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SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS—SERIES III

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Structures and Materials Research
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Division of Structural Engineering
and Structural Mechanics

SHEAR STRENGTH
OF REINFORCED CONCRETE BEAMS
SERIES III

A Report of an Investigation

by

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A. C. Scordelis, Professor of Civil Engineering

to the

REINFORCED CONCRETE RESEARCH COUNCIL
BUREAU OF YARDS AND DOCKS, DEPARTMENT OF THE NAVY
OFFICE OF CHIEF OF ENGINEERS, DEPARTMENT OF THE ARMY
ENGINEERING DIVISION, DEPARTMENT OF THE AIR FORCE

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I. INTRODUCTION

1. General Remarks

The factors which influence the behavior and strength of concrete beams subjected to shear have been studied by numerous investigators. From 1897 to 1961 approximately 450 papers dealing with shear in reinforced and prestressed concrete beams were published*. From 1961 to 1965 more than 100 additional papers were published** on this subject. Yet the solution to the "riddle of shear failure" is not yet available, although considerable insight into the behavior of beams failing in shear has been gained through the numerous analytical and experimental studies.

In the absence of a general theoretical solution to the problem a number of empirical design criteria have been formulated, and these periodically undergo substantial revision. Some of the early developments were described by Hognestad,+ and the more recent developments were described in the 1962 report of the Joint ASCE-ACI Committee on Shear and Diagonal Tension.++

Examination of the U. S. design criteria (e.g. ACI Code for Building Design), as well as of the codes of other countries, reveals that numerous shortcomings still exist. For example, in the ACI Code, provisions for the effect of longitudinal compression on shearing strength of reinforced concrete elements are inconsistent with the prestressed concrete provisions. Other inadequacies in many of the design codes are found, such as:

(a) shearing strength of beams with partially cut-off longitudinal bars

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* "Shear and Diagonal Tension and Torsion in Structural Concrete," ACI Bibliography No. 4, Annotated, 1962.

** Bibliography, see Appendix of this report, pp. A-1 - A-10.

\+ E. Hognestad "What Do We Know About Diagonal Tension and Web Reinforcement in Concrete?" Circular Series No. 64, Univ. of Ill, Eng. Exp. Sta., 1952, 47 pp.

++ "Shear and Diagonal Tension," Report of the Joint ASCE-ACI Committee on Shear and Diagonal Tension, Journal of Amer. Concr. Inst., Proc. v. 59, Jan., Feb., March 1962.

in the tension zone is taken to be equal to that of beams with continuous reinforcement;

(b) nominal unit shearing strength (V/bd or V/bjd) in beams without web reinforcement is considered to be primarily proportional to the compressive strength of a standard concrete cylinder;

(c) contribution of the web reinforcement to the shearing strength of a beam is taken to be directly additive to the shear capacity of a beam without such reinforcement.

To a greater or lesser degree experimental evidence contradicts these assumptions. Furthermore, the problem of limiting the extent (width and propagation length) of diagonal cracking is generally ignored.

Development of consistent and rational design criteria based solely on experimental data would require an immense amount of testing, which, in the absence of a general theoretical solution, may be difficult to interpret. On the other hand, a general theoretical solution to the problem requires the treatment of a three-dimensional, non-linear, non-homogeneous problem with changing boundary conditions due to cracking under load. Furthermore, such a solution requires knowledge of numerous relationships and phenomena which are difficult to determine with sufficient precision. Among these are:

(a) deformation and failure in concrete under various states of stress;

(b) stress distribution - overall and local - in cracked beams with a given arrangement of reinforcement;

(c) influence of slip between reinforcement and concrete on the stresses in cracked beams;

(d) deformations and forces resulting from dowel action of both longitudinal and transverse steel reinforcement.

It is believed that availability of the computer as a powerful analytical tool on the one hand and increased emphasis in research on fundamental phenomena in reinforced concrete on the other will soon result in the development of a general rational solution to the "riddle of shear failure." Some of the recent attempts to obtain such a solution and an outline of the major factors which must be incorporated in the solution have been presented recently by Bresler and McGregor*.

Further consideration of the theoretical solutions for shear problems is beyond the scope of this report, which is limited to the description of the test program and the test results of a study on shear strength of twelve reinforced concrete beams and to the correlation of these with two earlier test programs carried out at the University of California, Berkeley. However, it is believed that the results of these tests will prove useful both in the verification of theoritical solutions which are under development, and in the formulation of empirical design criteria.

2. Background of the Test Program

An initial phase of the experimental investigation of shear strength of reinforced concrete beams, conducted at the University of California in 1960-61, comprised a series of 12 beam specimens. The objectives of these tests were to observe the general behavior and to determine the cracking load and strength of beams subjected to a single concentrated load at midspan. The beams had shear-span to depth ratios (a/d) between 4 and 7, corresponding to spans of 12, 15, and 21 feet, and had vertical stirrup reinforcement with rf_y values of 0, 50, 75 and 100. Most of the tests carried out prior to 1960 used short beams with heavy web reinforcement, and data on beams with low and moderate amounts of web reinforcement were scarce. The results of this first study were useful in partially filling this gap in experimental data. This investigation has

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* B. Bresler and J. McGregor, "Concrete Beams Failing in Shear - A Review," paper presented at ASCE Structural Conference, Miami Beach, February 1966 (to be published).

been reported in "Shear Strength of Reinforced Concrete Beams" by B. Bresler and A. C. Scordelis, Structures and Materials Research Report, Department of Civil Engineering, University of California, Series 100, Issue 13, June 1961, and a condensation of this report was subsequently published in the January 1963, ACI Journal. A comparison of the observed beams of this first series which failed in shear with calculated values, based on formulas which subsequently were adopted as a basis for the 1963 ACI Code, indicated that the test values were 27 to 49 percent higher than the calculated values.

Since the results of this initial investigation indicated a need for additional experimental data, a second series of tests was conducted in 1963. It included a series of 10 additional beam specimens, all with a 12 ft. span (a/d = 4) and also loaded at midspan. The principal objectives of these tests were to determine the effect of changing the size of longitudinal bars from No. 9 to No. 7 throughout the entire beam length while maintaining the same total area of tension reinforcement, and to determine the effect of partial cut-off of longitudinal tensile reinforcement, in a zone of high shear, on the shearing strength. The amount of vertical stirrup reinforcement in these beams was varied as before using rf_y values of 0, 50, 75, and 100.

To investigate the contribution to shear strength of the Howlett grip anchor nuts, which were used to prevent bond failures at the beam ends, two specimens similar to beams OB-1 and B-1, tested in the first series, but omitting the grips were made and tested. Also to obtain additional information on shear strength of beams without web reinforcement for comparison with companion specimens with web reinforcement, two specimens without web reinforcement but otherwise similar to beams tested in the first series were included in the program.

The results of this study have been reported in "Shear Strength of Reinforced Concrete Beams-Series II" by B. Bresler and A. C. Scordelis, Structures and

Materials Research Report No. 64-2, December 1964, Department of Civil Engineering, University of California. Based on the experimental data obtained in the tests of the beams in Series II, it was observed that longitudinal tensile bar cut-offs in a zone of high shear without supplementary web reinforcement in the cut-off zone reduced the shearing strength by an amount varying from 20 to 30 percent, and that reduction in the bar sizes of the longitudinal tensile steel, while maintaining the same total tensile steel area, decreased the shearing strength, probably as a result of the reduction in the contribution of dowel action to shearing strength. For the beams tested in this series the contribution of web reinforcement to shearing strength was substantially greater than that obtained by the equations used in the 1962 Report of the Joint ASCE-ACI Committee on Shear and Diagonal Tension. It was also noted that omission of Howlett grip anchors had negligible effect on shearing strength of the beams tested in Series II.

3. Objectives and Scope of Tests - Series III

As the cut-off of longitudinal tensile steel bars in the region of high shear and the reduction in the bar sizes of the tensile steel reinforcement showed significant reductions in the shearing strengths, additional tests were indicated. A third series of tests was carried out in the summer of 1964 and is reported herein. This investigation included testing 12 additional beam specimens with the following objectives:

1. To determine the effect of combined reduction in bar size and of partial (50 percent) cut-off of longitudinal tensile reinforcement on the shearing strength.

2. To determine the effect of additional vertical stirrup reinforcement in the zone of bar cut-off. Several arrangements of additional stirrups were

to be studied, some in accord with the 1963 ACI Building Code Sec. 918-(c)-2, and others deviating from the code requirement.

3. To explore the effect of reducing the size of the longitudinal tensile steel bars while maintaining constant sectional area on the shearing strength of beams without any web reinforcement.

This test program is reported below, and the test results are compared with those of test series I and II.

II. EXPERIMENTAL PROGRAM

1. Description of Test Beams

Because this program is an extension of two previous test series, similar cross-sections, span lengths, reinforcement and concrete were used so that the results of the tests - Series III could be directly related to those obtained in the first and second series.

Cross-sectional properties for each of the 12 beams finally selected and tested to failure are shown in Figs. 1-A and 1-B; beam elevations are shown in Figs. 2-A through 2-D; and actual beam dimensions obtained by measurements prior to each test are listed in Table 8. All beams were of rectangular cross-section with the same over-all depth of 21-3/4 inches and had an effective depth to the centroid of reinforcement of 18 inches. The main longitudinal reinforcement consisted of from two to four No. 9 or from three to six No. 7 high strength steel deformed (ASTM A-305) bars placed in the bottom of the beams at two levels. All stirrups were made from No. 2 intermediate grade steel deformed bars, bent, lapped, and welded to form box-type stirrups. Two No. 4 longitudinal reinforcing bars of intermediate grade steel were placed at the top of the beams to facilitate the spacing of stirrups. These reinforcing bars acted as compression steel. Percentages of steel reinforcement and stirrup spacing are shown in Figs. 2 and in Table 8.

The test beams were grouped into three series -- A, B, C -- using three beam widths -- 6, 9 and 12 inches -- to obtain the desired variations in rf_y values. One single span length of 12 feet was used, and beams were subjected to a concentrated load at midspan. The nominal strength of concrete used in the beams was 3500 psi.

The beam designations are summarized below:

BEAM DESIGNATIONS

BEAM WIDTH Inches	MAIN REINFORCING STEEL					
	No. 7 bars			No. 9 bars		
12	CRA-1	1WCRA-1	*ROA-1	1WCA-1	2WCA-1	3WCA-1
9	CRB-1	1WCRB-1		1WCB-1		
6	CRC-1	1WCRC-1		1WCC-1		

* No web reinforcement

To prevent bond failures due to possible insufficient anchorage after the formation of diagonal tension cracks, Howlett grip anchor nuts were attached to the No. 9 or No. 7 longitudinal bars which protruded about 6 inches from the ends of the specimens. Steel plates 1-3/8 inches thick were used at the ends of the beams to provide bearing for these nuts.

Group CR

Specimens CRA-1, CRB-1, and CRC-1 were identical to specimens RA-1, RB-1, and RC-1 of the second series, except that some of the No. 7 longitudinal bars were cut-off at a distance of 24 inches, L/6, from each support. Uniformly spaced web reinforcement was used throughout, but no additional stirrups were provided in the region of cut-off as required in the 1963 ACI Building Code Sec. 918-(c)-2.

Group 1W

Specimens 1WCRA-1, 1WCRB-1, 1WCRC-1, 1WCA-1, 1WCB-1, and 1WCC-1, were identical to the corresponding beams in series CR and series C (described in the report on the second test series) except that additional web reinforcement was provided in an amount equal to, or greater than, that required by the 1963 ACI Building Code Sec. 918-(c)-2. The additional web reinforcement, approximately doubling the amount provided outside of the cut-off region, was extended a distance approximately 0.75d on each side of the cut-off point.

Specimens 2W and 3W

These two beams, 2WCA-1 and 3WCA-1, see Figs. 1-A and 2-D, included additional web reinforcement arranged in a manner different from that in beam 1WCA-1, and not complying fully with Sec. 918-(c)-2 of the ACI Code. In beam 2WCA-1 the same spacing as in 1WCA-1 was used but was extended approximately 0.35d each side of the cut-off. In beam 3WCA-1 a stirrup spacing in the cut-off region approximately 1.5 times that in 1WCA-1 was used, and this spacing was extended through the same distance as in 1WCA-1.

Specimen ROA-1

Specimen ROA-1, see Figs. 1-B and 2-D, was similar to specimen OA-1 (in series 1) except that No. 7 reinforcing bars instead of No. 9 bars were used for longitudinal tensile reinforcement throughout the entire length.

2. Fabrication

Reinforcing steel was thoroughly cleaned, assembled into a cage, and then placed into the forms. The steel assembly was securely held in the proper location in the forms by means of specially fabricated chairs which were spaced 2 feet apart throughout the length of the specimen. Lifting lugs were also provided to facilitate transportation of the finished beam specimens.

The beams were cast in wooden forms made of plywood with a plastic coating to give a smooth and impervious surface. The forms were designed so that they could be adjusted to the desired width for each specimen.

The concrete was mixed in a 6 cubic feet capacity horizontal non-tilting drum-type mixer. Each batch averaged about 6.0 cubic feet while the total number of batches required for a single beam together with the control specimens varied between 3 and 5. Aggregates were blended and moisture contents were determined the day prior to casting. The dry materials were first blended in the mixer for one minute; then the water was added and the entire contents mixed for three additional minutes.

The concrete was transported to the forms in buggies and placed into the forms in three to five layers (depending on the number of batches). Each layer was vibrated internally with a high frequency vibrator (8000 to 10,000 cycles per second). Forms were stripped 3 days after casting. All specimens were cured moist for 7 days under wet burlap covers and then were air dried until testing at the age of 13 days.

3. Materials and Control Specimens

Concrete mixes were designed by trial-batch method to achieve 3500 psi mix. Type 1 Portland cement and locally available Elliot sand, Antioch sand and Fair Oaks gravel were used in all of the mixes.

The cement was purchased in one lot from a single mill run, was blended in 7 sack batches as needed, and stored in steel drums. A chemical analysis of the cement is given in Table 1.

Petrographic analyses of the aggregates are given in Table 2 and the results of sieve analyses on the aggregates are given in Table 3. The maximum size of the coarse aggregate was 3/4 inch.

The 3500 psi concrete mix had a cement factor of 5.85 sacks per cubic yard. The water-cement ratio was 0.577 by weight or 6.50 gallons per sack. Relative mix proportions by weight -- cement: Elliot sand: Antioch sand: Fair Oaks gravel -- were 1.00:2.74:0.137:3.04. The aggregate weights were based on a saturated surface dry condition. Consistency was measured by means of a Kelly-ball; it averaged about 4 inches slump-equivalent.

Concrete control specimens included from fifteen to twenty-five 6 x 12 inch cylinders and four 6 x 6 x 20 inch beams for each test specimen. The control specimens were cured in the same manner as the test beams. Values of compressive strength f'_c are given in Table 4. Values of modulus of rupture f_r obtained by loading the 6 x 6 x 20 inch beams at the third points of an 18 inch span, values of the splitting tensile strength f_t obtained by loading 6 x 12 inch cylinders, and values of secant modulus of elasticity E_c obtained from the compression tests of 6 x 12 inch cylinders at 1000 psi stress are shown in Table 5. Figure 3 depicts typical stress-strain curves for the concrete.

Three reinforcing bar sizes were used in the beams. The bottom tension steel was made up of No. 9 high strength deformed bars having an average yield point of 96.2 ksi or No. 7 high strength deformed bars having an average yield point of 101.1 ksi. Number 2 intermediate grade deformed steel bars were used for the stirrups, and two No. 4 intermediate grade steel bars were placed at the top of each of the beams with stirrups. Control specimens for each bar size were tested in tension to determine the yield strength f_y, ultimate strength f_u, and the modulus of elasticity E_s. The mechanical properties based on nominal areas are tabulated in Tables 6 and 7, and typical stress-strain diagrams for each bar size are shown in Fig. 4.

4. Method of Loading and Instrumentation

The loading arrangement and instrumentation are shown in Fig. 5. The center-point load was applied by means of a 200,000 pounds capacity Olsen universal testing machine. An 8 inch spherical loading block was utilized at the load point. One end of the beam was supported on a 6 inch spherical bearing block while the other end was supported on a 3 inch diameter roller.

Absolute midspan deflections were obtained by two methods. In the first method a simple dial gage with a least count of 0.001 of an inch, supported by a floor stand and bearing on the load plate of the beam at midspan, was used together with two similar dial gage setups over the end support points. In the second method a scale graduated in 0.01 of an inch and a mirror were glued to the beam on each face at midspan. A piano wire was then stretched between the support points on each face to obtain deflection readings.

Changes in the over-all depth of the beam due to diagonal cracking were measured by means of specially designed yoke extensometers. These measurements were taken at six separate stations on each beam. The yoke extensometers consisted of two 1/4 x 1/2 x 16 inch steel bars clamped to the beam, one across the top and one across the bottom. These two bars were connected vertically by means of a 3/8 inch diameter steel rod and a dial gage. Relative movements between the top and bottom surfaces of the beam were registered on the dial gages which read to the nearest 0.0001 inch. Details of the extensometers are shown in Fig. 6.

To facilitate the recording of cracks and the visual observation of the beam bahavior during testing, the entire beam was first whitewashed and a ruled grid was then marked on the two sides of the beam. For beams with stirrups, vertical grid lines were placed at stirrup locations so that during testing the number of stirrups being crossed by a particular crack would be discerned.

5. Test Procedure

Seven days after casting, the beam to be tested was placed in position under the testing machine after which it was whitewashed and the yoke extensometers, and deflection gages were installed. All beams were tested under centerpoint load at an age of 13 days.

The beams were first loaded to about 30 percent of ultimate in two or three increments and then the load was removed. The load was then reapplied in 10 kip increments until failure occurred.

Deflection and yoke extensometer readings were taken at the beginning and end of each load increment. Cracks were plotted at the end of each load increment directly on the beam and also on specially prepared data sheets. After failure a careful visual inspection of the beam was made and several photographs were taken. Total testing time for a single beam varied between 2-1/4 and 3 hours.

III. EXPERIMENTAL RESULTS AND ANALYSIS OF DATA

1. General Behavior

Beam behavior generally agreed with that described by the authors in the reports on test series I and series II, as well as by numerous other investigators. Typical initial flexural cracks appeared first, followed by the appearance of diagonal tension cracks, usually in the middle third of the over-all beam depth and at various sections along the span. With further increase in load this primary cracking, usually called flexural-shear cracking, was followed by secondary cracking in the zone of the tensile longitudinal reinforcement. The secondary cracks formed as a result of splitting forces developed by the deformed bars when slip between concrete and steel reinforcement occurred or as a result of "dowel action" forces in the longitudinal bars transferring shear across a crack.

Two general modes of failure were observed in this series of tests. These may be differentiated as shear-compression (V-C) failures and diagonal-tension (D-T) failures. Shear-compression failures were observed in all the beams with web reinforcement, and diagonal tension failure was observed in the beam without web reinforcement.

The general behavior of the various test specimens may be interpreted through a study of the crack patterns, Fig. 7A to 7L, the load deflection curves, Fig. 8, and the yoke data, Fig. 9A to 9L.

2. Shear Compression Failures

This type of failure occurred in all beams with web reinforcement. In beams CRA-1, CRB-1 and CRC-1 (Fig. 7A, 7B, 7C) in which no additional web reinforcement was provided in the region of cut-off, the diagonal crack causing failure commenced at the cut-off point for the main reinforcement at approximately 50 percent of the ultimate load. At approximately 80 percent of the ultimate value (when the diagonal crack was within about 2-1/2 inches of the top of the beam) extensive secondary cracking became evident at the cut-off point and was accompanied by an increase in the width of the main crack. The width of the main crack at failure was approximately 1/8th inch, failure being caused by fairly rapid crushing at the load point.

The beams with added web reinforcement in the cut-off region (Fig. 7D through 7K) behaved quite differently from those without. While the diagonal crack at the cut-off point appeared as before at approximately 50 percent of the ultimate load, this crack did not cause the final failure. Another diagonal crack appeared at a distance of about 6 inches inward from the point of cut-off at approximately 75 percent of the ultimate load and the propagation of this crack proceeded faster than that of the diagonal crack which formed earlier at the cut-off. This new crack came within about 3 inches of the load point and then a crushing of the concrete took place above the crack, the crushing progressing from the load point down to the crack at ultimate failure.

It was clearly evident that at loads approaching ultimate the diagonal cracks in the beams with added web reinforcement in the cut-off region were considerably narrower and the failure developed more gradually than in beams without the added stirrups.

The propagation of vertical flexural cracks in the center portion of the beams tested in this series ceased after the load reached about 40 percent of the ultimate, leading one to believe that the flexural capacity far exceeded the shear capacity of these beams. For all cases but two this is confirmed by calculated values of flexural capacity based on the Hognestad-McHenry* concrete stress block (Table 9). For beams 1WCRB-1 and 1-WCC-1 the calculated flexural capacities are slightly lower than the actual failure loads.

However, with the curvilinear shape of the stress-strain diagram for the high-strength steel reinforcement and with only slight increase in the ultimate strain in concrete over the assumed value of $\epsilon_u = 0.0031$, a significantly larger flexural capacity would be obtained by calculations more accurately representing the actual flexural strength.

3. <u>Diagonal Tension Failures</u>

This type of failure occurred in beam ROA-1 (Fig 7L) which had no web reinforcement and failed shortly after the formation of the "critical diagonal tension crack." The failure occurred as a result of longitudinal splitting in the compression zone near the load point, and was accompanied by splitting along the tensile reinforcement near the end of the beam. Failure was sudden; the critical crack formed at a load of approximately 80 percent of the ultimate load. Although the beam carried additional load after the formation of the primary crack, the ultimate failure was preceded by extensive secondary cracking in the region of the main tensile reinforcement, Fig. 7-L.

4. <u>Load-deflection Relationship</u>

Load deflection relationships for the beams tested are shown in Figure 8. The deflections plotted are the average values of readings on two faces recorded during the final loading.

Comparison of deflections at a given load reveals that beams with additional web reinforcement (series 1W, 2W, 3W) have consistently smaller deflections than similar beams without such reinforcement. Taking the deflection of a beam with continuous no. 9 tensile reinforcement and web reinforcement (Series A. B. C)* as a base 100 percent magnitude, and comparing this with the deflection of a similar beam with cut-off tensile reinforcement (series CA, CB, CC) and without additional web reinforcement, and also with the deflection of a beam where additional reinforcement is provided in the region of cut-off (Series 1WCA, 1WCB,

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* B. Bresler and A. Scordelis, "Shear Strength of Reinforced Concrete Beams," Structures and Materials Research Report, Department of Civil Engineering, Univ. of California, Berkeley, Series 100, Issue B. June 1961.

1WCC), reveals the following:

(1) Cutting-off main tensile bars in the tension zone and in the region of high shear without providing additional web reinforcement increases the deflection from about 10 to 60 percent of base value. The relative increase in deflection increases with increase in load and is due to larger crack-widths in the cut-off region.

(2) Providing additional web reinforcement in the zone of bar cut-off, in accordance with the 1963 ACI Building Code Requirements Sec. 918-c-2, markedly decreases the deflection which would occur without additional web reinforcement. This decrease varies from about 10 to 47 percent of base value. The effectiveness of the additional web reinforcement in decreasing the deflection is greatest at the intermediate load level when the diagonal crack has developed, but the beam is capable of resisting significant increase in load before failure. At low loads, prior to cracking, the role of additional web reinforcement is insignificant.

5. Yoke Extensometer Data

Vertical displacements of the bottom of the beam with respect to the top surface at selected sections for each of the specimens are shown in Figs. 9-A through 9-L. The sections for yoke placement were selected to correspond approximately to the sections similarly instrumented in Series I and II. Exceptions were made where it was thought that more significant data could be obtained at locations different from those previously selected. Average values for the displacements observed on the north and south faces are plotted in the figures. The maximum displacement shown on the figure represents the largest value recorded in the test but does not always correspond to the displacement at the ultimate load. Because of danger of impending failure at loads approaching ultimate, it was not always possible for the observers to read the dial gages at the ultimate load.

As the yokes were placed at the stirrup locations, the observed displacements could be related to elongation of web reinforcement. It may be assumed that in an "ideal stirrup" at insipient yielding: (1) the elongation takes place entirely as a result of extensional strain in the 18 inch leg of the stirrup; (2) that for a stirrup at incipient yielding the maximum stress at some point reaches f_y = 50 ksi; (3) that the stress varies linearly from the maximum value to zero at the top and bottom of the stirrup leg; and (4) that elastic modulus of stirrup steel is 30,000 ksi. Then the total elongation of such an "ideal stirrup" is calculated to be 0.015 inches. In a stirrup undergoing general yielding it may be assumed that the strain everywhere along the full length of 18 inches reached at least the initial yield strain value. In this case the total elongation would be 0.03 inches. On this basis, it is seen that for beams tested in series III the stirrups at critical sections have yielded in practically all beams; the only exception is beam CRA-1 in which four of the stirrups have not reached the assumed yield elongation value.

6. Ultimate Load

Table 9 presents a summary of test results including values of the diagonal tension cracking load P_{cr}, ultimate load P_u, maximum deflection Δ_{max}, and failure mode for each of the beams tested in Series III. Calculated values of flexural capacity P_f, cracking load P_{cr}, and shear capacity P_v are also included in the Table.

The value of P_f for each beam was determined by trial and error using the Hognestad-McHenry-Hanson stress block with an assumed ultimate compressive unit strain of 0.003 and using experimentally determined stress-strain characteristics for the top and bottom longitudinal reinforcement.

The values of P_{cr} and P_v were calculated using two approximate methods proposed in the ASCE-ACI Joint Committee on shear and Diagonal Tension 1962 Report, Equations 5-1 and 6-8. Applying these equations and neglecting the effect of weight of specimen:

$$P_{cr} = 2V_c = 2bd \left[1.9 \sqrt{f'_c} + 2500 \frac{pVd}{M}\right] = 2bd \left[1.9 \sqrt{f'_c} + 2500 \frac{pd}{a}\right] \qquad \text{1-a}$$

and

$$P_v = 2\left(V_c + V_s\right) = 2bd \left[1.9 \sqrt{f'_c} + 2500 \frac{pd}{a} + rf_y\right] \qquad \text{1-b}$$

The other method is a modification of the first, proposed by Bresler and Scordelis in the 1961 Report on the initial phase of this test program. This modification leads to somewhat simpler equations as follows:

$$P_{cr} = 2V_c = 2bd \left[2 \sqrt{f'_c}\right] \qquad \text{2-a}$$

and

$$P_v = 2\left(V_c + V_s\right) = 2bd \left[2 \sqrt{f'_c} + rf_y\right] \qquad \text{2-b}$$

The test results of Series III may be evaluated by a study of Tables 8 and 9 and Figures 10 and 11.

1. In table 9 it is seen that in all cases the values calculated by Equations 1a and 1b differ only slightly from those calculated by Equations 2a and 2b.

2. In Table 9 and Fig. 10 it is seen that almost all the beams tested had some reserve strength varying from 8 to 42 percent of the calculated value. Only one specimen, beam CRA-1, $rf_y = 50$, failed at 98 percent of the calculated capacity.

3. Although CRA-1 did not quite develop full calculated capacity, two other beams (CRB-1 and CRC-1, $rf_y = 75$ and 100 respectively), with part of the longitudinal bars cut-off in the tension zone and without additional web reinforcement in the cut-off region, developed significant reserve strength--8 and 20 percent respectively--over the calculated capacity.

4. The premature failure of CRA-1 is likely due to the fact that some of the stirrups in that beam did not develop their yield capacity (see discussion under Yoke Extensometer Data). It suggests that with development of longitudinal

(secondary) cracking and with consequent reduction of dowel action forces carried by the longitudinal reinforcement, the concrete compression zone and the web reinforcement were not adequate to sustain the load transfer, and sudden failure resulted.

5. In Fig. 11 test values are compared to the calculated values using two interpretations of the term (rf_y) in Eq. 1-b. The first interpretation is based on the designer's procedure which sets the amount of web reinforcement without reference to longitudinal bar cut-off, and then provides additional web reinforcement in the region of cut-off. Here the values of rf_y are taken nominally as 50, 75, and 100 (see Table 8 for actual values). The second interpretation is based on the analyst's procedure and strict interpretation of "truss analogy" in the failure zone whereby all the stirrups in the cut-off region are included in the calculation of rf_y, using values almost double the nominal values of 50, 75 and 100 (see Table 8). It is seen in Figure 11 that the additional stirrups at the cut-off region should not be included in the calculation of shear capacity, as otherwise the calculated value may over-estimate the strength by as much as 11 percent.

IV CONCLUSIONS

1. Corelation of Series I, II and III Test Results

The preceding parts of this report focused attention on the third test series. A preliminary corelation of the test results for the entire program will be given in this concluding section. A full corelation and evaluation is difficult without an adequate theory which would permit precise analysis of the differences observed in individual specimens. Nevertheless, it is believed that a number of important observations have been made; some substantiating the present concepts of evaluating shearing strength, and others either contradicting or supplementing these concepts.

2. Mechanism of Failure

Inclined shear cracks form as an extension of a previously developed flexural crack; both the initiating flexural crack and the inclined shear crack are called primary cracks.

As the load is increased, the primary crack extends along the longitudinal reinforcement as a result of splitting forces partly caused by "lug" splitting action" of the deformed bars and partly caused by "dowel action" of longitudinal bars transferring shear across a crack.

Finally, failure takes place by crushing under a combined state of stress in the shear compression zone beyond the tip of the primary crack and by longitudinal splitting along the tensile reinforcement. The two modes of failure may occur practically simultaneously, in which case it is difficult to identify the principal cause of failure.

In a beam with an inclined primary crack, relative rotation of beam segments separated by the crack will take place. This rotation will result in a relative transverse displacement of longitudinal reinforcement across the crack, a displacement which is associated with dowel action forces.

When the inclination of the primary crack is small, the transverse displacement is small. The dowel forces are also small, and they in turn depend on transverse stiffness of longitudinal reinforcement.

As the inclination of the primary crack increases, the transverse displacement increases and the dowel forces increase until a critical value is reached. These forces initiate longitudinal splitting (secondary crack) and unless failure takes place prior to full development of longitudinal splitting, the stiffness of the "dowel" decreases and the amount of shear carried by the reinforcement is reduced. The propagation of longitudinal splitting can be effectively arrested by the transverse reinforcement. As the dowel shear load may be decreased with longitudinal splitting, the shear in the compression zone may be increased causing redistribution of stresses and possible propagation of primary cracks.

3. Evaluation of Web Reinforcement Effectiveness

Evaluation of shearing strength P_v (see Eqs. 1-b and 2-b) assumes validity of "superposition" for determining the contribution of stirrup reinforcement to the beam's shear capacity. If P_{vc} denotes the capacity of a beam without web reinforcement, and P_v denotes the capacity of a beam with web reinforcement, (consisting of stirrups having total steel area A_v and yield strength f_y, spaced a distance s apart) then it is assumed that:

$$(P_v - P_{vc}) = 2\ (d/s)\ A_v f_y = P_{vs} \qquad 3$$

In calculating the contribution of web reinforcement to shear strength as $2(d/s)\ A_v f_y$ it is assumed that the critical diagonal tension crack has a horizontal projected length equal to the effective depth d and that all stirrups are yielding.

In the present program four pairs of beams which differed from each other in the amount of web reinforcement were tested. Four beams had no web reinforcement and provided test values of P_{vc}. Four similar specimens had varying amounts of web reinforcement and provided test values of P_v. Thus, the value of P_{vs} -- contribution of web reinforcement to shear strength -- could be determined directly from test results by taking the difference between the ultimate loads of corresponding beams and comparing this to the calculated value for $P_{vs} = 2(d/s)\ A_v f_y$.

This comparison is shown below:

	Values P_{vs}		
	Test Value, Kips $(P_v - P_c)$	Calculated Value, Kips $2(d/s)A_v f_y$	Test-to-Calculated Ratio
A-1 and OA-1	30.0	22.5	1.33
B-1 and OB-1	42.3	22.8	1.85
C-1 and OC-1	35.1	21.2	1.65
RA-1 and ROA-1	50	21.8	2.29

It is clearly seen that for the particular cases considered here, "superposition" does not represent an accurate approximation, although it errs on the safe side.

A study of the maximum "yoke" elongations observed in all the beams in the test program shows that the test beams can be divided into three groups. In the first groups the "yoke" elongations are small, between 0.0005 and 0.005 inches. This group included 7 beams without web reinforcement which failed in shear and 3 beams with web reinforcement which failed in flexure. In the second group the "yoke" elongations vary between 0.005 and 0.02 inches. This group included 7 beams with web reinforcement in which the main reinforcement consisted of No. 9 bars continued throughout the length of the beam. In the third group the "yoke" elongations vary between 0.02 and 0.175 inches. This group includes 17 beams with web reinforcement in which the main reinforcement is partly cut-off in the tensile zone (14 beams) or in which continuous main reinforcement is reduced from No. 9 to No. 4 bars.

Of particular interest in this study are the "yoke" elongations in the beams of second and third groups which are representative of stirrup elongations. It was suggested in the section dealing with yoke extensometer data that initial yielding of a stirrup would correspond to yoke elongation of approximately 0.015 inches and that general yielding of a stirrup would correspond to yoke elongation of 0.03 inches. It may be assumed that in an "ideal stirrup" at insipient yielding: (1) the elongation takes place entirely as a result of extensional strain in the 18 inch leg of the stirrup; (2) that for a stirrup at incipient yielding the maximum stress at some point reaches $f_y = 50$ ksi; (3) that the stress varies linearly from the maximum value to zero at the top and bottom of the stirrup leg; and (4) that elastic modulus of stirrup steel is 30,000 ksi. Then the total elongation of such an "ideal stirrup" is calculated to be 0.015 inches. Examination of the data reveals that shear failures in beams of the second group occurred without general yielding of the stirrups, while failures in beams of the third group definitely involved considerable yielding in the stirrups. This can be attributed to influence of bar cut-off and reduced bar size which resulted in more extensive diagonal cracking in the third group.

4. Influence of Longitudinal Bar Cut-Off on Shearing Strength of Beams Without Additional Stirrups

Before 1962 little recognition was given to the possible reduction in shear strength resulting from cut-off of longitudinal reinforcement in the tension zone and in a region of high shear. Tests carried out by Ferguson* to evaluate bond effectiveness of deformed bars under certain conditions focused attention on the possibility of this reduction. Also, tests conducted by Leonhardt and Walther** indicated such a reduction. In the program reported here six pairs of beams were included to investigate the reduction in the strength due to cut-off. These results are summarized below.

	Beams With Continuous Longitudinal Bars					
	A-1	B-1	C-1	RA-1	RB-1	RC-1
Test P_u, Kips	105	100	70	90	90	62
	Beams With Cut-Off Longitudinal Bars					
	CA-1	CB-1	CC-1	RCA-1	RCB-1	RCC-1
Test P_u, kips	74	79	50	75	77	53

Comparison of the values indicates that for beams with No. 9 longitudinal reinforcement a reduction in capacity of 20-30 percent results from the cut-off. For beams with No. 7 longitudinal reinforcement a reduction of 14-17 percent

* P. Ferguson and N. Thompson, "Development Length of High Strength Reinforcing in Bond", ACI JOURNAL, Proceedings, V. 59, No. 7, July 1962, pp. 887 - 917

** F. Leonhardt, "On The Reduction of Shear Reinforcement As Derived from the Stuttgart Shear Tests 1961-1963," International Assoc. for Bridge & Structural Engineering 7th Congress, Rio de Janeiro, Aug. 1964.

results from the cut-off. An average of these reductions is 20 percent and in order to account for such reduction in capacity either the permissible shear value must be reduced accordingly or additional web reinforcement provided to offset this reduction.

5. Influence of Additional Stirrups in the Cut-Off Zone on Shearing Strength

The 1963 ACI Building Code, recognizing the adverse effect of bar cut-off in the tension zone in the region of high shear, included provisions for additional web reinforcement to compensate for this effect, Sec. 918 (c) as follows:

No flexural bar shall be terminated in a tension zone unless one of the following conditions is satisfied:

1. The shear is not over half that normally permitted, including allowance for shear reinforcement, if any.
2. Stirrups in excess of those normally required are provided each way from the cut-off a distance equal to three-fourths of the depth of the beam. The excess stirrups shall be at least the minimum specified in Section 1206 (b) or 1706 (b). The stirrup spacing shall not exceed - $d/8r_b$, where r_b is the ratio of the area of bars cut-off to the total area of bars at the section.
3. The continuing bars provide double the perimeter required for flexural bond.

In the program reported here six pairs of beams were included to evaluate the effectiveness of additional reinforcement provided in accordance with the ACI Code below:

Beams Without Additional Web Reinforcement in Cut-Off Zone						
	CA-1	CB-1	CC-1	RCA-1	RCB-1	RCC-1
Test Pu, kips	74	79	50	75	77	53

Beams With Additional Web Reinforcement in Cut-Off Zone						
	1WCA-1	1WCB-1	1WCC-1	1WRCA-1	1WRCB-1	1WRCC-1
Test Pu, kips	99	91	64	96	92	65

It can be seen that the increase in shear capacity resulting from the additional web reinforcement varies from 15 to 34 percent -- essentially compensating for the adverse effect of cut-off, and raising the shear capacity to within a few percent of the capacity of beams with continuous longitudinal steel.

Beams 2WCA and 3WCA not complying with the ACI Code, were designed to explore alternate arrangements of additional web reinforcement. Although these beams performed equally as well as 1WCA, no conclusive result has been obtained here because of the small number of tests. It is merely noted that beam 2WCA, providing two less stirrups within the critical zone than beam 1WCA, had slightly greater shear capacity. While the increase in capacity with reduction in the number of additional stirrups may not be signiticant, an investigation of an optimum additional reinforcement in the cut-off region may lead to liberalization of the present ACI Code provision.

6. Influence of Reducing Longitudinal Bar Size on Shearing Strength

Previous investigators recognized the influence of the amount of longitudinal reinforcement, expressed in the usual terms of $p = (A_s/bd)$, on the shearing strength, and this effect was included* in the expression:

$$v_c = 1.9\sqrt{f'_c} + 2500p\ (d/a) \qquad 4$$

This suggests that for given f'_c and (d/a) shear capacity increases with p. On the other hand, this expression does not account for the possibility of the shear capacity being adversely affected by the reduction in the dowel capacity of longitudinal reinforcement. In the present program ten pairs of beams which differed only in the size of the longitudinal reinforcement were tested. In one set of 10 beams four No. 9 bars were used and in the other

* "Shear and Diagonal Tension," Report of the Joint ASCE-ACI Committee on Shear and Diagonal Tension, Journal of the Amer. Concr. Inst. Proc. V 59, Jan., Feb., Mar., 1962.

set six No. 7 bars were used to give essentially the same total area of reinforcement.

The results are summarized below.

Beams With No. 9 Longitudinal Bars										
Beam	A-1	B-1	C-1	OA-1	CA-1	CB-1	CC-1	1WCA-1	1WCB-1	1WCC-1
Test P_u, Kips	105	100	70	75	74	79	49	99	91	64

Beams With No. 7 Longitudinal Bars										
Beam	RA-1	RB-1	RC-1	ROA-1	RCA-1	RCB-1	RCC-1	1WCRA-1	1WCRB-1	1WRCC-1
Test P_u, Kips	90	90	61.9	73	75	77	53	96	92	65

It can be seen that in the first three pairs of beams which had web reinforcement and continuous longitudinal reinforcement, reductions in strength of the order 10-14 percent result with just the reduction of bar size from No. 9 to No. 7. The fourth pair of beams, which had no web reinforcement and continuous longitudinal reinforcement, show little difference in the strength for the two bar sizes used probably due to the sudden shear failure at initial diagonal cracking. In the remaining beams, all of which had bars cut-off, the differences in test values are considerably smaller, 2-8 percent and the strengths of beams with No. 7 bars are either smaller or greater than the strengths of corresponding beams with No. 9 bars. It is clear that the dominant factor in determining shear strength in these beams is the bar cut-off, and the influence of bar size in beams with cut-off is supressed.

7. Concluding Remarks

The principal findings of the test program, within the variables included, can be summarized as follows:

a. Evaluation of the results of the 34 tests carried out in this total program shows that the empirical procedures presently used in U.S.A. are fully justified. Figure 12 summarizes the comparison of calculated test values of shearing strength of beams in Series I, II, and III. All but three tests show appreciable reserve strength over calculated capacity. The three understrength specimens (CA-1, CC-1, and CRA-1) are only 2 to 6 percent understrength, and do not fully conform to the present ACI Code requirements in that no additional web reinforcement is provided in the region of longitudinal bar cut-off.

b. A distinction should be made between quantitative substantiation of an empirical procedure, and a quantitative substantiation of the simplifying assumptions usually made in justifying the empirical procedures. The substantiation of the present criterion can not be considered good when tests show results varying from 94 percent to 160 percent of calculated capacity. Such scatter emphasizes the need for improvement in the rationale of the design criteria.

c. Based on a limited number of tests in this study the contribution of web reinforcement to the shear capacity of reinforced concrete beams with continuous longitudinal steel is substantially greater than that indicated by simply adding ($rf_y bd$) to the shear capacity of a similar beam without web reinforcement.

d. Longitudinal tensile bar cut-offs in a zone of high shear, without adding supplementary web reinforcement in the cut-off zone, substantially reduce the shearing strength of reinforced concrete beams. For a given total steel area this reduction tends to be greater for beams with No. 9 bars than for beams with No. 7 bars.

e. The addition of a small amount of web reinforcement in the region of the tensile steel cut-off substantially eliminates the adverse effect of the cut-off.

f. A reduction in the bar size of the continuous longitudinal steel, while maintaining the same total tensile steel area tends to decrease the shear capacity of reinforced concrete beams with web reinforcement. In beams without web reinforcement, or with partial cut-off of the longitudinal steel this reduction does not seem to be significant.

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VI. NOTATION

The letter symbols used in this report are usually defined when they are introduced. They are listed below alphabetically for convenient reference:

a = Shear span = L/2 for beam under center point load

A_s = Area of longitudinal tension reinforcement

A'_s = Area of longitudinal compression reinforcement

A_v = Area of web reinforcement

b = Width of beam

d = Effective depth of beam

E_c = Secant modulus of elasticity of concrete at 1000 psi

E_s = Modulus of elasticity of steel

f'_c = Compressive strength of 6 x 12 in. concrete cylinder

f'_t = Modulus of rupture of concrete

f_s = Stress in longitudinal tension reinforcement

f_v = Stress in web reinforcement

f'_y = Yield point of compresssion steel reinforcement

f_y = Yield point of tension steel reinforcement

f_u = Ultimate strength of steel reinforcement

h = Over-all depth of beam

K = Constant depending on angle of inclination of web reinforcement; K = 1 for vertical stirrups

L = Span length

M = Bending moment at a section

N = Number of stirrups crossing a diagonal crack

p = Tension steel reinforcement ratio = A_s/bd

p' = Compression steel reinforcement ratio = A'_s/bd

P_{cr} = Load producing initial diagonal tension crack

P_f = Calculated ultimate load as governed by flexure

P_v = Calculated ultimate load as governed by shear

P_u = Ultimate test load

q = Longitudinal reinforcement index = $(pf_y - p'f'_y)/f'_c$

r = Web reinforcement ratio = A_v/bd

s = Longitudinal spacing of web reinforcement

v_c = Ultimate shearing stress for beams without web reinforcement

v_u = Ultimate shearing stress for beams with web reinforcement

V = Total shear at a section

V_s = Shear assumed taken by web reinforcement

α = $v_c/\sqrt{f'_c}$

β = f_v/f_y

Δ = Midspan deflection

λ = Ratio of the length of the horizontal projection of a diagonal crack to the effective depth.

TABLE 1 CHEMICAL ANALYSIS OF CEMENT[1]

Chemical	Percent
SiO_2	23.3
Fe_2O_3	2.4
Al_2O_3	4.7
CaO	64.2
MgO	1.7
SO_3	2.1
Ignition loss	1.2
Alkalis plus undetermined	0.8

[1]Type I, Portland cement, mill analysis supplied by Pacific Cement and Aggregate Company, Davenport, California.

TABLE 2 PETROGRAPHIC ANALYSES OF AGGREGATES

Elliot Sand[1]		Fair Oaks Gravel[2]	
Mineral	Percent	Mineral	Percent
Graywacke	64	Basic igneous rocks	24
Metaigneous rocks	16	Basic metaigneous rocks	42
Gabbro	2	Andesite	14
Jasper	16	Sandstone	11
Vein quartz	2	Quartzite	5
		Slate	2
		Vein quartz, chert, schist	1

1. Data supplied by Pacific Cement and Aggregate Company, Pleasanton, California.

2. Ref. "Test Data Concrete Aggregates in Continental United States," Corps of Engineers, U.S.A., TM No. 6-370.

TABLE 3 SIEVE ANALYSES OF AGGREGATES

Sieve Size	Percentage Retained on Sieve		
	Elliot Sand	Fair Oaks Gravel	Antioch Sand
3/4 in.		2.2	
1/2 in.		(44.6)	
3/8 in.		71.1	
No. 4		98.9	
No. 8	14.3	100.0	
No. 16	45.7		
No. 30	72.0		
No. 50	90.4		24.0
No. 100	97.6		96.0
No. 200			99.0
Fineness Modulus	3.20	6.72	1.20

TABLE 4 - COMPRESSIVE STRENGTH f'_c OF CONCRETE

3500 psi mix; 6 x 12 in. cylinders
All values given in ksi.
All tests at 13 days

	CRA-1	CRB-1	CRC-1	1WCRA-1	1WCRB-1	1WCRC-1	1WCA-1	1WCB-1	1WCC-1	2WCA-1	3WCA-1	ROA-1
1-A	3.62	3.53	3.42	3.71	3.38	4.07	3.62	3.74	3.84	3.54	3.61	2.76
1-B	3.64	3.53	3.58	3.57	3.33	4.08	3.76	3.98	3.73	3.71	3.93	3.52
1-C	3.46		3.40	3.79	3.35	4.02	3.69	3.80	3.58	3.64	3.77	3.50
2-A	3.88	3.46	3.69	3.84	3.42	3.72	3.76	3.72	3.48	3.64	3.86	3.28
2-B	3.81	3.62	3.56	3.58	3.28	3.78	3.90	3.78	3.56	3.82	3.90	
2-C	3.76	3.54	3.52	4.08	3.42	3.82	3.58	3.76	3.70	3.55	3.72	3.31
3-A	3.70	3.22	3.53	3.71	3.22	3.80	3.78	4.00	3.62	3.66	3.83	3.25
3-B	3.57	3.34	3.45	3.97	3.34	3.77	3.60	3.83	3.52	3.95	3.78	3.52
3-C	3.92	3.42	3.72	4.04	3.40	3.88	3.48	4.07	3.46	4.25	3.80	3.58
4-A	3.48	3.47		3.80	3.39		3.70	3.85		3.66	3.83	3.85
4-B	3.46	3.36		4.15	3.36		3.52	3.82		4.07	3.90	4.38
4-C	3.57	3.44		3.86	3.42		3.64	3.90		3.88	3.81	3.96
5-A	3.44	3.36		3.63	3.31		3.58	3.81		4.09	3.88	3.74
5-B	3.60	3.53		3.94	3.41		3.59	3.73		3.86	3.75	3.56
5-C	3.69	3.11		3.64	3.41		3.60	3.84		4.00	3.80	
6-A												3.24
6-B												3.35
6-C												3.48
Avg	3.64	3.43	3.54	3.82	3.36	3.88	3.65	3.84	3.61	3.82	3.81	3.50

TABLE 5 MODULUS OF RAPTURE, SPLIT-CYLINDER TENSILE STRENGTH, AND MODULUS OF ELASTICITY

Values of f_r and f_t in psi; values of E_c in 10^6 psi

All tests in 13 days

	CRA-1	CRB-1	CRC-1	1WCRA-1	1WCRB-1	1WCRC-1	1WCA-1	1WCB-1	1WCC-1	2WCA-1	3WCA-1	ROA-1
					MODULUS OF RUPTURE f_r							
1	513	527	553	636		520	547	583	533	541	546	591
2	597	674	574	605	545	533	605	679	520	577	588	596
3	602	575	535	694	660	507	584	635	584	584	611	627
4	390	560		575	578		532	590		575	565	555
Avg.	525	584	554	627	594	520	567	621	545	569	577	592
					SPLITTING TENSILE STRENGTH f_t							
1	478	502	526	507	446	531	456	534	424	441	494	473
2	489	507	483	480	470	475	490	496	493	485	480	485
3	500	510	502	528	427	508	500	445	462	529	468	497
4	477	485	499	496	492	453	476	478	487	577	497	462
5	510	500	510	540	462	474	492	436	497	457	498	515
6	485	492	458	514	470	492	460	468	504	490	501	518
7	447	482		482	406		469	473		441	505	522
8	474	488		482	465		466	504		550	443	538
Avg.	468	496	496	493	451	488	475	480	477	505	485	501
					SECANT MODULUS E_c							
1	3.45	3.60	3.77	3.70	3.45	3.70	4.00	4.00	3.84	3.57	3.92	3.85
2	3.70	3.33	3.45	3.85	3.70	4.17	3.85	3.70	3.70	3.57	3.70	3.85
3	3.70	3.33	3.77	3.85	3.71	4.00	4.00	4.00	3.84	3.34	3.85	3.57
4	3.70	3.33		4.00	3.70		4.00	3.70		3.63	3.84	3.00
5	3.45			3.57	3.57		3.92	3.85				3.00
Avg.	3.60	3.40	3.66	3.79	3.62	3.95	3.94	3.85	3.79	3.53	3.83	3.45

TABLE 6-A PROPERTIES OF NO. 9 HIGH STRENGTH STEEL REINFORCING BARS

Sample	1	2	3
Yield strength f_y, ksi	97.0	91.5	100.0
Ultimate strength f_u, ksi	149.0	138.1	149.6
Modulus of Elasticity E_s, ksi	27.2×10^3	28.0×10^3	27.0×10^3
% elongation in 8 inches	6.32	4.62	4.75
Weight per lineal ft., lb.	3.40	3.58	3.39
Nominal area, $in.^2$	1.02	1.07	1.02
Average deformation height, in.	0.05	0.046	0.059
Average deformation spacing, in.	0.52	0.52	0.52

a. f_y computed on basis of 0.2% offset.

b. Nominal bar areas computed from the weight including that of the deformations.

c. Heat 3069, Chemical analysis supplied by Inland Steel Co., % by weight: 0.41C; 0.92Mn; 0.015P; 0.0235S; 0.285Si; 0.93Cr; 0.24 Mo.

TABLE 6-B PROPERTIES OF NO. 7 HIGH STRENGTH STEEL REINFORCING BARS

Sample	1	2	3
Yield strength f_y, ksi	102.0	101.2	100.0
Ultimate strength f_u, ksi	157.0	145.1	157.0
Modulus of elasticity E_s, ksi	26.7×10^3	28.7×10^3	28.2×10^3
% elongation in 8 inches	5.87	5.63	8.00
Weight per lineal ft., lb.	2.07	2.07	2.07
Nominal area, $in.^2$	0.608	0.603	0.608
Average deformation height, in.	0.051	0.063	0.067
Average deformation spacing, in.	0.45	0.45	0.46

a. f_y computed on basis of 0.2% offset.

b. Nominal bar areas computed from the weight including that of the deformations.

c. Chemical composition similar to that of No. 9 bars above. Steel classified as A-431-Modified.

TABLE 7-A PROPERTIES OF NO. 4 INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength, f_y, ksi	49.7	52.0	49.5
Ultimate strength f_u, ksi	76.3	81.6	76.6
Modulus of elasticity E_s, ksi	27.8×10^3	29.9×10^3	29.0×10^3
% elongation in 8 inches	18.1	20.5	16.3
Weight per lineal ft., lb.	0.686	0.666	0.665
Nominal area, in.^{2}	0.206	0.200	0.200
Average deformation height, in.	0.027	0.032	0.036
Average deformation spacing, in.	0.31	0.30	0.30

a. Nominal bar areas computed from the weight including that of the deformations.

TABLE 7-B PROPERTIES OF NO. 2 INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength f_y, ksi	50.5	49.0	49.5
Ultimate strength f_u, ksi	64.8	66.6	66.8
Modulus of elasticity E_s,ksi	29.2×10^3	26.7×10^3	27.5×10^3
% elongation in 8 inches	20.5	16.9	13.7
Weight per lineal ft., lb.	0.170	0.166	0.173
Nominal area, in.^{2}	0.051	0.050	0.052
Average deformation height, in.	0.010	0.011	0.010
Average deformation spacing, in.	0.175	0.177	0.176

a. Nominal bar areas computed from the weight including that of the deformations.

TABLE 8 SUMMARY OF TEST PROGRAM

	Concrete			Beam Dimensions					Reinforcement					
Spec. No.	f'_c psi	f_r psi	f_t psi	b in	h in	d in	L ft	a/d	Main Tens. Reinf.	p %	Top Bars #4	p' %	Stirrup Spacing in.	rf_y - psi
CRA-1	3640	525	486	12.0	21.85	18.10	12.0	3.98	* 6-#7	1.69	2	0.179	8-1/4	50.7
CRB-1	3430	584	496	9.0	21.74	17.99	12.0	4.01	* 6-#7	2.28	2	0.241	7-1/2	74.5
CRC-1	3540	554	496	6.1	21.78	18.03	12.0	4.00	** 3-#7	1.67	2	0.354	8-1/4	100.0
1WCRA-1	3820	627	493	12.0	21.73	17.98	12.0	4.01	* 6-#7	1.71	2	0.179	8-1/4 - 3-1/4	50.8 - 129.0
1WCRB-1	3360	594	451	9.0	21.82	18.07	12.0	3.99	* 6-#7	2.26	2	0.231	7-1/2 - 3-5/8	74.5 - 154.0 x
1WCRC-1	3880	520	488	6.0	21.84	18.09	12.0	3.98	** 3-#7	1.69	2	0.359	8-1/4 - 4-1/2	101.5 - 186.5 x
1WCA-1	3650	567	475	12.0	21.96	18.21	12.0	3.95	* 4-#9	1.76	2	0.178	8-1/4 - 3-1/4	50.8 - 129.0 x
1WCB-1	3840	621	480	9.1	21.85	18.10	12.0	3.97	* 4-#9	2.34	2	0.236	7-1/2 - 3-5/8	73.7 - 152.6 x
1WCC-1	3610	545	477	6.1	21.86	18.11	12.0	3.97	* 2-#9	1.75	2	0.352	8-1/4 - 4-1/2	100.0 - 183.5 x
2WCA-1	3820	569	505	12.0	21.89	18.14	12.0	3.96	* 4-#9	1.77	2	0.179	8-1/4 - 3-1/4	50.9 - 129.0 x
3WCA-1	3810	577	485	12.0	21.87	18.11	12.0	3.97	* 4-#9	1.77	2	0.177	8-1/4 - 4-1/2	50.8 - 93.2 x
ROA-1	3500	592	501	12.0	21.86	18.11	12.0	3.98	6-#7	1.69	0	0	-	0

* One-half of tensile reinforcement cut-off 24 inches from each support.
** One-third of tensile reinforcement cut-off 24 inches from each support.
x Values of rf_y in the cut-off region.

TABLE 9 ANALYSIS OF RESULTS

All load values shown in kips.

Spec. No.	Test Values				Calculated Values					Test Value/Calculated Value			
	P_{cr} 1)	P_u 1)	Δ Max in.	Failure Mode	P_f 2)	P_{cr}		P_v 3)		P_{cr}		P_v	
						Eq. 1	Eq. 2	Eq. 1	Eq. 2	Eq. 1	Eq. 2	Eq. 1	Eq. 2
CRA-1	40	75.3	0.69	V-C	125.0	54.5	52.5	76.5	74.5	0.73	0.76	0.98	1.01
CRB-1	45	77.3	0.50	V-C	89.8	40.6	37.9	64.6	62.0	1.11	1.19	1.20	1.25
CRC-1	25	53.2	0.30	V-C	61.2	27.2	26.2	49.1	48.1	0.92	0.95	1.08	1.11
1WCRA-1	55	96.4	0.65	V-C	116.0	55.0	53.2	73.5 / 101.7*	71.5 / 99.6*	1.00	1.03	1.31 / 0.95*	1.35 / 0.97*
1WCRB-1	40	91.6	0.59	V-C	90.5	40.4	37.7	64.5 / 90.4*	61.9 / 87.8*	0.99	1.06	1.42 / 1.01*	1.48 / 1.04*
1WCRC-1	30	64.6	0.53	V-C	65.5	28.0	27.0	50.8 / 68.4*	49.0 / 67.4*	1.07	1.11	1.27 / 0.95*	1.32 / 0.96*
1WCA-1	50	99.0	0.61	V-C	120.0	55.1	51.9	77.4 / 111.1*	75.0 / 109.0*	0.91	0.97	1.28 / 0.89*	1.32 / 0.91*
1WCB-1	50	90.8	0.63	V-C	99.1	43.6	40.8	68.0 / 93.6*	65.0 / 91.0*	1.15	1.23	1.34 / 0.97*	1.40 / 1.00*
1WCC-1	30	64.0	0.51	V-C	60.8	27.6	26.5	49.7 / 68.2*	48.5 / 67.0*	1.09	1.13	1.29 / 0.94*	1.32 / 0.97*
2WCA-1	50	108.5	0.65	V-C	123.8	55.9	53.7	78.1 / 112.0*	75.9 / 110.0*	0.90	0.93	1.38 / 0.97*	1.43 / 0.99*
3WCA-1	50	93.1	0.43	V-C	120.0	55.9	53.8	77.9 / 96.2*	75.9 / 94.4*	0.90	0.93	1.20 / 0.97*	1.23 / 0.99*
ROA-1	40	72.7	0.45	D-T	111.0	53.4	56.0	53.4	56.0	0.75	0.72	1.36	1.30

(1) Applied loads - exclusive of weight of specimen. P_{cr} - cracking load; P_u - ultimate load.
(2) Critical section at midspan, adjustment made for weight of specimen. P_f - calculated.
(3) Critical section at midspan, requires no adjustment for weight of specimen. P_v - shear capacity.
* Values include shear capacity of additional stirrups at cut-off section.

Equation 1: $P_{cr} = 2bd \left(1.9\sqrt{f'_c} + 2500 \frac{pVd}{M}\right)$; $P_v = P_{cr} + 2rf_y bd$

Equation 2: $P_{cr} = 2bd \left(2\sqrt{f'_c}\right)$; $P_v = P_{cr} + 2rf_y bd$

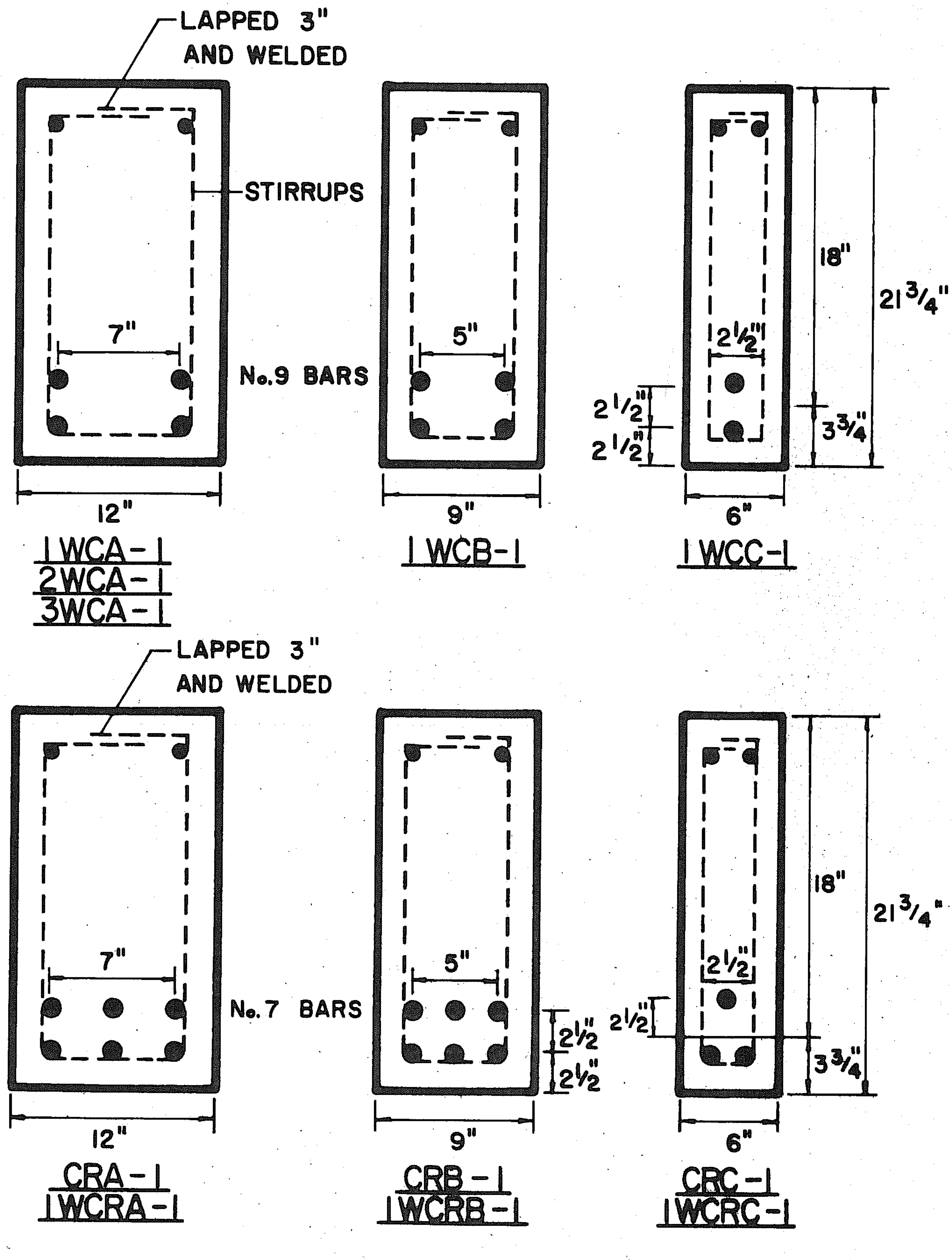
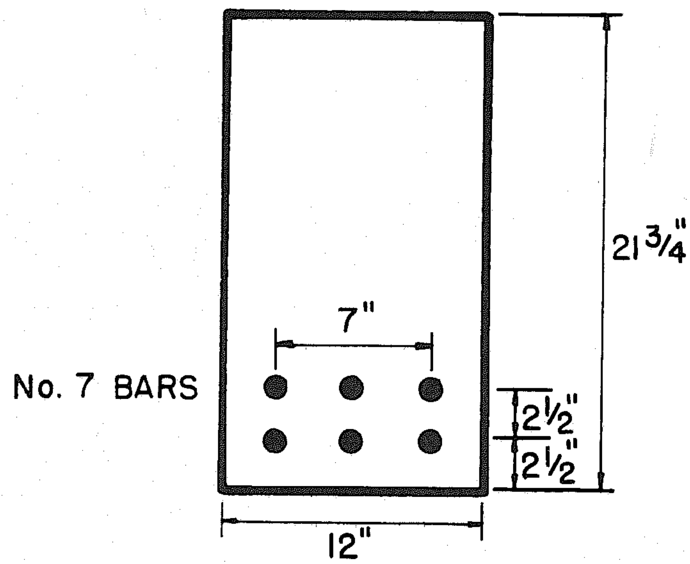


FIG. 1-A BEAM CROSS-SECTIONS WITH WEB REINFORCEMENT

(1) ALL DIMENSIONS ARE NOMINAL; SEE TABLE FOR MEASURED DISTANCES.
(2) TOP BARS ARE No. 4; STIRRUPS ARE No. 2
(3) ONE-HALF OF TENSILE REINFORCEMENT CUTOFF 24" FROM EACH SUPPORT.



ROA - 1

FIG. 1-B BEAM CROSS-SECTION WITHOUT WEB REINFORCEMENT

(1) ALL DIMENSIONS ARE NOMINAL; SEE TABLE FOR MEASURED DISTANCES

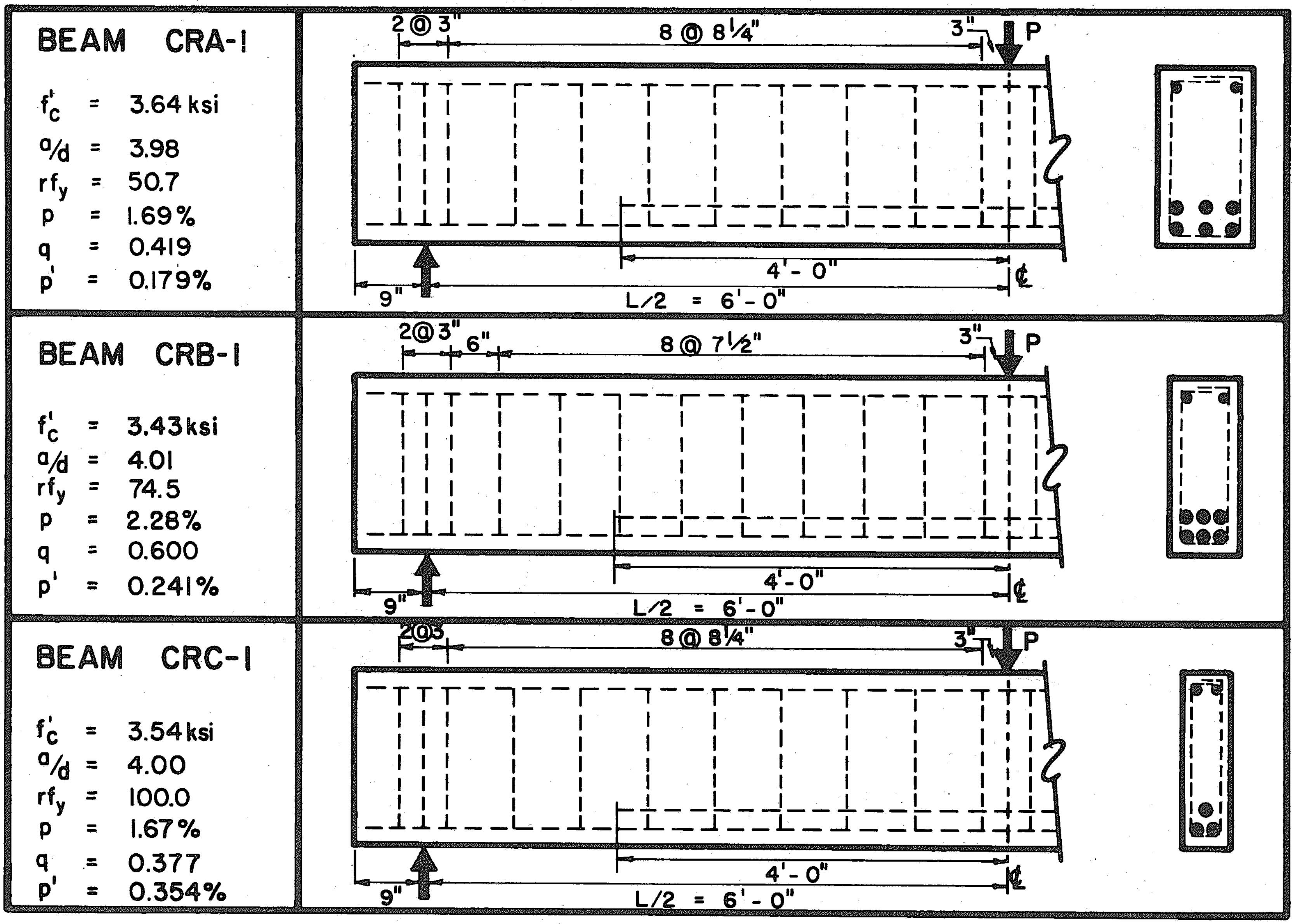


FIG. 2A SERIES CR BEAM ELEVATIONS

BEAM IWCRA-1

f'_c = 3.82 ksi
a/d = 4.01
rf_y = 50.8
p = 1.71%
q = 0.404
p' = 0.179%

BEAM IWCRB-1

f'_c = 3.36 ksi
a/d = 3.99
rf_y = 74.5
p = 2.26%
q = 0.600
p' = 0.231%

BEAM IWCRC-1

f'_c = 3.88 ksi
a/d = 3.98
rf_y = 101.5
p = 1.69%
q = 0.346
p' = 0.359%

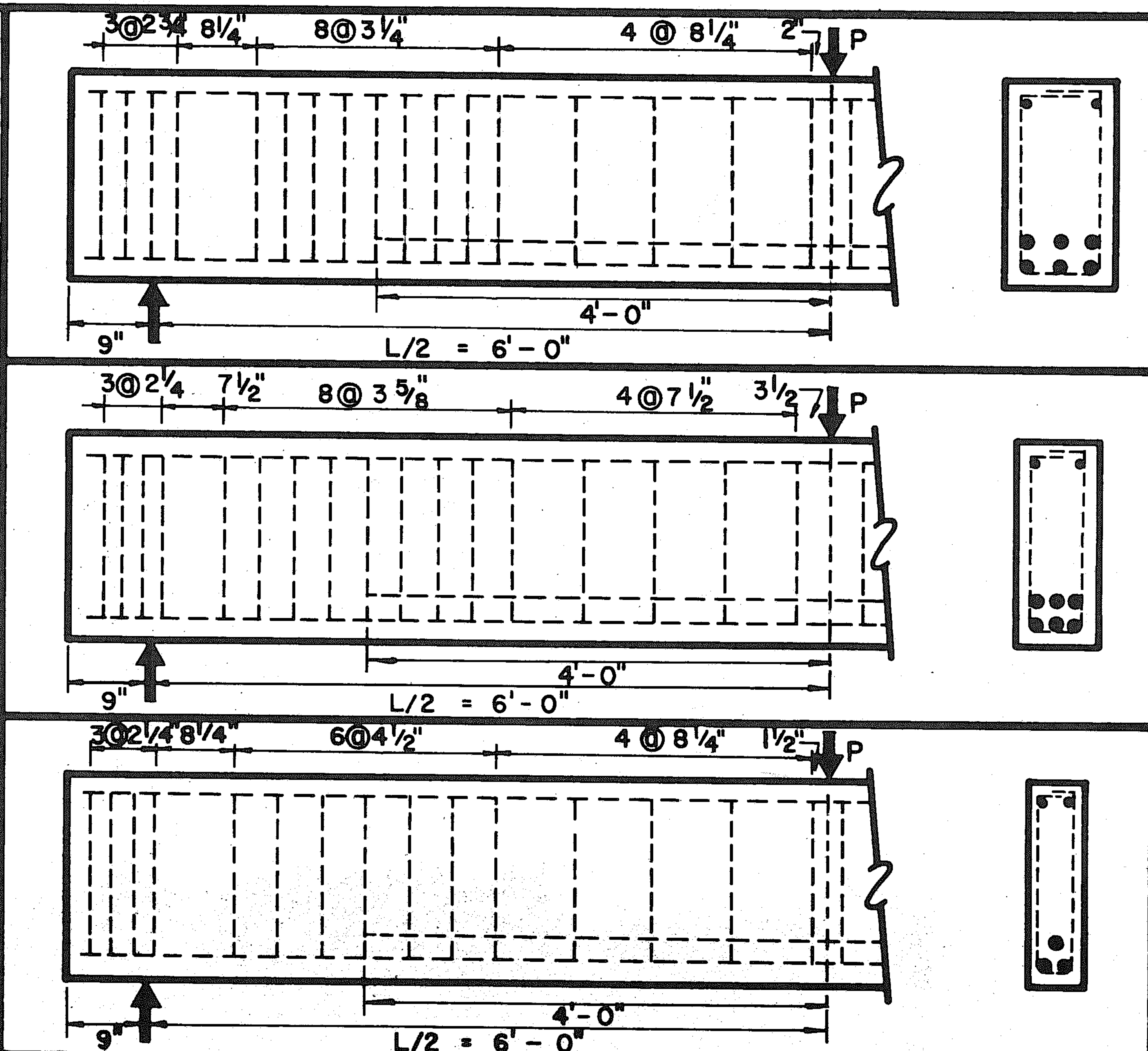


FIG. 2B SERIES IWCR BEAM ELEVATION

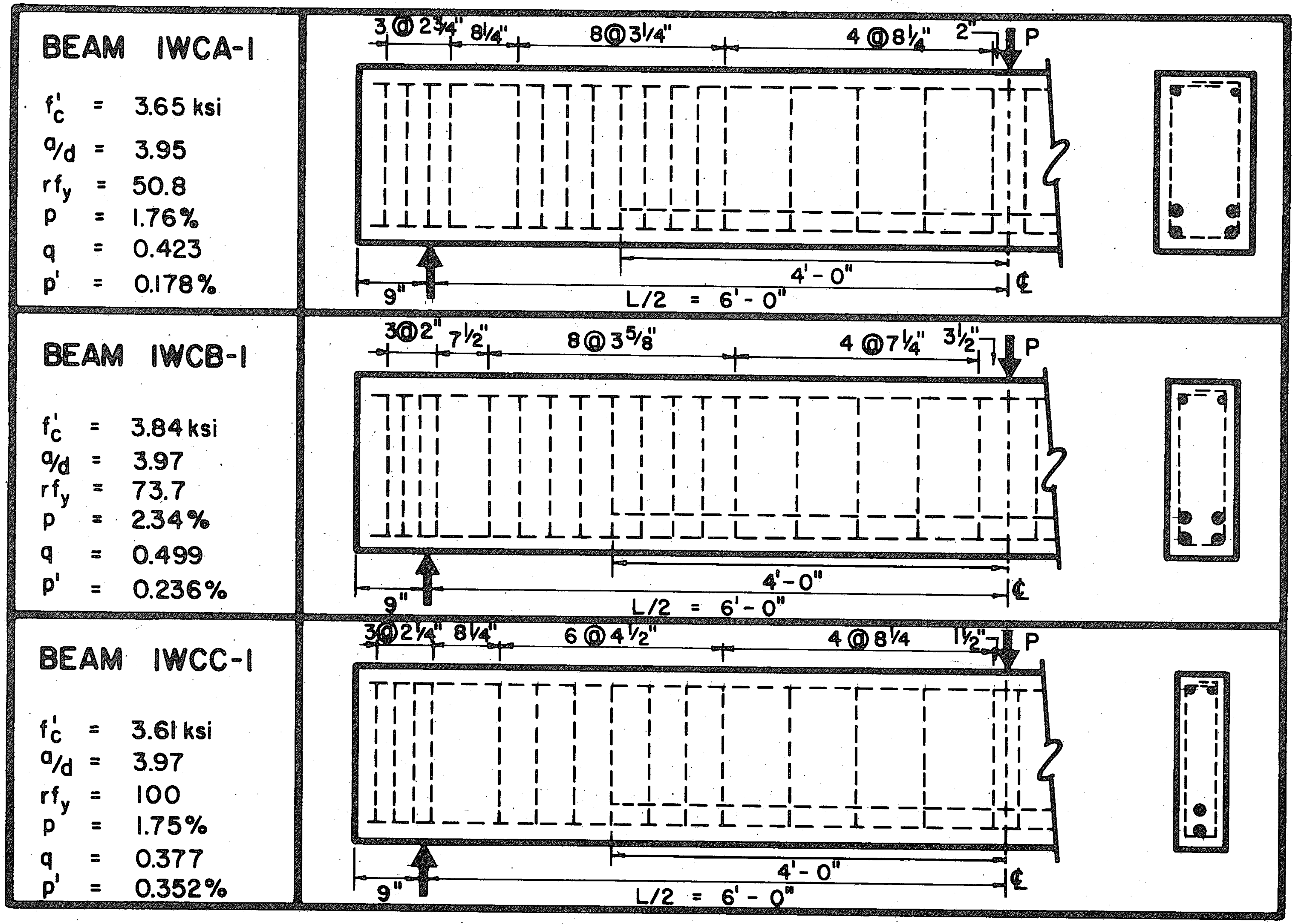


FIG. 2C SERIES IWC BEAM ELEVATIONS

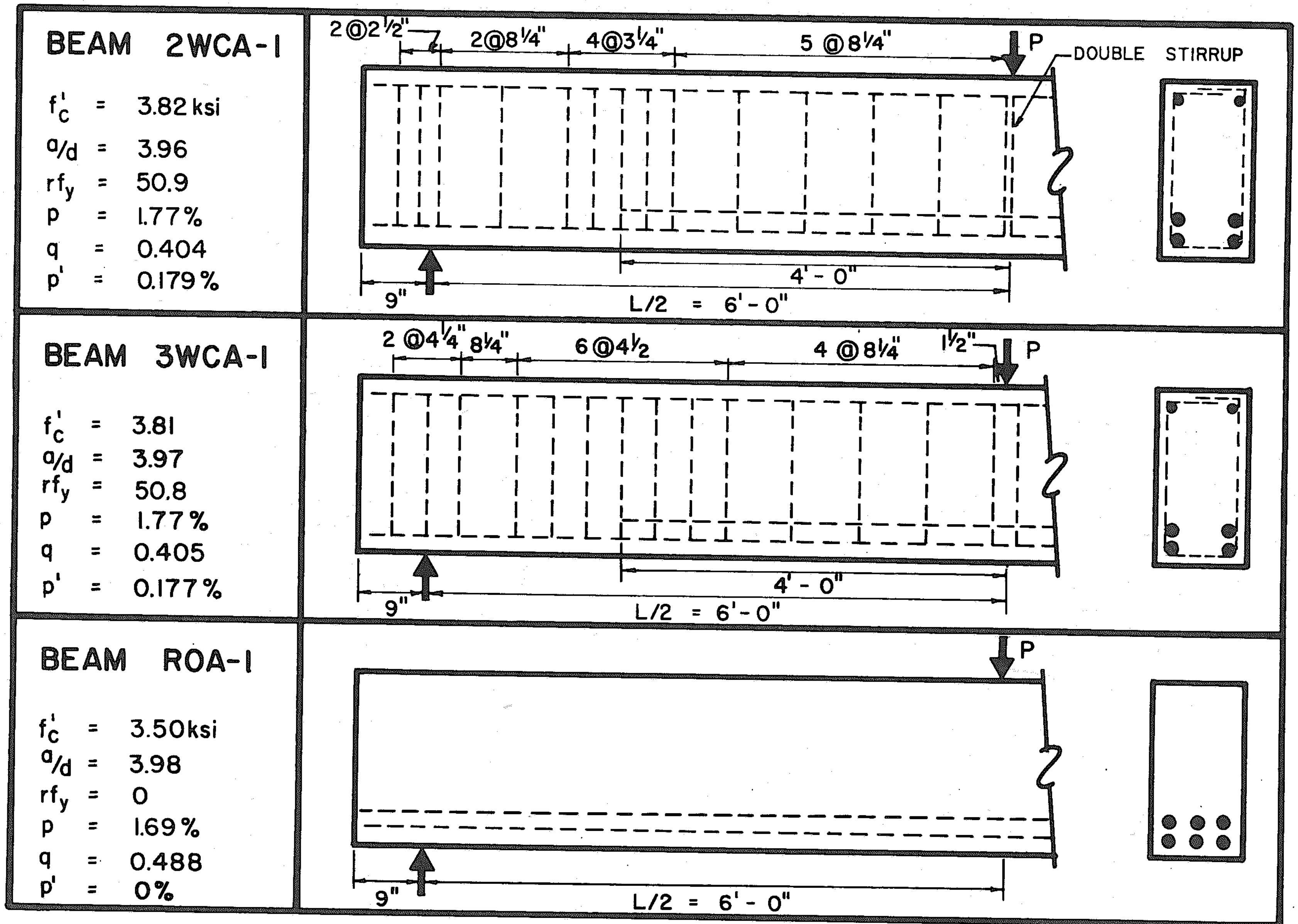


FIG. 2D BEAMS 2WCA-1, 3WCA-1, ROA-1 ELEVATIONS

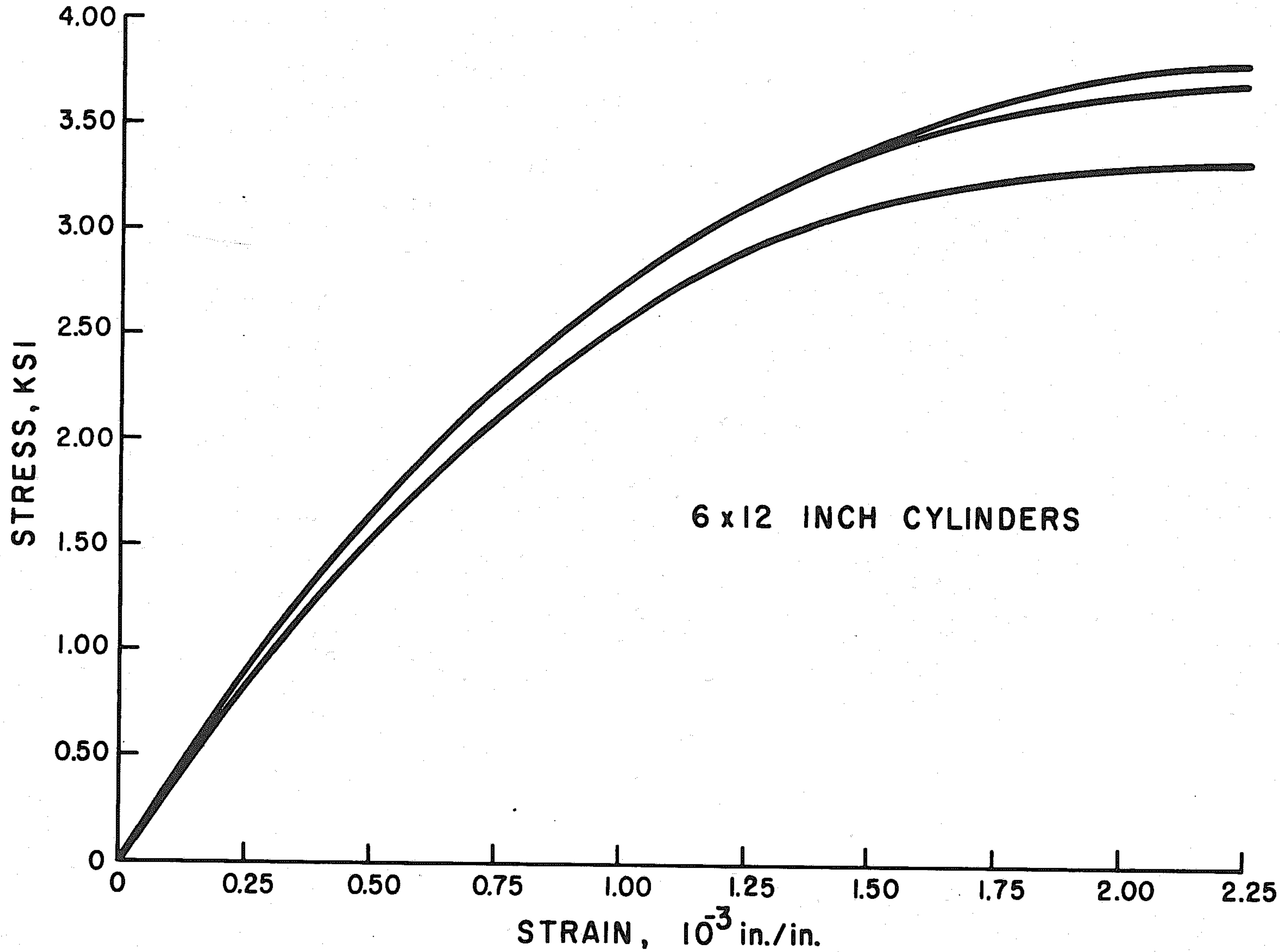


FIG. 3 TYPICAL STRESS-STRAIN RELATIONSHIPS FOR CONCRETE BEAM IWCC-I

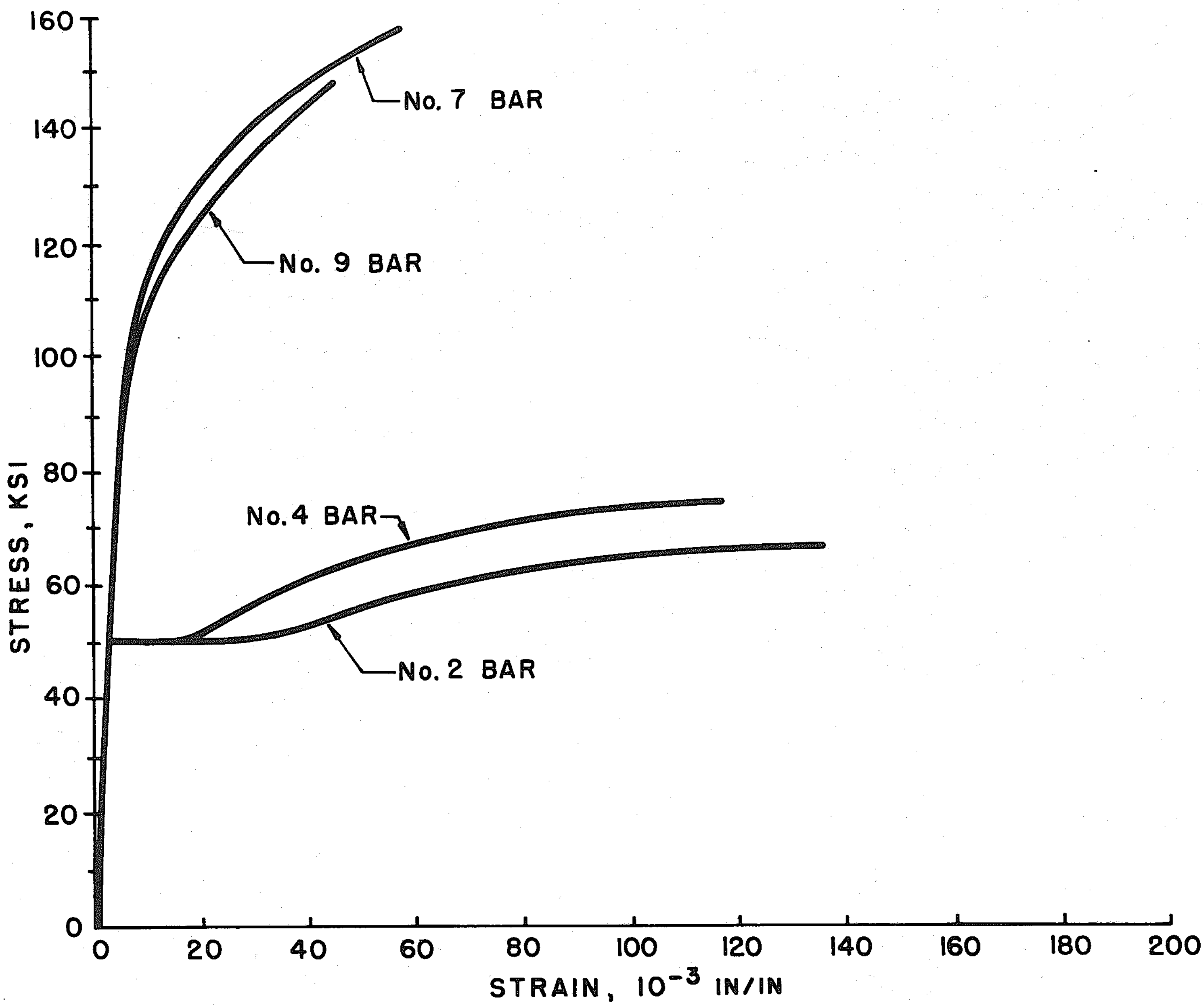


FIG. 4 TYPICAL STRESS-STRAIN DIAGRAMS FOR STEEL REINFORCEMENT

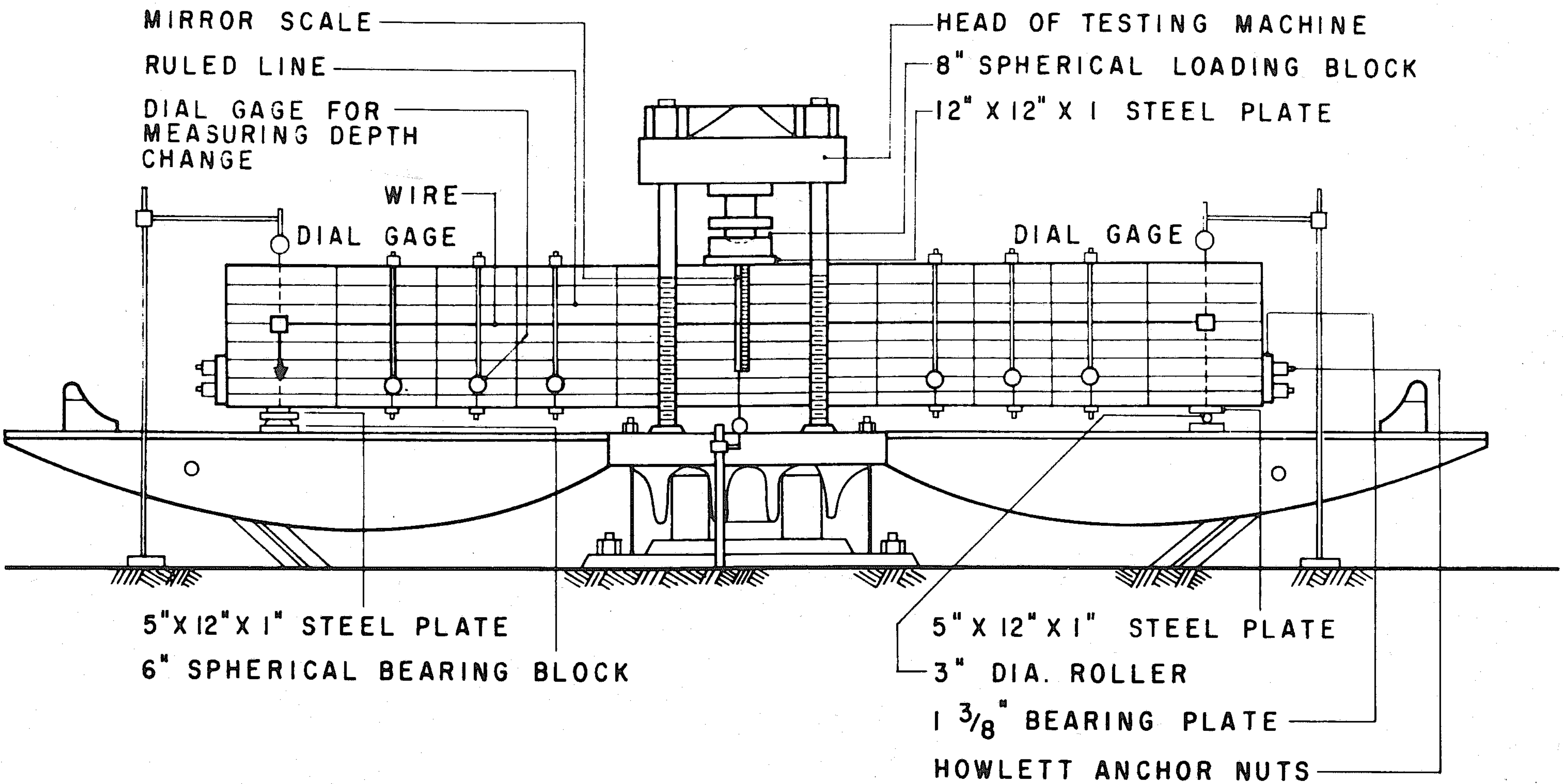


FIG. 5 LOADING ARRANGEMENT AND INSTRUMENTATION

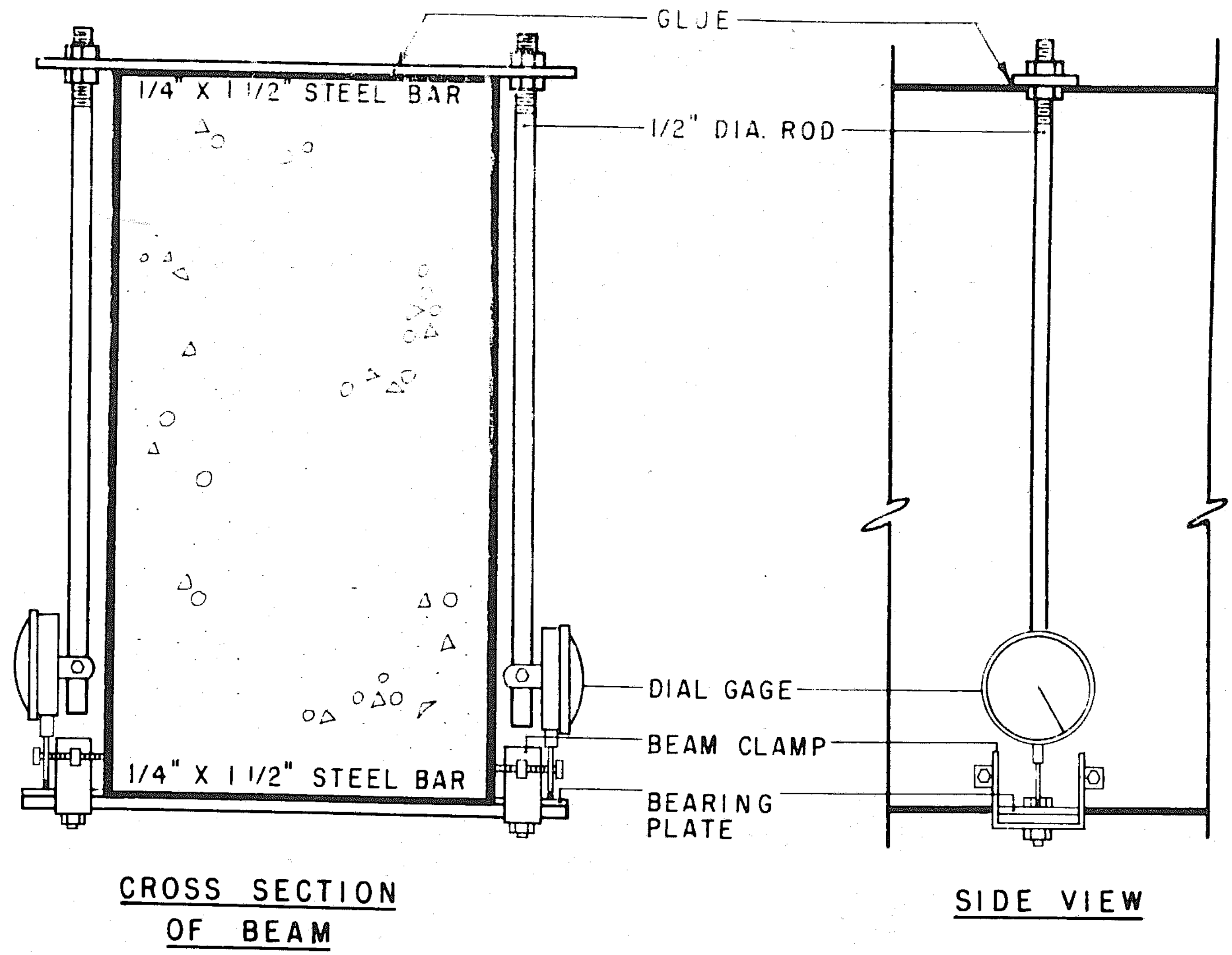


FIG. 6 DETAILS OF DIAL GAGE FOR MEASURING DEPTH CHANGE.

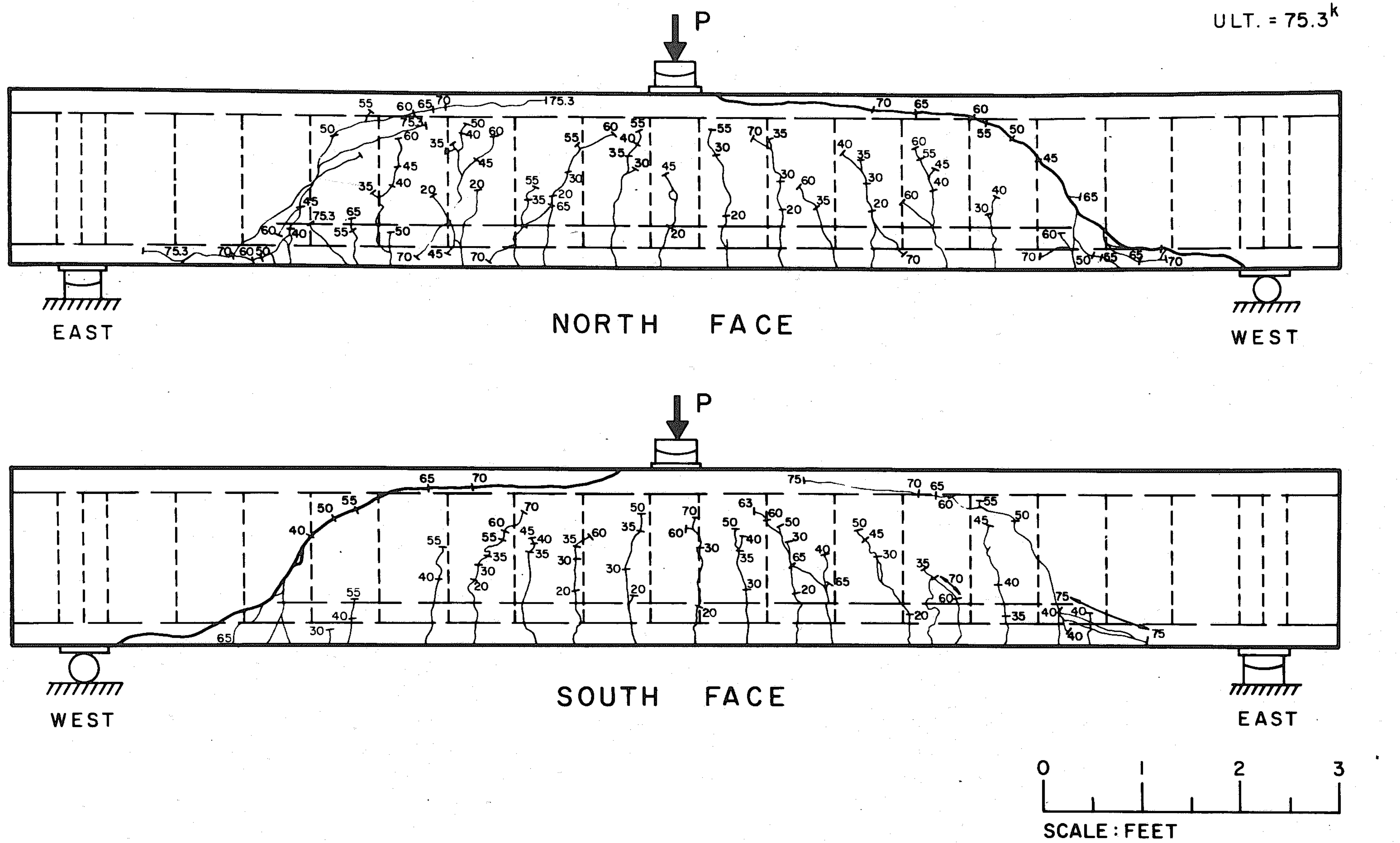


FIG. 7A BEAM CRA-I CRACK PATTERN

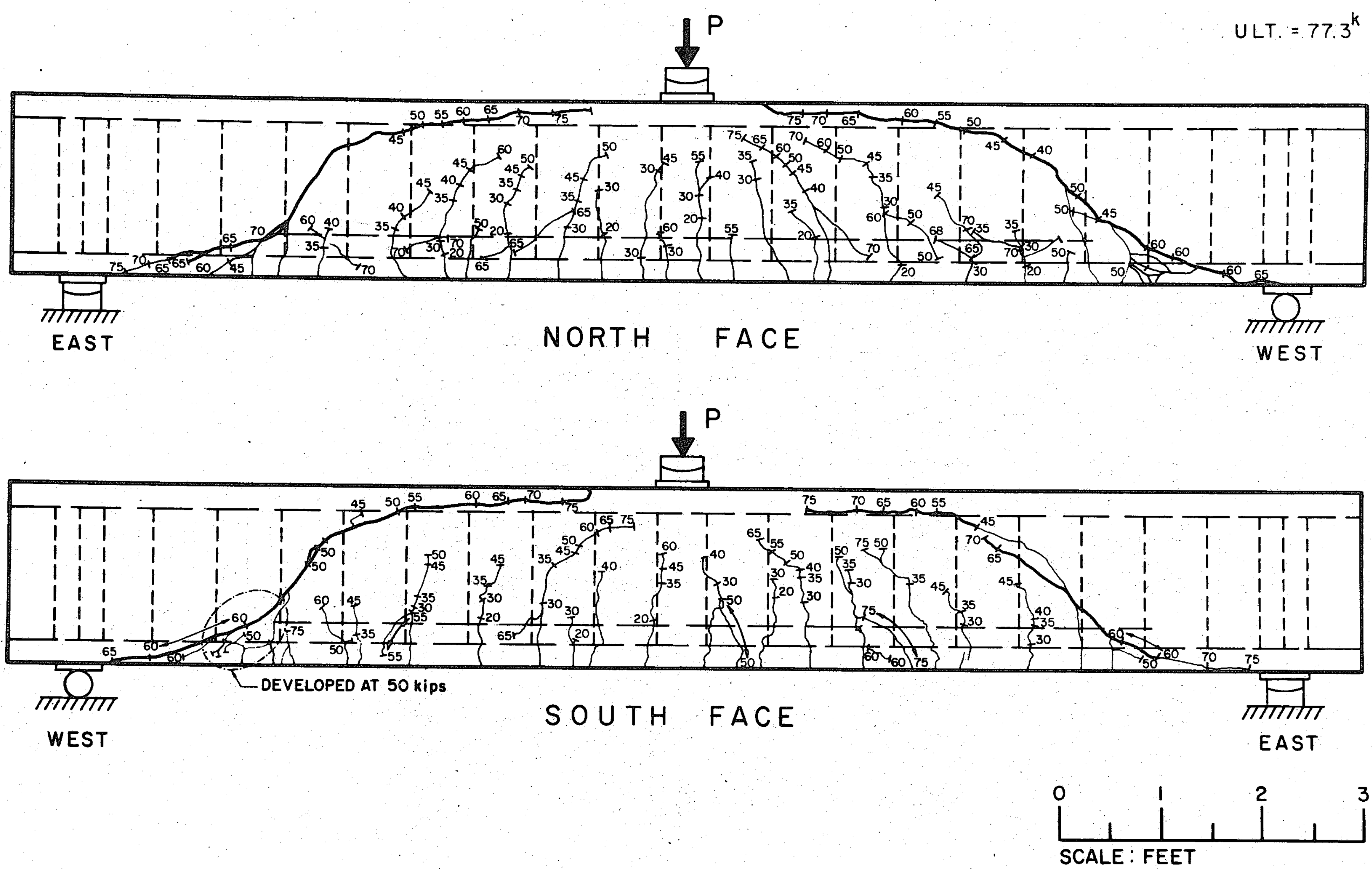


FIG. 7B BEAM CRB-I CRACK PATTERN

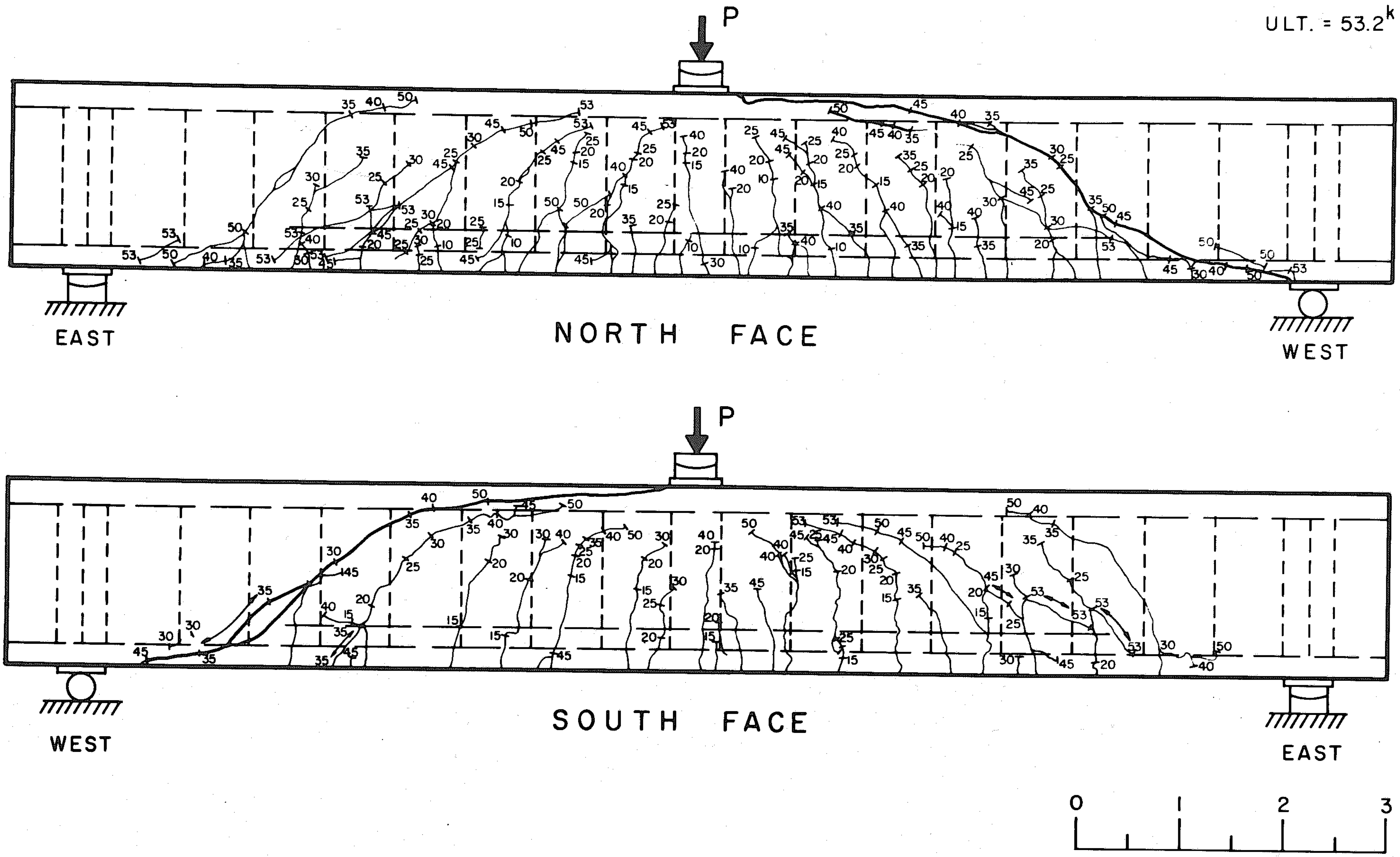


FIG. 7C BEAM CRC-1 CRACK PATTERN

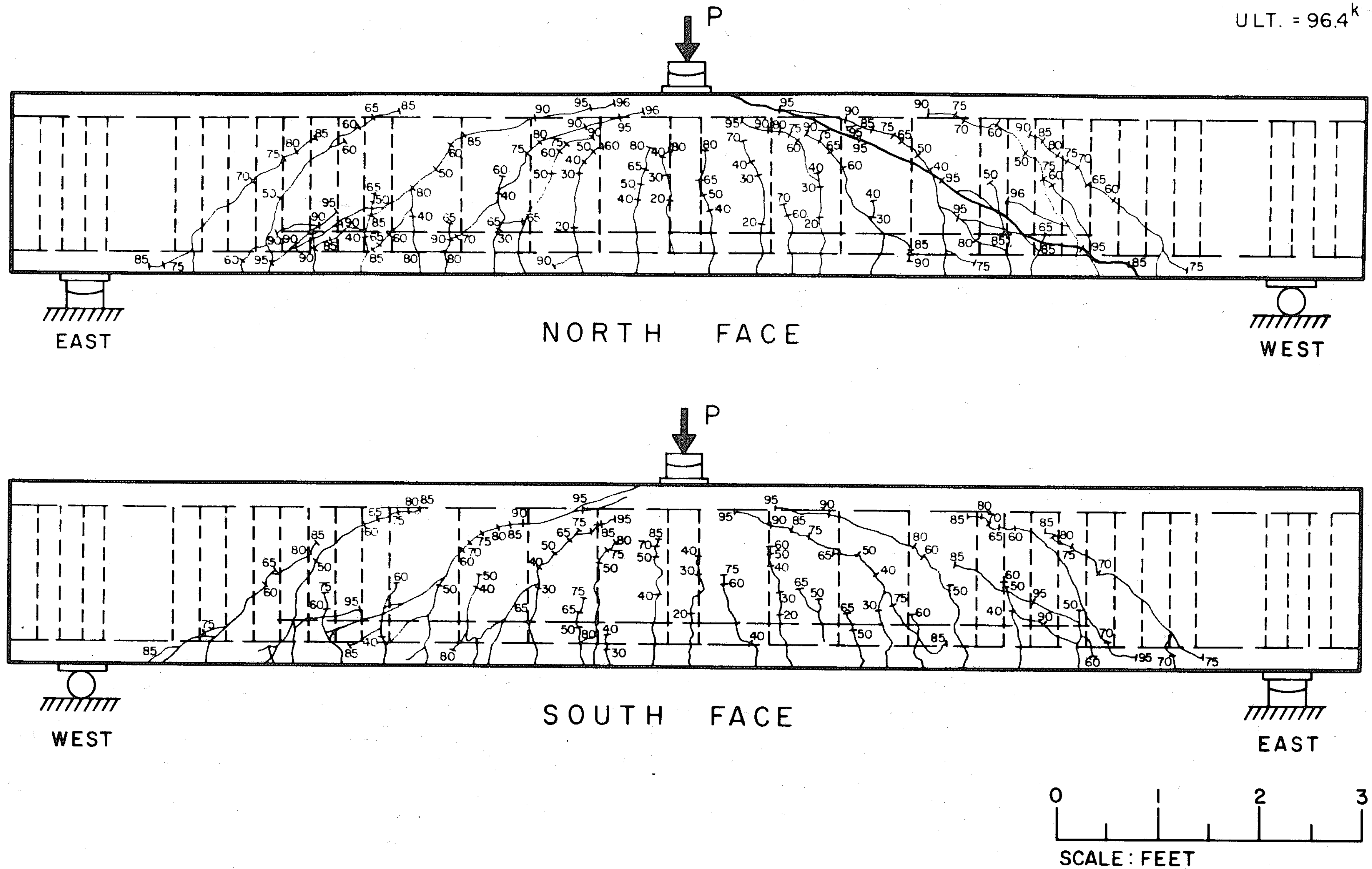


FIG. 7D BEAM IWCRA-I CRACK PATTERN

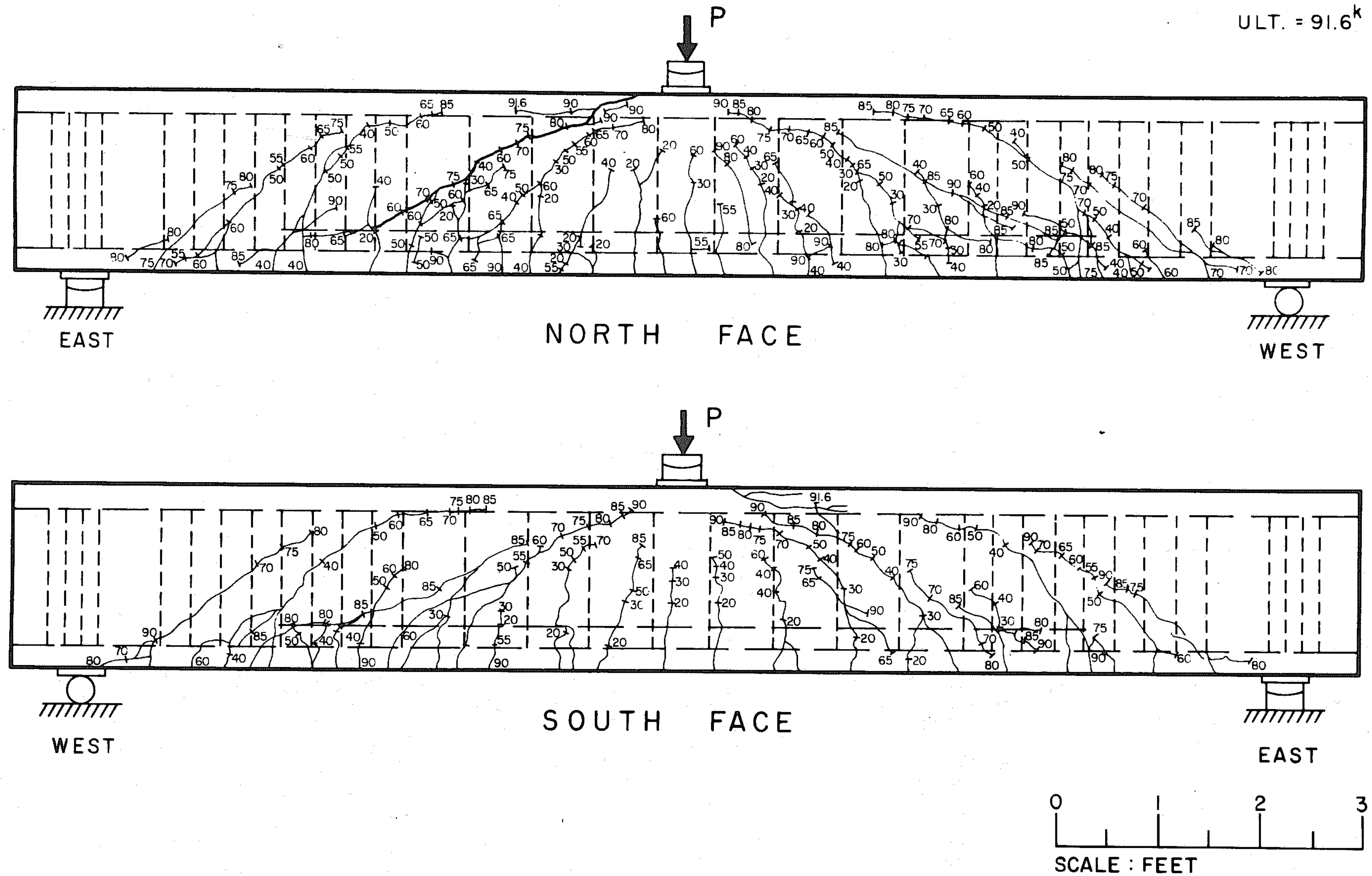


FIG. 7E BEAM I WCRB-I CRACK PATTERN

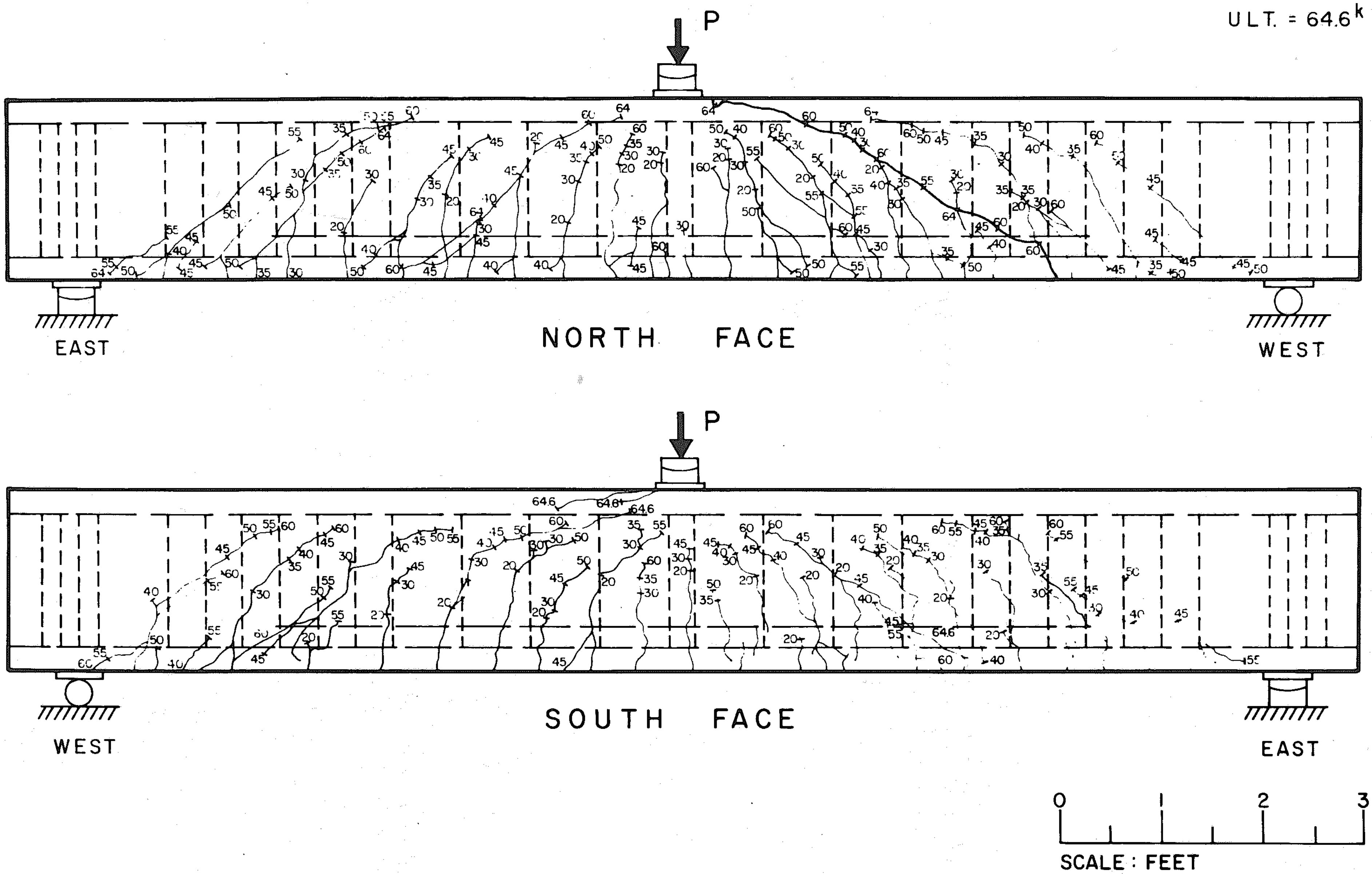


FIG. 7F BEAM I WCRC-I CRACK PATTERN

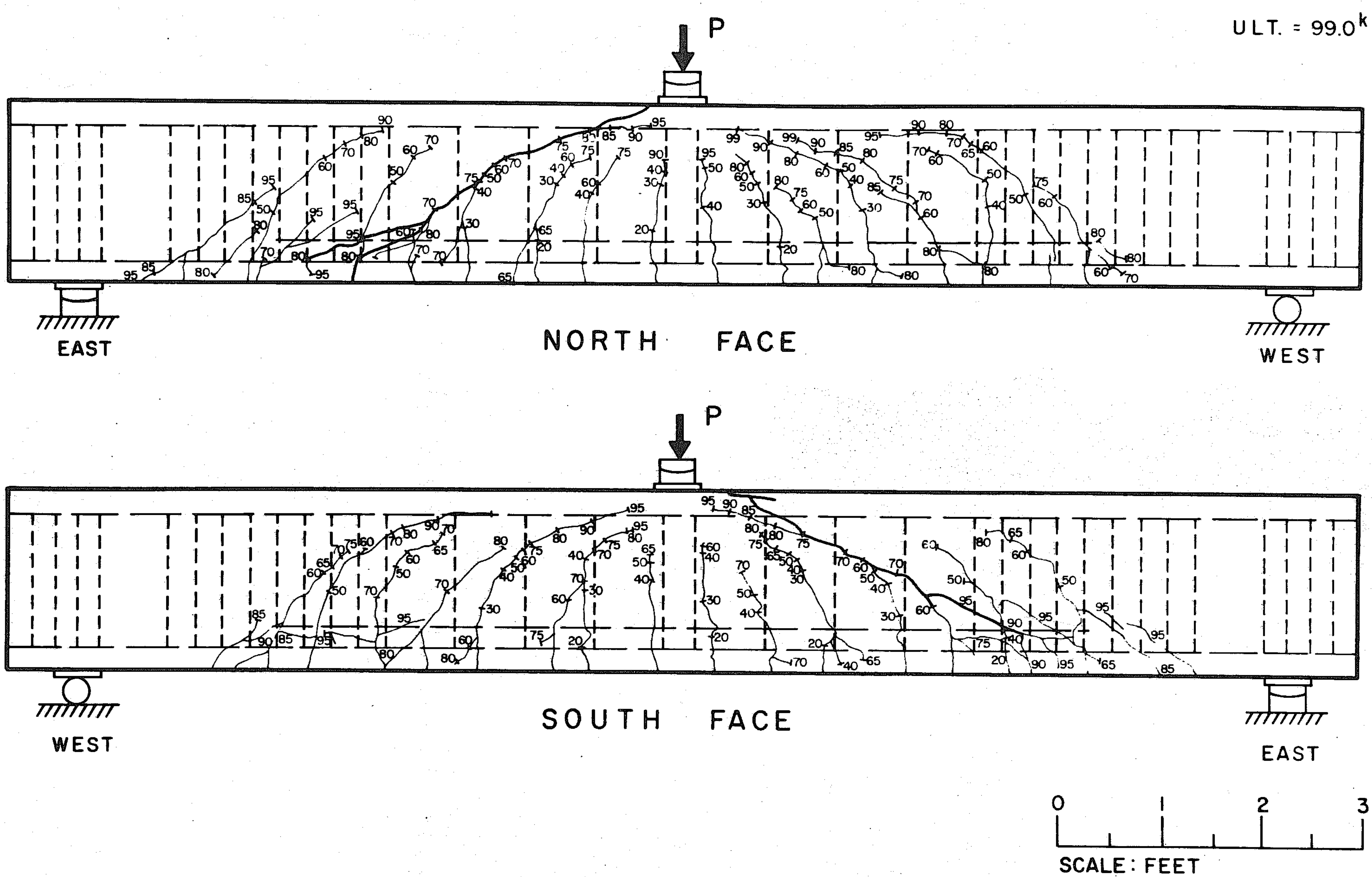


FIG. 7G BEAM IWCA-1 CRACK PATTERN

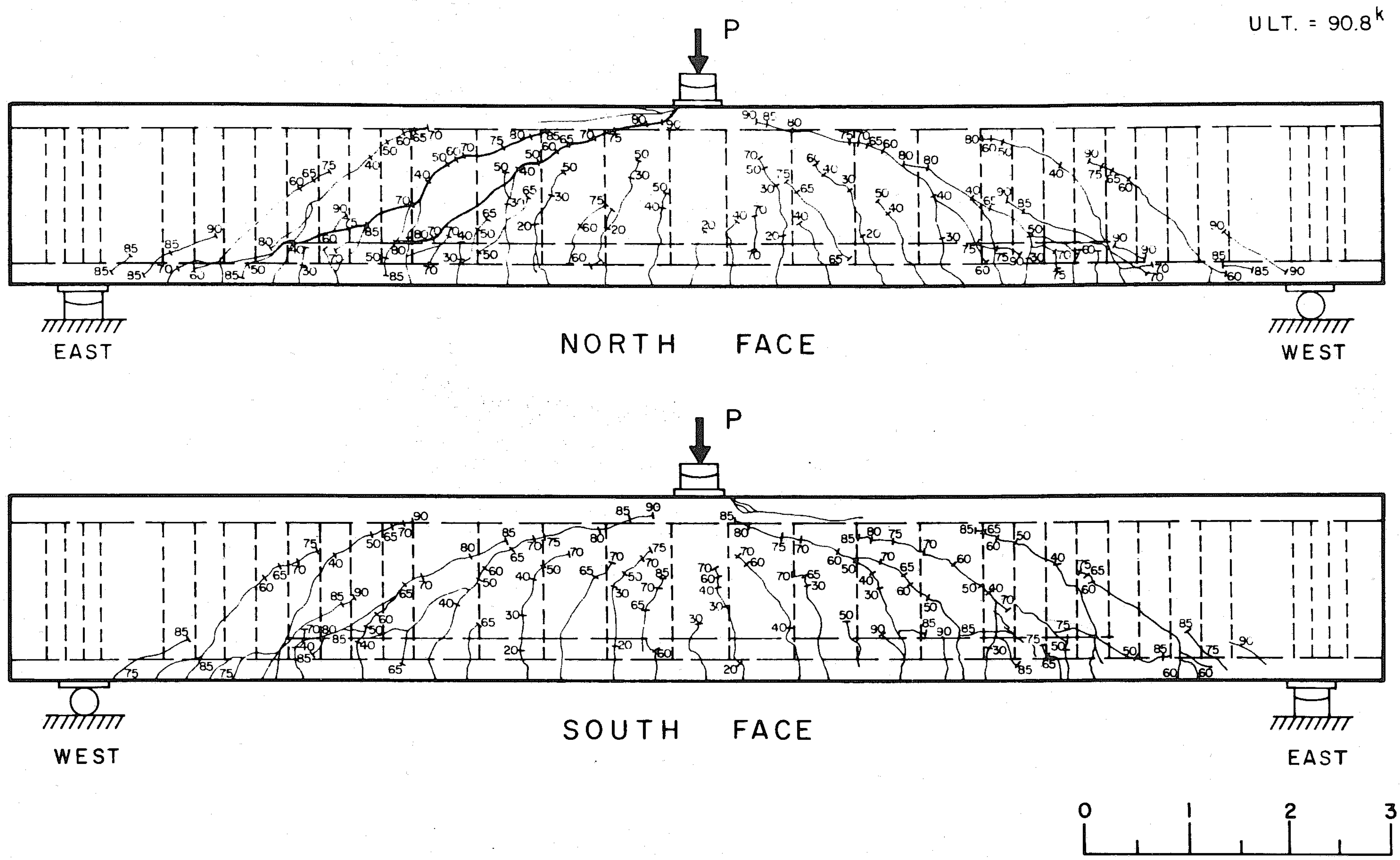


FIG. 7H BEAM I WCB-I CRACK PATTERN

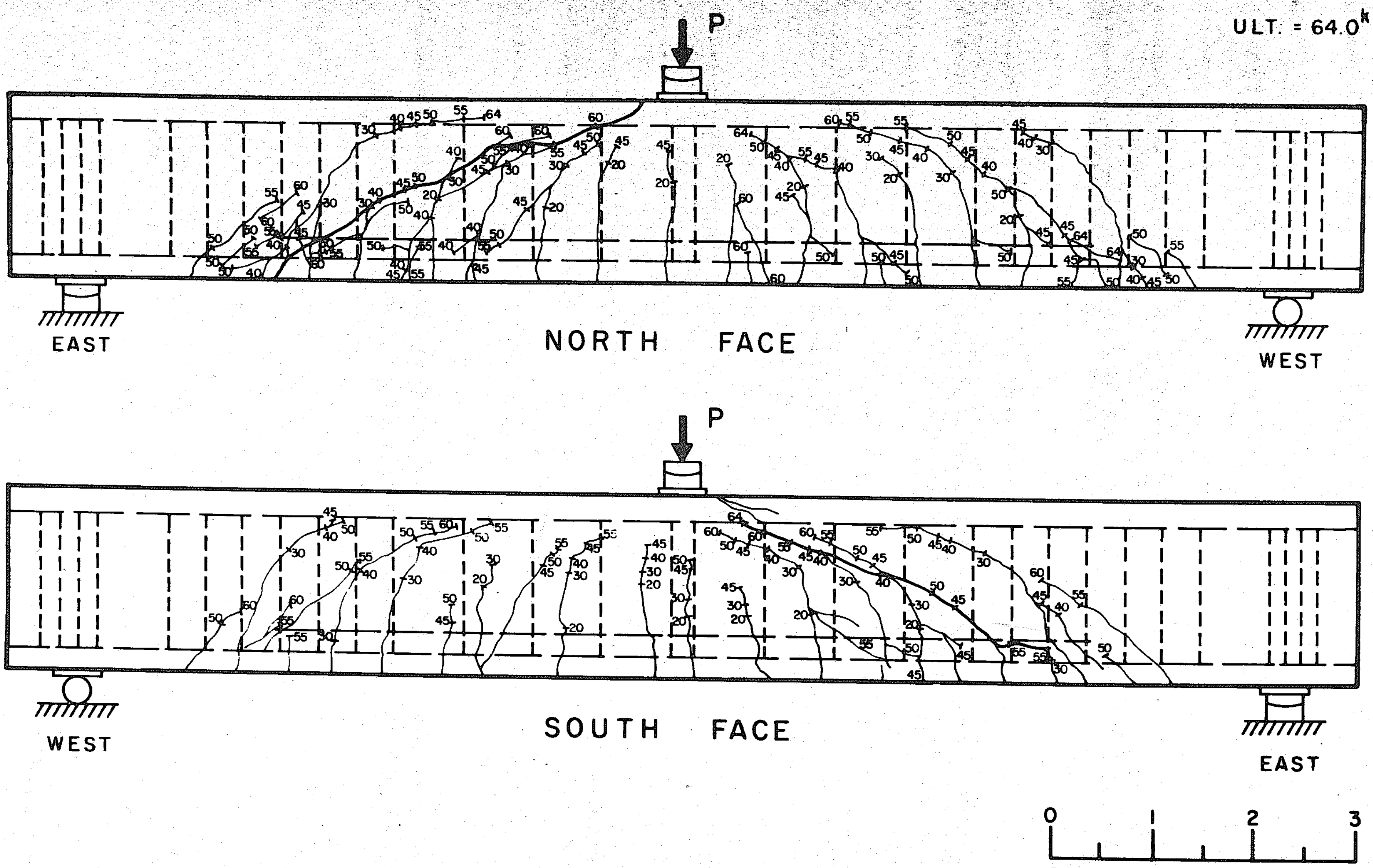


FIG. 71 BEAM IWCC-I CRACK PATTERN

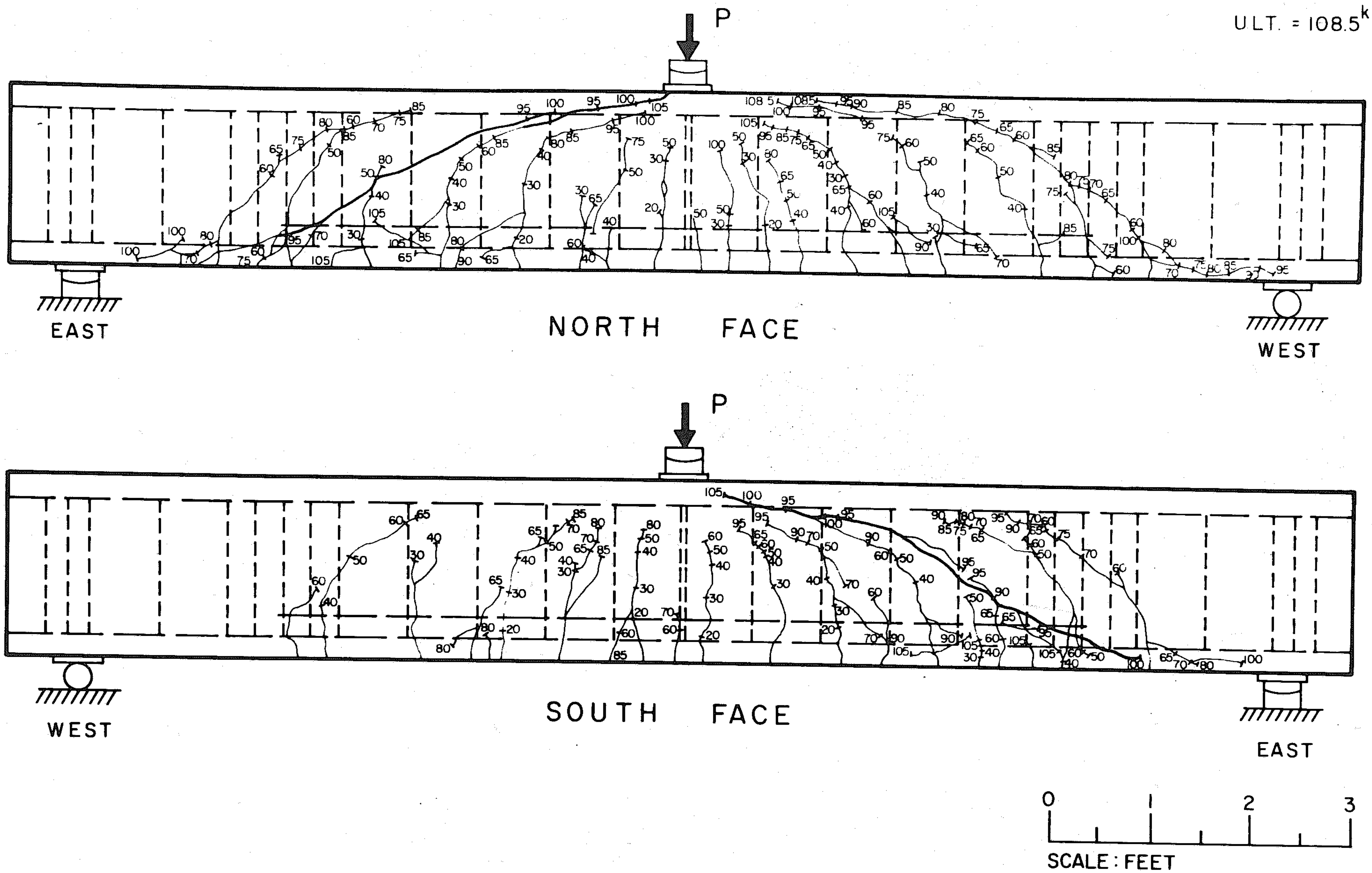


FIG. 7J BEAM 2WCA-1 CRACK PATTERN

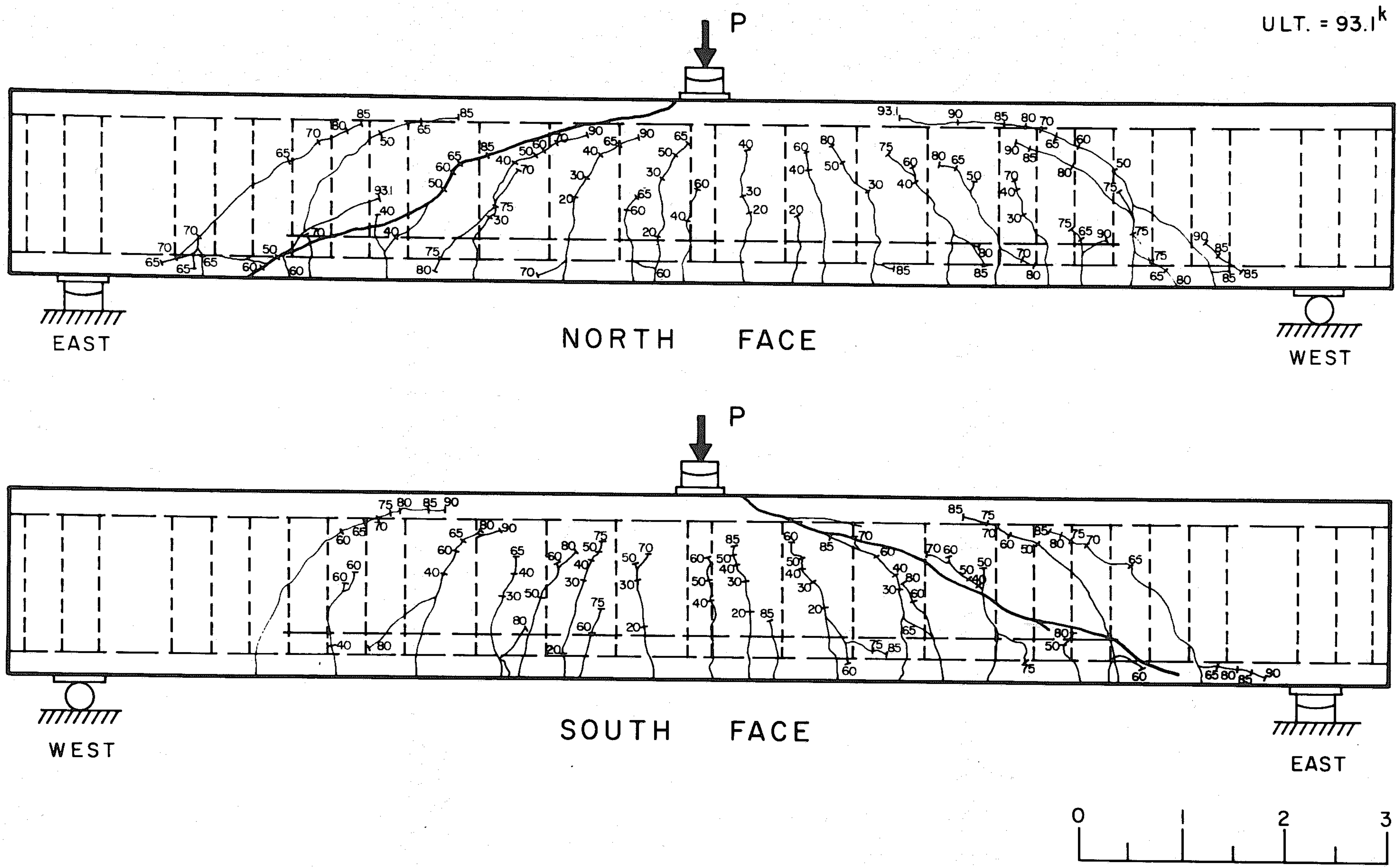


FIG. 7K BEAM 3WCA-1 CRACK PATTERN

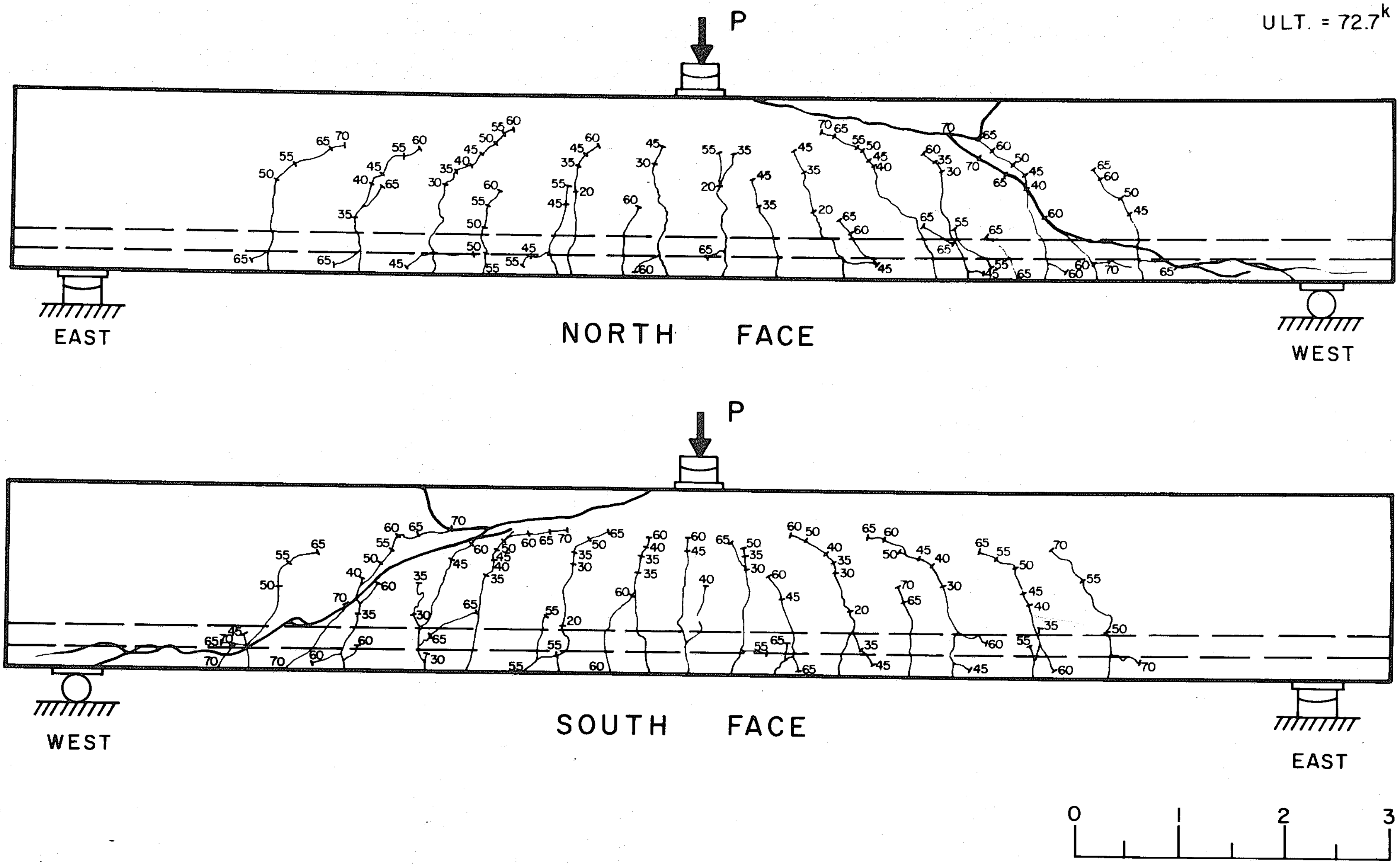


FIG. 7L BEAM ROA-1 CRACK PATTERN

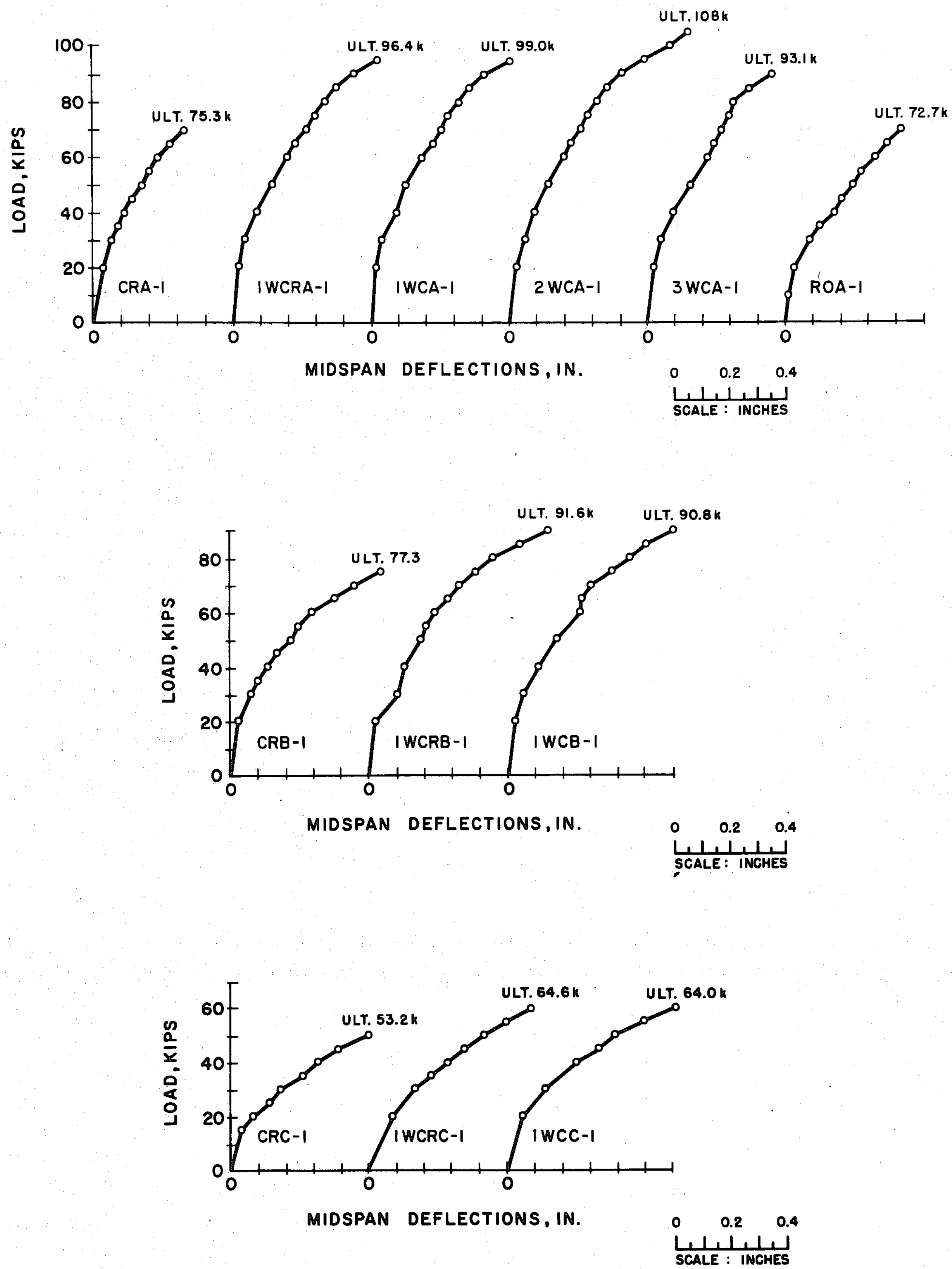


FIG 8. LOAD DEFLECTION CURVES

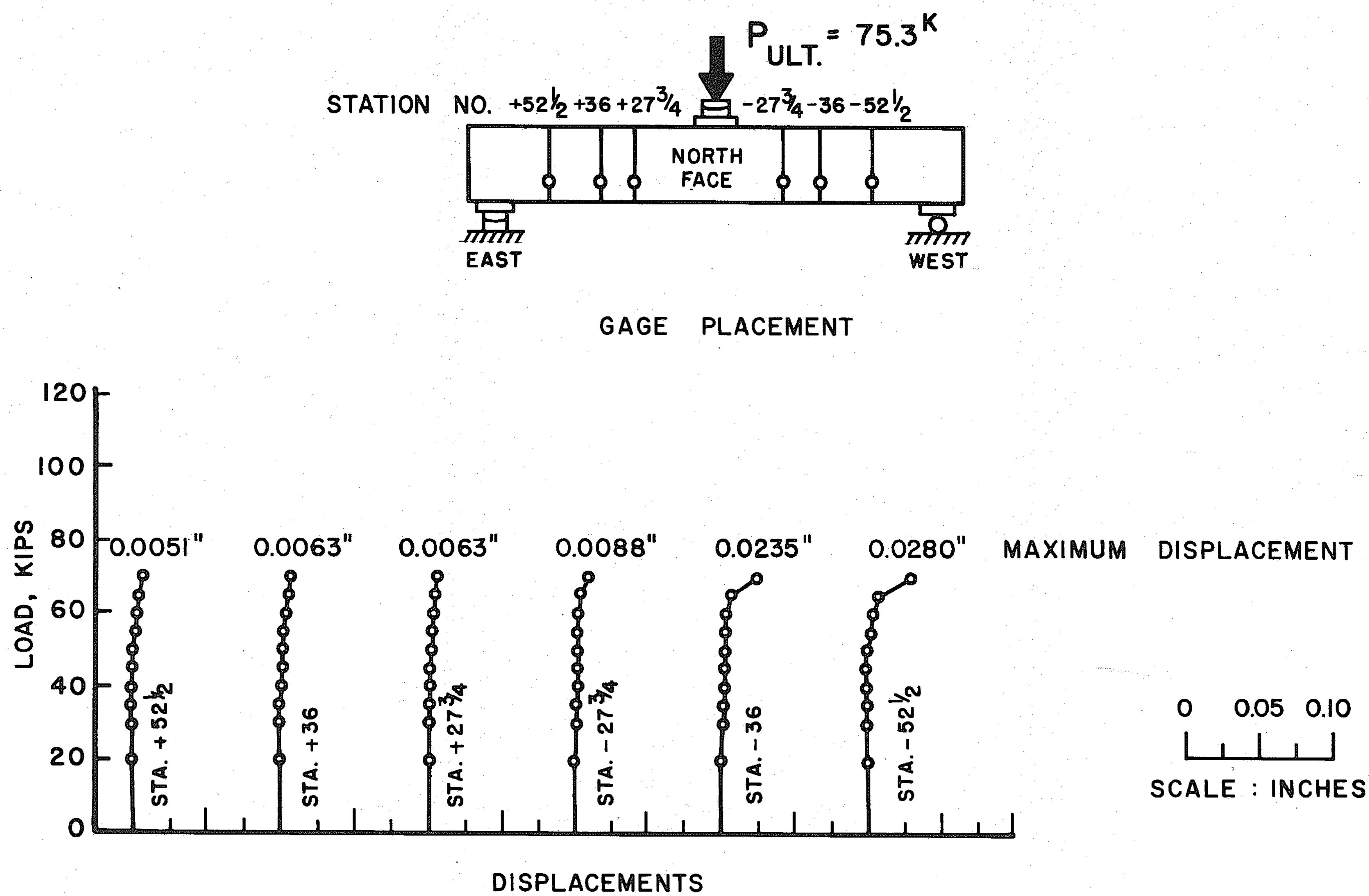


FIG. 9-A YOKE DATA, BEAM CRA-I

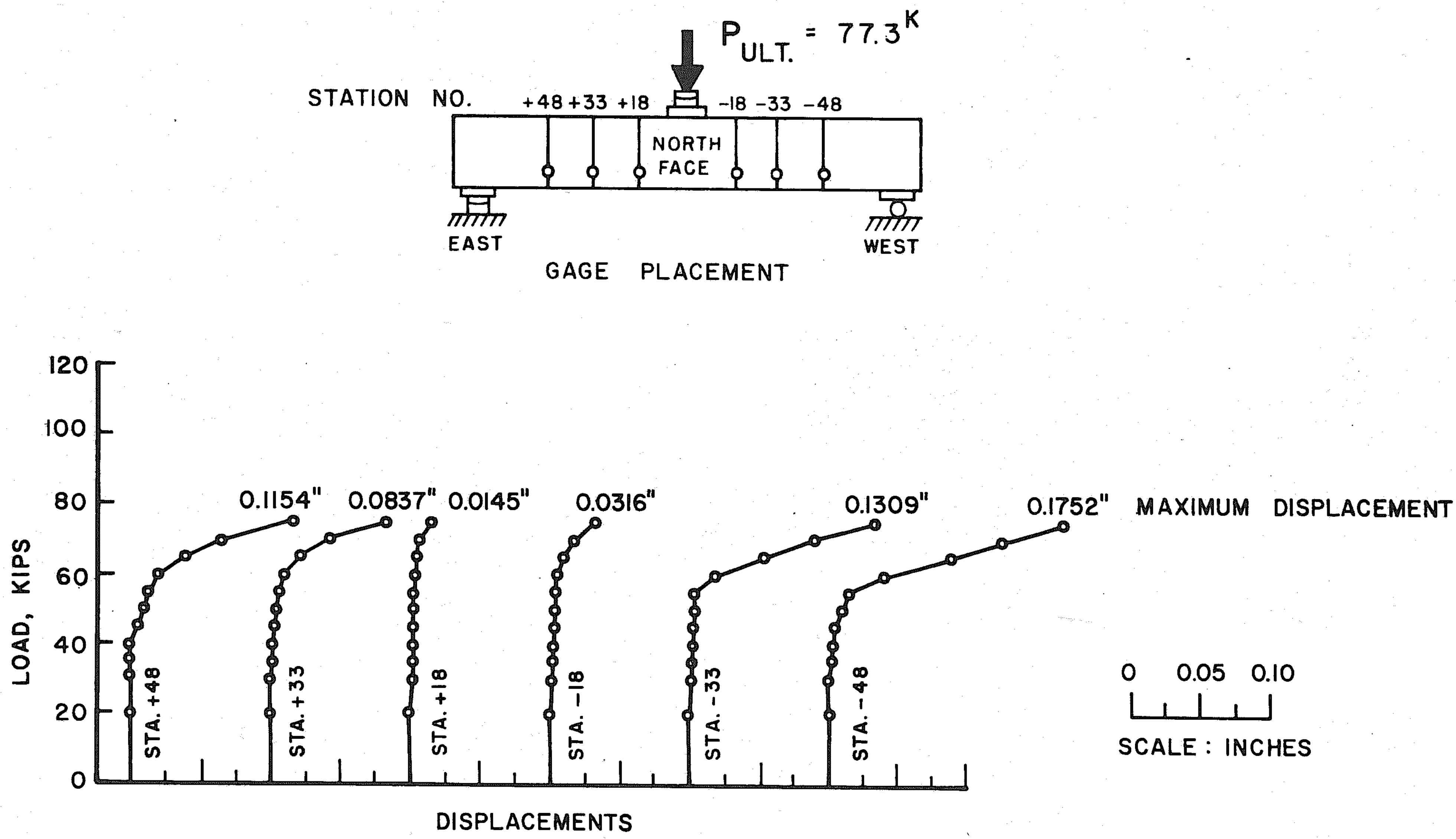


FIG. 9-B YOKE DATA, BEAM CRB-1

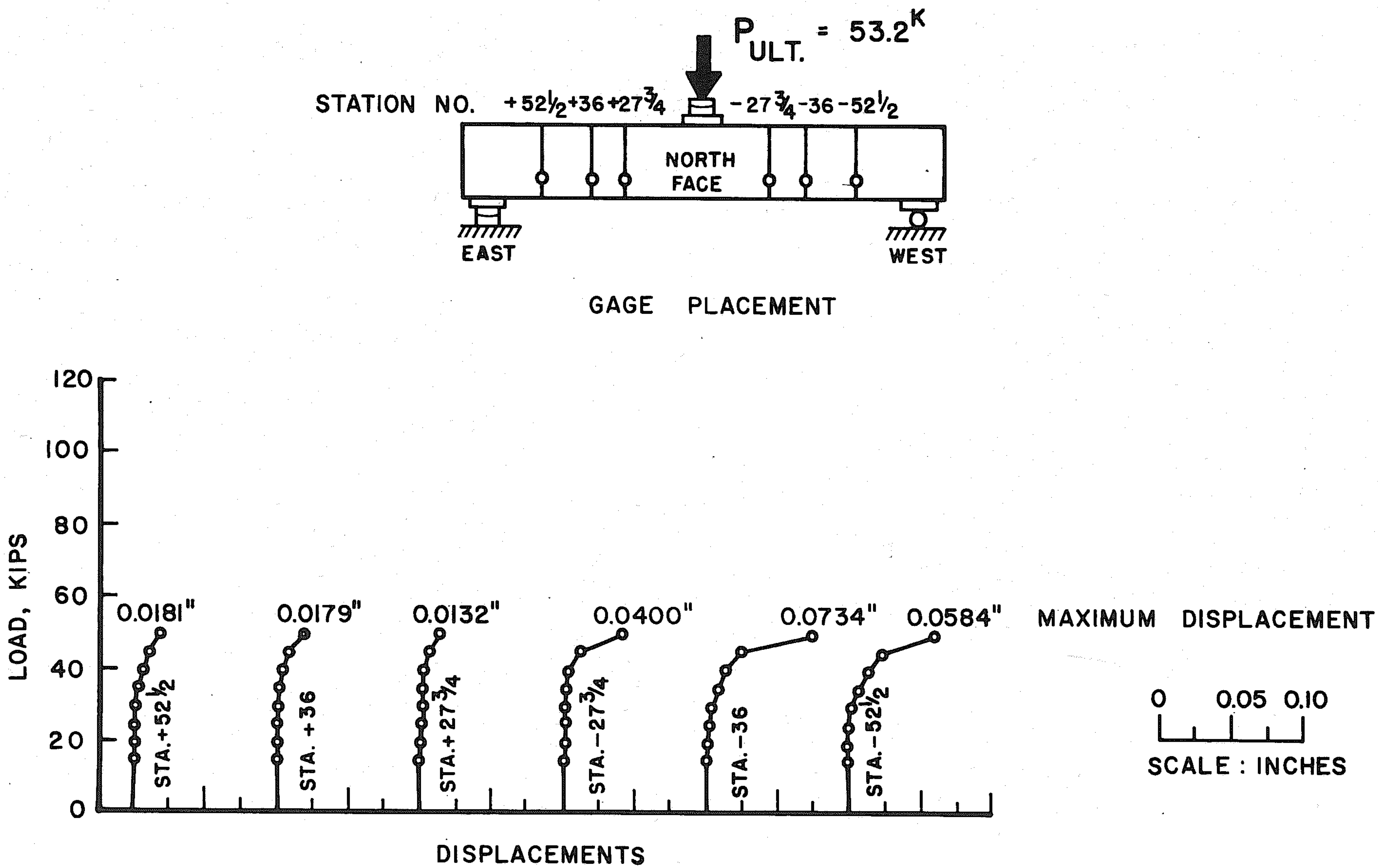


FIG. 9-C YOKE DATA, BEAM CRC-1

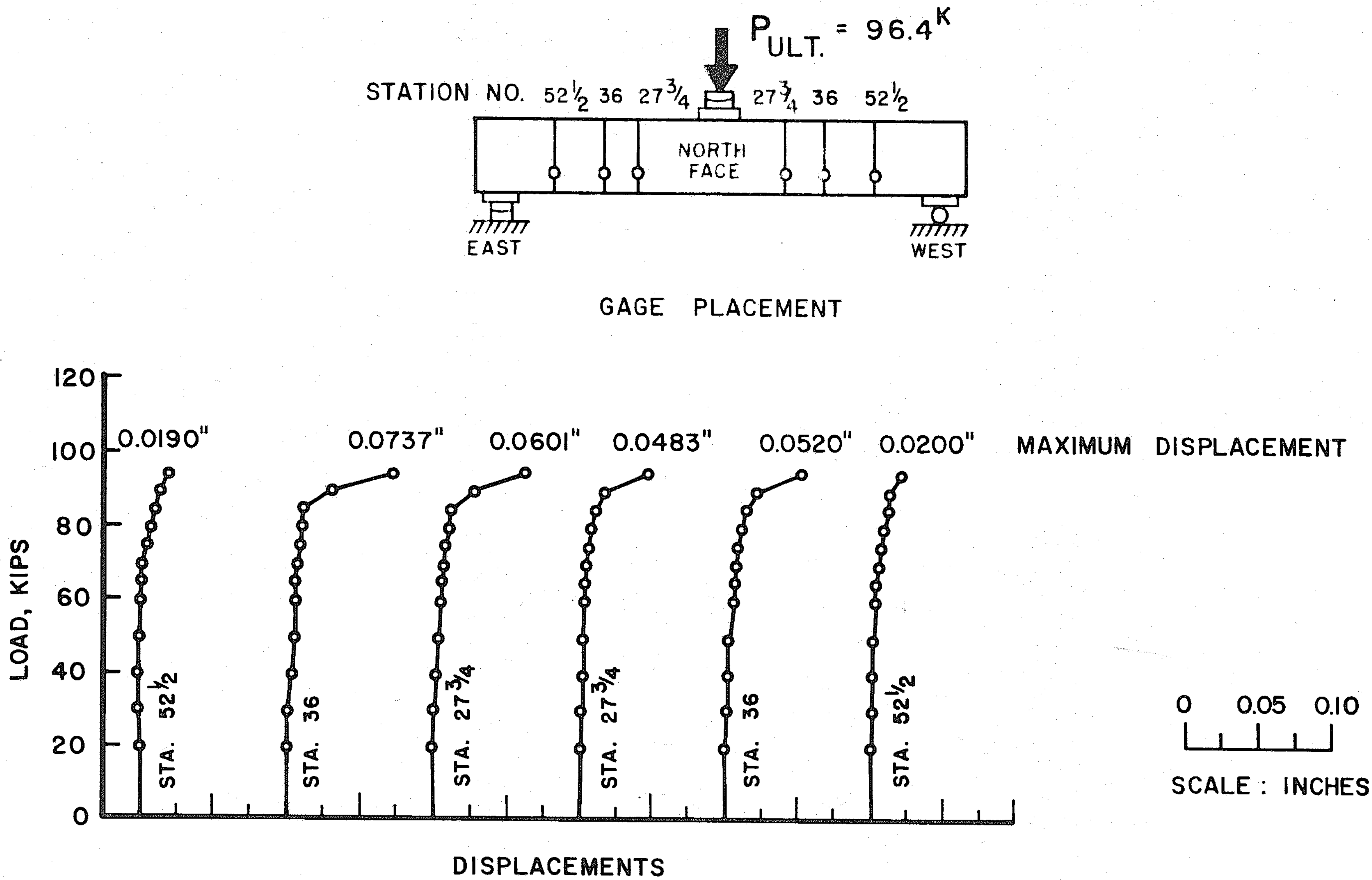


FIG. 9-D YOKE DATA, BEAM IWCRA-I

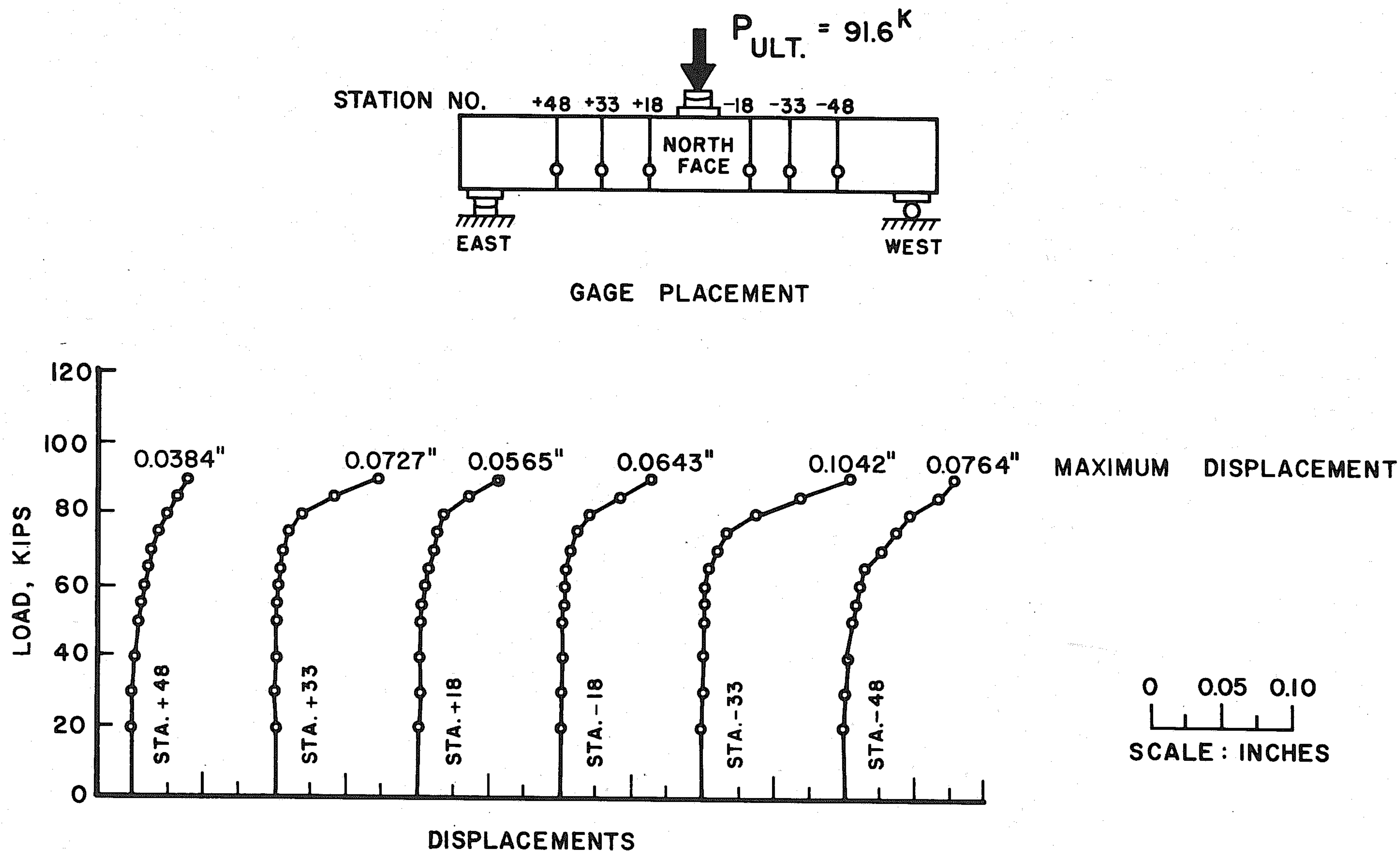


FIG. 9-E YOKE DATA, BEAM IWCRB-I

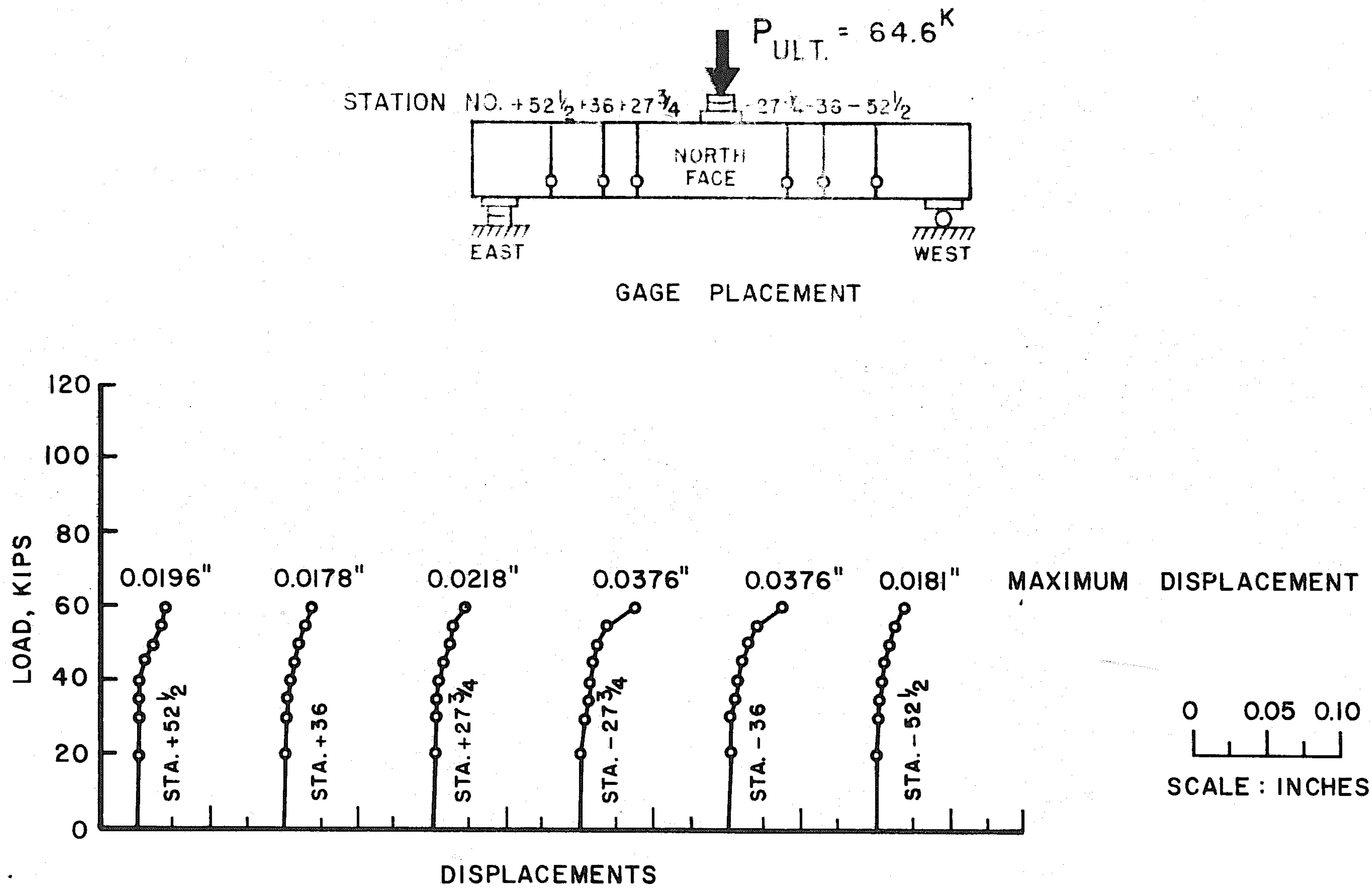


FIG. 9-F YOKE DATA, BEAM IWCRC-I

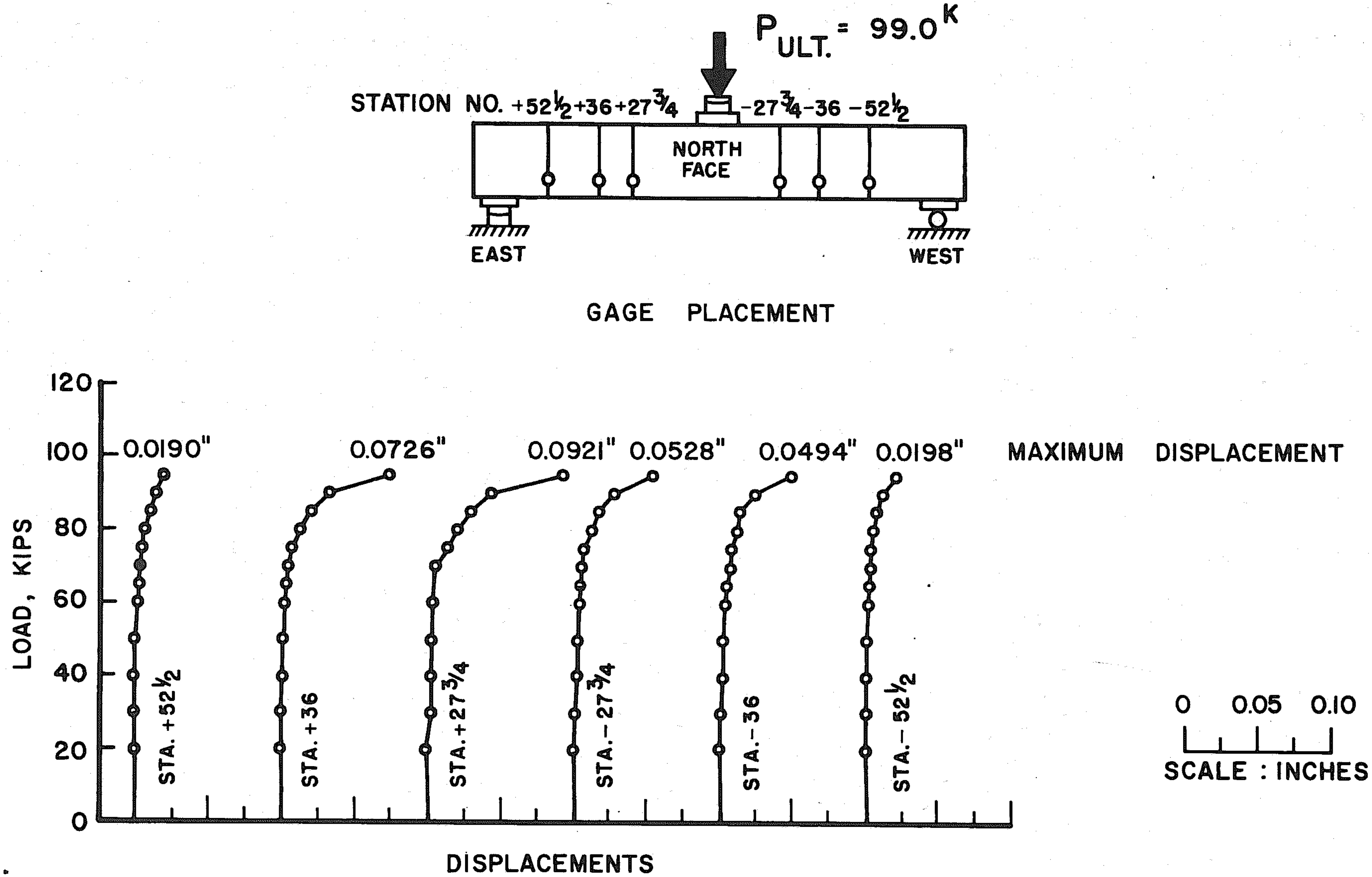


FIG. 9-G YOKE DATA, BEAM IWCA-I

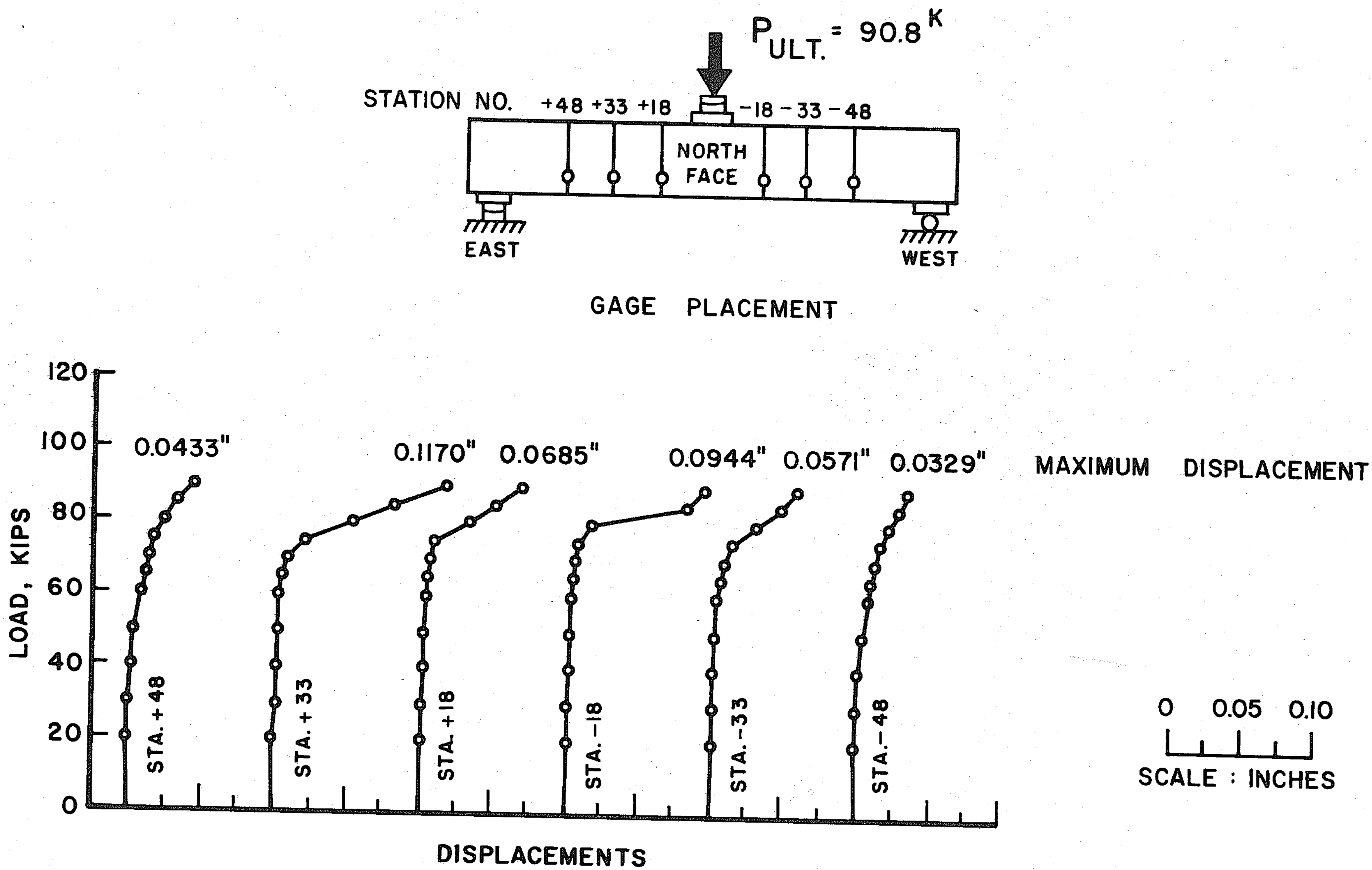


FIG. 9-H YOKE DATA, BEAM IWCB-I

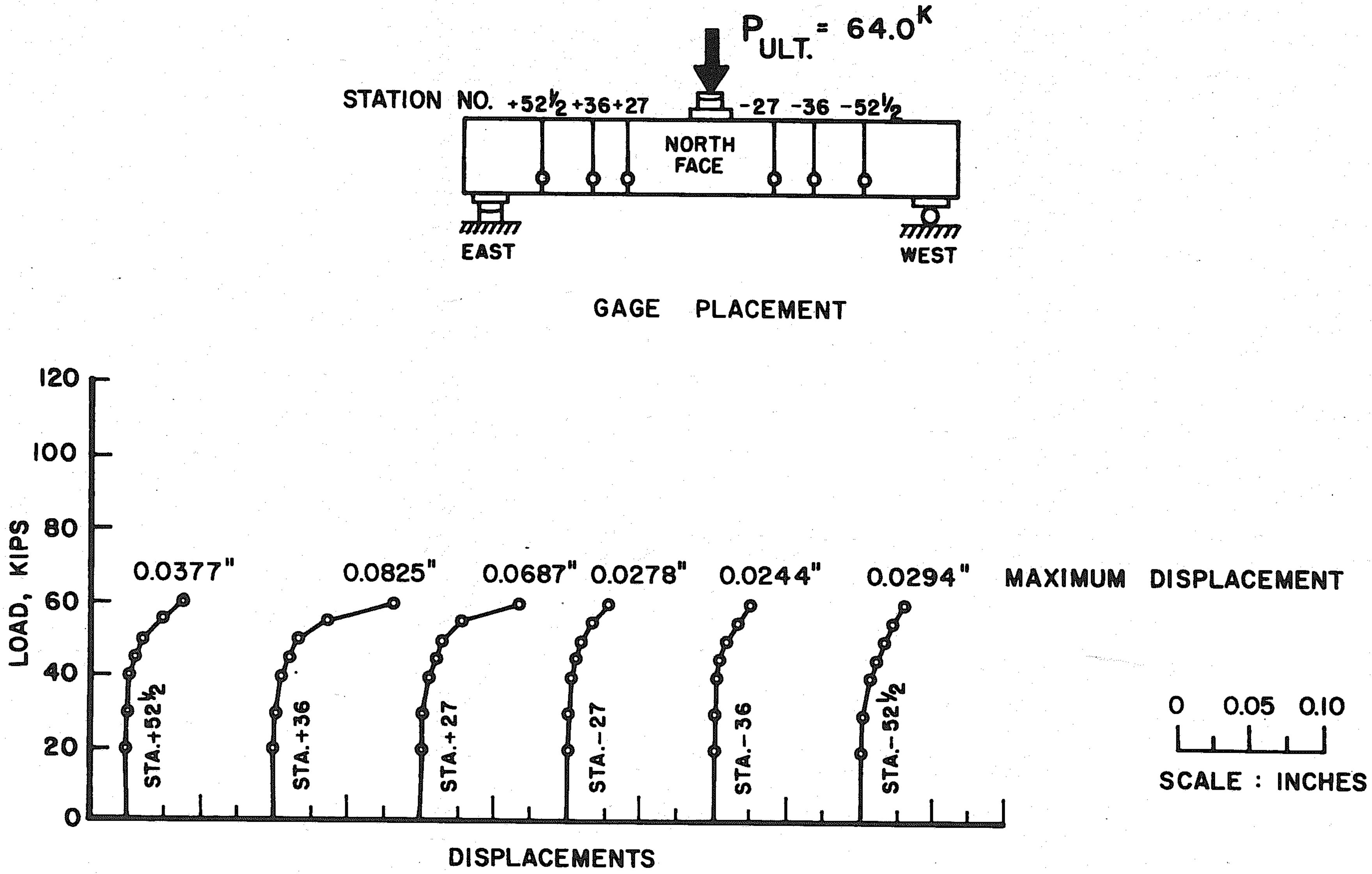


FIG. 9-1 YOKE DATA, BEAM IWCC-1

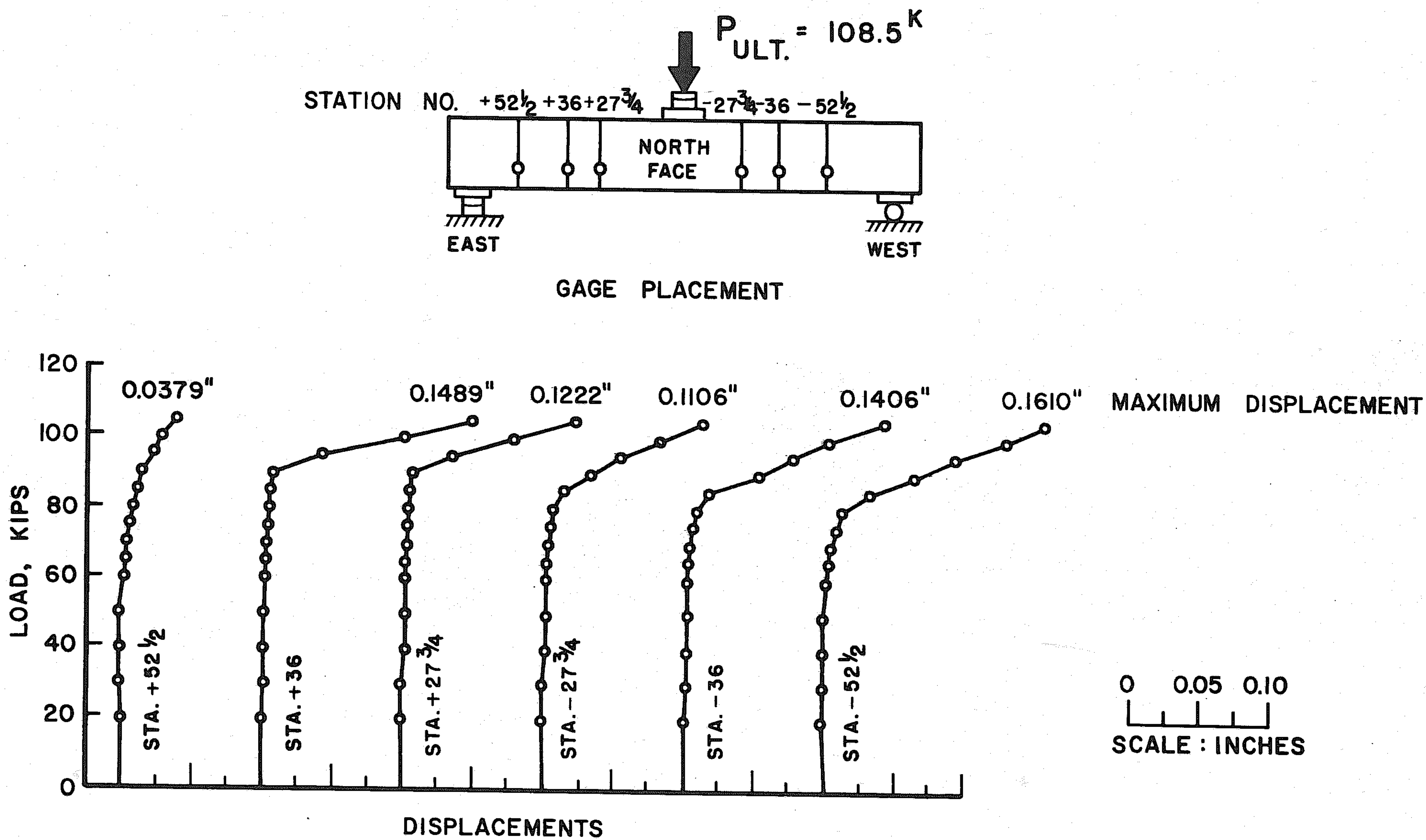


FIG. 9-J YOKE DATA, BEAM 2WCA-1

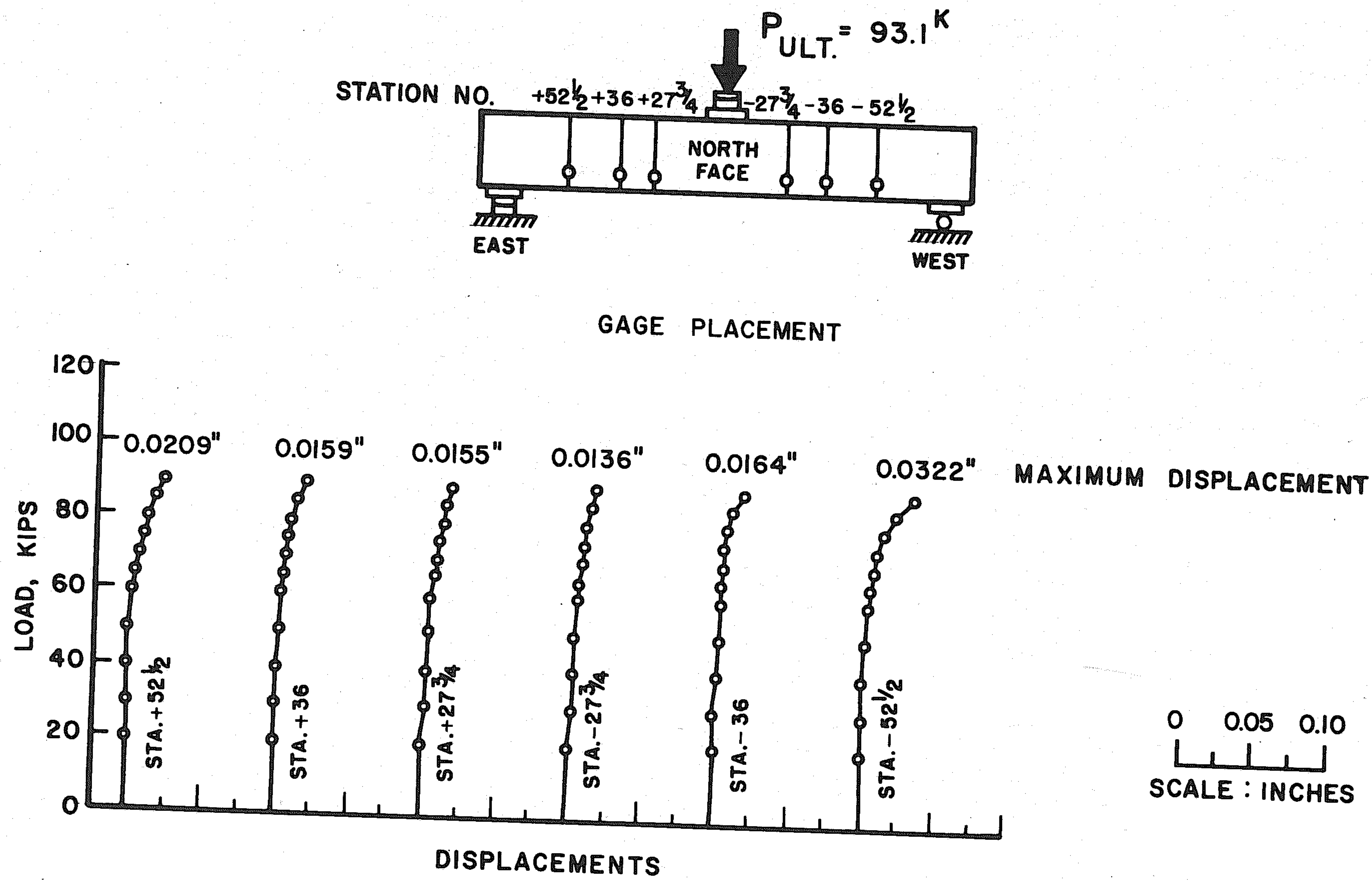


FIG. 9-K YOKE DATA, BEAM 3WCA-1

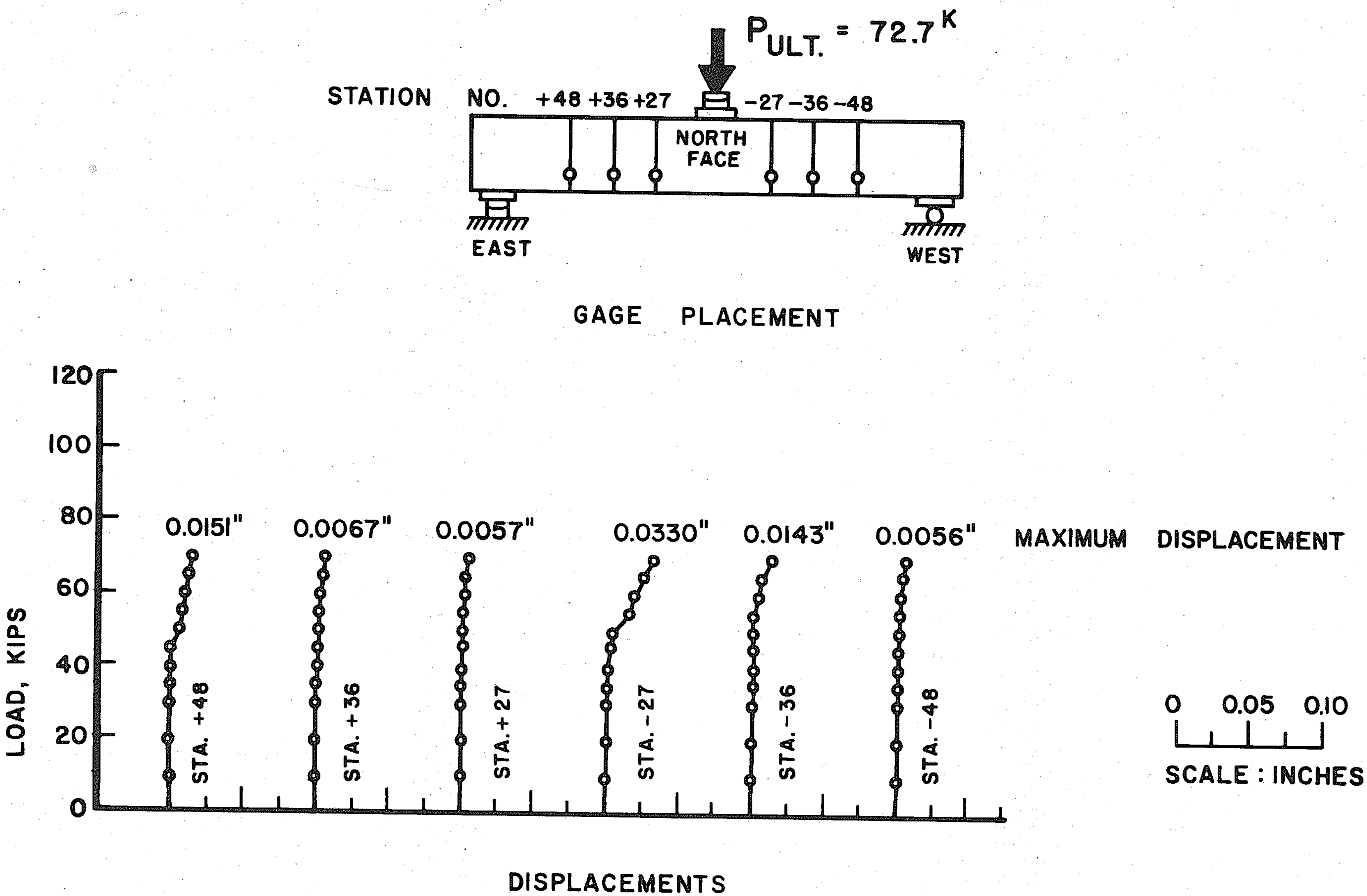


FIG. 9-L YOKE DATA, BEAM ROA-1

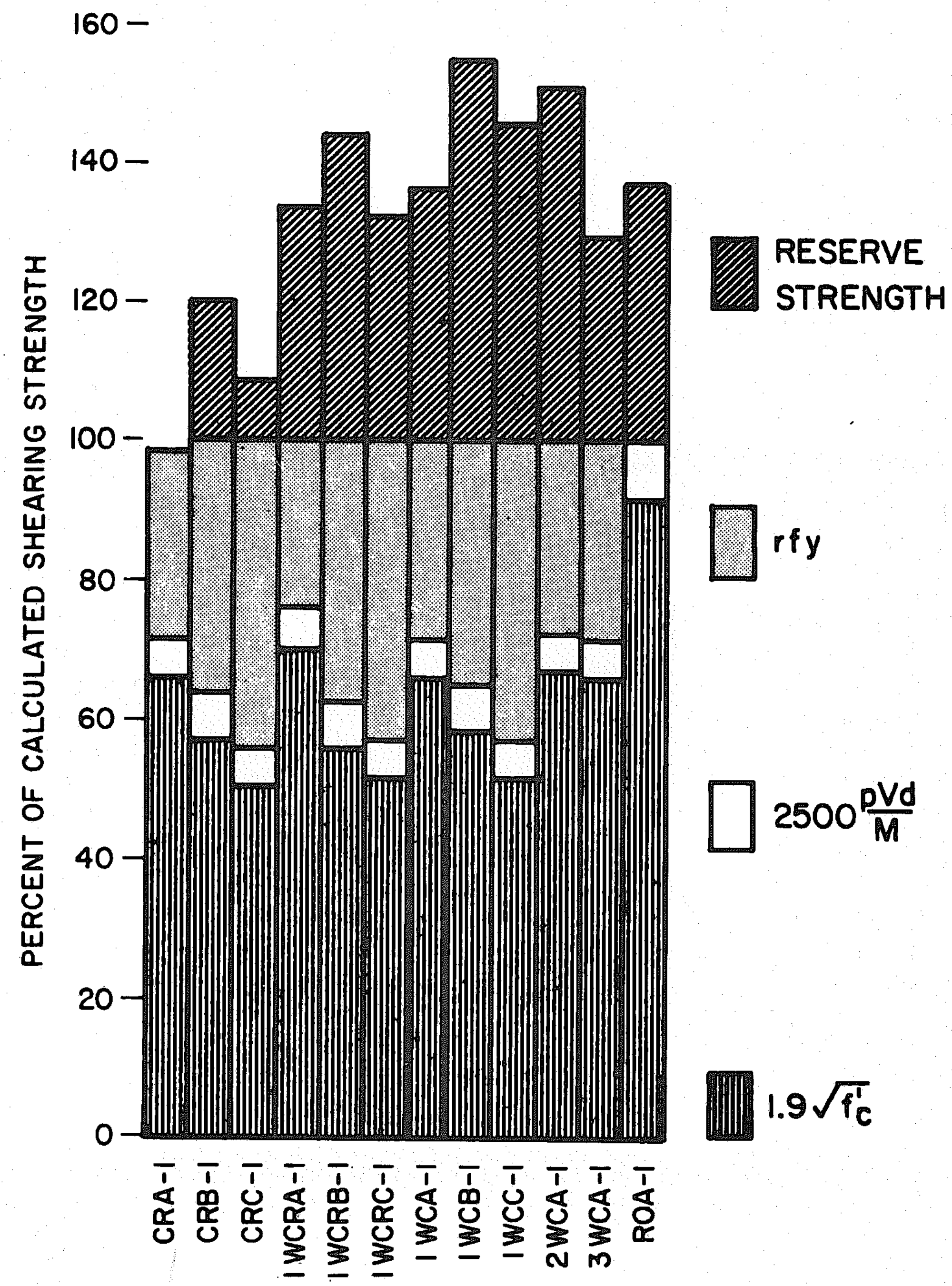


FIG. 10 COMPARISON OF CALCULATED AND TEST VALUES OF ULTIMATE SHEARING STRENGTH SERIES III

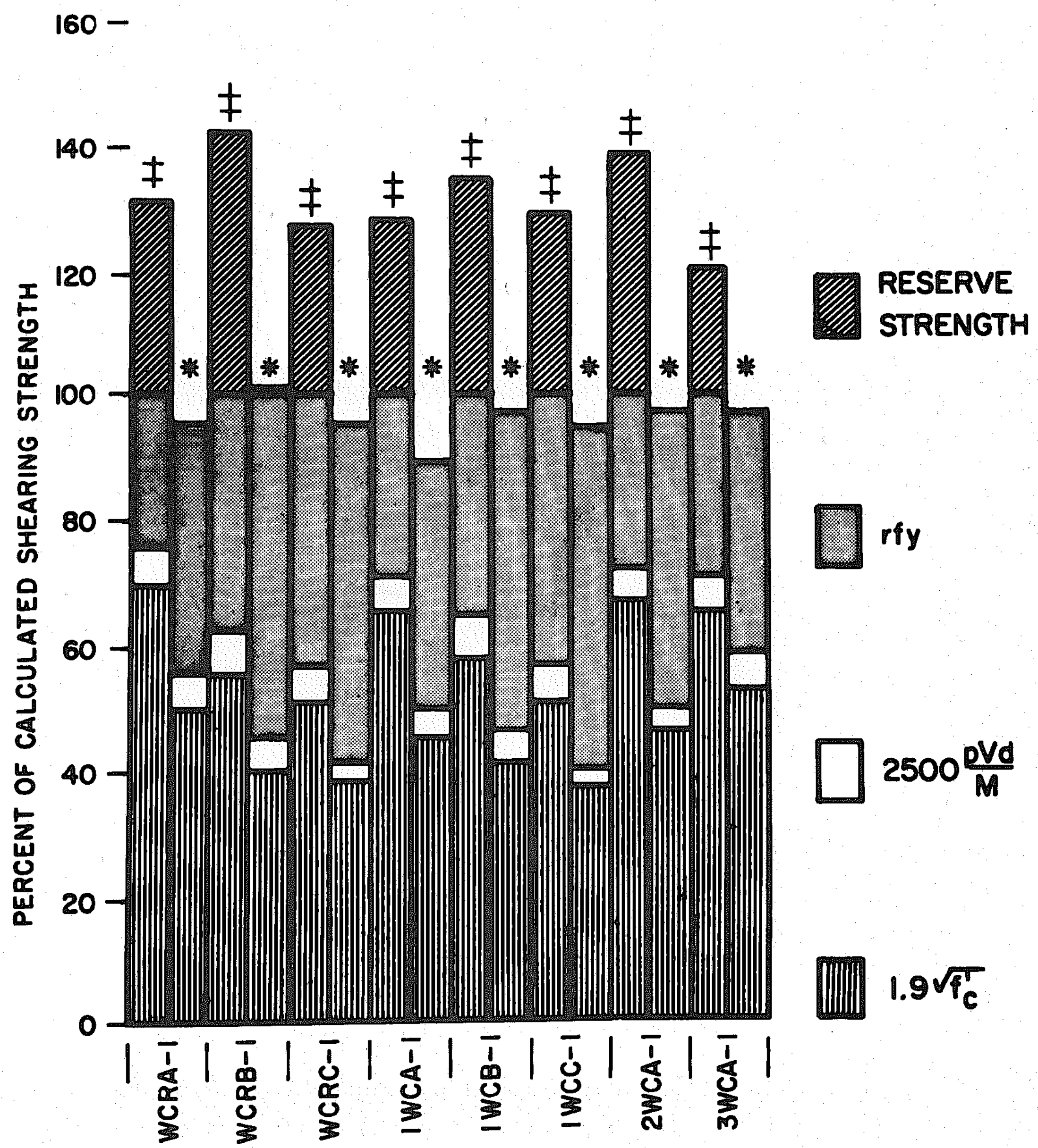


FIG. 11 INFLUENCE OF ADDITIONAL WEB REINFORCEMENT ON COMPARISON OF CALCULATED AND TEST VALUES

ULTIMATE SHEARING STRENGTH SERIES III

‡ BASIC VALUE - 100% - EXCLUDES ADDITIONAL WEB REINFORCEMENT REQUIRED BY ACI CODE.

✱ BASIC VALUE - 100% - INCLUDES ADDITIONAL WEB REINFORCEMENT REQUIRED BY ACI CODE.

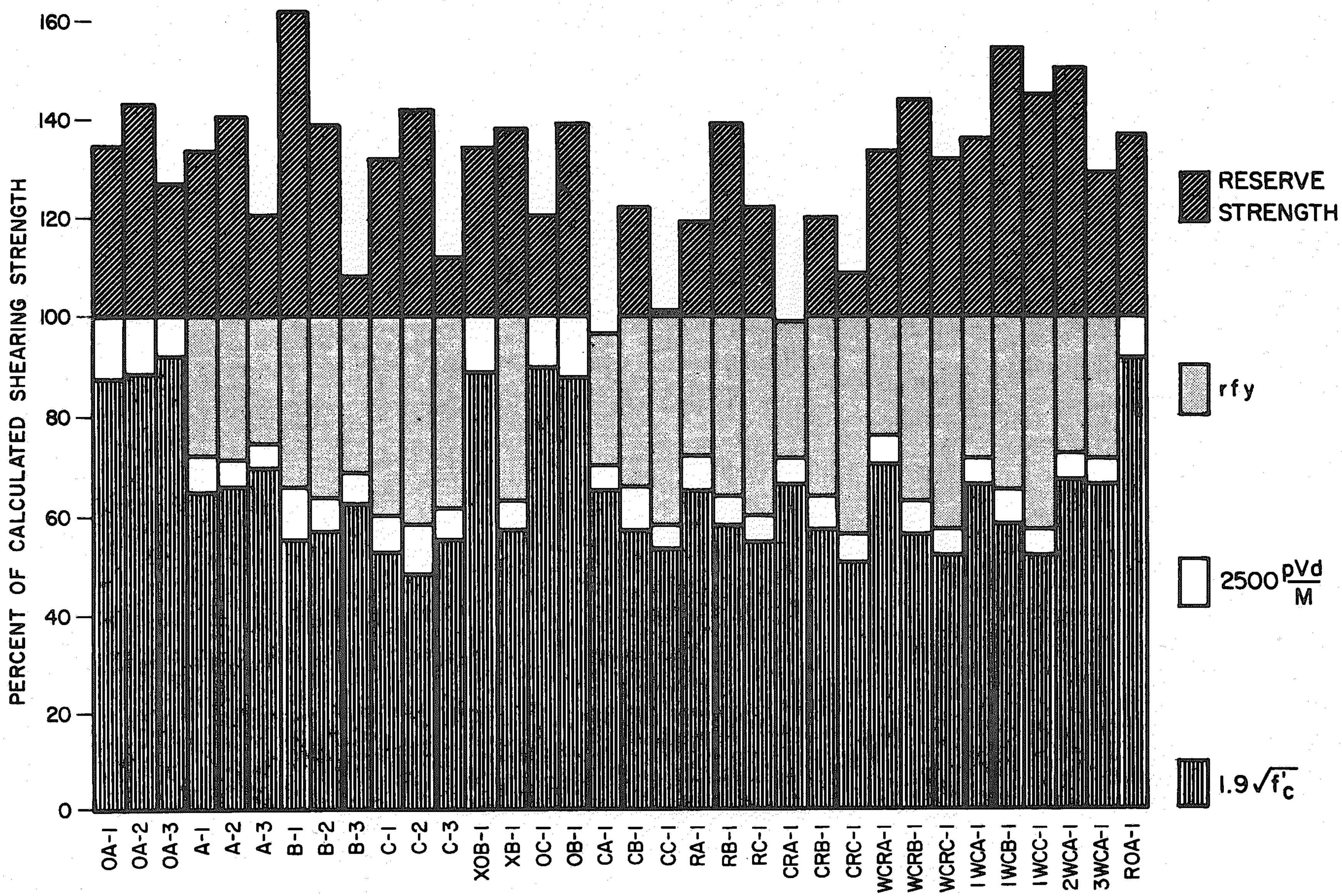


FIG.12 COMPARISON OF CALCULATED AND TEST VALUES OF ULTIMATE SHEARING STRENGTH SERIES I, II, AND III

APPENDIX

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Papers listed cover the period of January 1961 - June 1965; they are arranged chronologically by year, alphabetically by author within each year.

1961

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13. Tapia, E. U., and DeCossio, R.D. "Diagonal Tensile Strength of Slabs Reinforced in One Direction. II: Investigation Into the Effects of Size and Shape on the Shear Strength of Concrete Members," (in Spanish), Universidad Nacional Autonoma de Mexico, Publication No. 50, Oct. 1961.

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