Title
Noninvariance of space/time-scale ranges under a Lorentz transformation and the implications for the study of relativistic interactions

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Noninvariance of Space/Time-Scale Ranges under a Lorentz Transformation. Implications for the Study of Relativistic Interactions.

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Special relativity

Lorentz transformation (v along x)

$$t' = \gamma\,(t - vx/c^2) \qquad \gamma = (1-v^2/c^2)^{-1/2}$$

$$x' = \gamma\,(x - vt)$$

$$y' = y$$

$$z' = z$$

Time dilation/space contraction

at rest: $\Delta t,\ \Delta x=0$ $\rightarrow$ in motion: $\Delta t'=\gamma\Delta t$

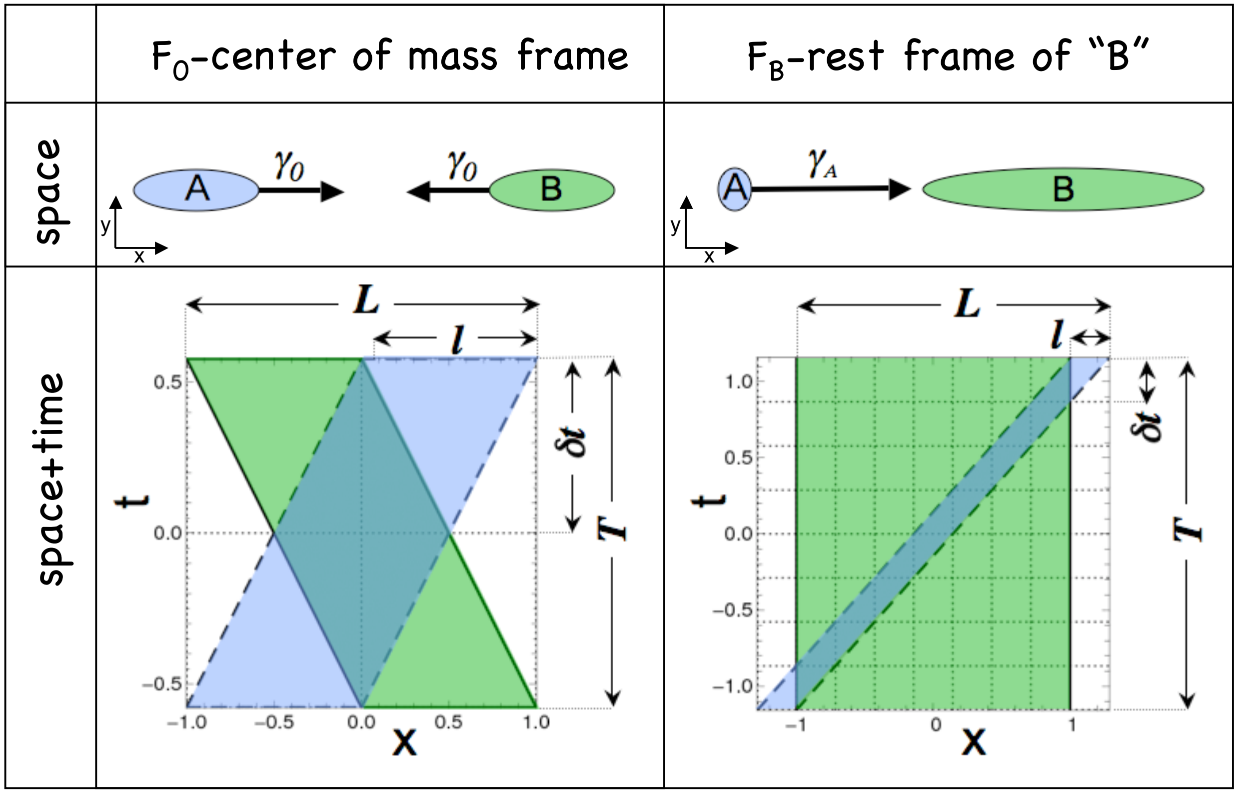
$\Delta x,\ \Delta t=0$ $\qquad \Delta x'=\Delta x/\gamma$

Lorentz invariant (invariant to change of reference frame)

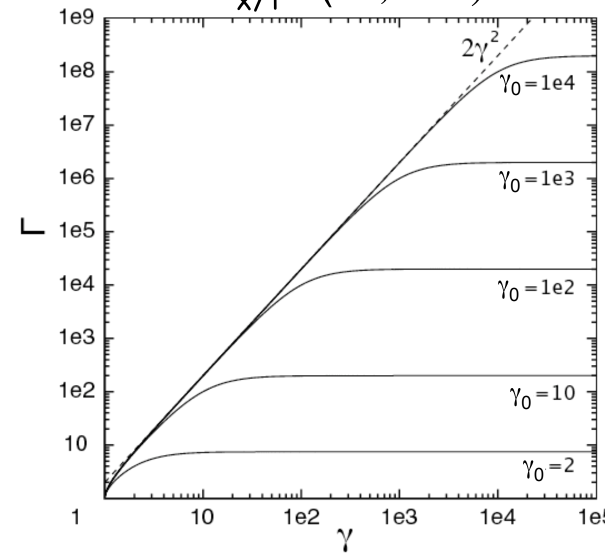
$$\Delta s^2 = \Delta x^2+\Delta y^2+\Delta z^2-c^2\Delta t^2 = \Delta x'^2+\Delta y'^2+\Delta z'^2-c^2\Delta t'^2$$

Range of space and time scale of a simple system
two identical objects crossing each other

same event as seen in two frames



$\Gamma_{x/t} = (L/l,\ T/\delta t)$*



$$\Gamma_{x/t} \propto \gamma^2$$

$$\Gamma = \Gamma_x \cdot \Gamma_t \propto \gamma^4$$

- Γ is not invariant under the Lorentz transformation.
- There exists an optimum frame which minimizes ranges.
- For PIC, Vlasov, fluid methods, cost$\propto\Gamma \Rightarrow$ huge penalty if calculation not performed in optimum frame!

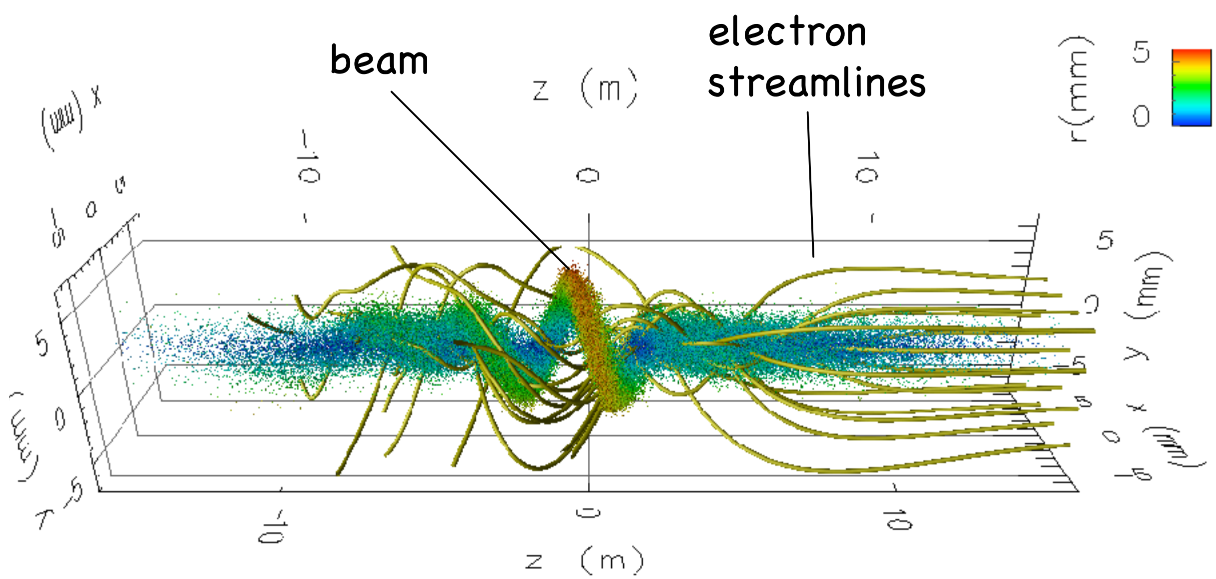
*J.-L. Vay, PRL 98, 130405 (2007)

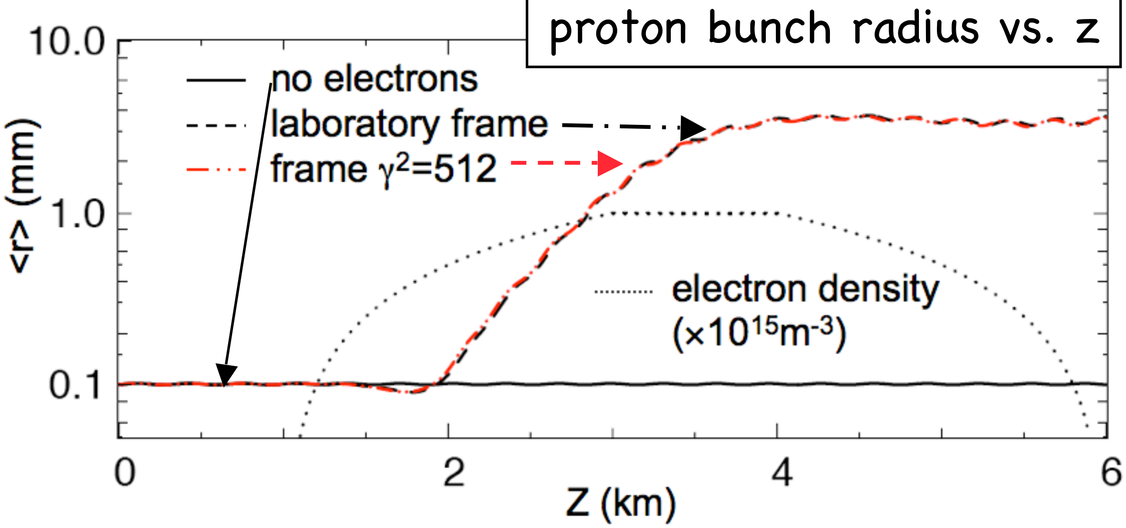
Boosted frame calculation sample
proton bunch through a given e^- cloud*

This is a proof-of-principle computation: hose instability of a proton bunch

Proton energy: γ=500 in Lab
• L= 5 km, continuous focusing

Code: WARP (Particle-In-Cell)





CPU time:
- lab frame: >2 weeks
- frame with γ^2=512: <30 min

Speedup x1000

*J.-L. Vay, PRL 98, 130405 (2007)

"This sounds like a free lunch". How is this possible?

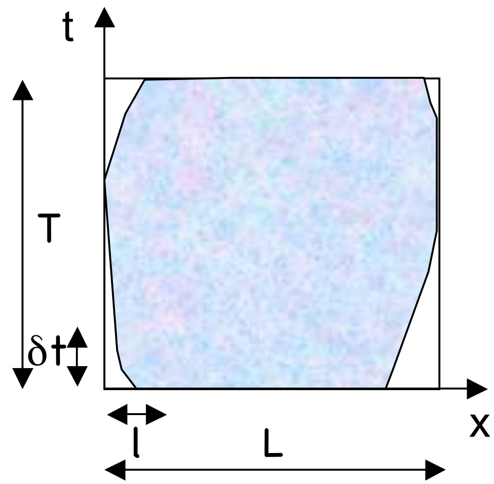
Conventional scientific wisdom: the "complexity" of a system is invariant to a change of reference frame. Is that so?

In order to respond to this question, one needs a definition of complexity which allows quantification, i.e. units of complexity.

One possible definition

Complexity = range of spatial scales x range of time scales

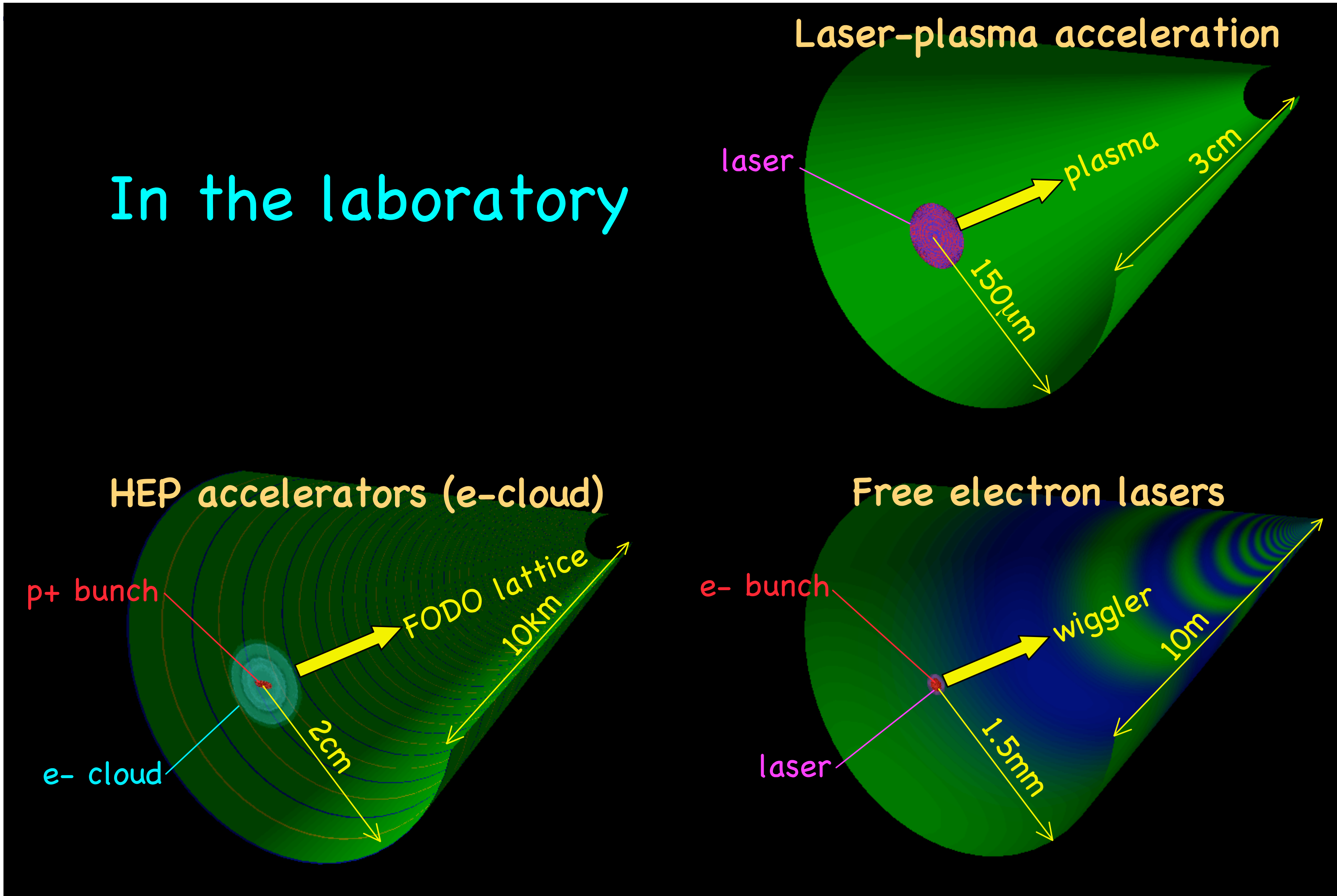
The complexity (Γ) of a system of
total length L,
shorter space scale l,
shorter time scale δt,
evolving for a total time T,
is then given by



$$\Gamma = L/l \cdot T/\delta t$$

This definition has some practical merit: difficulty in experiments or calculations often scales with ratio of scales to cover.

A few systems of interest



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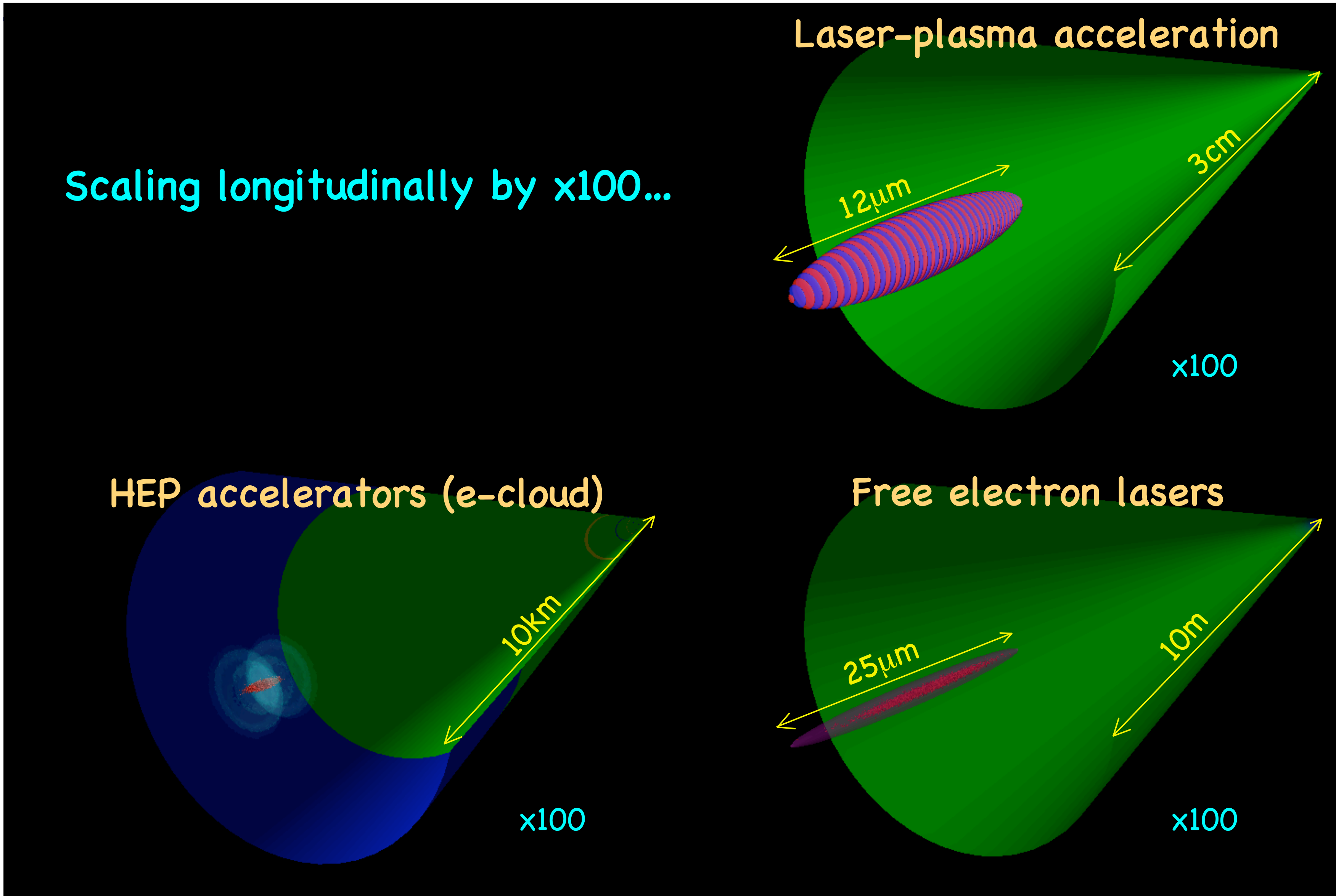

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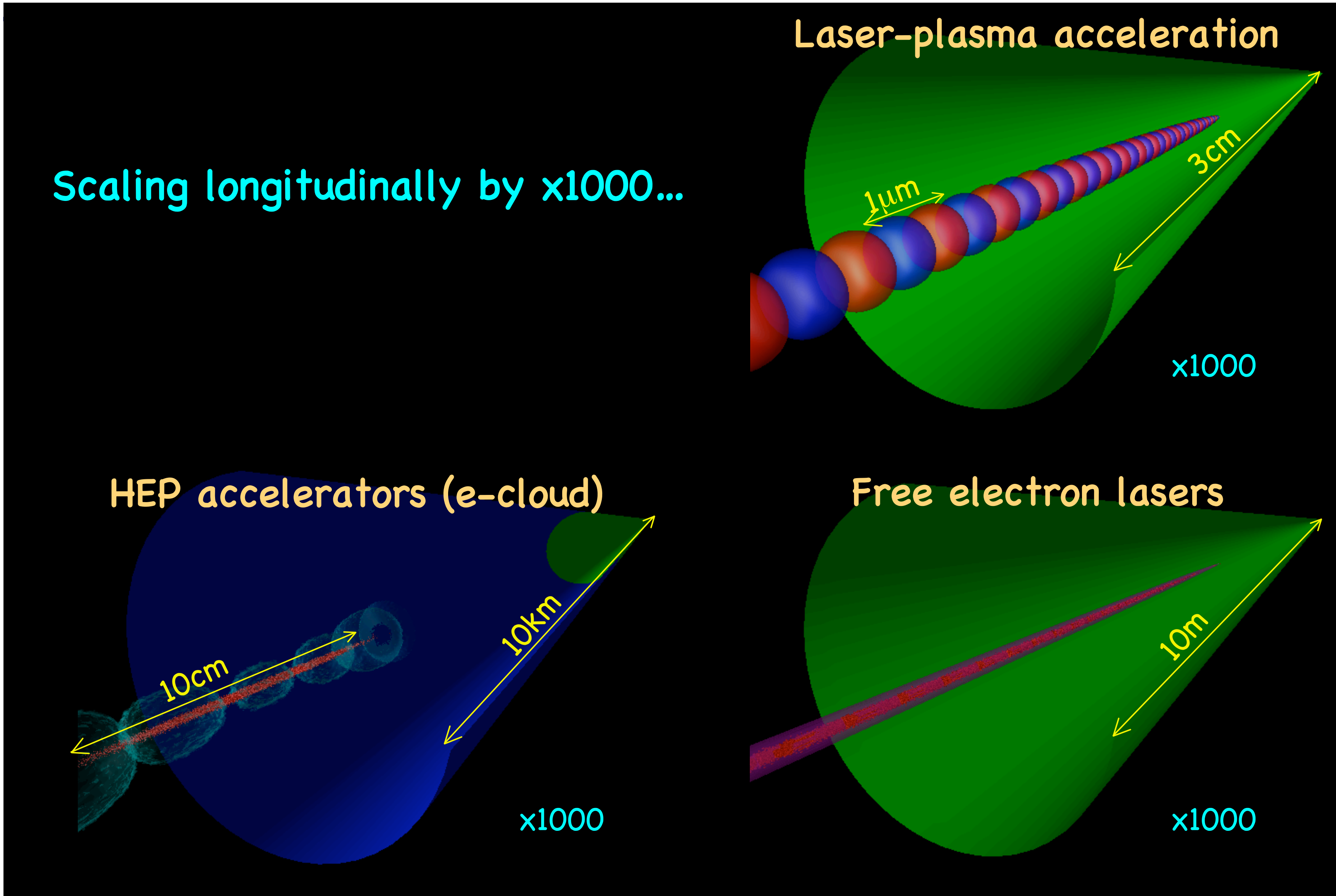
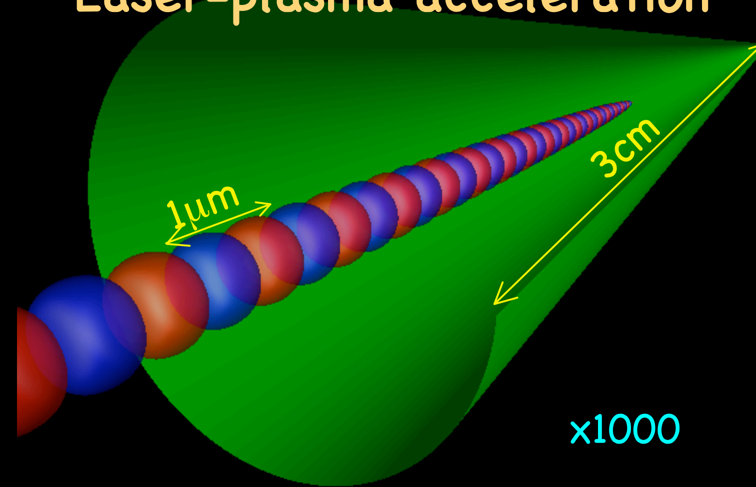


A few systems of interest

... so-called "multiscale" problems
= very challenging to model!
Use of approximations
(quasi-static, eikonal, ...).

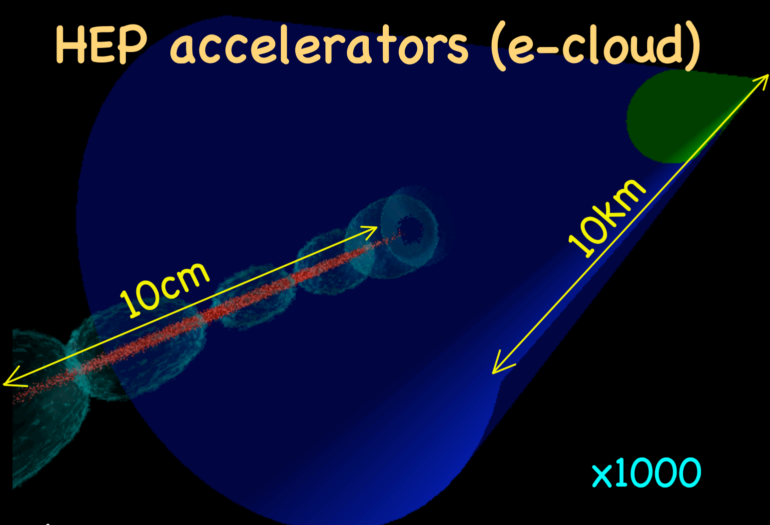
Laser-plasma acceleration



x1000

3cm/1μm=30,000.

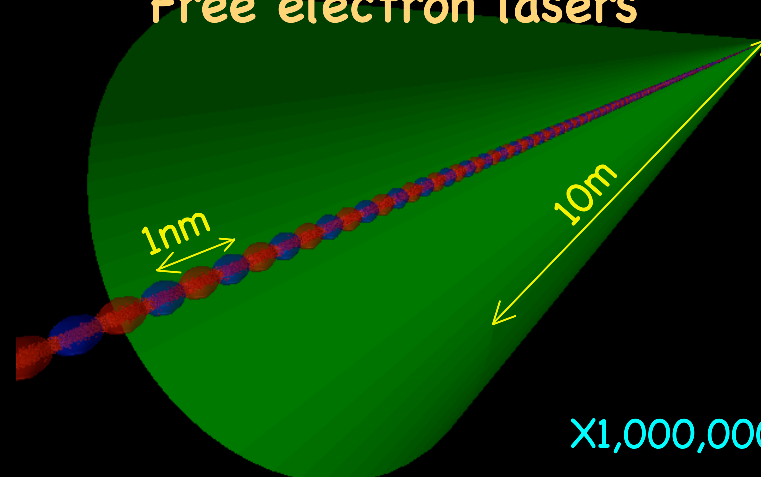
HEP accelerators (e-cloud)



x1000

10km/10cm=100,000.

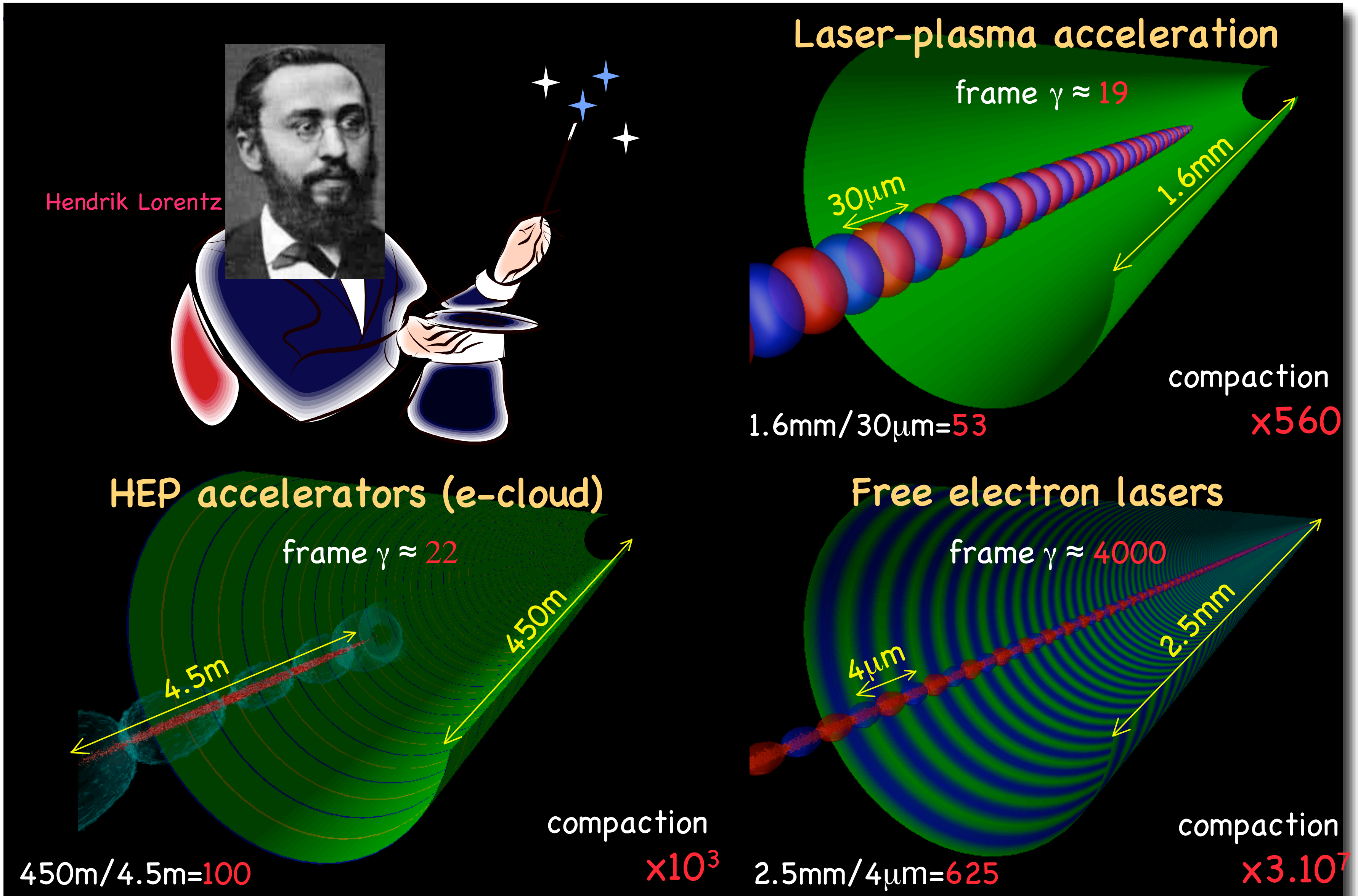
Free electron lasers



X1,000,000

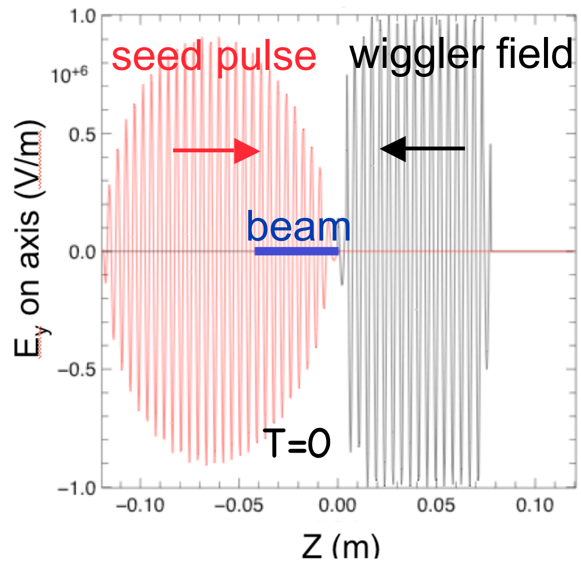
10m/1nm=10,000,000,000.

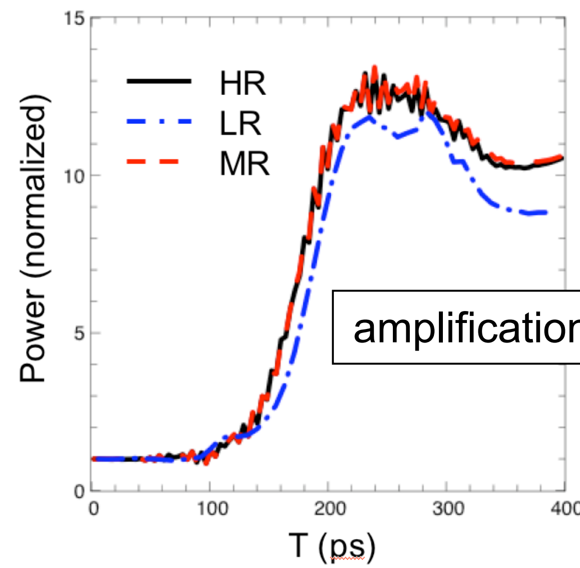
Optimum frame => large level of compaction of scale ranges

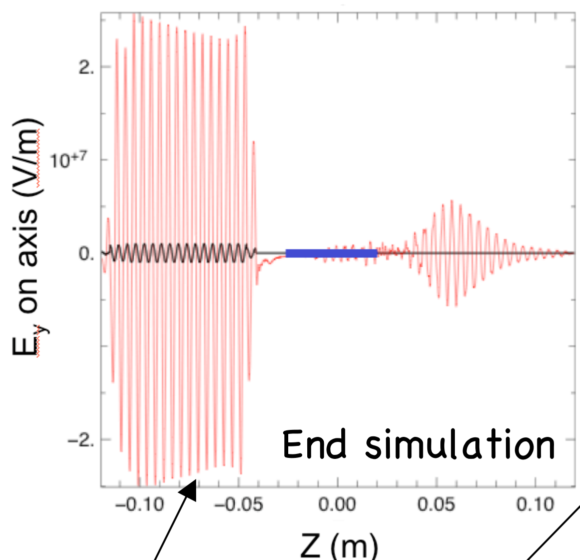


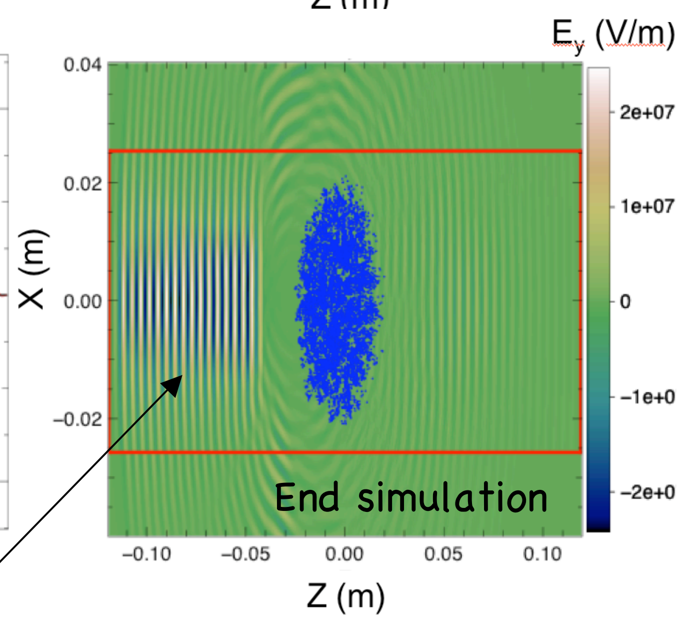
Demonstration on the modeling of a toy 2-D FEL problem

- e- and e+ beams
  - o current: 160kA
  - o energy: 76MeV ($\gamma \approx 150$)
- wiggler in lab frame:
  - o period: 43.6cm
  - o field: 37.5mT
- seed pulse (in bucket frame):
  - o wavelength: 4.3mm
  - o length: 12.cm
  - o Max electric field: 1.e6V/m









Calculation required 1000 time steps in the "bucket" frame (performed on this laptop). In the lab frame, the same calculation would have required $1000 \times 2\gamma^2 \approx 45{,}000{,}000$ time steps.

Speedup ≈ 45000

Backward emitted wave is neglected in Eikonal approximation: we get more physics!

Demonstration on the modeling of a toy 1-D LPWA problem

self-modulation instability in capillary discharge channels.

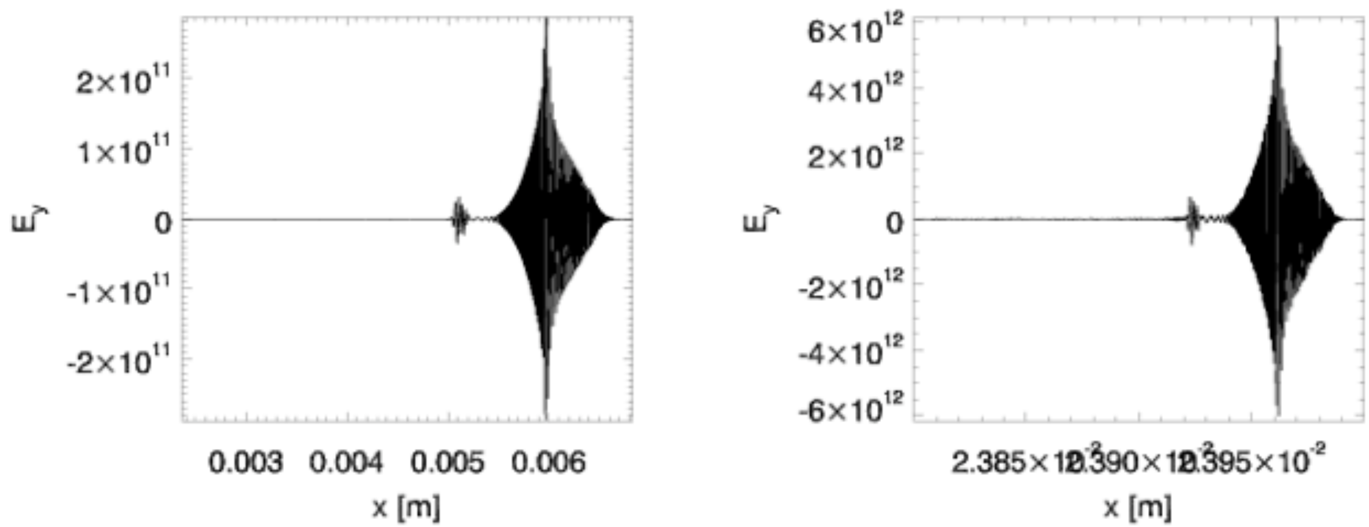


Fig. 1. (left) Self-modulated laser pulse from 1D PIC simulation in boosted frame, with 100x speedup; (right) self-modulated laser pulse from 1D PIC simulation in lab frame.

(courtesy D. Bruhwiler, J. Cary, Tech-X)

Summary

- The range of scales Γ of a system is not a Lorentz invariant and can vary greatly for some systems.

- There exists an optimum frame which minimizes Γ.

- We demonstrated speedup of x1000 for PIC simulation of relativistic beam interacting with electron background.

- It is not in contradiction with the conventional scientific wisdom that "complexity" is an invariant.

- We identified three domains of application (laser-plasma acceleration, e-cloud in HEP accelerators, free electron lasers) for which speedup ranging from 2 to 4 orders of magnitude were demonstrated on toy problems.

Outlook

- Update codes to accommodate calculations independently of the choice of frame.

- Apply to the modeling of actual experimental facilities for the three identified cases.

- Develop methods which costs do not depend on the range of scales, based on argument that "complexity", based on a more topological definition, can be made Lorentz invariant.

- Explore other applications: astrophysics,...