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Z pinches with multi-ion species: Ion separation and stability

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Recent experiments on gas-puff Z pinches using various ion mixtures have demonstrated that the plasma shell separates into two distinct annuli which implode concentrically. This phenomenon is quantitatively explained with the use of a semihydrodynamic model in which the electrons are treated as a collisional fluid and the ions are considered cold and collisionless. Stability in this model is investigated and an expression for the growth rate of a Rayleigh–Taylor type instability is presented. This growth rate is found to be independent of the degree of plasma compressibility and somewhat reduced in the presence of an ion mixture. Comparison of these results with the University of California, Irvine, Z-pinch experiment is discussed.

I. INTRODUCTION

The classic models of pinch dynamics are the snowplow for a collisional plasma and the Rosenbluth–Ferraro sheath for a collisionless plasma.[1] As neither model is able to explain the large ion separations observed in the UCI gas-puff Z pinch,[2] a semihydrodynamic model has recently been proposed to qualitatively understand this phenomenon.[3] In this model, the electrons are treated as a highly collisional fluid which is pushed by a self-magnetic force in a current-carrying layer on the outer boundary of the plasma shell. The ions are treated as free particles unaffected by this magnetic force. A small radial charge separation is induced in the current shell, resulting in an electric field which is redistributed through the entire plasma shell on a very short time scale. The ions are thus accelerated by this collective electrostatic field which will lead to an observable separation of the ion species if a significant difference in charge to mass ratio between these ion species is present.

The time evolution of the pinch may be divided into three stages: (1) ionization and slow heating of the hollow, cylindrical gas puff to a few eV, (2) a slow implosion phase lasting about 50 nsec, (3) and the final pinch phase followed by disruptions from instabilities. The slow implosive stage exhibits the separation phenomena that we wish to quantitatively investigate in this study.

The stability analysis of an accelerating, conducting plasma shell has previously been investigated using MHD models.[4,5] With the semihydrodynamic model used throughout this analysis, the stability of the imploding plasma shell must be analyzed anew.

In Sec. II, we present the basic equations to be used in the separation (Sec. III) and stability analyses (Sec. IV). Section V is devoted to a discussion of the principal results and conclusions of this paper.

II. BASIC EQUATIONS

The equations which underlie the semihydrodynamic model are the continuity equation and equation of motion for the ions ($i = 1, 2$) together with number, momentum, and energy conservation for electrons, respectively:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla\right)\mathbf{v}_i = \frac{\bar{Z}_i e}{m_i}\mathbf{E}, \tag{2}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_e \cdot \nabla\right)\mathbf{v}_e = \frac{-e}{m_e}\left(\mathbf{E} + \frac{\mathbf{v}_e \times \mathbf{B}}{c}\right) - \frac{1}{m_e n_e}\nabla P_e, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_e \cdot \nabla\right)P_e(m_e n_e)^{-\gamma} = 0, \tag{5}$$

where $n_i(n_e)$ is the ith ion species (electron) number density, $\bar{Z}_i$ is the ith ion species average charge number, $m_i(m_e)$ is the ion (electron) mass, $\mathbf{v}_i(\mathbf{v}_e)$ is the ion (electron) fluid velocity, P_e is the electron kinetic pressure, $\mathbf{B}_0$ is the magnetic field, $\mathbf{E}$ is the electric field, e is the elementary electric charge, c is the speed of light, and γ is the adiabatic exponent. We make the simplifying assumption that the ion kinetic pressure and the ion Lorentz force can be ignored.

Equations (1)–(5) must be supplemented with Ampere's law,

$$\nabla \times \mathbf{B} = (4\pi/c)\mathbf{j} \simeq -(4\pi/c)en_e\mathbf{v}_e, \tag{6}$$

where $\mathbf{j}$ is the electric current density, and the charge neutrality condition,

$$n_e = n_1 + n_2. \tag{7}$$

Substituting Eq. (6) into Eq. (4) and neglecting electron inertia yield the following expression for the electric field:

$$\mathbf{E} = \frac{-1}{en_e}\nabla\left(\frac{B^2}{8\pi} + P_e\right) + \frac{1}{4\pi en_e}(\mathbf{B} \cdot \nabla)\mathbf{B}. \tag{8}$$

Finally, one may derive the following useful expression for the unmagnetized region of the plasma by combining Eqs. (1), (3), and (7) to give

$$n_e\mathbf{v}_e = n_1\mathbf{v}_1 + n_2\mathbf{v}_2. \tag{9}$$

Equations (1), (2), and (7)–(9) complete our required set of equations and together with appropriate boundary conditions will fully describe the dynamics of our semihydrodynamic system.

III. ION SEPARATION

The physical mechanism which gives rise to a separation can easily be described qualitatively. During the run in

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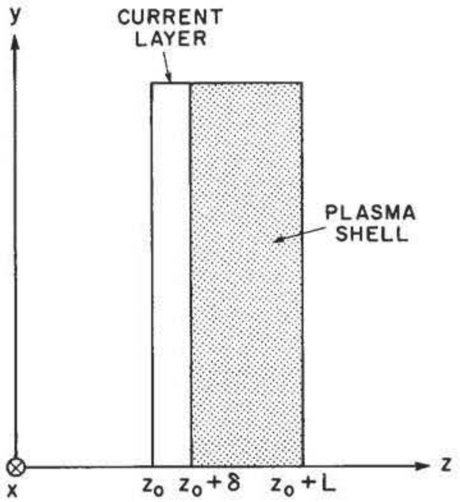

FIG. 1. Coordinate system used for the slab model of an imploding plasma shell.

phase, where the separation is observed, the electron temperature is a few eV and density is of the order of 10^{17} cm^{-3} so that $\tau_{ee} \sim 10^{-12}$ sec. Also the propagation time for the shocks across the slab, which propagate at a speed proportional to the electron sound speed $\sim (T_e/m_e)^{1/2}$, is of the order of $\sim 10^{-10}$ sec. These times are much shorter as compared to the energy exchange time between electrons and ions $\tau_{ei} \sim 50$ nsec. We can thus treat the electrons as a collisional fluid and ions as a collisionless fluid in the sense that they do not acquire energy from the electrons by collisions. Thus the electrons are pushed inward like an incompressible fluid by the external magnetic piston resulting in a small charge separation which gives rise to an electric field within the plasma. The ions will be accelerated inward by this collective electrostatic field, at a rate depending on the charge to mass ratio, assuring quasineutrality.

In order to study ion separation from a quantitative standpoint, we consider a planar approximation for the plasma shell, i.e., $L/R \ll 1$, where L is the thickness of the plasma shell and R is the radius of the cylindrical plasma. We choose the z axis to be the direction of acceleration for the imploding shell, the y axis as the direction of current flowing on the outer boundary of the plasma shell, and the x axis as the direction of the self-magnetic field (Fig. 1). For simplicity, we shall assume that the current flows in a very narrow boundary layer so that the magnetic pressure is zero inside. Its surface value supplies the necessary boundary condition. Under this assumption, the collective electrostatic field can be estimated by integrating the z component of Eq. (8) across the plasma shell:

$$E_z \cong B_{0x}^2 / 8\pi n_e e L. \tag{10}$$

The question of motion for each ion species in the presence of collisions follows from Eq. (2):

$$\frac{dv_{z1}}{dt} = \frac{\bar{Z}_1}{m_1} \frac{B_{0x}^2}{8\pi n_e L} - \frac{v_{1z} - v_{2z}}{\tau_{12}}, \tag{11}$$

$$\frac{dv_{z2}}{dt} = \frac{\bar{Z}_2}{m_2} \frac{B_{0x}^2}{8\pi n_e L} - \frac{v_{2z} - v_{1z}}{\tau_{12}}, \tag{12}$$

where Eq. (10) is used and a phenomenological collision term is included to describe the effect of collisions in reducing the separation. The collision time τ_{ij} are defined as

$$\tau_{ij}^{-1} \approx 6.8 \times 10^{-8} [(\mu_j)^{1/2}/\mu_i] \bar{Z}_i^2 \bar{Z}_j^2 \times \ln \Lambda_{ij} n_j (1 + \mu_j/\mu_i) T^{-3/2}, \tag{13}$$

where μ_i is the mass of the ith ion species in units of the proton mass m_p, $\ln \Lambda_{ij}$ is the Coulomb logarithm, and T is the ion temperature in eV. Subtracting Eq. (12) from Eq. (11), we obtain an equation of motion for the ion seperation velocity $v_z^{(1,2)}$.

$$\frac{dv_z^{(1,2)}}{dt} = \frac{B_{0x}^2}{8\pi n_e L}\left(\frac{\bar{Z}_1}{m_1} - \frac{\bar{Z}_2}{m_2}\right) - \frac{v_z^{(1,2)}}{\tau}, \tag{14}$$

where $\tau^{-1} = \tau_{12}^{-1} + \tau_{21}^{-1}$. Defining $d\Delta Z^{(1,2)}/dt = v_z^{(1,2)}$ and integrating twice Eq. (14) with respect to time gives the general formula for the ion separation as

$$\Delta Z^{(1,2)} = \left(\frac{\bar{Z}_1}{m_1} - \frac{\bar{Z}_2}{m_2}\right) \frac{B_{0x}^2}{8\pi n_e L \tau^2} \times \left[\exp\left(-\frac{\Delta t}{\tau}\right) + \frac{\Delta t}{\tau} - 1\right], \tag{15}$$

which for the case $\Delta t / t \gg 1$ leads to a simple form:

$$\Delta Z^{(1,2)} \cong \left(\frac{\bar{Z}_1}{\mu_1} - \frac{\bar{Z}_2}{\mu_2}\right) \frac{B_{0x}^2}{8\pi n_e m_{pL}} \Delta t \tau. \tag{16}$$

For a 90%D_2–10%Ar mixture with $n_e \sim 10^{17}$ cm^{-3} and $T \sim 10$ eV, τ is on the order of 1 nsec. Using this value for τ in Eq. (16) along with $B_{0x} \sim 10^5$ G, $\bar{Z}_{D_2} = 1$, $\bar{Z}_{Ar} \cong 2$, $\mu_{D_2} \sim 2$, $L \sim 0.2$, $\mu_{Ar} \sim 40$, and $\Delta t \sim 50$ nsec gives an average ion separation ~ 0.28 cm. This agrees with the separations observed in the interferograms of Ref. 3.

IV. STABILITY ANALYSIS

As we have assumed, the electrons behave as a fluid; therefore, from the perspective of a comoving observer with the accelerating slab, the electron kinetic pressure $p_e = n_e T_e$ inside the slab will satisfy the condition

$$\frac{\partial p_e}{\partial z} = -\rho_e g_e, \tag{17a}$$

and at the boundary $z = z_0$,

$$p_e(z = z_0) = B_{0x}^2 / 8\pi, \tag{17b}$$

where $\rho_e = m_e n_e$ and $g_e = B_{0x}^2 / 8\pi n_e L$. Upon integrating Eq. (17a) subject to the boundary condition (17b), we can write the pressure profile within the plasma shell as

$$p_e(z) \cong (B_{0x}^2/8\pi) \exp[-(z - z_0)/\beta L], \tag{18}$$

where $\beta = 8\pi n_e T_e / B_{0x}^2$.

Linearizing Eqs. (1), (2), (5), and (9) to describe small perturbations around the equilibrium state yields

$$\frac{\partial}{\partial t}\delta n_i + \nabla \cdot (n_i \delta \mathbf{v}_i) = 0, \tag{19}$$

$$m_i n_e \frac{\partial}{\partial t}\delta \mathbf{v}_i = \nabla\left(\frac{\delta B_x}{4\pi} B_x + \delta P_e\right) + \frac{1}{4\pi}\left(B_x \frac{\partial}{\partial x}\delta \mathbf{B} + \delta B_z \frac{\partial}{\partial z} B_x \hat{x}\right) - m_i \delta n_e g_i \hat{z}, \tag{20}$$

$$\frac{\partial}{\partial t}\delta P_e - C_s^2 m_e \frac{\partial}{\partial t}\delta n_e - (\gamma - 1)\delta v_{ez}\frac{\partial}{\partial z}P_{e0} = 0, \quad (21)$$

$$\delta v_{ez} = (n_1/n_e)\delta v_{1z} + (n_2/n_e)\delta v_{2z}, \quad (22)$$

where $C_s^2 \equiv \partial P_{e0}/\partial(m_e n_e)|_s$ is the adiabatic electron sound speed and $g_i = B_{0x}^2/8\pi m_i n_e L$. We assume all perturbed quantities are of the form

$$\delta f(x,y,z,t) = \hat{f}(z - z_0)\exp[i(\omega t + k_x x + k_y y)], \quad (23)$$

and the perturbed velocity $\delta \mathbf{v}_j$ $(j = e,1,2)$ is expressible in terms of the Lagrangian displacement:

$$\delta \mathbf{v}_j = \frac{\partial}{\partial t}\delta \xi_j = i\omega \delta \xi_j. \quad (24)$$

Within the plasma shell, i.e., $z_0 \leqslant z \leqslant z_0 + L$, $B_x \equiv 0$, and using Eq. (6), Eqs. (19)–(22) reduce to

$$\hat{n}_i = -in_i(k_x\hat{\xi}_{ix} + k_y\hat{\xi}_{iy}) - \frac{\partial}{\partial z}(n_i\hat{\xi}_{iz}), \quad (25)$$

$$m_i n_e \omega^2 \hat{\xi}_{ix} = ik_x \hat{P}_e, \quad (26)$$

$$m_i n_e \omega^2 \hat{\xi}_{iy} = ik_y \hat{P}_e, \quad (27)$$

$$m_i n_e \omega^2 \hat{\xi}_{iz} = \frac{\partial}{\partial z}\hat{P}_e + m_i \hat{n}_e g_i \quad (28)$$

$$\hat{P}_e - C_s^2 m_e \hat{n}_e - (\gamma - 1)\hat{\xi}_{ez}\frac{\partial}{\partial z}P_{eo} = 0, \quad (29)$$

$$n_e\hat{\xi}_{ez} = n_1\hat{\xi}_{1z} + n_2\hat{\xi}_{2z}. \quad (30)$$

Combining Eqs. (25)–(27), using the charge neutrality condition, and eliminating $\hat{n}_e$ by Eq. (29) and $\hat{\xi}_{iz}$ by Eq. (30), we obtain

$$\hat{P}_e = \frac{\rho_{12}\omega^2}{(k^2 - \omega^2/C^2)}\left(\frac{\partial}{\partial z}\hat{\xi}_{ez} - \frac{g_{12}}{C^2}\hat{\xi}_{ez}\right), \quad (31)$$

where

$$\rho_{12} = n_e(n_1/m_1 n_e + n_2/m_2 n_e)^{-1},$$
$$g_{12} = B_{0x}^2/8\pi\rho_{12}L,$$
$$C = \gamma C_s m_e(n_1/m_1 n_e + n_2/m_2 n_e),$$

and Eq. (17a) is used as an equilibrium condition. If we now combine Eq. (28) and Eq. (30) and eliminate $\hat{n}_e$ with Eq. (29), we obtain

$$\frac{\partial}{\partial_z}\hat{P}_e + \frac{g_{12}}{C^2}\hat{P}_e = \rho_{12}\left(\omega^2 - \frac{g_{12}^2}{C^2}(\gamma - 1)\right)\hat{\xi}_{ez}. \quad (32)$$

Finally, substituting Eq. (31) in Eq. (32) yields

$$\frac{\partial^2}{\partial z^2}\hat{\xi} - \frac{\gamma g_{12}}{c^2}\frac{\partial}{\partial z}\hat{\xi} - \left(k^2 - \frac{\omega^2}{C^2} - (\gamma - 1)\frac{k^2 g_{12}^2}{C^2\omega^2}\right)\hat{\xi} = 0, \quad (33)$$

where the subscripts on $\hat{\xi}_{ez}$ are dropped for convenience. Equation (33) has the general solution

$$\hat{\xi} = D_+ e^{k_+(z - z_0)} + D_- e^{k_-(z - z_0)}, \quad (34)$$

where

$$k_\pm = \frac{\gamma g_{12}}{2C^2} \pm \frac{1}{2}\left[\left(\frac{\gamma g_{12}}{C^2}\right)^2 + 4\left(k^2 - \frac{\omega^2}{C^2} - (\gamma - 1)\frac{k^2 g^2{}_{12}}{C^2\omega^2}\right)\right]^{1/2}, \quad (35)$$

and $D_\pm$ are arbitrary constants to be determined by the imposition of boundary conditions. These conditions will proceed from allowing small perturbations of the equilibrium conditions at $z = z_0$ and $z = z_0 + L$,

$$P_{e0}(z_0) = B_{0x}^2/8\pi; \quad P_{e0}(z_0 + L) = 0,$$

to give

$$\hat{P}_e(z_0) = -\hat{\xi}\frac{\partial P_{e0}}{\partial z}\bigg|_{z = z_0} + \frac{B_{0x}B_x(z_0)}{4\pi}, \quad (36)$$

and

$$\hat{P}_e(z_0 + L) = -\hat{\xi}\frac{\partial P_{e0}}{\partial z}\bigg|_{z = z_0 + L}. \quad (37)$$

All that remains is to find the perturbed magnetic field $\hat{B}_x(z_0)$.

The vacuum magnetic field satisfies the equations

$$\nabla \times \delta\mathbf{B} = 0; \quad \nabla \cdot \delta\mathbf{B} = 0,$$

so that $\delta\mathbf{B} = -\nabla\delta\Psi$, with $\delta\Psi$ satisfying $\nabla^2\delta\Psi = 0$. Writing $\delta\Psi$ as

$$\delta\Psi = \hat{\Psi}(z - z_0)\exp[i(\omega t + k_x x + k_y y)],$$

$\nabla^2\delta\Psi = 0$ explicitly becomes

$$\left(\frac{\partial^2}{\partial z^2} - k^2\right)\hat{\Psi}(z - z_0) = 0, \quad (38)$$

whose solution (which vanishes at $z = -\infty$) is given by

$$\hat{\Psi} = Ae^{k(z - z_0)}.$$

At the interface between vacuum and a perfect conductor $(z = z_0)$,

$$\nabla\phi \cdot \mathbf{B} = 0, \quad (39)$$

where $\nabla\phi$ is a surface normal. With the equation for the surface interface satisfying

$$\phi(x,y,z) = z - z_0 - \delta\xi_z(\mathbf{x},t) \equiv 0, \quad (40)$$

Eq. (39) reduces to $\delta B_z = B_{0x}\delta\xi_z/\partial x$. This in turn gives $A = -(ik_x/k)B_{0x}\hat{\xi}(z_0)$ and δB_x in the vacuum region:

$$\delta B_x = (k_x^2/k)B_{0x}\hat{\xi}_z(z_0)\exp[k(z - z_0)] \times \exp[i(\omega t + k_x x + k_y y)]. \quad (41)$$

Using Eqs. (31) and (41), Eqs. (36)–(37) now become

$$\frac{\partial}{\partial t}\hat{\xi}\bigg|_{z = z_0} = \frac{g_{12}}{C^2}\left[1 - \left(1 - \frac{C^2k^2}{\omega^2}\right)\left(1 + \frac{2K_x^2C^2}{kg_{12}\gamma}\right)\right]\hat{\xi}(z_0),$$

$$\frac{\partial}{\partial z}\hat{\xi}\bigg|_{z = z_0} + L = \frac{g_{12}k^2}{\omega^2}\hat{\xi}(z_0 + L).$$

Applying Eq. (34) for $\hat{\xi}$ yields the dispersion relation

$$k_- = \frac{g_{12}}{C^2}\left[1 - \left(1 - \frac{C^2k^2}{\omega^2}\right)\left(1 + \frac{2k_x^2C^2}{kg_{12}\gamma}\right)\right]. \quad (42)$$

For $\mathbf{k}\perp\mathbf{B}$, i.e., $k_x = 0$, Eq. (42) reduces to

$$(\omega^4 - g_{12}^2k^2)(\omega^2 - C^2k^2) = 0, \quad (43)$$

which has the roots

$$\omega^2 = C^2k^2$$

(hybrid species acoustic modes),

$$\omega^2 = \pm g_{12}k \quad (44)$$

(hybrid species interchange mode).

The solution $\omega^2 = -g_{12}k$ yields a Rayleigh–Taylor type growth rate Γ:

$$\Gamma = (g_{12}|k_y|)^{1/2}. \quad (45)$$

For a D_2–Ar mixture, Eq. (45) explicitly becomes

$$\Gamma = \left\{\frac{B_{0x}^2|k_Y|}{8\pi n_e m_{D_2} L}\left[1 - \frac{n_{Ar}}{n_e}\left(1 - \frac{\mu_{D_2}}{\mu_{Ar}}\right)\right]\right\}^{1/2},$$

where it is noted that the presence of argon somewhat reduces the growth rate since $\mu_{Ar}/\mu_{D_2} \gg 1$. As this growth rate is only mildly dependent on the presence of argon, the interchange instability is little affected by the presence of high Z ions. It becomes clear that the main reason for the experimentally observed smaller-pinch radii and increased densities with a D_2–Ar mixture is the copious radiative cooling from argon[6] and not a significant change in stability characteristics.

The form of the dispersion relation [Eq. (43)] explicitly shows that the Rayleigh–Taylor solution is mathematically distinct from the ion-acoustic solution. This corroborates the claim first made by Parks[5] that the interchange instability is unaffected by the compressibility of the plasma.

V. SUMMARY AND CONCLUSIONS

Large ion separations are characteristic of pinches using a mixture of ion species. A semihydrodynamic model is used to quantitatively investigate the ion-separation phenomenon. This method consists of treating the dynamics of the ions and electrons separately. Such a procedure is valid provided a well-defined hierarchy of collision time scales is present. Implicit in our estimate is the assumption that electron–ion collision times τ_{ei} far exceed the electron–electron collision time τ_{ee} throughout the slow implosion phase. Indeed, with $\tau_{ee} \sim 10^{-12}$ sec and $\tau_{ei} \sim 10^{-8}$ sec, the electron dynamics are largely unaffected by the presence of ions over a significant portion of the slow implosive time scale (5×10^{-8} sec). This is the underlying physical mechanism for the ion separations observed. Our calculated value of ~ 0.28 cm for a 90%D_2–10%Ar mixture is in good agreement with the experimental results of the UCI gas-puff Z pinch.

The presence of an ion mixture has led to more stable, efficient, and uniform pinches. For instance, the same D_2–Ar mixture has led to a confinement product $n\tau_E \sim 2\times10^{12}$ sec cm^{-3}, which is an order of magnitude higher than for a pure D_2 pinch. The stability analysis conducted here has indicated that radiative cooling from high-Z ions has more to do with the improved pinches that are observed than with a reduction of the Rayleigh–Taylor growth rate.

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