Title
Shell model calculation for the Ga71(ν,e-)71Ge solar neutrino detector

Permalink
https://escholarship.org/uc/item/3vx4698q

Journal
Physical Review C, 32(3)

ISSN
2469-9985

Authors
Mathews, GJ
Bloom, SD
Fuller, GM
et al.

Publication Date
1985-09-01

DOI
10.1103/physrevc.32.796

Peer reviewed

PHYSICAL REVIEW C VOLUME 32, NUMBER 3 SEPTEMBER 1985

Shell model calculation for the ^{71}Ga(ν,e$^-$)^{71}Ge solar neutrino detector

G. J. Mathews and S. D. Bloom
Lawrence Livermore National Laboratory, University of California, Livermore, California 94550

G. M. Fuller
Lick Observatory, Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz, Santa Cruz, California 95064

J. N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 14 March 1985)

Gamow-Teller allowed neutrino capture transitions between the ground state of ^{71}Ga and accessible excited states in ^{71}Ge are calculated in a model space of the $2p$ and $1f$ shells. The total (p,n) Gamow-Teller strength function is also calculated. The effects of model-space truncation are systematically investigated and detailed comparisons are made between calculated and experimental spectroscopic data for states in ^{71}Ga and ^{71}Ge. The calculated effect of excited-state transitions is in good agreement with a previous phenomenological estimate (based upon beta decay measurements in the vicinity of mass 71) except for the higher energy ^{8}B neutrinos. For these rare high energy neutrinos, there is not yet sufficient experimental information to make a complete phenomenological estimate. The total cross section for ^{8}B neutrinos is calculated to be about ten times larger than the ground-state—to—ground-state capture rate. The overall uncertainty in the capture rate for a ^{71}Ga detector that is caused by excited state transitions is estimated to be about 5 percent for a standard solar model and about 1 percent for nonstandard models that are consistent with the ^{37}Cl solar neutrino experiment.

I. INTRODUCTION

The ^{71}Ga(ν,e$^-$)^{71}Ge reaction has been much discussed as an experimental way to resolve the solar neutrino problem. The main nuclear physics reason that ^{71}Ga is a good solar-neutrino detector is that the capture reaction has a low Q value for the allowed ground-state to ground-state transition (about 233 keV). This low Q value implies[1] that a ^{71}Ga detector should be sensitive mostly to neutrinos from the primary $p+p+{}^2H+e^+ +\nu$ reaction (see Fig. 1). The existing ^{37}Cl detector,[2] on the other hand, involves a larger Q value (814 keV) and is not sensitive to p-p neutrinos. The large capture probability to the analog state state in ^{37}Ar implies[1] that the chlorine experiment is sensitive mainly to neutrinos from the decay of ^{8}B. Calculations of the production of ^{8}B are dependent upon details of the solar models and are subject to larger uncertainties than the primary p-p reaction, both in the input nuclear physics and in the calculated stellar physics.

Recently, there has been considerable discussion[1,3–6] concerning the contributions from neutrino captures which lead to excited states in ^{71}Ge (see Fig. 2). In order to use ^{71}Ga as a solar neutrino detector, one must know the cross sections for neutrino captures to excited states in ^{71}Ge or show that they are unimportant. The larger Q values for excited state transitions imply that these states are predominantly populated by neutrinos from the decay of ^{7}Be and ^{8}B (see Fig. 2). If the cross sections for capture

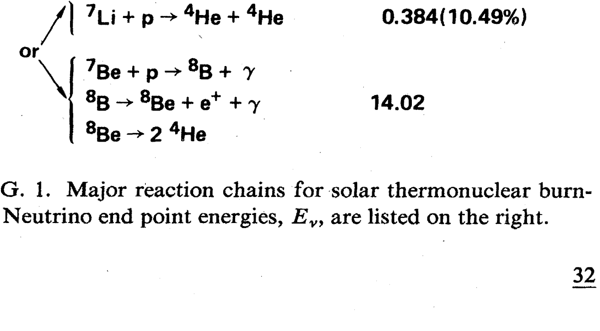

FIG. 1. Major reaction chains for solar thermonuclear burning. Neutrino end point energies, E_ν, are listed on the right.

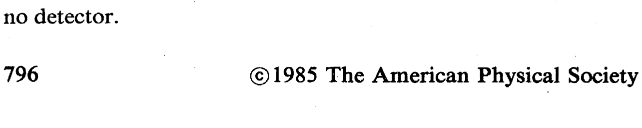

FIG. 2. Transitions of interest for the ^{71}Ga(ν,e$^-$)^{71}Ge neutrino detector.

to excited states were extremely large, the count rate in the ^{71}Ga detector could, in principle, include a large (and uncertain) contribution from ^{7}Be and ^{8}B neutrinos.

Different estimates for the cross sections for capture to excited states in ^{71}Ge have been obtained using either the systematics of similar transitions in the same mass range[1] or the initial interpretation of the recent (p,n) data obtained at 35 MeV.[3] The systematics imply that the total contribution of excited states to the capture rate is rather small (of order 10 percent). The 35 MeV (p,n) measurement, on the other hand, was interpreted to mean that there is substantial Gamow-Teller (GT) strength to the excited states in ^{71}Ge (particularly the first excited state at 175 keV). However, the initial calculation[3] of the effect of the cross sections inferred from the (p,n) data was probably a considerable overestimate.[4-6] The value inferred from the measurement at 35 MeV may be too large due to the contributions[4] from $l > 0$ transitions at this energy.

In order to clarify what the count rate in a ^{71}Ga detector will be, it is important to understand as well as possible, both theoretically and experimentally, the reasons for the differences between estimates based on beta-decay systematics and estimates based on the low-energy (p,n) reactions. In particular, we want to investigate if, within the context of calculable nuclear models, the transition to the first excited state in ^{71}Ge can be as large as or greater than the ground-state to ground-state transition (as the 35 MeV p-n reactions were originally interpreted to imply). We also want to determine whether or not the nuclear models predict GT strengths that are consistent with estimates based upon the systematics of beta-decay rates in the vicinity of mass 71.

With regard to the first question, our calculated GT strength to excited states in ^{71}Ge is lower than the value inferred by Orihara *et al.*[3] from the low-energy (p,n) data. Thus, our results support the analysis of Baltz *et al.*[4] which indicates that the inferred $l = 0$, zero-degree GT strength is overestimated in the 35 MeV experiment because of contributions from higher l waves. Our results for the total transition strength to the low lying levels in ^{71}Ge, which are most important for solar neutrino experiments,[1] are generally in good agreement with the previous phenomenological estimates. For the dominant sources, p-p and ^{7}Be neutrinos, the total cross sections are practically identical when they are calculated with the nuclear model and with the phenomenological estimate. We do, however, calculate a somewhat larger capture rate to the lowest $\frac{5}{2}^-$ state of ^{71}Ge than was inferred from the beta-decay systematics and a somewhat smaller rate to the lowest $\frac{3}{2}^-$ excited state. In addition, our nuclear model calculations suggest that there may be a large increase in the capture cross section for ^{8}B neutrinos that is caused by GT strengths to excited states with energies between 2–6 MeV above the ground state of ^{71}Ge. A similar result was obtained by Grotz *et al.*[6] using a more schematic nuclear model.

II. CALCULATION

A schematic illustration of the model space available to $A = 71$ nuclei is shown in Fig. 3(a) which displays the simplest configurations for the four states of interest, i.e., the ground state of ^{71}Ga and the three lowest states in ^{71}Ge. The detailed configurations for these states are probably complicated, although there is evidence from ^{72}Ge(p,d)^{71}Ge pickup-reaction data[7] that the one-hole (1h) configurations listed in Fig. 3(a) are significant. There is also some evidence from the pickup reaction data[8] that

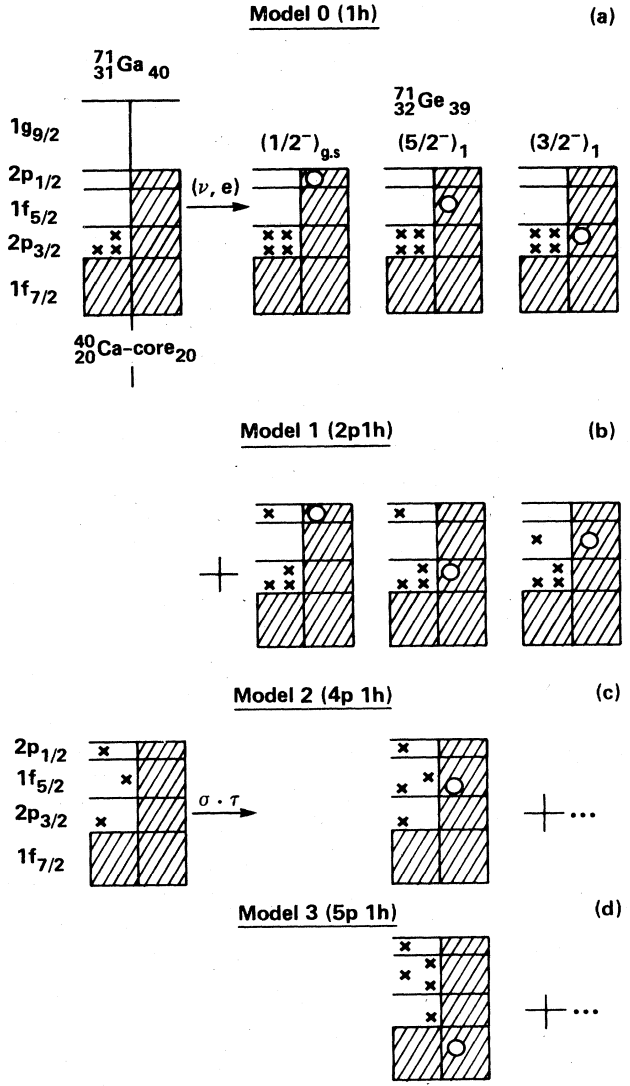

FIG. 3. Schematic illustrations of the model spaces described in the text. These are (a) zero-order shell model one-hole (1h) configurations for the ^{71}Ga ground state and the three lowest states in ^{71}Ge. Outside of the closed ^{40}Ca core, ^{71}Ga has a closed neutron fp shell, a closed proton $1f_{7/2}$ shell, and one proton hole in $2p_{3/2}$. The ^{71}Ge ground state has a filled proton $2p_{3/2}$ shell and one neutron hole in $2p_{1/2}$; (b) two-particle one-hole (2p1h) states generated by the operation of the Gamow-Teller operator on the 1h ground state for ^{71}Ga; (c) the space of four-particle one-hole (4p1h) configurations in ^{71}Ge resultant from including all three-particle proton correlations in the p-$f_{5/2}$ shells for ^{71}Ga; (d) the five-particle one-hole (5p1h) space generated when the $\nu f_{7/2} \rightarrow \pi f_{5/2}$ spin-flip transition in is allowed to proceed from the correlated ^{71}Ga ground state.

the $1g_{9/2}$ neutron orbital is about 10% occupied in the ground state of ^{71}Ga, and there is a low-lying $\frac{9}{2}^+$ excitation in ^{71}Ge at only 150 keV. Thus, this state should also be included in the model space. Unfortunately, however, the inclusion of such configurations into the problem renders the size of the calculation too large ($\sim$250 000 uncoupled Slater determinants) to obtain accurate results with our present technology. We have attempted several calculations in this space using various approximations to truncate the basis but without success since such truncations are at the expense of obtaining states with good angular momenta. Without this information, however, it is impossible to identify the transitions of interest. On the other hand, we expect that the dominant configurations due to including the $1g_{9/2}$ orbital are seniority-zero neutron excitations which probably mix similarly with the low-lying states of interest. These correlations, therefore, may not affect the *relative* Gamow-Teller strength for the neutrino capture transitions to low-lying states other than via an overall normalization constant. Therefore, for the purposes of simplicity we ignore the $1g$ shell in the calculations. Within the context of the $1f$, $2p$ shells we investigate the importance of model-space truncation in three illustrative calculations described below. As far as the relative Gamow-Teller strengths are concerned, we assume that this mode space is adequate.

A. Hamiltonian

Our aim has been to reproduce the properties of states in ^{71}Ga and ^{71}Ge with a minimum number of free parameters. Therefore we have chosen to use a realistic, finite-range *ab initio* two-body effective interaction derived from free nucleon-nucleon scattering. The simplest such interaction which is adequate for our purposes is the Kallio-Kolltveit force[9] which approximates a G-matrix effective interaction by applying the Scott-Moszkowski cutoff procedure[10] to the singlet-even and triplet-even components of an exponential-plus-hard-core nucleon-nucleon force. The parameters of this force are summarized in Table 1. Having selected the two-body interaction we then adjust the one-body Hamiltonian (single-particle energies) to reproduce the three lowest states in ^{71}Ga. These states are known from ^{70}Zn(^{3}He,d)^{71}Ga stripping reaction data[11] to have significant single-particle character. A summary of the single-particle energies for the different model spaces is given in Table I.

B. Model 0 (1h)

Figure 3(a) represents what might be considered a zeroth-order shell-model description for the transition to states in ^{71}Ge. From these simple configurations one can already see an argument as to why the transition to the first excited state, $(\frac{5}{2}^-)_1$, should have a lower strength than the g.s.$\rightarrow$g.s. transition. Whereas the spin-flip $\nu p_{1/2} \rightarrow \pi p_{3/2}$, g.s.$\rightarrow$g.s. transition is Gamow-Teller allowed, the $\nu f_{5/2} \rightarrow \pi p_{3/2}$ transition to the first excited state requires a change in angular momentum and thus could not be mediated by the allowed GT operator.

If these configurations correctly represented the problem, the ratios of GT strength to the g.s./$(\frac{5}{2}^-)_1$/$(\frac{3}{2}^-)_2$ states in ^{71}Ge would be $(\frac{4}{3})/0/(\frac{5}{3})$. As we shall see, enlarging the shell-model basis will change these ratios, but not the basic argument that the GT strength to the $(\frac{5}{2}^-)_1$ state in ^{71}Ge should be less than the g.s.$\rightarrow$g.s. transition.

C. Model I (2p1h)

For the first increase in complexity we add to the basis for ^{71}Ge the configurations listed in Fig. 3(b), i.e., the states which are directly connected to the zeroth-order ^{71}Ga ground state by the Gamow-Teller operator. For this model we omit the configurations due to the $\nu f_{7/2} \rightarrow \pi f_{5/2}$ spin flip. It is of course necessary to eventually add this transition to produce the correct sum rule and we do this below.

The model space to diagonalize in this approximation consists of 95 (uncoupled) Slater determinants. This calculation produces the spectrum labeled 2p1h in Fig. 4. The GT strengths are given in Table II. These configurations include a number of two-particle one-hole (2p1h) states which can mix with the three configurations of Fig. 3(a). The net effect is to substantially increase the GT strength to the $(\frac{5}{2}^-)_1$ state. In this regard, the

TABLE I. Summary of Hamiltonian parameters.

Two-body interaction:

$$V_{12} = \begin{cases} -A_k \exp[-\alpha_k(r-c_k)] & r > d_k \\ 0 & r < d_k \end{cases}$$

	A_k (MeV)	α_k (fm^{-1})	c_k (fm)	d_k (fm)
Triplet even	475.0	2.5214	0.4	0.925
Singlet even	330.8	2.4021	0.4	1.025

Single-particle energies (MeV)

Model	2p1h	4p1h	5p1h
Orbit			
$2p_{1/2}$	1.120	0.823	0.946
$1f_{5/2}$	-1.012	-0.985	2.712
$2p_{3/2}$	0.000	0.000	0.000
$1f_{7/2}$			-2.835

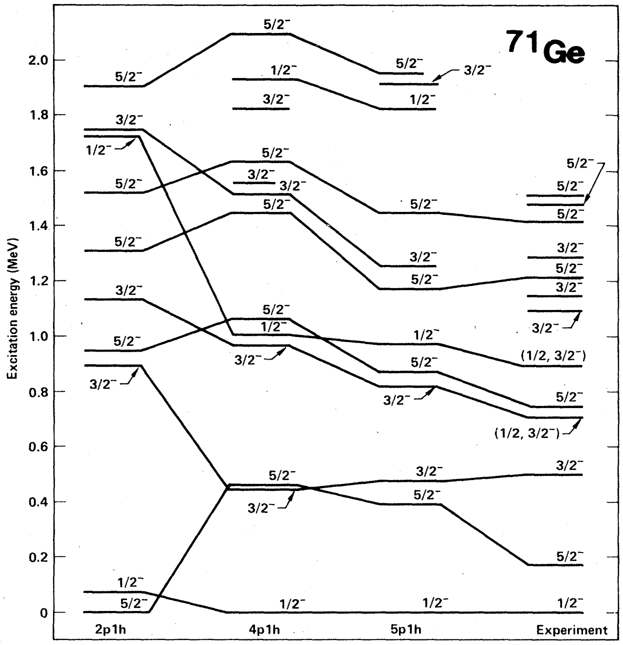

FIG. 4. Spectrum of excited states in ^{71}Ge calculated in each of the model spaces.

$$[\pi(p_{3/2})^3(p_{1/2})^1\nu(p_{1/2})^{-1}]_{5/2}$$

is particularly important due to the large attractive $[(p_{1/2})^2]_{T=0,J=1}$ two-body matrix element which causes this configuration to be strongly mixed with the zeroth-order configuration for the $(\frac{5}{2})_1$ state. This conclusion does not change as the model space is enlarged. This is one of the reasons why the neutrino-capture transitions to the $(\frac{5}{2})_1$ state are probably not as small as the value inferred from the systematics of lighter nuclei which an empty $2p_{1/2}$ neutron orbital would imply. We discuss this further in Sec. V.

We also note that these configurations lead to a depletion of GT strength to both the ground state and the $(\frac{3}{2}^-)_2$ states in ^{71}Ge as the bulk of the sum rule is mixed upward into the giant GT resonance. As we shall see, this trend continues as the size of the model space is increased.

D. Model II (4p1h)

At the next level of complexity, we consider the role of proton correlations in the parent, ^{71}Ga, ground state. We include all three-particle configurations in the p and $f_{5/2}$ shells (37 Slater determinants). These are connected via the GT operator to a basis of 794 Slater determinants represented schematically as four-particle one-hole (4p1h) configurations in Fig. 3(c). With this enlarged basis, there there is a dramatic improvement in the predicted spectrum of ^{71}Ge as shown in the second column of Fig. 4. In particular, the eigenvalues for the first two excited states in ^{71}Ge are now at about the right place. This improvement indicates the importance of proton correlations in the spectrum of ^{71}Ge. On the other hand, the effect on the relative GT strengths (as shown in the fourth column of Table II) to these states is much smaller. There is a continuation of the trend toward depletion of strength due to coupling with the giant resonance. The strength to the $(\frac{5}{2}^-)_1$ state actually increases slightly, however, due to additional configurations which can couple to this state and carry GT strength.

E. Model III (5p1h)

Finally, we add the effects of the occupied $1f_{7/2}$ shell. These configurations lead to a maximum of five-particle one-hole (5p1h) excitations in ^{71}Ge. The uncoupled basis becomes 7 969 Slater determinants. The eigenvalues labeled 5p1h in Fig. 4 are in good agreement with the observed values for both ^{71}Ge and ^{71}Ga as shown in Fig. 5. The GT strengths in Table II are again smaller than the strengths calculated in the other model spaces due to the build up of the giant resonance. Although the absolute strengths have varied considerably as the model space was increased, the ratio of GT strengths to the $(\frac{1}{2}^-)_{\text{g.s.}}$ and $(\frac{5}{2}^-)_1$ states appears to be about 2—4. It is plausible that the relative strengths calculated for these transitions would not change much if the basis were further enlarged. The strength to the $(\frac{3}{2}^-)_1$ state continues to decrease, however, relative to the g.s.

The amount of depletion of strength to the GT resonance is a curious feature of the calculations. The strength of the g.s.→g.s. transition is already less than the experimentally observed β-decay strength even before the expected effects of GT quenching[12,13] have been considered which could further reduce the strength by about a factor of 2. Our suspicion is that this low strength may be due to the absence of the $1g_{9/2}$ configurations in the model space. In a previous work,[14] we have shown that the effects of such correlations do not influence the gross distribution of GT strength in the resonance, but only introduce an increased spreading width due to the coupling

TABLE II. Gamow-Teller strengths to final states in ^{71}Ge calculated in various model spaces compared with values derived from (p,n) data (Ref. 3) obtained at 35 MeV and the calculation of Ref. 4.

Final state	1h	2p1h	4p1h	5p1h	Orihara *et al.* (Ref. 3)	Baltz *et al.* (Ref. 4)
$(\frac{1}{2})_{\text{g.s.}}$	1.33	0.48	0.10	0.052	0.0832	0.238
$(\frac{5}{2})_1$	0.00	0.028	0.030	0.012	0.080	0.011
$(\frac{3}{2})_1$	1.67	7.5×10^{-3}	6.4×10^{-4}	7.5×10^{-5}	0.019	0.058
$(\frac{3}{2})_2$		1.9×10^{-4}	3.9×10^{-4}	4.0×10^{-4}		

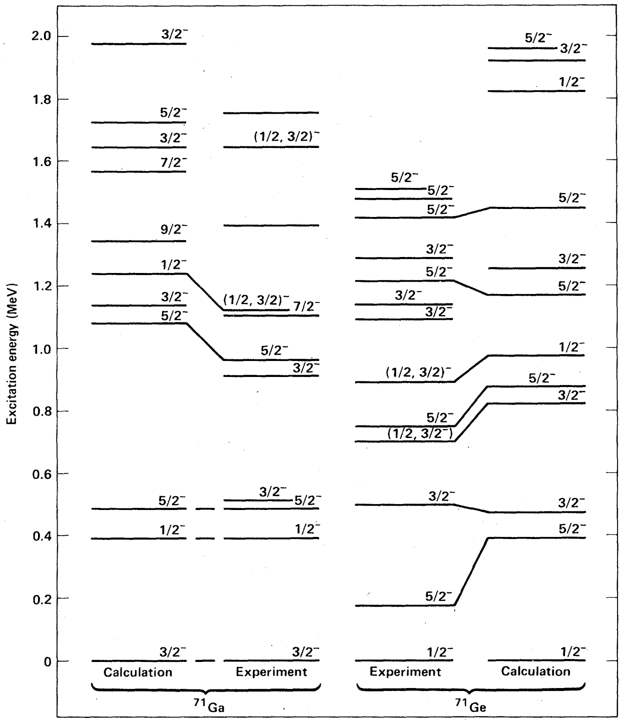

FIG. 5. Comparison of calculated and observed eigenvalues for states in ^{71}Ga and ^{71}Ge from the 5p1h model space.

of the resonance with the background of multiparticle configurations. This increased width could increase the GT strength to weakly populated states in the extrema of the strength function. We speculate here that this increased width causes an overall increase in the strength to the three lowest-lying states in ^{71}Ge which then is roughly cancelled by the effects of GT quenching. We assume that the mixing with these three states will be about the same and therefore renormalize the GT strength to these states by a constant factor determined from the known beta-decay strength for the g.s.→g.s. transition. As mentioned in the beginning of this section, a confirmation of this role of the $g_{9/2}$ correlations would require a much larger and more difficult calculation which is not possible at this time.

In Fig. 6 we show our best estimate of the total computed GT strength function for ^{71}Ga→^{71}Ge. This strength function is represented analytically as a sum of Gaussians the parameters for which are shown in Table III. The three low-lying states are plotted with the experimental energies and with strengths normalized to give the correct ground-state strength. The widths of these states are plotted with a Gaussian resolution of 0.125 MeV (FWHM) for the low-lying states which corresponds to good resolution for (p,n) reaction data.[12,13] The higher lying states are plotted with a larger width due to the dispersion introduced by the Lanczos[14] algorithm used to diagonalize this space in 30 iterations. This continuum distribution is normalized to a total strength of 13.5 corresponding to a GT quenching factor of 0.5 which is typical[12,13] for heavy nuclei. This figure exhibits at least part of the potential problem with determining the GT strength for the low-lying states from (p,n) reaction data. The structure due to different states is difficult to see at this resolution. It would be useful to compare this calculation with high-energy measurements of the total (p,n) strength function to test the reliability of the calculation, and also to determine the value of the quenching factor for this nucleus. Such measurements have recently been performed[15] and the analysis of the data will soon be complete.

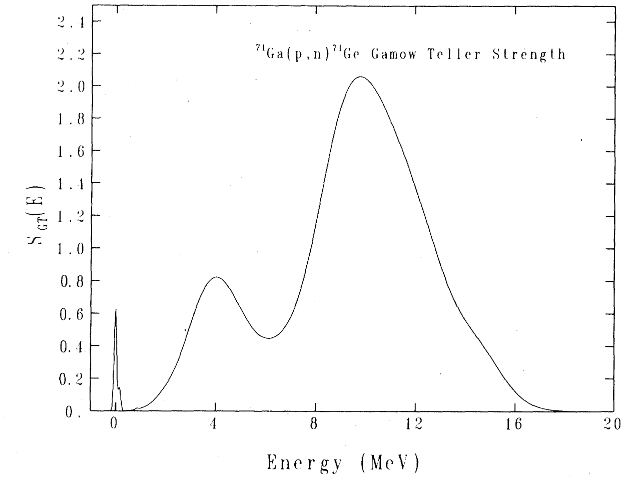

FIG. 6. Calculated total GT strength function for ^{71}Ga→^{71}Ge from the 5p1h model space. The three low-lying transitions have been normalized to give the correct experimental ground-state strength. The overall distribution is normalized to a quenched strength of 0.5 times the GT sum rule.

TABLE III. Summary of Gaussian parameters for the ^{71}Ga→^{71}Ge Gamow-Teller strength function.

$$S_{\rm GT}(E)=\frac{1}{\sqrt{2\pi}\sigma}\sum_i B_{\rm GT}(i)\exp[-(E-E_i)^2/2\sigma_i^2] \ .$$

E_i	σ_i	$B_{\rm GT}(i)$	E_i	σ_i	$B_{\rm GT}(i)$
0.0	0.053	0.0832	8.928	1.020	2.241
0.175	0.053	0.0190	9.689	1.127	1.881
0.399	0.053	0.0001	10.61	1.044	1.805
0.871	0.053	0.0010	11.43	1.120	1.334
1.156	0.261	0.0040	12.29	1.020	1.130
1.396	0.450	0.0050	13.07	1.089	0.545
1.907	0.370	0.0315	14.01	1.266	0.324
2.413	0.529	0.0131	14.57	0.901	0.443
3.092	0.760	0.355	15.28	1.019	0.0782
3.685	0.816	0.782	15.95	0.919	0.0197
4.483	0.833	0.748	16.62	0.921	0.0027
5.290	1.043	0.252	17.34	0.692	0.0006
6.144	1.135	0.421	17.90	0.492	0.0003
7.047	1.304	0.364	18.30	0.259	0.0002
8.055	1.261	0.637			

III. SPECTROSCOPIC FACTORS AND TRANSITION RATES

Before considering the effect of these calculations on the neutrino capture rates it is useful to compare first the predicted properties of these wave functions with experimental measurements of properties of these states in ^{71}Ga and ^{71}Ge. In particular, spectroscopic factors have been measured[8,11] for ^{70}Zn(^{3}He,d)^{71}Ga and ^{72}Ge(p,d)^{71}Ge. These two reactions are especially useful. The (^{3}He,d) stripping reaction on ^{70}Zn samples the single-particle properties of the protons in the ground state of ^{71}Ga (and excited states) which will undergo neutrino-capture reactions. The ^{72}Ge(p,d)^{71}Ge pickup reaction will similarly sample the hole states which will also be produced by the GT transitions to states in ^{71}Ge.

We have calculated spectroscopic factors for these reactions by diagonalizing the ground state of the even-even parent nuclei (^{70}Zn and ^{72}Ge) in the same model space as the 5p1h calculation of Sec. II D. The spectroscopic strength for the pickup reaction is then simply given as,

$$C^2S_j=\sum_{mj} |\langle 71\,|\,a_{mj}\,|\,72\rangle|^2(2j+1)\;, \tag{1}$$

where $|72\rangle$ is the state vector for ^{72}Ge with angular momentum, j, and a_{mj} is the neutron annihilation operator for magnetic substate mj. A similar expression can be written for the (^{3}He,d) stripping reaction.

Table IV summarizes the calculated and observed spectroscopic data for the ^{70}Zn(^{3}He,d)^{71}Ga reaction. Table V is for the pickup reaction on ^{72}Ge. In both cases the agreement with the observations is reasonable for most states. There is a tendency to overestimate the strength for the low states which is a consequence of the limited model space employed. Even so it is clear that the state vectors calculated in the present work give a fair representation of the single-particle structure of the low-lying states in ^{71}Ga and ^{71}Ge.

There is one other probe of the wave functions of interest which is the $E2$ lifetime for the $(\frac{5}{2}^-)_1\rightarrow(\frac{1}{2}^-)_{\text{g.s.}}$ transition in ^{71}Ge. This level has a measured[16] half-life of 79±2 ns which corresponds to a reduced ($E2$) matrix element of

$$|\langle(\tfrac{1}{2})||E2||(\tfrac{5}{2})_1\rangle|=16\;e\;\text{fm}^2\;.$$

This corresponds to a collective enhancement over the Weisskopf single-particle rate by a factor of 2.5. We calculate an $E2$ lifetime of 1600 ns,

$$|\langle(\tfrac{1}{2})||E2||(\tfrac{5}{2})_1\rangle|=3.6\;e\;\text{fm}^2\;.$$

This calculated slower transition rate suggests that we have underestimated the collective contribution. This is not surprising given that we have chosen a limited model space which introduces little collective strength. However, since the collective component of the state vectors probably do not participate much in the GT transition rate, this omission should not introduce a significant uncertainty in our calculated one-body GT strengths.

TABLE IV. Comparison of experimental and calculated spectroscopic factors for the ^{70}Zn(^{3}He,d)^{71}Ga reaction.

$(j)^\pi$	E_{exp} (MeV)	E_{calc} (MeV)	C^2S_{exp}	C^2S_{calc}
$(\frac{3}{2})_{\text{g.s.}}$	0.0	0.0	1.88	1.67
$(\frac{1}{2})_1$	0.390	0.393	1.61	0.68
$(\frac{5}{2})_1$	0.487	0.489	3.18	2.11
$(\frac{3}{2})_2$	0.512	1.148	0.40	0.11
$(\frac{5}{2})_2$	0.965	1.076		2.63
$(\frac{1}{2})_2$	1.109	1.242	1.41	0.67

TABLE V. Comparison of experimental (Ref. 11) and calculated spectroscopic factors for the ^{72}Ge(p,d)^{71}Ge reaction.

$(j)^\pi$	E_{exp} (MeV)	E_{calc} (MeV)	C^2S_{exp}	C^2S_{calc}
$(\frac{1}{2}^-)_{\text{g.s}}$	0.0	0.0	1.04	1.11
$(\frac{5}{2}^-)_1$	0.175	0.391	3.64	2.17
$(\frac{3}{2}^-)_1$	0.500	0.474	2.32	1.59
$(\frac{3}{2}^-)_2$	0.708	0.820	0.17	0.06
$(\frac{5}{2}^-)_2$	0.747	0.871	0.31	0.78
$(\frac{3}{2}^-)_3$	1.096	1.259	0.37	0.02
$(\frac{3}{2}^-)_4$	1.166	1.919	0.05	0.005
$(\frac{5}{2}^-)_3$	1.210	1.168	0.40	0.35
$(\frac{3}{2}^-)_5$	1.288	2.405	0.18	0.04
$(\frac{5}{2}^-)_4$	1.410	1.444	0.26	0.30
$(\frac{5}{2}^-)_5$	1.506	1.925	0.53	0.10
$(\frac{3}{2}^-)_6$	1.599	2.991	0.09	0.10
$(\frac{1}{2}^-)_2$	1.743	0.972	0.09	0.16
$(\frac{5}{2}^-)_6$	1.792	2.507	0.42	0.01
$(\frac{1}{2}^-)_3$	1.965	1.819	0.18	0.01
$(\frac{3}{2}^-)_7$	2.354	3.774	0.18	0.17

IV. NEUTRINO CAPTURE RATES

Table VI summarizes the corrections to the solar neutrino absorption cross sections due to excited state transitions. We have also computed, and show in Table VI, the corrections for the possible calibrating sources, ^{51}Cr and ^{65}Zn. The calculations of the cross sections have been carried out including all of the usual atomic and nuclear physics corrections.[1] The second column of Table VI shows the results obtained with the aid of the nuclear model discussed in the present paper and the third column shows the cross sections that were inferred earlier with a purely phenomenological analysis.[1] The two sets of numbers are in generally good agreement. The most important cross section, which refers to the p-p capture rate, has no significant excited state contribution because the maximum energy of the p-p neutrinos is small. The next most important neutrino source is ^{7}Be. The ratio of total to ground-state cross section for ^{7}Be neutrinos is about 1.16 according to the present estimate and about 1.18 according to the phenomenological estimate. The only large difference between the present work and the phenomenological estimate is for the higher energy neutrinos from ^{8}B

TABLE VI. Calculated correction factors for solar neutrino capture rates.

Neutrino source	Maximum neutrino energy (MeV)	$(\sigma_{\text{total}}/\sigma_{\text{g.s.}})$ Present work	$(\sigma_{\text{total}}/\sigma_{\text{g.s.}})^{a}$ Bahcall (1978)
p-p	0.420	1.00	1.00
pep	1.442	1.20	1.45
^{7}Be	0.862	1.16	1.18
^{7}Be	0.384	1.00	1.00
^{8}B	14.02	10.4[b]	1.86
^{13}N	1.198	1.15	1.18
^{15}O	1.737	1.18	1.34
^{51}Cr	0.746	1.15	1.15
^{51}Cr	0.426	1.12	1.04
^{65}Zn	1.342	1.19	1.42
^{65}Zn	0.330	1.00	1.00

[a] Includes only states at excitation energies of 0.0, 0.17, 0.5, and 0.7 MeV.

[b] Includes only the contributions from particle stable states in ^{71}Ge up to the neutron separation energy. The excited states up to and including 0.75 MeV contribute $(\sigma_{\text{total}}/\sigma_{\text{g.s.}})=1.25$.

decay. In this case, the low-lying excited states give a correction factor of 1.25 in the present calculation and a correction of 1.86 in the phenomenological estimate. However, the nuclear model we have used predicts large Gamow-Teller strengths in the region 2 to 6 MeV, yielding a correction factor from the total cross section to particle stable states in ^{71}Ge of 10.4. Because of the lack of experimental information, the phenomenological estimate[1] did not include contributions from transitions to states above 0.7 MeV.

The cross section for a ^{51}Cr source is predicted to be 6.35×10^{-45} cm^2 using either the nuclear matrix elements calculated in this paper or the phenomenological estimates; this cross section is about 15 percent larger than would be obtained if all excited state transitions were ignored. The cross section for a ^{65}Zn source is calculated to be about 8.4×10^{-45} cm^2 using the nuclear model considered here and is estimated to be about 10.0×10^{-45} cm^2 with the phenomenological estimates. About 30±10 percent of the ^{65}Zn rate is expected to come from excited state transitions.

How uncertain do excited state transitions make the total capture rate for solar neutrinos? We answer this question in Table VII for the standard solar model[7] and for two representative nonstandard solar models[1] that are consistent with the ^{37}Cl solar neutrino experiment. For the standard solar model, the total capture rate is expected to be 124 solar neutrino units (SNU) if we use the excited state transitions computed in the present work and 115 SNU if we use the phenomenological estimates. The capture rate that is computed using the standard solar model and ignoring all transitions to excited states of ^{71}Ge is[7] 107 SNU. We conclude that present uncertainties in the values of the excited-state transitions cause about a ±4 percent uncertainty in the total capture rate for the standard solar model, much smaller than the total estimated uncertainty (which is about ±10 percent[7]). For nonstandard models, like the low heavy element (Z) or the fully mixed model, the expected uncertainty is even less, corresponding to only of order 1 SNU (see the last four columns of Table VIII) or about ±1 percent.

In Table VIII we summarize the results of our calculations for the GT log(ft) to the four lowest states in ^{71}Ge. These are compared with the measurements of Orihara *et al.*[3] We find an increase in the strength to the $(\frac{5}{2}^-)_1$ state relative to the systematic estimates, but a decrease in strength to the $(\frac{3}{2}^-)_1$ and $(\frac{3}{2}^-)_2$ states. The reason for this decrease can largely be traced to the fact that $\frac{3}{2}^-$ states share only a small fraction ($\sim7\%$) of the total sum rule, and of this fraction, most of the strength is mixed into the GT resonance. The calculations of Baltz *et al.*, however, do not exhibit this phenomena, suggesting that this effect is sensitive to the two-body interaction employed. As noted in Sec. II A, we utilize a realistic effective interaction derived from free nucleon scattering which approximates the true G matrix. The calculations of Baltz *et al.* employ a modified surface-delta interaction with different parameter sets for the $f_{7/2}$ orbital and the rest of the fp shell. It will be interesting to see whether high-energy (p,n) measurements can make a distinction

TABLE VII. Calculated solar neutrino capture rates (in SNU) from the present work compared with previous estimates (Ref. 1). This table indicates the uncertainties in the capture rate due to excited states.

Neutrino source	Standard solar model excited state cross sections: This work	Bahcall (1978)	Low Z solar model excited state cross sections: This work	Bahcall (1978)	Mixed solar model excited state cross sections: This work	Bahcall (1978)
p-p	70.2	70.2	72.5	72.5	74.2	74.2
pep	3.0	3.6	3.2	3.9	3.2	3.9
^{7}Be	31.2	31.7	10.9	11.1	11.2	11.4
^{8}B	11.6	1.2	1.6	0.3	2.4	0.4
^{13}N	3.3	3.1	0.1	0.1	0.6	0.6
^{15}O	4.6	4.7	0.0	0.1	1.0	1.0
Total	124	115	88	88	93	92

TABLE VIII. Comparison of estimated log(ft) values for ^{71}Ga$\rightarrow$^{71}Ge neutrino-capture transitions.

Final state	Present	Orihara *et al.* (Ref. 3)	Baltz *et al.* (Ref. 4)
$(\frac{1}{2})_{\text{g.s.}}$	4.676	4.676	4.22
$(\frac{5}{2})_1$	5.3	4.69	5.6
$(\frac{3}{2})_1$	7.5	5.31	4.8
$(\frac{3}{2})_2$	6.8		

as to which of these effective interactions leads to a better prediction of the GT strength to the low-lying excited states in ^{71}Ge.

V. SYSTEMATICS

It is an interesting outcome of the present calculations and the (p,n) measurements that the $(\frac{3}{2}^-)_{\text{g.s.}}\rightarrow(\frac{5}{2}^-)_1$ transition from ^{71}Ga is faster by a factor of 2 or 3 compared to analogous transitions in this region. It is not certain whether this is a result of approximations made in our nuclear model calculations or whether it indicates something is special for these transitions in ^{71}Ga-^{71}Ge. In this section, we discuss some plausible reasons as to why the ^{71}Ga-^{71}Ge$(\frac{3}{2}^-)_{\text{g.s}}\rightarrow(\frac{5}{2}^-)_1$ transition may be different from related transitions in neighboring nuclei.

The transitions utilized[1] to generate an estimate of the ^{71}Ga(ν,e$^-$)^{71}Ge, $(\frac{3}{2}^-)_{\text{g.s.}}\rightarrow(\frac{5}{2}^-)_1$ capture rate from systematics can be divided into two groups as follows:

(1) Electron-capture (or β^+) transitions. These include $^{69}_{32}$Ge$_{37}$$\rightarrow$$^{69}_{31}Ga_{38}$ [log(ft)=6.3], $^{67}_{31}$Ga$_{36}$$\rightarrow$$^{67}_{30}Zn_{37}$ [log(ft) $>$6.4], $^{65}_{30}$Zn$_{35}$$\rightarrow$$^{65}_{29}Cu_{36}$ [log(ft)=7.5], $^{65}_{31}$Ga$_{34}$$\rightarrow$$^{65}_{30}Zn_{35}$ [log(ft)$>$5.9], and $^{65}_{32}$Ge$_{33}$$\rightarrow$$^{65}_{31}Ga_{34}$ [log(ft)$>$6.0].

(2) Beta transitions in lighter nuclei. These include $^{67}_{29}$Cu$_{38}$$\rightarrow$$^{67}_{30}$ Zn$_{37}$ [log(ft)=6.3] and $^{65}_{28}$Ni$_{37}$$\rightarrow$$^{65}_{29}Cu_{36}$ [log(ft) =6.6].

These two groups lead to an estimate which is somewhat biased toward a low value for the ^{71}Ga(ν,e$^-$)^{71}Ge, $(\frac{3}{2})_{\text{g.s.}}\rightarrow(\frac{5}{2}^-)_1$ transition.

The first set of transitions are probably hindered by the effects of Pauli blocking in neutron-rich nuclei which tends to slow electron-capture transitions. This situation we exemplify in a simple calculation for ^{69}Ge$\rightarrow$^{69}Ga shown in Fig. 7(a). This calculation is equivalent to the 2p1h (model I) calculation for ^{71}Ga(ν,e$^-$)^{71}Ge. The $\pi p_{3/2}\rightarrow\nu p_{3/2}$ transition is Pauli blocked, and the $f_{5/2}$ orbital can not participate in the transition. Only the $\pi p_{3/2}\rightarrow\nu p_{1/2}$ spin-flip transition can contribute. We calculate an absolute GT transition rate of $B_{\text{GT}}=7.3\times10^{-3}$ [log(ft)=6.0] which is an order of magnitude slower than the same transition in the analogous 2p1h calculation for ^{71}Ga.

The beta decays for the analogous transition in lighter nuclei are probably slower for a different reason. This situation is sketched in Fig. 7(b) for the case of ^{67}Cu$\rightarrow$^{67}Zn. The neutron $2p_{1/2}$ orbital is empty in the simplest ground state for ^{67}Cu. Consequently, there are fewer configurations which can contribute to the $\frac{3}{2}^-\rightarrow\frac{5}{2}^-$ transition. Furthermore, the lowest $\frac{5}{2}^-$ state in ^{67}Zn should contain a substantial admixture of $(\pi 2p_{3/2})^2(\nu f_{5/2})^{-1}$ which can not be reached via a GT transition from the ^{67}Cu ground state.

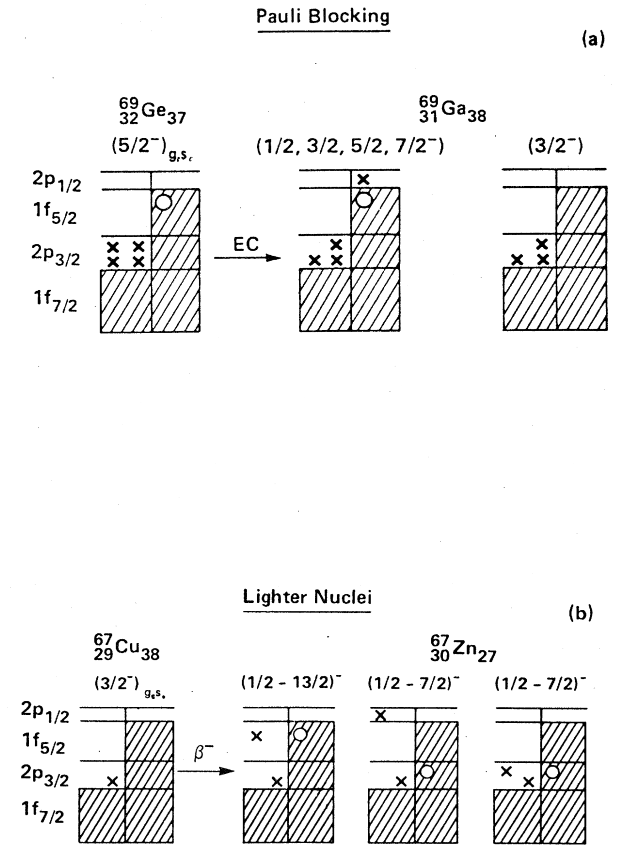

FIG. 7. Schematic illustrations of the calculations for transitions used to derive systematic estimates; (a) illustrates the Pauli blocking effect in neutron-rich nuclei; (b) illustrates the configurations which can be formed when the $2p_{1/2}$ neutron orbital is empty.

We have calculated this transition starting from the simple ^{67}Cu ground state shown in Fig. 7(b). We find that there are two calculated low-lying $\frac{5}{2}^-$ states in ^{67}Zn separated by only 0.125 MeV. Either one of these states could be taken as the $(\frac{5}{2}^-)_{\text{g.s.}}$ final state of interest. Assuming that the state with the largest $(\pi 2p_{3/2})^2(\nu f_{5/2})^{-1}$ amplitude is the best approximation to the actual ground state, we calculate a transition strength of $B_{\text{GT}}=1.3\times10^{-3}$ [log(ft)=6.5] which is 20 times slower than the analogous transition calculated for ^{71}Ga$\rightarrow$^{71}Ge.

VI. CONCLUSION

We have calculated the Gamow-Teller-allowed neutrino-capture transitions from the ground state of ^{71}Ga to the ground state and excited states in ^{71}Ge. A model space of the $2p$ and $1f$ shells has been diagonalized with the Kallio-Kolltveit effective interaction.[9] The effects of model-space truncation have been systematically investigated and it has been shown that a reasonable reproduction of properties of the low-lying levels in ^{71}Ga and ^{71}Ge is possible in the confines of the chosen model space. The possible contributions to the count rate in a ^{71}Ga solar neutrino detector from transitions to excited states in ^{71}Ge were investigated and it was found that the transition strength to the low-lying excited states is probably not as large as recent (p,n) measurements[3] at 35 MeV were interpreted to imply.

Our calculations suggest that there may be a large increase in the capture cross section for ^{8}B neutrinos that is caused by Gamow-Teller strength to excited states with energies 2–6 MeV above the ground state of ^{71}Ge. In particular, we find (see Table VII) that for the nuclear model we are using the total cross section for capture of ^{8}B neutrinos , including excited state transitions, is about ten times the ground state transition (see especially Table VII). This result is in qualitative agreement with a similar calculation made by Grotz *et al.*[6]

The total solar neutrino capture rate to excited states in ^{71}Ge is expected to be of order 10 percent of the rate to the ground state of ^{71}Ge. We estimate in Sec. IV (see especially Table VIII) that the overall uncertainty in the capture rate for a ^{71}Ga detector that is caused by excited state transitions is of order 5 percent for a standard solar model and is about 1 percent for nonstandard solar models that are consistent with the ^{37}Cl experiment. If neutrino oscillations occur, excited state transitions would typically cause the same uncertainty (5 percent) as in the case of a standard solar model (since the net decrease caused by oscillations is in many situations essentially independent of neutrino source).

ACKNOWLEDGMENTS

The authors acknowledge useful discussions and encouragement from Prof. William A. Fowler during the course of these calculations. This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract Number W-7405-ENG-48.

---

[1] J. N. Bahcall, Rev. Mod. Phys. **50**, 881 (1978).

[2] R. Davis, Brookhaven National Laboratory Report No. BNL 50879, 1978, Vol. 1, p. 1; J. N. Bahcall and R. Davis, Science **191**, 264 (1976).

[3] H. Orihara *et al.*, Phys. Rev. Lett. **51**, 1328 (1983).

[4] A. J. Baltz, J. Weneser, B. A. Brown, and J. Rapaport, Phys. Rev. Lett. **53**, 2078 (1984).

[5] J. N. Bahcall, Indiana University Cyclotron Facility Report (for experiment 181), 1984 (unpublished); J. Rappaport and E. Sugarbaker (private communication).

[6] K. Grotz, H. V. Klapdor, and J. Metzinger, *Capture Gamma-Ray Spectroscopy and Related Topics—1984 (Knoxville, Tennessee),* Proceedings of the Fifth International Symposium on Capture Gamma-Ray Spectroscopy and Related Topics, AIP Conf. Proc. No. 125, edited by S. Raman (AIP, New York, 1985), p. 793.

[7] J. N. Bahcall, W. F. Huebner, S. H. Lubow, P. D. Parker, and R. K. Ulrich, Rev. Mod. Phys. **54**, 767 (1982).

[8] R. Fournier, J. Kroon, T. H. Hsu, B. Hird, and G. C. Ball, Nucl. Phys. **A202**, 1 (1973).

[9] A. Kallio and K. Kolltveit, Nucl. Phys. **53**, 87 (1964).

[10] B. L. Scott and S. A. Moszkowski, Nucl. Phys. **29**, 655 (1962).

[11] A. Riccato and P. David. Nucl. Phys. **A228**, 461 (1974).

[12] G. Gaarde, J. Rapaport, T. N. Taddeucci, C. D. Goodman, C. C. Foster, D. E. Bainum, C. A. Goulding, M. B. Greenfield, D. J. Horen, and E. Sugarbaker, Nucl. Phys. **A369**, 258 (1981).

[13] C. D. Goodman, Nucl Phys. **A374**, 241 (1982).

[14] G. J. Mathews, S. D. Bloom, and R. F. Hausmann, Phys. Rev. C **28**, 1367 (1983).

[15] E. Sugarbaker (private communication).

[16] J. Morgenstern, J. W. Schmidt, G. Flügge, and H. Schmidt, Phys. Lett. **27B**, 370 (1968).