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Author

Kayser, Boris

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UNIVERSITY OF CALIFORNIA

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Section 2.1. Particle-Antiparticle Properties of Neutrinos

B. Kayser

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Section 2.1. PARTICLE-ANTIPARTICLE PROPERTIES OF NEUTRINOS *

Boris Kayser

*Theoretical Physics Group
Physics Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720*

and

*Division of Physics
† National Science Foundation
Washington, D.C. 20550*

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†Permanent address.

2.1.1. Motivation for Considering the Possibility that $\bar{\nu} = \nu$

Electrons and protons are obviously not their own antiparticles, since they are electrically charged. Similarly, neutrons are clearly not their own antiparticles, since they carry baryon number. By contrast, it is possible that neutrinos are their own antiparticles, since they carry neither electric charge, nor, as far as we know, any other charge-like attribute. It might be objected that neutrinos carry “lepton number”, the quantum number that distinguishes an antilepton from a lepton. However, as we shall see, there is in reality no evidence that any such quantum number exists. Thus, it is indeed possible that neutrinos, unlike all the other known fermions, are their own antiparticles.

From the theoretical standpoint, this possibility is a very attractive one. To see why, let us first note that, in general, grand unified theories lead us to expect that neutrinos are massive. In any grand unified theory, the neutrino of a given generation is placed in a multiplet together with the charged lepton and the quarks of the same generation (and sometimes together with additional particles as well). Now, the charged lepton and quarks of any generation are all known to be massive. Thus, being in a multiplet with them, the neutrino would have to be exceptional to be massless. Nevertheless, we know that the neutrino in each generation is, at the heaviest, much lighter than the corresponding charged lepton and quarks. Assuming that the neutrino is indeed massive, we have to understand why its mass is so much smaller than the masses of these other particles. The most popular explanation of this fact is the “see-saw mechanism” [GEL 79, YAN 79, MOH 80, MOH 81]. This predicts that each neutrino mass M_ν obeys a “see-saw relation” of the form $M_\nu M \approx [\text{Typical quark or charged lepton mass}]^2$, where M is a very large mass scale. Very importantly, the see-saw mechanism also predicts that neutrinos are their own antiparticles. [For a discussion of neutrino mass terms in gauge field theories, and a detailed explanation of the see-saw mechanism, see KAY 88a.]

2.1.2. The Precise Meaning of $\bar{\nu} = \nu$: Dirac and Majorana Neutrinos

What, precisely, do we mean when we say that a neutrino ν is its own antiparticle? We do not mean that $C|\nu\rangle = \bar{\eta}_C|\nu\rangle$, where C denotes charge-conjugation and $\bar{\eta}_C$ is the C -parity of ν . After all, the weak interactions which dress the state

$|\nu\rangle$ are maximally C -nonconserving. Hence, if $|\nu\rangle$ has some definite C -parity at one instant, it will not have this same C -parity at a later instant. Thus, a neutrino which is its own antiparticle must be defined by its transformation properties under CPT, which presumably is completely conserved. Under CPT, any neutrino $|\nu(\vec{p}, h)\rangle$ of momentum \vec{p} and helicity h transforms according to

$$\text{CPT } |\nu(\vec{p}, h)\rangle = \tilde{\eta}_{\text{CPT}}^h |\bar{\nu}(\vec{p}, -h)\rangle. \quad (2.1.1)$$

Here the helicity reversal is due to the P operation, and the phase factor $\tilde{\eta}_{\text{CPT}}^h$ depends on the helicity, as we shall see. If the neutrino is not its own antiparticle, then the particles ν and $\bar{\nu}$ in Eq. (2.1.1) differ. That is, the particles we call the “neutrino” and the “antineutrino” interact differently with matter. When this is the case, ν is referred to as a Dirac neutrino ν^D . Obviously, in its rest frame such a neutrino consists of four states: two spin states for the neutrino, and an additional two for the antineutrino. By contrast, when the neutrino ν is its own antiparticle, the particles ν and $\bar{\nu}$ in Eq. (2.1.1) are identical. That is, for given momentum and helicity, the particles we call the “neutrino” and the “antineutrino” have identical interactions with matter. When this is the case, ν is called a Majorana neutrino ν^M . In its rest frame, such a neutrino consists of only two states: one with spin up, and one with spin down.

2.1.3. Why We Do Not Know If $\bar{\nu} = \nu$

Why is it that we do not know whether neutrinos are their own antiparticles? The reason is that the experimentally available neutrinos are always polarized, and, in particular, the “neutrinos” are polarized oppositely from the “antineutrinos”. The particles we call “neutrinos” are always left-handed, while those we refer to as “antineutrinos” are always right-handed. As a result, we have not been able to compare the interactions with matter of neutrinos and antineutrinos of the same helicity. To be sure, we know very well that the left-handed neutrinos interact very differently from the right-handed antineutrinos. However, there is no way of knowing whether this difference is due simply to the difference in polarization in the two cases, or to a real distinction between neutrinos and antineutrinos that goes beyond mere polarization.

A good illustration of this state of affairs is provided by the neutrinos from pion decay. The neutral lepton emitted in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$, which

by convention we call a neutrino rather than an antineutrino, is always of left-handed (*i.e.*, negative) helicity. Let us indicate this fact by labelling it $\nu_\mu(-)$. By contrast, the neutral lepton emitted in the decay $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$, which by convention we call an antineutrino, is always of right-handed (positive) helicity. We shall indicate this fact by labelling it $\bar{\nu}_\mu(+)$. Now, it is observed that when a $\nu_\mu(-)$ strikes a nucleon N , the reaction $\nu_\mu(-) + N \rightarrow \mu^- + X$ may occur, but the reaction $\nu_\mu(-) + N \rightarrow \mu^+ + X$ will not. By contrast, when $\bar{\nu}_\mu(+)$ strikes a nucleon, the reaction $\bar{\nu}_\mu(+)+N \rightarrow \mu^+ + X$ may occur, but $\bar{\nu}_\mu(+)+N \rightarrow \mu^- + X$ will not. Unfortunately, this difference in interaction patterns has two possible explanations: (1) The difference may be due simply to the fact that $\nu_\mu(-)$ and $\bar{\nu}_\mu(+)$ have different polarizations. (2) It may be that there exists a conserved lepton number L , with $L(\nu_\mu) = L(\mu^-) = +1$ but $L(\bar{\nu}_\mu) = L(\mu^+) = -1$, so that the unobserved reactions are forbidden, and ν_μ and $\bar{\nu}_\mu$ are genuinely different.

To settle the issue of whether ν_μ and $\bar{\nu}_\mu$ differ, we must find out how the interactions of a ν_μ and a $\bar{\nu}_\mu$ of the same helicity compare. Suppose, for example, that we could somehow reverse the helicity of a $\bar{\nu}_\mu(+)$ created in π^- decay. We could then ask whether the resultant left-handed particle, $\bar{\nu}_\mu(-)$, interacts with nucleons in the same way as the left-handed $\nu_\mu(-)$ born in π^+ decay. If the answer is yes, then $\bar{\nu}_\mu(+)$ and $\nu_\mu(-)$ differ only in helicity; that is, ν_μ is a Majorana neutrino. If the answer is no, then $\nu_\mu(-)$ and $\bar{\nu}_\mu(+)$ evidently differ in a way that goes beyond helicity; that is, ν_μ is a Dirac neutrino. Regrettably, the reversal of neutrino helicity is very difficult, and has not been done.

Indeed, when a neutrino is massless, the reversal of its helicity is completely impossible, assuming there are no right-handed currents. For a massless neutrino, the helicity cannot be reversed by viewing the neutrino from a frame in which the direction of its momentum is reversed, since for a massless particle there is no such frame. It is not hard to show that, in addition, if all weak currents are left-handed, the helicity of a massless neutrino cannot be reversed by interactions between the neutrino and matter. Thus, in the massless case there is no way to produce a particle such as $\bar{\nu}_\mu(-)$, so it becomes meaningless to ask how this particle behaves. Consequently, the distinction between a Majorana neutrino and a Dirac one disappears. Furthermore, the approach to

the massless limit is a smooth one, so that even if, as we suspect, neutrinos have nonzero masses, it is nevertheless very difficult to tell whether they are Majorana or Dirac particles because their masses are so tiny compared to their energies and other mass scales [KAY 88b]. This difficulty has been referred to as the “practical Dirac-Majorana confusion theorem” [KAY 82].

2.1.4. CP and CPT Properties of Majorana Neutrinos

We have defined a Majorana neutrino ν^M as one which is its own mirror image under CPT:

$$CPT |\nu^M(\vec{p}, h)\rangle = \tilde{\eta}_{CPT}^h |\nu^M(\vec{p}, -h)\rangle. \quad (2.1.2)$$

To the extent that CP is conserved, such a neutrino is also an eigenstate of CP:

$$CP |\nu^M(\vec{p}, h)\rangle = \tilde{\eta}_{CP} |\nu^M(-\vec{p}, -h)\rangle. \quad (2.1.3)$$

Here the momentum and helicity reversals are due to the P operation, and the phase factor $\tilde{\eta}_{CP}$ is the intrinsic CP-parity of the neutrino ν^M . Different neutrinos can have different values of $\tilde{\eta}_{CP}$, but the permissible values of this quantum number are $\pm i$, rather than ± 1 . An easy way to see this is to consider the decay of the neutral weak boson into a pair of identical Majorana neutrinos: $Z^0 \rightarrow \nu\nu$. In the standard model, this decay conserves CP. To find the consequences of this conservation, it suffices to suppose that the outgoing neutrinos are nonrelativistic. Then, since they are identical fermions, they must be in a 3P_1 state, since this is the only antisymmetric nonrelativistic state with total angular momentum equal to the spin of the Z^0 . Now, from Eq. (2.1.3) it follows that if the intrinsic CP-parity of ν is $\tilde{\eta}_{CP}(\nu)$, then our $\nu\nu$ final state, with orbital angular momentum $L = 1$, obeys

$$\begin{aligned} CP |\nu\nu; {}^3P_1\rangle &= \tilde{\eta}_{CP}^2(\nu)(-1)^L |\nu\nu; {}^3P_1\rangle \\ &= -\tilde{\eta}_{CP}^2(\nu) |\nu\nu; {}^3P_1\rangle. \end{aligned} \quad (2.1.4)$$

Since the Z^0 has $CP = +1$, conservation of CP in $Z^0 \rightarrow \nu\nu$ then implies that $-\tilde{\eta}_{CP}^2(\nu) = +1$. Hence, the allowed values of the intrinsic CP-parity of a Majorana neutrino are [KAY 84]

$$\tilde{\eta}_{CP}(\nu) = \pm i. \quad (2.1.5)$$

To illustrate the consequences of $\tilde{\eta}_{CP}$, and of the fact that it is imaginary, let us consider the process $e^-e^+ \rightarrow N_1N_2$, where N_1 and N_2 are two distinct heavy Majorana neutral leptons [PET 86]. Assuming that the process is engendered by W boson exchange, the only incoming helicity configuration that couples is $e^-(-)e^+(+)$. In the e^-e^+ c.m. frame, this state is a CP eigenstate, and it is not hard to show that it has $CP = +1$. Now, consider the process just above N_1N_2 production threshold, and suppose that the final particles are in a state with definite orbital angular momentum L . Then the final state is also a CP eigenstate, and from Eq. (2.1.3) its CP is $\tilde{\eta}_{CP}(\nu_1)\tilde{\eta}_{CP}(\nu_2)(-1)^L$. Thus, if CP is conserved in our reaction,

$$+1 = \tilde{\eta}_{CP}(\nu_1)\tilde{\eta}_{CP}(\nu_2)(-1)^L. \quad (2.1.6)$$

Bearing in mind that the possible values of $\tilde{\eta}_{CP}$ are imaginary, we see that if $\tilde{\eta}_{CP}(\nu_1) = \tilde{\eta}_{CP}(\nu_2)$, the allowed partial wave near N_1N_2 threshold is the p wave, while if $\tilde{\eta}_{CP}(\nu_1) = -\tilde{\eta}_{CP}(\nu_2)$, it is the s wave. Had the values of $\tilde{\eta}_{CP}$ been real, it would have been the other way around.

Now what can be said about the CPT phase factor $\tilde{\eta}_{CPT}^h$ in Eq. (2.1.2)? With $\zeta \equiv \text{CPT}$, in the rest frame of ν^M this equation reads

$$\zeta \left| \nu^M(s) \right\rangle = \tilde{\eta}_\zeta^s \left| \nu^M(-s) \right\rangle, \quad (2.1.7)$$

where $s = \pm \frac{1}{2}$ is the projection of the spin of ν^M along some reference direction. This equation implies that, as long as we act only on the states $\left| \nu^M(s) \right\rangle$, $\zeta \vec{J} = -\vec{J} \zeta$, where \vec{J} is the angular momentum operator. It follows that $\zeta J_+ = -J_- \zeta$, where $J_\pm = J_x \pm iJ_y$ are the raising and lowering operators, and we have used the fact that ζ is antiunitary. If we apply this anticommutation relation to the state $\left| \nu^M(-\frac{1}{2}) \right\rangle$, we obtain [KAY 84]

$$\begin{aligned} \zeta J_+ \left| \nu^M(-\frac{1}{2}) \right\rangle &= \tilde{\eta}_\zeta^{+1/2} \left| \nu^M(-\frac{1}{2}) \right\rangle \\ &= -J_- \zeta \left| \nu^M(-\frac{1}{2}) \right\rangle = -\tilde{\eta}_\zeta^{-1/2} \left| \nu^M(-\frac{1}{2}) \right\rangle. \end{aligned} \quad (2.1.8)$$

Thus, $\tilde{\eta}_\zeta^s$ does indeed depend on the direction of the spin:

$$\tilde{\eta}_\zeta^{+1/2} = -\tilde{\eta}_\zeta^{-1/2}. \quad (2.1.9)$$

However, apart from this constraint, $\tilde{\eta}_\zeta^2$ is arbitrary, because the states $|\nu^M(s)\rangle$ and $|\nu^M(-s)\rangle$ appearing in Eq. (2.1.7) can always be redefined through multiplication by arbitrary phase factors.

2.1.5. Electromagnetic Properties of Majorana Neutrinos

How do the electromagnetic properties of neutrinos depend on whether they are Dirac or Majorana particles? From Lorentz invariance and current conservation, it follows that for any spin-1/2 fermion f , the matrix element of the electromagnetic current J_μ^{EM} has the form

$$\langle f(p_f, h_f) | J_\mu^{EM} | f(p_i, h_i) \rangle = i\bar{u}_f [F\gamma_\mu + G(q^2\gamma_\mu - 2m_f i q_\mu)\gamma_5 + M\sigma_{\mu\nu}q_\nu + E i\sigma_{\mu\nu}q_\nu\gamma_5] u_i. \quad (2.1.10)$$

Here p_i, h_i are the initial momentum and helicity of f , p_f, h_f are the final ones, $q = p_i - p_f$, m_f is the mass of f , and F, G, M , and E are form factors which depend on q^2 . If f is a Majorana neutrino ν^M , then the electromagnetic matrix element obeys the CPT constraint

$$\begin{aligned} \langle \nu^M(p_f, h_f) | J_\mu^{EM} | \nu^M(p_i, h_i) \rangle &= -\langle \zeta \nu^M(p_i, h_i) | J_\mu^{EM} | \zeta \nu^M(p_f, h_f) \rangle \\ &= -\tilde{\eta}_\zeta^{h_i} \tilde{\eta}_\zeta^{h_f} \langle \nu^M(p_i, -h_i) | J_\mu^{EM} | \nu^M(p_f, -h_f) \rangle. \end{aligned} \quad (2.1.11)$$

The minus sign in this relation arises from the fact that J_μ^{EM} is CPT-odd, and the interchange of the initial and final states from the fact that $\zeta \equiv \text{CPT}$ is antiunitary. Using the relation $\tilde{\eta}_\zeta^{h_i} \tilde{\eta}_\zeta^{h_f} = (-1)^{h_i - h_f}$ which follows from Eq. (2.1.9), and writing both the first and third “sides” of the constraint (2.1.11) in the form (2.1.10), one can show that this constraint implies that $F = M = E = 0$ [NIE 82, KAY 83, MCK 82]. That is, for a Majorana neutrino, the most general form of the electromagnetic matrix element is [KAY 82, NIE 82, SCH 81]

$$\langle \nu^M(p_f, h_f) | J_\mu^{EM} | \nu^M(p_i, h_i) \rangle = i\bar{u}_f G(q^2) (q^2\gamma_\mu - 2m_\nu i q_\mu)\gamma_5 u_i, \quad (2.1.12)$$

involving only a G -type form factor. By contrast, for a Dirac neutrino there is no analogue of the constraint (2.1.11), and the electromagnetic matrix element can have the full structure of Eq. (2.1.10), with all four form factors.

The magnetic and electric dipole moments of any fermion are, respectively, the values of its M and E form factors at $q^2 = 0$. Thus, a Majorana neutrino has no dipole moments. The electric charge radius of any fermion is, apart from a numerical factor, the derivative of its F form factor at $q^2 = 0$. Thus, a Majorana neutrino has no charge radius either.

The absence of dipole moments is easy to understand. Suppose that some Majorana neutrino has a magnetic dipole moment $\mu_{Mag}\vec{s}$ and an electric dipole moment $\mu_{El}\vec{s}$, where \vec{s} is the neutrino spin. Then, when this neutrino is at rest in static, uniform magnetic and electric fields \vec{B} and \vec{E} , it has a dipole interaction energy $-\mu_{Mag}\vec{s}\cdot\vec{B} - \mu_{El}\vec{s}\cdot\vec{E}$. Now, in the CPT-reflected state, the spin \vec{s} is reversed, but (as one may easily show) \vec{B} and \vec{E} are unchanged. Thus, the dipole interaction energy is reversed. Hence, if the world is to be invariant under CPT reflection, μ_{Mag} and μ_{El} must vanish.

The absence of a charge radius is also easy to understand. Suppose, for example, that some Majorana neutrino has a charge radius arising from the presence, in the (neutral) neutrino, of a positively-charged core surrounded by a compensating negatively-charged shell. Under CPT, this charge distribution transforms into a negative core surrounded by a positive shell, something quite different from its original self. However, a Majorana neutrino must transform into itself under CPT, apart from a spin-reversal. Thus, a Majorana neutrino actually cannot contain a positive core and negative shell. This illustrates why, more generally, such a neutrino cannot have a charge radius.

Despite the absence of dipole moments and a charge radius, a Majorana neutrino can couple to a photon. It does this through its G -type form factor. The electromagnetic structure to which this form factor corresponds [RAD 85] may be pictured as a torus formed by bending a flexible straight solenoid into the shape of a circle and joining the ends. The \vec{B} field formerly present inside the solenoid will now circulate around the interior of the torus.

Unfortunately, it is extremely unlikely that we will be able to determine whether neutrinos are Dirac or Majorana particles by studying their electromagnetic properties. Indeed, the insensitivity of electromagnetic studies to the Dirac-Majorana distinction is an example of the practical Dirac-Majorana con-

fusion theorem referred to earlier. It is true that, while a Majorana neutrino can never have a magnetic dipole moment, the standard model (with neutrino masses added) predicts that a Dirac neutrino of mass M_ν will have a dipole moment $\mu_{Mag} = 6 \times 10^{-19} (M_\nu/1eV) \mu_B$, where μ_B is the Bohr magneton [LEE 77]. However, for M_ν below the existing upper bounds, this moment is far too small to be detected experimentally [SHR 82]. It is also true that, while a Majorana neutrino can never have an F -type form factor, the standard model predicts that a Dirac neutrino will have one [Section 2.6 of this volume and DEG 88]. However, this model also predicts that both a Dirac and a Majorana neutrino will have a G -type form factor. Now, the only experimentally available neutrinos are highly-relativistic and left-handed. For such neutrinos, the F and G form factors lead to electromagnetic matrix elements which are helicity-preserving and of identical structure. Furthermore, the standard model (or any model with no right-handed currents) predicts that for any highly-relativistic left-handed neutrino, the matrix element arising from the G form factor if the neutrino is of Majorana character is identical, not only in structure but also in size, with that arising from the F and G form factors together if the neutrino is of Dirac character [KAY 82, KAY 88b. See also BAR 88].

For further discussion of the electromagnetic structure of neutrinos, see Section 2.6 of this volume.

2.1.6. CP Violation When $\bar{\nu} = \nu$

In the standard model, CP violation in the weak interactions of quarks arises from complex phase factors in the quark mixing matrix. However, unless there are at least three generations, all phase factors in this matrix can be rotated away, and so have no physical significance. Thus, in the standard model, the quark interactions could not violate CP at all if there were fewer than three generations [KOB 73].

In analogy with the quark interactions, the leptonic interactions can violate CP (in the standard model) as a result of complex phase factors in the leptonic mixing matrix. However, if neutrinos are their own antiparticles, then, for a given number of generations, fewer of the phases in the leptonic mixing matrix than of those in the quark matrix can be rotated away [BIL 80, SCH 80, DOI

81, KOB 80]. In particular, one phase already survives when there are only two generations. As a result, even if only two of the three known lepton generations mix appreciably, so that in effect there are only two generations, there can still be sizeable CP-violating effects in the leptonic sector.

One can understand why more lepton phases than quark phases have physical significance when neutrinos are their own antiparticles by noting that when this is the case, certain leptonic processes have more Feynman diagrams than do the analogous quark processes [KAY 88c]. Now, complex phase factors in the lepton or quark mixing matrix can lead to physical CP-violating effects only when Feynman diagrams, to which these phase factors have imparted complex overall phases, interfere with one another. If some leptonic processes involve more Feynman diagrams than the corresponding quark processes, there can be additional interferences between diagrams in the leptonic case. These additional interferences can allow phase factors which have no consequences when they occur in the quark mixing matrix to lead to physical CP-violating effects when they occur in the lepton matrix.

As an illustration, let us compare the radiative decay $\nu_2 \rightarrow \nu_1 + \gamma$ of a heavy Majorana neutrino into a lighter one with the analogous decay $c \rightarrow u + \gamma$ of the charmed quark into the up quark. We shall suppose for simplicity that only the first two generations exist. Then the quark decay is engendered by diagrams in which the c quark turns either into a virtual dW^+ pair, or into a virtual sW^+ pair, and the photon is radiated by one of the particles in the pair. The pair then coalesces into the daughter u quark. It is very easy to show that the interferences between the various diagrams are completely insensitive to any complex phase factors in the (two-by-two) quark mixing matrix. The related neutrino decay arises from diagrams in which the ν_2 turns either into a virtual e^-W^+ pair, or into a virtual μ^-W^+ pair, and the photon is radiated by one of the charged particles in the pair. The pair then coalesces into the daughter ν_1 . So far, everything is in complete analogy with the quark decay. However, if the ν_2 is its own antiparticle, then, "confused" about whether it is a lepton or an antilepton, it can turn not only into the virtual pairs already mentioned, but also into e^+W^- and μ^+W^- . Thus, there are additional diagrams in which one

of these new pairs replaces e^-W^+ or μ^-W^+ . These additional diagrams, which have no analogue in the quark case, interfere with the diagrams containing e^-W^+ or μ^-W^+ . It is not difficult to show that these new interferences are sensitive to a complex phase factor in the lepton mixing matrix. Through these added interferences, this phase factor, if present, can lead to a physical CP-violating effect [KAY 88c].

2.1.7. Neutrinoless Double Beta Decay

In spite of the difficulty of telling whether neutrinos are Majorana or Dirac particles, there is one reaction which could provide evidence that they are Majorana particles even if their masses are of order 1 eV or less. This reaction is the nuclear decay $(A, Z) \rightarrow (A, Z + 2) + 2e^-$, known as neutrinoless double beta decay ($\beta\beta_{0\nu}$). This decay can arise from a diagram in which the parent nucleus emits a pair of virtual W bosons, and then these W bosons exchange a neutrino ν_m , of mass M_m , to produce the outgoing electrons. The amplitude is a sum over the contributions of all the ν_m that may exist.

At the vertex where it is emitted, the exchanged ν_m is created together with an e^- . Thus, should there be a difference between leptons and antileptons and lepton number be conserved, this " ν_m " would have to be a $\bar{\nu}_m$. However, at the vertex where it is absorbed, this same particle creates a second e^- , so it must be a ν_m . Thus, the diagram vanishes unless $\bar{\nu}_m = \nu_m$. Even then, it is suppressed by a helicity mismatch at the two vertices touched by the virtual ν_m . Where this particle is emitted, it is behaving like an antineutrino.. Hence, assuming the leptonic weak current is left-handed, the ν_m will be emitted in a predominantly right-handed state. On the other hand, where it is absorbed, it is behaving like a neutrino, so the current prefers to absorb it from a left-handed state.

Now, there is an amplitude of order $M_m/[\text{Energy of } \nu_m]$ for the ν_m to be emitted left-handed. If it is a Majorana particle, it can then be reabsorbed without further suppression. Thus, in effect, $\beta\beta_{0\nu}$ is a realization of the type of gedanken experiment we described when discussing neutrinos from pion decay. In $\beta\beta_{0\nu}$, we produce a particle—the exchanged ν_m —which is identified as an antineutrino by the fact that it is emitted together with an e^- . However, at least some of the time, this "antineutrino" is produced left-handed. We can

then see whether this left-handed “antineutrino” interacts as would a left-handed neutrino at the vertex where it is absorbed.

If the leptonic weak current contains a small right-handed piece, then this piece will lead to emission of a virtual $\bar{\nu}_m(-)$ in $\beta\beta_{0\nu}$, just as does the ν_m mass. As before, if $\bar{\nu}_m(-) = \nu_m(-)$, this particle can then be reabsorbed without suppression.

The process $\beta\beta_{0\nu}$ can provide evidence that neutrinos are Majorana particles even if their masses are much smaller than those required by any other process that has been considered. The primary reason for this special sensitivity is that the decays which can in principle compete with $\beta\beta_{0\nu}$ are highly-suppressed. So long as one chooses a parent nucleus which is stable against single beta decay, this competing mode is totally absent. Of course, competition with $\beta\beta_{0\nu}$ can always come from decay by emission of two electrons and two antineutrinos, a mode which can occur whether or not neutrinos are Majorana particles. However, this mode is phase-space suppressed, typically by six orders of magnitude, relative to $\beta\beta_{0\nu}$.

The amplitude $A[\beta\beta_{0\nu}]$ for $\beta\beta_{0\nu}$ can be written in the form

$$A[\beta\beta_{0\nu}] = M_{eff}N, \quad (2.1.13)$$

where N is a very nontrivial nuclear matrix element [HAX 84], and M_{eff} , the effective neutrino mass for neutrinoless double beta decay, contains the particle physics of the process. Assuming that there are no right-handed currents, and that all neutrino masses are small compared to the typical momentum transfer in $\beta\beta_{0\nu}$ (~ 10 MeV), M_{eff} is given by [DOI 81, WOL 81, KAY 83, KAY 84, BIL 84]

$$M_{eff} = \sum_m \omega_{em} |U_{em}|^2 M_m. \quad (2.1.14)$$

In this sum over neutrino exchange contributions, the contribution of ν_m is proportional to its mass M_m because of the helicity considerations we have discussed. The quantity U_{em} is an element of a unitary mixing matrix describing the coupling of neutrinos to charged leptons, and ω_{em} is a phase factor.

Suppose that $\beta\beta_{0\nu}$ were actually to be observed. From the observed decay rate, and a calculated value for the nuclear matrix element N , one could then

obtain an experimental value for M_{eff} . Since $\sum_m |U_{em}|^2 = 1$, we see from Eq. (2.1.14) that this experimental value could not exceed the largest of the actual neutrino masses M_m . That is, the observation of $\beta\beta_{0\nu}$ would imply a lower bound on neutrino mass: at least one neutrino would have to have a mass no smaller than the measured M_{eff} . By contrast, the observed absence of $\beta\beta_{0\nu}$ at some level does not imply an upper bound on the masses of any neutrinos. This absence only limits M_{eff} , and M_{eff} can be much smaller than the actual neutrino masses M_m , due to the possible cancellations in Eq. (2.1.14).

If right-handed currents, and/or W bosons beyond the known one, do exist, then M_{eff} can become much more complicated than the expression in Eq. (2.1.14). In particular, the contribution to M_{eff} of a given ν_m exchange need no longer vanish with M_m . Nevertheless, a simple argument shows that it is still true that the observation of $\beta\beta_{0\nu}$ would imply nonzero neutrino mass, even if the origin of this decay is not neutrino exchange but some more exotic mechanism [SCH 82, TAK 84]. To be sure, the nonzero mass which would be implied according to this argument is of very high order in the weak interaction, and consequently could be extremely infinitesimal. However, if one does assume that $\beta\beta_{0\nu}$ is caused (at least primarily) by neutrino exchange, then, for a broad class of gauge theories, the observation of this reaction would imply an experimentally interesting lower bound on neutrino mass [KAY 87, KAY 89]. Namely, even if right-handed currents and numerous W bosons exist, the bound discussed previously assuming their absence would still hold. That is, at least one neutrino would have to have a mass no smaller than the experimentally measured M_{eff} defined by Eq. (2.1.13) and determined from the observed decay amplitude and a calculated nuclear matrix element. Now suppose, for example, that ${}^{76}\text{Ge}$ should be seen to undergo neutrinoless double beta decay with a lifetime τ_{Ge} . If we express the observed decay amplitude in terms of τ_{Ge} , and use for the calculated nuclear matrix element N a popular value [HAX 84], then Eq. (2.1.13) implies that $M_{eff} \simeq 1 \text{ eV} [10^{24}\text{yr}/\tau_{Ge}]^{1/2}$. Thus, at least one neutrino must have a mass M obeying

$$M \gtrsim 1 \text{ eV} [10^{24}\text{yr}/\tau_{Ge}]^{1/2}. \quad (2.1.15)$$

Now, the present lower bound on τ_{Ge} is approximately 10^{24}yr [see Section 2.2 of

this volume and CAL 87]. Hence, the observation of neutrinoless double beta decay of ^{76}Ge with a lifetime not far beyond the present limit would imply that at least one neutrino has a mass exceeding ~ 1 eV. A mass of order 1 eV is large enough to be sought in neutrino oscillation experiments, and perhaps even in future tritium beta decay experiments.

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LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
1 CYCLOTRON ROAD
BERKELEY, CALIFORNIA 94720