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Proximity-driven Social Interactions and Their Impact on the Throughput Scaling of Wireless Networks

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***Abstract*—We present an analytical framework to investigate the interplay between a communication graph and an overlay of social relationships. We focus on geographical distance as the key element that interrelates the concept of routing in a communication network with the dynamics of interpersonal relations on the corresponding social graph. We identify classes of social relationships that let the ensuing system scale—*i.e.*, accommodate a large number of users given only finite amount of resources. We establish that geographically concentrated communication patterns are indispensable to network scalability. We further examine the impact of such proximity-driven interaction patterns on the throughput scaling of wireless networks, and show that, when social communications are geographically localized, the maximum per-node throughput scales approximately as $1/\log n$, which is significantly better than the well-known bound of $1/\sqrt{n \log n}$ for the uniform communication model.**

I. Introduction

Computer networks can be conceptually organized into several distinct layers that, though logically separate, are operationally interconnected. Within this framework, such constructs are often referred to as *composite (or complex) networks* in which a *communication network* represents the physical communication infrastructure, computing servers, and clients, while a *social network* defines the communication patterns among end users collaborating with one another through applications running on end systems (host computers). An *information network* captures the distribution and relationships among information objects throughout the network.

The reciprocal interactions among the communication, social, and information layers of complex networks have an undeniable impact on performance. However, due to the complexity of characterizing complex networks, prior work has focused on the performance of networks from unidimensional viewpoints of communication, social, or information. Examples of studies on communication networks neglecting the latent social relationships are [1]–[5]. In contrast, several interaction patterns and social paradigms [6]–[8] are independently studied while the restrictions imposed by realistic underlying communication networks are neglected. Unfortunately, neglecting the interaction among the layers of a complex network renders overly simplified models with implications that are limited in scope and cannot be extended to more sophisticated real-world scenarios.

We present an analytical framework to investigate the interplay among the communication and social layers of complex networks. Particularly, we study how the spatial diversity of social connections affects the scalability of a wireless network. Section II provides a formal description of our model. We focus on *proximity-driven social models* according to which social relations are established with respect to the geographical vicinity of nodes. In this model, nodes are inclined to communicate with parties that are geographically closer to them more often than with the ones at farther distances. This is characterized with a clustering parameter α, such that nodes show higher tendency to communicate within their proximal neighborhood for larger values of α. The relevance of this model to real-world social behaviors of people has been widely studied and verified in both online and offline domains [9]–[12].

Prior work on the scaling limits of wireless networks has relied on coarse approximations of the way in which information is routed in a network. Examples of these approximations include routing along the straight line (see for example [1], [13]) and grid-based routing (see [3], [14], [15] for instance). Although these approaches make the models easier to evaluate, they can hardly represent the complexities of the routing process in real networks. Secondly, while the resulting models can safely be applied to networks of sufficiently high density, they cannot directly be used in analysis of sparser networks. To address these shortcomings, Sections III and IV discuss a framework for characterizing routing dynamics in random networks more accurately. We focus in particular on *the geographic greedy forwarding* (see for example GPSR [16] and GOAFR$^+$ [17]), because it can be used to represent the routing process in both dense and extended network models. Furthermore, greedy forwarding can scale with the network size [18], which is important to address the scaling properties of random networks.

Section V examines scalability conditions, and Section VI provides upper-bounds on the throughput capacity of wireless networks under various classes of social communication and the greedy forwarding scheme we introduced. The two extremes of this analysis are the traditional uniform communication model ($\alpha = 0$) in which nodes choose their destinations uniformly and at random; and the geographically concentrated interaction patterns ($\alpha > 3$). For the former case, as the number of nodes (n) goes to infinity, we retrieve the well-known upper-bound of $\mathcal{O}(1/\sqrt{n \log n})$ (see [1], [3], [13]), while for the latter case, we show that a throughput order of

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no better than $\mathcal{O}(1/\log n)$ can be expected.

Our framework identifies two primary obstacles against the scaling of throughput in wireless multi-hop networks; namely *bandwidth depletion*, and *inordinate relaying load*. Bandwidth depletion is related to the communication layer of the network, and results from the node transmission range having to be sufficiently small to minimize destructive interference [19] with other receivers' signal, and large enough to prevent partitioning of network into isolated clusters. As a result, a *critical transmission radius* [20], [21], denoted by $r(n)$, has to be used to minimize interference while maintaining network connectivity at the same time. In a dense network, $r(n)$ must shrink with the number of nodes (see [20]); conversely, it has to be expanded (see [21]) with n in the extended model. In either case, the value of $r(n)$ makes the available bandwidth per node gradually diminish to zero.

The problem of inordinate relaying load has its roots in the social aspect of internode interactions—deals with the unlimited accumulation of relaying traffic in the network as more nodes join. We demonstrate that this problem can be avoided if nodes have an inherent tendency to favor social contacts that are geographically closer to them. This model of communication concentrates social interactions within a logical cluster of certain radius around nodes. For a specific range of clustering exponent, *i.e.*, $\alpha > 3$, we show that the radius of such a cluster becomes finite. In that case, though the problem of immoderate load is remedied, the best throughput scaling becomes of order $\mathcal{O}(1/\log n)$, because of the bandwidth depletion problem.

Section VII provides an overview of related work, and Section VIII concludes the paper and discusses some avenues for future research. In particular, while hosts cannot be brought physically closer to one another in the network, the content that they share can be. Our results indicate that caching of information near the consumers of such information can be used to emulate localized communication patterns that render better scaling of the network.

The key contributions of this work are the following.

- Presenting an analytical framework to capture the interplay between a communication network and an overlay of social relationships.
- Providing a new perspective on the characterization of wireless networks through decoupling the function of social interactions from the natural limitations of the physical aspect of communication.
- Exploring the impact of geographical diversity of social interactions on the scalability and throughput enhancement of wireless multi-hop networks.

II. Model and Assumptions

Ordinarily, the term *scalable* refers to the systems capable of handling a large number of users without incurring significant loss in performance. Here, we need to present a more objective definition of scalability. We introduce a cost measure that reflects the average amount of resources needed to accommodate a user. In the context of communication networks, a reasonable cost measure is the average number of times a packet needs to be transmitted until delivery to its intended destination. There are three key factors that influence this measure, namely: *topological factors*, such as the physical connectivity among nodes and the number of hops separating a source-destination pair on the communication graph; *social factors*, such as the governing patterns according to which nodes interact with one another, and how a source node chooses its destinations; and *unrestrained factors* related to physical-layer effects, such as interference, fading, noise, and congestion, which might result in loss of packets and incur re-transmissions.

In this paper, we focus on topological and social factors only, which can be modeled under a minimal and general set of assumptions discussed below.

A. The Connectivity Graph Model

We assume a Random Geometric Graph (RGG) as our model for the network topology. Thanks to their simplicity and generality, RGG's have become a de facto standard in the research community to represent the underlying topology of wireless networks. A definition of RGG is provided in the following for future reference.

Definition 1. *$\mathcal{G}(\mathcal{X}; r)$ represents a random geometric graph in which $\mathcal{X}$ is a point process on $\mathbb{R}^k$ that describes the distribution of nodes. Further, an undirected edge connects every pair u and v iff $\|\mathcal{X}_u - \mathcal{X}_v\| \leq r$ for a given $r \in \mathbb{R}^+$.*

Here, $\|.\|$ is a norm of choice on $\mathbb{R}^k$. For simplicity, we use the Euclidean norm in this paper. We consider a Poisson point process (P.P.P.), $\mathcal{X}$, to describe the geographical distribution of nodes' in the network. The physical connectivity between nodes is defined according to a Boolean model that assumes nodes as being connected if and only if they are within a distance r from one another.

For simplicity, we assume that nodes are distributed on the surface of a sphere. This assumption has been commonly used to alleviate the network edge effect (see [1], [13] for example). It has been shown [1] that similar results can be derived when nodes are distributed on the plane at the expense of more tedious and unwieldy computations.

Two distinct models are usually considered when studying asymptotic behaviors of RGG's: the *extended model*, in which the node density is fixed, and the network dimensions go to infinity; and the *dense model*, in which the network dimensions are fixed, and the node density goes to infinity. In this paper, we construct a general framework that can be used to analyze the scaling properties of these two network models. We shall use *random network* in arguments that can equally be applied to both extended and dense models.

B. The Social Model

The social model describes the quality and frequency of inter-node communications in the network, *i.e.*, how sources choose their destinations. In this paper, we consider a proximity-driven social model defined as follows.

Definition 2. *A communication network follows a proximity-driven social model if the probability of every node u and v communicating with each other is inversely proportional to $\|\mathcal{X}_u - \mathcal{X}_v\|^\alpha$ for some arbitrary but fixed exponent $\alpha \in \mathbb{R}_0^+$.*

Definition 2 implies a social model that is power-law distributed with distance. For a specific realization of the network, the probability of node u choosing v as destination, $P_u(v)$, is obtained as follows according to this definition.

$$P_u(v) = \frac{d(u,v)^{-\alpha}}{\sum_{w \neq u} d(u,w)^{-\alpha}}\,, \tag{1}$$

where $d(u,v) = \|\mathcal{X}_u - \mathcal{X}_v\|$. The denominator of (1) is in fact a normalizing constant (for that specific realization).

According to Equation (1), the closer two nodes are geographically, the more likely they are to communicate; except for the case of $\alpha = 0$ that results in a uniform communication model in which a source node is equally likely to choose any other node as its destination, irrespective of their distance. At the other extreme, when $\alpha \to \infty$, every node communicates with its closest neighbor almost surely. In fact, different ranges of α correspond to distinct classes of social relationships with identical scaling behaviors. Identifying such social classes is a primary objective of this paper.

Let us denote the number of nodes in the network by n. We want to obtain a probability distribution for the event of having a social contact at any given physical distance. For generality purposes, we choose to calibrate the distance measure by scaling it by the nodes' critical transmission radius $r(n)$. This allows our social model to be equally applicable to both cases of dense and extended topologies. Particularly, for the case of a dense network in which the geometric diameter of the network is fixed and the critical transmission range approaches zero, this adjustment allows for having social contacts that are spaced infinitely far away.

Let $X_{\min} \leq x \leq X_{\max}(n)$ be such a range-adjusted distance measure, where $X_{\min}$ and $X_{\max}(n)$ respectively denote the minimum and maximum range-adjusted distances between two social contacts. Without loss of generality, we assume that nodes' social contacts are at least one transmission distance away from them; hence, $X_{\min} \triangleq 1$. Also, $X_{\max}(n) \triangleq d/r(n)$, where d is the geometric diameter of the network (the longest possible physical distance between any pair of nodes).

Define $F_\alpha(x) = \Pr\{\text{having a social contact at distance } \leq x\, r(n)\}$. According to Definition 2, assume that such density function is a power-law on distance. Also, by the Poisson approximation, we know that the number of potential social contacts at any distance x is linearly proportional to x. Therefore, we define the corresponding p.d.f. as follows.

$$f_\alpha(x) = C_\alpha\, x\, r(n) \cdot \big(x\, r(n)\big)^{-\alpha} = C_\alpha \big(x\, r(n)\big)^{1-\alpha}\,,$$

in which C_α is a constant independent of x. To obtain the value of C_α note that

$$1 = \int_1^{X_{\max}(n)} f_\alpha(x)\,\mathrm{d}x = \begin{cases} \dfrac{C_\alpha}{r(n)} \times \log x \Big|_1^{X_{\max}(n)} & \text{if } \alpha = 2\,, \\ \dfrac{C_\alpha\, r(n)^{1-\alpha}}{2-\alpha} \times x^{2-\alpha} \Big|_1^{X_{\max}(n)} & \text{if } \alpha \neq 2\,. \end{cases}$$

Solving for C_α yields

$$C_\alpha = \begin{cases} \dfrac{r(n)}{\log X_{\max}(n)} & \text{if } \alpha = 2\,, \\ \dfrac{(2-\alpha)\, r(n)^{\alpha-1}}{X_{\max}^{2-\alpha}(n) - 1} & \text{otherwise}\,. \end{cases} \tag{2}$$

The p.d.f. $f_\alpha(x)$ provides a description of our proximity-driven social model. We use this model to define our cost measure as discussed in the following subsection.

C. Expected Social Path Length

Recall that the cost that every packet imposes on the network is measured by the expected number of times it has to be transmitted in the network for its delivery. Knowing the average number of hops each packet travels considering the underlying social relations, we define the expected social path length as follows.

Definition 3. *The expected social path length (*ESPL*) is the expected number of hops, $\bar{h}(x)$, separating a source-destination pair on a proximity-driven social network identified by $f_\alpha(x)$ and is computed as*

$$\mathbb{E}[\mathscr{L}_\alpha] = \int_1^{X_{\max}(n)} f_\alpha(x)\, \bar{h}(x)\,\mathrm{d}x\,. \tag{3}$$

Definition 3 exploits the notion of geographical distance to combine the routing on the connectivity graph of the network with the concept of social relations. In view of that, ESPL is, in fact, a cost measure reflecting the amount of resources that every node consumes on average, taking into account both topological and social considerations.

Evidently, ESPL is a non-decreasing function of the network size; nevertheless, the network cannot sustain a continuously increasing load forever as more nodes join in. Hence, we present the following definition for the class of social relations that allow the underlying communication network scale appropriately without significant loss in performance.

Definition 4. *A communication network with proximity-driven social relations exhibits scalability if $\mathbb{E}[\mathscr{L}_\alpha] < \infty$ when $n \to \infty$.*

Based on Definition 4, a necessary condition for scalability is that the network performs, on average, a finite number of transmissions per packet, no matter how large the network is. In the sequel, we address the impact of different values of α on the growth of ESPL as the network grows larger. To that end, the next section introduces a methodology to compute the average number of hops, $\bar{h}(x)$, that a routing algorithm takes over any given distance x.

III. Progressive Walks

It follows from Definition 3 that an accurate evaluation of ESPL depends at least in part on the performance of the routing algorithm used in the network. Conventionally, it is preferred to characterize the behavior of the system under idealistic conditions to obtain a reasonable upper-bound on the achievable performance limits. As such, the underlying routing algorithm is assumed to be optimal for modeling purposes. Although finding optimal paths on deterministic graphs is algorithmically straightforward, in the context of random graphs, it turns out to be a highly challenging problem. Most of this complexity stems from the random nature of the underlying topology. In essence, an optimal routing algorithm requires global and exact information about the network structure and state, which is virtually non-existent when speaking of RGG's.

Several approximations of optimal routing have been studied in the literature, such as routing along the straight line [1], [13], and grid-based routing [3], [14]. These approximations are often accurate enough when studying a random dense network, but are less useful for the analysis of extended networks with finite density. Moreover, the internal mechanisms of such routing schemes are remarkably different from how distributed routing algorithms work in real networks.

Despite the theoretical difficulties in the analysis of optimal routing in random configurations, more tractable solutions with near-optimal performance can still be conceived. One such routing strategy is known as *greedy (geographical) forwarding* in which every relay attempts to push the packet some distance closer to the destination. With this policy, even though the global structure of the routes need not be necessarily optimized, a sub-optimal path can still be found by making locally optimized decisions when choosing subsequent relays along the path.

Various criteria for optimizing local decisions have been studied in the literature, and this is, essentially, what makes different variations of geographical forwarding. We abstract away such functional details by introducing the notion of *progressive walk* that captures the essence of greedy forwarding.

Definition 5. *We say a walk $\langle s, \dots, t\rangle$ on $\mathcal{G}(\mathcal{X}; r)$ is a progressive walk from s to t and denote it with $s \rightsquigarrow t$ iff $\|\mathcal{X}_u - \mathcal{X}_t\| \geq \|\mathcal{X}_v - \mathcal{X}_t\|$ for all ordered pairs (u, v) on $s \rightsquigarrow t$.*

For a given source-destination pair, a greedy forwarding algorithm attempts to output a progressive walk on the communication graph. The expected number of hops on a greedy route is equivalent to the expected length of the corresponding walk. To succeed, a greedy forwarding algorithm requires that a physical path does exist for the intended source-destination pair; however, the algorithm may not possibly succeed even though a path does exist.

The existence of a walk is trivial when the expected length of the walk is a parameter of interest. However, the ability of a greedy forwarding algorithm to always be able to find a progressive walk with high probability (w.h.p.) need not be true because of possible dead-ends, and hence we make this assumption to simplify our modeling problem. Later, however, we will relax this assumption by slightly modifying the routing algorithm to circumvent dead-ends, should they be encountered. Let us first have a closer look at the core mechanism of the greedy routing algorithm, *i.e.*, progressive forwarding, through the following definition. Here, $\mathcal{B}(\mathcal{X}_c; r)$ denotes the ball of radius r centered at $\mathcal{X}_c$.

Definition 6. *The hand-off region of a relay u for a final destination t is $\mathcal{H}_t(u) \triangleq \mathcal{B}(\mathcal{X}_u; r) \cap \mathcal{B}(\mathcal{X}_t; x)$, where $x = \|\mathcal{X}_u - \mathcal{X}_t\|$. Further, we say that node v is a potential next-hop for $u \rightsquigarrow t$ iff $\mathcal{X}_v \in \mathcal{H}_t(u)$.*

According to Definition 5, the hand-off region defines the subset of nodes that can be considered by a relay as potential next hops to further the packet towards its destination. The convergence of the progressive walk relies upon having at least one potential next-hop in each and every hand-off region along the walk. If the packet comes at a relay with a void hand-off region, *i.e.*, a dead-end, the progressive walk stalls as no further progress towards destination is made. For the time being, we *assume* that the greedy algorithm converges w.h.p.

A. Greedy Forwarding with Almost Sure Convergence

The key element of a progressive walk according to Definition 5 is to progressively reduce the remaining distance to the destination along the walk. In fact, at every stage of the walk, the packet is pushed some distance closer to the destination on the Euclidean plane. In view of this, a progressive walk can be perceived as a drifted random walk on the communication graph. The distance traveled by the packet at every hop is a random variable determined by the process specifying the topology of the communication graph as well as the optimization criteria of the greedy routing algorithm. Exploiting results from the theory of martingales, Theorem 1 provides a useful model that describes the relationship between the physical distance and the average hop-count on a RGG, under certain conditions when a greedy forwarding algorithm is considered.

Assume that node s sends a packet to t through multiple intermediate hops employing a geographical greedy forwarding algorithm. Let ξ_δ be a random variable denoting the progress towards destination if a transmission at distance δ from destination takes place. The following theorem provides bounds on the expected number of hops under a greedy forwarding algorithm given an initial physical distance x.

Theorem 1. *Consider a source s and a destination t at distance $x = \|\mathcal{X}_s - \mathcal{X}_t\| > r$ in a RGG $\mathcal{G}(\mathcal{X}; r)$. Provided that ξ_δ's are independent, and the routing algorithm converges w.h.p.,*

$$\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta] < \frac{x}{h(x)} < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta]\,.$$

Proof. Let $S_\delta(t) = \sum_{i=1}^{t} \xi_\delta(i)$ be a random walk where $\xi_\delta(i)$ is a stochastic process with respect to i representing the progress towards destination when at distance δ from it.

In fact, S_δ is a progressive walk that assumes all relays have similar-sized hand-off regions as if they are all at distance δ from destination.

Let $T_\delta = \inf\{t : S_\delta(t) \geq x\}$ be the first time $S_\delta(t)$ hits the target distance x. Note that $0 \leq \xi_\delta(i) \leq r$ and $\mathbb{E}[\xi_\delta] > 0$; thus, $P(T_\delta < \infty) = 1$. Also, $\{t < T_\delta\} = \{S_\delta(1), \ldots, S_\delta(t) < x\}$ which is clearly independent of $S_\delta(t')$ for $t' > T_\delta$. Therefore, T_δ is a stopping time with respect to $S_\delta(t)$.

Fix a δ such that $r < \delta < x$, and consider a relay at distance δ from destination. The measure of hand-off region is a monotonically decreasing function of δ (see Fig. 1); therefore,

$$\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta] < \mathbb{E}[\xi_\delta] < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta] \quad \text{for all } \delta > r\,. \tag{4}$$

Now, consider the process $M_\delta(t) = S_\delta(t) - t\,\mathbb{E}[\xi_\delta]$. Note that,

$$\begin{aligned}
\mathbb{E}[M_\delta(t)] &= \mathbb{E}\Big[S_\delta(t) - t\,\mathbb{E}[\xi_\delta]\Big] = \mathbb{E}\Big[\sum_{i=1}^{t} \xi_\delta(i) - t\,\mathbb{E}[\xi_\delta]\Big] \\
&= \mathbb{E}\Big[\sum_{i=1}^{t} \Big(\xi_\delta(i) - \mathbb{E}[\xi_\delta]\Big)\Big] = \sum_{i=1}^{t} \mathbb{E}\Big[\xi_\delta(i) - \mathbb{E}[\xi_\delta]\Big] \\
&= \sum_{i=1}^{t} \Big(\mathbb{E}[\xi_\delta] - \mathbb{E}[\xi_\delta]\Big) = 0 < \infty\,.
\end{aligned}$$

Also, $\mathbb{E}[M_\delta(t+1) - M_\delta(t)] = \mathbb{E}[M_\delta(t+1)] - \mathbb{E}[M_\delta(t)] = 0$. Therefore, $M_\delta(t)$ is a martingale with respect to ξ_δ. According to the optional stopping theorem, $M_\delta(T_\delta \wedge t)$ is also a martingale with respect to ξ_δ, where $(T_\delta \wedge t)$ is the minimum of T_δ and t. Hence,

$$\begin{aligned}
\mathbb{E}[M_\delta(T_\delta)] &= \mathbb{E}\Big[S_\delta(T_\delta) - T_\delta\,\mathbb{E}[\xi_\delta]\Big] \\
&= \mathbb{E}[S_\delta(T_\delta)] - \mathbb{E}[T_\delta] \cdot \mathbb{E}[\xi_\delta] = 0\,,
\end{aligned}$$

which yields

$$\mathbb{E}[S_\delta(T_\delta)] = \mathbb{E}[T_\delta] \cdot \mathbb{E}[\xi_\delta]\,. \tag{5}$$

Now, consider the process $S(t) = \sum_{i=1}^{t} \xi_y(t)$, where $y = \max\big(x - S(t-1), r\big)$ and $S(0) = 0$. Let $T = \min\{t : S(t) \geq x\}$ be a stopping time. From Equation (4), for all $y > r$ we have that

$$\begin{aligned}
&\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta] \cdot \mathbb{E}[T] < \mathbb{E}[\xi_y] \cdot \mathbb{E}[T] < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta] \cdot \mathbb{E}[T] \quad \Rightarrow \\
&\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta] \cdot \mathbb{E}[T] < \quad \mathbb{E}[S(T)] \quad < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta] \cdot \mathbb{E}[T] \quad \Rightarrow \\
&\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta)] < \quad \frac{\mathbb{E}[S(T)]}{\mathbb{E}[T]} \quad < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta]\,.
\end{aligned}$$

Having $\mathbb{E}[S(T)] = x$ and noting that $\mathbb{E}[T] = \bar{h}(x)$ is in fact the average number of hops over distance x, we obtain that

$$\lim_{\delta \to r^+} \mathbb{E}[\xi_\delta] < \frac{x}{\bar{h}(x)} < \lim_{\delta \to \infty} \mathbb{E}[\xi_\delta]\,,$$

which completes the proof. □

As mentioned earlier, a problem that limits the accuracy of the given bounds in Theorem 1 is the assumption that the routing algorithm converges w.h.p. This assumption might be true when studying dense networks, but it is not applicable to networks of finite node density in which a dead-end might be encountered. In the following, we extend the case studied in Theorem 1 to account for such possibilities as well.

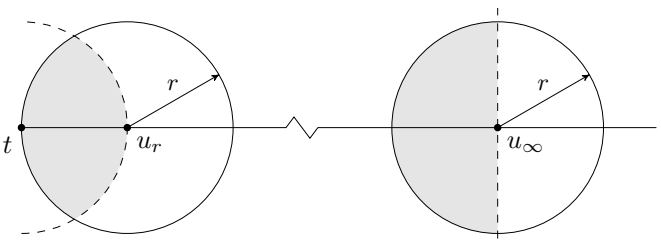

Fig. 1: A 2-D illustration of how the hand-off region, *i.e.*, shaded area, shrinks as the remaining distance to the destination is reduced. Here t is the destination, and u_r and u_∞ represent relays at distances r and ∞ from destination, respectively.

B. Greedy Forwarding with Backtracking

We analyze a modified greedy forwarding algorithm which works as follows. At every stage t of the walk, the packet either makes a progress of $+\xi(t)$ towards destination with probability p, or backtracks for a random step size of $-\xi(t)$ with probability $1-p$ in the event of encountering a dead-end. Considering the underlying P.P.P., the probability p is then

$$p = 1 - \exp\big(-\rho\,|\mathcal{H}(\cdot)|\big)\,,$$

where ρ is the intensity of the P.P.P., and $|\mathcal{H}(\cdot)|$ denotes the Lebesgue measure of the hand-off region. The corresponding random walk, hence, is formalized as follows.

$$S(t) = \begin{cases} S(t-1) + \xi(t-1) & \text{with probability } p\,, \\ S(t-1) - \xi(t-1) & \text{with probability } 1-p\,, \end{cases}$$

and $S(0) = 0$. Therefore, $\mathbb{E}[S(t)] = S(t-1) + (2p-1)\,\xi(t-1)$. Consider the process $M(t) = S(t) - t\,(2p-1)\,\mathbb{E}\xi$ for $t > 0$. We first verify that $M(t)$ is a martingale.

$$\begin{aligned}
&\mathbb{E}[M(t+1)\,|\,M(t)] \\
&\quad = \mathbb{E}\big[S(t+1) - (t+1)(2p-1)\mathbb{E}\xi \,\big|\, S(t) - t(2p-1)\mathbb{E}\xi\big] \\
&\quad = \mathbb{E}\big[S(t) + (2p-1)\xi(t) - (t+1)(2p-1)\mathbb{E}\xi \,\big|\, \cdot\,\big] \\
&\quad = S(t) - t\,(2p-1)\,\mathbb{E}\xi = M(t)\,.
\end{aligned}$$

Define a stopping time $T = \inf\{t : S(t) \geq D\}$. By the optional stopping theorem, $\mathbb{E}[M(T)] = \mathbb{E}[M(0)] = 0$. Thus,

$$\begin{aligned}
\mathbb{E}[S(T)] = (2p-1)\,\mathbb{E}[T]\,\mathbb{E}[\xi] \quad &\Rightarrow \\
x = (2p-1)\,\mathbb{E}[T]\,\mathbb{E}[\xi] \quad &\Rightarrow \\
\mathbb{E}[T] = \frac{x}{(2p-1)\,\mathbb{E}[\xi]}\,. \quad &
\end{aligned} \tag{6}$$

The natural constraint of $\mathbb{E}[T] > 0$ requires that $p > 1/2$ in order for Equation (6) to make sense. As $p \to \frac{1}{2}^+$, $\mathbb{E}[T]$ diverges, which is an intuitive behavior. Also, when $p = 1$, (6) simplifies to (5) which is also expected.

The bounds given in Theorem 1 are expressed in terms of the expected progress the greedy forwarding algorithm makes per hop when at a limiting distance of ∞ or r from destination.

As seen from Fig. 1, in 2-D space, the hand-off region shrinks from a half-disk at the former distance to a shape resembling a biconvex lens at the latter. Aside from the size of the hand-off region, the expected progress per hop does also depend on the forwarding policy of the greedy algorithm, *i.e.*, the criteria by which the next relay is chosen from within the set of potential next-hops. In the next section, we examine the tightness of the suggested bounds in Theorem 1.

IV. Expected Progress Per Hop

Several next-hop selection policies have been proposed in the context of greedy routing algorithms. A widely used policy is to always choose the next-hop with the least remaining distance (LRD) to the destination. Even though this strategy does not guarantee that the packet would necessarily travel the fewest number of hops, it ensures the maximum possible progress towards destination at every hop.

An issue with the LRD policy is that it violates the required condition on the independence of per-hop progresses. To clarify, observe that the hand-off regions of subsequent hops are not disjoint. For instance, in Fig. 2, the hand-off region of node u overlaps that of node v on $v \rightsquigarrow t$ in the crosshatched region. Therefore, if node v is chosen as next-hop for $u \rightsquigarrow t$ under LRD, then v cannot logically have a potential next-hop in the crosshatched region. This implies that when LRD is used as the forwarding policy, the information from the past history of the walk can, in fact, affect the future decisions.

In order to use Theorem 1, we must make sure that the adopted forwarding policy does not violate the independence of succeeding progresses as described above. One such compliant policy is random greedy forwarding (RGF) by which a current relay forwards the packet to a randomly chosen next hop. Such a next-hop could clearly be located anywhere within its hand-off region, and its election as the next relay does not impose any restriction on the location of subsequent hops. As such, RGF satisfies the required conditions of Theorem 1.

In the following, we quantify the expected progress per hop under RGF policy. It is worth mentioning that although RGF is not an optimal forwarding strategy, it can still serve as a lower-bound for more aggressive policies such as LRD.

A. A Lower-bound on Per-Hop Progress

Consider the case when the source and destination are located at a distance $\delta + \epsilon$ for a small positive $\epsilon \to 0$. In that case, the hand-off region for the source can be approximated by a symmetrical biconvex lens, as illustrated in the left-hand-side of Fig. 1. For the moment, assume $r = 1$ and define the boundaries of the hand-off region as follows.

$$|\omega| = \begin{cases} \sqrt{1-(1-\delta)^2} = \sqrt{2\delta - \delta^2} & \text{if } 0 \le \delta \le \frac{1}{2}\,, \\ \sqrt{1-\delta^2} & \text{if } \frac{1}{2} \le \delta \le 1\,. \end{cases}$$

Due to the symmetry of the region, the enclosed area can be calculated as

$$A(r) = 4 \cdot \int_0^{\frac{1}{2}} \int_0^{\sqrt{2\delta-\delta^2}} \mathrm{d}\omega\,\mathrm{d}\delta = 4 \cdot \int_0^{\frac{1}{2}} \sqrt{2\delta-\delta^2}\,\mathrm{d}\delta\,.$$

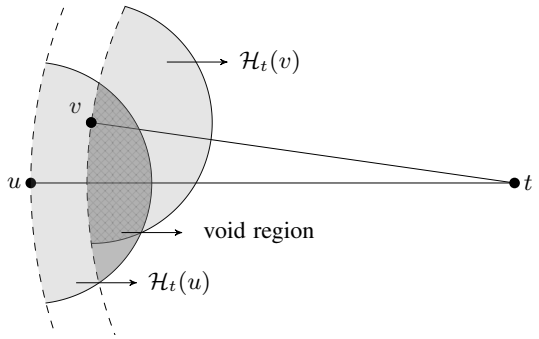

Fig. 2: Succeeding hand-off regions may overlap. Here, the darker shaded region is empty, and part of it, *i.e.*, the crosshatched area, overlaps the hand-off region of v on $v \rightsquigarrow t$.

Since the next-hop can be located anywhere within the hand-off region with equal probability, $\mathbb{E}[\xi(r)]$ is the expected distance from the relay over the region which can be calculated as follows.

$$\mathbb{E}[\xi(r)] = \frac{2}{A(r)} \left(\int_0^{\frac{1}{2}} \int_0^{\sqrt{2\delta-\delta^2}} \sqrt{\delta^2+\omega^2}\,\mathrm{d}\omega\,\mathrm{d}\delta + \int_{\frac{1}{2}}^{1} \int_0^{\sqrt{1-\delta^2}} \sqrt{\delta^2+\omega^2}\,\mathrm{d}\omega\,\mathrm{d}\delta \right).$$

Using numerical methods and noting that $\mathbb{E}[\xi(r)]$ is linear in r, for a general case, we obtain that

$$\lim_{\delta \to r^+} \mathbb{E}[\xi(\delta)] \approx 0.643\, r\,. \tag{7}$$

B. An Upper-bound on Per-Hop Progress

Consider the right-hand side of Fig. 1. The hand-off region is defined as follows.

$$|\omega| \le \sqrt{1-\delta^2} \quad \text{for } 0 \le \delta \le 1\,.$$

The area of the hand-off region is clearly $A(\infty) = \pi/2$. Hence,

$$\mathbb{E}[\xi(\infty)] = \frac{2}{A(\infty)} \left(\int_0^{1} \int_0^{\sqrt{1-\delta^2}} \sqrt{\delta^2+\omega^2}\,\mathrm{d}\omega\,\mathrm{d}\delta \right) = \frac{2}{3}\,.$$

By analogy to the previous case, we obtain that

$$\lim_{\delta \to \infty} \mathbb{E}[\xi(\delta)] \approx 0.667\, r\,. \tag{8}$$

From Theorem 1 and Equations (7) and (8), we obtain that, under a routing with RGF policy, the average hop count over any given distance $x \gg r$ is bounded as

$$1.50 \left(\frac{x}{r}\right) < \bar{h}(x) < 1.56 \left(\frac{x}{r}\right). \tag{9}$$

Note that x/r is the theoretical lower-bound on the number of hops under any routing scheme, which, of course, can almost never be attained on a RGG.

V. Scalability Analysis

We now examine the scalability conditions of random networks under proximity-driven social models, taking advantage of the mathematical models developed in previous sections. The following theorem identifies a large family of social relationships that allow a communication network to scale.

Theorem 2. *Under a proximity-driven social model identified by the power-law p.d.f. $f_\alpha(\cdot)$, a random network exhibits scalability if $\alpha > 3$.*

Proof. Recall from Definition 4 that the required condition on scalability is to maintain $\mathbb{E}[\mathcal{L}_\alpha] < \infty$ when the network size grows infinitely large. From Equation (9) and for a range-adjusted distance measure x we established that $C'_{\min}\, x < \bar{h}(x) < C'_{\max}\, x$ for constants $C'_{\min}, C'_{\max} > 0$ independent of x. Plugging $C'_{\max}$ into Equation (3) and expanding yields

$$\begin{aligned}\mathbb{E}[\mathscr{L}_\alpha] &< \int_1^{X_{\max}(n)} C_\alpha \left(x\, r(n)\right)^{1-\alpha} \cdot C'_{\max}\, x \,\mathrm{d}x \\ &= C'_{\max}\, C_\alpha\, r(n)^{1-\alpha} \int_1^{X_{\max}(n)} x^{\,2-\alpha}\,\mathrm{d}x \\ &= \begin{cases} C'_{\max}\, C_\alpha\, r(n)^{-2}\, \log X_{\max}(n) & \text{if } \alpha = 3\,, \\ \dfrac{C'_{\max}\, C_\alpha}{3-\alpha}\, r(n)^{1-\alpha}\Big(X_{\max}^{3-\alpha}(n) - 1\Big) & \text{otherwise}\,. \end{cases}\end{aligned}$$

Replacing C_α from Equation (2) results in the following upper-bounds on ESPL.

$$\mathbb{E}[\mathscr{L}_\alpha] < \begin{cases} C'_{\max}\, \dfrac{X_{\max}(n) - 1}{\log X_{\max}(n)} & \text{if } \alpha = 2\,, \\ C'_{\max}\, \dfrac{X_{\max}(n) \log X_{\max}(n)}{X_{\max}(n) - 1} & \text{if } \alpha = 3\,, \\ C'_{\max}\, \dfrac{\alpha - 2}{\alpha - 3} \times \dfrac{X_{\max}^{3-\alpha}(n) - 1}{X_{\max}^{2-\alpha}(n) - 1} & \text{otherwise}\,. \end{cases}$$

Likewise, replacing $C'_{\max}$ by $C'_{\min}$ results in similar lower-bounds. Note that $\lim_{n\to\infty} X_{\max}(n) = \infty$. Therefore, we can express ESPL in terms of $X_{\max}(n)$ as follows.

$$\mathbb{E}[\mathscr{L}_\alpha] = \begin{cases} \Theta\big(X_{\max}(n)\big) & \text{if } 0 \le \alpha < 2\,, \\ \Theta\left(\dfrac{X_{\max}(n)}{\log X_{\max}(n)}\right) & \text{if } \alpha = 2\,, \\ \Theta\big(X_{\max}^{3-\alpha}(n)\big) & \text{if } 2 < \alpha < 3\,, \\ \Theta\big(\log X_{\max}(n)\big) & \text{if } \alpha = 3\,, \\ \Theta(1) & \text{if } 3 < \alpha\,. \end{cases} \tag{10}$$

As seen from Equation (10), ESPL becomes a constant independent of n only when $\alpha > 3$ and the theorem follows. □

Fig. 3 provides a graphical view of the relations provided in Equation (10). The figure illustrates how ESPL grows against an increasingly growing network diameter ($X_{\max}(n)$) for various degrees of clustering (α). A linear relationship is evident for $0 \le \alpha < 2$. When $\alpha = 3$, ESPL is still growing but the growth rate is very slow with respect to the network geometric diameter. For any $\alpha > 3$, ESPL becomes constant and decreases with increasing α to a limiting value of 1 as $\alpha \to \infty$. At that point, each node only communicates to its closest neighbor (that is one hop away) almost surely.

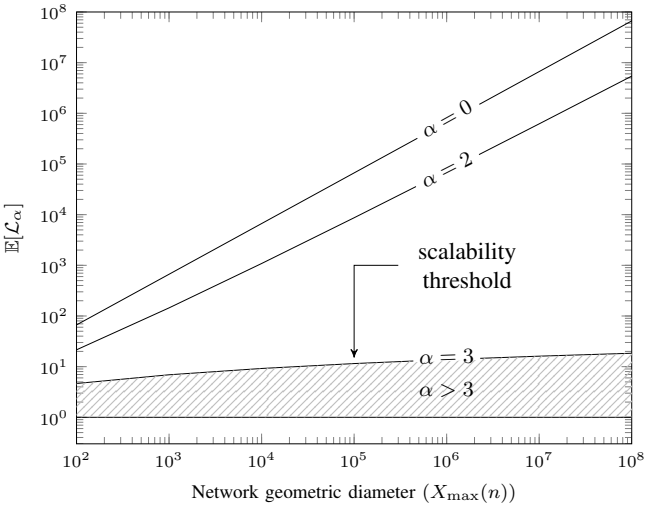

Fig. 3: The growth rate of ESPL for various clustering exponents (α) as the geometric diameter of the network increases. $\alpha > 3$ is the scalability threshold and the shaded region reflects networks with bounded average social path lengths.

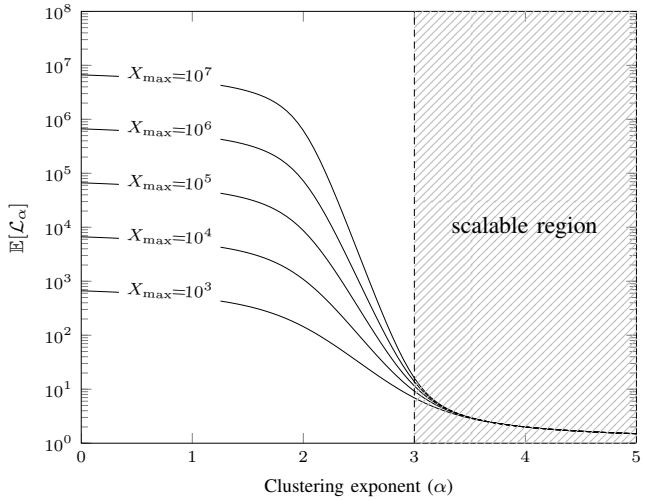

Fig. 4: The decaying rate of ESPL versus an increasing clustering exponent (α). The shaded area, *i.e.*, $\alpha > 3$, illustrates the scalable region in which a finite expected social path length is attained.

An alternative view of the relationship between ESPL and the clustering exponent α is depicted in Fig. 4. As seen, $\alpha > 3$ is the scalable region where ESPL demonstrates a stable behavior. As soon as α drops below the threshold of 3, ESPL demonstrates a rapid initial growth which gradually flattens out around $\alpha = 2$. The monotonic relationship between ESPL and $X_{\max}(n)$ is apparent for $0 \le \alpha < 2$.

VI. Upper-bounds on Throughput

Our scalability analysis can be extended to a characterization of the throughput capacity of random networks when various social interaction models are applied. The analysis presented below is in fact a generalization of the upper-bound calculation in [1]. Assume a network consisting of n nodes each capable of transmitting W bits per second. Each node chooses a social contact at random according to some proximity-driven social model with parameter α as described in Section II-B. Let $R_\alpha(n)$ be the rate at which each node transmits (including both original and relaying traffic) and each packet goes through a path consisting of $\mathbb{E}[\mathcal{L}_\alpha]$ hops on average. The network, hence, carries a total of $n\,R_\alpha(n)\,\mathbb{E}[\mathcal{L}_\alpha]$ bits per second.

Due to the shared nature of the wireless medium, some distributed medium access control protocol must be in place to avoid multiple access interference. For that purpose, a simple TDMA scheme similar to the *protocol model* in [1] can be conceived. According to this model, a transmission from node u to v is considered successful if (1) $\|\mathcal{X}_u - \mathcal{X}_v\| \leq r(n)$, and (2) $\|\mathcal{X}_v - \mathcal{X}_w\| \geq (1+\Delta)\,r(n)$ for every node w transmitting simultaneously with u over the same (sub)channel.

Lemma 1. *The maximum number of simultaneous transmissions the network can handle grows as* $\Theta\big(X^2_{\max}(n)\big)$.

Proof. The proof shares a similar logic with the proof of Lemma 5.4 in [1]. By requirement (2) of the protocol model and the triangle inequality, simultaneous receivers on the same (sub)-channel must be at least $\Delta\,r(n)$ distance away from one another (See Fig. 5). Thus, the surface of the network can be covered with disjoint disks of radius $\Delta\,r(n)/2$ centered at each receiver. The area of each such disk is $\Theta\big(r(n)^2\big)$. The total area of the network is $\Theta\big(d^2\big)$. The maximum number of simultaneous receivers is thus $\Theta\big(d^2/r(n)^2\big) \equiv \Theta\big(X^2_{\max}(n)\big)$. Every receiver corresponds to an identical transmitter and hence, the lemma follows. □

From Lemma 1, it follows that the maximum accumulative traffic in the network cannot grow faster than $\Theta\big(X^2_{\max}(n)\big)$. In symbols,

$$n\,R_\alpha(n)\,\mathbb{E}[\mathcal{L}_\alpha] \leq \Theta\big(X^2_{\max}(n)\big)\,. \tag{11}$$

We use this result to derive the theoretical maximum throughput capacity per node.

Lemma 2. *In a random network,* $X_{\max}(n) = \Theta\Big(\sqrt{\frac{n}{\log n}}\Big)$.

Proof. In Section II, we defined $X_{\max}(n) \triangleq d/r(n)$. We prove the lemma for the cases of dense and extended networks separately, referring to some known results from the random networks literature.

a) The case of dense network: The critical transmission range to ensure connectivity in dense graphs is derived by Gupta and Kumar [20] as $r(n) = \Theta\Big(\sqrt{\frac{\log n}{n}}\Big)$. Noting that $d = \Theta(1)$ for dense networks, the result follows immediately.

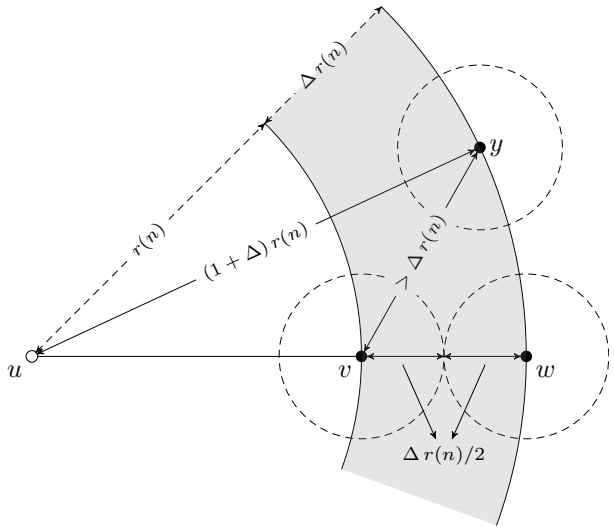

Fig. 5: To avoid multiple access interference, the protocol model demands concurrent receivers over the same (sub)channel to maintain a distance of at least $(1+\Delta)\,r(n)$ from an irrelevant active transmitter. Here, u is transmitting to v. The shaded region (cropped to save space), with a width of $\Delta\,r(n)$, is the guard zone in which no other node can simultaneously receive. w and y are at distance $(1+\Delta)\,r(n)$ from u and thus, can be simultaneous receivers over the same (sub)channel without being affected by u's signal. By triangle inequality, such simultaneous receivers cannot be closer than $\Delta\,r(n)$ to v. Therefore, imaginary disks of radius $\Delta\,r(n)/2$ centered at all simultaneous receivers are disjoint.

b) The case of extended network: Santi and Blough [21] derive the critical transmission range for extended networks as $r(d) = \Theta(\sqrt{\log d})$, where d is the geometric diameter of the network. For an extended network, $d = \Theta(\sqrt{n})$ and thus, $r(n) = \Theta(\sqrt{\log n})$. The lemma follows. □

Using Lemma 2 and the bounds obtained in Equation (10), we derive the theoretical upper-bounds on the per-node throughput.

Theorem 3. *In a random network and under a proximity-based social model with clustering exponent* α*, the theoretical maximum per-node throughput* $R_\alpha(n)$ *is bounded above as*

$$R_\alpha(n) = \begin{cases} \mathcal{O}\Big(\dfrac{1}{\sqrt{n\,\log n}}\Big) & \text{if } 0 \leq \alpha < 2\,, \\ \mathcal{O}\Big(\sqrt{\dfrac{\log n}{n}}\Big) & \text{if } \alpha = 2\,, \\ \mathcal{O}\Big(\Big(\dfrac{1}{n}\Big)\Big(\dfrac{n}{\log n}\Big)^{\frac{\alpha-1}{2}}\Big) & \text{if } 2 < \alpha < 3\,, \\ \mathcal{O}\Big(\dfrac{1}{\log^2 n}\Big) & \text{if } \alpha = 3\,, \\ \mathcal{O}\Big(\dfrac{1}{\log n}\Big) & \text{if } 3 < \alpha\,. \end{cases} \tag{12}$$

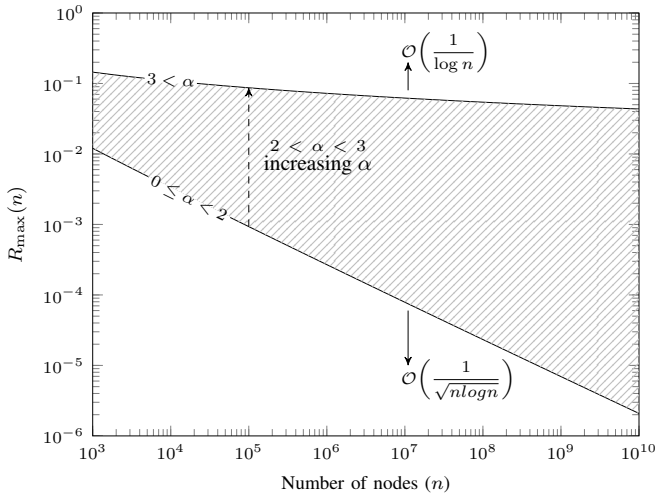

Fig. 6: An illustration of how the order throughput capacity decays versus the network size when various classes of social relationships are in effect. Curves are to only exhibit the approximate decaying rates and should not be interpreted as being accurate.

Proof. From Lemma 2 and Equation (11), it is immediate that

$$R_\alpha(n) \leq \Theta\big(\frac{1}{\log n\, \mathbb{E}[\mathcal{L}_\alpha]}\big)\,.$$

The theorem follows by replacing $\mathbb{E}[\mathcal{L}_\alpha]$ with corresponding bounds given in Equation (10). □

Fig. 6 illustrates the bounds derived by Theorem 6. The drastic decay in per-node throughput can be observed when $0 \leq \alpha < 2$. The shaded region depicts the transition into a slowly decaying order of $1/\log n$ when α crosses over the threshold of 3.

The order throughput of $1/\sqrt{n \log n}$ was first derived in the pioneering paper by Gupta and Kumar [1] under the assumption of uniform communication model when nodes choose their destinations randomly and uniformly, *i.e.*, $\alpha = 0$. The result in Theorem 3 is consistent with this well-known result. In fact, Theorem 3 makes a stronger statement that this bound indeed holds for a larger range of α between 0 and 2. With increasing α, the order throughput improves and eventually gets bounded above by $1/\log n$ when α exceeds 3.

This suggests that the maximum throughput scaling to be expected is of order $\mathcal{O}(1/\log n)$ for $\alpha > 3$ when the expected social path length remains constant. In that case, even though every packet is transmitted a finite number of times along its path to the destination, still, a constant throughput per node cannot be attained. The fundamental reason for this limitation is the restrictions imposed by the MAC layer and the connectivity requirement. The area consumed by every transmission is quadratic in the nodes' transmission radius and this prohibits other nodes within the interference range from being able to receive their intended packets at the same time. On the other hand, in order to maintain a giant connected component in both the extended and dense network models, the nodes' critical transmission radius must be adjusted such that an increasing number of nodes are covered within the one-hop neighborhood as the network grows larger. This natural requirement inevitably contributes to larger interfering groups of nodes and hence, increasingly limits the available capacity per node.

Though even highly localized interaction patterns cannot guarantee a decaying rate of better than $\mathcal{O}(1/\log n)$ in per-node throughput, in practice, this can still serve as a reasonable upper-bound should it be realized. Theorem 3 does not provide insights as to whether or not this bound is achievable. Nonetheless, a constructive approach to study the feasibility of this bound, similar to the lower-bound analysis in [1], can be developed. We leave this analysis as a subject of future research.

VII. Related Work

Originated by the seminal paper of Gupta and Kumar [1] and followed by a handful of subsequent works (*e.g.*, [3]–[5], [13], [22]) in the past decade, it was revealed that the asymptotic per-node throughput in wireless multi-hop networks rapidly decays as the network grows in size. This unfavorable behavior was primarily attributed to the effect of interference caused by the shared nature of wireless medium that could only be avoided at the expense of some sort of spatial or temporal sacrifice of bandwidth. Shortly after, a flurry of research discussed potential mechanisms to mitigate the wireless channel impairments and thereby, improve the throughput capacity through leveraging mobility [2], [13], hybridization [23], [24], directional antennas [25], [26], or cooperative transmission [27], [28].

All these proposals have correctly identified a root cause of throughput deterioration in wireless networks by focusing on the physical aspect of communication. Nonetheless, there also exists a social aspect of communication which has received much less attention. The social model, indeed, defines the patterns according to which nodes interact with one another. For this side of the problem, the mainstream literature has generally resorted to a naïve uniform interaction model—see for example [1]–[3], [13], [15]. As it turns out, not only does such a simplistic model fail to reflect a realistic picture of interaction paradigms in the network, but it also yields an overly pessimistic view of the network scaling limits. A remarkable body of research [9]–[12] has been undertaken to explore geographical dynamics of social interactions in real networks, but no notable piece of work has been devoted to investigating the impact of applying realistic social models on the scalability and throughput capacity of communication networks. A primary objective of this paper has been to provide additional insight in the exploration of this cross-domain problem.

VIII. Conclusions and Outlook

We investigated how geographical diversity of social interactions can affect the scalability of communication networks. Particularly, we identified a threshold on the spatial diversity

of social interactions beyond which the majority of inter-node communications become statistically concentrated within a finite neighborhood around nodes. We showed that this phenomenon enables the underlying communication graph to scale as the number of nodes in the network increases.

We further examined how the upper-bound on the throughput capacity can be improved if the social interactions among nodes are fueled by geographical proximity. According to our analysis, an upper-bound of $\mathcal{O}(1/\log n)$ can be expected on the maximum per-node throughput if communication patterns are highly concentrated. Although more promising than the well-established bound of $\mathcal{O}(1/\sqrt{n \log n})$ under uniform communication model, our results does not yet guarantee the existence of networking mechanisms to realize this bound. In fact, the feasibility of this limit depends largely on the agreement of our model description with the actual communication patterns among nodes.

Even if the spatial diversity of social contacts does not naturally meet the necessary localization conditions as mandated by our model, similar bounds may be attainable by employing content replication and caching mechanisms in the network in order to bring content closer to consumers. We believe that this is a promising avenue for future research.

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