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Publication Date
1957-07-01

UCRL 3874

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BERKELEY, CALIFORNIA

UCRL-3874

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Contract No. W-7405-eng-48

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Printed for the U. S. Atomic Energy Commission

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ABSTRACT

By solution of the appropriate hydrodynamic and heat transfer equations it is possible to predict the observed heat transfer coefficients. The average deviation is 25% and the maximum 71%. An equation for optimum rotor speed is derived. For a certain desired amount of product the rotor speed should be such that the power supplied to the vapor compressor is $3(1 + \frac{BPE}{\Delta t})$ times that supplied to the rotor. BPE is the mean boiling point elevation in the evaporator and Δt is the total temperature drop for heat transfer.

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K. C. D. Hickman[1] has described and given operating data on a centrifugal boiler still in which the liquid to be evaporated flows in a thin film outward along the inside of a rapidly rotating cone. The vapors generated are compressed and returned to the other side of the cone where they condense to supply heat for the evaporation taking place on the inside.

Flow Regime in the Film

Whether the flow along the cone is viscous or turbulent can be predicted by calculation of the Reynolds Number. For flow along a flat plate

$$Re = \frac{4\Gamma}{\mu} \tag{1}$$

where Γ = flow rate per unit length normal to flow, lb/hr ft

μ = viscosity of the liquid.

If the Reynolds number is less than about 2000 one should expect to have viscous flow. For this case Eq. (1) may be rewritten

$$Re = \frac{2W}{\pi r \mu} \tag{2}$$

where W = total weight flow, lb/hr

r = any radius, ft.

By inspection it is observed that the highest Reynolds number will occur near the hub of the cone. Let us calculate this radius at which the flow would change from turbulent to viscous. The largest feed rate reported is 1500 lbs/hr total on the two 54-inch c.d. rotors at about 125°F. The critical radius is then

$$r_{crit} = \frac{2W}{2000 \times \pi \times \mu} = \frac{750}{1000 \times \pi \times 0.55 \times 2.42} = 0.18 \text{ ft} \tag{3}$$

or about 2 inches. Thus for all reported tests the flow was viscous. It might possibly be turbulent near the hub of a larger cone at very high feed rates.

Theory

To proceed with the derivation, the following assumptions are made.

1. Viscous flow in both evaporating and condensing films.
2. No nucleation (bubble, or drop formation).
3. No inert gas present.
4. Condensate is flung off only at outer edge.
5. Heat flow only by conduction.
6. No abrupt temperature drops at phase boundaries.

For flow down vertical walls it has been shown[2]

$$\Gamma = \frac{\rho^2 g y^3}{3\mu} \qquad (4)$$

In a centrifugal force field large compared to gravity on a cone inclined to the axis of rotation at an angle ϕ, g would be replaced by $4\pi^2 r N^2 \sin\phi$. It is implicitly assumed that the velocity profile is fully developed at any radius. Although this cannot be exact it is probably a good approximation and is similar to the approach used by Nusselt[3] for condensation. Hence Eq. (4) can be written for this case

$$y = \frac{1}{2\pi} \sqrt[3]{\frac{3W\mu}{r^2 \rho^2 N^2 \sin\phi}} \qquad (5)$$

But since we postulate heat flow by conduction only,

$$h = \frac{k}{y} = \left[2\pi \sqrt[3]{\frac{k^3 \rho^2 N^2 \sin\phi}{3\mu}} \right] \cdot \frac{r^{2/3}}{W^{1/3}} \qquad (6)$$

$$= E \frac{r^{2/3}}{W^{1/3}}$$

where E, the quantity in square brackets, is a constant quantity in any single experiment.

For the next part of the derivation the metal resistance to heat transfer will be neglected. The error introduced will be corrected for later. By a heat balance at any point r one may write for the flow W on the condensing side (outside) of the cone [the flow on the evaporating side is $(W_F - W)$]:

$$\frac{dW}{dr} = \frac{U2\pi r\Delta t}{\lambda \sin\phi} = \frac{2\pi r\Delta t}{(\frac{1}{h_e} + \frac{1}{h_c}) \lambda \sin\phi} = \frac{2\pi r^{5/3}\Delta t\ E}{[(W_F - W)^{1/3} + W^{1/3}]\ \lambda \sin\phi} \tag{7}$$

where W_F = feed rate to one cone of the evaporator. Integrating Eq. (7) for the amount of distillate W_D between r_i and r_o results in Eq. (8):

$$W_F^{4/3} - (W_F - W_D)^{4/3} + W_D^{4/3} = \frac{\pi\Delta t\ E}{\sin\phi\ \lambda}\ r_o^{8/3} \left[1 - \left(\frac{r_i}{r_o}\right)^{8/3}\right] \tag{8}$$

but we are interested in the average heat transfer coefficient $\bar{U}$ as defined by

$$W_D\lambda = \frac{\bar{U}\pi\ (r_o^2 - r_i^2)\ \Delta t}{\sin\phi} \quad . \tag{9}$$

Solving Eq. (9) for $\bar{U}$ and eliminating Δt and E by means of (8) and (6) one obtains

$$\bar{U} = \frac{\pi}{\sqrt[3]{3}} \cdot \frac{2W_D}{W_F\left[1 - \left(1 - \frac{W_D}{W_F}\right)^{4/3} + \left(\frac{W_D}{W_F}\right)^{4/3}\right]} \left(\frac{k^3\ \rho^2 N^2 r_o^2\ \sin\phi}{W_F\mu}\right)^{1/3} \frac{\left[1 - \left(\frac{r_i}{r_o}\right)^{8/3}\right]}{\left[1 - \left(\frac{r_i}{r_o}\right)^2\right]} \tag{10}$$

or

$$\bar{U} = 2.18\ \left[f\left(\frac{W_D}{W_F}\right)\right]\ \left[g\left(\frac{r_i}{r_o}\right)\right]\left(\frac{k^3\rho^2 N^2 r_o^2\ \sin\phi}{W_F\mu}\right)^{1/3} \tag{11}$$

where

$$f\left(\frac{W_D}{W_F}\right) = \frac{2W_D}{W_F\left[1 - \left(1 - \frac{W_D}{W_F}\right)^{4/3} + \left(\frac{W_D}{W_F}\right)^{4/3}\right]} \tag{12}$$

values for which are tabulated in Table 1. Also tabulated in Table 1 are values calculated for $f(\frac{W_D}{W_F})$ for $h_c \longrightarrow \infty$ (i.e., no condensate resistance or perfect dropwise condensation).

From Table 1 it is apparent that for practical purposes $f(\frac{W_D}{W_F}) \approx 1.0$ over the region of most usefulness, but could be increased considerably if the condensate could be removed (as in dropwise condensation or flung off by enough centrifugal force).

The function $g(\frac{r_i}{r_o})$, which is equal to the last terms in brackets in Eq. (10), is tabulated in Table 2.

Since in practice, r_i would be made as small as possible, $g(\frac{r_i}{r_o})$ would usually be nearly 1.0.

For most practical calculations Eq. (11) can be simplified. Let us replace the outer radius by the outer diameter, D_o, (adjusting the constant accordingly):

$$\overline{U} \approx 1.37 \left(\frac{k^3 \rho^2 N^2 D_o^{\ 2} \sin \phi}{W_F \mu} \right)^{1/3} \qquad (13)$$

If the metal wall has appreciable thermal resistance then the true value of the over-all coefficient U can be calculated from

$$\frac{1}{U} = \frac{1}{\overline{U}} + \frac{x_m}{k_m} \qquad (14)$$

Turbulent Flow

As noted before, at very high flows near the hub of a rotating cone the flow may be turbulent (if Re > 2000). For heating fluids in turbulent flow down vertical walls Drew[2] gives

$$h = 0.01 \left(\frac{k^3 \rho^2 g}{\mu^2} \right)^{1/3} \left(\frac{C_p \mu}{k} \right)^{1/3} \left(\frac{4\Gamma}{\mu} \right)^{1/3} . \qquad (15)$$

If one assumes that this would also be approximately valid for evaporation with g replaced by the centrifugal force then one obtains for the evaporation coefficient in turbulent flow

$$h_{e_{turb}} = 0.0293 \left(\frac{k^2 \rho^2 N^2 W_F C_p \sin \phi}{\mu^2} \right)^{1/3} \qquad (16)$$

W_F was used as the flow should change little between r_i and the critical radius. It is interesting to note that the coefficient is essentially independent of radius as long as turbulence persists. The condensation coefficient between r_i and r_{crit} may be calculated from

$$h_c = 4.35 \left(\frac{k^3 \rho^2 N^2 r_{crit} \sin \phi}{W_c \mu} \right)^{1/3} g\left(\frac{r_i}{r_{crit}} \right) \qquad (17)$$

where W_c is the amount condensed between r_i and r_{crit}.

Minimize Power Required

As pointed out by Hickman, power is used to turn the rotor, compress the vapor, and a small amount is used in auxiliary equipment, and some is lost as heat. Since the rotor and compressor use the major share of the power the optimum speed of rotation will be calculated that will minimize the power requirement for a desired amount of feed and product for a certain rotor.

Power to Rotor. The rotor must overcome the frictional loss caused by flow of liquid over the surface; per unit mass of distillate:

$$F = \frac{2\pi^2 N^2 r_o^2}{g_c} \frac{W_F}{W_D} \qquad (18)$$

The gain of kinetic energy in the radial direction is neglected.

In addition the kinetic energy of the leaving streams will either be degraded or perhaps some recovered in the form of pressure, but in any case it must be supplied to the rotor. Numerically it is equal to the above. There will be additional kinetic energy loss due to evaporation from a rotating surface and condensing from a relatively stagnet vapor. This will be allowed for in the efficiency, η_r.

There is also going to be energy lost due to drag of the scoop(s). If the scoop is streamlined and made as small as possible consistent with handling the flow then this energy need be perhaps only 10 to 20% of the kinetic energy loss. If, on the other hand, large scoops are used and if they are not streamlined, then this energy loss could easily be 1 to 10 times the kinetic energy loss and would represent a serious loss in energy. There will also be small losses due to windage and mechanical friction but these should both be small. These latter losses should be reduced to a minimum by proper design and will also be allowed for by an efficiency factor η_r.

$$\frac{\text{work to rotor}}{\text{lb of product}} = \frac{4\pi^2 N^2 r_o^2}{\eta_r g_c} \frac{W_F}{W_D} \tag{19}$$

Work delivered to compressor. For each pound of distillate this work is

$$\frac{\Delta P}{\eta_c \rho_v} = \frac{\lambda J}{\eta_c T} (\Delta t + \text{BPE}) . \tag{20}$$

This is true as long as the temperature drop for heat transfer, Δt, and the mean boiling point elevation, BPE, of the evaporating liquid are small compared to the absolute temperature, T, of evaporation. J is the mechanical equivalent of heat. If one neglects the rotor metal resistance to heat transfer one may eliminate Δt by means of Eqs. (11) and (9). The addition of Eqs. (19) and (20) then results in the equation for total work:

$$\frac{\text{Total work}}{W_D} = \frac{4\pi^2 N^2 r_o^2 W_F}{\eta_r\, g_c\, W_D} + \frac{3^{\frac{1}{3}}\, J\, \lambda^2\, W_D}{\pi^2 \eta_c T r_o^2 \left[1 - \left(\frac{r_i}{r_o}\right)^{8/3}\right]\left[f\left(\frac{W_D}{W_F}\right)\right]} \left[\frac{W_F \mu}{k^3 \rho^2 r_o^2 N^2 \sin\phi}\right]^{\frac{1}{3}} + \frac{J\lambda(\text{BPE})}{\eta_c T} \tag{21}$$

If all quantities in the above equation are fixed (or nearly so) except total work/W_D and rate of rotation N, Eq. (21) may be written

$$\frac{\text{total work}}{W_D} = AN^2 + \frac{B}{N^{2/3}} + C \qquad (22)$$

where A, B, and C will be considered independent of N. The optimum value of N is then

$$N_{opt} = \left(\frac{B}{3A}\right)^{\frac{3}{8}} = \frac{(g_c J)^{3/8}}{\pi^{3/2}\, 3^{1/4}\, 4^{3/8}} \left(\frac{\lambda^6\, W_D^{\ 6}\, \mu\ \ \eta_r^{\ 3}}{T^3 W_F^{\ 2} r_o^{14}\ k^3 \rho^2\ \sin\phi\ \eta_c^3 \left[1 - \left(\frac{r_i}{r_o}\right)^{8/3}\right]^3 \left[f\left(\frac{W_D}{W_F}\right)\right]^3} \right)^{1/8}$$

and the work at N_{opt} becomes

$$\left(\frac{\text{total work}}{W_D}\right)_{\text{at } N_{opt}} =$$

$$\frac{4^{5/4}\ J^{3/4}}{3^{1/2}\ \pi\ g_c^{1/4}} \left[\frac{\lambda^6\ \mu}{T^3 k^3 \rho^2\ \sin\phi \left[1 - \left(\frac{r_i}{r_o}\right)^{8/3}\right]^3 \left[f\left(\frac{W_D}{W_F}\right)\right]^3 \eta_c^{\ 3}\ \eta_r} \right]^{\frac{1}{4}} \left[\frac{W_F\ W_D}{r_o^{\ 3}}\right]^{\frac{1}{2}} + \frac{J\ \lambda\ (BPE)}{\eta_c T} \qquad (24)$$

and

$$\left(\frac{\text{work to compressor}}{\text{work to rotor}}\right)_{\text{at } N_{opt}} = 3\ \left(1 + \frac{BPE}{\Delta t}\right) \qquad (25)$$

It will be noted that power consumed per pound of product may be reduced by increasing the rotor size or, since W_F and W_D are for one rotor, the work may be reduced by dividing the total desired flow among a number of rotors. The temperature should be as high as possible (without the formation of scale). Although it is important to improve rotor efficiency it is even more important to improve compressor efficiency.

It is interesting to note that Hickman suggests that the ratio of work supplied to the compressor to work supplied to the rotor be 3.22 for a commercial still. This is nearly that predicted for optimum rotor speed.

Comparison of Heat Transfer Coefficients with Experiment

Table 3 compares the heat transfer coefficients reported by Hickman to those calculated by use of Eq. (11). It will be noted that the values range from +71% to -32%. The measured values with the greatest deviation are either those with very low Δt or very low W_D which tend to magnify any experimental errors. On the whole the agreement is satisfactory indicating that the proposed mechanism, viscous flow, is probably correct.

Conclusions

On the basis of the derived equations it is possible to predict the operating characteristics of the Hickman-Badger still. It is also possible to predict the optimum conditions of operation.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

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2. W. H. McAdams, "Heat Transmission," 3rd ed., McGraw-Hill (1954).
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Table 1

Values of $f(\frac{W_D}{W_F})$ for use in Eq. (11)

$\frac{W_D}{W_F}$	0	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	1.0
$f(\frac{W_D}{W_F})$	1.50	1.29	1.12	1.07	1.03	1.01	1.00	0.99	0.99	0.98	0.99	1.00	1.00
$f(\frac{W_D}{W_F})$ for $h_c \longrightarrow \infty$	1.50	1.50	1.53	1.55	1.58	1.62	1.66	1.70	1.75	1.81	1.89	1.98	2.00

Table 2

Values of $g(\frac{r_i}{r_o})$ for use in Eq. (11)

$\frac{r_i}{r_o}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$g(\frac{r_i}{r_o})$	1.00	1.00	1.03	1.06	1.09	1.12	1.16	1.20	1.25	1.29	1.33

Table 3

Comparison of calculated and Hickman's[1] experimental heat transfer coefficients

	Temperature °F	ΔP in H_2O	$2W_F$ #/hr	$2W_D$ #/hr	U_{calc} BTU/hr ft^2 °F	U_{exp} BTU/hr ft^2 °F	% error based on experiment
Still	78	0.73	50	13.5	3620	2120	+71
#2,	80	0.78	50	15	3620	2410	+50
1000	93	1.20	50	18	3900	2550	+53
RPM,	100	1.55	50	20.5	3940	2740	+44
City	110	1.85	50	22	4050	3180	+27
water	125	2.80	50	23	4270	3070	+39
	137	3.25	50	24	4510	3650	+24
	135.2	4.10	80	31	3850	3560	+ 8
	144	6.00	80	38	3890	3760	+ 3
	155	7.00	80	41	4000	4180	- 5
5.29	107	1.75	44	11	4150	3550	+17
NaCl	109	2.7	50	17.8	4080	3180	+28
Sea Water	99	1.60	44	12.9	4000	2720	+47
Still	118	6.3	958	378	1990	2360	-16
#4,	119	4.3	967	265	2070	2440	-15
400 RPM,	121	2.8	962	174	2120	2480	-15
54" OD,	121	1.5	968	91	2240	2520	-11
City water	117	8.5	968	510	1980	2350	-16
Ocean	119	6.5	946	327	2020	2450	-18
Water	116	9.3	931	462	2020	2250	-10
	120	4.3	938	198	2140	2480	-14
	122	1.6	962	42	2320	3380	-32

NOMENCLATURE

Symbol		Definition	Suggested Unit
BPE	=	Mean boiling point elevation of evaporating liquid	^{o}F
C_p	=	Heat capacity of vapor	BTU/lb ^{o}F
D_o	=	Outside diameter of rotor	Ft
E	=	Constant parameter in Eq. (5)	--
F	=	Friction loss on rotor per unit mass distillate	ft lbf/lbm
$f(\frac{W_D}{W_F})$	=	See Eq. (11) and Table 1	--
g	=	Acceleration of gravity	ft/hr^2
g_c	=	Gravitational constant	$4.18 \times 10^8 \frac{lbm}{lbf} \frac{ft}{hr^2}$
$g(\frac{r_i}{r_o})$	=	See Eq. (9) and Table 2	--
$g(\frac{r_i}{r_{crit}})$	=	Same as above with r_o replaced by r_{crit}	--
h	=	Coefficient of heat transfer	$\frac{BTU}{hr\ ft^2\ ^{o}F}$
h_e	=	Coefficient of heat transfer for evaporation	"
h_c	=	Coefficient of heat transfer for condensation	"
J	=	Mechanical equivalent of heat	$778 - \frac{ft\ lbf}{BTU}$
k	=	Thermal conductivity of the liquid	$\frac{BTU}{hr\ ft\ ^{o}F}$
k_m	=	Thermal conductivity of rotor metal	"
N	=	Rate of rotor rotation	Rev per hr (or min)
N_{opt}	=	Optimum rate of rotor rotation	"
ΔP	=	Pres. difference between condensing and evaporating sides of rotor	$\frac{lbf}{ft^2}$
r	=	Radius	ft
r_i, r_o	=	Inside and outside radius respectively	ft
r_{crit}	=	Radius at which flow changes from turbulent to viscous	ft
Re	=	Reynolds number	--
Δt	=	Total temperature drop for heat transfer	^{o}F
T	=	Absolute temperature of evaporation	^{o}R

NOMENCLATURE
(-2-)

			Suggested Unit
U	=	Over-all heat transfer coefficient (See Eq. (13))	Btu/hr ft^2 ^{o}F
$\overline{U}$	=	Over-all heat transfer coefficient (not including metal resistance)	"
U_{calc}, U_{exp}	=	Calculated and experimental values of U	"
W	=	Liquid flow on a rotor (condensate flow after Eq. (6))	lbm/hr
W_F	=	Feed flow to a rotor (one cone only)	"
W_D	=	Distillate rate from one rotor cone	"
W_c	=	Distillate rate at r_{crit}	"
X_m	=	Metal wall thickness of rotor	ft
Γ	=	Mass flow per unit periphery normal to flow	lbm/hr ft
η_r, η_c	=	Efficiency of rotor and compressor, respectively	--
λ	=	Latent heat of vaporization at temperature T	Btu/lbm
μ	=	Viscosity of liquid	lbm/hr ft
ρ, ρ_v	=	Density of liquid and vapor, respectively	lbm/ft^3
ϕ	=	Angle of rotor to its axis	--