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Josephson Weak Links: Two Models.* P. K. Hansma† and G. I. Rochlin, Dept. of Phys., Univ. of Calif., Berkeley and IMRD, Lawrence Berkeley Lab., Berkeley, Calif. 94720 --We have performed a series of experiments to investigate two "models" for studying the behavior of weak-link Josephson effect structures. The first is a low-inductance, externally shunted, oxide-barrier Josephson junction. This acts as a generalized weak link in that it can be adjusted, even at liquid helium temperatures, to replicate the I-V characteristics of widely varying types of weak links. The other model is a driven, damped pendulum whose equation of motion is equivalent to the second-order nonlinear current-phase equation of the links. Observations on both models not only indicate the correctness of the assumed equivalent circuit by verifying the solutions in detail, they provide new physical insight into the nonlinear behavior of the phase and thus of the current-voltage relationship of weak links in general.

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Josephson Weak Links: Two Models

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In 1968 Stewart[1] and McCumber[2] independently advanced a theory to explain why the observed current-voltage characteristics of one type of weak link device differed so markedly from that of other related devices. They proposed that the difference between the current-voltage characteristics of many of these devices could be explained by analyzing them in terms of a lumped circuit model that treated the distributed internal resistance and capacitance of the device as a single resistor and a single capacitor, both in parallel with a hypothetical idealized Josephson element having zero capacitance and an infinite resistance to quasiparticle tunneling. These ac and dc supercurrents obey Josephson's equations[3]

$$i = i_c \sin\phi \tag{1a}$$

and

$$\left(\frac{2e}{\hbar}\right) V = d\phi/dt, \tag{1b}$$

where i_c is the critical supercurrent, ϕ and V are the pair phase difference and instantaneous voltage across the element, and i is the instantaneous supercurrent. The observed time-averaged dc current-voltage characteristic of such a lumped circuit model is a function of the dimensionless parameter

$$\beta_c = (\frac{2e}{\hbar})\ i_c(\frac{C}{G^2}), \qquad (2)$$

which gives a normalized measure of the balance of influence of the capacitance C, shunt conductance G, and critical supercurrent i_c on the hysteresis.

Our first construction to verify the circuit model on which the theory is based was a generalized weak link.[4] This was constructed by modifying a conventional S-I-S evaporated film junction with an external resistive shunt in parallel with the junction as shown in Fig. 1. The shunting is through the Ag-I-Sn quasiparticle tunneling junctions adjacent to the Sn-I-Sn Josephson junctions. By placing the shunts this close to the junctions, we minimize the loop inductance between the junction and the shunt that prevented detailed comparison with theory for some earlier geometries.[5] With these shunts, the resistance that appears in the lumped circuit model of the junction is the parallel combination of the Josephson junction quasiparticle tunneling resistance and the resistance of the shunt. We were able to investigate large ranges of β_c for <u>each</u> shunted jucntion since β_c can be easily changed, while the junctions are in the dewar, by applying a small magnetic field (0 to 1 Oe).

For our mechanical model experiments we used a mechanical analog of the type developed by Sullivan and Zimmerman.[6,7] This model is not only a great aid to the intuition, but also can be used quantitatively to generate plots of torque vs time-averaged rate of rotation which are analogs of junction current-voltage characteristics.

ANALOGY MODEL

Using the lumped circuit model for a generalized Josephson weak link, the total current, I, is the sum of three terms: (1) the current through the capacitor, C(dV/dt); (2) the current through the resistor, GV; and (3) the Josephson supercurrent, $i_c \sin\phi$. That is:

$$I = C\frac{dV}{dt} + GV + i_c \sin\phi. \qquad (3)$$

In writing this equation we have used only elementary circuit theory and the dc Josephson equation. By using the ac Josephson equation we can eliminate the voltage, thus obtaining the equation

$$I = \frac{C\hbar}{2e}\frac{d^2\phi}{dt^2} + \frac{G\hbar}{2e}\frac{d\phi}{dt} + i_c \sin\phi. \qquad (4)$$

This equation can be reduced to dimensionless form by dividing through by the critical current and substituting a dimensionless time $\tau = (2e/\hbar)(i_c/G)t$. The resulting equation is

$$\frac{I}{i_c} = \beta_c \frac{d^2\phi}{d\tau^2} + \frac{d\phi}{d\tau} + \sin\phi \qquad (5)$$

where β_c is the dimensionless circuit parameter defined in Eq. (2).

This equation can be solved for the time-averaged voltage, $\langle V \rangle_t$ as a function of the current, I. The main difference between the solutions for different values of β_c is in the amount of hysteresis in the I-V characteristic, that is, the range of current over which there is both a zero bias and a finite bias solution. Following McCumber, we can define a hysteresis parameter, α, as the ratio of the minimum current as the voltage goes to zero to the critical supercurrent. Thus α ranges from 1 in the case of no hysteresis (low β_c) to 0 in the case of maximum hysteresis (high β_c).

Figure 2 shows a β_c vs α plot for the junctions of Fig. 1; the solid line is the theory. The capacitance was used as a fitting parameter. This fitting capacitance cannot, however, be equated to the Josephson junction capacitance alone because of the additional contributions from the Sn-Insulator-Ag junctions used for shunting. We can, however, note that the fitting capacitance is indeed larger, ~ 800 pf, than the Josephson junction capacitance alone, 500-600 pf.[12]

MECHANICAL MODEL

Consider a driven, damped simple pendulum, where m is the mass of the bob, g is the acceleration due to gravity, ℓ is the length of the pendulum, and θ is the angle of the bob from vertical. The total torque has three components: (1) the applied torque, T_a; (2) the opposing torque from the pendulum bob, $-mg\ell \sin\theta$; (3) the opposing torque from the magnetic damping, $-D(d\theta/dt)$, where D is the damping coefficient. Thus the fundamental equation is:

$$T_a - mg\ell \sin\theta - D\frac{d\theta}{dt} = M\frac{d^2\theta}{dt^2} . \tag{7}$$

By rearranging the terms we obtain:

$$T_a = M\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + mg\ell \sin\theta. \tag{8}$$

Note the exact analogy between the equation for the electrical model, Eq. (4), and this equation for the mechanical model. For the Josephson weak link we are most interested in the time-averaged voltage as a function of applied current. By analogy, for the mechanical model we are most interested in the time-averaged rate of rotation, $\langle d\theta/dt \rangle_t$ as a function of applied torque.

The mechanical model allows us to study the nonlinearities of the motion as it slows down the characteristic periods from on the order of 10^{-10} sec to 1 sec. Furthermore, we are able to acquire a better physical intuition from the behavior of this mechanical system than from the electrical system. By studying the motion of the mechanical model, we gain great insight into the behavior of a Josephson weak link.[7]

Figure 3 shows experimental results for "β_c" = $mg\ell(M/D^2)$ vs α. Here α is a hysteresis parameter, defined as the ratio of the minimum torque for which there is a rotating solution to the critical torque $mg\ell$, in exact analogy to the definition of α for the electrical system. We varied m and used M' as a fitting parameter, again in exact analogy to our experimental method for shunted junctions. The solid line gives the theoretical result obtained by McCumber transformed to solve the equation of motion for the mechanical model.

SUMMARY

The investigation of two models of a Josephson weak link has enabled us to gain new insight into the properties of such links. For the case of low device inductance and a constant current source, both models indicate that the theory of McCumber and Stewart provides a complete description of weak link behavior. Moreover, any device which behaves according to the predicitons of this theory can be assumed to be analyzable by an equivalent circuit similar to the one discussed here. This circuit can then be used to exactly predict the ac and dc response of the device with the only difficult parameter to measure, the device capacitance, determined by a fit of α vs β_c.

The use of such models, especially the mechanical one, is highly recommended for researchers in this area. It readily provides a simple visual impression of highly complex behavior and gives one a degree of physical intuition for the Josephson mechanism not easily acquired by other means.

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We wish to thank Ted Fulton for pointing out the importance of making shunted junctions in a low inductance geometry and for suggesting construction of a mechanical model. Paul Richards and Yuan Taur originally made the mechanical model that was modified for use in these experiments. It is a pleasure to acknowledge their good design and craftsmanship. This work was performed under the auspices of the U. S. Atomic Energy Commission. One of us (PKH) was supported by a National Science Foundation Fellowship during the research.

FIGURE CAPTIONS

Fig. 1. An externally shunted Josephson junction of the type used in our experiments. The outer two S-I-S Josephson junctions are shunted by Sn-Insulator-Ag quasiparticle tunneling junctions.

Fig. 2. Experimental results for the hysteresis parameter α as a function of β_c for a junction of the type shown in Fig. 1. β_c was varied by decreasing the critical current i_c with a small magnetic field. This field varied from zero for the uppermost point to $\approx$ 1 Oe for the lowest point. The solid line is McCumber's theoretical prediction.

Fig. 3. Experimental results for the mechanical model. For this model "β_c" = $mg\ell M/D^2$ while α was determined as described in the text. The solid line is McCumber's solution.

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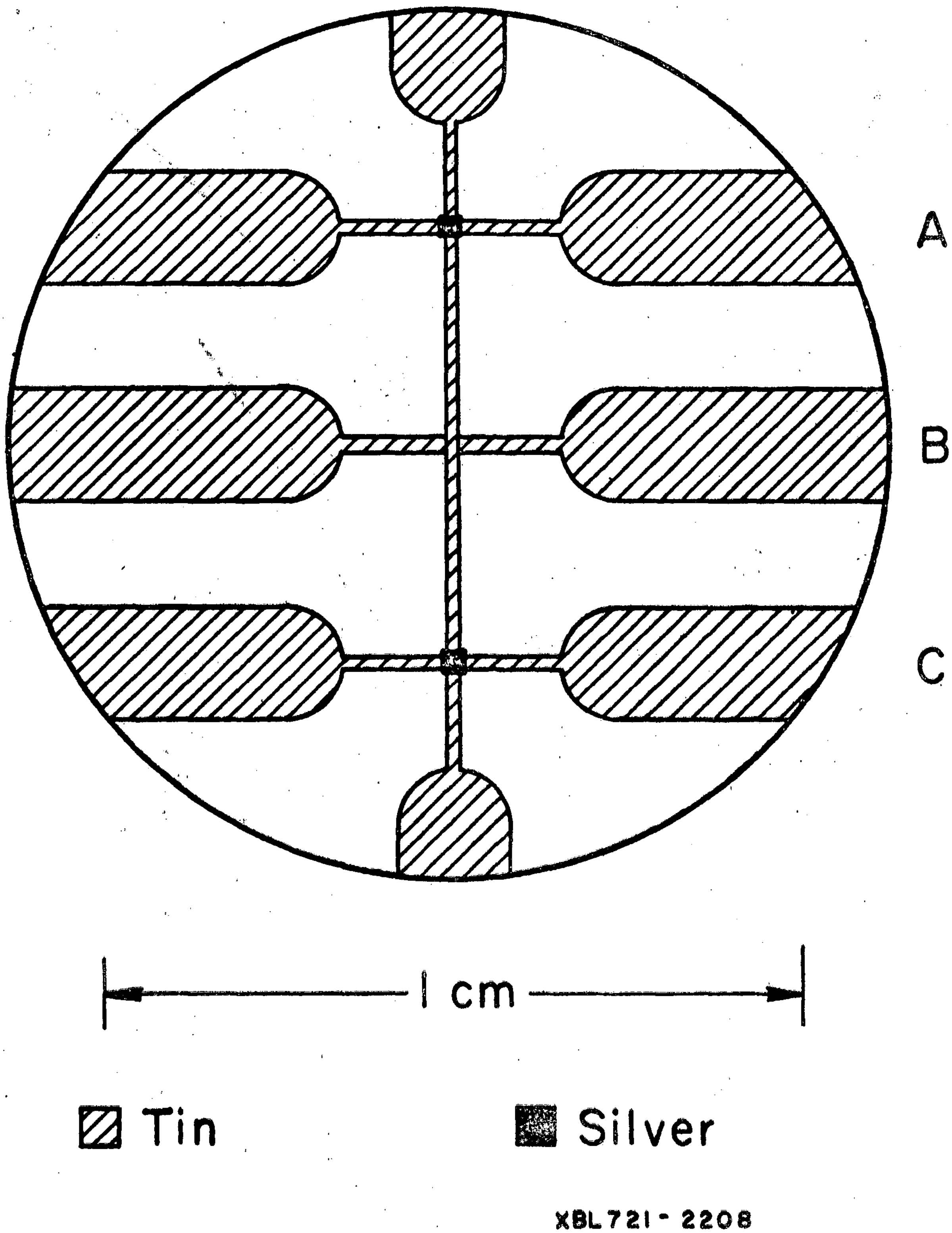

Fig. 1

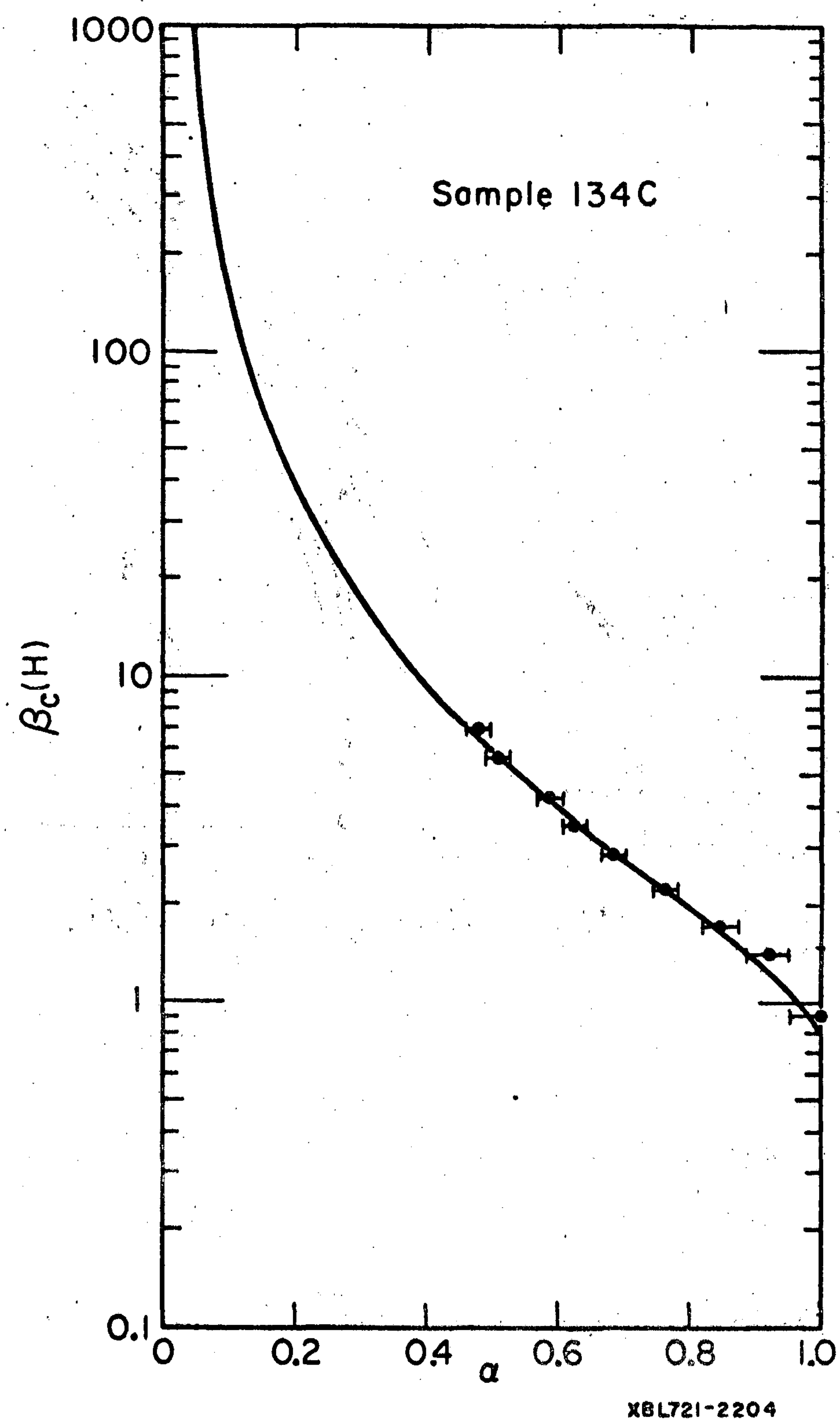
Sample 134C
1000
100
10
1
0.1
β_c(H)
0
0.2
0.4
0.6
0.8
1.0
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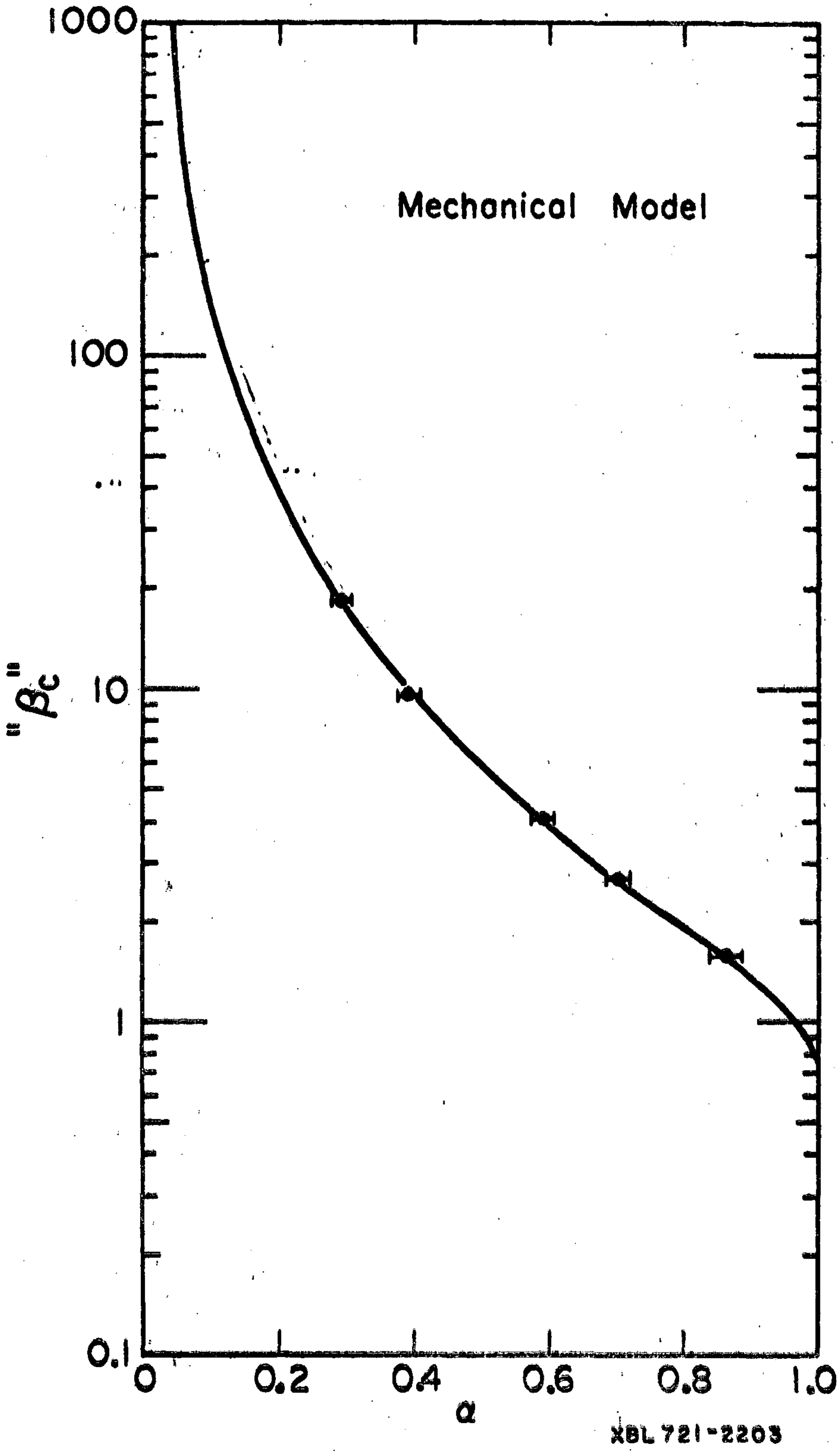
Fig. 2



Mechanical Model
1000
100
10
1
0.1
"β_c"
0
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Fig. 3

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