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It has long been known that in a compressible nondissipative hydromagnetic medium (continuum) there are three distinct propagation speeds of small-amplitude plane waves clearly corresponding to three different polarizations of the fluid displacement.[1] These phenomena have been investigated and discussed by many authors, a particularly rigorous treatment being included in the comprehensive work by Bazer and Fleischman.[2] It is readily shown that one of the three modes is purely transverse, i.e., the fluid displacement and the magnetic field disturbance are each normal to both the direction of propagation and the undisturbed field. This noncompressive (pure shear) mode is commonly called the intermediate or Alfvén wave.[3] It is also usually demonstrated that the other two types of propagating disturbances, frequently called the fast and slow waves,[3] consist of displacements that are coplanar with the propagation vector and the undisturbed field. Since, in general, these two modes involve compressions, they may be termed magnetoacoustic waves.

The physical difference between the slow and the fast wave, however, has not always been rendered transparent by the past analyses. It is therefore the purpose of this note to show, in a very simple and direct manner, how the polarizations of the two types of magnetoacoustic waves are related.

Our starting point is the familiar set of linearized equations of magnetohydrodynamics of a dissipationless fluid. With the help of either Fourier analysis[4] or the method of characteristics,[3] and using the continuity equation to eliminate the density as a variable, these are readily transformed into

$$c^2 \mu\rho \underline{v} = a^2 \mu\rho(\underline{n}\cdot\underline{v})\underline{n} + c\underline{B} \times (\underline{n} \times \underline{b}) \qquad (1)$$

$$c\underline{b} = \underline{n} \times (\underline{B} \times \underline{v}) \ . \qquad (2)$$

Here $\underline{v}$ and $\underline{b}$ signify the small disturbances of the fluid velocity and of the magnetic induction, respectively. The undisturbed medium is characterized by a density ρ and a speed of ordinary sound a, and is assumed to be permeated by a magnetic field B/μ. The signals are propagating at speed c in the direction of the wave normal $\underline{n}$.

Where $\underline{n}\cdot\underline{v} \neq 0$, i.e., in cases of magnetoacoustic waves, the coplanarity of $\underline{b}$, $\underline{v}$, $\underline{n}$, and $\underline{B}$ can immediately be deduced from (1) and (2). Inspection of these equations also reveals that the mutual orientation of the vectors in question must be of the type sketched in Fig. 1 for the domain $\underline{n}\cdot\underline{B} > 0$ and $\underline{b}\cdot\underline{B} > 0$. It will be recognized as an interesting physical feature that in fast waves $\underline{n}\cdot\underline{v}$ and $\underline{b}\cdot\underline{B}$ have the same sign, but in slow waves they have opposite signs.

A convenient quantitative relation between the direction of $\underline{v}$, defined by $\underline{v}\cdot\underline{B} = vB\cos\phi$, and the direction of propagation $\underline{n}$, defined by $\underline{n}\cdot\underline{B} = B\cos\theta$ (see Fig. 1), is readily obtained if we use elementary vector algebra to eliminate c, $\underline{b}$, and $|\underline{v}|$ from (1) and (2). One can show, for instance, that $(\underline{B}\cdot\underline{v})(\underline{B}\times\underline{v}) = \mu\rho a^2(\underline{n}\cdot\underline{v})(\underline{n}\times\underline{v})$. If we introduce the abbreviation $r = \mu\rho a^2/B^2$, this can be written very simply as $\sin 2\phi = r \sin 2(\phi - \theta)$.

In terms of the discriminant of magnetoacoustics, defined by $q^2 = r^2 - 2r \cos 2\theta + 1$, the relation between ϕ and θ takes the explicit form

$$\sin 2\phi = (r/q) \sin 2\theta. \tag{3}$$

Since the propagation speed in this notation is given by $c^2 = (1+r+q)B^2/2\mu\rho$, the positive root $q > 0$ obviously refers to the fast wave, while $q < 0$ belongs to the slow wave. At a given angle θ, then, Eq. (3) yields two admissible basic solutions for ϕ which differ by $\pi/2$. Evidently these are the two orthogonal directions labeled v_f and v_s in Fig. 1.

A plot of ϕ vs θ for various values of r is presented in Fig. 2. For $r >> 1$, the fast wave very clearly approaches pure compression ($\phi = \theta$), whereas for $r << 1$, $\underset{\sim}{v}_f$ is transverse to the magnetic field at all values of θ. It is remarkable that for values of r of only 4 and 1/4, respectively, the deviations from these limiting directions amount to only about 7 deg.

The angle ϕ for slow waves is indicated by broken lines in Fig. 2. At the point $\theta = \pi/2$, the direction of $\underset{\sim}{v}_s$ changes by π. This occurrence is connected with the fact that both c_s and $\underset{\sim}{n} \cdot \underset{\sim}{v}$ go to zero at this point.[3]

The magnitudes of $\underset{\sim}{v}$ and $\underset{\sim}{b}$ are, of course, uniquely related to each other. After some algebraic manipulation, this relation in terms of q and r is found to be

$$2b^2 = \mu\rho v^2 [1 + (1 - r)/q]. \tag{4}$$

The case $r = 1$ deserves special attention. Equation (4) states that as far as relative amplitudes are concerned, there is no distinction between fast and slow waves under this condition. And in Fig. 2 we see that in the directions $\theta = 0$ and $\theta = \pi$ the fast and slow waves are not only indistinguishable but in a certain sense exchange roles. This is also apparent when $c(\theta)$, for the case of $r = 1$, is shown in a polar plot.[2]

Footnotes and References

*Work done under the auspices of the U. S. Atomic Energy Commission.

1. N. Herlofson, Nature 165, 1020 (1950).

2. J. Bazer and O. Fleischman, Phys. Fluids 2, 366 (1959).

3. K. O. Friedrichs and H. Kranzer, in Nonlinear Wave Motion, Notes on Magnetohydrodynamics, VIII, U. S. Atomic Energy Commission, New York Operations Office Report NYO-6486, July 1958 (unpublished).

4. L. D. Landau and E. M. Lifschitz, Electrodynamics of Continuous Media (Pergamon Press, New York, 1960), p. 219.

Figure Captions

Fig. 1. Orientations of the vectors in magnetoacoustic waves.

Fig. 2. The angle ϕ as function of θ for various values or $r = \mu\rho a^2/B^2$. The solid curves refer to the fast wave, the broken curves to the slow wave.

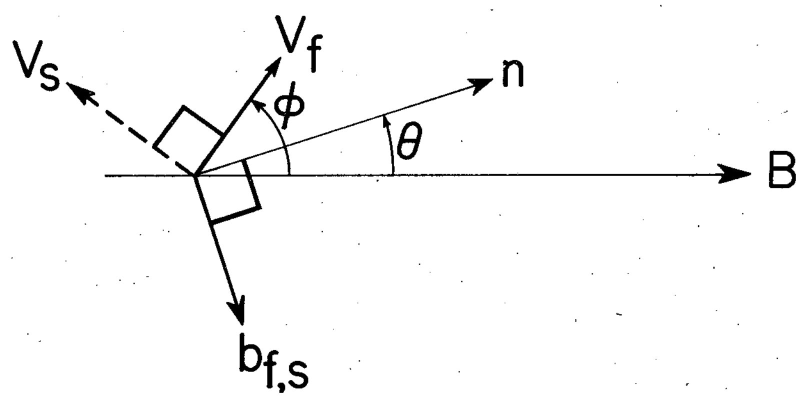

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Fig. 1

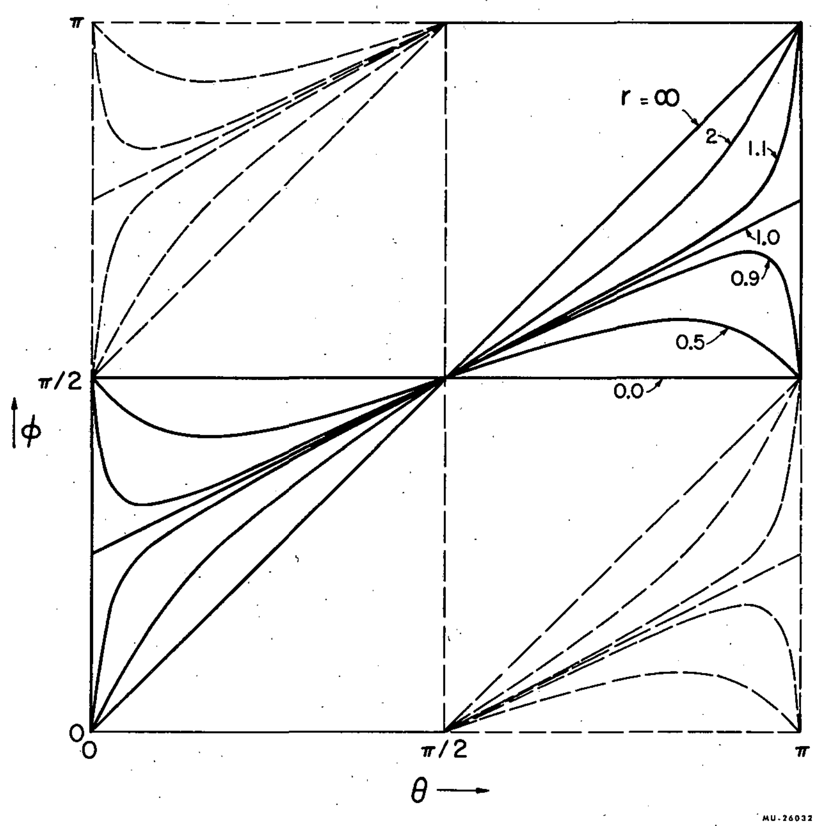

Fig. 2