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#### When Do Consumers Favor Price Increases: With Applications to Congestion and to Regulation

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# When Do Consumers Favor Price Increases: With Applications to Congestion and to Regulation

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## **1** Introduction

For a conventional good, an increase in price reduces the consumer surplus of both those who no longer buy the good, and of those who continue to buy it. If, however, consumers must spend real resources to obtain rights for the good, or if the quality depends on the number of other consumers trying to obtain the same good, then a price increase may have different effects. Both these characteristics apply to congestible goods: a consumer's utility decreases in the price he pays and in the number of other persons who use the good. Some users may gain from a price increase which reduces demand. Therefore, even if the revenue is not returned to the users, their welfare can increase. The change in welfare appears to be particularly important in assessing the political popularity of road tolls. We inquire here into the conditions under which imposing a congestion toll will prove politically acceptable. Political acceptance, we suppose, is greater if consumer welfare increases even when the toll revenue is not returned to users.

Users are not the only political actors. One may argue that the general population, which may not use the road with a toll, will certainly favor the toll—they enjoy the revenue. But on urban systems it can well be that a majority of voters use the road. Moreover, the populace may fear that the revenue will be wasted, or perhaps go to another jurisdiction. (For example, the Port Authority of New York is a joint authority of the states of New York and New Jersey. Revenues collected from commuters in New Jersey may go to projects in New York.) Finally, pressures by special interest groups may be especially important. The affected individuals, those who pay the tolls or who find that they must use alternative modes, immediately recognize their increased costs. Those who suffer may well lobby against tolls. The general population may not recognize that they gain added revenue from the tolls. Moreover, when the revenue per person is small, no person has much incentive to lobby for tolls.

Much work examines the distributional effects of a road toll. Strong results are obtained when all consumers have the same cost for a delay of unit time. For then a toll which is not returned to consumers necessarily reduces the welfare of all consumers (see Weitzman (1974)). If consumers differ in their value of time, then a congestion toll can increase aggregate consumer welfare, even if the revenue is not returned to users (Layard (1977), Glazer (1981), Niskanen (1987), Small (1992)). Evans (1992), while noting this possibility, considers it "unlikely." We disagree. Instead, we show the conditions under which it can happen.

Our analysis corrects another common view, that states that a congestion toll "will discourage journeys with low time values, and will probably encourage journeys with high time values" (Layard (1977), p. 297). We show that the opposite can hold: a toll can instead discourage journeys for persons with high time values. Like others, we suppose that persons with high values of time have high values from a trip. But unlike earlier studies, we consider two modes—a slow one and a fast one—which can both be congested. Under these conditions a toll need not induce those persons with a low value for the trip to be the ones who stop using the mode. Instead, a toll on a slow mode may also induce persons with a high value of time to switch to the fast mode.

#### 2 Assumptions

For concreteness, we shall mostly speak of commuters subject to road congestion. The essential analysis and results also apply whenever an increase in price which reduces the number of persons who use the product also has the effect of increasing quality.

Potential commuters can have different valuations of travel for a given travel time. We index them according to their decreasing willingness to pay for a completed trip. Let potential commuters be sufficiently numerous so that the aggregate demand for the trip can be represented by a twice continuously differentiable function defined over a continuum of commuters i: p = p(i, T). The value of p is the *i*th consumer's willingness to pay for a trip, given travel time T. The indexes i are chosen so that  $p_i(i, T) \leq 0$ : a low value of i indicates a consumer with a high willingness to pay for a trip. In principle the correlation between the valuations of time and travel can have any sign. But we shall follow the literature in assuming the correlation is either zero or positive. If all consumers have the same value of time,  $p_{T_i}(i, T) = 0$ . If  $p_{T_i}(i, T) > 0$ , a consumer values time more the more he values travel (given any travel time).

Such a positive correlation is often justified by referring to commuting trips: persons with high wage rates are willing to spend more getting to work, and willing to spend more to shorten the journey. The correlation need not hold, however, if workers have a choice of employment locations. For example, suppose that at all locations Smith could earn a uniformly higher wage than Jones. But suppose also that the difference between his and Jones' wage is the same at all locations of employment. Let Smith and Jones both work downtown because wages are higher there. Then the producer surplus from making the trip downtown is identical for the two persons. The benefit from making a trip will not be correlated with the value of time.

We let the trip in question be made on two alternative modes, s and f. Both modes are congestible. Travel time on each mode increases in the number of commuters,  $x^j$ , on that mode:  $T^j \equiv T^j(x^j)$ , with  $\partial T^j/\partial x^j \ge 0$ , for j = s, f. We let one mode, s, be the slow mode for all equilibrium values

of  $x^s$  and  $x^f$ :  $T^s(x^s) > T^f(x^f)$ .

For expository simplicity we suppose that the cost on a mode consists of a congestion cost and of a money cost (which can include a toll). This money cost on mode j is called  $r^{j}$ .

Consumer welfare is measured by the area under a demand curve. The measure is valid if the marginal utility of money is constant (see Just, Hueth, and Schmitz (1982) and Starrett (1988)).

#### 3 Equilibrium

Both modes will be used in equilibrium only if the money cost on the slow mode (mode s) is lower than on the fast mode (mode f), that is, only if  $r^s < r^f$ . We henceforth assume that this inequality holds. The slow mode is used by those persons who have the lowest value of time. Recall that a high index corresponds to a person with a low value of time. So persons indexed by  $x^f$  through  $x^s + x^f$  use the slow mode; persons indexed by 0 through  $x^f$ use the fast mode.

The simplest case arises when all commuters have identical valuations of time,  $p_{Ti} = 0$  in the relevant range of T. Then in equilibrium

$$p(x^{s} + x^{f}, T^{s}(x^{s})) = r^{s}$$
 (1)

$$p(x^{s} + x^{f}, T^{f}(x^{f})) = r^{f}.$$
 (2)

In this equilibrium, every consumer values the time difference  $T^* - T^f$  equally, and thus is indifferent between the two modes.

The more interesting case arises when consumers have heterogeneous time valuations. We follow the literature in supposing that  $p_{Ti} > 0$ . In equilibrium the value of  $T^s - T^f$  (the difference in travel times) unambiguously depends on  $r^s - r^f$  (the difference in money costs). Thus  $p(i, T^s) < p(i, T^f)$  for any individual only if  $r^s < r^f$ . Given the way we indexed consumers, the absolute value of this difference for any  $T^s$  and  $T^f$  must be smaller the greater is *i*.

The number of users on each mode are determined by two conditions: (1) the marginal consumer who makes the trip (necessarily one with a low value for the trip, and hence by assumption with a low value of time) obtains zero consumer surplus from using the slow mode; (2) the marginal user on the fast mode is indifferent between using the fast mode or the slow mode. Algebraically, these conditions mean:

$$p(x^s + x^f, T^s(x^s)) = r^s$$
(3)

$$p(x^{f}, T^{f}(x^{f})) - p(x^{f}, T^{s}(x^{s})) = r^{f} - r^{s}.$$
 (4)

The situation is depicted in Figure 1, where potential commuters are arrayed on the horizontal axis according to their decreasing valuation of travel.

#### 4 Consumer welfare

Aggregate consumer surplus (when equations (3)-(4) describe behavior) is

$$S = \int_0^{x^f} p(i, T^f(x^f)) di + \int_{x^f}^{x^s + x^f} p(i, T^s(x^s)) di - r^s x^s - r^f x^f.$$
 (5)

We first examine the effects of a toll imposed on the fast mode (f). The toll induces those users of f with the lowest valuations of travel and of time to switch to the slow mode (s). The increased use of s causes users on that mode with the lowest valuations of travel and of time to leave. The toll on f unambiguously decreases the welfare of those persons who initially used s. Consider next persons who initially used the fast mode. The toll unambiguously decreases the welfare of those with the lowest valuations of travel and of time. The welfare of persons with the highest valuations can either decrease or increase: they pay the higher tolls, but enjoy lower congestion. If their welfare increases by more than the welfare of others decreases, then the toll can increase aggregate consumer welfare.

We next examine a toll on the slow mode (s). (We assume that the toll is sufficiently small so that s continues to be the slow mode.) The toll makes users of s with the lowest valuations of travel and of time leave the mode. These non-users surely have lower welfare. Users of s with the highest valuations of travel and time can either gain or lose. If they lose, they will switch to the faster mode, f. The switch to f increases congestion on that mode; consumers on both modes thus lose. If some users on s gain, then some users of f (those with the lowest value of time of time) will switch to s.

Thus, when a toll induces users to switch to the losw mode, aggregate consumer surplus can increase. Note, however, that a toll on the slow mode increases consumer welfare only if usage on the fast mode declines. This provides a simple rule-of-thumb for determining the welfare effects of a toll.

In political evaluations of tolls, the short-run effects on welfare are likely to be the most important. For example, if persons vote retrospectively and if the term of office for the incumbent is short, then of greatest concern for the political popularity of a toll is its immediate effects. Similarly, in a community with large population movements, current voters will be concerned with their short-term welfare, rather than with the long-term effects of tolls. Especially for these reasons, it is important to consider the short-run effects of tolls.

For an instructive short-run analysis let the total number of users of the two modes be fixed. This assumption of inelastic aggregate demand may be especially plausible when applied to commuters who must get to work. Consider a toll on the faster mode. Some initial users of f (those with the lowest valuation of time on it) will switch to s. This switch increases congestion on s, and unambiguously decreases the welfare of all users of s. (This loss, however, may be very small if most users of s have low valuation of time). The remaining users of the fast mode now suffer from less congestion. If they have sufficiently high values of time, then they can gain. Thus, aggregate consumer surplus can increase following a toll on the fast mode.

Consider next a toll on the slower mode. The toll will induce some persons (the persons with the highest value of time) to switch to f. The switch to f increases congestion on that mode. The welfare of all users of the fast mode therefore declines. And by the principle of revealed preference, the welfare of those persons who switched from the slow mode to the fast mode necessarily declines. Since by assumption all persons who remain on s have a lower value of time than do persons who switched, the welfare of all consumers on the slow mode also decreased. Thus, in the short run a toll on the slow mode reduces the welfare of all users.

This effect of a toll on the slow mode appears to be consistent with casual observation. Turnpikes (which are presumably faster than alternative modes) are often tolled roads. Highly congested urban roads are not. Indeed, our approach says that, other things equal, the slower is a congested road compared to a faster mode, the less politically acceptable is a congestion toll on the road.  $^1$ 

To formally analyze the effects of tolls, define the fixed number of users as  $X \equiv x^* + x^f$ . Omit equation (3) (which is now redundant), and eliminate  $x^*$  from (4) to obtain

$$p(x^{f}, T^{f}(x^{f})) - p(x^{f}, T^{s}(x^{s})) = r^{f} - r^{s}.$$
 (6)

The equation can be solved for  $x^{f}$  as a function of  $r^{s}$  and  $r^{f}$ . The derivatives of  $x^{f}$  with respect to  $r^{s}$  and  $r^{f}$  are

$$\partial x^{f} / \partial r^{s}$$

$$= -\partial x^{f} / \partial r^{f}$$

$$= -1 / ((p_{i}(x^{f}, T^{f}) - p_{i}(x^{f}, T^{s})))$$

$$+ (p_{T}(x^{f}, T^{f})T^{f}_{x} + p_{T}(x^{f}, T^{s})T^{s}_{x}).$$

Since  $p_i(x^f, T^f) - p_i(x^f, T^s) \leq 0$ , the expression on the right-hand side is positive.

The aggregate consumer surplus (5) is

$$S = \int_0^{x^f} p(i, T^f(x^f)) di + \int_{x^f}^{x^s + x^f} p(i, T^s(x^s)) di - r^s x^s - r^f x^f.$$
(7)

Differentiate (8) with respect to  $r^*$  (the toll on the slow mode) and use (7) to obtain

$$\frac{\partial S}{\partial r^{s}} = (p(x^{f}, T^{f}) - p(x^{f}, T^{s}) - r^{f} + r^{s})(\partial x^{2}/\partial r^{1}) \\ + \int_{0}^{x^{f}} p_{T}(i, T^{f})T_{x}^{f}di \\ - \int_{x^{f}}^{x^{s} + x^{f}} p_{T}(i, T^{s})T_{x}^{s}di)(\partial x^{f}/\partial r^{s}) - x^{s}$$

This is negative, using (6) and (8). This proves that any toll on the slow mode decreases consumer welfare. In summary, if the total number of users is fixed, and if toll revenue is not returned to users, then they always suffer from a toll on the slow mode.

<sup>&</sup>lt;sup>1</sup>In contrast, Knight (1924) assumes the existence of a *fast congested* mode and a *slow* uncongested mode.

We can also show that a toll on both modes reduces the welfare of all users. For suppose otherwise, that both  $r^s$  and  $r^f$  are positive. Let  $d \equiv r^f - r^s$ . If d > 0, then the same effects on consumer behavior can be attained by letting  $r^s = 0$ , and letting  $r^f = d$ . The lower tolls necessarily increase consumer welfare. A similar argument applies when d < 0. Thus, users may gain from a toll only if it is imposed on the fast mode. These results may explain why we often see a toll on the faster mode but not on the slower mode.

If toll revenues from the fast mode can be used to subsidize users of the slow mode, then a toll on the fast mode becomes even more attractive. Also note that "fast" need not literally mean shorter travel time. It can also reflect other characteristics that make one mode more comfortable than another.

Another observation. Earlier analyses of two modes typically assumed the existence of a congested mode and of a noncongested mode. This is a natural and interesting case to analyze when users of these modes are homogeneous. In contrast, we consider a fast and a slow mode, which can both be congested. This is a natural and interesting situation to address when users differ in their valuations of both travel and of time.

Also note that a toll increase on the slow mode can cause users with the *highest* valuation of travel and time to leave that mode. The intuition for our result is as follows. Whether a person continues to use a road after a toll is imposed depends on his consumer surplus on that road compared to the best alternative. If his consumer surplus is low, then he will switch. But the welfare effects of a toll depend in large part on the marginal value a person places on a marginal reduction in travel time. The correlation between consumer surplus from using a mode and the marginal value of time savings can be positive or negative. That is, a person who uses a road may obtain low consumer surplus from it, and yet may place little or high value on a reduction in congestion.

#### **5** Social welfare

We saw above that consumers can benefit from a toll on the faster mode. In the short run they never gain from a toll on the slow mode or on both modes. We now examine the effect of tolls on social welfare. This, unlike consumer surplus, considers the toll revenue to be a transfer payment rather than a cost. We show that maximization of social welfare in the short run, unlike maximization of consumer welfare, can require a toll on the slow mode.

Let  $t^i$  be the toll on mode *i*. Interpret  $r^s$  and  $r^f$  as monetary costs of travel other than the tolls; let these also represent all social costs other than congestion. Replace (6) by

$$p(x^{f}, T^{f}(x^{f})) - p(x^{f}, T^{s}(x^{s})) = r^{f} + t^{f} - r^{s} - t^{s}.$$
 (8)

Social welfare, W, is then given by (8). Set  $t^f = 0$ , and differentiate with respect to t1. Use (10) to obtain

$$\partial W/\partial t^s = \left(-t^s + \int_0^{x^2} p_T(i, T^f) T_x^f di - \int_{x^f}^{x^s + x^f} p_T(i, T^s) T_x^s di\right) \frac{\partial x^f}{\partial t^s}.$$
 (9)

The sign of the expression on the right-hand side is ambiguous in general. Consumers may gain from a toll on the fast mode, but not from a toll on the slow mode. And this holds regardless of which mode is more congestible. Social efficiency instead requires a toll on the congested mode, regardless of whether it is the faster or slower mode.

### 6 Implications for transportation policy

Note that a small toll may be politically unpopular, while a larger one could be popular. If few persons have a low value of time, then a small toll will not induce many to leave the mode, and will thus do little to reduce congestion. A large toll can dincude some persons to leave. The reduced congestion can increase the welfare of persons remaining on the road.

Our story implies that it is better to subsidize slow rather than fast modes of transit. For a slow mode will entice drivers with a low value of time. This reduction in congestion will benefit the persons with a high value of time, who continue to use the fast mode. In contrast, subsidizing a fast mode draws away consumers with a high value of time. The reduced congestion on the road thus benefits persons who place little value on the time savings. If we suppose that persons with low values of time are the poor, then here we have a case where considerations of efficiency and of redistribution point in the same directions.

Transportation costs obviously influence property values. An increase in such costs should increase the value of living near employment, and thereby increase residential property values near downtown. (Commercial values, however, may decline because such a location becomes less attractive to employers.) Property values far from employment centers should decrease (Mohring and Harwitz (1962)). Our analysis shows that the effects of tolls on such property values will depend critically on whether the congestion toll is imposed on the fast or the slow mode. A toll on the fast mode decreases the transportation costs of those persons who care most about time, and therefore should decrease demand for housing near employment centers. In contrast, a toll on the slow mode should have the opposite effect, increasing property values. These effects can have political repercussions. Thus, we noted above that consumers would prefer a toll on a fast mode than on a slow mode. But a congestion toll on the fast mode will decrease property values. The interests of landowners will conflict with those of consumers.

### 7 Other applications

The analysis given above applies more generally. It occurs whenever there is some congestion, and whenever alternative modes exist. Consider downtown parking, or parking near busy commercial centers. For users of such parking, it is the fast mode. The alternative is to park further away, take the bus, walk, and so on. Now a parking fee reduces the length of time each person parks, and therefore reduces the length of time a person must search for parking. A fee thus induces persons with a low value of time to park elsewhere (or take the bus, etc.). The remaining persons who park will be those with a high value of time. Consumer welfare can therefore increase.

The same applies for medical care. The persons most sick are the ones who most highly value seeing a physician quickly. A fee for a visit to a physician will therefore induce persons who are least sick to use an alternative facility. The fee can thus be politically popular.

The analysis also applies when an increase in the number of users of a service does not affect the time cost, but instead affects quality. The critical relation is then between the marginal benefit from quality improvement and level of consumer surplus. The level of consumer surplus (taking into account the utility of using an alternative) determines which consumers will choose to stop using service when its price increases. If persons with high consumer surplus get the highest benefit from improvement of quality, then a price increase can increase aggregate consumer welfare. The welfare of a majority of consumers can also increase. If, instead, an increase in quality benefits most those persons who get little consumer surplus from the service, then these consumers may cease using it when price increases. Since the remaining customers gain little added benefit from the increased quality, the increased price will decrease aggregate consumer surplus.

Regulated industries sometimes face similar conditions. In a congested telephone network, a decrease in the number of callers increases the chances that any one call will go through. That means that under some conditions an increase in the price (even if the revenue goes to the monopolist) can increase aggregate consumer welfare, and increase the welfare of a majority of users. The same applies to electricity. An increase in demand may cause brownouts or blackouts, thus decreasing the quality of service to all. Once again, consumers may favor price increases. Under such conditions, which are most likely to appear in underdeveloped countries, Peltzman's (1976) justly famous analysis of the politics of regulation would have to be modified. In some range the politician does not face a tradeoff between prices and profits, but instead can gain support from both the firm and the consumers by increasing prices.

When low prices create shortages, an increase in price may decrease the wait until service is rendered. One may think of parents wishing to send their children to either an initially free day care center, or to a private center which charges a fee. We can thus think of public day care centers as the slow mode, and private centers as the slow mode. An increase in fees at the public centers will induce those parents most anxious about finding immediate day care to switch to private centers. For that reason, an increase in fees will decrease consumer welfare, and will probably be politically unpopular.

Finally, consider public and private schools. Suppose that initially private schools are better, and that public schools are free. Imposing a fee on public school makes those persons most concerned about quality switch to private schools. This reduces congestion at public schools. Nevertheless, students at public school worse off, and aggregate consumer welfare declines.

## 8 Notation

 $d r^f - r^s$ .

- p the *i*th commuter's willingness to pay for the trip.
- $r^j$  Money cost on mode j
- $T^j$  Travel time on mode j
- $t^i$  Toll on mode i
- $x^j$  Number of users on mode j

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