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Axial-vector meson mixing in orthocharmonium decays

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Abstract

In the light of the recent measurement by the BES Collaboration, the two-body decays of J/ψ and ψ' into an axial-vector meson and a pseudoscalar meson are analyzed in the framework of the $K_A - K_B$ mixing including substantial SU(3) and G parity violations due to one-photon annihilation. A somewhat puzzling pattern of the $K_1^+ K^-$ decay channel can be understood with no tight constraint on the mixing angle. The ratio of $K_1^0(1400)\overline{K}^0$ to $K_1^0(1270)\overline{K}^0$ will be the cleanest source of information to determine the mixing angle from the 1^+0^- decays in the presence of one-photon annihilation.

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1 Introduction

Mixing between the strange meson states of two axial-vector octets was established through their decay modes, mass splitting, and production in the τ decay [1, 2, 3]. In the zeroth order approximation, the mixing is maximal within large uncertainties according to the current experimental information. The BES Collaboration recently reported on the 1^+0^- decays of the orthocharmonia [4]. In their measurement, the branching fraction $\mathrm{Br}(\psi' \to K_1^+(1270)K^-)$ is far dominant over $\mathrm{Br}(\psi' \to K_1^+(1400)K^-)$, while it is in the other way in the J/ψ decay. If one ignores one-photon annihilation and assumes the maximal mixing, the $K_1(1270)\overline{K}$ and $K_1(1400)\overline{K}$ branching fractions would be equal to each other both in J/ψ and ψ' decays. We study here implications of the BES data on the $K_A - K_B$ mixing. The purpose of this paper is twofold: First to show that the BES measurement of the $K_1^+K^- + cc$ [4] is perfectly consistent with the $K_A - K_B$ mixing when one-photon annihilation is properly added to three-gluon annihilation, and second to show that the cleanest determination of the mixing angle from the 1^+0^- decay of orthcharmonia is to measure the neutral modes of $K_1\overline{K}$.

Our analysis is based on flavor SU(3) symmetry with the following standard assumptions.

(1) The decay occurs through three-gluon annihilation and also through one-photon annihilation. While the former is dominant over the latter, their relative magnitude is left as a parameter.

(2) The strange meson components of the axial-vector octets mix with each other. The mixing angle is roughly 45°, but with large uncertainties.

(3) Flavor SU(3) invariance is valid for strong interactions apart from the meson mixing. The electromagnetic current of the light quarks is an octet with negative charge conjugation.

2 SU(3) parametrization

Two axial-vector meson octets have been known. They form approximate nonets like the vector mesons though one state is still missing:

$$a_1(1260),\ K_A,\ \overline{K}_A,\ f_1(1285),\ f_1'(1420)\ \cdots 1_A^+(\mathbf{8}) - 1_A^+(\mathbf{1}), \tag{1}$$

$$b_1(1235),\ K_B,\ \overline{K}_B,\ h_1(1170),\ h_1'(?)\ \cdots 1_B^+(\mathbf{8}) - 1_B^+(\mathbf{1}), \tag{2}$$

where the numbers in boldface are SU(3) representations. K_A and K_B mix through SU(3) breaking to form mass eigenstates $K_1(1270)$ and $K_1(1400)$ as

$$\begin{aligned} K_1(1400) &= K_A \cos\theta - K_B \sin\theta, \\ K_1(1270) &= K_A \sin\theta + K_B \cos\theta. \end{aligned} \tag{3}$$

Similarly for $\overline{K_1}$ with $\theta \to -\theta$. The mixing coefficients in Eq.(3) are real provided that the dispersive part dominates over the absorptive part in the mass matrix. It should be noted that the signs of $\sin\theta$ and $\cos\theta$ can be absorbed into the phases of particle states or fields. Hereafter we shall choose the phase convention of states such that

$$0^\circ \leq \theta \leq 90^\circ. \tag{4}$$

Since $1^+_A(\mathbf{8})$, $1^+_A(\mathbf{1})$ or $1^+_B(\mathbf{1})$ plus a pseudoscalar octet $0^-(\mathbf{8})$ cannot form an SU(3) singlet of negative charge conjugation, the three-gluon annihilation allows only

$$J/\psi, \quad \psi' \to 1^+_B(\mathbf{8})\, 0^-(\mathbf{8}). \tag{5}$$

In contrast, the one-photon annihilation allows most of 1^+0^-:

$$J/\psi, \;\; \psi' \to 1^+_A(\mathbf{8})\, 0^-(\mathbf{8}), \;\; 1^+_B(\mathbf{8})\, 0^-(\mathbf{8}), \;\; 1^+_B(\mathbf{1})\, 0^-(\mathbf{8}). \tag{6}$$

Let us denote the decay amplitude of Eq.(5) by M_0, and the amplitudes for the first and second processes of Eq.(6) by M_A and M_B, respectively. Then we can describe the decay branching fractions of J/ψ or ψ' into $1^+(\mathbf{8})\,0^-(\mathbf{8})$ by three parameters,

$$\theta, \quad \xi \equiv M_A/M_0, \quad \eta \equiv M_B/M_0. \tag{7}$$

We distinguish the parameters ξ and η for ψ' from those for J/ψ by attaching them primes ($\xi \to \xi'$ and $\eta \to \eta'$).

The standard SU(3) analysis gives the decay amplitudes as shown in Table 1. For the channels involving h_1 or h_1', the $1^+_B(\mathbf{1})\,0^-(\mathbf{8})$ coupling has been related to the $1^+_B(\mathbf{8})\,0^-(\mathbf{8})$ coupling by the ideally mixed nonet scheme [6] or simply the naive quark model. Since the processes are two-body decays on mass shell, the amplitude ratios are actually coupling ratios. Therefore ξ, η, ξ', and η' can be chosen to be real numbers. We have only sign ambiguity instead of continuous phase ambiguity, when different terms interfere in squared matrix elements.

A few parameter-independent relations can be read off from Table 1.

$$|M(b_1^+\pi^-)|^2 = |M(b_1^0\pi^0)|^2. \tag{8}$$

This relation is actually a consequence of charge conjugation invariance and the isospin property of electromagnetic current alone, not of SU(3) symmetry: Since $b_1\pi$ is G odd, only the isoscalar part of electromagnetic current is capable of producing $b_1\pi$ in one-photon annihilation. Then the relation (8) follows immediately. Only photon-loop corrections on the light quarks can violate it. This is one of the special cases where even the photon interaction cannot violate isospin nor G parity invariance. Despite the very robust nature of the relation, the current data are only marginally consistent with it: $\mathrm{Br}(J/\psi \to b_1^0\pi^0) = (2.3 \pm 0.6) \times 10^{-3}$ *vs* $\mathrm{Br}(J/\psi \to b_1^+\pi^- + cc) = (3.0 \pm 0.5) \times 10^{-3}$[5]. It has been known that the similar equality $\mathrm{Br}(\rho^+ \to \pi^+\gamma) = \mathrm{Br}(\rho^0 \to \pi^0\gamma)$ is not well satisfied [5]. The $\rho - \omega$ mixing is probably responsible for the violation. The other parameter-independent relation from Table 1 is:

$$|M(K_1^+(1270)K^-)|^2 + |M(K_1^+(1400)K^-)|^2 = |M(b_1^+\pi^-)|^2 + |M(a_1^+\pi^-)|^2. \tag{9}$$

To test this sum rule, we need measurement of the $a_1^\pm\pi^\mp$ mode. More interesting is the relation,

$$|\mathrm{M}(K_1^0(1400)\overline{K}^0)|^2 = \tan^2\theta \times |\mathrm{M}(K_1^0(1270)\overline{K}^0)|^2. \tag{10}$$

It will be able to determine θ directly without referring to other parameters. In this paper we focus on $M(K_1^+(1270)K^-)$ and $M(K_1(1400)^+K^-)$ for J/ψ and ψ' on which BES Collaboration shed a light.

3 Ranges of parameters

The decay pattern of the K_1 mesons first alerted theorists of the $K_A - K_B$ mixing. Earlier theoretical works [1] pointed to the maximal mixing of $\theta = 45°$. The maximal mixing occurs if the diagonal elements of the $K_A - K_B$ mass matrix are exactly equal. Phenomenologically, however, the latest decay data still allow for sizable uncertainty [2]:

$$30° < \theta < 60°. \tag{11}$$

Since the masses of K_A and K_B are not directly measurable, they must be computed by theory. Therefore determination of the $K_A - K_B$ diagonal mass difference is subject to uncertainties of theoretical assumptions, some kinematical and others dynamical. In our analysis below, we treat the entire range of Eq.(11) as allowed.

We can make a crude estimate of magnitude of (ξ, η) and (ξ', η') by comparing the integrated decay rates of three-gluon and one-photon annihilation. From measurement [5] we know

$$\begin{aligned} \frac{\Gamma(1^-(\bar{c}c) \to \gamma \to \text{hadrons})}{\Gamma(1^-(\bar{c}c) \to ggg \to \text{hadrons})} &= 0.25 \pm 0.03 \quad \text{for } J/\psi, \\ &= 0.26 \pm 0.04 \quad \text{for } \psi'. \end{aligned} \tag{12}$$

The square roots of the right-hand sides give us an indication of the amplitude ratios. There is no compelling reason to equate these numbers to ξ and η, or ξ' and η' of exclusive channels. With no better clue at hand, however, we use Eq.(12) to set the ballpark ranges in which the parameter values are found. Our parameters are so normalized in Table 1 that the ratio of sum of the one-photon rates over all $1^+_B(8)\,0^-(8)$ channels to sum of the three-gluon rates is equal to $|\eta|^2$. The normalization of ξ is chosen in parallel to that of η. If we equate $|\eta|^2$ to the number in the first line of Eq.(12), we obtain $|\eta| \approx 0.5$. It is not unreasonable to expect that $|\xi|$ is in a range similar to that of $|\eta|$. Therefore our very crude estimate or guess is:

$$|\xi|, \quad |\eta| \leq 0.5 \quad \text{for } J/\psi. \tag{13}$$

By the same assumption we obtain for ψ'

$$|\xi'|, \quad |\eta'| \leq 0.5 \quad \text{for } \psi'. \tag{14}$$

In terms of amplitudes, the one-photon process is by no means a small correction. In fact, it is known that in some exclusive decay channels G parity and/or SU(3) violating amplitudes are comparable to corresponding conserved ones. For instance, we find in the Review of Particle Physics [5] the wrong-to-right G parity amplitude ratio $[\mathrm{Br}(J/\psi \to \rho\eta')/\mathrm{Br}(J/\psi \to \omega\eta')]^{1/2} \approx 0.8$. We shall keep Eqs.(13) and, (14) in mind in the following analysis.

4 Analysis of data

For J/ψ the average of two measurements on the decay $J/\psi \to b_1^{\pm}\pi^{\mp}$ is [5]:

$$\mathrm{Br}(J/\psi \to b_1^+\pi^- + cc) = (3.0 \pm 0.5) \times 10^{-3}, \tag{15}$$

The new BES measurements are:

$$\mathrm{Br}(J/\psi \to K_1^+(1270)K^- + cc) < 1.8 \times 10^{-3} \quad 90\%\mathrm{CL}, \tag{16}$$

$$\mathrm{Br}(J/\psi \to K_1^+(1400)K^- + cc) = (5.0 \pm 1.3) \times 10^{-3}. \tag{17}$$

In addition, the BES Collaboration measured three 1^+0^- decay modes of ψ':

$$\mathrm{Br}(\psi' \to b_1^+\pi^- + cc) = (7.3 \pm 1.9) \times 10^{-4}, \tag{18}$$

$$\mathrm{Br}(\psi' \to K_1^+(1270)K^- + cc) = (7.6 \pm 1.7) \times 10^{-4}, \tag{19}$$

$$\mathrm{Br}(\psi' \to K_1^+(1400)K^- + cc) < 2.7 \times 10^{-4} \quad 90\%\mathrm{CL}. \tag{20}$$

The most conspicuous is the pattern that $K_1(1270)^+K^-$ is suppressed in the J/ψ decay while $K_1(1400)^+K^-$ is suppressed in the ψ' decay. This is incompatible with the zeroth order picture of the maximally mixed $K_A - K_B$ combined with three-gluon annihilation dominance. We ask whether inclusion of one-photon annihilation and possibly a deviation of θ from 45° can explain this pattern or not.

4.1 J/ψ decay

The ratio of the two $K_1^+K^-$ amplitudes can be expressed in our parametrization as

$$\frac{\mathrm{M}(K_1^+(1270)K^-)}{\mathrm{M}(K_1^+(1400)K^-)} = \frac{\sqrt{2}\xi\tan\theta + (1 + \sqrt{2/5}\eta)}{\sqrt{2}\xi - (1 + \sqrt{2/5}\eta)\tan\theta}. \tag{21}$$

The amplitude ratio of $K_1^+(1400)K^-$ to $b_1^+\pi^-$ is subject to the experimental constraint from Eqs.(15) and (17):

$$\begin{aligned} \frac{\mathrm{M}(K_1^+(1400)K^-)}{\mathrm{M}(b_1^+\pi^-)} &= \frac{\sqrt{2}\xi\cos\theta - (1 + \sqrt{2/5}\eta)\sin\theta}{1 + \sqrt{2/5}\eta}, &(22)\\ &= -1.36 \pm 0.47. &(23)\end{aligned}$$

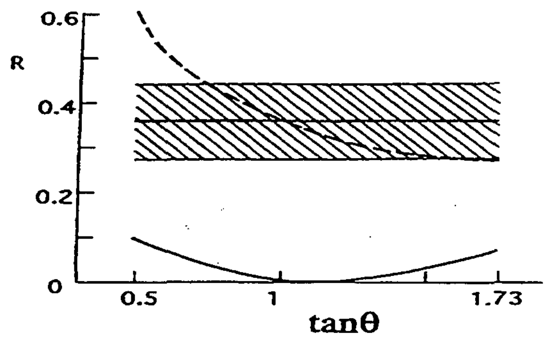

FIG.1

The number in the last line has been extracted with the s-wave decay assumption. Mixture of d-wave tends to raise the magnitude of the central value, for instance, from -1.36 to -1.67 for 50% mixture of d-wave. In order to suppress $\mathrm{Br}(J/\psi \to K_1^+(1270)K^-)$ relative to $\mathrm{Br}(J/\psi \to K_1^+(1400)K^-)$, the three-gluon and one-photon terms must interfere destructively in the former and constructively in the latter. Therefore ξ must be negative according to Eq.(21). Since ξ and η enter Eqs.(21) and (23) only through the ratio $\xi/(1+\sqrt{2/5}\eta)$, we can eliminate this ratio and express Eq.(21) in terms of θ and the experimental value of Eq.(23). In FIG.1 we have plotted the ratio $\mathrm{Br}(J/\psi \to K_1^+(1270)K^- + cc)/\mathrm{Br}(J/\psi \to K_1^+(1400)K^- + cc)$ as a function of $\tan\theta$.

We see that any value of θ between 30° and 60° produces a number much smaller than the experimental upper bound,

$$R \equiv \frac{\mathrm{Br}(J/\psi \to K_1^+(1270)K^- + cc)}{\mathrm{Br}(J/\psi \to K_1^+(1400)K^- + cc)} < 0.36 \pm 0.09. \tag{24}$$

We must make sure that we can find values for ξ and η in the acceptable range of Eq.(13) under the constraint of Eq.(23). It happens that this constraint is insensitive to θ ($= 30° \sim 60°$). For illustration we have shown in FIG.2 the range of (ξ, η) that correctly produces Eq.(23) for $\theta = 45°$.

Irrespective of values of θ, preferred values for ξ and η are always near one of the boundary corners ($\xi \approx -0.5$, $\eta \approx -0.5$) of Eq.(13). If we allow for an upward experimental error (making smaller in absolute value) in Eq.(23),

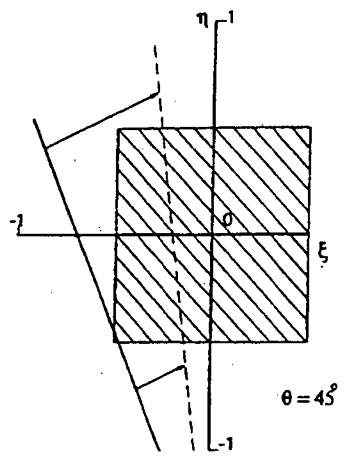

FIG.2

ξ and η can be made smaller in magnitude. An upward shift of one standard deviation of Eq.(23) would raise the prediction in Fig.1 to the thin broken curve.

Therefore the BES measurement of the decay $J/\psi \to K_1^+ K^-$ can be accommodated with theory once a right amount of the one-photon annihilation contribution is added.

4.2 ψ' decay

We want to be consistent with the measurement

$$R' \equiv \frac{\mathrm{Br}(\psi' \to K_1^+(1400)K^- + cc)}{\mathrm{Br}(\psi' \to K_1^+(1270)K^- + cc)} < 0.36 \pm 0.08, \tag{25}$$

with the matrix element ratio,

$$\frac{\mathrm{M}(K_1^+(1400)K^-)}{\mathrm{M}(K_1^+(1270)K^-)} = \frac{\sqrt{2}\xi' - (1 + \sqrt{2/5}\eta')\tan\theta}{\sqrt{2}\xi'\tan\theta + (1 + \sqrt{2/5}\eta')}. \tag{26}$$

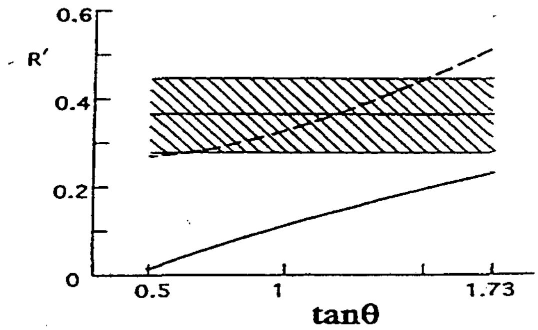

FIG.3

Positive ξ' suppresses the numerator by destructive interference. We have the experimental information on the ratio of $K_1^+(1270)K^-$ to $b_1^+\pi^-$:

$$\frac{\mathrm{M}(\psi' \to K_1^+(1270)K^-)}{\mathrm{M}(\psi' \to b_1^+\pi^-)} = \frac{\sqrt{2}\xi' \sin\theta + (1+\sqrt{2/5}\eta')\cos\theta}{1+\sqrt{2/5}\eta'} \tag{27}$$

$$= 1.03 \pm 0.34. \tag{28}$$

Again expressing the ratio Eq.(26) as a function of θ alone, we have plotted the ratio R' in FIG.3.

We obtain the solid curve for R' when we take the central value in Eq.(28). As in the case of J/ψ, any value of θ between 30° and 60° is consistent with the experimental upper bound on R'. Values of ξ' and η' are constrained by the ratio of $K_1^+(1270)K^-$ to $b_1^+\pi^-$ of Eq.(28).

FIG.4 depicts the allowed ranges for ξ' and η' when $\theta = 30°, 45°$, and $60°$. ξ' and η' must be on the solid straight line when the central value is taken in Eq.(28). This time the constraint on ξ' and η' are mildly dependent on θ. Though we can find ξ' and η' in the acceptable range (14) for any value of θ, ξ' can be smaller when $\theta < 45°$ than when $\theta > 45°$. If we make a downward shift of the number in Eq.(28), even smaller values would be allowed for ξ'. However we can make such a shift only by a half standard deviation or less, depending on θ, in order to be compatible with the upper bound on R'. The situation is shown in FIG.3 and FIG.4.

Let us summarize our numerical analysis: After a reasonable amount of the one-photon annihilation amplitudes is included, any value of θ between

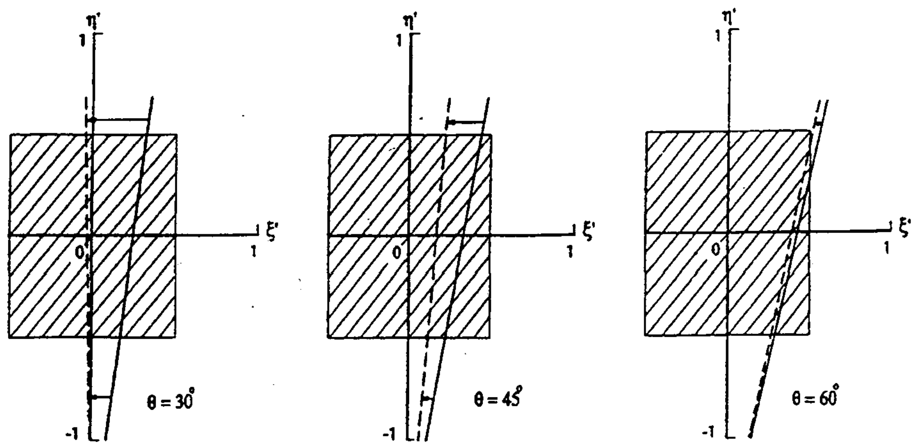

FIG.4

30° and 60° can be consistent with the 1^+0^- decay modes of both J/ψ and ψ' that have been so far measured. No stringent constraint is imposed on the one-photon annihilation amplitudes of those decay modes either.

Therefore the characteristic of the BES measurement that may look surprising at the first sight is not a surprise at all.

5 Outlook

When future experiment measures the neutral modes of $K_1\overline{K}$ and $\overline{K}_1 K$, they will allow us to restrict the ranges of parameters more stringently. Our SU(3) parametrization gives us

$$\frac{\mathrm{Br}(J/\psi \to K_1^0(1400)\overline{K}^0 + cc)}{\mathrm{Br}(J/\psi \to K_1^+(1400)K^- + cc)} = \left|\frac{(1-\sqrt{8/5}\eta)\tan\theta}{\sqrt{2}\xi - (1+\sqrt{2/5}\eta)\tan\theta}\right|^2, \tag{29}$$

$$\frac{\mathrm{Br}(\psi' \to K_1^0(1270)\overline{K}^0 + cc)}{\mathrm{Br}(\psi' \to K_1^+(1270)K^- + cc)} = \left|\frac{1-\sqrt{8/5}\eta'}{\sqrt{2}\xi'\tan\theta + (1+\sqrt{2/5}\eta')}\right|^2. \tag{30}$$

As parameters sweep in the currently allowed region, these ratios change over a wide range both above and below unity. For the purpose of fixing the mixing angle, the ratio $\mathrm{Br}(K_1^0(1400)\overline{K}^0)/\mathrm{Br}(K_1^0(1270)\overline{K}^0)$ is by far the cleanest source (cf. Eq.(10)). As for the $b_1\pi$ modes, $\mathrm{Br}(b_1^0\pi^0) = \mathrm{Br}(b_1^+\pi^-)$ is a robust prediction which follows from the isospin and charge conjugation property of the electromagnetic current. Violation of this equality would mean that emission and reabsorption of a photon somewhere inside the light quark sector is enhanced. If $\rho - \omega$ mixing should cause a substantial misidentification of a_1 and b_1, we would have to modify our analysis by including this $b_1 - a_1$ mixing.

To draw a definite conclusion on the $K_A - K_B$ mixing from the 1^+0^- decays of orthocharmonia, we shall have to wait until the neutral modes $J/\psi \to K_1^0\overline{K}^0 + cc$, $\psi' \to K_1^0\overline{K}^0 + cc$ [7] and $\psi' \to b_1^0\pi^0$ are measured. In addition, a more accurate measurement of the $b_1^0\pi^0/b_1^+\pi^-$ ratio in the J/ψ decay is desired.

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Table 1: SU(3) parametrization of decay amplitudes for $J/\psi \to 1^+0^-$. The decay modes not listed below are forbidden by charge conjugation invariance or the Okubo-Zweig-Iizuka rule. For the ψ' decay, replace ξ and η by ξ' and η', respectively.

$a_1^+\pi^-(=-a_1^-\pi^+)$	$\sqrt{2}\xi$
$K_A^+K^-(=-K_A^-K^+)$	$\sqrt{2}\xi$
$K_A^0\overline{K}^0(=-\overline{K}_A^0K^0)$	0
$b_1^+\pi^-(=b_1^-\pi^+)$	$1+\sqrt{2/5}\eta$
$b_1^0\pi^0$	$1+\sqrt{2/5}\eta$
$b_1^0\eta$	$\sqrt{6/5}\eta$
$K_B^+K^-(=K_B^-K^+)$	$1+\sqrt{2/5}\eta$
$K_B^0\overline{K}^0(=\overline{K}_B^0K^0)$	$1-\sqrt{8/5}\eta$
$h_1\eta$	$\sqrt{1/3}+\sqrt{2/15}\eta$
$h_1\pi^0$	$\sqrt{18/5}\eta$
$h_1'\eta$	$\sqrt{2/3}-(4/\sqrt{15})\eta$

Figure 1: $R = \mathrm{Br}(J/\psi \to K_1^+(1270)K^- + cc)/\mathrm{Br}(J/\psi \to K_1^+(1400)K^- + cc)$ against $\tan\theta$. The shaded band is the experimental upper bound when one standard deviation error is taken into account. The broken curve is for $\mathrm{M}(K_1^+(1400)K^-)/\mathrm{M}(b_1^+\pi^-) = -1.36 + 0.47$.

Figure 2: The range of ξ and η allowed by Eq.(23) for $\theta = 45°$. The shaded region is $|\xi|, |\eta| < 0.5$. ξ and η are constrained on the solid line when the central value is taken in Eq.(23). This solid line is virtually independent of θ between 30° and 60°. The broken line is for $\mathrm{M}(K_1^+(1400)K^-)/\mathrm{M}(b_1^+\pi^-) = -1.36 + 0.47$.

Figure 3: $R' = \mathrm{Br}(\psi' \to K_1^+(1400)K^- + cc)/\mathrm{Br}(\psi' \to K_1^+(1270)K^- + cc)$ against $\tan\theta$. The shaded band is the experimental upper bound with one standard deviation error. The solid curve below the band is the prediction when the central value is taken in Eq.(28). It rises to the broken curve passing through the band when the central value is lowered by a half standard deviation to $1.03 - 0.17$ in Eq.(28).

Figure 4: The ranges of ξ' and η' for $\theta = 30°, 45°$, and $60°$. The shaded region is $|\xi'|, |\eta'| < 0.5$. ξ' and η' are constrained on the solid line when the central value is taken in Eq.(28). We can reduce magnitude of ξ' and η' without conflicting R' by lowering the number in Eq.(28), but no more than a half standard deviation (cf. FIG. 3). The broken curve is the limit to which we can move the constraint without conflicting with R'.

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