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Authors

Stoianov, Ivilin

Umilta, Carlo

Zorzi, Marco

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Computational Investigations of the Simon and the SNARC Effects

Ivilin Stoianov (Ivilin.Stoianov@unipd.it)

Università di Padova, Dipartimento di Psicologia Generale, via Venezia 8, Padova, Italy

Carlo Umiltà (Carlo.Umiltà@unipd.it)

Università di Padova, Dipartimento di Psicologia Generale, via Venezia 8, Padova, Italy

Marco Zorzi (Marco.Zorzi@unipd.it)

Università di Padova, Dipartimento di Psicologia Generale, via Venezia 8, Padova, Italy

Abstract

This article presents computational investigations of the Simon (location-response) and the SNARC (numerical-response) correspondence effects, with the aim of shedding light on the issue of whether they arise from independent or shared spatial coding and spatial processing mechanisms. A neural network was trained to perform a parity-judgement task on lateralized numerals, following pre-training on location-response and numerical-response associations. Results showed independent effects of Simon and SNARC correspondences (i.e., without interaction between them), which supports the hypothesis that the two effects originate from different neural mechanisms.

Keywords: connectionist modelling; Simon effect; SNARC effect; Boltzmann machine

Introduction

Manual responses to lateralized stimuli are faster when the spatial location of the stimulus corresponds with that of the response, both when the spatial dimension is task-relevant (spatial-compatibility effect; see Umiltà & Nicoletti, 1990, for review) or task-irrelevant (the Simon effect; Simon & Rudell, 1967; Lu & Proctor, 1994, for review). In the latter case, the response is determined by a task-relevant specific stimulus dimension (e.g., color). In a typical Simon task, the subjects see lateralized color stimuli (e.g., red or green squares) and are instructed to respond with one hand to one color and with the other hand to the other color. One important finding about the Simon effect is that it decreases with increasing reaction times (e.g., Hommel, 1994). Hence, the Simon effect is assumed to originate from the processing of an automatically activated, but task-irrelevant, short-lasting positional code that in turn activates a response code competing at the response-selection level with the task-associated response (Zorzi & Umiltà, 1995). The effect is typically explained with dual-route processing models, in which one route processes the relevant stimulus dimension, whereas the second route processes the task-irrelevant dimension (location) (De Jong, Liang & Lauber, 1994; Tagliabue, Zorzi, Umiltà, & Bassagnani, 2000).

Response speed and accuracy in tasks where the relevant stimulus dimension is numerical information (e.g., parity) is also modulated by a specific spatial-numerical

correspondence, known as the Spatial-Numerical Association of Response Codes (SNARC) effect (Dehaene, Bossini & Giraux, 1993). The effect is culture-specific: in western countries, in which writing and reading direction is left-to-right, the effect appears as a faster response on small (big) numbers with the left (right) hand, and it reverses in Arabic cultures, in which reading direction is right-to-left. The effect is explained by an automatic activation of spatial-numerical representations (i.e., a mental number line; see Zorzi, Priftis, & Umiltà, 2002) that prime lateralized responses and interfere with the task-relevant response code. *Prima facie*, the SNARC effect and the Simon effect appear to be very similar. First, both depend on processing of task-irrelevant information within a spatial route; second, both depend on activation of spatial codes that in turn prime the response codes. Clearly, this raises the possibility that the two effects are produced by the same mechanisms (which would in turn suggest a common neural basis). However, in a study that aimed at producing both effects within the same experiment – where Arabic numerals were presented to the left or to the right of the centre of the screen and the subjects had to respond with the left or the right button according to the parity of the stimuli – Mapelli, Rusconi and Umiltà (2003) found additive Simon and SNARC effects. That is, parity judgments on lateralized numerals produced the two main effects but no interaction. The lack of interaction, according to the Additive Factors Method (AFM; Sternberg, 1969), suggests that the two effects rely upon distinct mechanisms. This result, however, was questioned by a recent study that found interaction between the two effects (Gevers, Caessens & Fias, *in press*).

There are relatively few attempts at modelling stimulus-response compatibility phenomena with connectionist simulations. Zhang, Zhang and Kornblum (1999) investigated various types of stimulus-response compatibility problems, among which the Simon effect, but the network models were completely hand-crafted. The Simon effect was explained with an indirect interference of the irrelevant stimulus dimension at the response level (output units), but no response times analysis was provided. Moreover, since the irrelevant positional code was not decayed, the model cannot accommodate the fundamental observation of a decreasing Simon effect with increasing response times. In contrast, the distribution of RTs and the

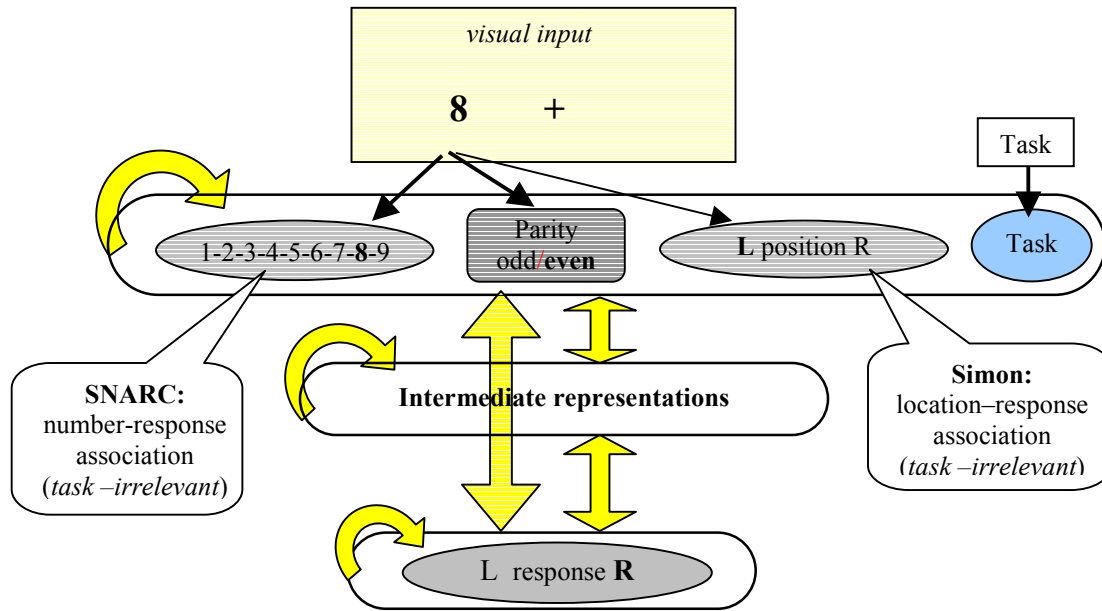


Figure 1. An associative model of spatial stimulus-response compatibility based upon the associative Boltzmann-machine trained on a parity-judgement task with lateralized numerals. Input is stimulus location, parity, magnitude, and task. Output is response code. The intermediate hidden layer allowed the development of internal representations.

time course of the Simon effect was of primary concern in the simulations of Tagliabue et al. (2000), which were based on variants of the network model of Zorzi and Umiltà (1995). The simulations revealed that a dual-route model comprising direct location-response connections and indirect feature-response and location-response connections could best explain their human RT-data. However, architecture and connection weights were set by hand. Erlhagen and Schoner (2002) modeled movement preparation with a dynamic field-theory model, in which stimulus-response compatibilities, and the Simon effect in particular, were also explained with an automatic modulatory input from the task-irrelevant dimensions. Their model, however, is very general and no learning and architecture were considered. To the best of our knowledge, no published study has computationally investigated the SNARC effect.

The current work aimed at investigating the computational bases of the two effects, by means of connectionist simulations of the parity-judgement task on lateralized numerals used by Mapelli et al. (2003). We employed cognitively plausible neural networks and minimal architectural assumptions. Our basic prediction was that both location-response (Simon) and numerical-response (SNARC) correspondences should significantly decrease response times with respect to non-corresponding trials. We particularly looked for an interaction between the effects, the occurrence of which would suggest common computational basis according to the AFM. We also

investigated the network connectivity to gather insights into the computational mechanisms underlying the two effects.

An associative model of stimulus-response compatibility

Stimulus-response compatibility effects were studied in the context of the parity-judgement task, in which odd and even numbers were associated with left- and right-hand responses, respectively. Stimuli were numbers 1...4 and 6...9, presented to the left or to the right of fixation. Location and number magnitude were task-irrelevant features, but associations between left (right) stimulus location and left (right) hand responses, as well as associations between small (big) numbers and left (right) hand responses were assumed to be present (i.e., pre-learned) before learning the parity task. These previous associations were the primary causes of the Simon and SNARC effects.

The model is shown in Figure 1. It is based upon the Boltzmann machine, an associative-memory neural network trained with a cognitively plausible learning mechanism (see below) that we have previously used for the simulation of various number processing tasks (e.g., Stoianov et al., 2002; Zorzi, Stoianov, & Umiltà, 2005, for review). One group of neurons in the visible layer provides the following input to the model: (i) location (left, centre, right), (ii) magnitude, (iii) parity, and (iv) task: response determined by location, magnitude, or parity. Other neurons in the visible layer are

output neurons that encode the response (left-hand, right-hand). The three locations were encoded with a distributed code: left [1 0], centre [1 1], and right [0 1]. Central location was encoded in order to examine location-neutral conditions. Numbers were encoded semantically with a linear *Number-line* code (see Zorzi et al., 2005), whereby a specific number is represented by the position of the hill of activity over a set of units labelled [1 ... 9]. Parity was localistically encoded with two features, standing for odd and even. The tasks were encoded by three dedicated units: location-based response, magnitude-based response, and parity-based response. All visible neurons were interconnected, which allowed the network to form any direct association within the visible layer. The network had 10 hidden units, which allowed indirect associations through intermediate representations.

Learning algorithm

Boltzmann Machines (Ackley, Hinton & Sejnowski, 1985) are associative networks of stochastic neurons that iteratively generate patterns according to the distribution of the data learned. They consist of a layer of visible neurons encoding the presented pattern and a layer of hidden neurons that learn complex statistical dependencies among the data observed at the visible layer. The networks are fully connected, without structural biases. To generate patterns, after initialisation of all neurons, the networks iterate until convergence by updating all neurons in parallel, or asynchronously. The number of steps to convergence can be readily interpreted as response time, to be matched against human RT data. Originally, Boltzmann Machines were trained with a contrastive Hebbian learning algorithm: in a positive phase corresponding to classical Hebbian learning, patterns were clamped to the visible layer; the hidden units were settled and the weights were augmented with the mean correlations between every coupled neurons. In a second, negative phase (anti-learning), the visible neurons were unclamped; all neurons settled again, and the weights were decreased with the mean correlations for this step. In this way, the visible neurons learned to reproduce the data. However, this stochastic learning algorithm is extremely slow.

Hence, we used the approximate deterministic learning algorithm of Welling and Hinton (2002) that dramatically speeds-up the simulations, maintaining biological plausibility (Contrastive Divergence Mean-Field learning). The algorithm operates upon the mean-field activities m_i of neurons $i=1...n$ and are given by the solution of a set of n coupled mean-field equations:

$$\| m_i = \sigma \left(\sum_{j=1..n} w_{ij} m_j + \theta_i \right) \quad (1)$$

where σ denotes the sigmoid function; w_{ij} is the weight from unit j to unit i , and θ_i is the bias of unit i . Note that these equations correspond to “classical” neuronal activation with recurrent connectivity. Then, the update Δw_{ij} of a weight connecting two units is proportional to the difference between the correlations between their mean-field activities

at time zero (positive phase) and one-pass data reconstruction (negative phase):

$$\Delta w_{ij} = \eta (m_i^0 m_j^0 - m_i^1 m_j^1) \quad (2)$$

where η denotes the learning coefficient. In our simulations we used this learning algorithm in an unsupervised mode, i.e. during learning there was no distinction between input and output neurons (for details, see Stoianov et al., 2002).

Training and testing procedure

The network was first *pre-trained* for 5.000 epochs on location-response and numerical-response associations, in proportion of 80% and 20%, respectively – a ratio that could roughly reflect the natural distribution of both types of associations. Each epoch included the combination of all numbers (1...9) and locations (L,C,R), resulting in a total of 27 patterns. In the spatial correspondence task, the left location was associated with left response, the right location with right response, and the central location was randomly associated with left or right responses. The number-correspondence task, in turn, associated small numbers with left response and large numbers with right response in a stochastic manner, linearly dependent on the distance from the halfway number of 5. During either of the tasks, the corresponding task-code was activated, simulating attention to location or number.

The network was then trained on the *parity task*, in which odd numbers were associated with left response and even numbers with right response. Stimuli were the same as in the previous learning tasks, with the exception of number 5, which was excluded in order to allow balanced distribution of left- and right-responses. During this phase all weights were free to change, which allowed the network to reallocate computational resources, in particular at the level of the hidden units. Learning in this phase was stopped at convergence of performance level.

Performance on the parity task was tested by presenting lateralized numerals, as in the Mapelli et al. (2003) study. Thus, the input patterns were defined by the combination of location (left or right) and identity of the numeral (1-9, excluding 5). To accommodate earlier psychological findings about relatively slower feature identification and fast decay of the positional code (see Tagliabue et al., 2000), the onset of numerical features was delayed by 30 iterations and the positional code was offset after a small delay of 7 iterations. Neuronal states were updated with a momentum of 0.97. Each testing condition was repeated 100 times, varying only the initial states of the hidden units and the Gaussian noise ($\mu=0$, $\sigma=0.1$) added to the response units. Response retrieval was stopped at convergence of all neurons, i.e., when the change in the neuron’s activation between two processing time-steps has no significance (being smaller than a constant $\epsilon=0.0001$). The number of iterations until response were recorded for statistical analysis.

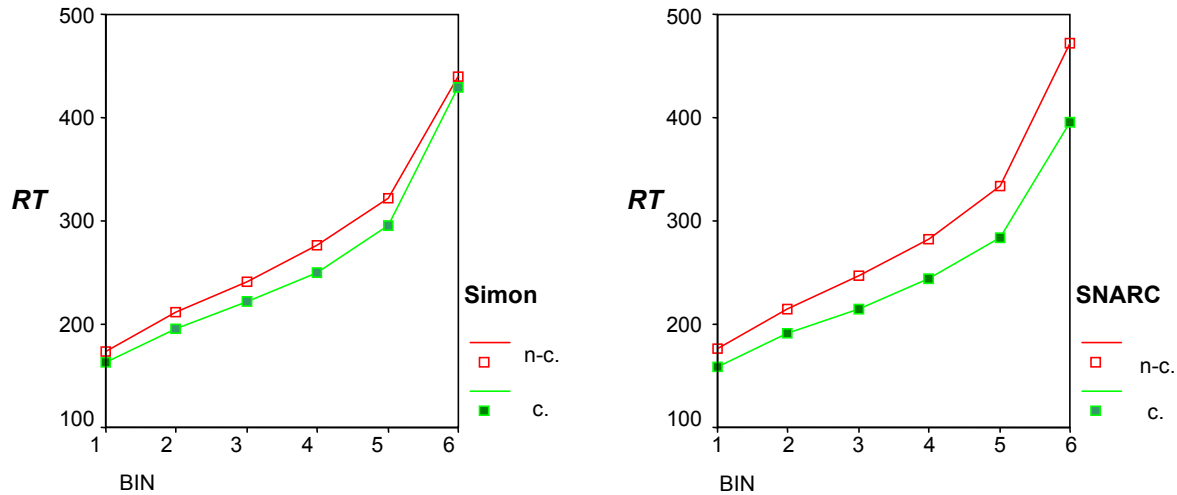


Figure 2. Network RTs as a function of Bin, Simon, and SNARC correspondences ('c.'=corresponding, 'n-c.'=non-corresponding).

Results

Pre-training: The network learned all location-response and numerical-response associations. The location-response associations were learned as both direct input-output and indirect associations. The numerical-response associations were encoded with direct links only, although some hidden units showed preference for certain numbers. Thus, the hidden units mainly encoded location-response associations. The two response units at the visible layer, as well as any other coupled neurons (e.g., location and task), developed strong inhibitory interconnections (similarly to the weights in Figure 3).

Parity-judgement: The parity-judgement task was learned for about 1000 epochs, relying mostly on direct parity-response connections, and preserving both the direct location-response and numerical-response associations (see Figure 3). Network performance on this task was then examined by multiple response retrieval. Erroneous responses were about 5% - a typical level of errors for humans in this task.

RT analysis: Response times on correct trials were first analysed for the SNARC and Simon effects with a 2x2 ANOVA. As predicted, both the location-response and numerical-response correspondences significantly influenced response times ($p < 0.001$), indicating faster responses for corresponding trials compared to non-corresponding trials. However, no interaction between the two effects was found ($p > 0.38$), which suggests their reliance on different computational resources. To examine the time-course of the two effects, the RTs for each combination of SNARC and Simon correspondences were divided into 6 bins, with equal number of items per bin. A three-way ANOVA – Simon (2) x SNARC (2) x BIN (6) – showed that the effects also depended on the BIN factor. In

particular, the Simon x BIN interaction ($p < 0.05$) shows that the Simon effect disappears at the slowest bin ($p > 0.40$).

Magnitude is another factor that generally affects number processing, so we examined its effect on the RTs. For this

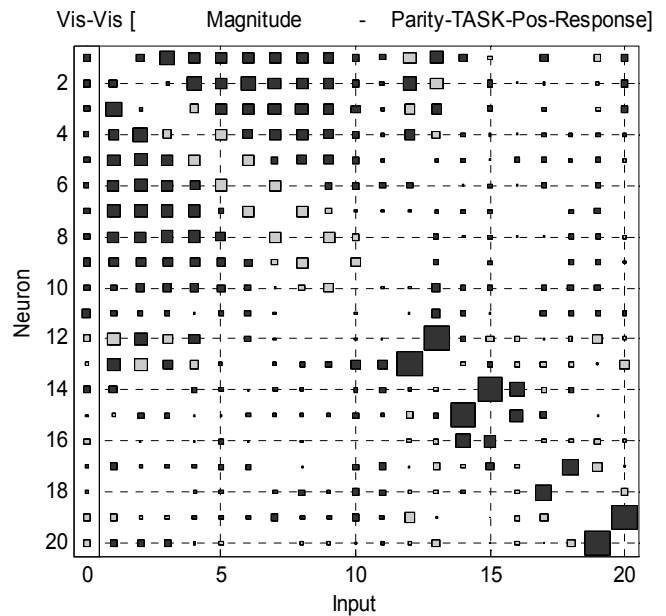


Figure 3. Direct connections among the neurons of the visible layer, after learning the parity task. The visible neurons encode: magnitude (neurons 1-11), parity (12-13), task (14-16), position (17-18), and response (19-20). Gray and black boxes represent positive, and negative values, respectively; box size represents absolute strength. We note the strong mutual inhibitory connections and position-response, and parity-response excitatory associations.

purpose, we first grouped odd and even numbers to produce four magnitude categories (1-2, 3-4, 6-7, 8-9). A three-way ANOVA on the RTs with predictors SNARC, Simon, and magnitude revealed significant main effect of each predictor ($p < 0.001$), but also an interaction between the SNARC effect and magnitude ($p < 0.001$). The latter shows that the size of the SNARC effect was bigger towards the extremes of the numerical interval 1-9.

Internal representations: Most of the hidden units were specialized for responding to parity, but preserving their prior response preference to location. Thus, a kind of intermediate feature-response code modulated by location was formed. Units responding to opposite features developed mutual inhibitory connections, which caused competition also at the intermediate levels. In contrast, units responding to the same features supported each other by means of positive interconnections (previous models did not include interaction among the internal code; e.g., Tagliabue et al., 2000).

In sum, partially maintained pre-learned spatial response preferences of the hidden units was one of the main determinants of the model's behaviour.

Discussion

This study investigated the computational basis of two stimulus-response compatibility phenomena: the Simon and the SNARC effects.

One source of the Simon effect in the model was a conflict between the task-relevant response code and the short-lasting positional signal coming from the direct location-response links that pre-activated or inhibited the target response. Indeed, the effect disappeared when the positional code offset was minimized: in a control simulation that used the same network and parameters, except for the positional code offset that was set to one, the network RTs again exhibited the SNARC effect ($p < 0.001$), but not the Simon effect ($p > 0.40$).

Therefore, a kind of intermediate location-response code was maintained active even after decaying the positional code at the input level, due to the positive feedback among hidden units responding to the same features. This signal interfered with the correct response in the non-corresponding trials, increasing RTs even in the case of relatively short positional code offsets. Thus, the typical pattern of the Simon effect (decreasing effect for bins containing slower RTs) was simulated only by means of Gaussian noise added to the input of the neurons (the noise gradually cancelled the intermediate positional signal).

In sum, the size of the Simon effect and its interaction with bin depended on the delay of the spatial code offset (no effect for a very fast offset; no cancellation of the effect for relatively long offset delays) and the size of the noise induced (a minimal level of noise was needed in order to cancel the effect for the slowest RTs).

The SNARC effect was relatively strong and it was caused by the direct associations between the number-line code and the response neurons that facilitated (or inhibited) response selection. The size of the effect was modulated by

the number magnitude, appearing stronger towards the extremes of the numerical interval. In effect, the change of response preference in humans is also gradual (Dehaene et al., 1993; Fias et al., 1996).

In agreement with the results of the Mapelli et al. (2003) study, the SNARC effect and the Simon effect did not interact (i.e., they were additive). According to the AFM, this result strongly suggests their reliance upon different neural mechanisms. From a computational point of view, the functional analysis of the network showed that (i) independent direct location-response and numerical-response associations had been formed during pre-training, and (ii) the hidden units tuned on parity had developed during pre-training a preference mainly for location, which allowed the existence of indirect parity-response associations modulated by location but independent of number. Thus, the functional analysis confirmed the RT-based finding that the two effects relied on different computational mechanisms, even if spatial and numerical signals necessarily meet at the response-selection (output) level.

We note that the distribution of intermediate resources reflects the distribution of location-response compatibility and numerical-response compatibility tasks during the pre-training (80% vs. 20%): the network has apparently used most of the indirect resources to encode location-response dependencies. The situation could be changed with more even distributions of the tasks (e.g., 50% - 50%), which would be however less plausible: spatial associations between stimuli and responses located on the same side (such as reaching for a right-located object with the right hand) are much more frequently experienced than spatial-numerical associations.

The model presented in this article is consistent with previous models of spatial stimulus-response compatibility, in particular with those developed by Zorzi and colleagues (Zorzi & Umiltà, 1995; Tagliabue et al., 2000). Dispensing with a hand-wired architecture represents a major advantage of the present model over its predecessors. However, our results stepped upon numerous assumptions: i) a simple uniform architecture; ii) limited intermediate computational resources (just 10 hidden units); iii) a simple positional code (only two neurons, encoding L, C, R); iv) a specific distribution of location-response and numerical-response associations during learning (80% vs. 20%). Therefore, further systematic simulations with a richer positional signal and a different hidden layer size are needed to strengthen the conclusions. We also plan to investigate in detail the modulation produced by the task signal upon the intermediate resources.

In conclusion, the simulations demonstrate that both the Simon and the SNARC effect arise at the response-selection (output) level, where all input signals converge, but they are based upon different mechanisms involving distinct processing pathways.

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