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INTRODUCING SITE QUALITY INTO RECREATION DEMAND FUNCTIONS

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Introducing Quality Into Demand Functions

Here we discuss some of the modelling issues that arise when one attempts to incorporate quality attributes into demand functions for recreation sites. We assume that A decision been made as to whether objective or subjective measures of site quality are used. Also, for simplicity we focus on the demand fct a single site; the extension of the ideas discussed here to a system of demand functions for many sites is straightforward.

Our starting point is thus the ordinary demand function for a recreation site, $x = h(p,y)$ where x is the number of visits to the site by an individual, p is the cost of visiting the site (possibly including time costs), and y is the individual's income, the last two variables being measured relative to the price of a numeraire good. We represent the quality of the site by the variable b which we shall treat as a scalar, although in practical applications it almost certainly will be a vector. The question at hand is: how should b be introduced into the demand function $h(p,y)$?

The simplest procedure is to add b to $h(p,y)$ in a fairly arbitrary manner, perhaps by making some coefficients of $h(\cdot)$ a function of b. For example, if $h(p,y) = \alpha + \beta p + \gamma y$, one could now write

(1) $x = h(p,b,y) = \alpha + \beta p + \gamma y + \delta b \quad \alpha, \delta > 0; \ \beta < 0.$

The advantage of building b directly into the ordinary demand function in this manner is that one can immediately discern the behavioral implications for consumer choices. In the case of (1) for example, an increase in quality shifts the graph of the demand curve in price-quantity space outward, in a parallel manner: demand increases by the same quantity regardless of price or the individual's income. The disadvantage of incorporating quality in this manner is that one cannot easily discern the implications for consumer preferences. It is not true, for example, that if quality is introduced into the demand function in a way such that $\partial h/\partial b > 0$, this necessarily implies that the consumer's welfare is increased by an improvement in quality (i.e. $\partial u(x,b,z)/\partial b > 0$, where z is the numeraire commodity and $u(\cdot)$ is the direct utility function). Conversely, if $\partial h/\partial b < 0$, this does not necessarily preclude $\partial u/\partial b > 0$.[1] Thus, in the case of (1), if $\gamma < 0$ (i.e. the commodity is an inferior good) while $\delta > 0$, it turns out that $\partial u/\partial b < 0$. The consumer's welfare is reduced by an increase in quality. This occurs because, as shown by LaFrance and Hanemann (1984), the direct utility function which gives rise to demand function (1) is

(2) $$u(x,b,z) = \left(\frac{\gamma x + \beta}{\gamma^2}\right) \exp\left[\frac{\gamma(\alpha + \delta b + \gamma z - x)}{\gamma x + \beta}\right].$$

Even if $\gamma > 0$ so that $\partial u/\partial b > 0$, the seemingly innocuous formulation (1) contains a peculiar implication about the consumer's preferences. For, as shown by Hanemann (1980, 1982b) and

Hausman (1982), the indirect utility function corresponding to (1) is

$$(3) \qquad v(p,b,y) = e^{-\gamma p}\left[y + \frac{1}{\gamma}\left(\beta p + \frac{\beta}{\gamma} + \alpha + \delta b\right)\right].$$

It follows from (3) that the compensated demand function, $x = g(\cdot)$, is independent of quality.[2] Moreover, the compensating and equivalent variations for a change in quality from b^0 to b^1, , defined respectively by

$$(4) \qquad v(p,b^1, y-C) = v(p,b^0, y)$$

and

$$(5) \qquad v(p,b^1,y) = v(p,b^0,y+E)$$

have the property that

$$(6) \qquad C = E = \frac{\delta}{\gamma}(b^1 - b^0);$$

i.e., there is no difference between the amount that the individual would be willing to pay to obtain the change and the amount of compensation he would require to forego it.[3]

To some extent, these results flow from the special structure of preferences associated with the linear demand function (1). Thus, for example, if one employs a semilog demand function of the form

$\ln x = \alpha + \beta p + \gamma y$ or $\ln x = \alpha + \beta p + \gamma \ln y$, $\beta < 0$, and makes the constant term, α, a function of b in such a way that $\partial h/\partial b > 0$, it can be shown that $C \neq E$ and $\partial u/\partial b > 0$. However, if one uses the log-log demand function $x = \alpha p^{\beta} y^{\gamma}$ or a demand function of the form $x = \alpha p^{\beta} e^{\gamma y}$ and makes α a function of quality in such a way that $\partial h/\partial b > 0$, it turns out that, while $C \neq E$, sign $(\partial u/\partial b)$ = - sign $(1+\beta)$; thus if $\beta > -1$ (i.e., the good is essential), $\partial u/\partial b < 0$. [4]

Out point is that mere inspection of the demand function $h(p,b,y)$ does not always provide a reliable indication of how quality affects the consumer's preferences. A procedure which avoids this uncertainty is to start with a utility function $\bar{u}(x,z)$ or $\bar{v}(p,y)$ and incorporate quality directly to obtain some formula $u(x,b,z)$ or $v(p,b,y)$, from which the ordinary demand function $h(p,b,y)$ can be obtained in the standard manner. If this route is followed one can simply make the coefficients of $\bar{u}(x,z)$ or $\bar{v}(p,y)$ functions of b or, alternatively, one can employ what might be called the "transformation method." In this method, one replaces the argument x (or z) in $\bar{u}(x,z)$ with some function $f(x,b)$ to obtain $u(x,b,z) = \bar{u}[f(x,b),z]$. The advantage of this method is that, for various transformations $f(x,b)$, the resulting demand function $h(p,b,y)$ can be directly related to the demand function $\bar{h}(p,y)$ associated with $\bar{u}(x,z)$. Also, the implications for preferences are readily discerned.

The simplest example is the transformation $f(x,b) = \psi(b)\cdot x$ where $\psi(\cdot)$ is some increasing function, which is known as the "scaling" or "repackaging" transformation. This was first introduced by Fisher and Shell (1971) and has been widely employed in the literature on quality and demand analysis. However, the utility function $u(x,b,z) = \bar{u}[\psi(b)\cdot x,z]$ has somewhat unusual implications which have not always been recognized by those who employ it. These stem from the fact that quality is a direct substitute for quantity in this formulation: a doubling of the quality index, ψ, has exactly the same impact on the consumer's welfare as a doubling of quantity, x. The consequences of this assumption may be observed in the ordinary demand function for x and the indirect utility function, which take the form

$$h(p,b,y) = \frac{1}{\psi(b)} \bar{h}\left(\frac{p}{\psi(b)}, y\right) \tag{7a}$$

$$v(p,b,y) = \tilde{v}\left(\frac{p}{\psi(b)}, y\right). \tag{7b}$$

Defining ε_b as the elasticity of demand for x with respect to its quality $(\partial h/\partial b)(b/x)$, ε_p, as the price elasticity of demand $(-\partial h/\partial p)(p/x)$, and η as the elasticity of the ψ function with respect to b, $(\psi'(b)\cdot b/\psi)$, it follows from (7a) that

$\varepsilon_b = -\eta(1-\varepsilon_p)$.

Thus the consumer's response to a change in quality is linked to his response to a price change, but in a somewhat peculiar manner. Even though we explicitly assume that utility is increasing in quality (i.e. $\eta > 0$),

if

$\varepsilon_p < 1$, then $\partial h/\partial b < 0$ —i.e., if demand is inelastic an increase in quality reduces consumer's demand.

An alternative transformation with somewhat more plausible behavioral implications is Willig's (1978) "cross-product repackaging" transformation $u(x,b,z) = \bar{u}\,[x, z + \psi(b)\cdot x]$. In this case

$$h(p,b,y) = \bar{h}[p - \psi(b), y] \tag{8a}$$

$$v(p,b,y) = \bar{v}\,[p - \psi(b), y]. \tag{8b}$$

Hence,

$$\frac{\partial h}{\partial b} = -\,\psi'(b)\,\frac{\partial h}{\partial p}.$$

Thus, as long as x is not a Giffen good (i.e. as long as $\partial h/\partial p < 0$), an improvement in quality raises the consumer's demand for the good, the response being proportional to the size of the price elasticity of demand. If demand is fairly inelastic (e.g. x is a necessity), a change in quality has a small effect, while if demand is very elastic, it is also very responsive to quality changes.

Another common transformation is $f(x,b) = x + \psi(b)$, which is known as "translation". In this case

$$u(x,b,z) = \bar{u}(x + \psi(b), z) \text{ and}$$

$$h(p,b,y) = \bar{h}[p, y + p\psi(b)] - \psi(b) \tag{9a}$$

$$v(p,b,y) = \bar{v}[p, y + p\psi(b)]. \tag{9b}$$

It follows from (9a) that the effect of a change in quality on the demand for x is tied to the effect of a change in income:

$$\partial h/\partial b = \psi'(b)\,[\sigma\varepsilon_y - 1]$$

where ε_y is the income elasticity of demand for x, $(\partial h/\partial y)\,(y/x)$, and σ is the budget share of x, (px/y). Thus while it is still true that $\partial u/\partial b > 0$, the demand function is increasing or decreasing in quality according to

$$\frac{\partial h}{\partial b} \gtrless 0 \quad \text{as } \varepsilon_y \gtrless \frac{1}{\sigma}. \tag{10}$$

In this context, it might be more convenient to apply the translation transformation to the <u>numeraire</u> good, and write $u(x,b,z) = \bar{u}(x, z + \psi(b))$. This generates the following demand function for x and indirect utility function [5, 6]

$$h(p,b,y) = \bar{h}\,[p, y + \psi(b)] \tag{11a}$$

$$v(p,b,y) = \bar{v}\,[p, y + \psi(b)]. \tag{11b}$$

Hence, $\partial u/\partial b > 0$ and

$$\partial h/\partial b = \psi'(b)\cdot[\partial h/\partial y] \gtrless 0 \text{ as } \partial h/\partial y \gtrless 0,$$

i.e. if x is a normal good, an increase in quality raises its demand, the increase being proportional to the income responsiveness of demand.[7]

While the method of transformations is a convenient procedure for generating general utility functions that incorporate quality and has the advantage of immediately exposing the implications for both the structure of consumer preferences and the nature of consumer choices, we do not mean to suggest that it must always be employed in preference to other methods of introducing quality into models of consumer demand. In the last analysis, the issue of the best formula for a demand function that includes quality variables, h(p,b,y), is an empirical question that can be resolved only by testing alternative formulations against data on actual choices in the presence of quality variation.

One final issue should be addressed before concluding this section: The role of the structural property of consumer preferences known as "weak complementarity", that was introduced by Maler (1974):

$$(12) \qquad x = 0 \quad \Rightarrow \quad \partial u/\partial b = 0.$$

Maler employed this property in order to derive a relation between the welfare measures C and E, defined in (4) and (5), and areas under compensated demand functions. If $u(x,b,z)$ does not satisfy this property, the area

$$(13) \qquad \int_{p}^{p^*}[g(p,b^1,u^o) - g(p,b^o,u^o)]dp$$

<u>understates</u> the quantity C, where $u^0 = v(p,b^0,y)$ and p^* is the cutoff price (possibly infinite) at which max $[g(p^*,b^1,u^0), g(p^*,b^0,u^0)] = 0$. If $u(x,b,z)$ <u>does</u> satisfy the property, the area in (13) measures C <u>exactly</u>. However, regardless of whether C and E can be measured by areas under compensated demand functions, the property (12) has considerable intuitive appeal as an axiom of consumer behavior: it implies that the consumer does <u>not</u> care about a change in a good's quality when he is not consuming the good. Both the scaling and the cross-product repackaging transformations possess this property, while the two "translation" transformations do not.

As shown in Hanemann (forthcoming), Maler's analysis was based on the implicit assumption of a "smooth" relation between quality and welfare in the utility function $u(x,b,z)$. If this smoothness assumption is dropped, the link between weak complementarity and the equivalence of welfare measures with areas under compensated demand functions changes. Suppose that, instead of

(14) $$u(x,b,z) = \bar{u}[x + \psi(b), z],$$

one writes

(15) $$u(x,b,z) = \bar{u}[x + \xi(x)\cdot\psi(b), z]$$

where $\xi(x)$ is a switching function: $\xi(x) = 1$ if $x > 0$ and $\xi(x) = 0$ if $x = 0$. Then (15), unlike (14), satisfies weak complementarity, but the change in the area under the compensated demand function implied by (15) still understates the quantity C. The only difference between (14) and (15) occurs at the boundary of the non-negative orthant, where $x = 0$; in the interior where $x > 0$, the two indifference maps coincide. Thus over the space for which $h(p,b,y) > 0$ the behavioral implications of (15) are identical to those of (14) (e.g., the ordinary demand function generated by (15) satisfies (10)).

In short, by means of the simple device used in (15), <u>any</u> demand function which on its face appears to violate weak complementarity can, in fact, be reconciled with this property. Moreover, the only way to test empirically whether the use of this device is justified is to obtain data covering cases where x is not consumed at all.[8] If one only has data for cases where a positive quantity of x is consumed, it is impossible in practice to determine whether weak complementarity holds: one cannot discriminate between (14) and (15) as the true preference structure. For this type of data set, weak complementarity is a costless assumption.

FOOTNOTES TO CHAPTER 7

1. On this point see Proposition 2(d) and 3 in Hanemann (1982a) and the accompanying discussion.

2. When $g(p,b,u)$ is independent of b, Proposition 2(d) in Hanemann (1982a) implies that sign $(\partial u/\partial b)$ = sign $(\partial h/\partial b)$ $\cdot$ sign $(\partial h/\partial y)$; this is why $\gamma < 0$ and $\delta > 0$ in (1) leads to $\partial u/\partial b < 0$. It is important to emphasize that the mere fact the $g(p,b,u)$ is independent of b does not preclude the existence of significant positive benefits to the consumer when quality improves; see the discussion of weak complementarity below.

3. It is worth noting that if, instead of (1), the demand function were $h(p,b,y) = \alpha + \beta(p/b) + \gamma y$ the same result that $C = E$ would arise, but not if $h(p,b,y) = \alpha + \beta p + \gamma yb$.

4. Although a closed form expression cannot be obtained for the direct utility function which generates some of these demand functions, the indirect utility function can be obtained--see Hanemann, 1982b - and the sign of $\partial u/\partial b$ can be deduced from this because, as shown in Prop 1c of Hanemann 1982a, sign $(\partial u/\partial b)$ = sign$(\partial v/\partial b)$.

5. The construction of the demand function (1) from $\bar{h}(p,y) = \alpha + \beta p + \gamma y$ is in fact an example of this type of translation where $\psi(b) = \delta b/\gamma$.

6. Note the implication of both (9b) and (11b) that $C = E$. In the case of (9b), $C = E = p[\psi(b^1) - \psi(b^o)]$; in the case of (11b), $C = E = \psi(b^1) - \gamma(b^o)$. Another implication of (11b), but not (9b), is that the compensated demand function for x is independent of quality.

7. As Gorman (1976) has pointed out, the scaling and translation transformations can be combined to generate more complex transformations in which the sign of $\partial h/\partial b$ depends on the signs or magnitudes of both price and income elasticities.

8. This argument is spelled out in more detail in Hanemann (forthcoming).

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