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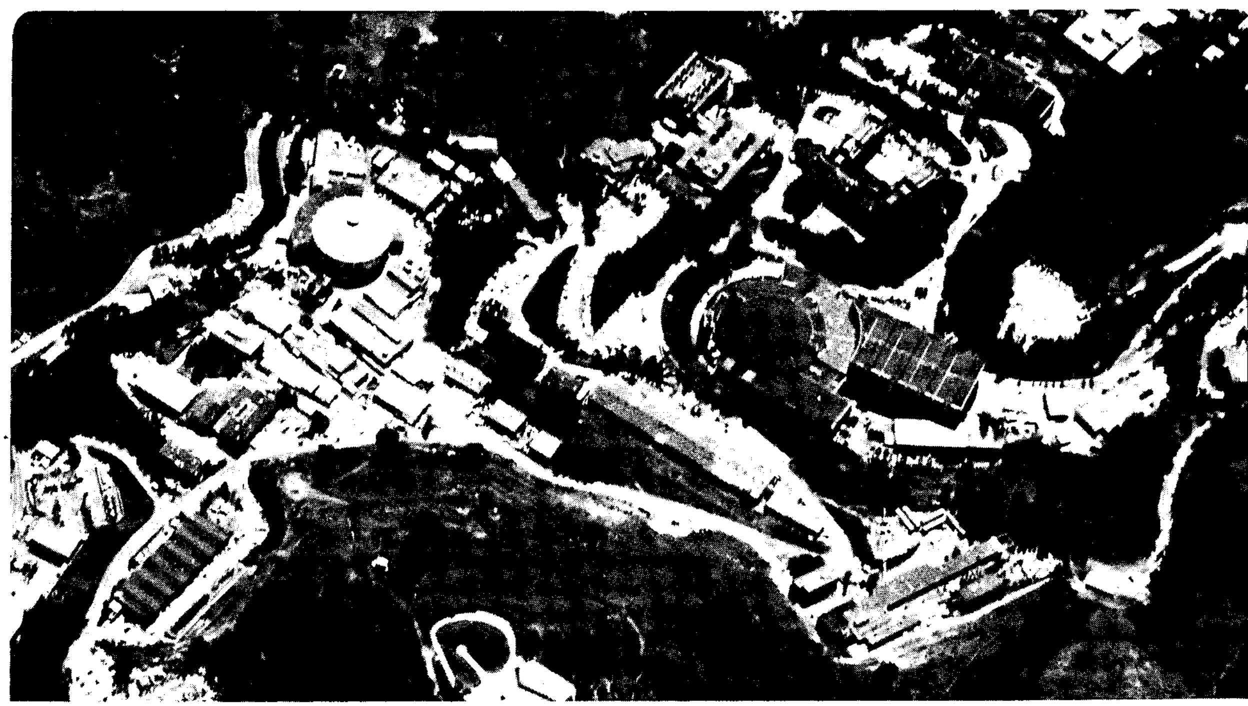

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A METHOD TO DETERMINE Z^0 PARAMETERS[1]

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A METHOD TO DETERMINE Z^0 PARAMETERS

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A method is suggested of determining the mass M_Z of the Z^0 resonance, the ratio $\frac{g_{Ve}}{g_{Ae}}$ of its vector and axial coupling constants to electrons, that same ratio for the coupling to τ's, and the product g^2 of the electron and τ coupling constants. The method requires only measurements near the Z^0 peak: of the τ-lepton forward-backward asymmetry; of average energies of τ-decay products; and of the forward-backward asymmetry of these average decay product energies. The method is model independent. It has the advantage of being essentially insensitive to errors associated with the luminosity measurement and of depending only a little on radiative corrections.

1. Background

With the high-statistics data we expect from LEP, measurement accuracy of key parameters may be limited more by systematic than statistical errors [1]. As a check of systematic errors, it will be useful to have different determinations of the same parameters using different techniques such that the biases due to systematic errors be small and not the same. One may also want to reduce to a minimum the need for theoretical assumptions (e.g. the standard model [2]) to validate the final results. The method described here is meant to complement measurements based on the line shape [3] and other means [4]. It has been designed with the intention of minimizing both systematic errors and theoretical assumptions.

The reaction that will be used here is

$$e^- + e^+ \to Z_0 + (\gamma) \to \tau^- + \tau^+ + (\gamma). \tag{1}$$

Our only assumptions are that the Z^0 resonance is made of a single boson of spin 1 and that reaction (1) can be analyzed using the six-step picture of Fig. 1, which we describe below.

a) The two incident electrons e^- and e^+, of energy $\frac{\sqrt{s}}{2}$ each, start interacting. At this time they may radiate photons by internal Bremsstrahlung. In that case, they lose energy and become virtual electrons. At the end of this step, the electrons now labeled e^{-*} and e^{+*} form an object of effective mass $\sqrt{s^*} \leq \sqrt{s}$.

b) The two electrons e^{-*} and e^{+*} annihilate into a Z^0 boson of mass $\sqrt{s^*}$.

c) The Z^0 boson decays into a pair of τ's. Each of the τ's makes an angle θ^* in the Z^0 rest frame with respect to the incident e^* of the same sign. We will pay special attention to the case where one of these τ's can be used for polarization measurements. At this step, that τ lepton may be off the mass shell. We label it τ^*. It has an energy $E_\tau^* = \frac{\sqrt{s^*}}{2}$ in the Z^0 rest frame.

d) If the τ lepton of interest is off the mass shell, it radiates γ's by final state Bremsstrahlung. In any event, the final τ in this step is real. It has an energy $E_\tau \leq \frac{\sqrt{s^*}}{2}$ in the Z^0 rest frame.

e) The τ we analyze for polarization measurements decays into neutrino(s) and a π or a μ . For the sake of simplicity, we will not consider other decay modes for measuring polarization. This π or μ may be off the mass shell. It will be called π^{**} or μ^{**}. It has an energy E_π^{**} or E_μ^{**} , $\leq E_\tau$, in the Z^0 rest frame.

f) If the decay product π^{**} or μ^{**} is off the mass shell, it emits γ's by internal Bremsstrahlung. In any event, it is now a real particle, π or μ, with an energy which we now define in the laboratory as E_π or E_μ . That π or μ may traverse the detector and be recorded.

This six-step picture is assumed to be approximately correct [5, 6], except for very forward and very backward production angles, i.e. in regions escaping detection by present LEP detectors [7]. Step a) produces a spectrum of effective masses $\sqrt{s^*}$, i.e. of masses of Z^0 states in steps b) and c). In the rest frame of these states, the angular distributions of the differential cross section $\frac{d\sigma}{d(\cos\theta^*)}$ and of the τ^- average helicity $P^*(\theta^*)$, at step c), averaged over all values of $\sqrt{s^*}$, are functions of four s-dependent parameters [6, 8], which we call the average form-factors F_0, F_1, F_2, and F_3.

$$\frac{d\sigma}{d(\cos\theta^*)} = F_0\left(1+\cos^2\theta^*\right) + 2F_1\cos\theta^* \tag{2}$$

$$P^*(\theta^*) = \frac{F_2\left(1+\cos^2\theta^*\right) + 2F_3\cos\theta^*}{F_0\left(1+\cos^2\theta^*\right) + 2F_1\cos\theta^*} \tag{3}$$

where θ^* is the τ-production angle in the Z^0 rest frame, defined at step c). The τ^+ and τ^- helicities are anticorrelated with a coefficient almost equal to -100%. The τ^+ average helicity is $-P^*(\theta^*)$. The average form-factors are related to averages of physical quantities known as the forward-backward asymmetry A_{FB}, the average τ polarization A_{Pol}, and the τ polarization forward-backward asymmetry A_{Pol}^{FB}.

$$2 < \cos\theta^* > = \frac{F_1}{F_0} = \frac{4}{3}A_{FB} \tag{4}$$

$$< P^*(\theta^*) > = \frac{F_2}{F_0} = A_{Pol} \tag{5}$$

$$2 < P^*(\theta^*)\cos\theta^* > = \frac{F_3}{F_0} = \frac{4}{3}A_{Pol}^{FB} \tag{6}$$

where the symbol $< >$ around a quantity denotes an average of this quantity. In Eqs. (4) to (6) and only in these equations, it represents an average extended over the entire solid angle. The experimental determination of A_{FB}, A_{Pol}, and A_{Pol}^{FB} does not require luminosity measurements.

We describe how to determine the ratios of the vector and axial coupling constants from A_{FB}, A_{Pol}, and A_{Pol}^{FB} in Sec. 2. In Sec. 3 we show how to get the resonant mass M_Z and the product of the coupling constants g^2. Finally, in Sec. 4 we describe a method of measuring A_{FB}, A_{Pol}, and A_{Pol}^{FB} experimentally.

2. The ratio of the coupling constants

The vector and axial coupling constants to electrons (τ leptons) are called g_{Ve} and g_{Ae} ($g_{V\tau}$ and $g_{A\tau}$) and α is the fine structure constant $\left(\cong\frac{1}{137}\right)$. For the sake of simplicity, as in ref. [6], the coupling constants are expressed in units of the electron charge, instead of units involving the weak interaction Fermi coupling constant. It follows that the standard model predictions for our coupling constants are larger by the factor $\frac{1}{\sin 2\theta_W}$ than the expression given in ref. [9], calling the weak angle θ_W. The average form factors F's satisfy the following equations

$$F_0=\frac{\pi\alpha^2}{2s}\left[U+2g_{Ve}g_{V\tau}R+(g_{Ve}^2+g_{Ae}^2)(g_{V\tau}^2+g_{A\tau}^2)Q\right] \tag{7}$$

$$F_1=\frac{\pi\alpha^2}{2s}[2g_{Ae}g_{A\tau}R+4g_{Ve}g_{Ae}g_{V\tau}g_{A\tau}Q] \tag{8}$$

$$F_2=\frac{\pi\alpha^2}{2s}\left[2g_{Ve}g_{A\tau}R+2\left(g_{Ve}^2+g_{Ae}^2\right)g_{V\tau}g_{A\tau}Q\right] \tag{9}$$

$$F_3=\frac{\pi\alpha^2}{2s}[2g_{Ae}g_{V\tau}R+2g_{Ve}g_{Ae}(g_{V\tau}^2+g_{A\tau}^2)Q] \tag{10}$$

where U, R, and Q are averages over the spectrum of Z^0 masses $\sqrt{s^*}$, taking into account e^- and e^+ Bremsstrahlung (i.e. radiative corrections in step a) of Fig. 1) and the beam energy spread. Let ρ be a function expressing the spectrum of values of s* for a given value of s. It is essentially a function of $(1-\frac{s^*}{s})$ [6, 9]. Then:

$$\frac{U}{s}=\int\rho\left(1-\frac{s^*}{s}\right)\frac{ds^*}{s}\left(\frac{1}{s^*}\right) \tag{11}$$

$$\frac{R}{s}=\int\rho\left(1-\frac{s^*}{s}\right)\frac{ds^*}{s}\frac{\mathrm{Re}\{\chi(s^*)\}}{s^*} \tag{12}$$

$$\frac{Q}{s}=\int\rho\left(1-\frac{s^*}{s}\right)\frac{ds^*}{s}\frac{|\chi(s^*)|^2}{s^*} \tag{13}$$

$$\chi(s^*)=\frac{1}{1-\frac{M_Z^2}{s^*}+i\frac{\Gamma}{M_Z}} \tag{14}$$

where Γ is the width of the Z^0 resonance.

In the lowest order Born approximation the function ρ is the Dirac distribution $\delta(\frac{s^*}{s}-1)$. Thus U, R and Q are simply equal to 1, $Re\{\chi(s)\}$, and $|\chi(s)|^2$, respectively. With radiative corrections, $\rho(1-\frac{s^*}{s})$ is a weighting function for which an approximate expression will be given later in Sec. 3.

We restrict our analysis to data taken near the top and on the slopes of the Z^0 peak, where $\chi(s^*)$ is of the order of $\frac{M_Z}{\Gamma}$ and thus much larger than one. This means the contribution of $U \approx 1$ is small in Eq. (7) and we will neglect it from now on. Equations (7) to (10) become four homogeneous linear equations for the variables R and Q, so R and Q can be eliminated using two of the four equations. Two constraints remain. They are homogeneous relations between the average form factors F_0, F_1, F_2, and F_3. Dividing by F_0, the terms F_1, F_2, and F_3 become A_{FB}, A_{Pol}, and A_{Pol}^{FB} defined in Eqs. (4) to (6).

$$\left[\left(\frac{4}{3}A_{FB}\right)\frac{g_{Ve}}{g_{Ae}}-A_{Pol}\right]\frac{1+\left(\frac{g_{Ve}}{g_{Ae}}\right)^2}{1-\left(\frac{g_{Ve}}{g_{Ae}}\right)^2}=\left(\frac{g_{V\tau}}{g_{A\tau}}A_{Pol}-1\right)\frac{2\frac{g_{V\tau}}{g_{A\tau}}}{1-\left(\frac{g_{V\tau}}{g_{A\tau}}\right)^2} \tag{15}$$

$$\left[\left(\frac{4}{3}A_{Pol}^{FB}\right)\frac{g_{Ve}}{g_{Ae}}-1\right]\frac{1+\left(\frac{g_{Ve}}{g_{Ae}}\right)^2}{1-\left(\frac{g_{Ve}}{g_{Ae}}\right)^2}=\left(\frac{g_{V\tau}}{g_{A\tau}}A_{Pol}-1\right)\frac{1+\left(\frac{g_{V\tau}}{g_{A\tau}}\right)^2}{1-\left(\frac{g_{V\tau}}{g_{A\tau}}\right)^2} \tag{16}$$

The physical quantities A_{FB}, A_{Pol}, and A_{Pol}^{FB} depend on the incident energy $\sqrt{s}$, but the coupling constants g_{Ve}, g_{Ae}, $g_{V\tau}$, and $g_{A\tau}$ do not. It follows that Eqs. (15) and (16) define a straight line in the three-dimensional space of the observables A_{FB}, A_{Pol}, and A_{Pol}^{FB}. If data have been taken at several energies $\sqrt{s}$ near the Z^0 peak, it can be checked that they lie, within errors, on a straight line in the space of the variables A_{FB}, A_{Pol}, and A_{Pol}^{FB}, and also that this line is one of those defined by Eqs. (15) and (16). That straight line test is a test of the consistency of all the values of A_{FB}, A_{Pol}, and A_{Pol}^{FB} obtained at different energies. Any set of measurements of A_{FB}, A_{Pol}, and A_{Pol}^{FB} at any energy permits us to compute $\frac{g_{Ve}}{g_{Ae}}$ and $\frac{g_{V\tau}}{g_{A\tau}}$ by solving Eqs. (15) and (16) for these ratios. The results obtained at different $\sqrt{s}$ can be averaged to get the best accuracy. It is remarkable that this determination of the ratio of coupling constants and the straight line test are independent of the value of the initial energy $\sqrt{s}$, of the form of the function $\rho\,(1-\frac{s^*}{s})$ used in Eqs. (12) and (13) to describe radiative corrections in the initial state, and of the Z^0 width Γ. This is true as long as U is negligible, i.e. as long as we use only data near the Z^0 peak.

If the values of A_{Pol} and of $\frac{4}{3}A_{Pol}^{FB}$ are equal to one another within experimental errors, then the ratios of coupling constants $\frac{g_{Ve}}{g_{Ae}}$ and $\frac{g_{V\tau}}{g_{A\tau}}$ are also equal to one another within errors and electron-τ universality can be envisaged. We can determine a more accurate value of A_{Pol} by combining the measurements of the average and of the forward-backward asymmetry of the τ polarization. This more accurate value can then be introduced in Eq. (15) where, at the same time, we set $\frac{g_{Ve}}{g_{Ae}} = \frac{g_{V\tau}}{g_{A\tau}} = \frac{g_V}{g_A}$. With that assumption of electron-τ universality, the equation for $\frac{g_V}{g_A}$ reads

$$A_{Pol}\left(\frac{g_A}{g_V}+3\frac{g_V}{g_A}\right)-\left(\frac{4}{3}A_{FB}\right)\left[1+\left(\frac{g_V}{g_A}\right)^2\right]=2. \qquad (17)$$

It can be used for estimating $\frac{g_V}{g_A}$. For purpose of illustration, Fig. 2 shows the relation between A_{Pol} and A_{FB} deduced from Eq. (17) for different values of the ratio $\frac{g_V}{g_A}$. Given a value for A_{FB} and a value for A_{Pol} for the same $\sqrt{s}$, a point can be plotted on Fig. 2 and a corresponding value of $\frac{g_V}{g_A}$ can be determined.

If A_{FB} determined using this method with reaction (1) is equal, within experimental errors, to the forward-backward asymmetry obtained from μ pair production, μ-τ universality may be envisaged. Then, also using the muon data, a more accurate value for A_{FB} can be introduced in Eqs. (15) and/or (17).

3. The Z^0 mass and the product of coupling constants

The mass M_Z and the product g^2 of the coupling constants to electrons and τ's can be extracted from the s-dependence of the data. This time one needs an approximation for the function $\rho\,(1-\frac{s^*}{s})$. That function can be introduced in Eqs. (12) and (13) to obtain an s- and M_Z-dependent expression of R and Q. Then the ratio $\frac{R}{Q}$ gives a prediction, which depends on the coupling constants, for the quantity A_{FB}, thus for A_{Pol} and A_{Pol}^{FB}, since the latter are bound to A_{FB} by Eqs. (15) and (16).

$$\frac{4}{3}A_{FB}=\frac{1}{g^2}\frac{4g_{Ve}g_{Ae}g_{V\tau}g_{A\tau}+2k(s)g_{Ae}g_{A\tau}g^2}{g^2+2k(s)g_{Ve}g_{V\tau}} \tag{18}$$

$$k(s)=\frac{1}{g^2}\frac{R}{Q} \tag{19}$$

$$g^4=\left(g_{Ve}^2+g_{Ae}^2\right)\left(g_{V\tau}^2+g_{A\tau}^2\right) \tag{20}$$

Since the computation of $\frac{R}{Q}$ using Eqs. (12), (13) and (14) depends on M_Z and Γ, and since the quantity k(s) of Eq. (19) depends also on g defined by Eq. (20), a general fit of the predictions to the experimental data would allow us to determine all or some of these Z^0 parameters. We will use a more analytic approach instead, to demonstrate the limited influence of radiative corrections on M_Z and g.

After the ratios $\frac{g_{Ve}}{g_{Ae}}$ and $\frac{g_{V\tau}}{g_{A\tau}}$ have been determined using the method described in Sec. 2., the quantity k(s) can be obtained experimentally for each initial energy $\sqrt{s}$ at which A_{FB} has been measured. Solving Eq. (18) for k(s), one gets the formula for the experimental value of k(s)

$$k(s) = \frac{1}{2} \frac{\frac{4}{3}A_{FB} - 4\frac{g_{Ve}g_{Ae}g_{V\tau}g_{A\tau}}{g^4}}{\frac{g_{Ae}g_{A\tau}}{g^2} - \frac{g_{Ve}g_{V\tau}}{g^2}\left(\frac{4}{3}A_{FB}\right)} \tag{21}$$

which depends only on A_{FB} and on the ratios of coupling constants.

In the Born approximation, if beam energy spread can be neglected,

$$\frac{R}{Q} = 1 - \frac{M_Z{}^2}{s} \ . \tag{22}$$

Thus, in the plane of the variables $\frac{1}{s}$ and k(s), the theoretical prediction of Eq. (19) for k(s) represents a straight line that crosses the k = 0 axis at $\frac{1}{s} = \frac{1}{M_Z{}^2}$ and whose slope is $-\frac{M_Z{}^2}{g^2}$. At the point where k(s) = 0, $\frac{dk}{d\sqrt{s}} = \frac{2}{g^2 M_Z}$. If the Born approximation were satisfactory, an estimate of M_Z and g by a fit to the data points would be straightforward. Note that the Z^0 width, Γ, would not enter the theoretical prediction of k(s) and then could not be determined from these data alone.

To be more accurate, but still neglecting the effect due to beam energy spread, radiative corrections as given by ref. [6] for step a) of Fig. 1 (i.e. in the initial state), can be introduced to compute $\frac{R}{Q}$. For data taken near the Z^0 peak, in lowest order of the ratio $\frac{\Gamma}{M_Z}$, only the form of the function near $s^* = s$ is important. Thus we make the following approximation

$$\rho(v) \sim \gamma v^{\gamma-1} \tag{23}$$

where

$$v = 1 - \frac{s^*}{s} \tag{24}$$

$$\gamma = 2\frac{\alpha}{\pi}\ (\ln\frac{s}{m_e{}^2}\ -\ 1)\ \cong .108 \tag{25}$$

and m_e is the electron mass. Using v as the integration variable, the integrals of Eqs. (12) and (13) become integrals from v = 0 to v = 1.

$$R - i\frac{\Gamma}{M_Z}Q = \int_0^1 \gamma\ v^{\gamma-1}\,\frac{\chi(s^*)}{1-v}dv \tag{26}$$

In lowest order of $\frac{\Gamma}{M_Z}$, the integral of Eq. (26) can be equated to the same integral but from v = 0 to v = ∞, and then evaluated using standard integration techniques in the complex plane of the variable v.

$$R - i\frac{\Gamma}{M_Z}Q = \frac{-\gamma}{1+i\dfrac{\Gamma}{M_Z}}\quad \frac{1}{1-e^{2i\pi\gamma}}\quad 2\pi i\left[\chi(s)(1+i\frac{\Gamma}{M_Z})\right]^{1-\gamma} \tag{27}$$

From Eqs. (14), (19), and (27), we derive a relation between k(s) and $\sqrt{s}$. In lowest order of $\frac{\Gamma}{M_Z}$, this relation is

$$(1-\gamma)\arctan\left\{2\frac{\sqrt{s}-M_Z}{\Gamma}\right\} = \arctan\left\{k(s)g^2\frac{M_Z}{\Gamma}\right\} + \frac{\gamma\pi}{2}. \tag{28}$$

Given M_Z, g, and Γ, Eq. (28) defines a relationship between the measurable quantities k(s) and $\sqrt{s}$. One can expect that a plot of k(s) versus $\sqrt{s}$ defines a curve that intersects the k(s) = 0 axis at a value $\sqrt{s_0}$ of $\sqrt{s}$

$$\sqrt{s_0} = M_Z + \frac{\Gamma}{2}\tan\frac{\gamma\pi}{2(1-\gamma)} \cong M_Z + 0.192\frac{\Gamma}{2}. \tag{29}$$

From $\sqrt{s_0}$, M_Z can be extracted accurately, even if we know a much less good approximation for Γ. The value of Γ obtained from the line shape should be satisfactory [3, 10]. The constant g can be extracted from the slope of k(s) at that intersection $\sqrt{s} = \sqrt{s_0}$ with the $\sqrt{s}$ axis. That slope is equal to

$$\frac{dk}{d\sqrt{s}} = \frac{2(1-\gamma)}{g^2 M_Z\left[1+\tan^2\frac{\gamma\pi}{2(1-\gamma)}\right]} \cong \frac{1.72}{g^2 M_Z}. \tag{30}$$

The experimental determination of $\frac{dk}{d\sqrt{s}}$ yields g with very little dependence on the value of Γ. There are non-linearities as soon as the quantity

$$\Delta = 2\frac{\sqrt{s}-M_Z}{\Gamma}\tan\frac{\gamma\pi}{2} \cong 0.34\frac{\sqrt{s}-M_Z}{\Gamma} \tag{31}$$

is not negligible with respect to 1. These are essentially generated by radiative corrections. Note that data near the Z^0 peak are very valuable precisely because not only the term U in Eq. (7) is negligible, but also radiative corrections are less important for small values of the quantity Δ of Eq. (31). Note also that Eqs. (28), (29), and (30) reduce to the Born expression for $\gamma = 0$.

To illustrate these properties, the relationship between $\frac{\sqrt{s}}{M_Z}$ and $k(s).g^2$, as predicted by Eq. (28), has been plotted on Fig. 3. The parameter $\frac{\Gamma}{M_Z}$ for this curve is the measured value 0.028 and the parameter γ is 0.108, i.e. the value derived from Eq. (25). The curve does not depend strongly on $\frac{\Gamma}{M_Z}$ as can be seen from Fig. 3, where the variation due to ± 10 standard deviations from the measured value has also been plotted.

The best estimates for M_Z and g will be achieved by a general fit of Eq. (28) to the values of k(s) obtained experimentally using Eq. (21). The fit corresponds to the following operations on Fig. 3. Using trial values for M_Z and g, plot points of ordinate $k(s)\cdot g^2$ and of abscissa $\frac{\sqrt{s}}{M_Z}$. Then submit them to a horizontal translation to fit M_Z and a vertical scaling adjustment to fit g so as to get the points superposed to the solid curve. The width Γ plays a role in Eq. (28) only when $\gamma \neq 0$, which shows that Γ gets involved only via radiative corrections. That width would not be accurately estimated by a fit without using luminosity measurements. It should probably be obtained from the line shape [3, 10].

A similar but less elaborate method of determining M_Z from the forward-backward asymmetry and τ polarization measurements has been described in ref. [11]. The determination of $\frac{g_{Ve}}{g_{Ae}}$, $\frac{g_{V\tau}}{g_{A\tau}}$, and g was not detailed there as it is in this paper.

4. Measurement of A_{FB}, A_{Pol}, and A_{Pol}^{FB}.

To determine A_{FB}, A_{Pol}, and A_{Pol}^{FB} experimentally at any $\sqrt{s}$, we cannot simply use the quantities $\cos\theta^*$ and $P^*(\theta^*)$ defined at step c) in the Z^0 rest frame and appearing in Eqs. (4), (5), and (6), because they are not part of the data provided by the detector. Other quantities, experimentally accessible and correlated to $\cos\theta^*$ and P^*, will have to be substituted. Corrections for distortions introduced by that substitution will have to be applied, corrections which can be computed by Monte Carlo techniques. Our goal is to use experimentally accessible quantities that are strongly correlated to $\cos\theta^*$ and P^* because then corrections will be small and, in principle, flaws and model dependence of the Monte Carlo generator will be less important.

The parameter A_{FB} defined in Eq. (4) is equal to an average of $\cos\theta^*$. There are several good approximations for $\cos\theta^*$. In the detector, the data for one event of reaction (1) includes two energies E_- and E_+ and two angles θ_- and θ_+ in the laboratory. The energy E_- (E_+) is the energy of the charged decay products of the negative (positive) τ lepton. The angle $\theta_-(\theta_+)$ is the angle made by the sum of the momenta of these decay products from the $\tau^-(\tau^+)$ lepton with the negatively (positively) charged incident electron e^- (e^+). To take into account the fact that high-energy τ-decay products are more likely to travel in the same direction as their parent τ leptons than low-energy ones, one may want to use the following approximation $\cos\theta$ for $\cos\theta^*$

$$\cos\theta = \frac{E_-\cos\theta_- + E_+\cos\theta_+}{E_- + E_+}. \tag{32}$$

At colliders, all detectors are sensitive only to particles emitted in a limited solid angle that includes 90° from the beam axis. Let us call C_{lim} the limit for $|\cos\theta_-|$ and $|\cos\theta_+|$ below which efficiency of detection may be considered good and let us select only events with both θ_- and θ_+ within that limit. The reconstructed events will correspond to values of $\cos\theta$ given by Eq. (32) falling necessarily between $-C_{lim}$ and $+C_{lim}$. To determine A_{FB}, one can measure the average $< \cos\theta >$ of $\cos\theta$ between $-C_{lim}$ and $+C_{lim}$ and correct it to obtain an estimate of the average of $\cos\theta^*$ between $-C_{lim}$ and $+C_{lim}$. Note that the latter average is of course still different from the average over the entire solid angle which figures in Eq. (4). Using Eq. (2), and replacing $\frac{F_1}{F_0}$ by $\frac{4}{3}$ A_{FB},

$$< \cos\theta^* > = \frac{4}{3} A_{FB} \frac{2C_{lim}^2}{3+C_{lim}^2} \tag{33}$$

$$< \cos\theta > = \frac{4}{3} A_{FB} \frac{2C_{lim}^2}{3+C_{lim}^2} + \phi_{FB} (A_{FB}, A_{Pol}, A_{Pol}^{FB}) \tag{34}$$

where the term ϕ_{FB} $(A_{FB}, A_{Pol}, A_{Pol}^{FB})$ is a correction equal to the difference between $< \cos\theta >$ and $< \cos\theta^* >$. That term can be computed using a Monte Carlo program such as KORALZ [12]. For Monte Carlo events, both angles θ^* and θ are known. The distortions due to lack of detection efficiency for different angles θ can be taken into account by detector simulation. Thus $< \cos\theta^* >$ and $< \cos\theta >$, therefore the difference ϕ_{FB} $(A_{FB}, A_{pol}, A_{Pol}^{FB})$ can be computed. In principle, it is a function of all three parameters A_{FB}, A_{pol}, and A_{Pol}^{FB}, since these parameters affect the distributions of decay products in regions where detection efficiencies and distortions due to equipment limitations or experimental cuts may not be uniform. Of course, almost all modes of decay of the τ lepton can be used to determine a value for $\cos\theta$ according to Eq. (32) and thus can contribute to the measurement of A_{FB}.

As can be seen from Eqs. (3) and (5), A_{pol} is the average helicity of the τ^{-*} in the Z^0 rest frame over any symmetric interval of $\cos\theta^*$, between $-C_{lim}$ and $+C_{lim}$ in particular. The average helicity of τ leptons is related to the average energy of their decay products (see [6, 8] in particular) but, in our chain of steps described in Sec. 1 and Fig. 1, $P^*(\theta^*)$ is the τ^* helicity at the end of step c) and the τ-lepton decays only in step e). The changes in the τ polarization during step d) are small [13], but, to reduce them even further in the sample of events, it is advantageous to eliminate events with a γ of large energy in step d). Let E_{lim} be a small γ energy. We will make experimental cuts to reject most events for which a γ emitted in step d) has an energy larger than E_{lim} in the Z^0 rest frame and keep the bulk of those for which

$$E_\tau^* - E_\tau = \frac{\sqrt{s^*}}{2} - E_\tau < E_{lim} \tag{35}$$

where, as defined in Sec. 1., $\frac{\sqrt{s^*}}{2}$ and E_τ are the τ energies in the Z^0 rest frame before and after step d) respectively.

For the polarization measurements we further restrict the sample of events to τ decays into $\pi\nu$ and $\mu\nu\nu$. Assuming that the average helicity P^* is unchanged during step d), we deduce relations between the average polarization A_{Pol} of the sample and the average energies of the decay products π^{**} or μ^{**}. Neglecting the π and μ masses,

$$< E_\pi^{**} > = \frac{1}{6}(3 - A_{Pol}) < E_\tau > \tag{36}$$

$$< E_\mu^{**} > = \frac{1}{20}(7 + A_{Pol}) < E_\tau > \tag{37}$$

where $< E_\tau >$ is the average of the energy E_τ. This average depends on the initial energy $\sqrt{s}$ and on radiative corrections in the initial and final states, i.e. in steps a) and d) of Fig. 1. However the ratio of $< E_\pi^{**} >$ and $< E_\mu^{**} >$ does not depend on these quantities because it does not depend on $< E_\tau >$:

$$\frac{< E_\pi^{**} >}{< E_\mu^{**} >} = \frac{10}{3} \; \frac{3 - A_{Pol}}{7 + A_{Pol}}. \tag{38}$$

Note that the average energy of Eq. (36) is decreasing with A_{Pol} while the one of Eq. (37) is increasing. Because of that, the ratio defined in Eq. (38) is also more sensitive to A_{Pol} than either average energy of Eqs. (36) and (37).

For the data provided by the detector, the test of Eq. (35) can be approximated by the requirement that, in every event used to measure polarization, there be no γ-energy in the laboratory larger than E_{lim} detected in the solid angle covered by the detector electromagnetic calorimeter:

$$E_{\gamma\,lab} < E_{lim} \,. \tag{39}$$

Test (39) is a good approximation for test (35) because γs from Bremsstrahlung in step d) of Fig. 1 are emitted at a small angle with respect to the τ direction in the laboratory and have a good chance of being recorded in the electromagnetic calorimeter.

The energies E_π^{**} and E_μ^{**} in the Z^0 rest frame at the end of step e) of Fig. 1 will have to be approximated by the energies E_π and E_μ in the laboratory after the last step, since only E_π and E_μ are measured in the detector.

$$\frac{<E_\pi>}{<E_\mu>} = \frac{10}{3}\,\frac{3-A_{Pol}}{7+A_{Pol}} + \phi_{Pol}(A_{FB}, A_{Pol}, A_{Pol}^{FB}) \tag{40}$$

where ϕ_{Pol} (A_{FB}, A_{Pol}, A_{Pol}^{FB}) is a correction term, presumably small, equal to the difference between $\frac{<E_\pi>}{<E_\mu>}$ and $\frac{<E_\pi^{**}>}{<E_\mu^{**}>}$. That difference can be computed by generating Monte Carlo events [12]. In Monte Carlo events indeed, both E_π and E_π^{**}, (or E_μ and E_μ^{**}) are known, the test of Eq. (35) to select the sample for $<E_\pi^{**}>$ (or $<E_\mu^{**}>$) and the test of Eq. (39) to select the sample for $<E_\pi>$(or $<E_\mu>$) can be made. The effect of lack of efficiency can be simulated to account for the distortion of the averages due to the detector. Therefore both $\frac{<E_\pi>}{<E_\mu>}$ and $\frac{<E_\pi^{**}>}{<E_\mu^{**}>}$, and thus their difference, can be estimated.

Though $P^*(\theta^*)$ and so A_{Pol} have been defined as the τ^- average helicity, the τ^+ decays into π's and μ's can be used just as well. The τ^+ average helicity is opposite to the τ^- one but the decay asymmetry parameters are opposite too. The average energy of the τ^+

decay products should be the same as the τ^- ones. Thus τ's of both charges can be used to determine A_{Pol} and be included in the averages $< E_\pi >$ and $< E_\mu >$.

For A_{Pol}^{FB}, one can make a similar development as for A_{Pol}. One has to average $E_\pi \cos\theta$ and $E_\mu \cos\theta$ on the same samples of events as for A_{Pol}.

$$\frac{< E_\pi \cos\theta >}{< E_\mu \cos\theta >} = \frac{10}{3} \; \frac{3 - \frac{4}{3} A_{Pol}^{FB}}{7 + \frac{4}{3} A_{Pol}^{FB}} + \phi_{Pol}^{FB} (A_{FB}, A_{Pol}, A_{Pol}^{FB}) \tag{41}$$

where ϕ_{Pol}^{FB} $(A_{FB}, A_{Pol}, A_{Pol}^{FB})$ is another correction term, to be computed by Monte Carlo as the difference between $\frac{< E_\pi \cos\theta >}{< E_\mu \cos\theta >}$ and $\frac{< E_\pi^{**} \cos\theta^* >}{< E_\mu^{**} \cos\theta^* >}$.

Equations (34), (40), and (41) can be solved to get estimates of A_{FB}, A_{Pol}, and A_{Pol}^{FB}, which can be used in the analysis described in Secs. 2 and 3 of this paper. Note that luminosity measurements are not used to determine A_{FB}, A_{Pol}, and A_{Pol}^{FB}. In the analysis, they are used only in Sec. 3, indirectly, when a value of Γ measured with the line-shape is introduced into the computation of radiative corrections.

5. Summary and conclusion

Under the hypotheses that the Z^0 is a single spin 1 resonance and that the effect of radiative corrections can be approximated by the six-step picture of Fig. 1 and of Sec. 1, three quantities describe the data that can be obtained without luminosity measurement for reaction (1) at each value of the initial energy $\sqrt{s}$. They are the quantities A_{FB}, A_{Pol}, and A_{Pol}^{FB} of Eqs. (4), (5), and (6). They completely determine the angular distribution and the τ polarization properties in the Z^0 rest frame [6, 8].

Measurements of A_{FB}, A_{Pol}, and A_{Pol}^{FB} at any energy $\sqrt{s}$ are described and justified in Sec. 4. Let θ be the angle in the laboratory defined by Eq. (32) and let the interval between $-C_{lim}$ to C_{lim} be the range of $\cos\theta$ for which we accept τ pair events. Let θ^* be the angle in the Z^0 rest frame defined in Sec. 1 at step c). For experimentally obtained events, one can measure the average $<\cos\theta>$ in that interval and, for Monte Carlo events [12], one can compute both $<\cos\theta>$ and $<\cos\theta^*>$ in that same interval. The average $<\cos\theta>$ over the real events as well as the difference ϕ_{FB} (A_{FB}, A_{Pol}, A_{Pol}^{FB}) between the averages $<\cos\theta>$ and $<\cos\theta^*>$ over the Monte Carlo events can be introduced in Eq. (34).

Let E_π (E_μ) and E_π^{**} (E_μ^{**}) be the π (μ) energies defined at steps f) and e) of Sec. 1 respectively. The averages $<E_\pi>$, $<E_\mu>$, $<E_\pi \cos\theta>$, and $<E_\mu \cos\theta>$ can be measured for the experimental data while these same averages plus $<E_\pi^{**}>$, $<E_\mu^{**}>$, $<E_\pi^{**}\cos\theta^*>$, and $<E_\mu^{**}\cos\theta^*>$ can be computed for the Monte Carlo events. The averages $<E_\pi>$ and $<E_\mu>$ over the real events as well as the difference Φ_{Pol} (A_{FB}, A_{Pol}, A_{Pol}^{FB}) between $\frac{<E_\pi>}{<E_\mu>}$ and $\frac{<E^{**}{}_\pi>}{<E^{**}{}_\mu>}$ computed using the Monte Carlo events can be introduced in Eq. (40). Similarly, the averages of energies times production cosines can be introduced in Eq. (41). To minimize the size of the corrections, an experimental cut in γ energy detected at large angle from the beam direction (i.e. in the electromagnetic calorimeter) is suggested, as explicitly formulated in inequality (39). Note that such a cut also helps out in rejecting τ-decays involving π^0s.

From Eqs. (34), (40), and (41), A_{FB}, A_{Pol}, and A_{Pol}^{FB} can be obtained at various values of $\sqrt{s}$. Then, from these measurements, four s-independent parameters can be determined. They are the ratios $\frac{g_{Ve}}{g_{Ae}}$ and $\frac{g_{V\tau}}{g_{A\tau}}$ of the vector to the axial coupling constants of the Z^0 to electrons and τ leptons respectively, the mass M_Z of the Z^0 resonance, and the product of coupling constants g^2 (in units of electron charge squared) defined by Eq. (20). In the lowest order Born approximation, these four parameters contain all the information supplied by the quantities A_{FB}, A_{Pol}, and A_{Pol}^{FB} at all energies $\sqrt{s}$, regardless of the Z^0 width Γ. In the real world, in spite of the presence of radiative corrections, the ratios

$\frac{g_{Ve}}{g_{Ae}}$ and $\frac{g_{V\tau}}{g_{A\tau}}$ can be determined experimentally at each $\sqrt{s}$ using Eqs. (15) and (16) of Sec. 2, without having to use a value for Γ, or a value for $\sqrt{s}$, or an expression $\rho(1-\frac{s^*}{s})$ for the radiative corrections in the initial state. The mass M_Z and the product g^2 can be determined using the method justified in Sec. 3. For this purpose, the quantity k(s) of Eq. (21) can be computed at each value of $\sqrt{s}$ using A_{FB} and the ratio of coupling constants mentioned just above. Then Eq. (28) can be fitted to all the set of values of k(s). For this fit, some approximate estimate of Γ and of the distribution $\rho(1-\frac{s^*}{s})$ is needed in order to compute a correction. The line shape may be used for the determination of Γ. Determining Γ without using the data provided by the cross section line-shape does not seem advisable, in spite of the fact that the line-shape requires the data from the luminosity monitor.

In this paper, some approximations have been made but not corrected for. Among such approximations, there are the ones neglecting the term U in Eq. (7) and the remaining helicity changes in step d) of Fig. 1 [13]. Though these corrections can be computed by Monte Carlo when we test the standard model, they have not been spelled out for sake of simplicity. If electron-τ universality and/or μ-τ universality may be assumed, this determination of Z^0 parameters can be strengthened, as stated in Sec. 2.

As a check of both systematic errors and of the standard model [2], the values of Z^0 parameters obtained by the method described in this paper can be compared to those given by a fit of the standard model parameters to the data from the line shape [3]. To check systematic errors, they can also be compared to the measurements obtained by experiments using polarized beams.

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References

[1] Such a point of view is taken, for instance, by G. Altarelli et al. in: Physics at LEP, CERN 86-02 (1986) Vol. 1, p. 1.

[2] S. L. Glashow, Nucl. Phys. 22 (1961) 579; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264 and Phys. Rev. D5 (1972) 1412; A. Salam in: Elementary Particle Theory, Relativistic Groups and Analyticity, Proc. ed. N. Svartholm, (Wiley, New York, 1968) p. 367.

[3] Mark II Collaboration, G.S. Abrams et al., Phys. Rev. Lett 63 (1989) 2173; L3 Collaboration, B. Adeva et al., Phys. Lett. B231 (1989) 509; ALEPH Collaboration, D. Decamp et al., Phys. Lett. B231 (1989) 519; OPAL Collaboration, M. Z. Akrawy et al., Phys. Lett. B231 (1989) 530; DELPHI Collaboration, P. Aarnio et al., Phys. Lett. B231 (1989) 539.

[4] For instance, ALEPH Collaboration, D. Decamp et al., Phys. Lett. B234 (1990) 399.

[5] S. Jadach and Z. Was, First and higher order noninterference QED radiative corrections to the charge asymmetry at the Z resonance, preprint of MPI München (1989), MPI-PAE/PTh 33/89.

[6] S. Jadach and Z. Was in: Workshop in Z Physics at LEP, Proc. ed. G. Altarelli, R. Kleiss and C. Verzegnassi, (CERN Report 89-08, 1989) Vol I, p. 235.

[7] S. Jadach, private communication (1989).

[8] T. Riemann and M. Sachwitz, Polarization in tau lepton production at LEP, Berlin-Zeuten report 1988.

[9] Particle Data Group, Review of Particle Properties, Phys. Lett. B204 (1988) p. 103.

[10] R. N. Cahn, Phys. Rev. D36 (1987) 2666; D. Bardin et al., Phys. Lett B206 (1988) 539; F. A. Berends, W.L. Van Neerven, and G. J. H. Burgers, Nucl. Phys. B297 (1988) 429.

[11] P.H. Eberhard and Ph. Rosselet, in: A Method to Find M_Z With Little Dependence on Radiative Corrections, IPNL 89-4, Universite de Lausanne (1989), unpublished.

[12] KORALZ, S. Jadach et al. in: Workshop on Z Physics at LEP, Proc. ed. G. Altarelli, R. Kleiss and C. Verzegnassi, CERN Report 89-08, Vol. III, p. 69, to be published in Comp. Phys. Comm.

[13] Z. Was, Acta Physica Polonica B18 (1987) 1099.

FIGURE CAPTIONS

Fig. 1 A representation of the six steps constituting the reaction

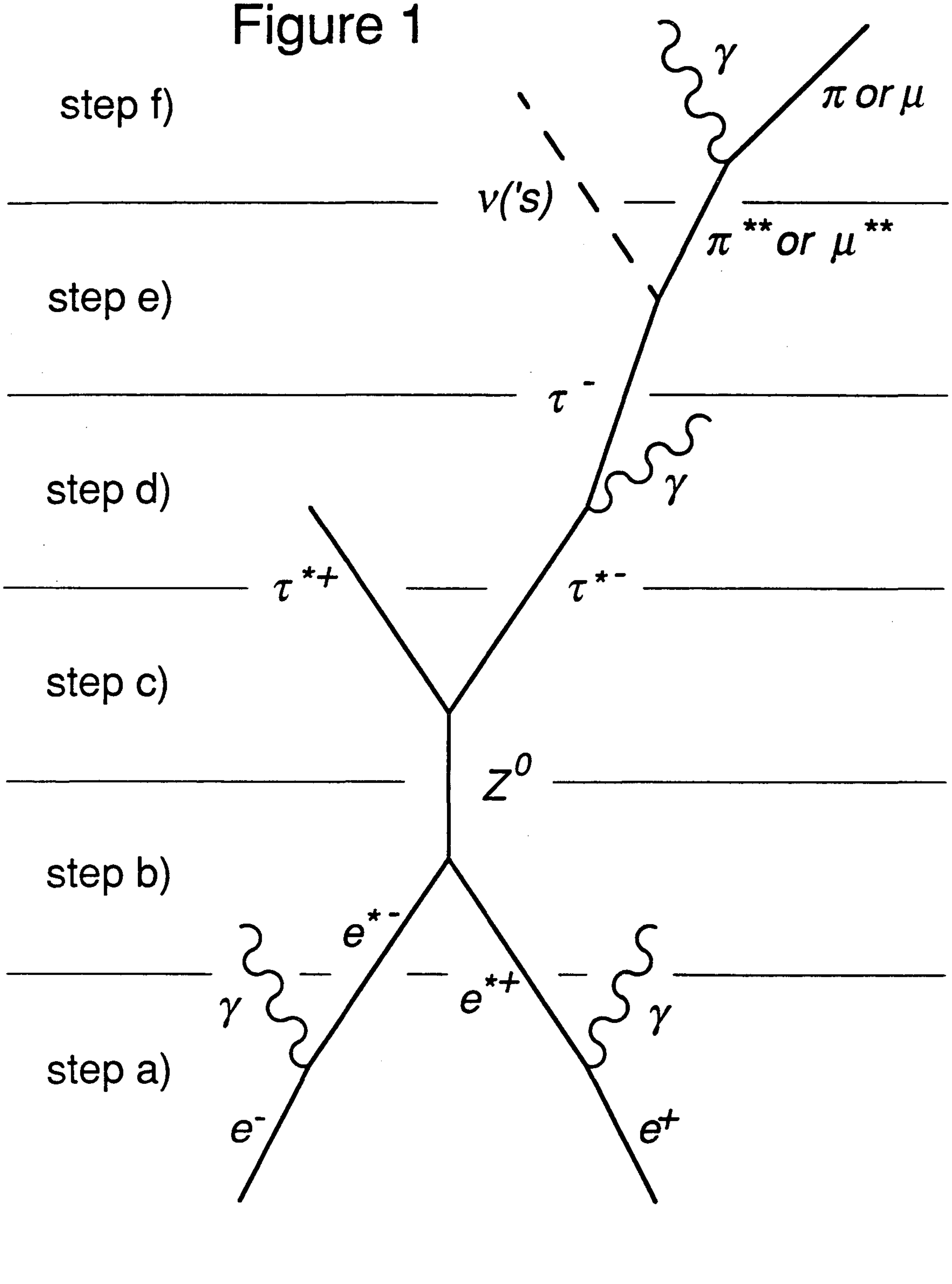
$$e^+e^- \rightarrow Z^0(\gamma) \rightarrow \tau^+\tau^-(\gamma) \hookrightarrow \begin{matrix} \mu\nu\nu(\gamma) \\ \pi\nu(\gamma) \end{matrix}$$

See Sec. 1.

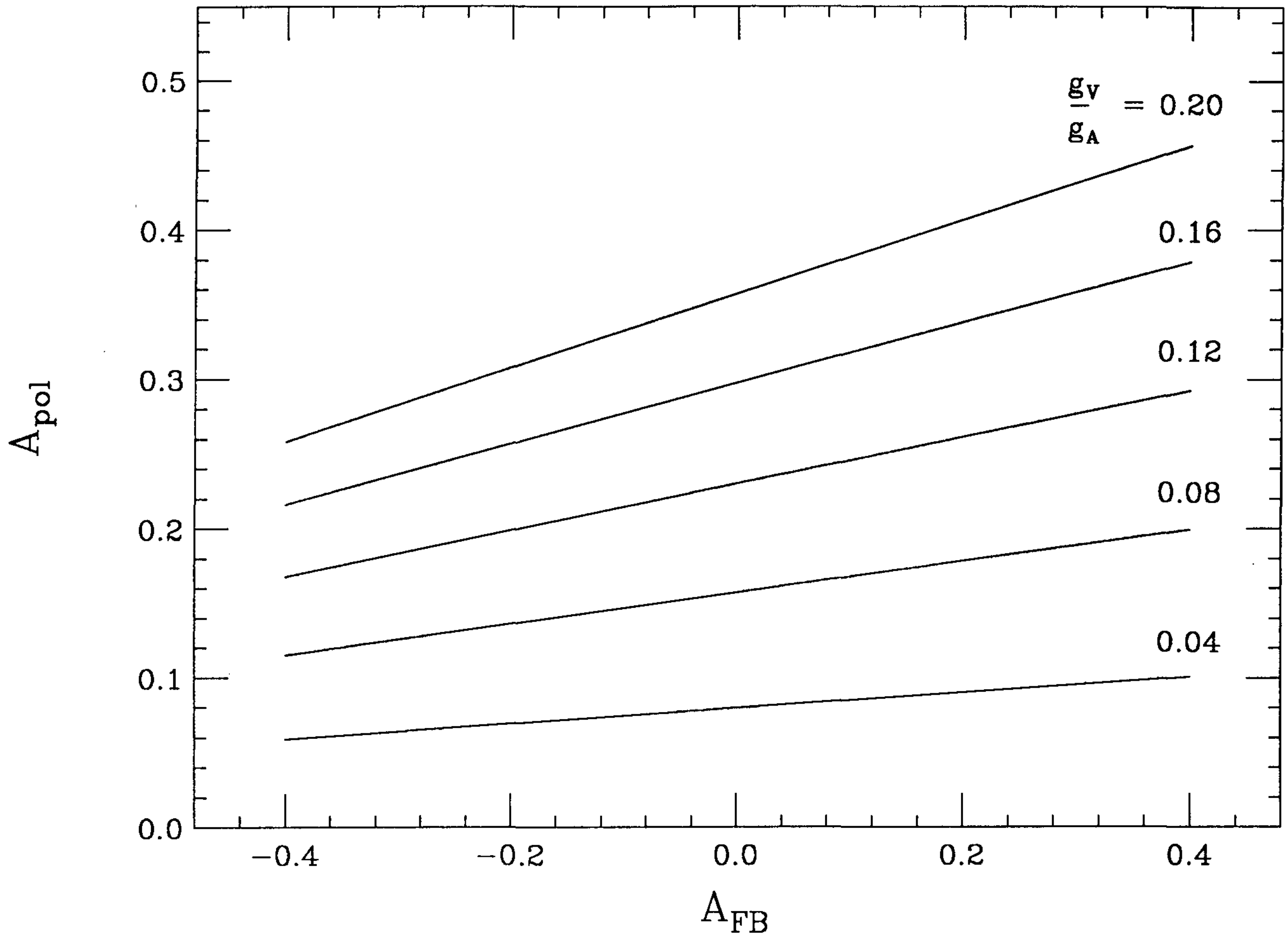
Fig. 2 The relationship between A_{Pol} and A_{FB} for several values of $\frac{g_V}{g_A}$ according to Eq. (17) that is, with τ-μ universality assumed.

Fig. 3 $k(s)\cdot g^2$ as a function of $\frac{\sqrt{s}}{M_z}$ as predicted by the Born approximation (dashed line) and taking radiative corrections into account (solid line) as predicted by Eq. (28). The dotted range around the solid curve represents the variation due to a Z^0 width which is $\pm$ 10 standard deviations away from the measured value of $\Gamma = 2.536 \pm 0.029$ GeV/c^2.

Figure 1

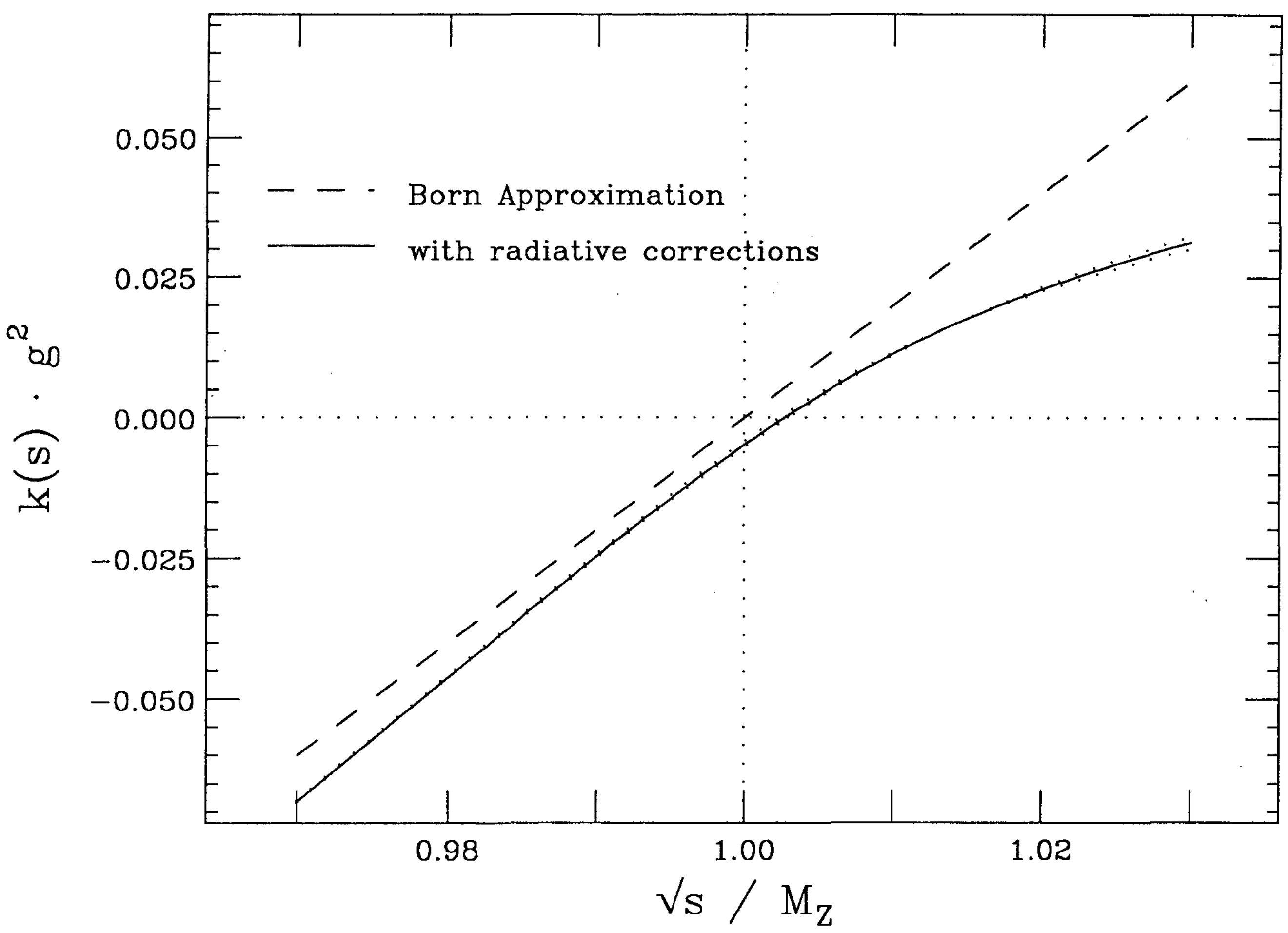


step f)
step e)
step d)
step c)
step b)
step a)
γ
π or μ
ν('s)
π^{**}or μ^{**}
τ^-
γ
τ^{*+}
τ^{*-}
Z^0
e^{*-}
e^{*+}
γ
γ
e^-
e^+



0.5
0.4
0.3
0.2
0.1
0.0
A_{pol}
$\frac{g_V}{g_A}$ = 0.20
0.16
0.12
0.08
0.04
−0.4
−0.2
0.0
0.2
0.4
A_{FB}

Figure 2



0.050
0.025
0.000
−0.025
−0.050
Born Approximation
with radiative corrections
$k(s) \cdot g^2$
0.98
1.00
1.02
$\sqrt{s}$ / M_Z

Figure 3

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