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MULTIPLICITY AND SCALING OF $e^+e^- \rightarrow$ HADRONS

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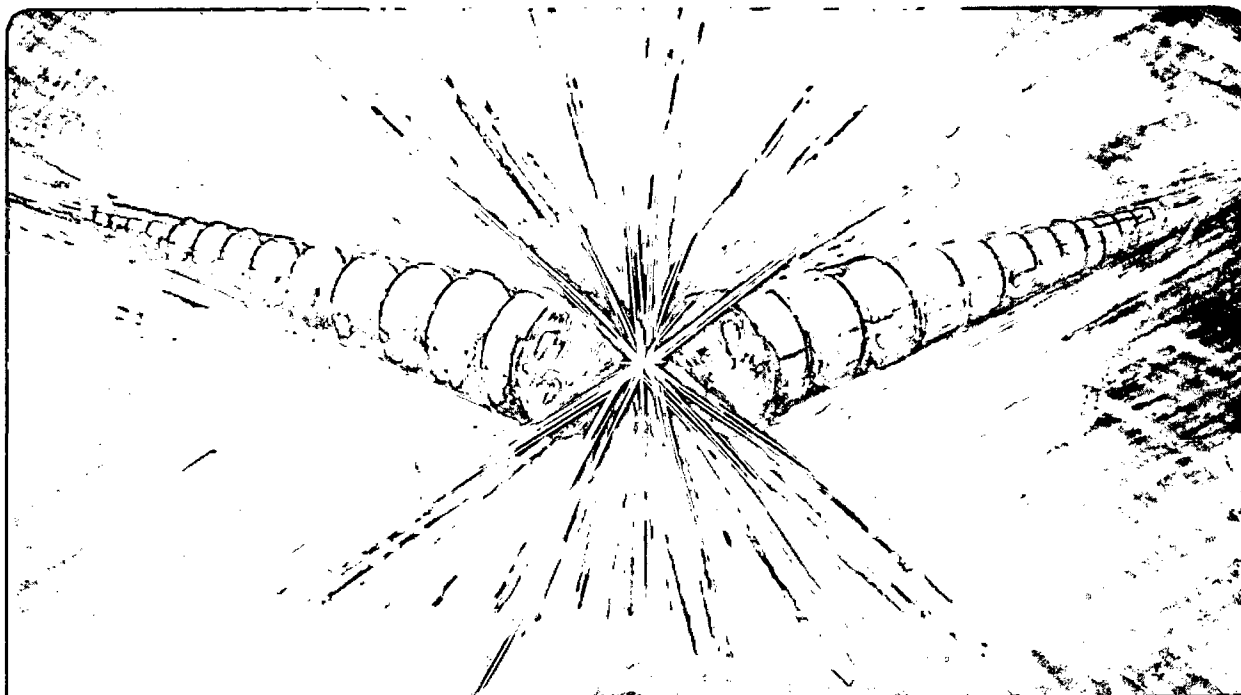
Bruce Cork and T. F. Hoang

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Multiplicity and Scaling of $e^+e^- \rightarrow$ hadrons

B. Cork and T.F. Hoang

Note added in proof:

After the completion of the present work we note that the SPEAR data of $\langle n_{\text{ch}} \rangle$ corrected for heavy leptons¹⁶ are $\sim 9\%$ higher, in better agreement with our fit, and that the JADE collaboration¹⁷ has used the Fermi model to fit their data of $\langle n_{\text{ch}} \rangle$ from $\sqrt{s} = 22$ to 31.6 GeV and other low energy data with $2 \cdot s^{1/4}$ as in our analysis.

Prof. G. Chew and Dr. I. Hinchliffe have called our attention to the relation predicted by QCD¹⁸

$$\langle n_{\text{ch}} \rangle = f(E_{\text{cm}}/\Lambda) / f(E_0/\Lambda)$$

where

$$f(E_{\text{cm}}/\Lambda) = \exp \left[2 \sqrt{\frac{N_c}{\pi b}} \ln(E_{\text{cm}}^2/\Lambda^2) \right]$$

with $b = (11 - 2 N_f/3)/4\pi$, $N_c = 3$ and $N_f = 4$ are numbers of colour and flavour, respectively, $\Lambda \cong 0.5$ GeV and $E_0 \cong 1$ GeV. We note that the abrupt rise of this prediction is ruled out by the experimental data; to be specific, the fit requires the coefficient b to be increased by a factor ~ 1.74 .

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Multiplicity and Scaling of $e^+e^- \rightarrow$ hadrons

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ABSTRACT

Meson production data from e^+e^- annihilations are investigated using the Bose-Einstein distribution modified for scaling. It is found that the average charged multiplicity increases as $E_{\text{cm}}^{1/2}$. A discussion is presented on the scaling behavior of e^+e^- annihilations compared with pp collisions.

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In the investigation of multiple meson production using Landau's hydrodynamical model,¹ it has been found that the general properties of the transverse and longitudinal momentum, P_T and P_L , as well as the kinematics of the fireball and the energy dependence of the average meson multiplicity $\langle n \rangle$ can be adequately interpreted by means of the following Bose-Einstein distribution modified to account for scaling:

$$\frac{d\sigma}{d^3p} \propto \frac{1}{e^{\varepsilon(\lambda)/T} - 1} \quad (1)$$

where

$$\varepsilon(\lambda) = \left(P_T^2 + \lambda^2 P_L^2 + m^2 \right)^{1/2}, \quad (2)$$

m being the mass of the produced meson under consideration. The two parameters are the temperature T and the scaling parameter λ ; the latter describes also the anisotropy of the meson angular distribution in the c.m. system.² Note that for simplicity, we set both the Boltzmann constant k and the velocity of light c equal to one. It is interesting to investigate certain properties of the meson production by e^+e^- annihilation:

$$e^+ + e^- \rightarrow \text{hadrons} \quad (3)$$

in terms of (1) using the currently available data, in particular results of recent experiments at PETRA with c.m. energy up to 27.4 GeV.

Consider first the temperature. We note that according to (1), the average P_T does not depend on λ but only on T , and that taking the leading term, i.e. dropping 1 in the denominator of the distribution (1), we find:

$$\langle P_T \rangle = m \sqrt{\frac{\pi T}{2m}} \frac{K_{5/2}(m/T)}{K_2(m/T)} \quad (4)$$

K_n being the modified Bessel function of the 2^d kind of order n .

We have estimated T for (3) using values of P_T measured by the PLUTO Collaboration³ and the TASSO Collaboration,⁴ assuming that the hadrons produced in the e^+e^- annihilation (3) are pions. The results thus obtained are shown in Fig. 1. Noting that T increases monotonically with the c.m. energy E_{cm} of the colliding e^+e^- beam, we try to fit the energy dependence of T with the following power law.

$$T \propto E_{cm}^\alpha \quad (5)$$

and find

$$\alpha = 0.30 \pm 0.06$$

consistent with $\alpha = 1/4$ predicted by Landau's model treating the pions as a photon gas. It should be mentioned that this behavior, namely Stefan's law, has been observed in pion production by pp, and $\pi^+ p$ collisions.⁵

Turn now to the average multiplicity $\langle n \rangle$. Note that most data deal with only charged pions and that a recent experiment at ADONE⁶ finds the behavior of neutral particles similar to that for charged ones for $E_{cm} = 1.4$ to 2.9 GeV. Consequently, we compare the charged multiplicity $\langle n_{ch} \rangle$ of the PLUTO and TASSO data with $E_{cm}^{1/2}$ according to Landau's model.

That this relationship holds also for e^+e^- annihilation (3) has been discussed by Cannuto and Lodenquai.⁷ Indeed, if c_s denotes the sound velocity (in units of $c = 1$) and V the volume in which the energy is distributed, then the average multiplicity is expected to be⁸

$$\langle n \rangle \propto E_{cm}^{1/(1+c_s^2)} V^{c_s^2/(1+c_s^2)}, \quad (6)$$

Therefore, with the assumptions that $V = \text{const.}$ and $c_s = 1$, we find

$$\langle n \rangle \propto E_{cm}^{1/2} \text{ as in the case of pion production by pp collisions.}$$

We assume the following empirical law for the average charged multiplicity:

$$\langle n_{ch} \rangle = a \cdot E_{cm}^{1/2} + b \quad (7)$$

a and b being two parameters to be determined by the experimental data. Note that in the case of meson production by pp collisions, this law⁹ gives actually a best-fit for $a_{pp} = 1.66 \pm 0.04$, $b_{pp} = -1.38 \pm 0.16$; this is because the term b accounts for the threshold effect (for a further discussion, see footnote 15).

We use e^+e^- annihilation data compiled by Wolf¹⁰ for $E_{cm} = 1.4$ to 27.4 GeV and find by least-squares fit

$$a = 2.04 \pm 0.04, \quad b = 0.15 \pm 0.43$$

The fit is shown in Fig. 2. A comparison with experimental points indicates that apart from $E_{cm} < 2.5$ GeV, the fit is in general satisfactory. Note that the estimate of b is consistent with zero, as should be under the assumption of a photon gas, i.e. zero pion-mass.

We now turn to the scaling of e^+e^- annihilation into hadrons as is described by the parameter λ of the Bose-Einstein distribution (1). Noting that $P_L \propto 1/\lambda$ according to (1) and that the Feynman-Yang scaling^{11,12} requires $\langle P_L \rangle \propto \sqrt{s}/2$, we may express in a general way, the scaling property by:

$$\lambda \sqrt{s}/2 \rightarrow \text{const.} \quad (8)$$

As for the parameter λ , we may estimate its value from the averages of P_L and P_T measured by the PLUTO collaboration³ and the TASSO collaboration.⁴ In this regard, we note that according to the modified Bose-Einstein distribution (1):

$$\frac{\langle P_T \rangle}{\langle P_L \rangle} = \frac{\pi}{2} \lambda . \quad (9)$$

The values of $\lambda \sqrt{s}/2$ thus obtained are shown by crosses in Fig. 3, the points being plotted as the available energy in the c.m. system denoted by $\sqrt{s} - 2M$ ($M = 0$ for the e^+e^- annihilation), and the broken curve being drawn to guide the eye.

It is to be noted that $\lambda \sqrt{s}/2$ increases monotonically with the c.m. energy $E_{cm} = \sqrt{s}$ from 3 to 27.4 GeV without reaching a constant value as is required by Feynman-Yang scaling. This behavior is in contrast with what has been observed in meson production by pp collisions¹³ shown by points in open circles in the same figure; here, we have plotted against the available energy in the c.m. systems, namely $\sqrt{s} - 2M$, M being the proton mass. We note that in this case, $\lambda \sqrt{s}/2$ increases from $P_{lab} = 12.5$ to 21 GeV, corresponding to $E_{cm} = 4.55$ GeV for the e^+e^- annihilation, then remains practically constant up to ISR energies equivalent to $P_{lab} = 1500$ GeV as shown by the straight line.

This striking difference in the scaling behavior between the two processes of meson production, the e^+e^- annihilation (3) on the one hand, and the pp collisions on the other, may be due to the fact that here, in the e^+e^- annihilation, no primary nucleons are involved in the reaction, and that mesons are produced in the central region of the rapidity distribution. Whereas in the pp collisions, besides this central region, there are in addition two fragmentation

regions of the colliding protons, which may be responsible for the observed scaling property.¹⁴ It would be interesting to compare the results with meson production by $\bar{p}p$ annihilations.

Finally, we mention that in the context of the fireball model, the kinematical properties such as the mass M^* and the velocity of the fireball are entirely determined by the scaling parameter λ , see Ref. 2(c), and that in the high energy limit, $M^* \propto E_{\text{cm}}^{1/2}$ in agreement with Landau's prediction for the meson multiplicity.¹⁵

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5. See Ref. 2(a), (c).
6. Roma-Bologna-Frescati Collaboration, C. Bacci, et al., *Phys. Lett.* 86B, 234 (1979).
7. See v.g. V. Cannuto and J. Lodenguai, *Phys. Rev.* D11, 233 (1975) and other references therein.
8. This relationship, which is a generalization of Landau's result $\langle n \rangle \propto E_{\text{cm}}^{3/4} V^{1/4}$, (Ref. 1), has been derived in the same way by assuming a general form for the equation of state:

$$p/\varepsilon = c_s^2,$$

where ε is the energy per unit volume, p the pressure and c_s the sound velocity. From the chemical potential of a photon gas, $\zeta = \varepsilon - Ts + p = 0$ (see Ref. 1), we obtain for the entropy density

$$s \propto \varepsilon^{1/(1 + c_s^2)}$$

Thus, the total entropy:

$$S = sV \propto E_{\text{cm}}^{1/(1+c_s^2)} V^{1-1/(1+c_s^2)},$$

which leads to (6), assuming $\langle n \rangle$ to be proportional to S . Note that for $c_s^2 = 1/3$ and $V \propto V_0/E_{\text{cm}}$ we find Landau's result:

$$S \propto E_{\text{cm}}^{1/2}.$$

9. It should be mentioned that this law was first derived by Y. Pal and B. Peters, *Mat. Fys. Medd. Dav. Vid. Seleskab.* 33, No. 15 (1964), in their analysis of meson production in cosmic rays.
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13. The values of λ are taken from Ref. 1(a), (b) and (c).
14. See v.g. M. Chaichian, et al. *Phys. Lett.* 50B, 362 (1974).
15. Referring to Ref. 1(c), we recall that the fireball velocity (in units of $c = 1$) is given by $b_F = 1 - \lambda$ and that in the case of pp collision, $M^* = M\gamma_{\text{cm}}/\gamma_F$, where M is the proton mass and $\gamma_F = 1/(1-b_F^2)^{1/2}$. For high energy, $M^* \approx 2M\gamma_{\text{cm}}^{1/2}$; thus $\langle n \rangle = c(M^* - M)$ becomes $-cM + a(2ME)^{1/2}$ where

$E = \sqrt{s} - 2M$ is the available c.m. energy. Note the negative sign for the constant term as discussed in the text. For the fit of pp data, we refer to T. F. Hoang, et al., Zeits. C, Particles and Fields, in press.

FIGURE CAPTIONS

1. Log plot of the energy dependence of the temperature. The straight line represents a least-squares fit with a power law.
2. Average charged multiplicity from e^+e^- annihilations. Data from Ref. 10. The curve represents the fit with

$$\langle n_{\text{ch}} \rangle = a \cdot E_{\text{cm}}^{1/2} + b, \quad \text{for } a = 2.03 \pm 0.04, \quad b = 0.15 \pm 0.43 .$$

3. Plots of $\lambda \sqrt{s}/2$ vs. available energy in the c.m. system $\sqrt{s} - 2M$: $M = 0$ for e^+e^- annihilations (shown by crosses) and $M =$ proton mass for pp collisions (shown by open circles). The lines are drawn to guide the eye. Feynman-Yang scaling requires $\lambda \sqrt{s}/2 \rightarrow \text{const.}$

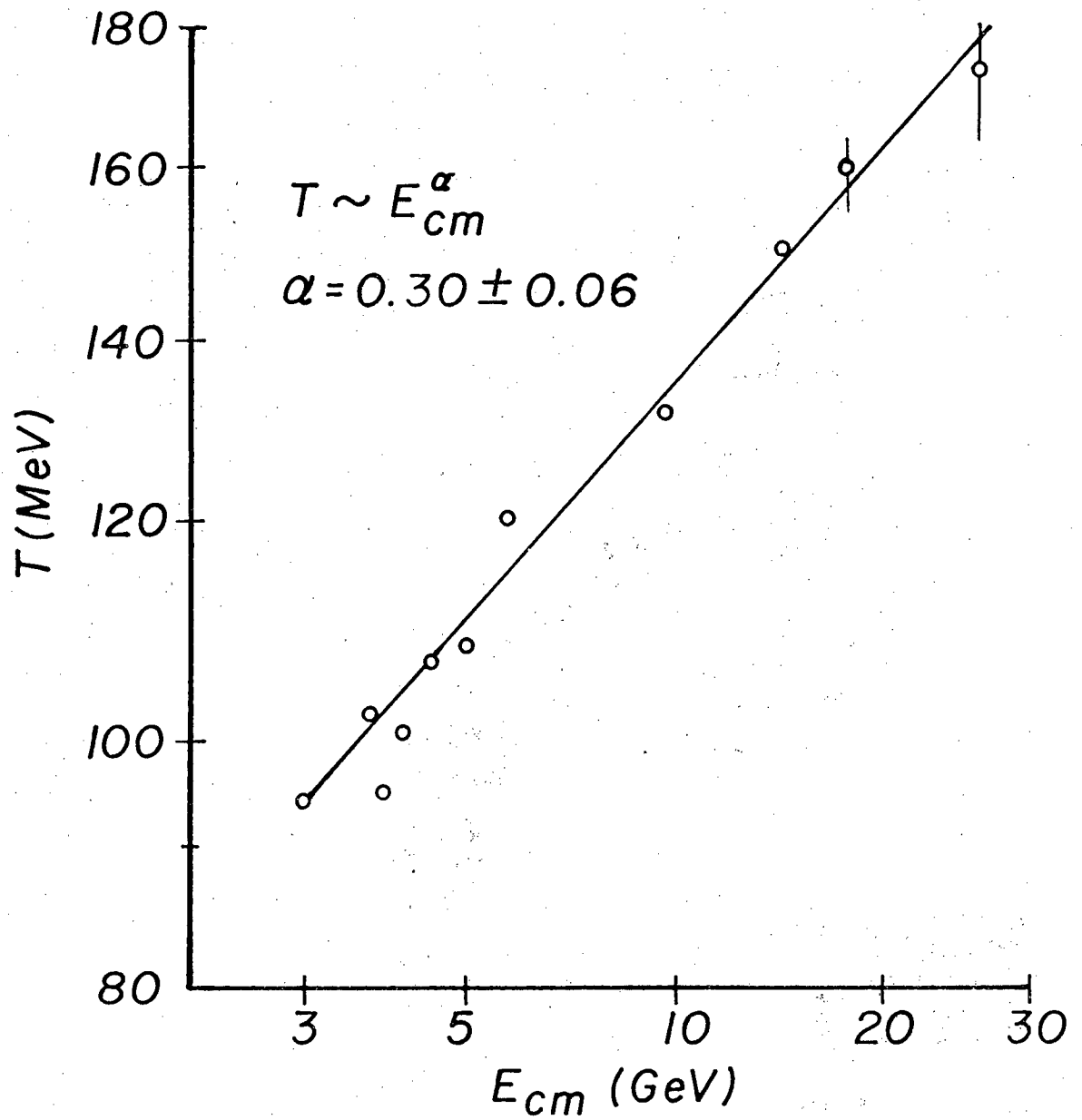
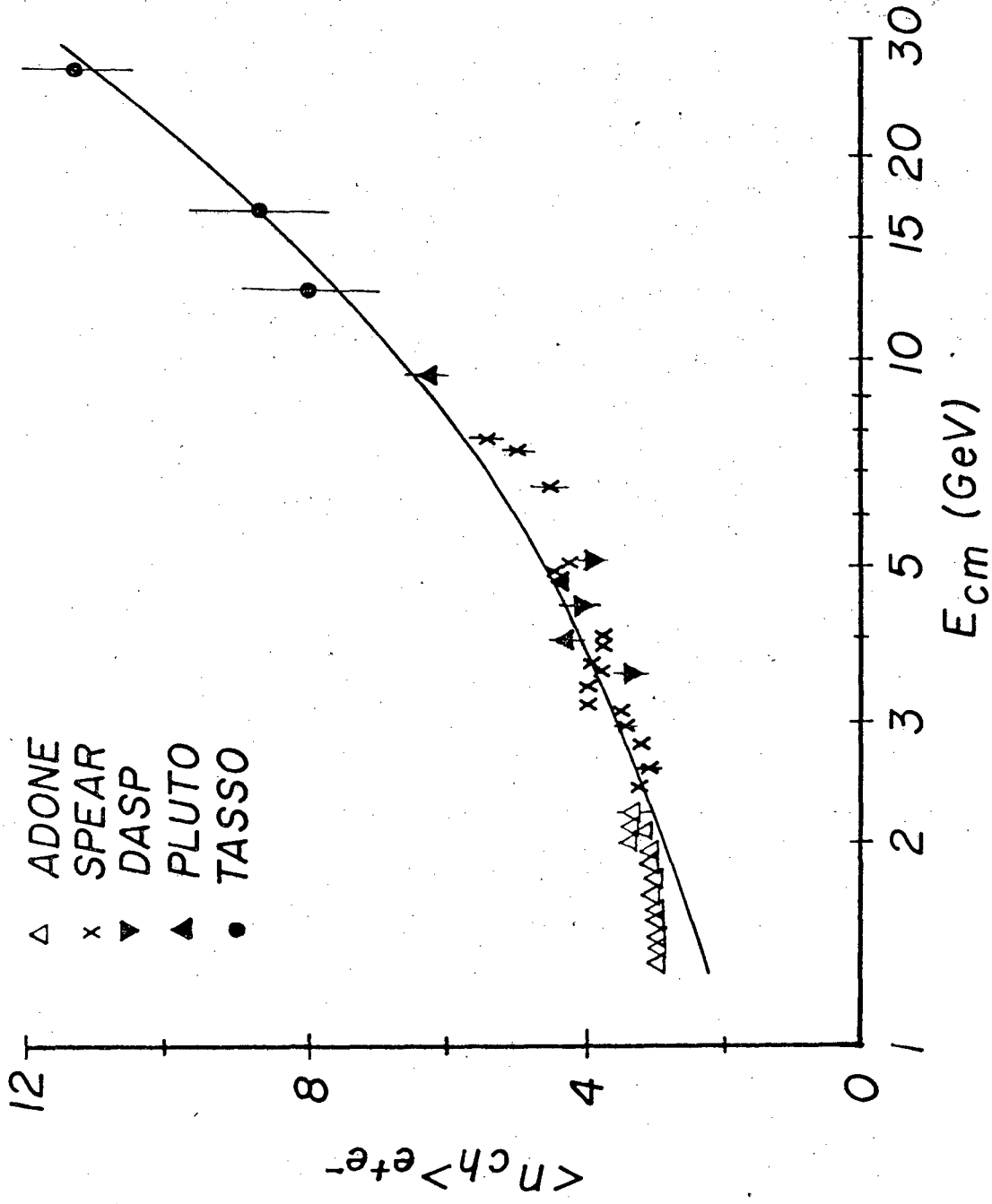


Fig. 1



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Fig. 2

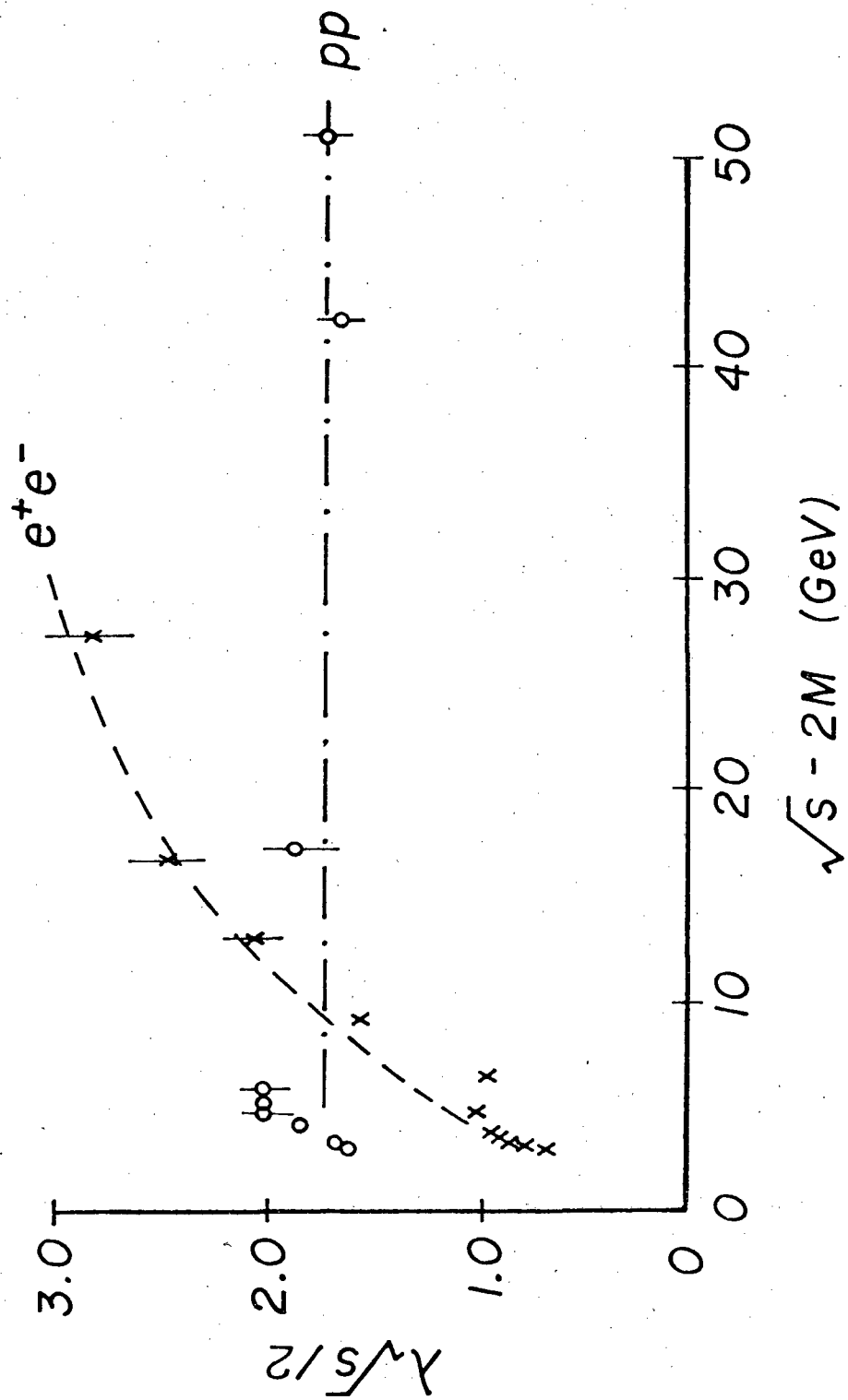


Fig. 3

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