to these concerns and are more fully discussed in section 2.5.4.

2.3 Empirical approach

My empirical approach focuses on the performance fee and the role of the high-water mark. The high-water mark is the threshold in the calculation of the performance fee. Managers share only in the returns of a fund above the high-water mark. It is calculated so that the manager is not paid a performance fee for recouping previous loses. At the end of each year managers are paid any performance fees they have earned and the high-water mark is adjusted for this payment. Figure 2.1 illustrates this. The red line identifies the cumulative return of a hypothetical hedge fund. At the end of 1994 this fund is 8% below its high-water mark and not paid a performance fee. In this event, the fund is considered to have a *Distance* of 8%. At the end of 1995, however, this fund is paid a performance fee because returns exceed the previous high-water mark and its high-water mark ratchets up. High-water marks are tracked individually for each investment into the fund, so each vintage of assets may have a different high-water mark. In the example of Figure 2.1 investments made at the end of 1994 are at their high-water mark when they are made, but older vintages are not. Because managers cannot make separate investment decisions for separate vintages, I use an asset weighted average of the distance to the threshold. So in the example of Figure 2.1, if the fund at the end of 1994 was composed equally of two vintages, one from the end of 1993 and one from the end of 1994, I would average the distance and treat this fund as if it is 4% below its high-water mark.

The implementation of this calculation depends on the returns the fund experiences as well as the flow of assets in and out of the funds. Returns are directly reported in the data, but funds do not report asset flows. Instead, I use reported assets to impute net flows of assets. I treat net inflows as new vintages and allocate net outflows proportionally

Figure 2.1: Calculation of High-Water Marks



The red line identifies the cumulative return of a hypothetical hedge fund. At the end of 1994 this fund is 8% below its high-water mark. Or in other terms the fund is 8% *Distant.* At the end of 1995, this fund is paid a performance fee and its high-water mark ratchets up.

across previous vintages. I discuss potential concerns of this source of measurement error in section 2.5.5.

The formal calculation of how far a fund is from its threshold follows. Let r_{it} represent the return of fund i in period t. Representing the initial period of fund i as t_{0i}, let $CR_it = \Pi_{j=t_{0i}}^{t}\left(1 + r_{ij}\right)$ be the cumulative return to period t. The high-water mark of investments of vintage v is $HWM_{ivt} = \max\left(CR_{iv}, CR_{iv+1}, \ldots, CR_{it}\right)$. To aggregate each vintage's high-water mark, I measure how far from the threshold as $Distance_{ivt} = \left(HWM_{ivt} - CR_{it}\right) / \left(CR_{it}\right)$ or what percentage growth is needed to bring that vintage to its high-water mark. To aggregate vintages, I weight each vintage by its share of assets. Let In_{iv} be the dollar inflows in period v. Let Out_{iv} be the outflows in period v as a percent of assets. So the assets remaining in vintage v at time t is $A_{ivt} = In_{iv}\frac{CR_{it}}{CR_{iv}}\left(\Pi_{j=v+1}^{t}\left(1 - Out_{ij}\right)\right)$. Note that by construction the assets of fund i at time t is $A_{it} = \sum_{v=t_{0i}}^{t} A_{ivt}$. Then, weighting vintages by assets: $Distance_{it} = \sum_{v=t_{01}}^{t}\left(Distance_{ivt} * \frac{A_{ivt}}{A_i}\right)$. How this depends on the underlying data is clearer with some expansion:

$$Distance_{it} = \sum_{v=t_{01}}^{t}\left(\frac{\max\left(CR_{iv}, CR_{iv+1} \ldots CR_{it}\right) - CR_{it}}{CR_{it}} * \frac{In_{iv}CR_{it}\left(\Pi_{j=v+1}^{t}(1-Out_{ij})\right)}{CR_{iv}A_i}\right) \tag{2.1}$$

Risk-taking of the funds is measured as the realized variance of the fund's monthly returns and return is the net return of the fund, both over the following year.[8] A year is the natural length of time over which to measure response because that is the period until the next payment of performance fees, but the results are robust to shorter and longer measurement periods of 6 months and 24 months. Using this calculated *Distance* suggests the basic, fixed effect regressions of the form:

$$Risk_{it+1} = \beta_1 Distance_{it} + \lambda X_{it} + \delta_i + \gamma_t + \epsilon_{it+1} \tag{2.2}$$

Where X_{it} are linear and curvature terms for assets and age, δ_i are fund fixed effects, γ_t are time fixed effects, and each period is a year. Empirically, the fund fixed effects are important. They make the analysis within a fund so that the effects are not driven in differences between funds or managers. Time varying age and asset controls are included to capture any systematic differences in risk or return that are due to fund age and size. In order to capture the non-linearities in these described in the literature, I include both linear and curvature terms for assets and age. Similarly, to examine the return consequences, I use:

$$Return_{it+1} = \beta_1 Distance_{it} + \lambda X_{it} + \delta_i + \gamma_t + \epsilon_{it+1} \tag{2.3}$$

However, there are several endogeneity concerns in both the return history of a fund

[8]This measure of risk-taking measures realized risk and not intended risk directly. See section 2.5.4 for some discussion.

and the fund flows that together go into calculating how far a fund is from the threshold. Indeed, more broadly, the literature has noted that risk levels influence contracts (e.g. Beatty and Zajac, 1994, Finkelstein and Boyd, 1998). Consider the return history of a fund chooses a higher return, higher volatility risk profile in period 2. This increases the probability that it is below its high-water mark at the end of period 2 and increases the distance below the high-water mark that it is. This also increases the realized volatility and performance in period 3. Because the fund pursued a different risk profile in period 1 these differences are not absorbed by the fund fixed effect.

To address this concerns, I use an instrumental variable approach in which I instrument for $Distance_{it}$ with how far from the threshold a hedge fund in the same strategy would be expected to be based on exogenous variation, $\overline{Distance_{it}}$. $\overline{Distance_{it}}$ is based on a synthetic high-water mark that does not depend on the time varying choices of the manager or the fund specific funding decisions of investors. As is shown in equation (1) above $Distance_{it}$ is a function of the return history of a fund and the flows it experiences. I calculate $\overline{Distance_{it}}$ using the same formula, but with exogenous return histories and flows.

Instead of the endogenous return history of a fund, I use the performance of 15 market "factors". These factors represent the returns to indexes of various market baskets. The factors reflect both the performance of equity markets (Fama and French 1993) as well as additional factors found to be important in explaining the returns of mutual funds (Carhart 1997) and hedge funds (Fung and Hsieh 2004). For each fund strategy I regress the monthly return of the funds in that strategy on the monthly performance of the market factors. That is, I estimate:

$$Return_{it} = \alpha_s + \beta_{sj} Factor_{jt} + \epsilon_{it} \tag{2.4}$$

Then, using the estimates $\widehat{\alpha_s}$ and $\widehat{\beta_{sj}}$ I calculate the predicted return of strategy s in time t, $r_{st:}$

$$r_{st} \equiv \widehat{\alpha_{st}} + \widehat{\beta_{sj}} Factor_{jt} \tag{2.5}$$

The predicted values from this regression capture the return of a hypothetical "passive" hedge fund in each strategy that does not make time varying investment decisions.

From r_{st} I calculate the return and variance of a passive hedge fund would experience in each outcome period. I use these to control for changes in the opportunity set of investments available, including, for example cyclicality in strategy returns. Additionally, I use this to estimate the risk increases that a passive fund would experience. If there is persistence or cyclicality in the performance and risk characteristics of underlying assets beyond that absorbed by time fixed effects, as for example Carhart (1997) demonstrated, then using the variation driven by these factors makes controlling for the risk and return that is driven by market factors particularly more important.

The fund flows that a fund experiences are another source of endogeneity. When investors decide to invest additional assets in a fund it experiences in-flows. When investors withdraw assets, a fund experiences redemptions, or out flows. Collectively, these additional investments and redemptions are called fund "flows." Suppose that the flows a particular fund experiences reflect investors' beliefs about the future performance of the fund. Also, suppose that investors believe that a fund with recent poor performance will experience low risk returns in the next period. If investors add funds to this fund at the end of this period then it will be less underwater than it would have been and, if those beliefs were correct, realized risk would be lower. Thus, the correct beliefs would produce a correlation between distance to the threshold and realized risk.

To replace flows, I use the flows of funds that identify themselves as "Fund of Funds", which I exclude from the analysis otherwise. These funds aggregate and allocate investments into other hedge funds. The flows they experience proxy for the general availability of funds to the industry that are not a consequence of the beliefs of investors about the future performance of particular funds or of their strategies. Indeed, these flows directly induce flows of specific strategy funds but are likely also to be correlated with general capital availability. Thus, these flows are correlated with flows into individual funds but are not correlated with flows that reflect beliefs about individual funds or strategies. For each of these funds I calculate the average percentage inflows In_{FoFt} and outflows Out_{FoFt}. Combining the two exogenous sources of variation, I calculate the synthetic $\overline{Distance_{it}}$ by replacing the actual return r_{it} with r_{st} (and CR_{it} with the analogue CR_{st}) and in and out flows with In_{FoFt} and Out_{FoFt}. That yields:

$$\overline{Distance_{it}} = \sum_{v=t_{0i}}^{t} \left(\frac{\max(CR_{sv}, CR_{sv+1}, \ldots, CR_{st}) - CR_{st}}{CR_{st}} * \frac{In_{FoFv} CR_{st} \left(\Pi_{j=v+1}^{t} \left(1 - Out_{FoFj}\right)\right)}{CR_{sv} \sum_{k=t_{0i}}^{v} \left(\frac{In_{FoFk} CR_{st}}{CR_{sk}} \left(\Pi_{j=v}^{t} \left(1 - Out_{FoFj}\right)\right)\right)} \right) \tag{2.6}$$

Despite the apparent complexity of this formula it has a simple interpretation. It is the Distance of a fund that had the same initial period as the fund, experienced the average flows of funds of funds, and had the returns that reflected the average exposure of its strategy to market factors.

With the calculated $\overline{Distance_{it}}$, I estimate the first stage regression:[9]

$$Distance_{it} = \beta_1 \overline{Distance_{it}} + \lambda X_{it} + \delta_i + \gamma_t + \epsilon_{it+1} \tag{2.7}$$

Which yields $\widehat{Distance}_{it}$ as its predicted value that I then use to estimate the second stage regressions:

[9]Note that because $Distance_{it}$ is a non-linear calculation based on the endogenous primitives (return history and flows) the exogenous primitives should not be used directly in an instrumental variables approach. Instead, I calculate $\overline{Distance_{it}}$ from plausibly exogenous instruments and use it in a linear first stage in the instrumental variables approach (see, e.g. Angrist & Pischke, 2008, Chapter 4).

$$Risk_{it+1} = \beta_1 \widehat{Distance}_{it} + \lambda X_{it} + \delta_i + \gamma_t + \epsilon_{it+1} \tag{2.8}$$

$$Return_{it+1} = \beta_1 \widehat{Distance}_{it} + \lambda X_{it} + \delta_i + \gamma_t + \epsilon_{it+1} \tag{2.9}$$

Indeed, it is illustrative to consider a few of the market factors that lead the instrument to be below from the threshold. The Managed Futures and Global Macro strategies were below their thresholds in 1994, presumably, because of the spike in interest rates.[10] Similarly, in 1998 emerging market funds were below their thresholds because of the crash in emerging market returns. However, some regional emerging market strategies were much more affected than others. The technology crash in 2000 and the market wide downturn in 2002 are also significant downward shocks that cause strategies to be below their thresholds. Again, despite affecting a wide range of strategies, the different exposures provide variation in how far different strategies are from their thresholds. Each of the listed market factors cause some strategies to be below their thresholds. By using these downturns as instruments I treat all funds in exposed strategies as being below their thresholds. One advantage of the instrumental variable approach is that it does not conflate the difference in the performance of funds that may have planned for the downturns and those that did not.

Beyond the average treatment effect the model also includes predictions of heterogeneous treatment responses. The empirical approach to examine these is to add interaction terms to the specifications in equations (2) and (3) interaction terms. In each of the following I interact some characteristic of the fund *Characteristic* with *Distance*. So to equations (2) and (3) I add $\beta_2 Characteristic_i \times Distance_{it}$. Because of the same endogeneity concerns in both $Distance_{it}$ and $Characteristic_i \times Distance_{it}$, I also use an instrumental variable approach in these regressions with $\overline{Distance_{it}}$ and $Characteristic_i \times \overline{Distance_{it}}$ as excluded instruments and refine (8) and (9) appropriately. If the heterogeneous characteristic is time varying, I use $Characteristic_{it}$ instead of $Characteristic_i$ and include the direct effect $Characteristic_{it}$ in the regressions.[11]

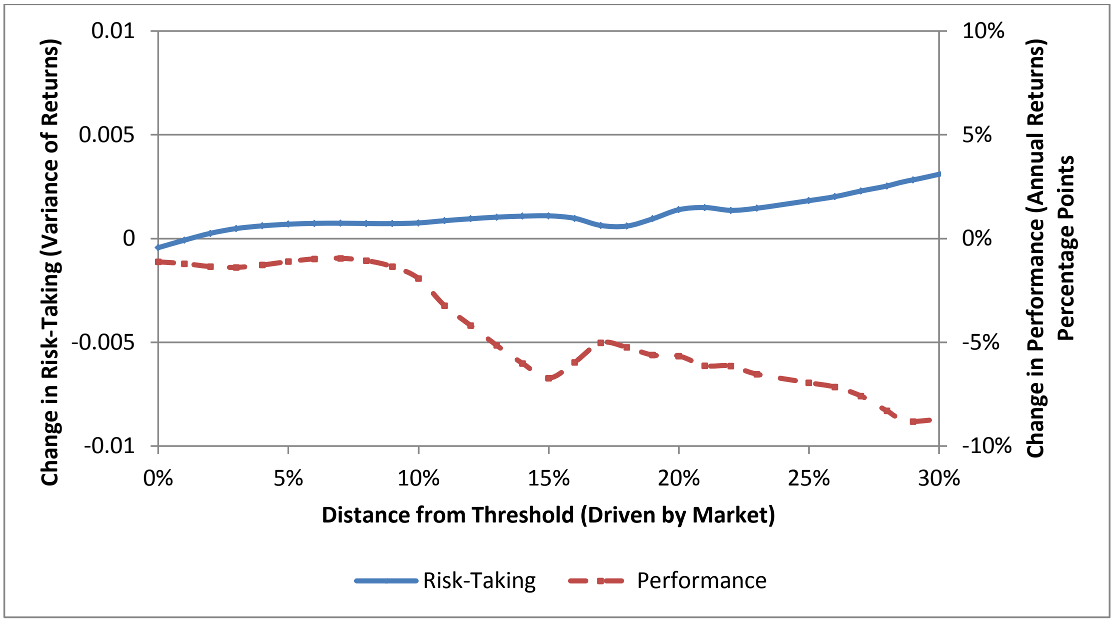
2.4 Primary Results

The first prediction is that the farther managers are from their threshold, the more risk they will take. Figure 2.2 shows the smoothed relationship between the risk-taking of a manager with the distance they are driven below their high-water mark, restricted to managers that are not extraordinarily far from their thresholds. The horizontal axis is the distance below the high-water mark the manager is driven by the market factors as described above. The left vertical axis measures the variance of fund returns in the

[10] http://money.cnn.com/magazines/fortune/fortune_archive/1994/10/17/79850/index.htm

[11] If $Characteristic_{it}$is not time varying the main effect is not included since it is absorbed by the fund fixed effects.

Figure 2.2: Risk-Taking and Performance with Distance to the Threshold.



The horizontal axis is the distance below the high-water mark the manager is as driven by the performance of market factors calculated above. The left vertical axis measures the variance of fund returns in the following year plotted in the blue solid line. The right vertical axis measures the annual return of funds in the following year plotted in the red dashed line. Both lines are non-parametric fitted values after including fund and year fixed effects and controls from age, age-squared, assets under management, and log assets under management. Includes only market driven distances up to 30%.

following year plotted in the blue line. The line plots non-parametric fitted values after including fund and year fixed effects and controls for age, age-squared, assets under management, and log assets under management. From this we see the fundamental result – the farther the manager is from the threshold the more they increase risk.

Table 2.1 takes this same approach on the effect of the distance a fund manager is below their high-water mark where we can put standard errors and control for endogeneity. The first four columns of Table 2 examine the question of increased risk taking. Columns (1) and (2) are OLS regressions, with the fund and time fixed effects and controls for age, age-squared, assets under management, and log assets under management. Column (2) includes controls for the return and variance of the passive comparison. With the fixed effects the interpretation of columns (1) and (2) is that a fund with assets equal to half of its high-water mark, and thus a distance of 100% has a variance of 0.0040 more (an amount equal to the average variance) than that fund has when it is at its high-water market. However, even when below their thresholds, funds are rarely that far from their thresholds. The mean distance for funds that are below their thresholds is 15%, implying that these coefficients suggest an increase in risk-taking of 15%.

As described above there are a number of endogeneity concerns with in this approach. Columns (3) and (4) implement the instrumental variables strategy described in section 4. The two stage least squares (2SLS) columns instrument for a fund's distance from its high-water mark using a calculation based on the history of returns for the same portfolio of market factors, the history of asset flows of Funds of Funds, and the fund's inception date as discussed in the empirical approach section above. In this sense, the instrument captures the distance the fund is expected to be below its high-water mark because of the performance of its strategy – not any time varying decisions of its own, or of its investors. The fund fixed effects absorb any time invariant effect of fund inception.

Column (3) shows that when instrumenting for high-water the estimated risk increase is 0.013, or more than three times the average variance. By a similar calculation, the estimated effect on the average underwater fund is an increase in risk of 50% of the average variance. Controlling for the variance of the strategy in that period in column (4) finds a significant, but smaller effect suggesting that market risk increases in periods when managers are below their thresholds. This difference is further discussed in section 2.5.4. Both specifications show sizable increases in risk.[12] The coefficient estimates from the 2SLS specifications are larger than the OLS estimates. One explanation for this difference is that the endogeneity of *Distance* causes some managers to appear to be below their threshold, but they act like they are at it. For example, if a manager borrows capital to arbitrage a mispriced asset they appear to perform poorly until markets adjust. Another potential cause of the larger estimates from the 2SLS specifications is that classical measurement error in the measurement of Distance is corrected for with the IV.

The second prediction is that the farther managers are from their thresholds the worse they will perform. The red dashed line in Figure 2.2 shows this. The right vertical axis measures the annual return of funds in the following year plotted in the red dashed line. The line plots non-parametric fitted values after including fund and year fixed effects and controls for age, age-squared, assets under management, and log assets under management. From this we see the next result – the farther the manager is driven from the threshold the worse they perform.

The next four columns of Table 2.1 look at performance in terms of annual return. The OLS results in Columns (5) and (6) show insignificant increases in performance before addressing endogeneity concerns. This is consistent with mean reversion where funds that have lost money (and thus are below their high-water mark) perform better the following period.[13] Considering the OLS results suggests that being underwater appears to have

[12] These specifications include all managers, even those that are very far from their thresholds. If the non-monotonic predictions hold, which I find in the next section, it suggests that these results underestimate the impact for most managers.

[13] This is also consistent with a mechanical effect of fees. A fund that is below its threshold will not asses performance fees until it reaches its high-water mark. Without the fee drag, performance is higher. Similarly volatility is also higher. See robustness check regarding net vs gross fees in section 2.5.5.

Table 2.1: Basic Returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance				Annual Return			
	OLS		2SLS		OLS		2SLS	
Distance	0.0040***	0.0043***	0.0131***	0.0071**	0.0200	0.0078	-0.0637*	-0.1414***
	[0.001]	[0.001]	[0.003]	[0.003]	[0.015]	[0.015]	[0.037]	[0.036]
Strategy Return		-0.0069		-0.0078		0.6522***		0.7020***
		[0.005]		[0.005]		[0.036]		[0.039]
Strategy Variance		0.2874***		0.2911***		0.1224		-0.0716
		[0.039]		[0.038]		[0.465]		[0.456]
Observations	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254
R-squared	0.20	0.20			0.47	0.49		
Number of funds			3,945	3,945			3,945	3,945
Kleibergen-Paap Wald rk F statistic on Excluded Variables			52.45	52.45			52.45	52.45

Robust standard errors clustered by fund in brackets

*** p<0.01, ** p<0.05, * p<0.10

All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.

“Distance” measures distant below the threshold. “Strategy Return” and “Strategy Variance” are the performance of a passive fund. 2SLS specifications include “Distance” that reflects strategy performance.

no correlation with performance or perhaps is even correlated with a very slight increase in performance. Columns (7) and (8) address the endogeneity concerns. In column (7) we see a small, but insignificant decrease in expected returns when below the high-water mark. However, given the potential for cyclical returns to strategies, controlling for the performance of the strategy is an import baseline. The effect of being underwater is larger in column (8). The difference between columns (7) and (8) implies that strategies have higher returns in periods following when the strategies are likely underwater.[14] However, unlike the risk-taking analysis there are several reasons to prefer specification (8) over specification (7). First there is cyclicality in the returns of various strategies and this should be controlled for. Second, from a performance evaluation perspective controlling for the passive opportunity set is important to distinguish actions taken by the manager from the market performance.[15] The coefficient in column (8) is large – it implies that annual returns are 14 percentage points lower for that fund with assets equal to half their high-water mark and 2.1 percentage points for the mean fund below its threshold. While these estimated effects are large, they are consistent with other findings in the literature. Agarwal, Daniel, and Naik (2009) find a cross-sectional correlation between hedge fund manager's marginal incentives including those provided by performance fees and future return.

The scale of these effects can also be converted to dollars. If we assume that the contracts were always readjusted so that the manager was always at the threshold, but that this adjustment does not affect performance when the manager is at the threshold, we can convert the coefficients estimates to the cost borne by investors for having misaligned incentives. The cost of the performance effect is straight forward. A fund is on average 7.3% below the threshold, so would perform an average of 1.03 percentage points better per year using column (8). Applied to the 2 trillion dollar hedge fund industry that is $20 billion per year. Valuing the risk requires assuming something about the utility function of investors. Suppose investors have a constant risk aversion coefficient of 1. If returns are normally distributed we can characterize the investors' utility functions as mean-variance utility. Using the coefficient estimate from column (3), the same 7.3% below the threshold, adjusting for the risk aversion coefficient that estimate is 0.0478 percentage points per month, 0.574 percentage points per year, or $11 billion per year across the hedge fund industry.

In sum, Table 2.1 suggests that funds that are below their high-water mark reduce their expected return and increase their risk – both because of increased underlying risk and additional risk-taking. These results are entirely consistent with the manger being increasingly likely to take the riskier, yet lower expected value project, the farther they

[14] This is consistent with mean reversion in strategy returns which is not captured by the time fixed effect: that is a correlation between recent poor performance by a strategy (which makes the instrument predict funds in that strategy are underwater) and positive subsequent performance.

[15] An alternative measurement approach that instead of using return as the dependent variable uses estimated alphas for the manager's performance contribution above asset allocation produces results that are comparable to specification (8).

are from their threshold. Indeed, this higher risk lower reward combination is consistent with Bowman's (1980) seminal observation that higher risk industries have lower returns. The effects of threshold incentives even provide a potential neoclassical micro foundation for this observation. Suppose all managers in all firms in all industries begin with the same threshold incentive schemes and risk levels. If some industries experience a negative shock, managers in those industries will respond to the threshold incentives by undertaking higher risk lower return projects. This mechanism could produce persistent differences between industries from transient shocks.

A similar logic provides a further cautionary consequence of threshold incentives. Threshold incentives lead managers to increase risk and reduce expected performance following a negative shock. However, if the shock is at the industry or economy wide level, this mechanism suggests that there would be systemic effects. Such a shock would lead to increase risk-taking and reduced performance across the industry or economy in question, essentially multiplying and sustaining the original shock. This magnification effect is important from a public policy perspective as it suggests incentive thresholds may have contributed to both the depth and duration of economic downturns.

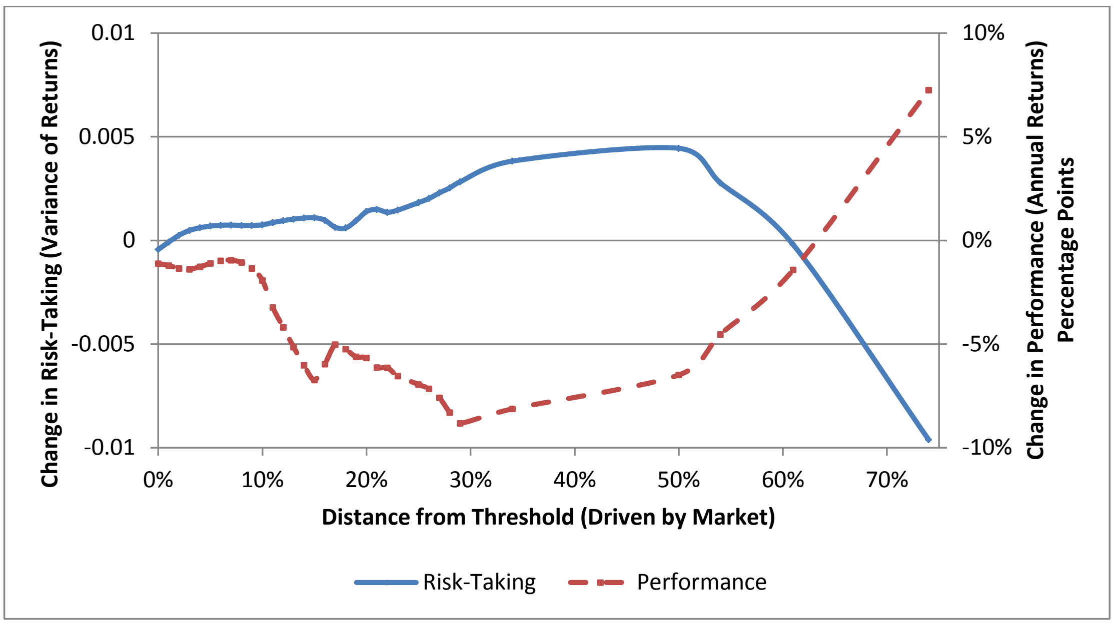
2.5 Extensions

2.5.1 Distant Managers

The next set of predictions follows from Proposition 1.3, which predicts that managers with distant thresholds would behave differently than those closer. The prediction is that those managers would no longer find risk-taking profitable – the fence is so unlikely to be reached it is not worth gambling to reach. However, the incentives for effort continue to decline as distance increases. Performance should continue to decline because of decreased effort, but the performance cost of risk-taking will no longer magnify the decline. Empirically, I estimate this by estimating a separate response to distance for managers far from their thresholds. The measure of "far from their threshold" is somewhat arbitrary. Here I present results using a dividing line of needing a return of 75% to reach the threshold, but the results are robust to other divisions. Approximately 2% of the observations reflect managers beyond this threshold. Figure 2.3 extends Figure 2.2, but now includes all managers, including the relatively few very distant ones. Looking at the solid blue risk-taking line we see that, consistent with the Propositions 1.3 and 1.4, it appears that risk-taking does not increase with distance for managers far from their thresholds and eventually decreases. Looking at the red dashed performance line we see that these same managers perform better at the same times they take less risk. This is consistent with Corollary 1.2 that the performance cost of risk-taking is large.

Table 2.2 presents these results on risk-taking and performance. These are similar specifications to those in Table 2.1, except the interaction of far from the threshold and

Figure 2.3: Risk-Taking and Performance with Distance to the Threshold including Distant Managers.



The horizontal axis is the distance below the high-water mark the manager is as driven by the performance of market factors calculated above. The left vertical axis measures the variance of fund returns in the following year plotted in the blue solid line. The right vertical axis measures the annual return of funds in the following year plotted in the red dashed line. Both lines are non-parametric fitted values after including fund and year fixed effects and controls from age, age-squared, assets under management, and log assets under management. Includes all managers.

Table 2.2: Distant Managers

	(1)	(2)	(3)	(4)
	Variance		Annual Return	
	2SLS		2SLS	
Distance	0.0336***	0.0158**	-0.3045***	-0.3694***
	[0.0085]	[0.0074]	[0.1143]	[0.0960]
Distance X More than 75%	-0.0270***	-0.0115*	0.3165***	0.3048***
	[0.0077]	[0.0066]	[0.1044]	[0.0896]
Strategy Return		-0.0071		0.6816***
		[0.0051]		[0.0391]
Strategy Variance		0.2937***		-0.1407
		[0.0386]		[0.4621]
Observations	20,254	20,254	20,254	20,254
Number of funds	3,945	3,945	3,945	3,945
Kleibergen-Paap Wald rk F statistic on Excluded Variables	66.61	66.60	66.61	66.60

Robust standard errors clustered by fund in brackets
*** p<0.01, ** p<0.05, * p<0.10
All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.

See notes to Table 2.1.
"More than 75%" indicates if the fund is more than 75% from the threshold.

distance is included and instrumented for.[16] The interaction term reflects the difference between the main effect – the responsiveness of near managers, and the responsiveness of distant managers. The net effect we see in columns (1) and (2) are that distant managers take much less risk than managers at moderate distances from their thresholds. Indeed, while the point estimate is that distant managers do take more risk than managers at their thresholds, this is not significant. In terms of magnitude, column (2) estimates that a manager 50% from their threshold increases risk-taking by twice the amount that a manager 100% from the threshold does.

Columns (3) and (4) show the same pattern. Managers far from their thresholds perform better than managers at moderate distances from their thresholds. The estimates in Column (4) suggest that a manager 50% from the threshold reduces performance by three

[16]Note that this creates potential endogeneity concerns because being far from the threshold is potentially endogenous. However, because the results suggest that risk-taking and performance are better beyond this threshold, the endogeneity concerns seem small.

times as much as a manger 100% from the threshold. These regressions allow an estimate of how much of the performance drop observed in managers moderate distances from their thresholds are due to reductions in effort, and how much is due to the performance cost of risk-taking. The logic behind this calculation is to assume that the reduction in performance observed by managers far from their thresholds is entirely due to effort reduction[17] and that the difference between this rate and the rate of reduction in performance by managers closer to the threshold is this reduction of effort combined with the performance cost of risk-taking. This calculation suggests that 83% of the performance drop observed in managers moderately below their thresholds can be attributed to the performance cost of risk-taking and only 17% to the reduced effort.

2.5.2 Fee Variation

The above fundamental results show that the average treatment effect of being below the threshold increases risk-taking and reduces performance. Yet, these average treatment effects include significant heterogeneity. In this section I examine the varying response of mangers depends on the details of their fee contracts. In addition to the direct interest in how the variation in fees affects this behavior, this investigation has several useful implications. First, these results provide significant additional confidence in the empirical strategy outlined above. One potential concern about the instrumental variable approach is that most of the variation I use is at the strategy level and I might be measuring something about the pattern of variance and performance of the underlying assets. The first method to address this concern is to directly control for the performance of the underlying assets as I do above. However, these results provide an additional test. Because the fees vary within strategy-year if the effect were driven by the pattern in the underlying assets all funds in a strategy, responses would not vary with fee structure. Because I find that they do, these results suggest that the instrumental variables approach is not finding an uncontrolled for relationship between factor performance and flows and the subsequent environment, except through the pathway of fee contracts. Second, this investigation has the potential to disentangle effects driven by the explicit fee contracts and other non-neoclassical behavior.

The first four columns of Table 2.3 explore the role of base or management fees. The predictions above are that higher management fees would lead to less risk-taking and smaller performance declines when below the threshold. About 40% of the funds have a management fee of 1%, and about 20% each have management fees of 1.5% and 2%, and the rest distributed at other values between 0 and 3%. Columns (1) and (2) show, consistent with the prediction, that funds with high base fees increased risk less when below the threshold. The magnitude of the interaction suggests that a fund without a base fee increases risk about 50% more than a fund with modal base fee of 1%. Columns

[17] As the point estimates in columns (1) and (2) suggest that risk-taking is still increasing slowly in managers far from their thresholds this may underestimate the performance cost of risk-taking.

(3) and (4) show that there is no effect of the base fee on the average return effect of being underwater. Not only are these results not statistically significant, the coefficient is small. Looking at column (4), the difference between 1% and 2% base fees implies a drop in performance of just 14 basis points. While the point estimate in column (4) is the predicted sign, the magnitude is small. Reconciling the different findings in columns (1) and (2) compared to (3) and (4) has several potential explanations. One is that risk-taking has no meaningful performance cost and that the base fee does not affect responsiveness to distance to the threshold. On the other hand, it maybe that risk-taking is costly, but that managers with higher base fees actually reduce the effort they place on improving performance when they are below their thresholds, perhaps to focus on soliciting investors.

The second fee of interest is the performance fee. This fee pays a manager a share of the profits the manager earns above the threshold. This fee is central to the empirical approach in this research. Indeed, funds without a performance fee have no threshold in their explicit incentives. The predictions of this fee developed above are straightforward. The higher the performance fee the larger the response we should see – both in increased risk-taking and reduced average return. Indeed if the linear functional form is right, we would expect that the direct measured effect of being below the threshold is zero if an interaction with the performance fee is included. Performance fee is the interaction in columns (5) through (8) of Table 2.3.[18] Approximately 80% of the funds have performance fees of 20%, with about 5% each having performance fees of 0%, 15%, and 25% and the rest distributed at other fees between 0 and 50%. Because of this distribution this test has somewhat limited power, and particularly limited support for the intercept of distance. Though not strongly significant, the results in columns (5) and (6) are consistent with about half of the increase in total risk-taking being driven by the performance fee. Indeed, column (6) estimates that funds with no performance fee do not have a statistically significant increase in active risk-taking when below the threshold and the coefficient estimate reflects an effect of just a 10% increase in risk for the average fund below its threshold. Despite this, looking at column (8) we see that all of the decrease in expected return is being driven by funds with performance fees. Indeed, this suggests that a fund with a 15% performance fee has a drop in return equal to about 75% of the drop that a fund with a 20% performance fee experiences. This suggests that performance fees provide strong incentives in these funds.

Taken together the heterogeneous response of managers depending on their contractual fee structures provides several interesting results. First, consistent with theory, bigger performance fees lead to more responsiveness to the threshold, and in contrast higher management fees serve to blunt the incentives to take extra risk. In other contexts, higher base and equity compensation can be an important moderator of option and other threshold compensation. Further, these results provide quite a lot of robustness to the empirical

[18]Theory would suggest that there might be an inverse relationship between base and performance fees. In this dataset, there is effectively no correlation between these fees and the results are substantially similar in a specification with interactions of both base and performance fees with distance. Empirically, this also means that the results are unchanged if both interactions are included in one specification.

Table 2.3: Explicit Incentives

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance		Annual Return		Variance		Annual Return	
	2SLS		2SLS		2SLS		2SLS	
Distance	0.0318***	0.0213**	-0.0412	-0.1623**	0.0068***	0.0030	0.1460	-0.0035
	[0.010]	[0.009]	[0.084]	[0.080]	[0.002]	[0.004]	[0.092]	[0.066]
Distance X Base Fee	-0.0124**	-0.0095*	-0.0149	0.0140				
	[0.005]	[0.005]	[0.044]	[0.045]				
Distance X Performance Fee					0.0004*	0.0003	-0.0138***	-0.0089**
					[0.000]	[0.000]	[0.005]	[0.004]
Strategy Return		-0.0074		0.7014***		-0.0077		0.6992***
		[0.005]		[0.039]		[0.005]		[0.039]
Strategy Variance		0.2983***		-0.0821		0.2941***		-0.1686
		[0.038]		[0.455]		[0.039]		[0.454]
Observations	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254
Number of Funds	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945
Kleibergen-Paap Wald rk F statistic on Excluded Variables	32.28	32.27	32.28	32.27	26.08	26.07	26.08	26.07

Robust standard errors clustered by fund in brackets

*** p<0.01, ** p<0.05, * p<0.10

All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.

findings in section 5. They suggest that the results are not driven by contamination of the instrument by some serial correlation in strategy performance. Furthermore, they give additional confidence that these behaviors are being driven by explicit incentives. Indeed, if these behaviors were the result of, for example, reference points, we would not expect differential responses for different fee structures. Applied more broadly these results suggest that top management teams with many options are more likely to increase risk and decrease effort when they are out of the money. Guaranteed compensation and equity holdings and compensation for firms that are not in bankruptcy risk serve a role similar to the base fee and should reduce the increased risk-taking.

2.5.3 Organizational Interactions

In this section I examine the predictions suggested by the organization economics literature. The first prediction is that direct capital investment would reduce responsiveness to the contractual incentives. Unfortunately, direct measures of manager's capital are not available. Agarwal, Daniel, and Naik (2009) use a proxy for additional manager's investments based on past performance fees. They argue that managers are likely to invest performance fees earned into the fund. Because this measure does not capture the manager's initial capital investment it is somewhat limited. It is closer in concept to the manager that amasses equity holdings over time, rather than the founder as manager. Nonetheless, I recreate their proxy in the data and interact it with the distance variable.

Columns (1) through (4) of Table 2.4 show this interaction. "New Manger's Capital" is this proxy. It is scaled in percent of fund assets. Though this proxy is incomplete - managers often make significant capital investments at fund inception - they provide some evidence of the role of manager's capital. This specification also has potential endogeneity concerns in that actual investments of additional capital by managers are endogenous decisions. This proxy, however, assumes that a fixed share of performance fees is reinvested. Since actual additional or initial investments are not observed which reflect the endogenous investment decisions of managers, this is resembles an instrumental variable approach. However, because the actual investments are not observed, the "instrument" is used directly. Given these concerns, this specification is merely suggestive. Interpreting columns (1) and (2), the scale suggests that once a manager's ownership of the fund reaches 10% there is no effect of being below their threshold on risk-taking. While the interactions are not significant for the return effects, the coefficient estimate suggests about 20% ownership is sufficient. These estimates are consistent with the theory – managers with more of an ownership stake respond to contractual explicit incentives less.

Reputation is the next organizational characteristic I explore. Connecting to the model, reputation functions similarly to the base rate in that a manager with a more valuable reputation has more to lose and gain, without regard to the threshold. However, because investors may be risk-averse, we might expect the risk-taking to be even more responsive than performance. While there are many facets to reputational value, I use two measures: one which reflects industry perceptions of reputation and a second which

Table 2.4: Implicit Incentives

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Variance		Annual Return		Variance		Annual Return		Variance		Annual Return	
	2SLS		2SLS		2SLS		2SLS		2SLS		2SLS	
Distance	0.0190***	0.0123**	-0.0521	-0.1798***	0.0265***	0.0147***	-0.2678***	-0.2946***	0.0168***	0.0096***	-0.0282	-0.1205**
	[0.005]	[0.005]	[0.055]	[0.057]	[0.007]	[0.005]	[0.056]	[0.057]	[0.004]	[0.004]	[0.050]	[0.047]
Distance X New Manager's Capital	-0.1816**	-0.1523*	-1.2617	0.0984								
	[0.085]	[0.079]	[1.085]	[1.054]								
Distance X Age					-0.0030***	-0.0016***	0.0454***	0.0331***				
					[0.001]	[0.000]	[0.011]	[0.010]				
Distance X Max Funds									-0.0005***	-0.0003**	-0.0043	-0.0025
									[0.000]	[0.000]	[0.003]	[0.003]
Strategy Return		-0.0088*		0.6962***		-0.0081		0.7076***		-0.0079		0.7011***
		[0.005]		[0.040]		[0.005]		[0.038]		[0.005]		[0.039]
Strategy Variance		0.2820***		0.2252		0.2835***		0.0815		0.2912***		-0.0712
		[0.038]		[0.471]		[0.037]		[0.455]		[0.038]		[0.457]
Observations	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254	20,254
Number of Funds	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945	3,945
Kleibergen-Paap Wald rk F statistic on Excluded Variables	6.807	6.806	6.807	6.806	19.62	19.61	19.62	19.61	17.54	17.53	17.54	17.53

Robust standard errors clustered by fund in brackets
*** p<0.01, ** p<0.05, * p<0.10
All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.

captures ability to profit from reputation. The first measure reflects industry perceptions that age is a good measure of reputation because of the value of having a long track record. Columns (5) through (8) of Table 2.4 look at age and show that the effect of being below the threshold decreases with age. The estimates are all consistent with the effects of being below the threshold being totally dissipated by the time the fund is 10 years old. While suggestive of reputational effects, there are many possible drivers of the age results. These results are consistent with the predictions of reputation discussed above. But age is also correlated with increases in a manager's capital. Similarly, older funds may have more experience (as in Simsek, 2007), other incentives, or different institutional characteristics such as structure inertia (de Figueiredo et al., 2012). Note, however, that this result is not driven by one apparent explanation: It cannot be that older funds reflect the funds' fixed quality as time invariant firm-specific effects are absorbed by the vector of fund fixed effects. Therefore, it must be that the funds either improve with age or that quality only differentiates performance in the types of environments that persist after poor strategy performance.

The second measure draws on existing literature. de Figueiredo and Rawley (2011) develop a model of diversification which shows how firms profit from a reputation. In their model, manager's face a privately observed cost of diversification. Those with low diversification costs find it more profitable to diversify if investors believe they are high quality. Managers with high diversification costs are unlikely to diversify, regardless of investors beliefs. As such, high cost managers have less to gain from investor's beliefs about their quality, that is, their reputation. Following this logic, realized diversification is then correlated with having a low diversification cost, and thus having a high value for one's reputation. As such, I use the number of funds I observe a fund to ever have as a measure of its ability to diversify and thus value of reputation. We should expect those who eventually have many funds to increase risk less. Note, because of the potential reverse causality – that eventual number of funds a firm has might be caused by their relative success when underwater, the effect on returns can be a test of this endogeneity. If diversifiers have comparably better average returns when below the threshold this concern would be most plausible.

Columns (9) through (12) examine the eventual scope the firm will achieve as a proxy of their reputational value. This is perhaps a cleaner test of the value of reputation following the logic discussed above. Though the firm's eventual scope may capture many things, one interpretation is to capture the value to the fund of its reputation. Empirically, there are several concerns. First, the proxy used is imperfect because low cost, high quality and/or lucky managers are those that we observed diversify but many of the non-diversifiers may be low cost (and thus high reputation value), but unlucky or low quality managers. If true, the results will be biased towards zero and reduce the power of the measure. Second, the specification has a potential endogeneity concern in that it describes a fund today with a future characteristic, the firm's future scope, which reflects among other things the performance of the fund today. If good performance leads to increase diversification, as de Figueiredo and Rawley (2011) find, we should be particularly concerned if future scope is correlated with better performance when below the threshold. However, in columns (11) and (12) we see a negative coefficient on the interaction term, suggesting that, if anything, diversifiers experience larger return drops. However, columns (9) and (10) do show less risk-taking among those with valuable reputations. Much like those with high management fees, those with valuable reputations appear to take fewer gambles.

This measure also allows some examination of the value of a reputation. Here we observe managers with valuable reputations taking less risk. Thus, they sacrifice short term contractual compensation for unobserved returns from reputation. The value of their reputation, then, must be higher than the compensation we observe them forgo. Using the estimated reduced risk-taking from column (9) we can calculate this lower bound on the value of reputation. To do this I estimate fee realizations assuming returns are normally distributed and using the mean values for returns and variances. With those assumptions this should be considered a lower bound not just because the benefits of reputation are not observed, but also because the use of a proxy biases the measure towards zero. A manager who is the average distance below the threshold with a 20% performance fee and a reputational value one standard deviation higher than average earns fees approximately 0.37% of the assets of the fund, or $410,000 for an average sized fund, less than a fund with an average reputation. [19]

The scope of a fund's management firm has also connects to the multi-tasking prediction. A firm with more funds has more incentivized tasks to which the manager may allocate effort. This leads to the prediction that firms with more funds will have larger decreases in average return. While not quite significant in this specification the coefficients in column (12) suggests that one reason a fund performs less well might have to do with the manager's alternative areas of work. Those managers with more funds, which may not be below their thresholds, are those who show the biggest decreases in expected outcome – these managers are likely shifting effort to where incentives are stronger. Considering the internal organization of the hedge fund – even if the manager of a particular fund is not a formal participant in the investment process of a separate fund the attentions of others in the firm as well as the allocation of investment ideas to funds may shift away from the fund below its threshold.

The interactions with the firm scope are also informative on the whether these behaviors are responses of the management firm as an organization or of the managers, and other individuals in the firm, responding to their incentives. The performance feedback literature suggests that when organizations direct resources in response to underperformance by entities within that organization. Essentially, it argues that organizations "put fires out". Being under the incentive threshold is indeed such an underperformance. This suggests that firms would redirect resources to the underperforming entity and those with more available resources would deploy them to the underperforming entity and improve its subsequent performance. The results in columns (11) and (12) have the opposite sign

[19] Including the reduced performance from column (11) results in an estimate of 0.44% of the fund, or $490,000.

than this prediction. Certainly, this is not a strong test of performance feedback theory, but it suggests that the responses by managers in this context are more concordant with individual behavior or the aggregation of individuals in an organization with closely aligned incentives.

The interactions in Table 2.4 are suggestive, but subject to some potential bias and measurement issues. However, these results show that age is correlated with smaller decreases in returns and smaller increases in risk. Eventual scope (reputation value) leads to less increase in risk. Scope is also correlated with bigger decreases in return, consistent with managers reallocating effort. Taken more broadly, these results provide some evidence that reputation can restrain managers and multi-tasking concerns can magnify the effort effects. These results have implications to organizational design. Creators of organizations can manage scope and use reputation and ownership incentives to balance contractual incentives.

2.5.4 Mechanisms

The previous sections have characterized that managers take more risk and reduce performance when they are farther below their thresholds and that this behavior is driven by explicit contracts and moderated by organization factors. In this section I use the data to explore the nature of this risk-taking. Second, I use the results of the conceptual framework to shed light on the relative importance of effort versus risk-taking on the performance of financial managers.

How are managers taking risk? Realized volatility as a measure of risk-taking does not measure the choices the manager directly makes. Instead it measures the realizations of their decisions. If managers take actions that they believe are riskier but do not result in riskier outcomes, those risk-taking decisions would not be captured. On the other hand if managers believe they are not taking different risks, but the realized environment is riskier that may be captured as risk-taking. Indeed, this distinction allows some understanding of the mechanisms the managers take to increase risk. Indeed, the difference between columns (3) and (4) in Table 2.1 provide some insight. The difference between the coefficients suggests that about half of the increase in variance is because of increased variance in the underlying markets, and about half is due to explicit increases in risk. To the extent that a hedge fund manager aims to maintain an absolute risk profile – that is, they endeavor to have the same level of risk despite the riskiness of the environment, then column (3) is the appropriate comparison. These managers should reduce their exposure to the market when the market is riskier. However, if hedge fund managers aim to maintain a relative risk profile, that is, have a constant exposure to market risks then column (4) is the right comparison. These managers should have riskiness that increases with the market risk.

The results in Table 2.3 actually provide suggestive evidence of whether managers account for increases in market-risk in their responses. To the extent that they do, it suggests that managers both have correct expectations about market risk and are con-

Table 2.5: Mechanisms of Risk-Taking

	(1) Beta	(2) Alpha Risk
	2SLS	
Distance	0.4205***	0.0032*
	[0.129]	[0.002]
Observations	20,254	20,254
Number of Funds	3,945	3,945
Kleibergen-Paap Wald rk F statistic on Excluded Variables	52.45	52.45

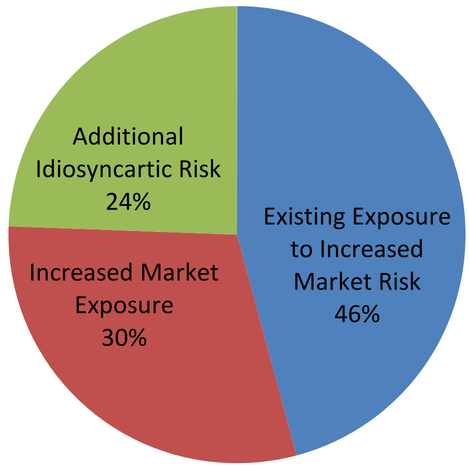
Robust standard errors clustered by fund in brackets
*** p<0.01, ** p<0.05, * p<0.10
All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.

cerned about total, not relative risk-taking. Consider column (5) in Table 2.3. Assuming the functional form is correct, the direct effect of Distance reflects the risk increases by a manager with no performance fee, and thus no reason to vary risk-taking with distance to an arbitrary threshold. The positive and significant coefficient suggests that managers are either surprised by the market risk or intend to maintain some exposure to market risks. Comparing column (5) to column (6) is also informative. If managers were surprised by the market risk, controlling for the market risk should not change the responsiveness of managers to their performance fee, and the coefficient on the performance fee interaction in columns (5) and (6) should be the same. While the difference is not statistically significant, the smaller coefficient in column (6) is consistent with managers expecting and compensating for the market risk.

To further decompose the nature of risk-taking, Table 2.5 contains several specifications with different dependent variables. These dependent variables are the results of a set of regressions. For each fund-year, I regress the performance of the fund on the performance of the passive market index for that fund's strategy. The first column has as the dependent variable the fund-year beta. This shows that increasing exposure to the market was an important part of the risk-taking. The second column has as its dependent variable the variance of returns not explained by the exposure to market risk. This "alpha risk" shows that not only do managers increase market risk they also increase their idiosyncratic risks. The coefficient suggests that about a quarter of the total increase increased in risk is due to idiosyncratic risks. Figure 2.4 aggregates the estimates of the mechanisms by which managers increase risk.

Another mechanism worth exploring is the importance of managerial effort and risk-taking to influence performance. The conceptual framework shows that both manager's risk decisions and effort can affect the average performance. Indeed, there is debate about

Figure 2.4: Sources of Increased Risk-Taking



the importance of effort in incentive contracts both because it is not entirely clear how much impact manager's effort can have on performance and because most managers appear to exert so much effort so that only minimal additional effort might be induced. This research provides additional evidence that effort, or at least effort allocation, responds to incentives in meaningful ways. First, the multi-tasking results above suggest that the opportunity to exert effort elsewhere reduces performance when incentives fall. Beyond that the framework allows us to isolate the performance impact of risk-taking separately from the effort choice. This is through the non-linearity result in the framework. As calculated above this leads to an estimate that 80% of the reduction in performance observed may be caused not by reduced effort, but by increased risk-taking. Taken together, both of these results suggest that incentives are important drivers of performance, even among individuals and managers with complex jobs. Second they both suggest that simple, one-dimensional, models of tasks and incentives may miss important responses of agents.

This evidence that additional risk-taking is not associated with higher realized return can also inform a debate about the role of convex incentives to encourage risk-taking by risk-averse agents. Implicit in that debate is that because of risk-aversion, a manager would not undertake risky, but profitable, projects for the firm because their disutility from exposing their compensation to risk. This optimal convexity depends on the interaction of the manager's risk aversion, the responsiveness of the firm to the manager's decisions and the firm's ability to insure. However, with no additional return for additional risk-taking the answer is simpler. Regardless of the degree of risk aversion, when managers are below their thresholds the compensation schemes induce too much risk-taking.

2.5.5 Robustness Concerns

The data used in this analysis has many potential limitations. In this section I address several of these and propose robustness tests. The first set of challenges comes from the fact that these data are self-reported and represent only a subset of the hedge fund industry. The funds included in this dataset represent approximately a quarter of the hedge funds during this period and are believed to be broadly representative. However, no comprehensive database of the hedge fund industry exists to evaluate this sort of selection bias. Furthermore, because this study looks at how these managers respond to their incentives, it is unclear that any systematic difference between this set of funds and the remainder would reduce the implications for risk-taking consequences of being below their thresholds.

Beyond the question of coverage, the voluntary reporting nature leads to two additional kinds of selection discussed in the literature. These selection come from the fact the funds decide when to report data. Intermittent reporting of data is not problematic. Few funds appear to report intermittently, and those that do are excluded. However, when a fund first begins to report to the data vendor, it generally reports not only current and future performance, but past performance as well. This "instant history" bias tends to include funds with particularly good initial performance. To account for this bias, the standard approach is to exclude the first two years of a fund's data. Doing this does not have a qualitative change to the results reported. Additionally, for a subset of this data, I have access to information about when a fund first began reporting. This allows robustness checks by limiting only to funds that began reporting immediately or to more precisely exclude the instant history.

The second selection question is that exiting the data set (stopping to report) is also voluntary. One way to test this is to restrict the analysis to a set of funds that as of a particular point are actively reporting, and only including those funds before that time. Again, the results are not qualitatively changed by this restriction. Additionally, for a subset of the funds that have exited the data set I have the reason the fund has left. While prior literature has assumed that exiting these data because of extreme success and failure were both common, this data suggests that the vast majority of fund exits are due to fund liquidation (45%) or firm failure (18%). Less than one percent of exits are closures to new investments (a sign of success). 26% are voluntary decisions to stop reporting for an unspecified reason, and 5% are mergers into other funds. These last three groups would be the potential sources of this reporting bias.

Survivor bias is another concern in this style of research, but is not a limitation of this data. Funds are included in this analysis regardless of whether or not they exit. Indeed, the robustness check described to address voluntary ending of reporting induces selection bias, but does not change the qualitative nature of the results, suggesting that survivor bias, if it existed, would not be a major problem.

There are, additionally, a variety of sources of measurement error in the data and in the calculation of high-water marks. Calculation of the high-water marks and distance depends on accurate reporting of returns and assets under management. While returns are regularly reported there are some funds for which assets are occasionally not reported. Returns before the first reporting of assets are excluded from the calculation of the high-water mark. This produces a bias in that some of these funds may be below their threshold, but they are treated at their threshold. This would, in general, bias the estimates towards zero. Similarly, any funds for which returns are not available from inception would produce the same bias. A robustness check restricting to funds for which assets are available at inception would check for this. Any funds which stop reporting assets but continue to report returns are treated as exits and are addressed in the robustness checks for voluntary reporting. For funds for which assets are not reported for some intermediate period, net flows over that period are distributed evenly over the period. This particular measurement error may in fact be corrected for by the instrumental variable approach.

Another measurement issue comes from the fact that the return of a fund in a particular period may have several definitions and the reporting practices are unclear. Some funds may report gross returns. Other funds may report the after fee returns. However, since different vintages have different fees, some funds may report after the fee of the oldest vintage, while other funds may report an average. Returns net of fees also depend on when fees are accounted for out of assets – returns are monthly, but fees are often accounted for quarterly. In this analysis, I treat all the returns in the data as gross returns, however the results are qualitatively similar treating the raw data as net returns, imputing gross returns, and analyzing those.

An additional complication in the calculation of the high-water mark is the role of hurdles. A hurdle is a base rate of return that a fund must earn before earning a performance fee. Effectively, it moves the high-water mark every year, regardless of performance. Unfortunately, while the data indicates whether a fund has a hurdle, it rarely indicates what this hurdle is. As such, I do not account for hurdle rates in this analysis. Generally, a hurdle will make a fund farther from its threshold than I estimate. The only time this would not be the case is when the hurdle is the rate of return of some asset that has experienced a loss. However, these sorts of hurdle rates are rare.

Another source of measurement error is the self-categorization. This realizes in two ways. First, a category might be too broad, incorporating funds with strategies that differ substantially. Additionally, categorization error might lead for a fund not to be categorized with like funds. Both of these errors will lead to estimates of a passive hedge fund that is a mix of the strategies employed in the category. This would lead to an instrument which is weak because of the low correlation between the “passive” returns and actual performance. This would also lead to less informative predictions about the performance of the passive fund in the year after being underwater. Both of these would lead to biases towards not finding any effect of being below the threshold and not finding a difference when controlling for market performance.

Serial correlation in returns is another potential source of error. There are several sources of serial correlation. First, funds which hold illiquid assets may use valuation measures that induce serial correlation. Second, assets that the funds own may exhibit momentum. The second source is partially addressed by the inclusion of Carhart's (1997) momentum factor for equity. However a broader robustness check is to estimate for each fund an AR(1) process and use a measure of return which is net of this AR(1) process. This does not change the qualitative results.

Finally, the calculation of high-water marks depends on vintages of investments into funds. However, I do not observe actual flows. Instead, I observe net flows each month. Net flows are a subset of actual flows. That is, there are always weakly more fund inflows and weakly more fund outflows than I observe. Effectively, this leads me to assume that the assets that are in a fund are from older vintages than they actually are. As older funds always have the highest high-water mark, this means that my measure of distance is biased towards the assets that are most distant. Thus, some funds are closer to their threshold than I measure. However, this is only a problem of scale. This is because the vintages do not matter unless a fund is some distance below its threshold. This scaling issue, however, is further complicated by not observing which assets are those that have flowed out of the fund. Instead, I apply the outflows proportionally among all funds. An alternative assumption is to apply exits on a first-in first-out basis that assumes that the funds that leave are the always the oldest vintages. Neither is a perfect representation of actual flows. Similarly, this mis-weighting of vintages introduces bias in the scale of distance. The biggest consequences of this source of error is that one should be careful in comparing the absolute levels of distance with those measures created from more detailed flows.

Beyond the robustness and measurement concerns, the question of what pathways lead to these risk and return changes is worth additional discussion. While, I emphasize the role of effort and risk-taking in response to incentives as the main drivers there are additional pathways worth exploring. One potential driver of changes in fund performance would be flows. For example, if a fund is experiencing significant net outflows it may change the composition of the fund as the fund sells liquid assets. Similarly, as a fund experiences in flows it may acquire liquid assets faster than illiquid assets. If illiquid and liquid assets have different risk and return profiles, flows, which may be correlated with distance, might be potential drivers of the changes I estimate. Further, flows may also change the concentration of a fund's assets, thus changing its risk. Empirically, because flows are potentially endogenous, this is not a simple robustness check to perform. However, the results in Table 2.3 suggest that the effects are being driven by the contracting terms. For the results in Table 2.3 to be spurious there must be a correlation not only between flows and distance, but also differential correlations between those flows as the contracting terms. The first is quite plausible, the second less so.

Further contemplation about the instrument used also suggests that some other pathways are possible. The instrument represents strategy specific performance, particularly, recent strategy specific losses. One concern is that strategy specific losses capture something relevant to the performance of the funds. From discussions with industry participants on of the main internal consequences of not earning a performance fee is employee retention. Hedge fund principals have to decide whether to invest additional capital in the firm to retain talent when performance fees are not earned, or risk losing that talent. If the entire strategy has performed poorly that risk may be lower. However, there are several limits to this possible pathway. First, industry participants were thinking partially of the financial crisis. This analysis does not include the financial crisis and the impacts of earlier macro-economic conditions on employee mobility were much smaller. Second, because the instrument is strategy specific performance that is not captured in the time fixed effect the potential scope of employee mobility would have to be not to some other place in the hedge fund industry, but restricted to strategies that are exposed to similar assets. However, the direction of this bias would seem to be against finding an effect. If the effect is driven by talent leaving funds that are distant, then when mobility is reduced because all the funds in a strategy are below their thresholds we should see smaller changes in risk and return than we would if mobility was unaffected.

Another pathway would be if the strategy specific performance changed the nature of competition, either between hedge funds in some fashion or between hedge funds and their trading partners. The results would be consistent with something leading to more competition for trades when funds are below their thresholds. If hedge funds face more competition the return of any particular trade would be lower, and may cause increases in concentration, leverage, or risk-taking. However, it is not obvious that distance is correlated with increased competition. Indeed, being below their thresholds suggests that these strategies have fewer assets than before so that there should be less "money chasing deals" and thus less competition. Further, the fact that these decreases in return and increases in risk are relative to the passive portfolio make the competition pathway less plausible – a passive portfolio is not subject to these competitive pressures and the funds do not choose to remain passive.

2.6 Conclusion

The empirical results of this research are clear. Managers in firms below their incentive threshold take on more risk and generate lower expected returns. However, those very far from their thresholds take less risk and perform better than managers closer to their thresholds, which is consistent with the added risks being negative expected value. The interactions of distance to the threshold with the management and performance fees strongly suggest that these results are driven by the contractual fee and incentive structure. An examination of organizational incentives suggests that reputation and direct ownership mitigate misbehavior induced by thresholds. Finally, the multitasking result suggests that much of the return effect observed is not driven by a direct reduction in effort, but instead, these managers reallocate firm resources and attention. Analysis of the mechanisms of risk-taking suggest that increases in risk are driven by a combination of accounted for market risks, increased exposure to market risks, and idiosyncratic risk-taking.

Hedge funds provide an empirical context to measure and observe incentives and risk-taking behavior that sheds light not only on hedge fund managers, but on any decision maker facing similar threshold incentives. While at first blush, the “2 and 20” contract seems to be an extremely powerful threshold incentive, firm managers, on average, earn a larger share of their compensation from threshold incentives than hedge fund managers. This research uses the hedge fund context to measure behavior that should be applicable in other contexts.

The results do not suggest that threshold-based incentive compensation should be avoided – the comparisons above are comparisons within a threshold based compensation scheme. Instead, the results suggest the importance of setting thresholds correctly. Very high thresholds appear to have particularly significant downside potentials. For example, following the recent stock market crash, Google and about 100 other publicly traded companies went through option repricings where they exchanged low value, out of the money options, for higher value in the money options, presumably to avoid the distortionary impact of setting thresholds too high. If the risks of high thresholds are indeed significant, then threshold effects imply a concise answer to Hall and Murphy’s (2000, 2002) puzzle about why almost all options are given at the money: the downside of options far out of the money is increased risk-taking.

Chapter 3

Relational Influence Buying

3.1 Introduction

> And it's not just favors, per se, but it's a circle: get money, do favors, get money, do favors.[1]

The influence of money in politics is a perennial question. Our society has developed a number of institutions to limit the influence of special interests. Bribery of elected officials is illegal. There are rules and regulations governing the disclosure of political contributions and often a variety of limitations of the amounts and uses. Yet there is much public debate over the scope and influence of money. Existing empirical research has attempted to connect campaign contributions to influence with puzzling results. Empirical research connecting campaign contributions to actual policy outcomes to date shows a large effect and a high return to campaign contributions (e.g. de Figueiredo and Edwards (2007)). [2]In contrast, and consistent with Tullock's (1972) observation, campaign contributions are well below 0.5% of the U.S. Federal budget, and the ratio is similarly small comparing explicit federal subsidies to contributions by the industries that receive them. If campaign contributions have high returns, why do we not observe more exchange? Finally, as Snyder (1992) points out, even if we cannot empirically connect contributions to outcomes, the clear consistent pattern of campaign contributions suggests a relationship. This paper develops a model of relation influence buying that rationalizes these three empirical predictions: observed campaign contributions may provide high returns, campaign contributions are relatively rare, and there are long-term repeated relationships between politicians and contributors.

Theoretical works including Ben-Zion and Eytan (1974), Bernheim and Whinston (1986), Baron (1989), Snyder (1991), and Grossman and Helpman (1994) have developed a range of models to understand the exchange of money for political favors. All of these models either omit or leave unspecified the source of commitment. Describing many of these models, Austen-Smith (1997) notes, "the structure of the (necessarily) implicit contracts between candidates and groups is unspecified." Yet, this is an important omission because these agreements rest outside of courts, external enforcement, or spot markets. This research begins by explicitly considering repeated interaction and studies the design of self-enforced relational influence buying contacts. This model reflects some institutional realities: campaign contributions are given before elections, the campaign funds are spent

[1]Joan Claybrook, President of Public Citizen, The NewsHour interview, May 20, 2008. Transcript at: http://www.pbs.org/newshour/bb/politics/jan-june08/lobbying_05-20.html

[2]There is a much larger literature that attempts to connect campaign contributions with votes. Ansolabehere et al. (2003) surveyed 40 studies on the influence of campaign contributions on votes, found weak evidence of links between contributions and votes, and argue contributions are consumption. As has been noted elsewhere, the use of voting behavior as the dependent variable ignores any other activity of legislators, including influencing the bills that are eventually voted on, and further, votes may not be informative about influence if bills pass with more than minimum votes necessary. Beyond this, without identifying the change in the policy outcome it is impossible to value contributors received for their contributions, so it does not speak to the question of whether campaign contributions have high returns.

by the politicians to influence the outcome of the election, once elected, politicians can provide favors, and the cycle repeats. In the self-enforcing contracts firms give campaign contributions which the candidate values because they increase (re)election probabilities. Politicians provide favors that cost electoral support but provide monetary value to the firm.

The design of these contracts provides several valuable insights into the role of campaign contributions and buying influence. First, joint surplus from transacting, that is, a favor which is more valuable to the firm than the cost of the electoral support to the candidate is not a sufficient condition for trade to occur. Favors and the balance of bargaining power have to leave enough benefits to each party so that neither has incentive to deviate. These are effectively a sort of individual enforcement constraint. Second, horizon effects are such that contracting, and thus both influence and contributions, are lower when there is less of a future. I show that even in the case of finite lived politicians, such as due to term limits, contracting is reduced, but not eliminated completely because the value the firm receives from contracting with future politician may provide enough incentives to stop unraveling. Third, the nature of the relational contract provides a justification for lobbying by lobbying firms who can profit from their relationships and contract over a longer stream of favors. This provides predictions about what type of favors and firms would lobby directly, or via a lobbying firm. Similarly, this logic also predicts that legislation may be structured in ways to ease contracting constraints. Finally, exploring self enforcement directly, it clarifies the distinction between changing a politician's behavior, which requires self-enforcing contracts, and supporting a politician with which one agrees, where contracts are not necessary.

The application of relational contracting to interactions between a firm and politician is an important use of self-enforcing contracts. In this environment enforceable contracts are unavailable, even if the relevant information is both observable and verifiable. A willing enforcer is simply unavailable. The model in this research draws on the existing relational contracting literature (e.g. Levin (2003)), but differs from it in two important ways. First, I restrict transfers between the parties to be only campaign contributions and favors. This precludes direct transfers between the parties (e.g. bribes or posting a compliance bond). Second, I model the relational contract as the outcome of a bargain between the two parties. Related to the second point, I find that some exchanges are not supportable under extreme bargaining power are supportable in cases of more balanced bargaining power. The underlying intuition of the model is the same as in other relational contracts. The future value of campaign contributions from the firm to the politician can induce the politician to provide the costly favor; the value of future favors to the firm can induce the firm to contribute. Extreme bargaining power may make one of these too small to self-enforce compliance.

The results of the model also speak to the empirical literature on influence buying. The first result provides an explanation for the puzzle of high value, yet relatively rare, campaign contributions. The self-enforcing constraints may preclude contracting on favors even when the value of the favor to the firm exceeds the cost of providing the favor to the politician. With this constraint, we should expect to see abnormally high returns from campaign contributions, yet not expect additional contributions and favors to be exchanged. The results regarding the path of contributions over the course of a politician's career allow the empirical detection of influence buying without the challenge of measuring the behavior of politicians and the explicit provision of favors. Indeed, in section 3, using age among US Congress people and explicit term limits in US states with legislative term limits as measures of horizons, I find that politicians raise fewer campaign contributions as their horizons approach. Further, consistent with another prediction of the model, I show that controlling for strength of challengers, candidates raise less funds in their first election. While there are alternative explanations for each of these correlations individually, they are consistent with the predictions of the model. Finally, the model's predictions about the structure of lobbying and legislation provide additional empirical predictions that could detect influence buying without observing the favor provision itself.

This research develops a relational model of campaign contributions as influence buying. This model has many important implications. It provides an answer to the conflicting evidence on contributions: contributions may provide high returns but still be small compared to potential subsidies. It provides a natural distinction between contributions that influence politicians' actions and election outcomes from those that just influence election outcomes. It provides firm level implications for the process of influence buying, such as the role of lobbyist firms. It provides empirical tests of influence buying that do not depend on any measure of policy outcomes.

Section 3.2 develops the basic model and discusses the assumptions. Section 3.3 extends the model considering candidates aligned with firms, horizons, and the model the implications on horizons and implications for lobbying firms and the structure of legislation. Section 3.4 provides consistent empirical evidence from US Congress people and State legislatures on the contributions raised by candidates over their careers. Section 3.5 concludes.

3.2 Relational Influence Buying

3.2.1 Model Setup

This model focuses on the potential influence of a firm on candidates through campaign contributions. In the model, there is a repeated election cycle where in each cycle: candidates are selected, contributions are raised, elections are won probabilistically, and policies are selected and implemented. The model abstracts away from the process by which the politician provides the favor and assume the politician can simply choose to provide the favor. Thus the favor can be thought of as shorthand for the value of shaping legislation, influencing other legislators, legislative bargaining, actual votes, administrative rule making, political appointments, or constituent services. Similarly, while the model is written in the language of a firm and a candidate, it could equally be viewed as a model of an interest group and an entity providing favors mediated by elections including political parties, elected judges, and others.

The model assumes there is a matching between the single firm and candidates such that the firm is only able to interact with one candidate at a time, and this matching persists across elections. This allows focus on the relationship itself. The single firm simplifies the model in that it precludes both competition among firms as well as concerns of common agency. The matching of candidates to firms precludes competition among candidates. One view is that there might be an ex-ante alignment between a candidate and a firm. For example, because of ideological views, a firm may only be willing to contribute to candidates of one political party. Baron (1994) makes a similar assumption in the discussion of "particularlistic" policies. He assumes an ex-ante alignment between a lobbyist and one of the candidates. In "particularlistic" policies opposing groups cannot organize to contest the lobbyist. Helpman and Persson (2001) make a similar simplifying assumption in their analysis of lobbying contributions impact on legislative bargains. The impact of this assumption precludes the firm from offering contingent contracts to all the candidates and thus rendering the election irrelevant. Finally, even if the firm were restricted to contract with at most one candidate in an election it still precludes the firm from credibly threatening to support the other candidate in the election following a deviation. The threat of this punishment would make it easier to ensure compliance by the candidate. However, because I consider the full range of bargaining power the extent of the substitutability, and thus competition between candidates can be reflect in the firm's share of bargaining power. Similarly, competition between firms asking for mutually exclusive favors is also captured in the bargaining power.

The model requires modeling the probability of candidates winning elections. Importantly, to be a model of campaign contributions and not bribes, campaign contributions have to improve election probabilities. This abstracts away from the behavior of voters, who might choose to punish candidates that provide favors, and the path through which campaign contributions influence elections. Beyond this, the model assumes a flexible form of the relationship between campaign contributions and election probabilities. The model also allows flexible timing of campaign contributions in the contract between the parties. Candidates and firms may want contributions to be given before the election and thus improving the probability of the candidate's election. Or, they may want contributions to be given after the favor done, providing contingent incentives. The model allows both the advance payment and the contingent bonus. One consequence of the flexible timing of contributions allows the parties to endogenously choose when to negotiate. Negotiating after the election is essentially equivalent to not offering an advance contribution.[3]

Because the model does not explicitly model voter preferences, the model assumes candidates are office motivated, and thus the cost of providing favors is reduced election

[3]Negotiation after the election, but before provision of the favor would payments after the election up to the firm's IR constraint rather than its IC constraint. But it does not change the role of the firms IC constraint in limiting the contingent bonus and the contractability of favors.

probability. In order to make the cost of the favor to the candidate and the benefit it provides to the firm directly comparable, the model measures the cost of the favor in terms of the campaign contributions that would provide the same amount of political support the candidate would receive for not providing the favor.

To emphasize the role of contributions and favors and not general relational contracting, the model does not allow payments between firms and candidates except through policies and campaign contributions. If side payments were available, it would be straight forward to avoid these contracting hazards and contract through side payments. I discuss this further below.

The chief contracting constraint in this setting is lack of external enforcement mechanisms, not a lack of information. Campaign contributions are publicly reported, accessible, and analyzed by many groups. Many policy and legislative actions are also publicly documented. Bills, amendments, votes, speeches, etc... are all a matter a public record. As such, I assume perfect monitoring. Particularly, candidates are aware of all past interactions. This informational assumption will ensure that the firm has a reputation beyond the current relationship.

3.2.2 Formally

For each cycle t there are the following steps:

1. Drawing Candidates Step: There are 2 candidates in each election.[4] If there is an incumbent running for reelection, nature draws one candidate. If there is no incumbent running for reelection, nature draws 2 candidates. I will call this possibility an "open seat" election. Each candidate is indexed by i. Exactly one of these candidates is amenable to interaction with the firm.

2. Negotiation Step. With probability $\lambda \in [0, 1]$ the firm makes a take-it-or-leave-it offer of a contract to the politician. With probability $1 - \lambda$ the politician makes a take-it-or-leave-it offer of a contract to the firm.[5] λ represents the bargaining power of the firm.

3. Election Step. Nature selects a winner of the election probabilistically according to the contributions c raised by each candidate. Candidate i wins with probability $\rho(c_i, c_{-i})$. where $\rho_1 > 0$, $\rho_{11} < 0$, $\rho_2 < 0$.

4. Implementation Step. The elected politician chooses to provide the favor. $x = 0$, or withhold it $x = 1$.

[4]Because of the matching between the firm and the candidate this assumption is without loss. The unmatched candidate represents all unmatched candidates.

[5]Standard principal-agent models make the assumption that the principle makes a take-it-or-leave-it offer to the agent. Absent private information and with direct transfers between parties who makes the offer is unimportant in standard models because the surplus can be divided with a direct transfer without affecting the rest of the contract. This specification of bargaining power is similar to that in Halac (2012)

Candidates have utility functions $U = \sum_t \delta^t E_t \gamma$ where $E_t \in 0, 1$ is an indicator of whether politician i is in office in time t; $E_t = 1$ if and only if i is elected in period t and γ is a constant scaling factor representing the utility of being in office. The firm's profit function $\prod = \sum_t \delta^t \left(\beta\left(1 - x_t\right) - \left(a_t + E_{t-1} b_{t-1}\right)\right)$, where $a_t$is the advance in the current period, and $b_t$is the contingent bonus paid after implementation. Note that absent favors and contributions the firm makes zero profits. Also note that $a_t + E_{t-1}b_{t-1}$is the firm's current period contributions.

In addition to the campaign contributions candidates receive from the firm, they receive political support if they did not provide the favor equivalent to$\alpha$of campaign contributions. I restrict attention to cases with benefit to trade, $\beta \geq \alpha$. I assume that all candidates also receive k contributions. These contributions ensure ρ is not degenerate and all candidates win and lose with positive probability. This is consistent with the literature that some contributions are not strategic, but best reflect consumption by the contributors (e.g. as Ansolabehere et. al. argue). These can be thought of campaign contributions made without regard to the positions of the politicians or the base level of support that politicians receive. Importantly, these contributions are independent of past or future agendas and policies and are constant for each politician. One way to interpret k is to think of it as the value in contributions of the politician's charisma or base level of support.

The information history is complete and common knowledge. Particularly, all candidates know if the firm has fulfilled past contracts. No external enforcement is available; any agreements between candidates and firms must be self-enforcing.

I restrict contracts to be Pareto Optimal Subgame Perfect Nash Equilibrium subject to the contracting limitations. Use of SPNE as the solution concept restricts solutions to equilibria consistent with self-enforcement. That is, if an equilibrium is supportable, it is supportable by best responses at all future decision nodes or credible threats and promises. Further restricting the analysis to Pareto optimal contracts avoids the generic problem of multiple equilibria in repeated games.

3.2.3 Contracts

Lemma 3.1. *If a contract including trade exists, the equilibrium contracts are uniquely specified as follows:*

If the firm makes the offer, the contributions are $a_t = max\left(0, a_f - b_{t-1}\right)$*, where* a_f *is the contribution that balances the cost of the contribution against the value of the policy, and* $b_t = \underline{b}$ *the minimum non negative contribution necessary to provide the candidate incentives.*

If the politician makes the offer, the contributions are a_t*, such that given* b_t *the firm is indifferent in accepting, and* $b_t \in \left[\underline{b}, \overline{b}\right]$*, where* $\overline{b}$ *is the largest contribution the candidate would not default on.*

Proof. See appendix. □

The intuition of this follows from the different maximization and incentive constraints. $\underline{b}$ and $\overline{b}$ are set by the incentive compatibility constraints of the candidate and firm respectively. Each of these represents an individual enforcement constraint. On the firm side, the benefit the firm receives from future contracts must be high enough to give incentive not to renege on the bonus payment. From the candidate's perspective the promised bonus plus the future contributions must give enough incentive to provide the favor.

a_f results from the firms maximization of the value of contributions. Increasing a_f increases the probability of the candidate's election (and thus the provision of the favor and future periods) at the cost of additional contributions. The politician, however, extracts the maximum surplus from the firm, but may divide the surplus between elections. Because the division of the surplus depends partially on the previous bonus this implies a certain non-stationarity in the contract. If the politician made the offer in the last period the bonus in that offer might be different that that in either the first election or the lobbyist's offer. The candidate will again optimize and possibly choose a new division of surplus between the current election and the future election. In contrast, regardless of previous offers the lobbyist always offers the same bonus. In this sense the current bonus is the relevant state variable to identify the offer the firm and the candidate will make in this period. However, the current bonus only evolves when the candidate makes the offer and is reset anytime the lobbyist makes the offer. From that we get the following Lemma.

Lemma 3.2. *The current contract is a function of how many periods it has been since the firm made the offer and who makes the offer today. Further, following an open or incumbent election the contingent contribution offered by the candidate is weakly increasing in the number of periods.*

Proof. See appendix. □

Proposition 3.1. *Mutual gain from trade is not sufficient for trade to be contractible. There is a threshold $\underline{\beta} > \alpha$ above which trade is feasible. Above a higher threshold $\overline{\beta} > \underline{\beta}$ no contingent contributions are necessary for contracting.*

Proof. See appendix. □

This result supports one of the main empirical complexities. We would expect campaign contributions to have high returns when an exchange is possible, but there may be a large range of favors which cannot be bought, despite value to both parties. Further, we can think of the contract over high value favors $\beta > \overline{\beta}$as an efficiency wage and the contract over favors in the intermediate range $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ as essentially a pay-for-performance bonus contracts. Consider high value favors first. These are favors that have high value to the firm. This leads the firm to want to make contributions in the current election if the firm expects that the candidate will follow through after the election when campaign contributions impact election outcomes. Once the candidate is elected he faces the prospect of these advances for the subsequent election. If those advances are large enough they alone provide enough incentive to provide the favor. In this range the individual enforcement constraints do not bind on the parties. In the intermediate range, at the candidate's individual enforcement constraint binds. The promise of future contracts is not enough to induce the candidate to follow through so the firm must offer a bonus. At the low range, $\beta < \underline{\beta}$, the firm's individual enforcement constraint binds at a level too low to satisfy the candidate's constraint.

It is important to note that while these thresholds describe the value of the favor they are contingent upon the split of the surplus implied by the bargaining power and the candidate's cost of providing the favor. It is easy to see the importance of bargaining power by considering the extreme case when the firm has no bargaining power, for example, if we assumed perfect competition between firms for a single favor. In this case, no matter how valuable the favor is bonus contracts are not self enforceable. Because candidates are unable to commit to leave value on the table in future periods no contingent payments are incentive compatible. Because of that the intermediate range is empty.

Lemma 3.3. *No contingent contributions are made when the candidate has all the bargaining power $\lambda = 0$.*

Proof. If $\lambda = 0$, the firm expects no future surplus because the full value of the favor is extracted in contributions. Given no future surplus, the firm would not follow through on a contingent contribution. □

More broadly, this points to the impact of the inability to transfer wealth directly between the parties. It leads to situations where a party may prefer to have less bargaining power than they do, because less bargaining power is a substitute for commitment.

3.2.4 Existing Contracting Literature

The previous lemma points out one important difference between this model and the standard contracting literature. The model I develop builds on the formal relational contracting literature developed in the context of an employment relationship (e.g. MacLeod & Malcomsom (1989), Levin (2003)), but with a few important modifications to the ability of the parties to transfer between them and that those transfers impact election probabilities. Kroszner and Stratmann (1998) is perhaps the closest to the goal in this research of connecting the relational contracting literature to campaign contributions. However, instead of modeling the process explicitly they draw on the intuitions in existing models and discuss implications for endogenous committee structures. Ishihara (2013) extends menu auctions, often used to describe competition between lobbyists, to repeated self-enforcing settings, but does not consider contributions impacting elections.

First, the nature of payments between the parties is restricted. Firms may only make payments to candidates in the form of campaign contributions. Unlike the existing literature, standard lump sum payments are not available. Further, like a limited wealth employee, candidates may only provide payments to firms in the form of policies (effort in the employee context) and not through lump sum transfers. Finally, I explicitly model that payoffs from future contracts are the result of future negotiation between the parties.

Second, separate from their incentive consequences, payments made by firms to campaigns impact the probability of winning an election, and thus the expected surplus from the relationship. This is unlike the standard models where the transfers between the principal and the agent have incentive consequences, but do not change total surplus directly.

Together, these modifications have an important effect in linking the total surplus and the division of that surplus. Unlike the standard approach in principal agent problems, it is not sufficient to look first at total surplus to determine incentives, and then construct payments to provide incentives and divide the surplus. In this model relational surplus is dependent on not only the actions taken, but on the payments between the players. If we were to maximize the relational surplus subject only to incentive constraints, as in the standard method, we would pin down not only the surplus maximizing actions, but also the surplus maximizing transfers. We would then not be able to, as in the standard approach, use lump-sum transfers between the parties to ensure the participation constraints are met.

Further, these modifications put this model outside of the scope of relationships discussed in the existing relational contracting literature. MacLeod and Malcomson (1989) characterize the set of contracts in a broadly defined principal agent problem. However, they consider a problem where the period surplus is a function of only the action taken (performance in the employment context), and not the payments between the parties. Similarly, in Levin (2003) surplus is a function only of the effort levels chosen, not the wage payments. Together, the restriction on payments and the productive affect of the payments mean that the results of these papers do not directly apply.

For example, consider the proof of Levin (2003) Theorem 1. This theorem says that if a self-enforcing contract exists that produces surplus s^* greater than the sum of the principal's and agent's outside options, any split of surplus $s \leq s^*$ between them such that both receive at least their outside option is supportable by a self-enforcing contract. Levin's proof notes that they can simply shift surplus through the fixed payments without changing the incentives. However, in this model, changing the fixed payments changes the surplus, so unlike in Levin, it impacts the incentive compatibility constraints.

3.3 Extensions

3.3.1 Supporting candidates vs Influencing candidates.

An important implication of the model is that there is a distinction between contributions a firm may make for a candidate who's position it supports compared to contributions to a candidate it is influencing. In the first case, there is no incentive compatibility problem on the candidate's side, thus no need for contingent payments, and thus no incentive problem on the firm's side. Effectively, no contract is necessary and the firm simply gives campaign contributions to improve the probability of election of the candidate is already agrees with. In contrast, a firm influencing a candidate faces requires a self-enforcing contract and this contract influences not just the election of the candidate, but the policies they choose. We may think of the first kind of support as akin to speech and the second as a sort of bribe through the campaign contribution system. Society may have more of an interest in limiting the second than the first.

Some have correctly suggested that there might be other payments beyond those that influence election outcomes between firms and politicians. For example, a payment after the candidates final term, such as the "revolving door" between public office and private industry or other favors, would support additional contributions in the presence of term limits. Not surprisingly, the presence of a sufficiently valued final payment fixes the contracting problem between the firm and the candidate in both term limited and no term limits scenarios. However, as this model demonstrates to the extent that campaign contributions are feasible beyond what is described here, those excess contributions support a politician whose behavior is influenced by the final payment. In fact, that final payment to the politician drives their behavior and is effectively the "bribe", while the contributions are "speech" in favor of the bribable. So, if we are thus concerned with contributions that distort the policy preference of elected officials rather than contributions that support candidates groups already agree with, regulation should be directed towards these non-campaign transactions between special interests and politicians.

Similarly some suggest that candidates receive direct benefits from firms and their contributions. Particularly, some note that candidates receive direct benefit from campaign funds, not only that it increases their election probabilities. If we adjust the model so that candidates can choose how to allocate their collected campaign contributions between the election and their personal benefit, the model is largely robust. The firms's constraints are unchanged. Similarly, the candidate's incentive constraint is unchanged. The main impact is that at each election the candidate can chose to direct some funds away from the election, and thus the timing preferences of both parties would take this in to account. Under similar conditions used to develop Lemma 2, this would actually strengthen the incentives to shift contributions as early as possible, and thus not change the main results.

3.3.2 Horizon Effects

Implicit in any relational model of influence buying is that contributions in each period are not only with regards to current favors, but part of a longer sequence of contributions and favors. Snyder (1992) shows that older candidates raise less contributions from "investors" than younger candidates. That research uses age as a proxy for the horizon of the candidate. In this section, I further explore the dynamics of contributions raised by candidates over their careers.

The model developed in this research provides the same comparative static as motivated Snyder (1992):

Proposition 3.2. *The thresholds $\underline{\beta}$, and $\overline{\beta}$ are decreasing in δ.*

Proof. See appendix. □

The more likely a future period for the relationship, the more favors are contractible. This result leads to one of the empirical predictions of the research. Decreased horizons for the candidate causes fewer favors to be contractible, reducing a candidate's fund raising ability.

Proposition 3.3. *Expected contributions are lower in the first election than subsequent elections.*

Proof. See appendix. □

The intuition behind this result is that in the first election agents receive only the advance contribution, but no contingent bonus contributions.

Term limits are an extreme version of reduces horizons. One in which the parties know that the probability of a future is zero. However, because firms are long lived and care about their reputation with future candidates, term limits do not completely unravel the contract in this setting. If the value of the favor is sufficiently large, the firm can commit to providing a contingent contribution, and stop the unraveling that would otherwise occur.

Proposition 3.4. *If β is sufficiently large, term limits of 2 or more terms (a finite number of elections a candidate can win) do not cause complete unraveling.*

Proof. See appendix. □

3.3.3 Structure of Legislation and Lobbying

This section explores how changing the structure of the favors available for exchange can increase the ability to contract. The above section took the favor as fixed and asked how it might be contracted for. However, firms and politicians can also choose the nature of the favor they wish to contract over. The literature has considered a wide range of favors that might be chosen. Here, I focus on intertemporal combination and division of favors. To begin with, it is straight forward that a firm with a one time need for a favor, no matter how valuable, cannot contract for it. However, if this favor can be spread out over time and converted into a sequence of favors contracting improves. Formally, consider a one time favor that provides one time payoff $\Omega$to the firm, at a policy salience G to the voters in this period, and both are zero in future periods. This favor is not contractible. Suppose, however, a surplus destroying conversion to a stream of favors that provide benefit $(1-\delta)\,\Omega - L$ to the firm each period and costs the politician goodwill $(1-\delta)\,G$. This sequence of favors is contractible if $\Omega$is large enough relative to L and G.

Proposition 3.5. *Regardless of the potential mutual gain one time favors are not contractible. But a joint surplus destroying conversion to a stream of one period favors may be.*

Proof. One time favors are never contractible, even if the favor is extremely high value. Converting it to a lower valued stream may make it contractible if the resulting stream has sufficient surplus. □

L could be the cost of suboptimal contracting or planning to the firm of the disruption. Or, redefined, could reflect increased electoral costs to the politician of repeatedly reminding voters of this favor. The next proposition defines L as the profit wedge a lobbying firm would have to aggregate favors from different firms. The lobbyist could maintain a relationship with a politician an then provide the favors to firms.

Proposition 3.6. *Aggregating non contractible favors can make them contractible.*

Proof. See appendix. □

The logic of this proof follows from Proposition 1. There needs to be sufficient surplus for the relationship to be contractible. Aggregating favors can cross these thresholds. This result provides a rationale for firms to profit by aggregating the exchange between many individual firms with small favors and politicians. These firms would then be the repository for the relationship, creating a firm who's source of profits is their reputation and relationships only.

These results, like the intuition of Kroszner & Stratmann (1998), have many implications for the structure of legislation and lobbying. Proposition 5 suggests that politicians will inefficiently commit when provide favors. Rather than making permanent changes or binding contracts that provide certainty, the politician prefers the uncertainty that allows the relational contract. This effect, however, may be counteracted by the incentives of a politician to bind future politicians as Glazer (1989) explores. Proposition 6 explores how lobbying firms have the potential for sustained profits and can use that wedge - the value of their relationship with a politician to sell favors in an environment to firms. Essentially, they can arbitrage around contract limitations with politicians and contract with firms in ways politicians cannot. These proposition leads to a number of predictions about lobbyists and which firms use them. First, it provides a way to distinguish informational from influence lobbying as we might expect informational lobbying to follow subject matters, while influence lobbying would follow individuals over time. It also suggests characteristics that make firms more likely to use outside lobbyists. Instability of the firm, for example, firms under receivership or high leverage, should be more likely to be seen as transient to the politician and unable to contract. Likewise, even firms with significant lobbying experience, when they need an occasional favor from a new body of politicians should be more likely to go to outside help.

3.4 Time patterns in Campaign Contributions

The predictions of time patterns in campaign contributions developed in propositions 2 and 3 can be explored in data on campaign contributions without knowledge of the favors exchanged. This is helpful because it potentially allows evidence detecting influence buying without requiring knowledge of the actual favors and estimating the counter-factuals of what the political process would have produced absent some contribution. However, this strategy is complicated by other potential reasons campaign contributions might vary with the horizon of the candidate. None the less, I take these predictions to data and find evidence consistent with the propositions. First, using data from Representatives to the U.S. House, and consistent with Snyder's result, find that older candidates raise less money, and candidates raise less money their first election then they do once they are in office. The data are candidate campaign contributions raised in a two year election cycle as reported in the Federal Election Committee (FEC) data from 1992 to 2006 for each candidate who was or became a member of congress. Candidate characteristics are merged from Congressional Biographical Directory. We can see these patterns in the follow specification:

$$LogDeflatedCampaignReciepts_{it} = \beta_1 * age_i t + \beta_2 * firstelection_{i,t} + year_t + FE_i$$

Proposition 2 predicts that $\beta_1 < 0$, Proposition 3 would predict that $\beta_2 < 0$.

Table 1, Column (1) reports this regression, and finds that $\beta_1<0$. This effect is sizable and significant. For each year older, a candidate raises approximately 1.4% less. However, there are two important omitted variables. First, challengers to an incumbent are chosen endogenously. If this selection is correlated to both age and the contributions a candidate raises it maybe this effect which is being measured. Regression (2) includes a measure of the amount of contributions raised by a candidate's challengers. We do see that this weakens the impact of age. However, by using age as a proxy for retirement, we bias our results against finding an effect significantly; as a candidate ages they also become more senior in congress. We would expect that their ability to provide favors increases with seniority. However, to include seniority and age we cannot use candidate fixed effects. Regressions (3) reports the same regression as (2) except it replaces the candidate fixed effects and year controls with state cross party cross year fixed effects. Regression (4) then adds seniority measured as the number of years a candidate has been in office. Comparing these two regressions we find that by not separately identifying seniority we underestimate the impact of age by about half a percentage point. Regressions (5) and (6) include non-linear specifications of age where we can see the acceleration of the decline past age 65.

Turning to first election effects, regression (1) does not find that $\beta_2 < 0$. This is not entirely surprising. There are a number of confounding effects. First, if fund raising increases the probability of winning and there are incumbency effects, there will be selection for those who raise more when they win, by definition their challenger election.

Table 3.1: Campaign Funds Raised by US House Representatives

	(1)	(2)	(3)	(4)	(5)	(6)
Log Funds Raised Discounted by Election Cycle Average						
First Election	0.087***	-0.113***	-0.157***	-0.100**	-0.146***	-0.080*
	[0.034]	[0.031]	[0.040]	[0.042]	[0.039]	[0.042]
Age	-0.008**	-0.004	-0.010***	-0.014***		
	[0.004]	[0.003]	[0.002]	[0.002]		
Competitors'		0.084***	0.120***	0.120***	0.120***	0.121***
Funds (Log)		[0.005]	[0.008]	[0.008]	[0.008]	[0.008]
Age 35-44					0.000	-0.012
					[0.083]	[0.085]
Age 45-54					-0.087	-0.122
					[0.079]	[0.082]
Age 55-64					-0.177**	-0.248***
					[0.080]	[0.086]
Age 65-74					-0.286***	-0.418***
					[0.094]	[0.104]
Age 75-84					-0.406**	-0.643***
					[0.182]	[0.188]
Age 85+					-0.657*	-1.028***
					[0.338]	[0.366]
Seniority				0.009**		0.009**
				[0.004]		[0.004]
Year Controls	YES	YES	FE	FE	FE	FE
Fixed Effect	Candidate		State $\times$ Party$\times$Year			
Observations	3222	3222	3222	3222	3222	3222
R-squared	0.714	0.769	0.428	0.435	0.428	0.434
Robust standard errors in brackets clustered by candidate						
*** p<0.01, ** p<0.05, * p<0.1						

Table 3.2: Effect of Age on First Election Change

	(1)	(2)	(3)	(4)	(5)
Log Funds Raised Discounted by Election Cycle Average					
Age Group	Less than 40	40 to 50	50 to 60	60 to 70	Interaction
First Election	-0.325***	-0.077	-0.092	0.057	-0.620***
	[0.108]	[0.070]	[0.063]	[0.144]	[0.135]
Age	-0.013	0.002	-0.005	-0.015	-0.004
	[0.024]	[0.009]	[0.006]	[0.010]	[0.003]
Competitors'	0.106***	0.084***	0.074***	0.078***	0.084***
Funds (Log)	[0.022]	[0.013]	[0.008]	[0.015]	[0.005]
Age X First					0.011***
Election					[0.003]
Year Controls	YES	YES	YES	YES	YES
Fixed Effect	Candidate	Candidate	Candidate	Candidate	Candidate
Observations	245	914	1194	693	3222
R-squared	0.861	0.797	0.846	0.845	0.771
Robust standard errors in brackets clustered by candidate					
*** p<0.01, ** p<0.05, * p<0.1					

With an incumbency effect, we should expect this bias to survive. Second, challengers are most likely in more competitive elections. They are either in open seat elections where competition is thought to be fierce, or running against an established incumbent. We see this effect, because controlling for competitiveness, as in regression (2) shifts this effect as predicted. Finally, the prediction of Proposition 3 holds the end effects constant. However, if the end of the relationship was near, it is possible that the end effects dominate the beginning effects. Table 2 shows regressions (1)-(4) show the same regression as regression (2) in table 1, but restrict the sample by age groups. Here we see strong and significant effects among the youngest candidates, exactly when their horizon is not meaningfully different between their first and second campaign. Regression (5) estimates this on a continuous basis, and we see the expected effect: A negative effect of being the first election, and a positive effect of age on the impact of being a first election. By dividing the coefficient of first election by the interacted coefficient, we estimate that the first election effect dominates the age effect until about age 56.

While age is certainly an imperfect measure it is a transparent measure available to both firms and politicians regarding the potential length of the relationship. However, this correlation could be caused by other correlates to age other than future life of the relationship. For example, infirmness or feebleness limiting the ability to fund raise could produce results that old age leads to lower campaign contributions. Yet, this explanation seems less appealing to describe differences in fund-raising in middle-ages where age is less likely to be informative of infirmness. Nor would this alternative explanation cause

Table 3.3: State Legislatures

State	Legislature	Term Limits	Term Length	Campaign Cost	Data Range		Candidates
AR	House	3	2 years	\$22,000	2000	2008	82
AZ	House	4	2 years	\$26,000	1996	2008	25
AZ	Senate	4	2 years	\$33,000	1996	2008	17
CA	Assembly	3	2 year	\$291,000	1998	2008	78
CO	House	4	2 years	\$22,000	1996	2008	34
FL	House	4	2 years	\$84,000	1998	2008	72
LA	House	6	2 years	\$55,000	1999	2009	4
ME	House	4	2 years	\$4,700	1996	2008	71
ME	Senate	4	2 years	\$20,000	1996	2008	16
MI	House	3	2 years	\$32,000	1996	2008	138
MO	House	4	2 years	\$28,000	1996	2008	88
MT	House	4	2 years	\$5,500	1992	2008	46
NV	Assembly	6	2 years	\$56,000	1998	2008	8
NV	Senate	3	4 years	\$107,000	1998	2008	11
OH	House	4	2 years	\$74,000	1996	2008	56
SD	House	4	2 years	\$7,500	2000	2008	18

the evidence of reduced fund raising in a candidate's first campaign.

Empirically, term limits provide cleaner tests of a candidate's horizon. To test this, I use data from US state legislatures that have implemented a term limit. I restrict the selection of legislatures to those that have a term limit of at least 3 terms and I observe the fund raising of candidates who will be termed out of their current office if they win. The restriction to 3 or more terms ensures that incumbency can be separated from final term. The second restriction simply ensures that candidate's final election is in my data. This leaves me with 16 state legislatures. Table 3 summarizes the legislatures and data used.

Because of the wide range in years and cost of campaigns, I deflate the campaign receipts of each candidate by the average campaign receipts of all candidates for that year for that legislature. I then estimate the following

$$LogDeflatedCampaignReciepts_{it} = \beta_1 * FinalElection_{it} + FE_i$$

where the sample has been restricted to politicians for which their termed out election and whose entire career is in the data I have collected. To exclude first election effects and multiple tries to win office, I restrict attention to incumbents. I also remove a handful of candidates who have non-contiguous service. These results are shown in Table 4. As predicted, candidates raise 7% less, relative to other candidates when they are termed out as when they are incumbents. These results do demonstrate that term limits do

Table 3.4: Fund Raising with Term Limits

Log Funds Raised Discounted by Election Cycle Average	
	(1)
Final Election	-0.072*
	[0.038]
Fixed Effect	Candidate X State X Legislature
Clusters	1380
Observations	3074
R-squared	0.75
*** p<0.01, ** p<0.05, * p<0.1. Robust standard errors in brackets clustered by candidate	

matter, and is consistent with the relational contract I propose. Taken together these empirical results provide consistent, but not completely identified correlations that match the horizon effects predicted by the model.

3.5 Conclusion

This model provides a more realistic model for buying influence through campaign contributions that reflects the timing of the election cycle. The main results show a number of characteristics and consequences of the relational contract. Both parties have surplus in expectation and there must be sufficient mutual gain for contracting to be feasible. Horizons – both discount factor related or term limits further constrain, but do not eliminate feasibility of contracting. The results also suggest a role for lobbying firms and implications for firms wishing to influence candidates.

The first finding helps rationalize an open question in the political economy literature. The horizon finding is supported with correlations in fund raising among US congress people and state legislators with term limits. This also provides empirical evidence consistent with influence buying without the difficult empirical challenge of observing the provision of favors. The implications for lobbying and lobbyist behavior provide implications for future research.

The formal model also distinguishes between contributions that change a politicians behavior once in office and contributions that help elect the preferred candidate. Though both screening and incentives are in general effect ways to influence outcomes our political system often distinguishes between one as speech and the second as corruption. This distinction and the broader model also provides opportunities to consider different methods to limit the potential for corruption without limiting speech.

Conclusion

This thesis provides theoretical and empirical evidence on a variety of contracts. Explicit threshold incentive contracts in wide use are shown to have significant implications for effort and risk-taking depending on their alignment. These findings are economically significant, casually identified, and describe incentive structures in wide use. Further, these evidence speak to a question in the personnel economic literature - is the response of managers to high power incentives an important consideration in giving them? Or are high powered incentives just ways to transfer wealth or attract talent? The answer is clear: incentives may serve to transfer value and attract talent, but their impact on behavior is first-order. And, significantly, they may impact not only effort, but how managers choose to take risk. These contracts are not only the private concern of firms, but are important in public policy debates. These type of incentive contracts are the object of much regulatory attention and this attention is warranted.

In another area of public concern, understanding the implicit contracting underlying trade of influence is shown to be important. It explains existing puzzles in the empirical literature: just because there would be value for the politician and special interest group in trade does not mean they can agree on how to exchange the value. The contracting details also generate predictions about time paths of contributions and the structure of legislation and lobbying. Evidence consistent with these is shown. The nature of the implicit contract also speaks to potential public policy implications. Understanding the structure of influence buying can be used to design campaign contribution limits.

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Appendix A

Risk-Taking and Performance

A.1 Decision Maker's Problem

The (static) profit function of the decision maker is:

$$
\begin{aligned}
\Pi(e,r) &= b\left(e-q\left(r\right)\right) \\
&\quad +p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right)\left(-d+e-q\left(r\right)+\frac{\sqrt{r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}{1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}\right)-c\left(e\right)
\end{aligned}
$$

Expanding we get:

$$
\begin{aligned}
\Pi(e,r) &= b\left(e-q\left(r\right)\right)+p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right)\left(\frac{\sqrt{r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}{1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)}\right) \\
&\quad +p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right)\left(-d+e-q\left(r\right)\right)-c\left(e\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pi(e,r) &= b\left(e-q\left(r\right)\right)+p\sqrt{r}\phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right) \\
&\quad +p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right)\left(-d+e-q\left(r\right)\right)-c\left(e\right)
\end{aligned}
$$

A.2 First order conditions

A.2.1 Effort

Taking the FOC with respect to e we get:

$$
\begin{aligned}
\frac{\partial\Pi(e,r)}{\partial e} &= b+p\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)+p\left(1-\Phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)\right) \\
&\quad +p\left(-d+e-q\left(r\right)\right)\left(\frac{1}{\sqrt{r}}\right)\phi\left(\frac{d-e+q\left(r\right)}{\sqrt{r}}\right)-c'\left(e\right)
\end{aligned}
$$

let $m=-d+e-q\left(r\right)$

$$
\frac{\partial\Pi(e,r)}{\partial e}=b-p\frac{m}{\sqrt{r}}\phi\left(\frac{-m}{\sqrt{r}}\right)+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)+p\frac{m}{\sqrt{r}}\phi\left(\frac{-m}{\sqrt{r}}\right)-c'\left(e\right)
$$

$$= b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right) - c'(e)$$

A.2.2 Risk taking

Taking the FOC with respect to r we get:

$$
\begin{aligned}
\frac{\partial \Pi(e,r)}{\partial r} &= -bq'(r) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad -p\sqrt{r}\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad -pq'(r)\left(1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right) \\
&\quad -p\left(-d+e-q(r)\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&= -bq'(r) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad -p\sqrt{r}\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad -pq'(r)\left(1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right) \\
&\quad -p\left(-d+e-q(r)\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&= -bq'(r) - pq'(r)\left(1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad +p\sqrt{r}\left(\frac{-d+e-q(r)}{\sqrt{r}}\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&\quad -p\left(-d+e-q(r)\right)\frac{\partial\left(\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right)}{\partial r}\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&= -bq'(r) - pq'(r)\left(1-\Phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right)\right) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{d-e+q(r)}{\sqrt{r}}\right) \\
&= -bq'(r) - pq'(r)\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
&= \left[-b-p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q'(r) + p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
&= p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) - \left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q'(r)
\end{aligned}
$$

A.2.2.1 Shadow cost of risk

Note that at an interior solution (so that both FOCs are satisfied) the cost of risk-taking $q'(r)$faces price $b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)$. At an interior effort level, that is equal to $c'(e)$. Effort and risk-reduction are substitutes in the production of mean improvement, so they face the same margins.

A.2.2.2 If risk has no impact on performance, risk-taking would be unbounded.

Suppose that $q(r)=0$, $q'(r)=0$, and effort were fixed, then profits should increasing for all d,e in r

$$\frac{\partial \Pi}{\partial r}=\left[-b-p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q'(r)+p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\geq 0$$

$$p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\geq 0$$

Which is straight forward.

A.2.2.3 Lemma: Risk taking is on the strictly downward sloping part of the curve $q'(r)>0$.

At any interior effort level,

$$b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)=c'(e)>0$$

Risk-taking might be at a lower bound, so we have:

$$\frac{\partial \Pi}{\partial r}=p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)-\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q'(r)\leq 0$$

By the above, $p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\geq 0$.
From the cost of effort $\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]>0$.
So, it must be that $q'(r)\geq 0$.
Suppose that $q'(r)=0$, then we have

$$p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\leq 0$$

But this is strictly positive. A contradiction.

A.2.2.4 Conditions on $q(r)$

1. Single troughed. That is, $q'(r) \leq 0$ on range $[0, r_1)$, $q'(r) = 0$ on range $[r_1, r_2]$, $q'(r) \geq 0$ on range (r_2, ∞). Later, we need that
   (a) $r_2 > 0$. This merely ensures that zero-risk is not the optimal and we do not need to worry about degenerate normal distributions.
   (b) $q'' \geq 0$ on range (r_2, ∞).
2. Given the thin tails of the normal distribution no additional conditions are necessary to ensure interior risk taking.

A.3 Comparative statics

A.3.1 Implicit function theorem.

Subject to the full rank condition of the IFT, we have that:

$$\frac{\partial e^*}{\partial d} = \frac{-\frac{\partial^2 \Pi}{\partial^2 r}\frac{\partial^2 \Pi}{\partial e \partial d} + \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial d}}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

$$\frac{\partial r^*}{\partial d} = \frac{\frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial e \partial d} - \frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial r \partial d}}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

We know that

$$-\frac{\partial^2 \Pi}{\partial^2 e} - c''(e) = \frac{\partial^2 \Pi}{\partial e \partial d}$$

and

$$-\frac{\partial^2 \Pi}{\partial e \partial r} == \frac{\partial^2 \Pi}{\partial r \partial d}$$

So we can simplify:

$$\frac{\partial e^*}{\partial d} = \frac{-\frac{\partial^2 \Pi}{\partial^2 r}\left(-\frac{\partial^2 \Pi}{\partial^2 e} - c''(e)\right) - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}} = \frac{\frac{\partial^2 \Pi}{\partial^2 r}\frac{\partial^2 \Pi}{\partial^2 e} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e} + \frac{\partial^2 \Pi}{\partial^2 r}\left(c''(e)\right)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

$$\frac{\partial e^*}{\partial d} = 1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

$$\frac{\partial r^*}{\partial d} = \frac{-\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

A.3.2 Second order conditions and uniqueness.

$$\frac{\partial^2 \Pi}{\partial^2 e} < 0$$

$$\frac{\partial^2 \Pi}{\partial^2 e} = p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right) - c''(e)$$

Note that $p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right) \leq p\left(\frac{1}{\sqrt{r_2}}\right)(\phi(0))$. So a sufficient condition for this is enough convexity or high enough b such that $c''\left(c'^{-1}(b)\right) \geq p\left(\frac{1}{\sqrt{r_2}}\right)\phi(0) = \frac{p}{\sqrt{2\pi r_2}}$.

$$\frac{\partial^2 \Pi}{\partial^2 r} < 0$$

This is implied by the conditions below, so we'll hold off for now.

$$\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e} > 0$$

As a reminder, we have:

$$\frac{\partial \Pi}{\partial r} = p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) - \left[b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right] q'(r)$$

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial^2 r} &= -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) - p\left(\frac{-m}{2r}\right)\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\ &\quad - \left[b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right] q''(r) - \left[\frac{\partial^2 \Pi(e,r)}{\partial e \partial r}\right] q'(r)\end{aligned}$$

$$\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = -p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

So we have

$$\left[p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)-c''(e)\right]$$
$$*\left[-p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)+p\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right]$$
$$-\left[p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)-c''(e)\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)$$
$$-p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)$$

$$= \left[p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)-c''(e)\right]$$
$$*\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right]$$
$$-\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)\left[p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)-c''(e)\right]$$
$$-p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)$$

$$= p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)$$
$$*\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right]$$
$$-p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)$$
$$-c''(e)\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right]$$
$$+c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)$$
$$-p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)$$

$$\begin{aligned}
= \quad & -p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& -p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -c''(e)\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right] \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)
\end{aligned}$$

$$\begin{aligned}
= \quad & -p^2\left(\frac{1}{4r^2}-\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right) \\
& -p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -c''(e)\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right] \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)
\end{aligned}$$

$$\begin{aligned}
= \quad & p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right) \\
& -p^2\left(\frac{1}{4r^2}\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right)-p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -c''(e)\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right] \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -p^2\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)^2\phi^2\left(\frac{-m}{\sqrt{r}}\right)
\end{aligned}$$

$$
\begin{aligned}
= \quad & -p^{2}\left(\frac{1}{4r^{2}}\right)\phi^{2}\left(\frac{-m}{\sqrt{r}}\right)-p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& -c''(e)\left[-p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\right] \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^{2}\left(\frac{1}{4r^{2}}\right)\phi^{2}\left(\frac{-m}{\sqrt{r}}\right)-p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& +c''(e)\,p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^{2}\left(\frac{1}{4r^{2}}\right)\phi^{2}\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\,p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& -p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\
& +c''(e)\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^{2}\left(\frac{1}{4r^{2}}\right)\phi^{2}\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\,p\left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +\left[-p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)+c''(e)\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^2\left(\frac{1}{4r^2}\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right)+c''(e)\,p \\
& \left(\left(\frac{1}{4r\sqrt{r}}\right)-\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +\left[-\frac{\partial^2\Pi}{\partial^2 e}\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^2\left(\frac{1}{4r^2}\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right)+c''(e)\,p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\,p\left(\frac{m}{2r}+q'(r)\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +\left[-\frac{\partial^2\Pi}{\partial^2 e}\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -p^2\left(\frac{1}{4r^2}\right)\phi^2\left(\frac{-m}{\sqrt{r}}\right)+c''(e)\,p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\,p\sqrt{r}\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +\left[-\frac{\partial^2\Pi}{\partial^2 e}\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \left(-p\left(\frac{1}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)+c''(e)\right)p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +c''(e)\,p\sqrt{r}\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\
& +\left[-\frac{\partial^2\Pi}{\partial^2 e}\right]\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)
\end{aligned}
$$

$$\begin{aligned}
&= \left[-\frac{\partial^2 \Pi}{\partial^2 e}\right] p \left(\frac{1}{4r\sqrt{r}}\right) \phi\left(\frac{-m}{\sqrt{r}}\right) \\
&\quad + c''(e)\, p\sqrt{r}\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)^2 \phi\left(\frac{-m}{\sqrt{r}}\right) \\
&\quad + \left[-\frac{\partial^2 \Pi}{\partial^2 e}\right]\left[b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right] q''(r)
\end{aligned}$$

The first term is strictly positive if $\frac{\partial^2 \Pi}{\partial^2 e} < 0$. The second term is weakly positive if $c''(e) \geq 0$. The third term is strictly positive if $\frac{\partial^2 \Pi}{\partial^2 e} < 0, q''(r) > 0$ and weakly so if those hold weakly.

Taking these two together, $c''\left(c'^{-1}(b)\right) \geq \frac{p}{\sqrt{2\pi r_2}}$ and $q''(r) \geq 0$ are sufficient conditions for the Hessian to be negative semidefinite, and ensure that there is exactly one local maximum of the agent's problem. Because we also know that the solution is interior, that means that there is a unique solution to the maximization problem for any set of parameters d, b, p.

A.3.3 Result: risk taking is increasing near the threshold

1. Risk-taking has the same sign as $-\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)$

2. Let near the threshold be where $m = 0$

$$\frac{\partial \Pi(e,r)}{\partial e} = b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right) - c'(e)$$

$$\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = -p\frac{\partial \frac{-m}{\sqrt{r}}}{\partial r}\phi\left(\frac{-m}{\sqrt{r}}\right)$$

$$\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = -p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

At $m = 0$

$$\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = -p\left[\frac{q'(r)}{\sqrt{r}}\right]\phi(0)$$

By lemma, we know that $q'(r) > 0$. $c''(e) > 0$.so risk taking has the sign of

$$-\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = p\left[\frac{q'(r)}{\sqrt{r}}\right]\phi(0) > 0$$

A.3.4 Result: If distance is far, risk-taking is decreasing

If d is large, then

$$\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = -p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

$$\approx -p\phi\left(\frac{-m}{\sqrt{r}}\right)\frac{m}{2r^{2.5}}$$

d large implies m is very negative. So, as above, risk-taking has the same sign as:

$$-\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} \approx p\phi\left(\frac{-m}{\sqrt{r}}\right)\frac{m}{2r^{2.5}} < 0$$

A.3.5 Result: if effort is sufficiently costly, then risk-taking is single peaked.

Risk-taking takes the shape of

$$-\frac{\partial^2 \Pi(e,r)}{\partial e \partial r} = p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

So that the direction of risk-taking depends on

$$\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}$$

$$= m + 2rq'(r)$$

$$= -d + e - q(r) + 2rq'(r)$$

A sufficient condition for monotonicity is for this to have the same sign for all d at the optimum levels.

$$-d + e^* - q(r^*) + 2r^*q'(r^*)$$

$$\begin{aligned}\frac{\partial}{\partial d}\left(-d + e^* - q(r^*) + 2r^*q'(r^*)\right) &= -1 + 1 - \frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r} - \frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}} \\ &\quad + \left[-q'(r^*) + 2q'(r^*) + 2r^*q''(r^*)\right] \\ &\quad \times \frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r} - \frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\end{aligned}$$

$$= -\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r} - \frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}} + \left[q'(r^*) + 2r^*q''(r^*)\right]\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r} - \frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

Which has the opposite sign as:

$$\frac{\partial^2\Pi}{\partial^2 r} + \left[q'(r^*) + 2r^*q''(r^*)\right]\frac{\partial^2\Pi}{\partial e\partial r}$$

(dropping the asterisks)

$$\begin{aligned} = \quad & -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) + p\left(\frac{-m}{2r}\right)\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\ & -\left[b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) \\ & +p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)q'(r) - p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\left[q'(r) + 2rq''(r)\right] \end{aligned}$$

$$\begin{aligned} = \quad & -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) + p\left(\frac{-m}{2r}\right)\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\ & -\left[b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r) - p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\left[2rq''(r)\right] \end{aligned}$$

$$\begin{aligned} = \quad & -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) + p\left(\frac{-m}{2r}\right)\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) \\ & -c'(e)\,q''(r) - p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\left[2rq''(r)\right] \end{aligned}$$

$$= -\frac{1}{2r}c'(e)\,q'(r) - c'(e)\,q''(r) + p\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\left[\left(\frac{-m}{2r}\right) - 2rq''(r)\right]$$

$$= -\frac{1}{2r}c'(e)\,q'(r) - c'(e)\,q''(r) + p\phi\left(\frac{-m}{\sqrt{r}}\right)\left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}}\right)\left[\frac{-m}{2r} - 2rq''(r)\right]$$

$$\begin{aligned}
&= -\frac{1}{2r}c'(e)\,q'(r) - c'(e)\,q''(r) \\
&\quad + p\phi\left(\frac{-m}{\sqrt{r}}\right)\left(-\frac{m^2}{4r^2\sqrt{r}} + \frac{-m}{\sqrt{r}}q''(r) - m\frac{q'(r)}{2r\sqrt{r}} - 2\sqrt{r}q'(r)\,q''(r)\right)
\end{aligned}$$

Note that this is negative if $m > 0$, so risk-taking has at most one local extrema in that range.

Note that this is also negative is m is sufficiently negative

$$\begin{aligned}
&= -\frac{1}{2r}\left(c'(e) + \frac{m}{\sqrt{r}}p\phi\left(\frac{-m}{\sqrt{r}}\right)\right)q'(r) \\
&\quad -\left(c'(e) + \frac{m}{\sqrt{r}}p\phi\left(\frac{-m}{\sqrt{r}}\right)\right)q''(r) + p\phi\left(\frac{-m}{\sqrt{r}}\right)\left(-\frac{m^2}{4r^2\sqrt{r}} - 2\sqrt{r}q'(r)\,q''(r)\right)
\end{aligned}$$

This is negative if

$$c'(e) + \frac{m}{\sqrt{r}}p\phi\left(\frac{-m}{\sqrt{r}}\right) > 0$$

Note that $\frac{m}{\sqrt{r}}p\phi\left(\frac{-m}{\sqrt{r}}\right)$ has a minimum at $m = -\sqrt{r}$ of $-e^{-\frac{1}{2}}/\sqrt{2\pi}$.
if $c'(e) > e^{-\frac{1}{2}}/\sqrt{2\pi}$ then this is always negative.
A sufficient condition for this is that $c'(b) > e^{-\frac{1}{2}}/\sqrt{2\pi}$

A.3.6 Result: effort is decreasing in distance.

$$\frac{\partial e^*}{\partial d} = 1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r}c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}}$$

Effort is decreasing in distance iff:

$$1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r}c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}} < 0$$

$$\frac{\frac{\partial^2 \Pi}{\partial^2 r}c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}} < -1$$

$$\frac{\partial^2 \Pi}{\partial^2 r}c''(e) < -\frac{\partial^2 \Pi}{\partial^2 e}\frac{\partial^2 \Pi}{\partial^2 r} + \frac{\partial^2 \Pi}{\partial e \partial r}\frac{\partial^2 \Pi}{\partial r \partial e}$$

Note that

$$\frac{\partial^2 \Pi}{\partial^2 r} c''(e) < -\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r}$$

is a sufficient condition since $\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e} > 0$

$$c''(e) > -\frac{\partial^2 \Pi}{\partial^2 e}$$

As a reminder

$$\frac{\partial \Pi}{\partial e} = b + p\left(1 - \Phi\left(\frac{-m}{\sqrt{r}}\right)\right) - c'(e)$$

$$\frac{\partial^2 \Pi}{\partial^2 e} = p\left(\frac{1}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) - c''(e)$$

so we have:

$$c''(e) > -p\left(\frac{1}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right) + c''(e)$$

$$0 > -p\left(\frac{1}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

Which is satisfied.

A.4 Interactions

A.4.1 Derive second order IFT

$$\begin{aligned}
\frac{\partial \Pi}{\partial e}(e^*, r^*, d, p) &= 0 \\
\frac{\partial \Pi}{\partial r}(e^*, r^*, d, p) &= 0
\end{aligned}$$

So we have

$$\begin{aligned}
\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial e}(e^*, r^*, d, p) &= 0 \\
\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial r}(e^*, r^*, d, p) &= 0
\end{aligned}$$

Which expands to:

$$\begin{aligned}
\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial e}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial d}e^{*}\frac{\partial^{2}\Pi}{\partial^{2}e}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial d}r^{*}\frac{\partial^{2}\Pi}{\partial r\partial e}\left(e^{*},r^{*},d,p\right) &= 0\\
\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial r}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial d}e^{*}\frac{\partial^{2}\Pi}{\partial e\partial r}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial d}r^{*}\frac{\partial^{2}\Pi}{\partial^{2}r}\left(e^{*},r^{*},d,p\right) &= 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial p}\frac{\partial \Pi}{\partial e}\left(e^{*},r^{*},d,p\right) &= 0\\
\frac{\partial}{\partial p}\frac{\partial \Pi}{\partial r}\left(e^{*},r^{*},d,p\right) &= 0
\end{aligned}$$

which expands to

$$\begin{aligned}
\frac{\partial}{\partial p}\frac{\partial \Pi}{\partial e}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial p}e^{*}\frac{\partial^{2}\Pi}{\partial^{2}e}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial p}r^{*}\frac{\partial^{2}\Pi}{\partial r\partial e}\left(e^{*},r^{*},d,p\right) &= 0\\
\frac{\partial}{\partial p}\frac{\partial \Pi}{\partial r}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial p}e^{*}\frac{\partial^{2}\Pi}{\partial e\partial r}\left(e^{*},r^{*},d,p\right)+\frac{\partial}{\partial p}r^{*}\frac{\partial^{2}\Pi}{\partial^{2}r}\left(e^{*},r^{*},d,p\right) &= 0
\end{aligned}$$

Now, taking the cross derivative and suppressing arguments

$$\begin{aligned}
\frac{\partial}{\partial p}\left(\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial e}+\frac{\partial}{\partial d}e^{*}\frac{\partial^{2}\Pi}{\partial^{2}e}+\frac{\partial}{\partial d}r^{*}\frac{\partial^{2}\Pi}{\partial r\partial e}\right) &= 0\\
\frac{\partial}{\partial p}\left(\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial r}+\frac{\partial}{\partial d}e^{*}\frac{\partial^{2}\Pi}{\partial e\partial r}+\frac{\partial}{\partial d}r^{*}\frac{\partial^{2}\Pi}{\partial^{2}r}\right) &= 0
\end{aligned}$$

Which expands to:

$$
\begin{aligned}
&\frac{\partial}{\partial p}\frac{\partial}{\partial d}e^*\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial p}\frac{\partial}{\partial d}r^*\frac{\partial^2\Pi}{\partial r\partial e}\\
&+\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right) \quad = \quad 0\\
&\frac{\partial}{\partial p}\frac{\partial}{\partial d}r^*\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial p}\frac{\partial}{\partial d}e^*\frac{\partial^2\Pi}{\partial r\partial e}\\
&+\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right) \quad = \quad 0
\end{aligned}
$$

Which, with rearranging gives: $\frac{\partial}{\partial p}\frac{\partial}{\partial d}e^*$and $\frac{\partial}{\partial p}\frac{\partial}{\partial d}r^*$.
Those have the same signs as:

$$
\begin{aligned}
&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial p}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial p}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)
\end{aligned}
$$

So we have the following identities:

$$\frac{\partial^2\Pi(e,r)}{\partial e\partial r}=-p\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

$$\frac{\partial\Pi}{\partial r} = p\left(\frac{1}{2\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)-\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q'(r)$$

$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial^2 r} &= -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)-p\left(\frac{-m}{2r}\right)\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)\\
&\quad-\left[b+p\left(1-\Phi\left(\frac{-m}{\sqrt{r}}\right)\right)\right]q''(r)-\left[\frac{\partial^2\Pi(e,r)}{\partial e\partial r}\right]q'(r)
\end{aligned}
$$

$$\frac{\partial^2\Pi}{\partial^2 e}=p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(\frac{-m}{\sqrt{r}}\right)\right)-c''(e)$$

$$\frac{\partial e^*}{\partial d}=\frac{-\frac{\partial^2\Pi}{\partial^2 r}\frac{\partial^2\Pi}{\partial e\partial d}+\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial d}}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}=1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

$$\frac{\partial r^*}{\partial d}=\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial d}-\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial r\partial d}}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}=\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

$$\frac{\partial^3\Pi(e,r)}{\partial p\partial e\partial r}=-\left(\frac{m}{2r\sqrt{r}}+\frac{q'(r)}{\sqrt{r}}\right)\phi\left(\frac{-m}{\sqrt{r}}\right)$$

$$\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial e}=-\frac{\partial}{\partial r}\frac{\partial}{\partial e}\frac{\partial \Pi}{\partial e}$$

So

$$\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial \Pi}{\partial e}+\frac{\partial}{\partial d}e^{*}\frac{\partial}{\partial r}\frac{\partial^{2}\Pi}{\partial^{2}e}=-\frac{\partial}{\partial r}\frac{\partial^{2}\Pi}{\partial^{2}e}+\frac{\partial}{\partial d}e^{*}\frac{\partial}{\partial r}\frac{\partial^{2}\Pi}{\partial^{2}e}=\frac{\frac{\partial^{2}\Pi}{\partial^{2}r}c''(e)}{\frac{\partial^{2}\Pi}{\partial^{2}e}\frac{\partial^{2}\Pi}{\partial^{2}r}-\frac{\partial^{2}\Pi}{\partial e\partial r}\frac{\partial^{2}\Pi}{\partial r\partial e}}\frac{\partial}{\partial r}\frac{\partial^{2}\Pi}{\partial^{2}e}$$

Suppose that m is small and negative
Then:

$$\frac{\partial^{2}\Pi(e,r)}{\partial e\partial r}=-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)$$

$$\frac{\partial^{3}\Pi(e,r)}{\partial p\partial e\partial r}=-\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)$$

$$\frac{\partial^{3}\Pi(e,r)}{\partial p\partial d\partial r}=\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)$$

$$\frac{\partial^{2}\Pi}{\partial^{2}e}=p\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(0\right)\right)-c''\left(e\right)$$

$$\begin{aligned}\frac{\partial^{2}\Pi}{\partial^{2}r} &= -p\left(\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-p\left(0\right)\left(0+\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\\ &\quad -\left[b+p\left(1-\Phi\left(0\right)\right)\right]q''\left(r\right)+p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)q'\left(r\right)\end{aligned}$$

$$\frac{\partial^{2}\Pi}{\partial^{2}r}=-p\left(-\frac{\left(q'\left(r\right)\right)^{2}}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\left[b+p/2\right]q''\left(r\right)$$

(since $\partial\phi\left(m\right)=-m\phi(m)$) we have:

$$\frac{\partial^{3}\Pi}{\partial r\partial^{2}e}=0$$

$$\frac{\partial^{3}\Pi}{\partial d\partial^{2}e}=0$$

$$\frac{\partial^3 \Pi}{\partial p \partial^2 e} = \left(\frac{1}{\sqrt{r}}\right)(\phi(0))$$

$$\frac{\partial^3 \Pi}{\partial^3 e} = -c'''(e)$$

$$\frac{\partial^3 \Pi}{\partial r \partial e \partial d} = 0$$

$$\frac{\partial^3 \Pi}{\partial p \partial e \partial d} = -\left(\frac{1}{\sqrt{r}}\right)(\phi(0))$$

$$\frac{\partial^3 \Pi(e,r)}{\partial e \partial^2 r} = -p\left(\frac{q''(r)}{\sqrt{r}} - \frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)$$

$$\frac{\partial^3 \Pi(e,r)}{\partial d \partial^2 r} = p\left(\frac{q''(r)}{\sqrt{r}} - \frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)$$

$$\frac{\partial^3 \Pi}{\partial p \partial^2 r} = -\left(-\frac{(q'(r))^2}{\sqrt{r}} + \frac{1}{4r\sqrt{r}}\right)\phi(0) - \frac{1}{2}q''(r)$$

$$\frac{\partial^3 \Pi}{\partial^3 r} = -p\left(-\frac{2(q'(r))q''(r)}{\sqrt{r}} + \frac{(q'(r))^2}{2r\sqrt{r}} - \frac{3}{8r^2\sqrt{r}}\right)\phi(0) - [b + p/2]\,q'''(r)$$

Substituting in where m is close to zero, we have:

$$
\begin{aligned}
&\frac{\partial^2 \Pi}{\partial r \partial e}\left(\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)+\frac{\partial}{\partial d}r^*\left(-\left(-\frac{(q'(r))^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi(0)-\frac{1}{2}q''(r)\right)\right) \\
&\quad +\frac{\partial^2 \Pi}{\partial r \partial e}\left(\frac{\partial}{\partial d}e^*\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right) \\
&\quad +\frac{\partial^2 \Pi}{\partial r \partial e}\left(0+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)+0\right) \\
&\quad +\frac{\partial^2 \Pi}{\partial r \partial e}\left(\left(p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)+\frac{\partial}{\partial d}e^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right) \\
&\quad +\frac{\partial^2 \Pi}{\partial r \partial e}\left(\frac{\partial}{\partial d}r^*\left(-p\left(-\frac{2(q'(r))q''(r)}{\sqrt{r}}+\frac{(q'(r))^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)-[b+p/2]\,q'''(r)\right)\right) \\
&\quad -\frac{\partial^2 \Pi}{\partial^2 r}\left(-\left(\frac{1}{\sqrt{r}}\right)(\phi(0))+\frac{\partial}{\partial d}e^*\left(\frac{1}{\sqrt{r}}\right)(\phi(0))+\frac{\partial}{\partial d}r^*\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right) \\
&\quad -\frac{\partial^2 \Pi}{\partial^2 r}\left(0+\frac{\partial}{\partial d}e^*\left(-c'''(e)\right)+0\right) \\
&\quad -\frac{\partial^2 \Pi}{\partial^2 r}\left(0+0+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)
\end{aligned}
$$

and

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\left(\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)+\frac{\partial}{\partial d}r^*\left(-\left(-\frac{(q'(r))^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi(0)-\frac{1}{2}q''(r)\right)\right)\right.$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial d}e^*\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(0+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)+0\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\left(p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)+\frac{\partial}{\partial d}e^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial d}r^*\left(-p\left(-\frac{2(q'(r))q''(r)}{\sqrt{r}}+\frac{(q'(r))^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)-[b+p/2]\,q'''(r)\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(-\left(\frac{1}{\sqrt{r}}\right)(\phi(0))+\frac{\partial}{\partial d}e^*\left(\frac{1}{\sqrt{r}}\right)(\phi(0))+\frac{\partial}{\partial d}r^*\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(0+\frac{\partial}{\partial d}e^*\left(-c'''(e)\right)+0\right)$$

$$\left.+\frac{\partial^2\Pi}{\partial r\partial e}\left(0+0+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)\right)$$

Simplifying we have:

$$\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial d}r^*\left(-\left(-\frac{(q'(r))^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi(0)-\frac{1}{2}q''(r)\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\partial}{\partial d}r^*\left(-p\left(-\frac{2(q'(r))q''(r)}{\sqrt{r}}+\frac{(q'(r))^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)-[b+p/2]\,q'''(r)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\right)(\phi(0))+\frac{\partial}{\partial d}r^*\left(-\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial d}e^*\left(-c'''(e)\right)+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)$$

and

$$
\begin{aligned}
-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial d}r^*\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)\right) \\
-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right) \\
-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right) \\
-\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial}{\partial d}r^*\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)-\left[b+p/2\right]q'''\left(r\right)\right) \\
-\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right) \\
+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(0\right)\right)+\frac{\partial}{\partial d}r^*\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right) \\
+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial d}e^*\left(-c'''\left(e\right)\right)+\frac{\partial}{\partial d}r^*\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)
\end{aligned}
$$

And substituting in for r*

$$\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(0\right)\right)+\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-c'''\left(e\right)\right)\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)$$

and

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(0\right)\right)+\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-c'''\left(e\right)\right)\right)$$

$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)$$

Simplifying:

$$-\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)$$
$$-\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$-\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$-\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$
$$+\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)+\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\right)2\phi\left(0\right)$$
$$-\frac{\frac{\partial^2\Pi}{\partial^2 r}\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\phi\left(0\right)-c'''\left(e\right)\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(-c'''\left(e\right)\right)$$

and

$$\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{q'\left(r\right)}{\sqrt{r}}+p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\right)\phi\left(0\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\left(\frac{1}{\sqrt{r}}\right)\left(\phi\left(0\right)\right)-c'''\left(e\right)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{q'\left(r\right)}{\sqrt{r}}+p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\right)\phi\left(0\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(-c'''\left(e\right)\right)$$

Simplifying:

$$\frac{-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{\left(q'\left(r\right)\right)^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi\left(0\right)-\frac{1}{2}q''\left(r\right)\right)$$

$$\frac{-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$\frac{-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$\frac{-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$

$$+\frac{\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)+\left(-\frac{q'\left(r\right)}{\sqrt{r}}\right)\right)2\phi\left(0\right)$$

$$-\frac{\frac{\partial^2\Pi}{\partial^2 r}\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{1}{\sqrt{r}}\phi\left(0\right)-c'''\left(e\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 r}\left(-c'''\left(e\right)\right)$$

and

$$\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left(-\frac{(q'(r))^2}{\sqrt{r}}+\frac{1}{4r\sqrt{r}}\right)\phi(0)-\frac{1}{2}q''(r)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2(q'(r))q''(r)}{\sqrt{r}}+\frac{(q'(r))^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-[b+p/2]\,q'''(r)\right)$$
$$+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{q'(r)}{\sqrt{r}}+p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\right)\phi(0)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\left(\frac{1}{\sqrt{r}}\right)(\phi(0))-c'''(e)\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(\frac{q'(r)}{\sqrt{r}}+p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\right)\phi(0)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(-c'''(e)\right)$$

Which, if $c'''(e)$ is sufficiently large yield the signs of propositions 5 &7.
Similarly, with respect to b we have:
Those have the same signs as:

$$\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$
$$+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$
$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)$$

and

$$
\begin{aligned}
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
&+\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)
\end{aligned}
$$

Gives the following identities:

$$\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}=0$$

$$\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}=0$$

$$\frac{\partial}{\partial b}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}=0$$

$$\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 e}=0$$

$$\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial r\partial e}=0$$

$$\frac{\partial}{\partial b}\frac{\partial^2\Pi}{\partial^2 r}=-q''\left(r\right)$$

Substituting gives:

$$\begin{aligned}
&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial d}r^*\left(-q''\left(r\right)\right)\right)\\
+&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
+&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
-&\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
-&\frac{\partial^2\Pi}{\partial^2 r}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)
\end{aligned}$$

and

$$\begin{aligned}
&-\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial d}r^*\left(-q''\left(r\right)\right)\right)\\
-&\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
-&\frac{\partial^2\Pi}{\partial^2 e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial r}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 r}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
+&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial e}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial e}\frac{\partial^2\Pi}{\partial r\partial e}\right)\\
+&\frac{\partial^2\Pi}{\partial r\partial e}\left(\frac{\partial}{\partial r}\frac{\partial}{\partial d}\frac{\partial\Pi}{\partial e}+\frac{\partial}{\partial d}e^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial^2 e}+\frac{\partial}{\partial d}r^*\frac{\partial}{\partial r}\frac{\partial^2\Pi}{\partial r\partial e}\right)
\end{aligned}$$

Supposing m is small and negative and substituting from the previous set of identities gives:

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(-q''\left(r\right)\right)\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 r}\left(\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-c'''\left(e\right)\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 r}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

and

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-q''\left(r\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'\left(r\right)\right)q''\left(r\right)}{\sqrt{r}}+\frac{\left(q'\left(r\right)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''\left(r\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-\frac{\partial^2\Pi}{\partial^2 e}\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(\left(1+\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)\left(-c'''\left(e\right)\right)\right)$$

$$-p\left(\frac{q'\left(r\right)}{\sqrt{r}}\right)\phi\left(0\right)\left(\frac{-\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''\left(r\right)}{\sqrt{r}}-\frac{q'\left(r\right)}{2r\sqrt{r}}\right)\phi\left(0\right)\right)\right)$$

And simplifying gives:

$$-p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\,\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}q''(r)$$

$$-p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\left(p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}$$

$$+p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\,\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'(r)\right)q''(r)}{\sqrt{r}}+\frac{\left(q'(r)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)\right)$$

$$+p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\,\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''(r)\right)$$

$$p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\,\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)2\phi(0)$$

$$+\frac{\partial^2\Pi}{\partial^2 r}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}c'''(e)$$

$$+\frac{\partial^2\Pi}{\partial^2 r}c'''(e)$$

and

$$
\begin{aligned}
&-\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}q''(r)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\\
&+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(-\frac{2\left(q'(r)\right)q''(r)}{\sqrt{r}}+\frac{\left(q'(r)\right)^2}{2r\sqrt{r}}-\frac{3}{8r^2\sqrt{r}}\right)\phi(0)\right)\\
&+\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-\left[b+p/2\right]q'''(r)\right)\\
&-\frac{\partial^2\Pi}{\partial^2 e}\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\\
&+p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\left(\frac{\frac{\partial^2\Pi}{\partial^2 r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\right)c'''(e)\\
&+p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\,c'''(e)\\
&+p\left(\frac{q'(r)}{\sqrt{r}}\right)\phi(0)\left(\frac{\frac{\partial^2\Pi}{\partial e\partial r}c''(e)}{\frac{\partial^2\Pi}{\partial^2 e}\frac{\partial^2\Pi}{\partial^2 r}-\frac{\partial^2\Pi}{\partial e\partial r}\frac{\partial^2\Pi}{\partial r\partial e}}\left(-p\left(\frac{q''(r)}{\sqrt{r}}-\frac{q'(r)}{2r\sqrt{r}}\right)\phi(0)\right)\right)
\end{aligned}
$$

which is $q''(r)$is sufficiently large yields the signs of propositions 6&8.

Appendix B

Relational Influence Model Details and Solution

B.1 Firm's offer

B.1.1 Notation: Elections and contributions.

The current election is one of the following settings:

1. Open election with firm offer, OF
2. Open election with candidate offer, OC
3. Incumbent election with firm offer where the incumbent has $x contributions already, IF
4. Incumbent election with candidate offer where the incumbent has $x contributions already, IC
5. Challenger election with the firm offer, CF
6. Challenger election with the candidate offer, CC.

Other candidate's contributions: In challenger elections, the incumbent is the non contracting candidate type, and thus has raised $c = \alpha + k$ in goodwill and charisma. In open elections the other candidate is the non-contracting type and thus has $c = k$ contributions of charisma. In incumbent elections, the challenger has contributions $c = k$.

This candidate's contributions: Before negotiations. In Open or Challenger elections, the candidate has contributions k. In incumbent elections, the candidate has prior contributions of $c = b_{t-1} + k + g_{t-1}$

The contributions made to the candidate then, are defined as: these four pairs: $\left\{a^{OF}, b^{OF}\right\}$, $\left\{a^{OC}, b^{OC}\right\}$, $\left\{a^{CF}, b^{CF}\right\}$, $\left\{a^{CC}, b^{CC}\right\}$, and the functions $\left\{a^{IF}\left(x\right), b^{IF}\left(x\right)\right\}, \left\{a^{IC}\left(x\right), b^{IC}\left(x\right)\right\}$ where x is the incumbents already raised contributions.

B.1.2 Lemma: The firm's offered bonus is the minimum necessary to maintain incentive compatibility.

Suppose the firm's prefers to give a contribution in the next period greater than the minimum necessary for IC. If in the next election the firm again makes the offer, the firm could make that part of the next period's offer. If in the next election the candidate makes the offer the candidate will extract at least as much in the current period.

So $b^{OF} = b^{IF}(x) = b^{CF} = \underline{b}$

B.1.3 Lemma: If the agents contract in incumbent elections, then the agents contract in open and challenger elections.

Note that the future incentives, once the candidate is in the position to provide the favor or the firm is in the position to pay the bonus following an open or challenger election are exactly the same as if the current election were an incumbent election.

B.1.4 Lemma: If the firm has made the offer and the candidate wins the election, the future tree of the game is identical, regardless of whether this election was open, incumbent or challenger.

This follows from the above lemmas. The agents contract over the same favors in all elections and the same bonus is due and the subsequent election is an incumbent election with the same amount of contributions already raised.

B.1.5 Notation: Values.

We can note the firm's values from elections starting at the negotiation step as:
$\Pi^{OF}, \Pi^{CF}, \Pi^{IF}(x), \Pi^{OC}, \Pi^{CC}, \Pi^{IC}(x), \Pi^{ONC}, \Pi^{CNC}, \Pi^{INC}(x)$
We can similarly note the candidate's values from elections as:
$U^{OF}, U^{CF}, U^{IF}(x), U^{OC}, U^{CC}, U^{IC}(x), U^{ONC}, U^{CNC}, U^{INC}(x)$
Where O, C, and I represent open, challenger, and incumbent elections
and L, C, and NC represent Firm offer, Candidate offer, and No Contract offer.

B.1.6 Lemma: If the candidate makes the offer the firm has no expected surplus. $\Pi^{OC} = \Pi^{ONC}, \Pi^{CC} = \Pi^{CNC}, \Pi^{IC}(x) = \Pi^{INC}(x)$

That is, the firm's individual rationality constraint binds. Proof. Suppose the firm has expected surplus. The candidate could extract that surplus by increasing a with no change in the incentive constraints.

B.1.7 Firm's advances

The firm is solving the following maximization problem in an open election.

$$\rho(k+a,k)\,\delta\left[\beta - \underline{b} + \lambda\Pi^{IF}(\underline{b}) + (1-\lambda)\,\Pi^{INC}(\underline{b})\right] \\ + (1-\rho(k+a,k))\,\delta\left[\lambda\Pi^{CF} + (1-\lambda)\,\Pi^{CNC}\right] - a$$

in a challenger election:

$$\rho\left(k+a,k+\alpha\right)\delta\left[\beta-\underline{b}+\lambda\Pi^{IF}\left(\underline{b}\right)+\left(1-\lambda\right)\Pi^{INC}\left(\underline{b}\right)\right]$$
$$+\left(1-\rho\left(k+a,k+\alpha\right)\right)\delta\left[\lambda\Pi^{CF}+\left(1-\lambda\right)\Pi^{CNC}\right]-a$$

and in an incumbent election

$$\rho\left(k+a+b_{t-1},k\right)\delta\left[\beta-\underline{b}+\lambda\Pi^{IF}\left(\underline{b}\right)+\left(1-\lambda\right)\Pi^{INC}\left(\underline{b}\right)\right]$$
$$+\left(1-\rho\left(k+a+b_{t-1},k\right)\right)\delta\left[\lambda\Pi^{CF}+\left(1-\lambda\right)\Pi^{CNC}\right]-a$$

Which we can rearrange:

$$\rho\left(k+a,k\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+\left(1-\lambda\right)\Pi^{CNC}\right]-a$$

$$\rho\left(k+a,k+\alpha\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+\left(1-\lambda\right)\Pi^{CNC}\right]-a$$

$$\rho\left(k+a+b_{t-1},k\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+\left(1-\lambda\right)\Pi^{CNC}\right]-a$$

So we have as FOCs:

$$\rho_1\left(k+a,k\right)=\frac{1}{\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]}$$

$$\rho_1\left(k+a,k+\alpha\right)=\frac{1}{\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]}$$

$$\rho_1\left(k+a+b_{t-1},k\right)=\frac{1}{\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+\left(1-\lambda\right)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]}$$

Here we see that the previous period's bonus is becomes a substitute for this period's advance if the firm makes the offer.

Define $a_f = a^{OF}$

Note that:

1. $a^{IF}(b_{t-1}) = \max\left(0, a^{OF} - b_{t-1}\right)$
2. Because $\rho_{11} < 0$ we have this ordering: $a^{CF} \leq a^{OF} \leq a^{IF}(b_{t-1}) + b_{t-1}$ where the first holds strictly if k is small enough and the second holds strictly only if $b_{t-1} > a^{OF}$. Which also implies that$a^{IF}(b_{t-1}) = 0$.
3. if $a^{OF} \leq \underline{b}$ then $a^{IF}(b_{t-1}) = 0$

B.1.8 Firm's Payoffs

We can then define the firm's payoffs:

$$\Pi^{OC} = \Pi^{ONC}, \Pi^{CC} = \Pi^{CNC}, \Pi^{IC}(x) = \Pi^{INC}(x)$$

$$\begin{aligned}\Pi^{OF} &= \rho\left(k + a^{OF}, k\right)\delta\left[\beta - \underline{b} + \lambda\left(\Pi^{IF}(\underline{b}) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}(\underline{b}) - \Pi^{CNC}\right)\right] \\ &\quad + \delta\left[\lambda\Pi^{CF} + (1-\lambda)\Pi^{CNC}\right] - a^{OF}\end{aligned}$$

$$\begin{aligned}\Pi^{CF} &= \rho\left(k + a^{CF}, k + \alpha\right)\delta\left[\beta - \underline{b} + \lambda\left(\Pi^{IF}(\underline{b}) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}(\underline{b}) - \Pi^{CNC}\right)\right] \\ &\quad + \delta\left[\lambda\Pi^{CF} + (1-\lambda)\Pi^{CNC}\right] - a^{CF}\end{aligned}$$

$$\begin{aligned}\Pi^{IF}(b_{t-1}) &= \rho\left(k + b_{t-1} + a^{IF}(b_{t-1}), k\right)\delta \\ &\quad \left[\beta - \underline{b} + \lambda\left(\Pi^{IF}(\underline{b}) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}(\underline{b}) - \Pi^{CNC}\right)\right] \\ &\quad + \delta\left[\lambda\Pi^{CF} + (1-\lambda)\Pi^{CNC}\right] - a^{IF}(b_{t-1})\end{aligned}$$

$$\begin{aligned}\Pi^{ONC} &= \rho(k,k)\delta\left[\lambda\left(\Pi^{IF}(\alpha) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}(\alpha) - \Pi^{CNC}\right)\right] \\ &\quad + \delta\left[\lambda\Pi^{CF} + (1-\lambda)\Pi^{CNC}\right]\end{aligned}$$

$$\begin{aligned}\Pi^{CNC} &= \rho(k,k+\alpha)\,\delta\left[\lambda\left(\Pi^{IF}(\alpha)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}(\alpha)-\Pi^{CNC}\right)\right]\\ &+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]\end{aligned}$$

$$\begin{aligned}\Pi^{INC}(b_{t-1}) &= \rho(k+b_{t-1},k)\,\delta\left[\lambda\left(\Pi^{IF}(\alpha)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}(\alpha)-\Pi^{CNC}\right)\right]\\ &+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]\end{aligned}$$

where $a^{CF}, a^{OF}, a^{IF}(b_{t-1})$ are defined by the implicit functions from the FOCs and $\underline{b}$ is defined from the candidate's IC (below).

It is then straight-forward, though quite tedious, to invert the last six equations to find the firms profit as a function of the probability functions, β, and the contributions.

B.1.9 Firm's IC

The firm also has to have an incentive not to deviate and withhold the bonus b.

So it must be the case that

$$\beta - b + \lambda\Pi^{IF}(b) + (1-\lambda)\,\Pi^{IC}(b) \geq \beta$$

or that

$$\lambda\Pi^{IF}(b) + (1-\lambda)\,\Pi^{IC}(b) \geq b$$

note that left hand side is bounded by:

$$\lambda\Pi^{IF}(b) + (1-\lambda)\,\Pi^{IC}(b) \leq \Pi^{IF}(b) < \frac{\delta}{1-\delta}\left[\beta - \underline{b}\right]$$

So there must exist a maximum $\bar{b}$ so that

$$\lambda\Pi^{IF}\left(\bar{b}\right) + (1-\lambda)\,\Pi^{IC}\left(\bar{b}\right) = \bar{b}$$

[1]

if $\bar{b} > a^{OF}$ then we have

[1]Existence follows because the above upper bound gives a closed set $\left[0, \frac{\delta}{1-\delta}\left[\beta - \underline{b}\right]\right]$.

$$\lambda\Pi^{IF}\left(\bar{b}\right)+(1-\lambda)\,\Pi^{IC}\left(\bar{b}\right)=$$
$$\lambda\left(\rho\left(k+\bar{b},k\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]\right)$$
$$+\lambda\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]$$
$$+(1-\lambda)\left(\rho\left(k+\bar{b},k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]\right)$$
$$+(1-\lambda)\,\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]$$
$$=\bar{b}$$

$$=\lambda\rho\left(k+\bar{b},k\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]$$
$$+(1-\lambda)\,\rho\left(k+\bar{b},k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]$$

$$=\lambda\rho\left(k+\bar{b},k\right)\delta\left[\beta-\underline{b}+\lambda\left(\Pi^{IF}\left(\underline{b}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\underline{b}\right)-\Pi^{CNC}\right)\right]$$
$$+\lambda\rho\left(k+\bar{b},k\right)\delta\left[-\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)-(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$
$$+\rho\left(k+\bar{b},k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]$$

$$=\lambda\rho\left(k+\bar{b},k\right)\delta\left[\beta-\underline{b}+\lambda\Pi^{IF}\left(\underline{b}\right)+(1-\lambda)\,\Pi^{INC}\left(\underline{b}\right)-\lambda\Pi^{IF}\left(\alpha\right)-(1-\lambda)\,\Pi^{INC}\left(\alpha\right)\right]$$
$$+\rho\left(k+\bar{b},k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\,\Pi^{CNC}\right]$$

if $\bar{b}\leq a^{OF}$ then we have

$$\begin{aligned}
&\lambda \Pi^{IF}\left(\bar{b}\right) + (1-\lambda)\,\Pi^{IC}\left(\bar{b}\right) = \\
&\lambda\left(\rho\left(k + a^{OF}, k\right)\delta\left[\beta - \underline{b} + \lambda\left(\Pi^{IF}\left(\underline{b}\right) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}\left(\underline{b}\right) - \Pi^{CNC}\right)\right]\right) \\
&+\lambda\left(\delta\left[\lambda\Pi^{CF} + (1-\lambda)\,\Pi^{CNC}\right] - a^{IF}\left(b_{t-1}\right)\right) \\
&+(1-\lambda)\left(\rho\left(k+\bar{b}, k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}\left(\alpha\right) - \Pi^{CNC}\right)\right]\right) \\
&+(1-\lambda)\,\delta\left[\lambda\Pi^{CF} + (1-\lambda)\,\Pi^{CNC}\right] \\
&= \bar{b}
\end{aligned}$$

$$\begin{aligned}
\lambda\rho\left(k+b+a^{IF}\left(b\right), k\right)\delta\left[\gamma + \lambda U^{IF}\left(\underline{b}\right) + (1-\lambda)U^{IC}\left(\underline{b}\right)\right]& \\
+(1-\lambda)\,\rho\left(k+b+a^{IC}\left(b\right), k\right)\delta& \\
\times\left[\gamma + \lambda U^{IF}\left(b^{IC}\left(b\right)\right) + (1-\lambda)U^{IC}\left(b^{IC}\left(b\right)\right)\right] &\geq \frac{\rho(k+\alpha,k)\delta}{1-\rho(k+\alpha,k)\delta}\gamma
\end{aligned}$$

B.1.10 Firm's IR Constraint

The firm's IR constraint binds offers made by the candidate. That is: $\Pi^{OC} = \Pi^{ONC}, \Pi^{CC} = \Pi^{CNC}, \Pi^{IC}\left(x\right) = \Pi^{INC}\left(x\right)$

$$\begin{aligned}
\Pi^{OC} &= \rho\left(k + a^{OC}, k\right)\delta \\
&\quad\times\left[\beta - b^{OC} + \lambda\left(\Pi^{IF}\left(b^{OC}\right) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}\left(b^{OC}\right) - \Pi^{CNC}\right)\right] \\
&\quad+\delta\left[\lambda\Pi^{CF} + (1-\lambda)\,\Pi^{CNC}\right] - a^{OC}
\end{aligned}$$

$$\begin{aligned}
\Pi^{ONC} &= \rho\left(k,k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right) - \Pi^{CF}\right) + (1-\lambda)\left(\Pi^{INC}\left(\alpha\right) - \Pi^{CNC}\right)\right] \\
&\quad+\delta\left[\lambda\Pi^{CF} + (1-\lambda)\,\Pi^{CNC}\right]
\end{aligned}$$

$$\Pi^{OC} = \Pi^{ONC}$$

So we have

$$\rho\left(k+a^{OC},k\right)\delta\left[\beta-b^{OC}+\lambda\left(\Pi^{IF}\left(b^{OC}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{OC}\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\Pi^{CNC}\right]-a^{OC}$$
$$=\rho\left(k,k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$
$$+\delta\left[\lambda\Pi^{CF}+(1-\lambda)\Pi^{CNC}\right]$$

$$\rho\left(k+a^{OC},k\right)\delta\left[\beta-b^{OC}+\lambda\left(\Pi^{IF}\left(b^{OC}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{OC}\right)-\Pi^{CNC}\right)\right]-a^{OC}$$
$$=\rho\left(k,k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]$$

$$\begin{aligned} a^{OC} &= \rho\left(k+a^{OC},k\right)\delta \\ &\times\left[\beta-b^{OC}+\lambda\left(\Pi^{IF}\left(b^{OC}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{OC}\right)-\Pi^{CNC}\right)\right] \\ &-\rho\left(k,k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]\end{aligned}$$

$$\begin{aligned} a^{OC} &= \rho\left(k+a^{OC},k\right)\delta\left(\beta-b^{OC}\right)+\rho\left(k+a^{OC},k\right)\delta \\ &\times\left[\lambda\left(\Pi^{IF}\left(b^{OC}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{OC}\right)-\Pi^{CNC}\right)\right] \\ &-\rho\left(k,k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]\end{aligned}$$

Similarly, for $\Pi^{CC}=\Pi^{CNC}$ we have

$$\begin{aligned} a^{CC} &= \rho\left(k+a^{CC},k+\alpha\right)\delta\left(\beta-b^{CC}\right) \\ &+\rho\left(k+a^{CC},k+\alpha\right)\delta\left[\lambda\left(\Pi^{IF}\left(b^{CC}\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{CC}\right)-\Pi^{CNC}\right)\right] \\ &-\rho\left(k,k+\alpha\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]\end{aligned}$$

Similarly, for $\Pi^{IC}\left(b_{t-1}\right)=\Pi^{INC}\left(b_{t-1}\right)$ we have

$$\begin{aligned}
a^{IC}\left(b_{t-1}\right) &= \rho\left(k+b_{t-1}+a^{IC}\left(b_{t-1}\right),k\right)\delta\left(\beta-b^{IC}\left(b_{t-1}\right)\right) \\
&+\rho\left(k+b_{t-1}+a^{IC}\left(b_{t-1}\right),k\right)\delta \\
&\times\left[\lambda\left(\Pi^{IF}\left(b^{IC}\left(b_{t-1}\right)\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(b^{IC}\left(b_{t-1}\right)\right)-\Pi^{CNC}\right)\right] \\
&-\rho\left(k+b_{t-1},k\right)\delta\left[\lambda\left(\Pi^{IF}\left(\alpha\right)-\Pi^{CF}\right)+(1-\lambda)\left(\Pi^{INC}\left(\alpha\right)-\Pi^{CNC}\right)\right]
\end{aligned}$$

Which implicitly define $a^{OC}, a^{CC}, a^{IC}\left(b_{t-1}\right)$ as functions of $b^{OC}, b^{CC}, b^{IC}\left(b_{t-1}\right)$ respectively.

B.2 Candidate offers

B.2.1 Candidate's offer

The candidate makes an offer of a, b that maximizes in open elections:

$$U^{OC}=\rho\left(k+a,k\right)\delta\left[\gamma+\lambda U^{IF}\left(b\right)+(1-\lambda)U^{IC}\left(b\right)\right]$$

in challenger elections:

$$U^{CC}=\rho\left(k+a,k+\alpha\right)\delta\left[\gamma+\lambda U^{IF}\left(b\right)+(1-\lambda)U^{IC}\left(b\right)\right]$$

and in incumbent elections

$$U^{IC}\left(b_{t-1}\right)=\rho\left(k+b_{t-1}+a,k\right)\delta\left[\gamma+\lambda U^{IF}\left(b\right)+(1-\lambda)U^{IC}\left(b\right)\right]$$

B.2.2 IC constraint

First, it's useful to define the off equilibrium, deviation utility for an incumbent.

$$U^{ID}=\rho\left(k+\alpha,k\right)\delta\left[\gamma+U^{ID}\right]$$

$$\left(1-\rho\left(k+\alpha,k\right)\delta\right)U^{ID}=\rho(k+\alpha,k)\delta\gamma$$

$$U^{ID}=\frac{\rho(k+\alpha,k)\delta}{1-\rho(k+\alpha,k)\delta}\gamma$$

Note that the off equilibrium no contract open and challenger election candidates utilities aren't necessary since provided the contract meets IC conditions (which depend only on Incumbent no contract), the candidate prefers to contract in open or challenger elections. Because of the lack of transfers directly between the parties, and particularly the inability to extract transfer, before the election, surplus from the candidate to the firm, the candidate weakly prefers any IC compatible contract to no contract. The candidate does not have a potentially binding individual rationality constraint.

It's also useful to expand the candidate's utility in incumbent elections where the firm makes the offer:

$$U^{IF}\left(b_{t-1}\right)=\rho\left(k+b_{t-1}+a^{IF}\left(b_{t-1}\right),k\right)\delta\left[\gamma+\lambda U^{IF}\left(\underline{b}\right)+(1-\lambda)U^{IC}\left(\underline{b}\right)\right]$$

Then the candidate's IC constraint is:

$$\lambda U^{IF}\left(b\right)+(1-\lambda)U^{IC}\left(b\right)\geq U^{ID}=\frac{\rho(k+\alpha,k)\delta}{1-\rho(k+\alpha,k)\delta}\gamma$$

Expanding we have:

$$\begin{array}{c}\lambda\rho\left(k+b+a^{IF}\left(b\right),k\right)\delta\left[\gamma+\lambda U^{IF}\left(\underline{b}\right)+(1-\lambda)U^{IC}\left(\underline{b}\right)\right]\\ +(1-\lambda)\,\rho\left(k+b+a^{IC}\left(b\right),k\right)\delta\left[\gamma+\lambda U^{IF}\left(b^{IC}\left(b\right)\right)+(1-\lambda)U^{IC}\left(b^{IC}\left(b\right)\right)\right]\\ \geq\frac{\rho(k+\alpha,k)\delta}{1-\rho(k+\alpha,k)\delta}\gamma\end{array}$$

Note that since the left hand side is increasing in b there is one solution to this. Let $\hat{b}$ be that solution. Define $\underline{b}=\max\left(0,\hat{b}\right)$

B.2.3 Candidate's split of contributions between today and tomorrow

Optimization. Given that contributions by the firm are bounded by the firm's IR constraint, the candidate's optimization is to decide on the split of contributions between the advance and the bonus. Note that the size of the bonus may be against the upper or lower bounds given by the IC constraints.

1. Consider the optimization of $a^{IC}\left(b_{t-1}\right)$ and $b^{IC}\left(b_{t-1}\right)$

   (a) Suppose the bonus is at the upper bound for b', $b^{IC}\left(b'\right)=\bar{b}$ then for all $b''>b'$ then $b^{IC}\left(b''\right)=\bar{b}$ because b_{t-1} substitutes for a. Further $a^{IC}\left(b''\right)>a^{IC}\left(b'\right)$

   (b) Let b''' be the smallest such $b^{IC}\left(b'''\right)=\bar{b}$. Suppose $\bar{b}\geq b'''$ then if $b^{IC}\left(b_{t-1}\right)=\bar{b}$ then this is an absorbing sequence of offers.

   (c) Note that if $b_{t-1}=0$ then open elections are equivalent to incumbent elections. So that if$b^{IC}\left(0\right)=\bar{b}$, that is, $b'''\leq 0$ then the candidate always makes the absorbing offer in an open or any incumbent election.

   (d) If $b^{OC}=\bar{b}$ then $b^{CC}=\bar{b}$. Because $p_{11}<0$ and $p(k,k+\alpha)<p(k,k)$ both give incentives to shift contributions forward in challenger elections, there is even

more of an incentive in challenger elections to keep incentives against the upper bound.

2. If $b'' > b'$ then $b^{IC}(b'') \geq b^{IC}(b')$, and strictly so, if $b^{IC}(b')$ or $b^{IC}(b'')$ is interior.
   (a) This follows from both $p_2 < 0$ and because b_{t-1} substitutes for a
   (b) This implies that the sequence of b^{IC} is weakly (strictly, if interior) increasing
3. Suppose $b^{IC}(\underline{b}) = \underline{b}$, then following either any periods after a firm offer or any period after an open election the candidates offer is fixed with $b^{IC} = \underline{b}$
   (a) The firms offers $b = \underline{b}$,so that follows by assumption
   (b) $b^{IC}(\underline{b}) \geq b^{IC}(0)$ this also follows after an open election.
4. Bonuses might be decreasing in the sequence of candidate's offers following a challenger election.
   (a) Because $p_{11} < 0$ and $p(k, k+\alpha) < p(k, k)$ both give bigger incentives to shift contributions forward in challenger elections then otherwise.
   (b) This effect may persist because a large b_{t-1}gives incentives to shift to the future.

B.2.4 If the firm never offers, $\lambda = 0$, then there is no contingent payment. If so, then there is some lower bound to the βs that can be supported in trade.

Step 1: if $\lambda = 0$, then the no-contract profits are zero.

Step 2: If the no-contract profits are zero and $\lambda = 0$, then the candidate offer profits are zero.

Step 3: If there was a contingent payment, it would not satisfy the IC.

Step 4: If the present value of trade is high enough, i.e. β is high enough, and contributions make enough of a difference, then advances can support trade.

This is like an efficiency wage contract.

B.3 Contracts

B.3.1 For a sufficiently high β, for all λ , the firm offers an efficiency wage. Call this threshold $\overline{\beta}$.

Step 1: Increasing β increases the candidate's surplus from trade.

Step 2: Increasing surplus from trade increases the candidate's utility from making an offer.

Step 3: That relaxes the candidate's IC constraint.
Step 4: For a sufficiently relaxed IC constraint, the firm offers an efficiency wage.

B.3.2 For a small enough $\beta > \alpha$, trade is not possible. Call this threshold $\underline{\beta}$.

$\bar{b}$ is increasing in β and λ

If trade is not possible at $\beta = \alpha$, $\lambda = 1$ then trade is not possible at $\beta = \alpha$, $\forall \lambda$

Suppose $\lambda = 1$, then we need $a^{IC} + b^{IC} \geq \alpha$ to satisfy candidate's IC.

Suppose $\lambda = 1$, then we need $p\left(k + a^{IC} + b_{t-1}, k\right)\delta\beta \geq p\left(k + a^{IC} + b_{t-1}, k\right)\delta b^{IC} + a^{IC}$ to satisfy the firm's IR.

So we have $\beta \geq b^{IC} + a^{IC}/p\left(k + a^{IC} + b_{t-1}, k\right)\delta$

$b^{IC} + a^{IC}/p\left(k + a^{IC} + b_{t-1}, k\right)\delta > a^{IC} + b^{IC}$

Then we have $\beta > \alpha$, a contradiction, and the above follows by continuity.

B.3.3 For β in between and $\lambda > 0$, the firm offers a bonus wage.

This follows by a continuity argument. The above two results show that above a threshold the contract has $\underline{b} = 0$ and below a threshold $\underline{b} > \bar{b}$. It suffices to show those are not the same threshold.

Note that for all $\lambda > 0$ and $\beta > \alpha$ the firm has positive surplus of making an offer so that $\bar{b} > 0$. (By revealed preference - fix the bonus, the firm could offer a smaller advance and earn profit, so the firm must earn profit at the original offer. Suppose the firm does offer $a = 0$. Then $\beta > \alpha \geq \underline{b}$ implies positive profits.)

So, at the threshold, for $\lambda > 0$ we have $\bar{b} > \underline{b} = 0$. By a continuity argument forβ just below the upper threshold it must be the case that $\bar{b} > \underline{b} > 0$.

B.3.4 If the firm offers a bonus wage, so does the candidate.

This follows directly. The firm's offer is always weakly above $\underline{b}$. If $\underline{b} > 0$, then the candidate offers a bonus wage.

B.4 Horizon effects

B.4.1 $\overline{\beta}$ is decreasing in δ

$\overline{\beta}$ is the lower bound of the region where the value of future periods are enough to satisfy the candidate's IC constraint. Increasing δ increases the value of those future periods, so it must decrease $\overline{\beta}$.

In a more narrow sense, increasing δ

- increases the future advance the firm is willing to make because it increases the value of trade,
- increases the value of future advances in firm's offers to the candidate because the next period is discounted less, and
- increases the expected surplus available for the candidate to extract,

all of which reduce the temptation to deviate.

B.4.2 $\underline{\beta}$ is decreasing in δ

The logic above leads $\underline{b}$, the necessary bonus to be decreasing in δ. Showing that $\bar{b}$ is increasing in δ is more than sufficient to prove the claim. $\bar{b}$is the largest contribution that satisfies the firm's IC constraint. Increasing δ increases the value of the future periods, so relaxes the IC constraint.

B.4.3 The firm's offered contributions in the first election is smaller than the bonus and advance offered in incumbent elections.

This follows from $a^{CF} \leq a^{OF} \leq a^{IF}(b_{t-1}) + b_{t-1}$ developed in the firm's advances above.

B.4.4 The candidate's requested advance in the first election is smaller than the bonus and advance requested in incumbent elections.

Note that $a^{OC} = a^{IC}(0)$. Suppose the bonus is interior and the First order conditions are appropriate. Then it suffices to show that $\partial a^{IC}(x)/\partial x \leq 1$. But by an application of the envelope theorem $\partial a^{IC}(x)/\partial x = 0$.

Suppose the bonus is not interior and the bonus is equal to $\bar{b}$. Then increasing b_{t-1} does not change the advance, because the candidate is unable to shift contributions to the future.

Suppose the bonus is not interior and the bonus is equal to $\underline{b}$. Then increasing b_{t-1} does not change the advance, because the candidate prefers to shift as much contributions as possible early.

It is then sufficient to show that $a^{CC} \leq a^{OC}$. Since the firm has lower profits at challenger elections and contributions are less less valuable, the candidate has left value to shift between elections and lower value in contributing them to the current election it follows that $a^{CC} \leq a^{OC}$.

B.5 Term Limits

B.5.1 If β is sufficiently large, term limits of 2 or more terms (a finite number of elections a candidate can win) do not cause complete unraveling.

This follows from the complete public histories. Note that one component, even in the above infinite horizon case one, of the firm's IC constraint is the benefit the firm receives from contracting with future candidates.

$$\beta - b + \lambda \Pi^{IF}(b) + (1-\lambda)\Pi^{IC}(b) \geq \beta$$

$$\begin{aligned}
\lambda \Pi^{IF}(b) + (1-\lambda)\Pi^{IC}(b) \quad = \quad & \lambda \delta Prob(Elected)\left[\beta - b^{IF}(b)\right] \\
& +\lambda \delta Prob(Elected)\left[\lambda \Pi^{IF}(b) + (1-\lambda)\Pi^{INC}(b)\right] \\
& +\lambda \delta (1 - Prob(Elected))\left[\lambda \Pi^{CF} + (1-\lambda)\Pi^{CNC}\right] \\
& +(1-\lambda)\delta Prob(Elected)\left[\lambda \Pi^{IF}(\alpha) + (1-\lambda)\Pi(\alpha)\right] \\
& +(1-\lambda)\delta(1 - Prob(Elected))\left[\lambda \Pi^{CF} + (1-\lambda)\Pi^{CNC}\right] \\
& \geq b
\end{aligned}$$

Because the firm's contingent contribution is supported, partially, by value from contracting with other candidates, the firm will still be willing to make some campaign contribution even to help a candidate be elected to their final (Lame Duck) term.

If the firm can commit to a sufficiently large contingent contribution then contracting is possible. A sufficiently large β makes this commitment possible.

Two (or more) terms are required, because term limits mean that the candidate, once elected to their lame duck term, is not swayed by the firm.

B.6 Disaggregation over time

Consider a one time favor that provides one time payoff Ω to the firm, at a policy salience G to the voters in this period, and both are zero in future periods. This favor is not contractible.

Proposition 5: Suppose, however, a surplus destroying conversion to a stream of favors that provide benefit $(1-\delta)\Omega - L$ to the firm each period and costs the politician goodwill $(1-\delta)G$. This sequence of favors is contractible if $\Omega$is large enough relative to L and G.

Replace β with $\beta' = (1-\delta)\,\Omega - L$ and α with $\alpha' = (1-\delta)\,G$ in the model above. Note that increasing Ω increases β' without changing any other terms. As long as $\beta' > \alpha'$ all the results of proposition 1 and lemmas hold

B.7 Aggregation over firms

Suppose that in each period the favor is needed by a different firm. With no future, it is clear that the firms will never pay a contingent bonus. Absent contracting between firms, each of these favors is not contractible if β is small enough to require a bonus in the case of a single firm. However, if β is large enough then there exists an equilibrium with trade even among different firms. Suppose the firm expects the candidate to provide the favor, then the firm contributes such that $\rho_1\,(a) = 1/\delta\beta$. Note that because there is no future for this firm, this advance is smaller than the advance that would be given by a single long lived firm. Consequently, the threshold in β to suppose this is higher than $\bar{\beta}$. If future firms expect follow through only if past favors were provided, and this advance is large enough, then the candidate provides the favor.

Suppose that β is not that large. Then contracting between the firms is required to contract with candidates. Suppose the firms all contract with a third party, a "lobbyist" over the favors such that the lobbyist receives $\beta^l < \beta$ if a favor is provided. Then if $\beta^l > \underline{\beta}$ the favor is contractible. Note that other forms of inter-firm contracting (acquisition, joint venture, selling reputation, etc...) can also facilitate contracting.