STOCHASTIC PROPERTIES OF THE CHIRIKOV-TAYLOR MAPPING*

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The "standard" mapping

$$I_{n+1} = I_n + (k/2\pi) \sin 2\pi\theta_n$$
$$\theta_{n+1} = \theta_n + I_{n+1} \qquad \text{(mod 1)}$$

arises in many problems in plasma dynamics. We have studied the statistical properties of a single orbit, for various values of the parameter $k > 4$, i.e., well within the stochastic domain. For most tests, we code the orbit by coarse-graining I-space into 10 equal cells, and expressing the orbit as a "semi-infinite" sequence of ($N > 10^4$) digits, e.g. 8, 1, 4, 4, 7, 9, 7, 8, 7, 2, 2, 3, 5, 5, 1, 5, 5, 3, 2,... We then examine various joint and conditional probabilities, by counting the relative frequency of finite sequences. For example, P(3,5) is the relative frequency of the 2-digit sequence 3,5.

First we test for ergodicity by comparing P(a) for all a = 0,1...9; we find them all equal (to within expected fluctuations), demonstrating essential ergodicity, for $k > 4$. Next we test for statistical independence of I-values m iterations apart, and find it for $m \geq 8$ at $k = 5$, for $m \geq 6$ at $k = 7.5$, for $m \geq 1$ at $k = 50$. A related test is for the Markov amnesia time m_A: for $m \geq m_A$, the conditional probabilities obeys the Markov assumption for the mth iterate of the mapping. We find that $m_A = 5$ for $k = 7.5$. Under Markov and non-Markov conditions, we evaluate the transition matrix $P(b|a)$ and its associated entropy. We relate the latter to the Kolmogorov-Sinai entropy and to the Lyapunov exponent. Finally, without coarse-graining, we evaluate the autocorrelation function $\langle \exp 2\pi i(I_{n+m} - I_n)\rangle$ as a function of discrete m and continuous k.

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