Lawrence Berkeley National Laboratory

Recent Work

Title

RESONANCES AND KINEMATICAL PEAKS IN STRONG INTERACTIONS

Permalink

<https://escholarship.org/uc/item/3vs3m56b>

Author

Kramer, Ivan.

Publication Date

1967-05-12

cy. 2

University of California

Ernest O. Lawrence Radiation Laboratory

RESONANCES AND KINEMATICAL PEAKS IN STRONG INTERACTIONS

Ivan Kramer

May 12, 1967

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

Berkeley, California

UCRL-17554
cy. 2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

RESONANCES AND KINEMATICAL PEAKS IN STRONG INTERACTIONS

Ivan Kramer

May 12, 1967

RESONANCES AND KINEMATICAL PEAKS IN STRONG INTERACTIONS*

Ivan Kramer

Lawrence Radiation Laboratory
University of California
Berkeley, California

May 12, 1967

ABSTRACT

Two peripheral models involving pion-nucleon reactions leading to a four-body final state are studied with particular respect to the predictions of the models for various three-particle invariant mass distributions. These predictions are compared in detail to experimental results. The peaks in the mass distributions produced by the models, being accidents of the phase space, cannot be associated with true resonances. The first model considered assumes a double two-particle resonance intermediate state. The second model contains one two-particle resonance and one virtual nucleon and is used in fitting diffraction scattering data. The entire formalism contained herein assumes arbitrary values for the center-of-mass energy and all masses involved. Thus, the results can easily be applied to a variety of reactions at different energies.

-1-

I. INTRODUCTION

Recently, many experimental groups have investigated three-particle invariant mass distributions in production processes with four particles in the final state.¹⁻⁴ Once a peak in the distribution has been identified, the question as to whether the peak is a kinematical accident or a true resonance remains to be answered. Investigation of the two-particle invariant mass distributions has shown that these processes often proceed through intermediate resonances such as ρ , K^* , N^* . In many cases a multi-resonant intermediate state dominates the experimental data. The following question then arises: Can a peripheral diagram of the form shown in Fig. 1 explain the peaks observed in the three-particle invariant mass distributions in reactions in which a multi-resonant intermediate state is dominant? In many cases only one two-particle resonance is present, e. g. ρ , and the other vertex is consistent with a diffraction scattering model.

In this work we shall assume the dominance of the peripheral model. In addition to the model shown in Fig. 1 we shall put forth an analogous model to interpret diffraction scattering data. This latter model will essentially consist of replacing the spin-3/2 resonance in Fig. 1 by a virtual nucleon.

Assuming arbitrary values for the nine masses of Fig. 1, arbitrary incident energy, and the space-time properties of the particles, we shall work out general expressions for any three-particle invariant mass distribution. Expressions will also be given for distributions

-2-

where a cutoff on the momentum transfer, either to the intermediate or to the final state, was applied to the data. In every case the point at which form factors can be introduced will be indicated. We can summarize the results to be obtained as follows. Let m_x represent any three-particle invariant mass and let t represent the momentum transfer to this combination. Then, denoting the Breit-Wigner distribution for the two-particle resonances by $W(q')$ and $W(p')$ respectively, and using the notation in Fig. 1, we shall compute

$$\frac{d\sigma}{dm_x^2 dr^2} = \int K_r \cdot W(q') \cdot W(p') \cdot dm'^2 d\mu'^2$$

$$\frac{d\sigma}{dm_x^2} = \int K \cdot W(q') \cdot W(p') \cdot dm'^2 d\mu'^2$$

$$\frac{d\sigma}{dm_x^2 dt^2} = \int K_t \cdot W(q') \cdot W(p') \cdot dm'^2 d\mu'^2$$

where the kernels K_r , K , and K_t are determined exactly. Analogous expressions are obtained for our diffraction scattering model, but the integration over m' is not restricted by a Breit-Wigner distribution.

Calculations of this type have been tried before⁵ using Monte Carlo techniques,⁶ but such methods have suffered from the inordinate amount of computer time required on the 7090--on the order of hours--to produce statistically significant results. Recently, this technique has been greatly improved, thereby reducing the computer time for such calculations, but the time is still very much greater than the requirements of the exact result. We have been able to duplicate the distributions of

-3-

the Monte Carlo method with computer times on the order of a minute.

In proceeding with this calculation and in analyzing the results we are mindful of all of the failings of peripheralism insofar as agreement with experiment is concerned. The failure of the double intermediate resonance model to adequately explain the intermediate-state momentum-transfer distribution is well known.⁷ In most cases form factors must be introduced to force agreement with experiment. Since the expressions given here require the minimum possible numerical integrations, these results provide a quick and easy way to determine the effects of a wide variety of form factors.

For those who are not familiar with the technique of forming parity-invariant couplings involving the spin- $3/2$ baryon or the spin-1 meson we suggest reading Appendix A as a prelude to this work.

In the double intermediate resonance model we shall find that the model yields two peaks in any of the three-particle invariant mass distributions. This result can be traced to the spin-averaging factors of the intermediate resonances. More will be said on this subject in the Results, but because of this feature of the model we have also included a section in Appendix A on the bounds of any three-particle invariant mass in a four-particle final state. It is found that these two peaks produced by the model always occur near the lower and upper bounds of the three-particle invariant mass. Thus, a rough idea of the location of the peaks due to the double resonance model can be obtained in Sec. AIII of Appendix A.

-4-

II. THE S MATRIX, CROSS SECTION, AND PHASE SPACE

Many reactions considered here have identical particles in the four-particle final state. In these instances the S matrix consists of a direct term and an exchange term. Using the notation of Fig. 1, we set

$$S = M \delta(p + q - p'' - q'' - f - h), \quad (1a)$$

where $p = (\underline{p}, i p_0)$, etc., and

$$M = M_D + M_E. \quad (1b)$$

We then find

$$\sum_{sp} |M|^2 = \sum_{sp} |M_D|^2 + \sum_{sp} |M_E|^2 + \sum_{sp} \{M_D M_E^\dagger + M_D^\dagger M_E\}, \quad (1c)$$

where $\sum_{sp}$ is the sum over the spins. Because of the symmetry of the phase space and the matrix element under the interchange of any two particles in the final state, the first two terms on the right in (1c) yield identical distributions, and therefore we need consider only one of these. The last term on the right is the cross-term contribution. We shall show in a later section that, at the reaction energies of interest, the cross-term contribution is small with respect to the direct terms and hence can be neglected. In what follows we shall assume this result and therefore deal only with the direct-term contribution.

-5-

Assuming the spin and parity assignments given in Fig. 1 and adopting the appropriate couplings from Appendix A, we find

$$\sum_{sp} |M_D|^2 = \frac{m'^2}{9(2\pi)^{10}} \cdot \frac{|g|^4 |G|^4}{q_0 p_0 q_0'' f_0 p_0'' h_0} \cdot \frac{1}{(r^2 + \mu_r'^2)^2} \cdot \frac{\{ \underline{q} \cdot \underline{q}'' \}_{q'}^2}{(q'^2 + \tilde{\mu}'^2)^2} \cdot \frac{\{ (m + p_0) (m'' + p_0'') [\underline{p}^2 \underline{p}''^2 + 3 (\underline{p} \cdot \underline{p}'')^2] \}_{p'}}{(p'^2 + \tilde{m}'^2)^2} \quad (2a)$$

where the subscripts on the two expressions in braces indicate the rest frame in which the quantities within the braces are evaluated. We shall adhere to this notation throughout. The quantities μ' and m' are complex and are related to their respective mean masses and partial widths through

$$\tilde{\mu}' = \mu'_0 - i \frac{\Gamma_{\mu'}}{2}$$

and

$$\tilde{m}' = m'_0 - i \frac{\Gamma_{m'}}{2}$$

The cross section for this process is then given by

$$\sigma = \frac{(2\pi)^2 q_0 p_0}{[(q \cdot p)^2 - m^2 u^2]^{\frac{1}{2}}} \cdot \int \left\{ \sum_{sp} |M_D|^2 \right\} \delta(p + q - q'' - p'' - f - h) \cdot d\underline{f} d\underline{h} d\underline{q}'' d\underline{p}'' \quad (2b)$$

We now use the relationships between the coupling constants and the

-6-

partial widths as given in Appendix C, and the fact that each partial width is very much smaller than the value of its corresponding mean mass. Henceforth, we shall use the notation $\underline{q} \equiv |\underline{q}|$, etc. Then if we set

$$W(q') \equiv \frac{\mu'^2 \Gamma_{\mu'}^2}{[(\mu'^2 - \mu_0'^2)^2 + \mu_0'^2 \Gamma_{\mu'}^2]},$$

$$W(p') \equiv \frac{m'^2 \Gamma_{m'}^2}{[(m'^2 - m_0'^2)^2 + m_0'^2 \Gamma_{m'}^2]},$$

and

$$K(m', \mu') \equiv \frac{9 m'^2 \mu'^2}{(2\pi)^4 [(q \cdot p)^2 - m^2 \mu^2]^{\frac{1}{2}} \{\underline{q}''^6\}_q \{(m'' + p_0) \underline{p}''^6\}_{p'}}, \quad (3a)$$

the cross section takes on the form

$$\sigma = \int K(m', \mu') \cdot W(q') \cdot W(p') \cdot (r^2 + \mu_r'^2)^{-2} \cdot \{(m + p_0) [\underline{p}^2 \underline{p}''^2 + 3(\underline{p} \cdot \underline{p}'')^2]\}_{p'} \cdot \{\underline{q} \cdot \underline{q}''\}_q^2 \cdot dp, \quad (3b)$$

where the phase space factor is given by

$$dp \equiv 4 \delta(p + q - p'' - q'' - f - h) \frac{df}{f_0} \frac{dh}{h_0} \frac{dp''}{p_0''} \frac{dq''}{q_0''}. \quad (3c)$$

We first turn our attention to reducing dp to a form free of the delta function.

-7-

Using the result in Appendix D twice, we find

$$d\rho = dm'^2 d\mu'^2 \left[\frac{\underline{p}'' d\Omega_{\underline{p}''}}{D(\underline{p}', \underline{p}'')} \right] \cdot \left[\frac{\underline{q}'' d\Omega_{\underline{q}''}}{D(\underline{q}', \underline{q}'')} \right] \left\{ \frac{d\underline{p}'}{p_0'} \frac{d\underline{q}'}{q_0'} \delta(\underline{p} + \underline{q} - \underline{p}' - \underline{q}') \right\}.$$

The factor within the braces is just the usual two-body phase-space result for a final state consisting of particles $\underline{p}'$ and $\underline{q}'$. Evaluating this factor in the overall center-of-mass (c.m.) system, we obtain

$$d\rho = dm'^2 d\mu'^2 \left[\frac{\underline{p}'' d\Omega_{\underline{p}''}}{D(\underline{p}', \underline{p}'')} \right] \cdot \left[\frac{\underline{q}'' d\Omega_{\underline{q}''}}{D(\underline{q}', \underline{q}'')} \right] \left\{ \frac{\underline{q}' d\Omega_{\underline{q}'}}{s} \right\}_{\text{c.m.}}, \quad (4)$$

where s is the c.m. energy of the reaction. This reduction of the phase space to the form given in (4) is the pivot point of the entire discussion that follows.

-8-

III. THREE-PARTICLE INVARIANT MASS DISTRIBUTIONS

For the four-particle final state under consideration, we have four possible three-particle invariant mass distributions. However, two of these have exactly the same form as the other two and are obtained from them by merely interchanging the values of some masses. For example, the form of the mass distribution for both $m_x^2 = -(p' + q'')^2$ and $m_x^2 = -(p' + f)^2$ is identical, and to obtain the second from the first we have only to interchange the values of the masses associated with q'' and f in the first distribution. We shall consider each of the two distinct cases in turn.

$$A. \quad \underline{M_x^2 = -(P' + Q'')^2}$$

In the c.m. we have by definition and from energy-momentum conservation

$$m_x^2 = s^2 + \mu_f^2 - 2 q'_0 s + 2 q''_0 s$$

$$\cos(\underline{q}', \underline{q}'') = \frac{[\mu_f^2 - \mu''^2 - \mu'^2 + 2 q'_0 q''_0]}{2 \underline{q}' \underline{q}''},$$

where $(\underline{q}', \underline{q}'')$ is the angle between $\underline{q}'$ and $\underline{q}''$. Referring the first bracket in (4) to the p' rest frame and the second bracket to the c.m., we obtain

-9-

$$d\rho = dm'^2 d\mu'^2 \left\{ \frac{\underline{p}'' d\Omega_{\underline{p}''}}{m'} \right\}_{\underline{p}'} \left\{ \frac{\underline{q}'' \underline{q}' d\Omega_{\underline{q}'} d\Omega_{\underline{q}''}}{D(\underline{q}', \underline{q}'') \cdot s} \right\}_{\text{c.m.}}$$

Using the transformation in (E3) of Appendix E, we find

$$d\Omega_{\underline{q}'} d\Omega_{\underline{q}''} = d\psi d(\cos \theta) d\phi \cdot \left\{ \frac{Q dm_x^2}{2 \underline{q}' \underline{q}''} \right\} \quad (5a)$$

with

$$Q \equiv \left\{ \frac{(\mu''^2 + \mu'^2 - \mu_F^2) q_0'' - 2 \mu''^2 q_0'}{2 \underline{q}''^2 s} \right\}. \quad (5b)$$

Since our matrix element is independent of ϕ , we find effectively

$$d\rho = \frac{\pi Q}{s D(\underline{q}', \underline{q}'')} \cdot dm'^2 d\mu'^2 \cdot \left[\frac{\underline{p}'' d\Omega_{\underline{p}''}}{m'} \right] d\psi d(\cos \theta) dm_x^2. \quad (6)$$

From (6) and (3b) we can now straightforwardly compute $d\sigma/dm_x^2$. We have relegated all of the details to Appendix F and shall quote here merely the main results. The following functions heretofore undefined are defined in Appendix F and shall not be repeated here.

First, since in the c.m. we have

$$x \equiv \cos \theta = \frac{[(2p_0 p_0' - m^2 - m'^2) - r^2]}{2 \underline{q} \underline{q}'},$$

where $\underline{q} = |\underline{q}|$ etc., we obtain the distribution

-10-

$$\frac{d\sigma}{dm_x^2 dr^2} = \int \frac{K(m_x^2, m'^2, \mu'^2)}{2 \underline{q} \underline{q}'} \cdot \frac{[D_i x^i]}{(r^2 + \mu_r^2)^2} \cdot F(r^2) \cdot W(q') \cdot W(p') \cdot dm'^2 d\mu'^2, \quad (7)$$

where $i = 0, \dots, 5$, and $F(r^2)$ is an arbitrarily inserted form factor. The boundaries on this integral are shown in Fig. 2. This is a versatile result which enables investigation of a wide variety of model predictions. For example, it can be used to determine the predictions of the model due to an explicit form factor by inserting a particular expression for $F(r^2)$ and integrating over all kinematically allowed values of r^2 . Alternatively, we can use (7) to compare with experimental data in which an r^2 cut-off was applied to the data by restricting the r^2 integration to precisely this same region. Moreover, if the delta-function approximation (DF) of Appendix B is appropriate, then (7) takes on the simplified form

$$\left(\frac{d\sigma}{dm_x^2 dr^2} \right)_{DF} = \tilde{K}(m_x^2, m_0'^2, \mu_0'^2) \cdot F(r^2) \cdot \frac{[D_i x^i]}{(r^2 + \mu_r^2)^2}. \quad (8)$$

Since we shall be principally concerned with the predictions of the unadulterated peripheral model, we find, after setting $F(r^2) = 1$ and integrating (7) over the entire range of the r^2 variable, the following general result:

-11-

$$\frac{d\sigma}{dm_x^2} = \int K(m_x^2, m'^2, \mu'^2) \cdot I(m_x^2, m'^2, \mu'^2) \cdot W(q') \cdot W(p') \cdot dm'^2 du'^2. \quad (9)$$

In the delta function approximation defined in Appendix B this reduces to the very simple result

$$\left(\frac{d\sigma}{dm_x^2} \right)_{DF} = (\pi m'_0 \Gamma_{m'}) \cdot (\pi \mu'_0 \Gamma_{\mu'}) \cdot K(m_x^2, m_0'^2, \mu_0'^2) \cdot I(m_x^2, m_0'^2, \mu_0'^2). \quad (10)$$

$$B. \quad \underline{M_x^2} = -(Q' + H)^2$$

The calculations and results in this case are completely analogous to those of case A above. Indeed, the form of the results here is identical to that of case A. Relegating all of the definitions and final details to Appendix G, we merely sketch some of the initial steps that highlight the differences of this case from case A.

Analogous to the transformation in (5), we now have

$$d\Omega_h d\Omega_{p'} = d\psi d(\cos \theta) d\phi \cdot \left\{ \frac{Q dm_x^2}{2 \underline{p'} \underline{h}} \right\} \quad (11a)$$

with

$$Q \equiv \left[\frac{(\mu_h^2 + m'^2 - m''^2) h_0 - 2 \mu_h^2 p'_0}{2 \underline{h}^2 s} \right], \quad (11b)$$

where, of course, the Euler angles here have a different meaning from those in (5) because they refer to a different set of particles. Thus, we obtain the result analogous to (6):

-12-

$$d\rho = \frac{\pi Q}{s D(p', h)} \cdot \left[\frac{q'' d\Omega_{q''}}{\mu'} \right] \cdot d\psi d(\cos \theta) dm'^2 d\mu'^2 dm_x^2. \quad (12)$$

The remainder of this calculation proceeds in exactly the same way as in case A. The results in Eqs. (7) through (10) can be taken over directly here, provided we interpret the quantities appearing in these equations to have the redefined values given in Appendix G.

-13-

IV. MASS DISTRIBUTIONS FOR LOW MOMENTUM TRANSFER

In this section we are concerned with mass distributions with a special dependence on the momentum transfer to the three-particle invariant mass under consideration. This could be in the form of either a momentum transfer cutoff or of an inserted form factor. The object of this section, then, is to find a closed expression for $d\sigma/(dm_x^2 dt^2)$, where t^2 is the square of the momentum transfer to m_x . We have two distinct cases to consider corresponding to the two cases given in Sec. III above. We shall find that closed expressions for $d\sigma/(dm_x^2 dt^2)$ for all cases can be obtained, but any integrations of t^2 must be done numerically.

$$\text{A. } \underline{T^2 = (F - Q)^2 \text{ WITH } M_x^2 = -(P'' + H + Q'')^2}$$

In many respects the pattern of this calculation follows that of the $d\sigma/dm_x^2$ calculation in Sec. IIIA. We begin with Eqs. (6) and (3b). All quantities heretofore undefined are defined in Appendix H, where all computational details can be found.

From the definition t^2 in terms of the Euler angles is

$$t^2 = b_1 - b_2 \cos \theta + b_3 \sin \psi \sin \theta, \quad (13)$$

where the b_i are independent of the angles. We now impose the transformation $\psi \rightarrow t^2$, which yields the extra factor

-14-

$$\frac{\partial t^2}{\partial \psi} = (D_0 + D_1 x + D_2 x^2)^{\frac{1}{2}} \quad (x \equiv \cos \theta)$$

in the denominator of the cross-section integrand. Setting

$$I(m_x^2, t^2, m'^2, \mu'^2) = \int_{x_-}^{x_+} F(r^2) \frac{[A_1 x^i] [B_0 + B_1 x + B_2 x^2]}{(c_0 - c_1 x)^2 [D_0 + D_1 x + D_2 x^2]^{\frac{1}{2}}} dx \quad (14a)$$

where $i = 0, \dots, 3$, and $F(r^2)$ is an arbitrarily inserted form factor, we find

$$\begin{aligned} \frac{d\sigma}{dm_x^2 dt^2} &= \int G(m_x^2, m'^2, \mu'^2) \cdot I(m_x^2, t^2, m'^2, \mu'^2) \\ &\cdot W(q') \cdot W(p') \cdot dm'^2 d\mu'^2. \end{aligned} \quad (14b)$$

The integration in (14a) is carried out in Appendix H for $F(r^2) = 1$.

In the delta-function approximation, (14b) reduces to the simplified form

$$\begin{aligned} \left(\frac{d\sigma}{dm_x^2 dt^2} \right)_{DF} &= (\pi \mu'_0 \Gamma_{\mu'}) \cdot (\pi m'_0 \Gamma_{m'}) \cdot G(m_x^2, m'_0{}^2, \mu'_0{}^2) \\ &\cdot I(m_x^2, t^2, m'_0{}^2, \mu'_0{}^2). \end{aligned} \quad (15)$$

The results in (14) or (15) have great applicability. For example, suppose we are interested in the distribution $d\sigma/dm_x^2$ for events with $t^2 \leq 15 \mu_\pi^2$. Then if we set $H(m_x^2, t^2) \equiv d\sigma/(dm_x^2 dt^2)$, we must numerically compute

-15-

$$\left(\frac{d\sigma}{dm_x^2} \right)_{t^2 \leq 15\mu_\pi^2} = \int_{t_{\min}^2(m_x^2)}^{15\mu_\pi^2} H(m_x^2, t^2) dt^2,$$

where $t_{\min}^2(m_x^2)$ is the lower physical bound on t^2 .

$$\text{B. } \underline{T^2 = (P'' - P)^2 \quad \text{WITH } M_x^2 = -(Q'' + F + H)^2}$$

This calculation is completely analogous to the one in Sec. IVA above. By appropriately redefining the functions b_i , D_j , and all other functions appearing in case A, we obtain for this case results that are identical in form to those in (14) and (15). All such details and redefinitions are bound in Appendix I.

-16-

V. THE CROSS TERM

Suppose h and q'' are identical particles. We shall show that the cross term that results from the symmetrization of the S matrix is small in comparison to the direct terms at the reaction energies to be considered and hence be neglected.

If we write the S matrix for the exchanged diagram using the notation $q'_e = h + f$, $p'_e = p'' + q''$, and $r_e = q'_e - q = p - p'_e$, then the last term on the right in (1c) becomes

$$\begin{aligned} \sum_{sp} \{M_D M_E^+ + M_D^+ M_E\} &= \frac{4m^2}{9(2\pi)^{10}} \frac{|g|^4 |G|^4}{q_0 p_0 f_0 h_0 q_0'' p_0''} \\ &\cdot \{ \{ \underline{q \cdot q''} \}_{\underline{q}} \cdot \{ \underline{q \cdot q''} \}_{\underline{q}_e} \cdot \{ (\underline{p'' \cdot p})_{\underline{p}} \cdot (\underline{p'' \cdot p})_{\underline{p}_e} + \frac{1}{4} (\underline{p'' \times p})_{\underline{p}} \cdot (\underline{p'' \times p})_{\underline{p}_e} \} \\ &\cdot \frac{R}{(r^2 + \mu_r^2) (r_e^2 + \mu_r^2)} \cdot \{ [(\underline{m + p_0})(\underline{m'' + p_0''})]_{\underline{p}} \cdot [(\underline{m + p_0})(\underline{m'' + p_0''})]_{\underline{p}_e} \}^{\frac{1}{2}} \end{aligned} \quad (16a)$$

where the resonance propagator terms can be reduced to the simple form

$$R = 2 \frac{\{ (v v_e + a^2) (w w_e + b^2) - a b (v - v_e) (w - w_e) \}}{(v^2 + a^2) (v_e^2 + a^2) (w^2 + b^2) (w_e^2 + b^2)} \quad (16b)$$

Here we have set $v \equiv \mu'^2 - \mu_0'^2$, $v_e \equiv \mu_e'^2 - \mu_0'^2$, $w \equiv m'^2 - m_0'^2$, $w_e \equiv m_e'^2 - m_0'^2$, $a = \mu_0' \Gamma_{\mu'}$, and $b = m_0' \Gamma_{m'}$. Inserting (16a)

into the form given in (2b) yields the cross-term contribution to the cross section. From this, the cross-term contribution to all of the invariant mass distributions obtained in Sec. III and IV can be calculated.

-17-

The exact calculation of the cross-term contribution to the mass distribution is sufficiently tedious to warrant the use of the Monte Carlo method to evaluate it for some relatively low reaction energy. This was done by the Goldhaber group at Berkeley for the reaction $\pi N \rightarrow \rho N^* \rightarrow 3\pi + N$ at a beam momentum in the laboratory frame of 3.665 BeV/c. The conclusion was that at no time did the cross-term contribute more than 10% of the direct terms' contribution, and most of the time it was very much smaller than 10%. We would expect this percentage to decrease as the beam momentum is increased. Thus, the cross term can be safely ignored.

-18-

VI. DAMPENING OF THE MOMENTA

In this section we investigate the behavior of the results in Sec. III and IV when the proper dampening of the matrix element for high momenta is included. Up to this point we have only assumed the correct threshold dependence.

Consider the process $p + q = p' = p'' + q''$, where p' is a two-particle resonance of mass m' , p and q are in an angular-momentum state of unity, and the particles in the final state are identical to those in the initial state. We shall also assume the spin-parity assignments of Fig. 1. In the p' rest frame we have $k'' \equiv \underline{p}'' = \underline{q}''$. If k'' is small and if $R(p')$ is the characteristic range of the interaction, then the partial width for the decay process can be written as

$$\Gamma_{m'} = \tilde{\Gamma}_{m'} [k'' R(p')]^2 \cdot \{k'' R(p')\}, \quad (17)$$

where $\tilde{\Gamma}_{m'}$ is the reduced width. Although this expression has the correct threshold behavior as a function of k'' , it breaks down as k'' becomes large. The more accurate expression, valid for all p-wave resonances, is⁸

$$\Gamma_{m'} = \tilde{\Gamma}_{m'} \left[\frac{\{k'' R(p')\}^2}{1 + \{k'' R(p')\}^2} \right] \cdot \{k'' R(p')\}. \quad (18a)$$

Comparing (17) and (18a), we see that the sole difference is that of

-19-

the dampening factor in the denominator of (18a). From the derivations contained in Appendix C we can construct a very easy way of incorporating the dampening factor into our original formalism. We simply replace every momentum that appears in the coupling at each vertex by a dampened momentum:

$$k'' \Rightarrow \frac{k''}{[1 + \{k'' R(p')\}^2]^{\frac{1}{2}}} \quad (18b)$$

The net effect of (18) is to introduce the additional factor

$$\left[\frac{\{1 + [\underline{p}'' R(p')\}^2\}}{\{1 + [\underline{p} R(p')\}^2\}} \right]_{p'} \cdot \left[\frac{\{1 + [\underline{q}'' R(q')\}^2\}}{\{1 + [\underline{q} R(q')\}^2\}} \right]_{q'} \quad (19)$$

into the integrand of (3b), where $R(q')$ is the characteristic range of interaction at the q' vertices.

-20-

VII. DISTRIBUTIONS WITH A VIRTUAL NUCLEON IN THE INTERMEDIATE STATE

The main object of this section is to compute distributions of the types given in Sec. III and IV, but with p' now taken to represent a spin- $\frac{1}{2}$ nucleon. Aside from the alteration of our matrix element due to a different spin-averaging expression, we are also prevented from using the delta-function approximation for the $p' \rightarrow p'' + h$ vertex. We continue to use the notation given in Fig. 1. A model of this type is used to fit diffraction-scattering data.

The phenomenological coupling of baryons to pseudoscalar mesons can be taken to be

$$\bar{\psi}(x) \gamma_5 \psi(x) \phi(x) \Rightarrow \bar{U}(p') \gamma_5 U(p).$$

We confine our interest to the $m_x^2 = -(h + f + q'')^2$ combination.

Leaving all of the details to Appendix J and using the notation

$m_v^2 = -(p'' + h)^2$, the two main results are

$$\frac{d\sigma}{dm_x^2} = \int K(m_x^2, \mu'^2) \cdot I(m_x^2, m_v^2, \mu'^2) \cdot \frac{W(q')}{(m_v^2 - m'^2)^2} d\mu'^2 dm_v^2 \quad (20)$$

and

$$\begin{aligned} \frac{d\sigma}{dm_x^2 dt^2} &= \frac{G(t^2)}{\pi} \cdot \int K(m_x^2, \mu'^2) \cdot H(m_x^2, t^2, m_v^2, \mu'^2) \\ &\cdot \frac{W(q')}{(m_v^2 - m'^2)^2} \cdot d\mu'^2 dm_v^2. \end{aligned} \quad (21)$$

-21-

In both cases the integration over m_v^2 is nontrivial and will have to be done numerically. The function $G(t^2)$ is an arbitrarily inserted form factor.

-22-

VIII. RESULTS

A. The Double-Resonance Model1. The Goldhaber Reaction: $\pi^+ p \rightarrow \rho^0 N^{*++} \rightarrow \pi^+ \pi^- \pi^+ p$ at 3.665 BeV/c.

The original motivation for this work was the experimental analysis of the Goldhaber reaction.² To determine the possibility of two-particle intermediate resonances, two-particle invariant mass distributions were plotted. The reaction was found to produce ρ 's and N^* 's copiously. The events were then screened so that only those reactions that proceeded through a $\rho - N^*$ intermediate state remained. Then, using the screened events, were plotted the various three-particle invariant mass distributions. A pronounced peak in the $N^* \pi$ invariant mass distribution at 1.56 BeV was evident with $\Gamma = 0.2$ BeV. This experimental distribution is reproduced for convenience in Fig. 3a. It is perhaps significant that this peak at 1.56 BeV was almost completely obscured if all events were considered, i.e., if the $\rho - N^*$ screening was abandoned. The second, dominant peak at around 2.55-BeV appeared in both cases. The possibility that the three-particle invariant mass distribution for the screened events could be completely accounted for by the formalism developed in Sec. III must now be considered. The correlation between $\rho - N^*$ events with the peaking around 1.56 BeV for the $N^* \pi$ distribution makes this a distinct possibility. If, indeed, we can explain the $N^* \pi$ distribution in this manner, then this peaking around 1.56 BeV

-23-

is a result of a kinematical accident and not a three-particle resonance. We mention in passing that if such a three-particle resonance did exist, then it would have special importance. Such a resonance would have an isotopic spin of $5/2$ and therefore would be the first particle in a new SU_3 multiplet. (The 35-dimensional representation would be the lowest possibility.)

From now on we assume that the Goldhaber reaction is synonymous with screened $\rho - N^*$ events. A Treiman-Yang test performed on these events confirms that the data is consistent with a π -exchange model. Thus, the dominant diagram is that shown in Fig. 1. We now rephrase the question that we will endeavor to answer. Can the experimental three-particle invariant-mass distributions be qualitatively accounted for by the peripheral diagram of Fig. 1? We shall be principally concerned with reproducing the two peaks appearing in the experimental distribution for the $N^*\pi$ combination, although we will give results for all possible three-particle combinations in the four-particle final state.

The answer to the above question was pursued by the Goldhaber group using the Monte Carlo method to perform the weighted averaging over the phase space. Their results are contained in a paper by Dash and Goldhaber.⁵ The disadvantages of this method are well known. In the first place it involves a very large expenditure of computer time--on the order of hours on the 7040. Secondly, the statistics are generally poor, and the results are usually very spiked and rough. Lastly, a realistic Monte Carlo calculation can only be done if the

-24-

energy of the reaction is relatively low. Thus, if the c.m. energy goes to very high values, calculation by this method becomes a practical impossibility. In Sec. III and IV we have been able to develop analytic expressions for the main Goldhaber distributions and to reproduce these distributions using very small amounts of 7040 time, on the order of a minute.

The general features of the $p \pi^+ \pi^+$ distributions that we will obtain analytically can be guessed in advance on the basis of arguments presented in the introduction. Since this is our first encounter with a concrete example, we apply those arguments in detail to the case at hand.

If we go into the q' or the p' rest frame, the matrix element will yield either a $\cos^2 \beta$ or a $1 + 3 \cos^2 \beta'$ distribution, respectively, for the decay particle relative to the incident direction (the $\underline{q}$ or $\underline{p}$ axis, respectively). Here β represents the angle between $\underline{q}''$ and $\underline{q}$ as viewed from the q' rest system. Similarly, β' represents the angle between $\underline{p}''$ and $\underline{p}$ as viewed from an observer in the rest frame of p' . Thus, if we were seated in the q' (p') rest frame, we would observe most decay particles decaying into two symmetric, narrow cones whose axis is the $(\underline{q})_{q'}$, $[(\underline{p})_{p'}]$ direction. For the sake of argument let us go into the q' (the p) rest system. The decay pions follow the $\cos^2 \beta$ distribution, falling with equal probability into a forward or backward cone with the greatest probability of decaying along the cone axis itself, i.e. along the $(\underline{q})_{q'}$ direction (parallel or antiparallel). It is these two

-25-

relatively intense decay concentrations along the parallel and antiparallel directions that give rise to two possible peaks in the $N^*\pi$ distribution. To narrow things down, suppose $(\vec{p}')_q$ is relatively antiparallel to $(\vec{q})_q$, as shown in Fig. (4b). This assumption in no way compromises the conclusions of our arguments. The forward decay cone [axis parallel to $(\vec{q})_q$] then represents decay pions sent in a direction largely away from the $N^*[(\vec{p}')_q]$. Suppose q'' represents just such a pion. If we then form $m_x^2 = -(p' + q'')^2$, we find that the forward decay pions will contribute to the high m_x region. In the same way decay pions in the backward cone are sent largely toward the N^* , have small energies with respect to the N^* , and consequently contribute predominantly to the low m_x region. Thus, we expect two peaks in the $N^*\pi$ distribution--one at the high and one at the low end. If $(\vec{p}')_q$ were relatively parallel to $(\vec{q})_q$, we would obtain the identical result, except that it would be pions in the forward cone that contribute to the low m_x peak and pions in the backward cone that would contribute to the high m_x peak. The above arguments are perfectly general and can be applied to all other cases that we shall consider.

We have reproduced the experimental $N^*\pi$ invariant mass distribution obtained by Goldhaber et. al. in Fig. 3a. Notice that we have two peaks in this distribution, one at the low and one at the high end. Using the analytic formalism developed in the foregoing sections, we have obtained the analogous distribution given by the pure model

-26-

(i.e., no form factors) in the delta-function approximation. This curve appears in Fig. 3b and is labelled by "DF". Notice that the model predicts very strong peaks at the kinematical end points--in this case at 1.40 BeV and 2.60 BeV--and a pronounced trough centered around 2.10 BeV. Before we can compare these two distributions, we must note that the Goldhaber $N^* - \rho$ filter was much broader than our delta function approximation. The Goldhaber filter counted all events that fell within ± 0.100 BeV of the mean value of each of the two-particle resonances. In terms of our model this means that we would have to integrate the delta-function-approximation curve multiplied by the Breit-Wigner distributions over the range permitted by the Goldhaber filter to obtain a true analogous curve that can be compared with the Goldhaber distribution. This was done, and the result also appears in Fig. 3b. The curve labelled "NI" is the curve obtained by a numerical integration of the delta-function curve over the Goldhaber mass bands. We are now in a position to compare the predictions of the pure model to experiment.

From Fig. 3b we see that the principle difference that the numerical integration over the principle part of the Breit-Wigner forms produces with respect to the delta-function approximation occurs at the low end. The "NI" curve now has a definite peak at around 1.45 BeV. The secondary peak at 2.60 BeV and the trough at around 2.10 BeV are features of both the DF curve and the NI curve. Comparing the NI curve to the experimental distribution shown in Fig. 3a, we see that the model prediction very closely reproduces

-27-

the main features of the experimental result. Experiment obtains peaks at 1.56 BeV and 2.55 BeV, while the model predicts peaks at around 1.45 BeV and 2.60 BeV. The experimental distribution has a broad trough in the region around 2.0 BeV, and this is precisely what the model predicts. Thus, the pure model predictions are sufficiently close to the experimental result to invalidate drawing any conclusions about the existence of $p \pi^+ \pi^+$ resonances based solely on the above experimental distribution. We shall now investigate modifications of the pure model and compare these predictions with the ones obtained so far.

As shown by Goldhaber et al.,⁷ the momentum transfer to the intermediate-state distribution predicted by the pure model does not fall off sufficiently rapid to agree with the experimental result. To force agreement between the model and experiment, Goldhaber used the form factor

$$F(r^2) = \frac{(-\mu_\pi^2 + \Lambda^2)}{(r^2 + \Lambda^2)}, \quad \Lambda = 2.6 \mu_\pi.$$

Using the Goldhaber form factor in our formalism, we obtain the curves shown in Fig. 5. Comparing these results with the predictions of the pure model as shown in Fig. 3b, we see that there is remarkable agreement insofar as the location of the peaks and the trough are concerned. The form-factor result again predicts peaks at around 1.45 BeV and 2.60 BeV and a trough at around 2.10 BeV. However, these peaks are broader and these curves are flatter than the pure

-28-

model result. This appears to be the only difference between these two sets of curves.

We shall next consider the effects on our model produced by the inclusion of dampening of the momenta factors at each of the vertices. First of all we should mention that the central values for the intermediate resonances used in our computations are $m'_0 = 1.238$ BeV and $\mu'_0 = 0.750$ BeV for the N^* and ρ , respectively, with partial widths of $\Gamma_{m'} = \Gamma_{\mu'} = 0.120$ BeV. All curves labelled DF have been computed for the case where the intermediate masses have been held constant at these central values. Slight changes in the central values of the intermediate resonances and in their partial widths have no significant effect on the three-particle invariant mass curves. This remains true regardless of the model considered here.

We have considered models with dampening of the momenta for a wide variety of values for the range of the force at each vertex. For all cases we obtain essentially the same result, and hence we need consider only one example. We have produced such an example in Fig. 6 in the delta-function approximation and labelled it "D". For comparison purposes we have reproduced the Goldhaber form-factor result (G) and the result obtained from the pure model (P) on the same scale. The particular value for the ranges chosen for the D curve shown is

$$(R_{N^*}, R_{\rho}) = \frac{0.88}{\mu_{\pi}} \left(1, \frac{1.238}{0.750}\right).$$

-29-

The N^* range used above is the same as that used by Gell-Mann and Watson. The ρ range above then presents itself as a natural choice. Comparing all three curves, we see immediately that the curves agree in all three essential features and differ only in relative flatness. All three curves have peaks and trough at identical points. From the experience obtained from numerical integration over the Breit-Wigner forms in the cases of the pure model and modified model with the Goldhaber form factor, we see that a numerical integration of curve D would produce a curve very similar to the previous two cases and, in particular, would peak in the exact same places. Comparing all the model curves with the experimental distribution given in Fig 3a, we see that both of the modified models considered above improve the model's agreement with experiment insofar as flatness is concerned. By flatness we mean something like the ratio of the probability of the lower peak of the curve to the probability of the center of the trough. In summary, we see that the peripheral model and its modifications give good qualitative agreement with the experimental distribution.

We now wish to compare model predictions with experiment for distributions with a cutoff in the momentum transfer to the final state. The model's formalism for these curves is contained in Sec. IV. In Fig. 7a we reproduce the Goldhaber experimental result for all events with $t^2 \leq 15 \mu_\pi^2$ (no $N^* - \rho$ filter). Because no filter was used, a strictly valid comparison to the model's curves is not correct. However, because most Goldhaber events turned out to be

-30-

N^* - ρ events, it is useful to make this comparison anyway. In Fig. 7b we show the analogous curve for the pure model and for the modified model with the Goldhaber form factor in the delta-function approximation. Comparing the P curve shown here to the P curve without any t^2 cutoffs shown in Fig. 3b, we see that the portion of the events retained by the cutoff curve is just the opposite of what we would expect. Instead of dampening out the high invariant mass region, the cutoff curve in fact damps out the low mass region. Needless to say, the model prediction here bears very little correlation with reality and is in very poor agreement with the unfiltered experimental distribution. Use of the Goldhaber form factor greatly improves the result, and the lower peak is greatly enhanced. The high peak is still retained, however, although this peak is absent from the experimental distribution. A numerical integration over the Breit-Wigner forms for the intermediate resonances would undoubtedly yield a curve that once again peaked around 1.45 BeV. We should point out here that the delta-function curves given in Fig. 7b were themselves obtained by numerically integrating over the cut-off t^2 region.

We now want to investigate the other main three-particle invariant mass distribution predicted by the model for the Goldhaber reaction--the $\rho \pi$ distribution. The experimental results appears in Fig. 8a. A fairly strong peak is apparent at around 1.10 BeV. It is interesting to compare the predictions of the pure model to this experimental result. The results of the pure model appear in Fig. 8b, and we see that we have a pronounced peak at around 1.02 BeV, very close

-31-

to the position of the experimental peak. The model also predicts a smaller peak at around 1.50 BeV which is not clearly reflected in the experimental distribution. It is also interesting to compare the features of the model curves obtained here to the ones obtained for the $N^*\pi$ case. Notice that we again get two peaks, one at the high and one at the low end, and a pronounced trough between them. We find that these features are inherent characteristics of our model and are retained regardless of reaction, incident energy, and three-particle invariant mass under consideration.

The effects of the Goldhaber form factor and the inclusion of dampening of the momenta on the pure model are shown in Fig. 9 in the delta-function approximation. For comparison we have reproduced the DF curve of the pure model given in Fig. 8b. Notice that we obtain essentially the same result as we did for the $N^*\pi$ mass distribution. All curves have the identical features and differ only in the degree of flatness. The same parameters used in the previous case were used to obtain the D curve shown here. Thus, the NI curves corresponding to the DF curves of Fig. 9 would all peak in the region around 1.02 BeV and would differ only in the degree of relative flatness. Agreement with experiment is reasonable.

We next investigate the t^2 cut-off curves that the model produces. In Fig. 10 we have reproduced such curves for the case of the pure model and the modified model with the inclusion of the Goldhaber form factor. In both cases a low t^2 cutoff damps out the

-32-

high mass peak, while retaining the low one. This is what we intuitively expect should happen, although we did not get such a behavior for the $N^*\pi$ case. There was no experimental distribution available for this case to compare with the model predictions.

2. $\pi p \rightarrow \rho N^* \rightarrow 3\pi + p$ at 8.0 and 16 BeV/c.

In this section we investigate the behavior of curves obtained in the previous section when the c.m. energy of the reaction is increased.

The $N^*\pi$ curves produced by the pure model for an incident pion with a laboratory momentum of 8.0 BeV/c are shown in Fig. 11a. Comparing these results to the equivalent curves at 3.665 BeV/c given in Fig. 3b, we see that the principle effect of increasing the energy of the reaction is to shift the threshold and upper boundary of the three-particle invariant mass curve to higher values. In addition the physical region of the phase space of the three-particle invariant mass increases with an increase of the energy. Other than these changes, the features of the model curves remain remarkably constant regardless of energy. Thus, we still have two peaks in the mass distribution, but now they are centered around 1.52 BeV and 3.77 BeV. The trough in the curve now occurs at around 2.87 BeV.

The effects of the inclusion of dampening of the momenta factors were also looked into at this energy, and the results appear in Fig. 11b. These curves should be compared with those appearing in Fig. 6 for 3.665 BeV/c. As previously noted, modifications of the pure model in this manner change only the relative flatness of the curves and do not

-33-

alter the position of the peaks. The results in Fig. 11b confirm this result for 8.0 BeV/c.

The equivalent curves for the $\rho\pi$ case at 8.0 BeV/c are all drawn on one curve in Fig. 12. Stated tersely, an increase in energy mainly shifts the peaks to the higher regions of 1.25 BeV and 2.17 BeV.

The pure model curves for the $N^*\pi$ invariant mass at 16.0 BeV/c are shown in Fig. 13. The two peaks in the NI curve have now risen to 1.76 BeV and 5.31 BeV. From the results obtained at the lower energies it is clear that the modified model with dampening of the mementa factors would not alter the position of these peaks.

The pure model predictions for the $\rho\pi$ mass at this energy are shown in Fig. 14. The two peaks in the NI curve are now at around 1.48 BeV and 3.04 BeV.

3. $K + N \rightarrow K^* + N^* \rightarrow K + \pi + \pi + N$.

In this section we look at the pure model predictions for a $K^* N^*$ intermediate state.

In Fig. 15 we show the curves for the $K^*\pi$ invariant mass for four different values of beam momentum. For 2.30 BeV/c the NI curve yields two not very prominent peaks, one around 1.09 BeV and one around 1.29 BeV. By raising the beam momentum to 4.60 BeV/c we see that the peaks become more pronounced and are now centered around 1.14 and 1.73 BeV.

-34-

The results for 8.0 and 16 BeV/c appear in Fig. 15b and 15c respectively.

In Fig. 16 we have reproduced some preliminary data obtained from the Goldhaber group in Berkeley for the above reaction at 2.30 BeV/c. The experimental distribution has two fairly well-formed peaks--one at around 1.13 BeV and the other around 1.27 BeV. Since the model's peaks are not at all prominent at this momentum, no great victories can be claimed for the apparent correlation between the model's prediction and the location of peaks in the experimental distribution.

B. Diffraction-Scattering Model

In this section we concern ourselves with the predictions of our virtual model with a $\rho - N$ intermediate state. As we have previously noted for this model, the $p'' + h$ invariant mass is now no longer restricted by a Breit-Wigner form to a narrow region of the phase space. Thus, the delta-function approximation for p' is not valid in this case, for at no point does p'^2 take on the value $-m'^2$. Therefore, an extra, nontrivial integration is introduced in this model which does not lend itself to approximations. This integration over m_v^2 , where $m_v^2 = -(p'' + h)^2$, was performed numerically over the entire kinematically allowed region. In the results that follow we have set $\mu' = 0.750$ BeV.

One of the main aims of this section is to determine how much of the useful results obtained by Deck,⁹ with his diffraction scattering model, can be reproduced here. In the spirit followed thus far we shall start out with the pure model and then determine the minimum modifications

-35-

necessary to bring the results within reasonable agreement with experiment. We mention in passing that for the diffraction model to apply, of course, all N^* events must be excluded. Thus, the Deck model limited m_v such that $m_v^2 > 2.70 \text{ (BeV)}^2$. We shall investigate models with this restriction as well.

In Fig. 17a we have produced the results for the pure model for the $\rho\pi$ invariant mass. In one curve we have restricted the lower limit of the m_v^2 integration to be 2.70 (BeV)^2 . The other curve has no such restrictions placed on it, and the m_v integration is over the entire phase-space region. It is apparent that no real peaks result in either case, and the results are very reminiscent of a pure phase-space calculation.

In Fig. 17b we obtain some very interesting results when we consider the effects of t^2 form factors on the models in Fig. 17a. (We point out in passing that all three curves in Fig. 17b require double numerical integrations.) Curve (1) is just the pure-model result with t^2 restricted to values less than $15 \mu_\pi^2$. We see that the curve rises rapidly to its peak at around 1.02 BeV and then diminishes to zero fairly slowly. Curve (2) represents the pure-model curve with exponential dampening in place of the t^2 cutoff. We now get a curve with an entirely different character. The curve peaks very strongly in the region around 1.00 BeV and drops off to zero very rapidly. In the region of 1.30 BeV the curve has already fallen to less than half its peak value. Curve (3) is the same as curve (2)

-36-

except that, in addition, we have imposed the indicated m_v^2 cutoff. Of the three curves the one that comes closest to the Deck calculation is curve (3), where the peak has now shifted to around 1.10 BeV. Otherwise, this result is fairly similar to curve (2).

In Fig. 17c we have reproduced for convenience the principal result obtained by Deck in his attempt to explain the A_1 as a kinematical accident. Our results are very similar to Deck's, although we have changed our model less drastically. For one thing we have made no alterations in the spin-averaging factor of the vector meson. It is also clear that merely adding the single form factor involving exponential dampening is enough to bring the model to within reasonable agreement with experiment. Of course this is getting out of the model what is really put into it, and the use of any form factor to force agreement with experiment will affect all predictions of the model. Thus, it is not surprising that forcing the model to agree with the experimental t^2 distribution by inserting a form factor yields results in the three-particle invariant mass distributions that agree fairly well with experiment. Accordingly, we should not be too enthusiastic here, but should note that there appears to be many ways to get essentially the same result.

Maor and O'Halloran have attempted to improve Deck's calculation by extending the m_v integration to all values except those narrowly residing within the N^* band.¹⁰ They obtained results not startlingly different from Deck's. In our model changing the range of

-37-

integration of m_v merely produces a curve that lies somewhere between curves (2) and (3) in Fig. 17b. Thus, nothing drastic happens in our model either.

Following the Deck calculation, Shen et al. reexamined the original experimental results of the Goldhaber group in the light of the diffraction scattering model.¹¹ From the evidence presented they concluded that the A_1 bump at 1.08 BeV could indeed be explained by diffraction scattering of virtual pions. Thus, they conclude the A_1 to be a kinematical accident. Our own model calculation in Fig. 17b supports just this conclusion.

-38-

ACKNOWLEDGMENTS

I wish to thank Professor Charles Zemach for first suggesting the initial problem. I also wish to thank Mr. Jan Dash for teaching me how to write a decent program, and the members of the Goldhaber group at Berkeley for some spirited conversations.

-39-

APPENDIX A

A1. The Spin-3/2 Baryon and its Couplings

In this section we wish to describe the field-theory formalism for the spin-3/2 baryon in sufficient detail that we know how to unambiguously couple it to other fields. For the strong interactions this is equivalent to determining how the spin-3/2 field behaves under the parity transformation (hereafter denoted by P.T.).

The spin-3/2 field is described by $\psi_{\nu,\alpha}^{(j)}(x)$ which is a 16 component Lorentz vector - Dirac spinor. This means that each vector component ν is itself a four-spin whose components are labelled by α . The index j specifies the projection of the 3/2 spin on some reference axis. The spin-3/2 vector-spinor is constructed by forming linear combinations of direct products of a spin-1 field and a spin-1/2 field such that only the spin-3/2 part is projected out.

In the Takahashi and Umezawa formalism¹² the spin-3/2 field satisfies

$$\Lambda_{\mu\nu}(\partial) \psi_{\nu,\alpha}^{(j)}(x) = 0$$

$$\Lambda_{\mu\nu}(\partial) = \frac{1}{2} \left\{ \left(\delta_{\mu\nu} - \frac{1}{3} \gamma_{\mu} \gamma_{\nu} \right) (\gamma \cdot \partial + m') \right.$$

$$\left. + (\gamma \cdot \partial + m') \left(\delta_{\mu\nu} - \frac{1}{3} \gamma_{\mu} \gamma_{\nu} \right) \right\}. \quad (A1)$$

This single equation is exactly equivalent to the Rarita-Schwinger equations for this field¹³

-40-

$$(\gamma \cdot \partial + m') \psi_{\nu, \alpha}^{(j)} = 0; \quad \gamma_{\nu} \psi_{\nu, \alpha}^{(j)} = 0. \quad (A2)$$

From (A1) or (A2) it can be shown that

$$(\square - m'^2) \psi_{\nu, \alpha}^{(j)} = 0; \quad \partial_{\nu} \psi_{\nu, \alpha}^{(j)} = 0. \quad (A3)$$

The propagator for the spin-3/2 field is obtained by finding the inverse of the operator $\Lambda_{\mu\nu}$ given in (A1). All we really need for our purposes is the pole part of the propagator in momentum space, which is

$$P_{ij} = - \frac{1}{(2\pi)^4} \cdot \frac{1}{(p'^2 + m'^2)} \cdot \left\{ m' (1 + \gamma_4) \left(\delta_{ij} - \frac{1}{3} \sigma_i \sigma_j \right) \right\}_p, \quad (A4)$$

where $i, j = 1, 2, 3$. Here the quantities within the braces refer to the rest frame of the particle. The expression in braces divided by $2m'$ is the spin-3/2 positive energy-projection operator. We choose the same normalization for the spin-3/2 field as is usual to choose for the Dirac field. Henceforth, we suppress the indices j and α , their presence being understood.

We now pursue the question of coupling the spin-3/2 field to other fields. To do this we must first determine how the spin-3/2 field transforms under space inversion, so that we can form parity-conserving couplings. Let us first review the effects of the P.T. on other fields.

-41-

For spinless fields the P.T. is

$$\underline{x} \Rightarrow -\underline{x}; \quad \phi^p(\underline{x}, t) = \epsilon_0 \phi(-\underline{x}, t), \quad (A5)$$

where $\epsilon_0 = 1$ (-1) for scalar (pseudoscalar) fields. For the Dirac field we have

$$\underline{x} \Rightarrow -\underline{x}; \quad \psi^p(\underline{x}, t) = \epsilon_{\frac{1}{2}} \gamma_4 \psi(-\underline{x}, t), \quad (A6)$$

where by convention $\epsilon_{\frac{1}{2}} = 1$. Lastly, for the spin-1 case we have

$$\underline{x} \Rightarrow -\underline{x}; \quad A_\mu^p(\underline{x}, t) = \epsilon_1 T_{\mu\nu} A_\nu(-\underline{x}, t)$$

$$T_{\mu\nu} = -\delta_{\mu\nu} (-1)^{\delta_{\mu 4}}, \quad (A7)$$

where $\epsilon_1 = 1$ (-1) for a vector (pseudovector) field.

Now the spin-3/2 is formed by coupling a Dirac field with a spin-1 field in a manner that forms a pure 3/2 field. We can symbolically write

$$\psi_\mu(\underline{x}, t) = A_\mu(\underline{x}, t) \bigcirc \psi(\underline{x}, t),$$

provided we understand that the right side above may stand for a sum of such direct products, and furthermore these direct products are to

-42-

be combined by the proper Clebsch-Gordon coefficients to form a pure spin-3/2 field. It is manifestly apparent that the spin-3/2 field has mixed transformation properties under the Lorentz group. Thus, from (A6) and (A7) the P.T. for the spin-3/2 field is

$$\begin{aligned} \underline{x} &\Rightarrow -\underline{x} ; \quad \psi_{\mu}^p(\underline{x}, t) = A_{\mu}^p(\underline{x}, t) \bigcirc(\underline{x}) \psi^p(\underline{x}, t) \\ &= \epsilon_{3/2} \cdot T_{\mu\nu} A_{\nu}(-\underline{x}, t) \bigcirc(\underline{x}) \gamma_4 \psi(-\underline{x}, t). \end{aligned} \quad (A8)$$

Again, $\epsilon_{3/2}$ can take on the values ± 1 . For the spin-3/2 baryon resonances we shall consider we have $\epsilon_{3/2} = \epsilon_{1/2} = 1$. We now use this result to construct parity conserving couplings.

For the case of a vertex like $(3/2)^+ \rightarrow (0)^- + (1/2)^+ (p' \rightarrow p'' + h)$ we can only have the coupling

$$\bar{\psi}_{\mu}(\underline{x}) \psi(\underline{x}) \vec{\partial}_{\mu} \phi(\underline{x}) \Rightarrow \bar{U}_{\mu}(\underline{p}') p_{\mu}'' U(\underline{p}'') . \quad (A9)$$

It is a straightforward matter to show that this coupling is invariant under P.T. Couplings like $\bar{\psi}_{\mu} \gamma_{\mu} \psi \phi$ and $\bar{\psi}_{\mu} \overleftrightarrow{\partial}_{\mu} \psi \phi$, where the arrow on ∂_{μ} indicates the direction in which it operates, are zero because of the subsidiary conditions in (A2) and (A3). For such a vertex with scalar mesons, we would have to insert a γ_5 factor between the spinors in (A9).

Let us next consider a vertex like $(3/2)^+ \rightarrow (1)^- + (1/2)^+$. The most general parity-conserving coupling we can have is

-43-

$$\begin{aligned} & \{\bar{\Psi}_\mu [g_1 \delta_{\mu\nu} + g_2 \gamma_\nu \vec{\partial}_\mu + g_3 \vec{\partial}_\mu \vec{\partial}_\nu] \gamma_5 \Psi\} A_\nu \\ \Rightarrow & \bar{U}_\mu(p') \{h_1 \delta_{\mu\nu} + h_2 \gamma_\nu p'_\mu + h_3 p''_\mu p''_\nu\} \gamma_5 U(p'') \epsilon_\nu^{(h)}. \quad (A10) \end{aligned}$$

Since $p' = p'' + h$ and (A3) applies, we can rewrite these couplings in any number of equivalent forms, which essentially permits us to replace any of the p''_μ 's in (A10) by h_μ .

A11. The Vector-Meson and its Couplings

The equations for the massive vector-meson field are

$$\begin{aligned} \Lambda_{\nu\mu}(\partial) A_\mu(x,t) &= 0 \\ \Lambda_{\nu\mu}(\partial) &= (\square - \mu'^2) \delta_{\nu\mu} - \partial_\nu \partial_\mu. \quad (A11) \end{aligned}$$

From these we can obtain the usual subsidiary condition by applying ∂_ν from the left:

$$\partial_\mu A_\mu(x,t) = 0 \Rightarrow q'_\mu \epsilon_\mu^{(\lambda)}(q) = 0. \quad (A12)$$

The vector-meson propagator in momentum space is

$$P_{\sigma\nu} = -\frac{i}{(2\pi)^4} \cdot \frac{\left\{ \delta_{\sigma\nu} + \frac{q'_\sigma q'_\nu}{\mu'^2} \right\}}{(q'^2 + \mu'^2)}. \quad (A13)$$

-44-

Therefore, if $\epsilon_\mu^{(\lambda)}(q)$ represents the polarization four-vector of the meson, the completeness relation is then determined:

$$\epsilon_\sigma^{(\lambda)} \cdot \epsilon_\nu^{(\lambda)} = \delta_{\sigma\nu} + \frac{q'_\sigma q'_\nu}{\mu'^2}. \quad (A14)$$

We now wish to investigate the couplings of the vector meson to pseudoscalar mesons. The behavior of the meson fields under the P.T. are given in (A5) and (A7). The general form for the $(1)^- - (0)^- - (0)^-$ vertex (q', q'', h) is

$$\{K_1 (\partial_\mu \phi^{(1)}) \phi^{(2)} + K_2 \phi^{(1)} (\partial_\mu \phi^{(2)})\} \cdot A_\mu$$

$$\Rightarrow \{g_1 (q'' - h)_\mu + g_2 (q'' + h)_\mu\} \cdot \epsilon_\mu(q'). \quad (A15)$$

Thus, we obtain two different types of coupling in momentum space, depending on how four-momentum is conserved at the vertex. For $q' = q'' + h$, the g_2 term vanishes because of (A12) and the coupling becomes

$$g_1 (q'' - h)_\mu \cdot \epsilon_\mu(q'). \quad (A16)$$

For $q'' = q' + h$, the g_1 term vanishes, and the coupling is

$$g_2 (q'' + h)_\mu \cdot \epsilon_\mu(q'). \quad (A17)$$

-45-

AIII. Kinematical Boundaries of Three-Particle Invariant Masses

In this section we derive the kinematical bounds imposed on the three-particle invariant masses by the values of the masses in the four-particle final state and the value of the c.m. energy, s .

Consider the overall reaction

$$p + q \rightarrow p'' + h + q'' + f \quad (\text{A18a})$$

with the corresponding masses

$$(m, \mu) \rightarrow (m'', \mu_h, \mu'', \mu_f). \quad (\text{A18b})$$

Let us consider a particular three-particle invariant mass, say

$$m_x^2 = -(p'' + h + q'')^2. \quad \text{For this case we find}$$

$$\left\{ s(\mu_h + \mu'') + \mu_f^2 + \frac{s(m''^2 - \mu_f^2)}{(s - \mu_h - \mu'')} \right\} \leq m_x^2$$

$$\leq \left\{ (s - \mu_f)^2 \right\}. \quad (m'' \geq \mu'' \geq \mu_h) \quad (\text{A19})$$

By permutation of this result we can obtain the bounds for any three-particle combination.

For a given model with a particular intermediate state the range of variation of the m_x is smaller than the range given in (A19), but (A19) gives the absolute limits. Knowing these limits we can determine whether our peripheral-model calculation could conceivably

-46-

produce kinematical bumps at points of interest. For example, let us take the $p \pi^+ \pi^+$ combination for the Goldhaber reaction with

$(q)_{\text{lab}} = 3.665 \text{ BeV/c.}$ From (A19) we find

$$1.32 \text{ BeV} \leq m_x \leq 2.65 \text{ BeV.}$$

The observed kinematical bumps of the model do indeed occur close to the end points of this region. Thus, (A19) provides us with a quick and simple way of determining where the peripheral model will most likely have its peaks.

-47-

APPENDIX B

Consider the quantity

$$\frac{1}{(u^2 + b^2)} = \frac{1}{2ib} \left\{ \frac{1}{(u - ib)} - \frac{1}{(u + ib)} \right\}.$$

We now use the identity

$$\lim_{b \rightarrow 0^+} \frac{1}{(u \mp ib)} = \text{P.V.} \left(\frac{1}{u} \right) \pm i\pi \delta(u).$$

If the region of the u integration is symmetric about the origin, then the principal value term vanishes on integration, and we have the approximation

$$\frac{1}{(u^2 + b^2)} \approx \frac{\pi}{b} \delta(u). \quad (b \ll 1) \quad (\text{B1})$$

For a resonance propagator, if we assume that the partial width is very much smaller than its corresponding mean mass, then (B1) is applicable if we identify b with the product of the mean mass and the partial width. We shall refer to (B1) as the delta-function approximation.

-48-

APPENDIX C

From the results of Appendix AI, the S-matrix for the decay of the spin-3/2 baryon is given by

$$S = \frac{G}{i(4\pi)^{\frac{1}{2}}} \left\{ \frac{m' m''}{p_0' p_0'' h_0} \right\}^{\frac{1}{2}} \bar{U}(\underline{p}'') p_{\mu}'' U_{\mu}(\underline{p}') \cdot \delta(\underline{p}' - \underline{h} - \underline{p}'').$$

We evaluate everything in the $\underline{p}'$ rest frame. Using the positive energy projection operator for the resonance, averaging over the four initial spin states, and summing over the final states yields

$$\sum_{sp} |M|^2 = \frac{2}{3} \cdot \frac{|G|^2}{(4\pi)} \cdot \left\{ \frac{\underline{p}''^2 (m'' + p_0'')}{4 h_0 p_0''} \right\}_{p'}.$$

Now the total transition probability per unit time to all final states is given by

$$\Gamma_{m'} = \frac{1}{(2\pi)} \int \left\{ \sum_{sp} |M|^2 \right\} d\underline{p}'' d\underline{h} \delta(\underline{p}' - \underline{p}'' - \underline{h}). \quad (C1)$$

We then find

$$\Gamma_{m'} = \frac{1}{3} \cdot \left(\frac{|G|^2}{4\pi} \right) \cdot \left\{ \frac{\underline{p}''^3 (m'' + p_0'')}{m'} \right\}_{p'}. \quad (C2)$$

From the results of Appendix AII, the S matrix for the decay of the spin-1 meson is

$$S = \frac{g}{i(16\pi)^{\frac{1}{2}}} \cdot \frac{(q'' - f)_{\mu} \epsilon_{\mu}(\underline{q}')}{[q_0 f_0 q_0']^{\frac{1}{2}}} \cdot \delta(\underline{q}' - \underline{q}'' - \underline{f}).$$

-49-

Proceeding in exactly the same way, we find

$$\Gamma_{\mu'} = \frac{2}{3} \left(\frac{|g|^2}{4\pi} \right) \left\{ \frac{q''^3}{\mu'^2} \right\}_{q'} \quad (C3)$$

APPENDIX D

Consider the quantity

$$\begin{aligned}
 dy &\equiv \frac{d\mathbf{p}''}{p_0''} \frac{dh}{h_0} \delta^4(R - \mathbf{p}'' - \mathbf{h}) \\
 &= \frac{d\mathbf{p}''}{p_0''} \frac{dh}{h_0} \int \delta(p'^2 + m'^2) dm'^2 \cdot \delta^4(\mathbf{p}' - \mathbf{p}'' - \mathbf{h}) d^4\mathbf{p}' \cdot \delta^4(R - \mathbf{p}') \\
 &= \frac{d\mathbf{p}'}{2p_0'} \int dm'^2 \left[\frac{d\mathbf{p}''}{p_0''} \frac{dh}{h_0} \delta^4(\mathbf{p}' - \mathbf{p}'' - \mathbf{h}) \right] \delta^4(R - \mathbf{p}'). \quad (D1)
 \end{aligned}$$

Performing the $\mathbf{h}$ integration and setting

$$D(\mathbf{p}', \mathbf{p}'') \equiv p_0' - \frac{p_0''}{p''} \cos(\mathbf{p}', \mathbf{p}'') \quad (D2)$$

yields

$$dy = \frac{d\mathbf{p}'}{2p_0'} dm'^2 \left[\frac{p'' d\Omega_{\mathbf{p}''}}{D(\mathbf{p}', \mathbf{p}'')} \right] \delta^4(R - \mathbf{p}'). \quad (D3)$$

We note that in the $\mathbf{p}'$ rest system $[D(\mathbf{p}', \mathbf{p}'')]_{\mathbf{p}'} = m'$. By permuting the labelling, the result in (D3) can be used to obtain analogous expressions for any two-particle combination.

-51-

APPENDIX E

Consider the decay $q' \rightarrow f + q''$. All variables are referred to the overall c.m. frame. We now relate the angular variables defined in the production coordinate system to a system in the decay frame.

In the production system we take the $\underline{p} - \underline{q}$ direction as the polar axis. In the decay frame we take $\underline{q}'$ to define the new polar axis and assume that the decay plane is coincident with the $x' - z'$ plane. The matrix relating the description of a vector as seen by each of these systems is, in terms of usual three Euler angles ϕ , θ , and ψ ,

$$A = \begin{pmatrix} c\psi \cdot c\phi - c\theta \cdot s\phi \cdot s\psi & c\psi \cdot s\phi + c\theta \cdot c\phi \cdot s\psi & s\psi \cdot s\theta \\ -s\psi \cdot c\phi - c\theta \cdot s\phi \cdot c\psi & -s\psi \cdot s\phi + c\theta \cdot c\phi \cdot c\psi & c\psi \cdot s\theta \\ s\theta \cdot s\phi & -s\theta \cdot c\phi & c\theta \end{pmatrix}, \quad (E1)$$

where a c before the angle means cosine and an s means sine.

This matrix carries us from the production to the decay system. Using the fact that

$$\hat{\underline{z}} = s\psi \cdot s\theta \cdot \hat{\underline{x}}' + c\psi \cdot s\theta \cdot \hat{\underline{y}}' + c\theta \cdot \hat{\underline{z}}' \quad (\hat{\underline{z}} \cdot \hat{\underline{z}} = 1, \text{ etc.})$$

we find

-52-

$$\cos\theta_{q'} = c\theta,$$

$$\cos\theta_{q''} = s\psi \cdot s\theta \cdot \sin(q', q'') + c\theta \cdot \cos(q', q''),$$

$$\cos\phi_{q'} = s\phi,$$

$$\exp(i\phi_{q''}) = \frac{\exp(i\phi)}{\sin\theta_{q''}} \cdot \left\{ \sin(q', q'') \cdot [c\psi + ic\theta \cdot s\psi] - is\theta \cdot \cos(q', q'') \right\}. \quad (E2)$$

It is then a straightforward matter to show that

$$d\Omega_{q'} d\Omega_{q''} = d\psi d(\cos\theta) d\phi d[\cos(q', q'')], \quad (E3)$$

where the absolute value of the Jacobian of the transformation is unity. Through substitution, the result in (E3) can be applied to all other cases.

-53-

APPENDIX F

In this appendix we wish to define all of the functions appearing in Sec. IIIA. Without elaboration we merely quote the results in a form suitable for programming. All quantities appearing below are evaluated in the c.m. frame, and we will adhere to the notation $\underline{q} \equiv |\underline{q}|$ etc. here as well as in all subsequent appendices.

We define:

$$a_1 \equiv m m' + p_0 p'_0$$

$$a_2 \equiv - \underline{q} \underline{q}'$$

$$a_3 \equiv a_2^2 / m'^2$$

$$a_4 \equiv - (2 p_0 p'_0 \underline{q} \underline{q}') / m'^2$$

$$a_5 \equiv (p_0^2 p_0'^2 / m'^2) - m^2$$

$$A_0 \equiv a_1 a_5$$

$$A_1 \equiv a_1 a_4 + a_5 a_2$$

$$A_2 \equiv a_2 a_4 + a_1 a_3$$

$$A_3 \equiv a_2 a_3$$

-54-

$$b \equiv (\mu'^2 + \mu''^2 - \mu_f^2)/(2\mu'^2)$$

$$b_1 \equiv \underline{q} \underline{q}'' \sin(\underline{q}', \underline{q}'')$$

$$b_2 \equiv \underline{q} \underline{q}'' \cos(\underline{q}', \underline{q}'') - \underline{q} \underline{q}' b$$

$$b_3 \equiv -q_0 q_0'' + q_0 q_0' b$$

$$B_0 \equiv 2 b_3^2 + b_1^2$$

$$B_1 \equiv 4 b_2 b_3$$

$$B_2 \equiv 2 b_2^2 - b_1^2$$

whence

$$D_0 \equiv A_0 B_0$$

$$D_1 \equiv A_0 B_1 + A_1 B_0$$

$$D_2 \equiv A_0 B_2 + A_1 B_1 + A_2 B_0$$

$$D_3 \equiv A_1 B_2 + A_2 B_1 + A_3 B_0$$

$$D_4 \equiv A_2 B_2 + A_3 B_1$$

$$D_5 \equiv A_3 B_2$$

(F1)

-55-

From energy-momentum conservation we have

$$\begin{aligned}
 p' \cdot p'' &= \frac{1}{2} (\mu_h^2 - m''^2 - m'^2) \\
 \{\underline{p}''\}_{p'} &= \left\{ \frac{(p' \cdot p'')^2}{m'^2} - m''^2 \right\}^{\frac{1}{2}} \\
 \{(m'' + p_0'')\}_{p'} &= \frac{1}{m'} \cdot [m m' - p' \cdot p''] \\
 q' \cdot q'' &= \frac{1}{2} (\mu_f^2 - \mu''^2 - \mu'^2) \\
 \{\underline{q}''\}_{q'} &= \left\{ \frac{(q' \cdot q'')^2}{\mu'^2} - \mu''^2 \right\}^{\frac{1}{2}}, \quad (F2)
 \end{aligned}$$

whence

$$K(m_x^2, m'^2, \mu'^2) = \frac{9}{(2\pi)} \cdot \frac{\mu'^2 q}{\underline{q} s^2 D(q', q'') \{\underline{q}''^6\}_{q'} \{\underline{p}''^3(m'' + p_0'')\}_{p'}} \quad (F3)$$

From (F1) and (F3) all the quantities appearing in (7) are now defined.

From (F3)

$$\tilde{K}(m_x^2, m_0'^2, \mu_0'^2) \equiv (\pi m_0' \Gamma_{m'}) (\pi \mu_0' \Gamma_{\mu'}) \cdot \frac{K(m_x^2, m_0'^2, \mu_0'^2)}{2 \underline{q} \underline{q}'}, \quad (F4)$$

which defines everything in (8). Lastly, we have

$$c_0 \equiv 2 q_0 q_0' - \mu^2 - \mu'^2 + \mu_r^2,$$

-56-

$$c_1 \equiv 2 \underline{q} \underline{q}',$$

and

$$c \equiv c_0/c_1,$$

whence

$$\begin{aligned} I(m_x^2, m'^2, \mu'^2) &\equiv \frac{2}{c_1^2(c^2 - 1)} \cdot \left[\sum_{i=0}^5 D_i c^i \right] \\ &+ \frac{1}{c_1^2} \cdot \ln \left| \frac{(c - 1)}{(c + 1)} \right| \cdot \left\{ \sum_{i=1}^5 D_i \frac{d}{dc} c^i \right\} \\ &+ \frac{2}{c_1^2} \{ D_2 + 2c D_3 + \left(\frac{1}{3} + 3c^2\right) D_4 + 2c \left(\frac{1}{3} + 2c^2\right) D_5 \}. \end{aligned} \quad (F5)$$

With the result in (F5) everything in (9) and (10) is defined.

-57-

APPENDIX G

In this appendix we define all functions appearing in Sec. IIIB. Again, all quantities appearing below are evaluated in the c.m. frame:

$$B_0 \equiv (q_0^2 q_0'^2 / \mu'^2) - \mu^2$$

$$B_1 \equiv (2 q_0 q_0' \underline{q} \cdot \underline{p}') / \mu'^2$$

$$B_2 \equiv \underline{q}^2 \underline{p}'^2 / \mu'^2$$

$$a \equiv (m'^2 + \mu_h^2 - m''^2) / (2m'^2)$$

$$a_1 \equiv -\mu_h^2 + m'^2 a^2$$

$$a_2 \equiv \underline{q} \cdot \underline{h} \sin(\underline{p}', \underline{h})$$

$$a_3 \equiv \underline{q} \cdot \underline{h} \cos(\underline{p}', \underline{h}) - \underline{q} \cdot \underline{p}' a$$

$$a_4 \equiv p_0 h_0 - p_0 p_0' a$$

$$a_5 \equiv \underline{q}^2 \underline{p}'^2 / m'^2$$

$$a_6 \equiv (2p_0 p_0' \underline{q} \cdot \underline{p}') / m'^2$$

$$a_7 \equiv -m^2 + (p_0^2 p_0'^2 / m'^2)$$

-58-

$$a_8 \equiv m m' + p_0 p_0'$$

$$a_9 \equiv \underline{q} \underline{p}'$$

$$A_0 \equiv a_8 \{2a_1 a_7 + 6a_4^2 + 3a_2^2\}$$

$$A_1 \equiv a_8 \{2a_1 a_6 + 12a_3 a_4\} + a_9 \{2a_1 a_7 + 6a_4^2 + 3a_2^2\}$$

$$A_2 \equiv a_8 \{2a_1 a_5 + 6a_3^2 - 3a_2^2\} + a_9 \{2a_1 a_6 + 12a_3 a_4\}$$

$$A_3 \equiv a_9 \{2a_1 a_5 + 6a_3^2 - 3a_2^2\}.$$

Using the results in (F2), we make the following redefinition:

$$K(m_x^2, m^2, \mu^2) \equiv \frac{3m' \mu' Q}{4\pi \underline{q} s^2 D(p', h) \{ \underline{q}''^3 \}_{\underline{q}}, \{ \underline{p}''^6 (m'' + p_0'') \}_{\underline{p}'}}.$$

The results in (F1) and (F4) can be taken over directly now to apply to this case. Furthermore, by defining

$$c_0 \equiv 2 q_0 q_0' - \mu^2 - \mu'^2 + \mu_r^2$$

$$c_1 \equiv -2 \underline{q} \underline{p}',$$

the result in (F5) is directly applicable here.

-59-

APPENDIX H

We define here all functions appearing in Sec. IVA:

$$b_1 \equiv 2 f_0 q_0 - \mu^2 - \mu_f^2$$

$$b_2 \equiv 2 \underline{q} [\underline{q}' - \underline{q}'' \cos(\underline{q}', \underline{q}'')]]$$

$$b_3 \equiv 2 \underline{q}'' \underline{q} \sin(\underline{q}', \underline{q}'')]$$

$$D_0 \equiv b_3^2 - (b_1 - t^2)^2$$

$$D_1 \equiv 2 b_2 (b_1 - t^2)$$

$$D_2 \equiv - (b_2^2 + b_3^2)$$

$$d \equiv (\mu'^2 + \mu''^2 - \mu_f^2) / (2\mu'^2)$$

$$d_0 \equiv \frac{1}{2} b_3$$

$$d_1 \equiv \underline{q} \underline{q}'' \cos(\underline{q}', \underline{q}'') - \underline{q} \underline{q}' d$$

$$d_2 \equiv - q_0 q_0'' + q_0 q_0' d$$

$$B_0 \equiv \frac{1}{4} (t^2 - b_1)^2 + d_2^2$$

-60-

$$B_1 = \frac{1}{2} b_2 (t^2 - b_1) + 2 d_1 d_2$$

$$B_2 = \frac{1}{4} b_2^2 + d_1^2.$$

If we now use the same definitions for the A_i and the c_j given in Appendix F, then everything in (14a) is defined. Using the definition in (F3), if we now define

$$G(m_x^2, m'^2, \mu'^2) = \frac{2}{\pi} \cdot K(m_x^2, m'^2, \mu'^2),$$

then everything in (14b) and (15) is defined.

The limits of integration of the integral in (14a) are the two solutions of

$$D_0 + D_1 x_{\pm} + D_2 x_{\pm}^2 = 0. \quad (H1)$$

For $F(r^2) = 1$, the simplest way to evaluate this integral is to impose the transformation

$$y = c_0 - c_1 x$$

on every factor in the integrand. The solution can then be written as the sum of six integrals which can be found from any good integral table.

-61-

APPENDIX I

We define here all terms that apply to case IVB:

$$a \equiv (m'^2 + \mu_h^2 - m''^2)/(2m'^2)$$

$$a_1 \equiv (m'^2 a^2 - m''^2)^{\frac{1}{2}}$$

$$a_2 \equiv \underline{q} \underline{h} \sin(\underline{p}', \underline{h})$$

$$a_3 \equiv \underline{q} \underline{h} \cos(\underline{p}', \underline{h}) - a \underline{q} \underline{p}'$$

$$a_4 \equiv p_0 h_0 - a p_0 p'_0$$

$$a_5 \equiv [\underline{q} \underline{p}'/m']^2$$

$$a_6 \equiv 2 p_0 p'_0 \underline{q} \underline{p}'/m'^2$$

$$a_7 \equiv [p_0 p'_0/m']^2 - m^2$$

$$a_8 \equiv m m' + p_0 p'_0$$

$$a_9 \equiv \underline{q} \underline{p}'$$

$$b_1 \equiv 2 p_0 p''_0 - m^2 - m''^2$$

-62-

$$b_2 \equiv -2 \underline{q} [\underline{p}' - \underline{h} \cos(\underline{p}', \underline{h})]$$

$$b_3 \equiv -2 \underline{h} \underline{q} \sin(\underline{p}', \underline{h})$$

$$a_{10} \equiv a_1^2 a_7 + 3 a_4^2 + \frac{3}{4} (t^2 - b_1)^2$$

$$a_{11} \equiv a_1^2 a_6 + 6 a_3 a_4 - \frac{3}{2} b_2 (t^2 - b_1)$$

$$a_{12} \equiv a_1^2 a_5 + 3 a_3^2 + \frac{3}{4} b_2^2$$

$$A_0 \equiv a_8 a_{10}$$

$$A_1 \equiv a_8 a_{11} + a_9 a_{10}$$

$$A_2 \equiv a_8 a_{12} + a_9 a_{11}$$

$$A_3 \equiv a_9 a_{12} .$$

The functions B_i and c_j that are applicable here are those defined in Appendix G. The functions D_k for this case are those defined in Appendix H provided we use the values of the b_1 defined above.

-63-

APPENDIX J

We define here all terms appearing in the virtual nucleon model appearing in Sec. VII:

$$A_0 \equiv m m'' (m'^2 + m_v^2) - p_0 p_0'' (m_v^2 - m'^2) - m m' (m_v^2 + m''^2 - \mu_h^2) \\ + p_0 p_0' (m_v^2 + m''^2 - \mu_h^2 - 2m' m'')$$

$$A_1 \equiv \underline{q} \underline{p}' (m_v^2 + m''^2 - \mu_h^2 - 2m' m'') \\ + (m_v^2 - m'^2) \underline{q} [-\underline{p}' + \underline{h} \cos(\underline{p}', \underline{h})].$$

Then, using the same functions for the B_i and c_j defined in Appendix G, we define

$$I(m_x^2, m_v^2, \mu'^2) \equiv \int_{-1}^1 F(r^2) \frac{(A_0 + A_1 x) (B_0 + B_1 x + B_2 x^2)}{(c_0 - c_1 x)^2} dx,$$

where $F(r^2)$ is an arbitrary form factor. For $F(r^2) = 1$ this integral can be written immediately by using the definitions in (F1) with $A_2 = A_3 = 0$ and the result in (F5) with $c = c_0/c_1$. Using the definition for Q in (11), we define

$$K(m_x^2, \mu'^2) \equiv \frac{3}{2(2\pi)^3} \cdot \frac{Q m m'' \mu'}{\underline{q} s^2 D(p', h) \{q''^3\}_{q'}} \cdot |G|^4.$$

-64-

This defines everything appearing in (20). Lastly we define

$$H(m_x^2, t^2, m_v^2, \mu'^2) = \int_{x_-}^{x_+} F(r^2) \frac{(A_0 + A_1 x) (B_0 + B_1 x + B_2 x^2)}{(c_0 - c_1 x)^2 (D_0 + D_1 x + D_2 x^2)^{\frac{1}{2}}} dx,$$

where the D_i are given in Appendix I. Everything in (27) is now defined.

The limits are given by (H1).

-65-

FIGURE LEGENDS

- Fig. 1. Pseudoscalar exchange double-resonance peripheral model.
- Fig. 2. Kinematical boundaries of the two-particle invariant masses.
- Fig. 3. (a) Goldhaber experimental distribution for the $p \pi^+ \pi^+$ invariant mass. The reaction was $\pi^+ p \rightarrow \pi^- \pi^+ \pi^+ p$ at 3.665 BeV/c with an $(N^*)^{++} - \rho^0$ filter applied to the data. There was a yield of 565 events. The dashed curve represents the phase-space distribution. (b) Pure-model calculation for the distribution.
- Fig. 4. The reaction as viewed in (a) the c.m. frame and (b) the q' rest frame.
- Fig. 5. Model prediction with the Goldhaber form factor for the $N^* \pi$ combination at 3.665 BeV/c.
- Fig. 6. Model prediction with dampening of the momenta in the delta-function approximation at 3.665 BeV/c. The momenta dampening curve is labelled D, the Goldhaber form factor curve G, and the pure model curve P.
- Fig. 7. (a) Goldhaber experimental distribution for the $p \pi^+ \pi^+$ invariant mass for 315 $\pi^+ p \rightarrow \pi^- \pi^+ \pi^+ p$ events at 3.665 BeV/c with a t^2 cutoff of $15 \mu_\pi^2$. No resonance filters were applied to the data. (b) Model predictions in the delta-function approximation with the same t^2 cutoff and beam momentum as in Fig. 7a.
- Fig. 8. Goldhaber experimental distribution for the $\rho^0 \pi^+$ invariant mass for the reaction $\pi^+ p \rightarrow \pi^- \pi^+ \pi^+ p$ at 3.665 BeV/c with an $(N^*)^{++} - \rho^0$ filter applied to the data. The yield shown

-66-

has 555 events. The dashed curve represents the phase-space distribution. (b) Pure-model predictions for the same distribution as in Fig. 8a.

Fig. 9. Model predictions in the delta-function approximation for the $\rho \pi$ invariant mass at 3.665 BeV/c.

Fig. 10. Model predictions with an t^2 cutoff of $15 \mu_\pi^2$ for the $\rho \pi$ invariant mass in the delta-function approximation at 3.665 BeV/c.

Fig. 11. (a) Pure-model prediction at 8.0 BeV/c for the $N^*\pi$ invariant mass. (b) Model prediction for the $N^*\pi$ invariant mass with dampening of the momenta at 8.0 BeV/c in the delta-function approximation.

Fig. 12. Model predictions for the $\rho \pi$ invariant mass at 8.0 BeV/c.

Fig. 13. Pure-model predictions for the $N^*\pi$ invariant mass at 16.0 BeV/c.

Fig. 14. Pure-model predictions for the $\rho \pi$ invariant mass at 16.0 BeV/c.

Fig. 15. Pure-model predictions for the $K^*\pi$ invariant mass for the reaction $KN \rightarrow K^* N^* \rightarrow K \pi \pi N$ at laboratory beam momenta of (a) 2.30, (b) 4.60, (c) 8.0, and (d) 16.0 BeV/c.

Fig. 16. Experimental data for the $K^*\pi$ invariant mass for the reaction $KN \rightarrow K^* N^* \rightarrow K \pi \pi N$ at 2.30 BeV/c. A $K^* N^*$ filter was applied to the data which yielded 525 events.

Fig. 17. (a) Pure virtual-nucleon-model prediction for the $\rho \pi$ invariant mass in the reaction $\pi N \rightarrow \rho N \rightarrow 3\pi + N$ at 3.665 BeV/c. (b) Model predictions for various t^2 form factors of the form $F(t^2) = \exp(-\lambda t^2)$ for the same reaction. (c) The Deck result for the $\rho \pi$ invariant mass at the same energy. The dashed curve represents a fit to the

-67-

data obtained by combining the model calculation with phase space so as to obtain reasonable agreement at the upper and lower extremes of the spectrum.

FOOTNOTES AND REFERENCES

- * This work was done under the auspices of the U. S. Atomic Energy Commission.
1. G. Goldhaber, J. Brown, S. Goldhaber, J. Kadyk, B. Shen, and G. Trilling; Phys. Rev. Letters 12, 336 (1964).
 2. G. Goldhaber in Lectures in Theoretical Physics: Fundamental Particles and High Energy Physics, Wesley E. Brittin, editor (University of Colorado Press, Boulder, 1965), Vol. VIIB; Lawrence Radiation Laboratory Report UCRL-11729, July 1964.
 3. G. Goldhaber, S. Goldhaber, T. O'Halloran, and B. Shen in Proceedings of the XII International Conference on High Energy Physics, Dubna, 1964 (Atomizdat, Moscow, 1966); Lawrence Radiation Laboratory Report UCRL-11445, July 1964.
 4. S. Goldhaber, G. Goldhaber, J. Kadyk, B. Shen, and G. Trilling in Proceedings of the XII International Conference on High Energy Physics, Dubna, 1964 (Atomizdat, Moscow, 1966); Lawrence Radiation Laboratory Report UCRL-11464, July 1964.
 5. J. Dash and G. Goldhaber, UCRL Physics Notes, Memo No. 136, September 1965.
 6. G. Lynch, Lawrence Radiation Laboratory Report UCRL-10335, July 1962.
 7. G. Goldhaber, W. Chinowsky, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Letters 6, 62 (1963).
 8. M. Gell-Mann and K. Watson, Ann. Revs. Nucl. Sci. 4, 219 (1954).
 9. R. Deck, Phys. Rev. Letters 13, 169 (1964).

-69-

10. U. Maor and T. O'Halloran, Phys. Letters 15, 281 (1965).
11. B. Shen, G. Goldhaber, S. Goldhaber, and J. Kadyk, Phys. Rev. Letters 15, 730 (1965).
12. Y. Takahashi and H. Umezawa, Progr. Theoret. Phys. (Kyoto) 9, 14 (1953); See also H. Umezawa, Quantum Field Theory, (North Holland Press, Amsterdam, 1956). In addition, two papers that should be required reading for anyone interested in the subject of resonances of arbitrary spin are C. Zemach, Phys. Rev. 140, B97 (1965); *ibid.* B109 (1965).
13. W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

XBL675-3175

Fig. 1

XBL675-3176

Fig. 2

XBL 675-3177

Fig. 3

(a)

(b)

XBL675-3178

Fig. 4

XBL675-3179

Fig. 5

XBL675-3180

Fig. 6

XBL675-3181

Fig. 7

XBL675-3182

Fig. 8

XBL 675-3183

Fig. 9

XBL 675-3184

Fig. 10

XBL675-3185

Fig. 11

XBL675-3186

Fig. 12

XBL675-3187

Fig. 13

XBL675-3188

Fig. 14

XBL675-3189

Fig. 15

XBL675-3190

Fig. 16

XBL675-3191

Fig. 17

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.